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"DISTRESSED FIRM VALUATION: TRADITIONAL APPROACHES
AND A BINOMIAL OPTION PRICING PERSPECTIVE"

RELATORE:

CH.MO PROF. FABIO BUTTIGNON

LAUREANDA: MICHELA CASALE

MATRICOLA N. 1126634

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Firma dello studente

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Introduction

One of the most debated topics in corporate finance is the valuation of companies in operational and financial distress. These companies, in fact, present some specificities which prevent practitioners from valuing them as if they were healthy. For example, companies in operational distress are usually subject to declining sales and revenues, raising operating costs and, therefore, decreasing margins. This can be owed to an economic downturn, but may also follow from managerial or business difficulties for the single company. It usually follows from economic difficulties, but may also arise independently of it, that the firm gets financially distressed, i.e. it becomes unable to meet its debt obligations. Indeed, its cash flows may be insufficient to pay debt holders, and the firm is destined to file in bankruptcy, which implies either final cessation of the business and liquidation of assets or continuation of activities and restructuring.

When a company is distressed, implementing the usual valuation approaches may lead to an unreliable or even misleading outcome, because they rely upon overoptimistic forecasts or unrealistic assumptions. As far as the former are concerned, firms under valuation are expected to experience a large revenue growth for the explicit forecast period, which boils down to a lower steady-state growth for perpetuity. These dynamics are not followed by firms in distress: they usually experience losses in the short-term, and could end up in liquidation in the long-term. Furthermore, interest tax shields are expected to arise for the whole length of the projection, but, in case of losses, tax benefits disappear.

On the other hand, income approaches adopt a constant leverage and determine the cost of capital on that of comparable companies. However, a company in distress is expected to change leverage, but continuously adapting it is cumbersome and often useless. Moreover, if it suffers from idiosyncratic distress, other companies will experience different costs of capital and, more in general, different market ratios.

As a result, usual income and market approaches are often inconsistent.

A first solution is to adapt them for the change in conditions, assumptions and behaviours of the distressed firm. Although this renders the valuation feasible as well as reliable most of the times, this still prevents from valuing credibly equity and debt claims, where uncertainty should also be taken into account and which, therefore, deserve a more explicit valuation.

To deal with this issue, Merton (1974) first proposed to value equity and debt as options on the firm's assets value. At maturity, equity holders have the chance to choose between keeping the firm and continue the business by paying the debt in full, or to default on the debt and leave to

firm's control to debt holders. This decision by equity holders can be interpreted as if equity were a call option on the firm's assets value: if the assets' value exceeds the debt face value, the option is exercised and equity holders pay out the debt and keep control of the firm by obtaining the difference between the firm and the debt value. Otherwise, equity holders will get zero and debt holders will get control of the firm. This idea was very innovative and allowed to rely on option pricing theory to deal with real values. That is why, over time, it was developed by adding more realistic assumptions, as well as by increasing the choices available to debt holders before and at maturity.

This work aims at implementing a binomial option pricing method which follows the model implemented by Merton (1974) and its subsequent developments. This method will consider first the case when the firm is liquidated immediately after filing for bankruptcy. Then, restructuring will be included. In particular, the main advantage of making use of a binomial lattice is that the regulatory framework can be modelled more in detail. This will allow to analyse not only the values of claims, but also how they are affected by the determination of the main parameters and how the effect differs in the different situations assumed.

This work is structured as follows. The first Chapter will briefly define distress, its causes and consequences, and will introduce the U.S. Bankruptcy Code regulation, especially to distinguish between liquidation and restructuring. The second Chapter analyses how to adapt traditional valuation approaches for distress, while the third Chapter focuses on option pricing valuation and its developments. Finally, in the last Chapter, a binomial lattice method to value corporate debt and equity was implemented. Both immediate liquidation and restructuring frameworks were introduced, in accordance with the model presented by Broadie and Kaya (2007). Thanks to the implementation of a numerical approach, the main results and issues related to the model were thereafter analysed in detail, also in comparison with theoretical models.

Chapter I. Dealing with Decline and Distress

1. The features of companies in decline

Every company evolves during its life through several stages, each of which is characterized by different results, activities, managerial decisions, balance sheet and leverage. In the initial phase of start-up or development, the company's chances are almost just ideas, commercialization is not established yet and the firm is seeking for financing. This is followed by growth, in which intangible assets still prevail but are transformed into operations. Furthermore, revenues and margins increase, a capable management is hired and permanent sources of financing are granted to the firm. During the maturity phase, the company, although not growing, is stable and achieves revenues, margins and goes on in its business without incurring in financial problems. This phase can also last for a long period of time, when the firm can maintain stable revenues or continually reinvents its business or strategy. Otherwise, the firm falls into the last phase of its life cycle, defined as "decline", a stage in which, according to Damodaran (2010)¹, most assets are tangible and included in the balance sheet, growth assets are almost zero, and leverage is increasing. Declining firms are not all in the same trouble. On the contrary, they move along a continuum, since some of them are only experiencing a short-term liquidity shortage, whereas others face severe difficulties in maintaining their business model. Nevertheless, they all have some features in common, especially analysed by Damodaran (2010), which generate issues when using valuation techniques which are built for healthy companies, such as discounted cash flow and relative valuation models.

The main signal of decline is revenues, which are stagnant, declining or increasing at a rate lower than inflation. This is also accompanied by decreasing operating margins, explained by a drop in prices, necessary to increase quantity or due to the loss of pricing power. Shrinking revenues and margins can reflect operating and management difficulties or a crisis in the whole sector, but have the most dramatic consequences in the first case, since relative valuation based on multiples of comparable companies becomes inaccurate.

Furthermore, as return on invested capital becomes lower than the cost of capital, the firm also faces overwhelming debt burdens. Debt had in fact usually risen when the firm was healthy and thus at terms which cannot be matched in the decline phase. As a consequence, the firm cannot

¹ DAMODARAN, A., 2010. *The dark side of valuation: valuing young, distressed and complex businesses*. Second edition. New Jersey: Pearson Education.

meet its obligations on time and has either to liquidate its least profitable assets or to delay payments, assuming that it is not able to issue new equity or debt because of its over indebtedness.

Divestitures, another feature of declining firms, are usually worth when some assets are not sufficiently profitable, since they can generate additional cash flows. This holds especially when the buyer pays a premium for them, which however is not usually the case, as described by Schleifer and Vishny (1992)². In fact, an asset can be sold either to be converted for an alternative use, which in many cases is anyway worth less than the current one, it can be sold to competitors and maintain the same use, which is unlikely if decline hits the whole sector, since all competitors are in a similar situation of decline, or finally be sold to an outsider, who faces several difficulties in determining the fair price and additional costs to start running the business and could therefore hesitate to buy. Asset divestitures, however, can hardly be included in free cash flows computations, as they cannot be forecasted with certainty and they can affect the future going concern of the business.

On the other hand, when the firm cannot meet its debt obligations, the cost of equity will increase as the probability of default grows, whereas debt rating will drop. Moreover, if earnings are negative, the additional value arising from interest tax benefits will disappear.

As far as equity is concerned, if the firm does not face debt overhang problems and if it is able to get liquidity from divestitures, it will also credibly increase dividend payout or equity buyback. Alternatively, firms in financial trouble will also suffer higher stock volatility and therefore an increase in the cost of equity. As a consequence, valuing the cost of capital is particularly hard in case of decline.

Finally, estimating a stable growth rate can be difficult for these firms, not only because decline usually leads to a limited growth, negative or below the growth rate of the economy or the inflation rate, but also and especially because the future is uncertain and the firm could end up in liquidation, so that forecasts based on growth will get overoptimistic and unreliable.

2. The firm crisis path

Although firms in decline can have some features in common, these features do not characterize all declining firms in the same way, nor they are exclusive, since each declining firm has to cope with specific factors. In particular, there are different stages as well as types of decline.

² SCHLEIFER, A., VISHNY, R. W., 1992. Liquidation values and debt capacity: a market equilibrium approach. *The Journal of Finance*, vol. 47, no. 4, pp. 1343-1366.

Exhibit 1.1: A firm crisis path.

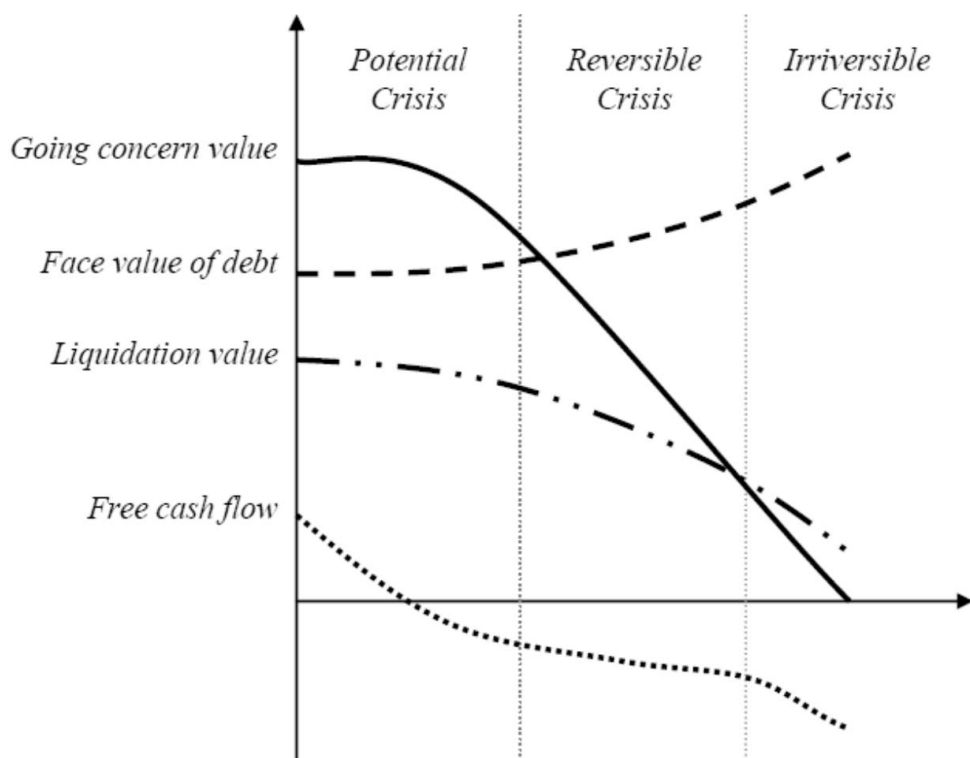


Exhibit 1.1, proposed in Buttignon (2015), shows the three main stages of a corporate crisis and focuses on the main indicators of its health: going-concern value, face value of debt, liquidation value and free cash flows.

In a healthy state, a firm's going-concern value is sufficiently high to repay the face value of debt, so that it is not convenient for a firm to file for bankruptcy and leave the company to debt holders, and it is higher than liquidation value, i.e. the value which would be paid by an outsider to buy all the assets of the company. Of course, indeed, when they produce cash flows they are more valuable if included in the business of the company than outside it. Moreover, debt is constant and its value does not depend on the going-concern value because a healthy firm has no difficulties in meeting its debt obligations. Free cash flows are positive and denote a safe operating context.

As we can observe in Exhibit 1.1, in the initial phase of distress, in which the crisis is only *potential*, shrinking free cash flows represent the main symptom of distress. Loss of competitiveness, market share and pricing power, a business model which is no longer viable, excessive operating and non-operating costs and even a decline of the whole sector reduce revenues and tighten margins, which decrease operating cash flows. This, in turn, leads to a decrease in the ongoing value of the company, although enough to repay the debt. However, in this first phase, as argued by Ratner, Stein, Weitnauer (2009)³, even most experienced managers

³ RATNER, I., STEIN, G. T., WEITNAUER, J. C., 2009. *Business valuation and bankruptcy*. New Jersey: John Wiley & Sons Inc.

incur in a denial phase, during which they “assume that the impairment is temporary and that performance will return to the norm” and act consequently. In the meantime, the value of debt starts increasing because the dropping cash flows reduce liquidity and force the firm to ask for new sources of financing.

When the going-concern value reaches the debt face value, the firm enters the *reversible crisis* phase. Distress is now perceived externally, and seriously damages the firm’s operations⁴. In this phase, the company loses customers and thus revenues, suppliers lose confidence and cease offering favourable terms or even short-term financing, best employees may switch to a safer job, thus leading to an increase in turnover and a parallel drop in know-how, skills and experience, new sources of financing and favourable terms get harder and harder to be obtained. If this happens, furthermore, managers mostly cope with obtaining debt refinancing and restructuring, rather than operating the business.

As before, the face value of debt follows an increasing trend. However, when it is independent from operating dynamics, such as in the case of long-term debt, the external perception is limited, and therefore negative effects on both sources of financing and ongoing business are reduced. This allows for a higher strategic and financial flexibility, but may also entail opportunistic behaviours and delay managerial intervention, which may exacerbate the crisis even more.

The liquidation value is slightly decreasing, due partially to asset divestitures and partially to the decreasing willingness to pay by potential buyers, as seen above.

A reversible crisis is not followed immediately by an irreversible one. Although not easily, some troubled companies may restructure (in one of the way described by Koh, Durand, Dai and Chang, 2015⁵) and turn around. As Damodaran (2010) argues, this is usually the case for cyclical companies, whose revenues trends follow economic booms and downturns, or for commodity firms, whose prices depend on the commodity market. Firms which show a history of ups and downs and have already recovered from decline in past cycles are more likely to turn around after a reversible crisis occurs as well. In general, however, reversibility is more likely when decline is idiosyncratic, which means when the industry or sector to which the firm belongs is healthy. In this situation, indeed, decline is the result of wrong management decisions which can be easily reverted.

⁴ DAMODARAN, A., 2010. *The dark side of valuation: valuing young, distressed and complex businesses*. Second edition. New Jersey: Pearson Education.

PRATT S. P., GRABOWSKI R. J., 2010. *Cost of capital: applications and examples*. 4th Edition. New Jersey: John Wiley & Sons Inc., pp. 313-346.

⁵ KOH, S., DURAND, R. B., DAI, L., CHANG, M., 2015. Financial distress: lifecycle and corporate restructuring. *The Journal of Corporate Finance*, vol. 33, pp. 19-33.

When the firm is not able to turn around and falls into the *irreversible crisis*, the ongoing value of assets might breach the liquidation value threshold. If this happens, the firm will find it more convenient to file for bankruptcy to liquidate.

3. Operational and financial distress

As mentioned, it is also possible to distinguish between operational and financial distress. Both can occur in all the phases of the firm crisis path, and, although it cannot be stated that one implies the other or they are linked by a causal link, they are for sure interrelated and most times they seem to go together. This section does not only focus on identification of distress, but also on the consequences of it.

Operational distress

According to Crystal and Mokal (2006)⁶, operational distress is the situation in which the business is no longer viable, and in particular, “the net present worth of the business as a going concern is less than the total value of its assets, were they to be broken up from the business and sold separately”. The main issue is therefore the comparison between the *going-concern value* and the *market value* of assets. On one hand, the former considers whether the following factors are favourable, in order to address the company valuation according to the usual valuation methods:

- The market conditions and outlook for a business segment, industry, or economy as a whole;
- The competitive environment for a business and industry;
- The financial history of the company;
- The historical operations of the company;
- Management’s track record;
- The ability of a business to secure adequate capital to move forward as an ongoing entity (Ratner, Stein and Weitnauer, 2009).

On the other hand, the market value of assets is the price at which each asset should be disposed of individually in a functioning market, after a suitably lengthy and extensive process of advertising, where reasonable efforts are made to identify potential purchasers, and after an appropriate level of negotiations with the identified parties in order to obtain the best price. However, this only works with essentially viable companies, whereas in case of insolvency the *liquidation value*, which will be discussed later, is a more appropriate approach.

⁶ CRYSTAL, M., MOKAL, R. J., 2006. The valuation of distressed companies- A conceptual framework. *International Corporate Rescue*, vol. 3 (2,3).

Operational distress is usually associated with periods of economic downturn, but also changes in jurisdiction or new impositions can negatively affect the whole industry or sector. Competition from other companies, competition from replacement products and services, the departure of key employees or management, rapid changes in raw material quality or availability, changes in cost structure that cannot be passed on to consumers, or a change in the demand for the company's products or services make up a non-exhaustive list, suggested by Pratt and Grabowski (2010)⁷, of possible causes of operational distress.

In this situation, the firm typically incurs in shrinking revenues and increased costs which usually reduce operating margins and thus cash flows (Pratt and Grabowski, 2010; Crystal and Mokal, 2006). Solving these issues usually requires some forms of restructuring, which are listed by Sudarsanam and Lai (2001)⁸: managerial, operational, asset and financial.

When managers are unable to deal with distress or deny it, they may be replaced to identify the sources of distress and to define better strategies for turnaround. In other cases, operational restructuring may be enough to restore profitability through cost control, maximization of output and reduction of overheads. Asset restructuring can be linked to divestitures, which allow to focus on the core business or at least to get liquidity in the short term.

Indeed, when the firm faces economic problems from which it is not able to recover, operational distress can quickly lead to financial distress and this, in turn, can exacerbate economic conditions and lead to economic costs of financial distress.

Financial distress

Financial distress refers to a too high leverage, such that the company finds it hard or even impossible to meet its scheduled debt interest or principal obligations. In other words, the firm faces a problem of liquidity.

Several factors can cause financial distress. As already mentioned, it can be the natural result of a reduction in cash flows due to a bad economic situation. It can also occur when capitalization is not adequate to the structure of operations, for instance when long-term assets are financed through short term liabilities, which may lead to a liquidity gap. Other reasons are explained by Hrdí and Šimek (2012)⁹ and include bad operating working credits, bad management and over indebtedness, which is not strictly related to economic problems: a firm could be economically viable but unable to fulfil its financial obligations.

⁷ PRATT S. P., GRABOWSKI R. J., 2010. *Cost of capital: applications and examples*. 4th Edition. New Jersey: John Wiley & Sons Inc., pp. 313-346.

⁸ SUDARSANAM, S., LAI, J., 2001. Corporate financial distress and turnaround strategies: an empirical analysis. *British Journal of Management*, vol. 12, issue 3, pp. 183–199.

⁹ HRDÍ, M., and ŠIMEK, B., 2012. Valuation of the company in financial distress. *E&M Economics and Management*, vol. 15, issue 4.

In such a variety of reasons, identifying financial distress is one of the main issues for economists. If only firms who filed for bankruptcy are included in distress, relatively few public companies will appear. On the contrary, including all firms who had troubles in fulfilling their commitments will make the sample much larger. Asquith et al. (1994)¹⁰ argue that a firm is in distress if in any two consecutive years its EBITDA is lower than its reported expenses. Sudarsanam and Lai (2001) use Taffler's Z-score and define a financially distressed firm as one which has positive Z-scores in the two previous years and a negative Z-score in the current year. Another way comes from the Merton model (1974), where distance to default was computed following Black and Scholes model (1973) and defined as the probability of default. According to Koh et al. (2015), the firm was included in the distress sample with two consecutive years of falling distance to default.

Whether the firm was previously economically distressed or not, the lack of liquidity may result in the inability to service its debt with proper cash flows or with refinancing, leading to insolvency. To solve this issue, managers could decide to liquidate part of the firm's assets and to focus on the core business only. If this is not enough, the firm will become insolvent, and, as suggested by Pratt and Grabowski (2010), its cost of capital and leverage will increase and its debt rating will drop to below-investment grade. When the crisis becomes irreversible, the best solution is to file for bankruptcy, and thus either liquidate the firm or restructure debt in-court or out-of-court with creditors. However, these are not always the best solutions, because they entail direct and indirect costs, which will be analysed in the following section.

4. The reaction to distress: Liquidation value

As explained above, a firm valuation can be based on two premises of value: the going-concern value, which relies on favourable assumptions about future developments of the firm, and the liquidation value, which is referred to as the “the sum of the net proceeds of the sale of assets and recoveries on bankruptcy claims, net of any expenses to recover those funds (such as legal expenses, commissions, or transaction fees)” (Ratner, Stein and Weitnauer, 2009). However, a distinction between orderly and forced liquidation is necessary. In the former, liquidation occurs in a reasonable time period and therefore assets are valued at a fair market price under normal market conditions. The latter, on the other hand, reflects the value of assets as if they were to be sold immediately (Damodaran, 2005) and is usually referred to as fire sale. As a result, the firm is discounted with respect to both its book value and its DCF valuation. Liquidation is usually computed by assigning to each balance sheet asset an estimate of the proceeds that

¹⁰ ASQUITH, P., GERTNER, R., and SCHARFSTEIN, D., 1994. Anatomy of financial distress: an examination of junk-bond issuers. *The Quarterly Journal of Economics*, vol. 109, pp. 625–658.

would be obtained by selling it under Chapter 7. The amounts obtained, once fees and expenses are paid out, can be used to satisfy claimholders according to their priority. Liquidation costs, such as the costs for compensation of a bankruptcy trustee to oversee the process, legal and other professional fees, asset disposition expenses, litigation costs, and claims arising from the operations of the debtor while the case is pending shall also be included. In particular, liquidation costs are estimated to be between 30% and 50% of the value of assets in a going concern¹¹, although this value varies depending on the characteristics of the potential buyer, as explained above and described by Schleifer and Vishny (1994).

The liquidation procedure which we will follow is described in Chapter 7 of the U.S. Bankruptcy Code. According to Chapter 7, when a firm files for bankruptcy it can choose to proceed with immediate liquidation. After this decision, the Office of the United States Trustee appoints an initial trustee, who can be substituted by a creditors' decision. His task is to sell the debtor's assets and then to distribute the proceeds, net of fees, to the claimants in accordance with their seniority, after investigating the claims to recover preferences and fraudulent conveyances. The main exception to this is represented by secured creditors that are owed more than their collateral: they can get the right to foreclose by seeking relief from the automatic stay, which usually prevents creditors' actions against the debtor.

Despite the discount explained by urgency, liquidation can be the most convenient choice when liquidation value is higher than the going-concern value, in order to maximize the value for stakeholders.

5. The reaction to distress: Restructuring

A valid alternative to liquidation when the firm is in a reversible crisis is to reorganize it. In this sense, four types of restructuring can be investigated.

As mentioned above, when the management is unable to understand the severity of the crisis or denies it, the best choice is to replace it in order to deal with distress and implement a new strategy. When distress is caused by reduced margins due to too high expenses or decreasing quantity or price of sales, which was previously incorporated into operational distress, then the best solution is the enhancement of efficiency, by controlling costs and reducing overheads through the sale of surplus-fixed resources, such as land or administrative offices. Asset distress stems from the focus on non-core line of businesses, which allow more diversification in the business but also disalign the strategic focus. In case of distress, some non-core businesses

¹¹ For further discussion about liquidation costs: DAMODARAN, A., 2005. *Valuation approaches and metrics: a survey of the theory and evidence*. Foundation and trends in finance, vol. 1, issue 8, pp. 693-784.

should therefore be divested, at least because if economic and financial distress go hand in hand, short-term liquidity obtained from asset divestitures surpluses may enhance liquidity.

In case of financial distress, equity-based or debt based strategies for restructuring can arise (Koh et al., 2015). If the first includes cuts in dividends or issuance of new shares as a source of financing (which has the disadvantage to decrease the value of shares, as Myers, 1977 explained), the second mostly refers to the adjustment in the terms of debt obligations. Among them, Crystal and Mokal (2006) listed:

- The postponement of imminent liabilities, e.g. the obligation to make a capital or interest payment due the following month is exchanged for an obligation to make the payment the following year;
- The conversion of fixed liabilities into fluid ones, e.g. a debt-for-equity swap, which replaces the obligation to pay over a specified amount with an obligation to repay whatever, if anything, is available;
- Debt write-downs, e.g. all creditors of a particular type agree a *pro rata* reduction in the value of their pre-distress claims.

However, this list is non-exhaustive and financial restructuring can be considered a very broad topic.

Before focusing on it, however, it is worth discussing the effects of restructuring, presented by Koh et al. (2015). Following the same procedure as in the distress case, they defined recovered any firm in the distressed sample which experienced two consecutive increases in its distance-to-default. Then, they used a logistic regression with bootstrapped standard errors of the dependent variable Recovery on the phases of the firms' life cycle and on the types of measures undertaken. According to these findings, managerial restructuring is more effective for start-ups, whereas operational restructuring constitutes a short-time solution by engaging in investment reduction, but is eventually ineffective. There is no evidence about the effects of financial restructuring. For financially distressed firms, cutting dividends or increasing equity do not lead to significant positive results. This is in line with the pecking-order theory: when a firm is in financial distress and its only alternative is the issuance of new equity, adverse selection by investors will discount the price of the share so much that it could even be inconvenient for the company to issue equity.

However, these results are only limited to the implementation of each restructuring strategy separately. The implementation of more than one strategy reverts the result and restructuring seems to lead to recovery. However, this is only limited to two to three strategies: implementing too many strategies can be detrimental and lead to insignificant coefficients for recovery.

If combining restructuring strategies seems to lead to the best solution, the regulator mostly focused on financial restructuring, since it also requires approval from third parties. Financial restructuring can occur in or out of court, depending on the costs and benefits arising from each of them as well as on the level of approval by claimants¹². The following focuses on a definition and a description of the procedures for these forms of restructuring and analyses strengths and weaknesses of each of them.

Out-of-court reorganization

Upon out-of-court restructuring, the debtor agrees new debt obligations terms with creditors through a private workout: delays of interest or principal payments, extension of maturity, debt-equity swaps and debt holidays are all examples of private restructuring, in which the court does not intervene (Fan and Sundaresan, 2000¹³). Although the overall firm value seems to be maximized by this choice with respect to both liquidation (which entails liquidation costs) and in-court reorganization (where the court's intervention is usually negatively perceived from an economic and operational perspective), there are a number of obstacles that could end up preventing such a solution, described by Crystal and Mokal (2006) and Gilson (2012). First of all, creditors may fail to agree on a plan because they cannot agree on an equitable division of the gains. Furthermore, certain influential creditors could strategically hold out for a higher recovery at the expense of other creditors or shareholders, or they could free ride by staying out of the agreement expecting others will sign out. If their strategy succeeds, they profit after recovery occurs and avoid more unfavourable terms on their debt. Moreover, there may be irresolvable disagreements about how much value is at stake (or even about which restructuring option is most likely to maximize overall value) or coordination costs, which arise when the necessary parties to a restructuring are so dispersed that it would be excessively costly to locate and bring them into the agreement.

Out-of-pocket administrative costs and fees represent additional "direct" costs in restructuring. Differently from in-court reorganizations, it is not required to report them, which causes an underestimation of them. Nevertheless, according to Gilson (2012), they account for less than 1% of the face value of the affected bonds.

In-court debt restructuring

Alternatively, the firm can file for bankruptcy under Chapter 11 of the U.S. Bankruptcy Code.

¹² GILSON, S., 2012. Coming Through in a Crisis: How Chapter 11 and the Debt Restructuring Industry Are Helping to Revive the U.S. Economy. *Journal of Applied Corporate Finance*, a Morgan Stanley Publication, vol. 24, no. 4.

¹³ FAN, H., and SUNDARESAN, S. M., 2000. Debt valuation, renegotiation, and optimal dividend policy. *Review of Financial Studies*, vol. 13, pp. 1057-1099.

A petition may be voluntary, if filed by the debtor, or involuntary, if petition is required by creditors who have to meet certain requirements¹⁴. Despite this difference, they both cause the court to grant the firm an observation period, or grace period, during which creditors are prevented from taking any actions against the debtor (with the exception of secured creditors, who follow a proper treatment) and the firm is allowed to make a plan of reorganization, which must eventually be approved by creditors. Chapter 11 reorganization is designed to preserve the business and continue operations if the firm is worth saving but it is currently suffering liquidity problems and is aimed at returning to a healthy state with as few as possible damages for the company business.

Unless a trustee is appointed by the court to take control and manage firm properties, the firm, which is defined in Chapter 11 as “debtor in possession”, “DIP” or simply “debtor”, remains in possession of its assets and can run the business. The debtor acts as a fiduciary, with all the rights and powers of a Chapter 11 trustee but the investigative functions and duties. He also has many other powers and duties usually entrusted to a trustee, such as hiring professionals to support the debtor during the bankruptcy procedure, if approved by the court, sell property or businesses, collect debt and filing tax returns and reports required by the court. Cash collateral¹⁵ may be also disposed after the approval of the court or the consent by the secured creditor. Absent consent, the court can authorize the disposal if the debtor can provide adequate protection, such as an equity cushion, to the secured creditor.

The directors of the DIP are required to maximize the value of assets and owe a duty of loyalty and faith to shareholders as well as, in case of insolvency, to creditors.

On the other hand, the task of the trustee appointed by the court is to monitor the bankruptcy case and supervise the administration. In fact, the trustee monitors operations as well as the submission of reports and fees, applications for compensation and reimbursement by professionals, plans and disclosure statements filed with the court, and creditors' committees. He may also impose reporting requirements to the debtor in possession and, if the debtor does not meet the requirements, the trustee may ask the court to dismiss the case or to convert it to another chapter. In the last situation, the trustee may be appointed to take over management and operations of the debtor's business.

¹⁴ US COURTS. Chapter 11: Bankruptcy Basics. Available on: <http://www.uscourts.gov/services-forms/bankruptcy/bankruptcy-basics/chapter-11-bankruptcy-basics> [accessed on 14th September 2017].

¹⁵ 11 U.S.C. § 363. Section 363 defines "cash collateral" as cash, negotiable instruments, documents of title, securities, deposit accounts, or other cash equivalents, whenever acquired, in which the estate and an entity other than the estate have an interest. It includes the proceeds, products, offspring, rents, or profits of property and the fees, charges, accounts or payments for the use or occupancy of rooms and other public facilities in hotels, motels, or other lodging properties subject to a creditor's security interest. (www.uscourts.gov)

A creditors' committee is usually also appointed by the court and ordinarily consists of the seven largest unsecured creditors eligible and willing to serve. It represents one of the major parties in the case. It may, among other things:

- consult with the debtor in possession on administration of the case;
- investigate the acts, conduct, assets, liabilities and financial condition of the debtor, the operation of the debtor's business and any other matter relevant to the case or to the formulation of a plan;
- participate in the formulation of a plan;
- request the appointment of a trustee or examiner with the court's approval.

Other figures, such as the case trustee or the examiner, may also be appointed, though this occurs only seldom.

One of the main benefits of Chapter 11 for debtors is the automatic stay of assets, a legal injunction that prevents creditors from seizing their collateral or taking any other action to collect their debt for the whole period from petition to the confirmation of the plan. In other words, the automatic stay of assets ensures that the company will be able to continue its operations without the threat of a "race" by creditors, which would eventually dismantle the firm business.

Of course, numerous exceptions apply to the automatic stay of assets. In particular, a secured creditor may obtain an order from the court granting relief from it. This happens if the debtor cannot provide "adequate protection" to the secured creditor when disposing of his collateral or when, in an act against property, the debtor both does not have an equity in the property and the property is not necessary to an effective reorganization.

A debtor has an exclusive period of at least 120 days, which may be prolonged up to 18 months under some circumstances, to file a plan of reorganization. Exclusivity ensures that negotiations with creditors will occur in respect of a particular plan or other course of action, and that creditors will not act opportunistically by presenting their whole plan. However, if the grace period expires without any plan being filed, the other parties in interest may file their own plan, often in competition among each other or with the plan filed by the debtor itself. If a trustee is appointed, the trustee must file a plan, a report explaining why the trustee will not file a plan, or a recommendation for conversion or dismissal of the case.

Once the plan is filed, the debtor has 60 days (which are prolonged in case of extension of the exclusivity period up to 20 months) to seek acceptance of the plan. Before any vote, however, the court has to approve a disclosure statement written by the debtor. The disclosure statement acts as a sort of prospectus, which is aimed at providing "adequate information" about the affairs of the debtor, especially its history and nature, books and records, to enable the holder

of a claim or interest to make an informed judgement when voting. No acceptance or rejection can be solicited without a previous approval of the disclosure statement.

Approval of the reorganization plan occurs if at least two thirds of the creditors in amount and more than one half in number have voted for it, excluding any vote not in good faith. Moreover, at least one impaired class of creditors must accept the plan, and those who did not accept it must obtain at least the amount they would have obtained or retained upon liquidation (best interest of creditor test). Moreover, the plan must be fair and equitable with respect to each impaired class of claimants who did not accept the plan (cram down test). Finally, the plan must be feasible and thus not lead to another restructuring or liquidation.

After the plan confirmation, the debtor is discharged from any debt arisen before the confirmation date, but at the same time new obligations become binding and supersede pre-bankruptcy contracts. This holds of course not only for those creditors who accepted the plan, but also for those who rejected it.

Benefits and costs of Chapter 11 Restructuring

When companies file for bankruptcy, they choose the method which is most likely to offset costs and benefits of the procedure. It is straightforward that Chapter 11 bankruptcy is usually costlier than out-of-court restructuring. Chapter 11 entails indeed out-of-pocket administrative expenses, court costs, and fees for legal, financial, and other professional services (Gilson, 2012). Although they partially coincide with those which are faced out of court, they are usually higher because they include legal expenses, fees and professional services, which can be avoided in out-of-court restructuring. Nonetheless, in-court restructuring can be considered a formal admission of financial distress, with all the subsequent negative impacts in investment decisions and business operations, which are classified as indirect costs. These costs mostly refer to (Damodaran, 2010; Pratt and Grabowski, 2010; Gilson, 2012):

- Inability of the firm to pay its suppliers on a timely basis, potentially leading to supply shortages or disruptions, or to the refusal to service the company;
- More bargaining power left to clients, which can reduce operating working capital and thus cash flows, but also hamper the ability to do business;
- Shift of customers to other safer suppliers due to concerns for service and warranty interruptions and cancellation of orders;
- Increasing employee turnover, leading to loss of experience, technical know-how and stability;
- Focus of the management on the renegotiation and refinancing procedure rather than on running the business;

- Higher competition due to the perception by competitors that bankrupt firms are easier prey.

Whereas direct costs make up less than 5% of the total firm value (usually between 3 and 5% according to Warner, 1977), indirect costs may even be disruptive and, according to Andrade and Kaplan (1998)¹⁶, they range from 10% to 23% of firm value before bankruptcy. Moreover, they strongly differ across firms and are normally higher for highly levered firms. Altman (1984)¹⁷ tried to estimate costs of bankruptcy as well. He observed that, on average, bankruptcy costs ranged from 11 to 17 percent up to three years prior, when measured based on a regression technique constructed to capture the unexpected lost profits of a firm in distress, but this value exceeded 20% in many cases, and using another method based on expert security analyst expectations of earnings versus actual lead to even more dramatic results.

Among other disadvantages, in-court resolution usually entails a lengthy procedure which could further deteriorate the value of the firm, and it can result in a sort of protection to economically inefficient companies which are only strategically defaulting¹⁸.

Nevertheless, many benefits arise under Chapter 11 restructuring, as Gilson (2012) pointed out. One obvious advantage refers to the automatic stay provision, which concentrates the management in running the business and preparing the plan, without being threatened by claimants' actions directed to forced assets liquidation procedures. Furthermore, Chapter 11 allows to raise cash in simpler ways: not only can the debtor stop pre-petition debt payments, but she can also dispose of assets in a much simpler way and she is given access to DIP financing. This form of financing is particularly convenient for new lenders of a bankrupt firm, because they are granted superior priority with respect to previous creditors and, as a result, their claims almost never fail insolvent and provide high fees. On the other hand, it also ensures the debtor sufficient liquidity and avoids the debt overhang problem, which could prevent the firm from undertaking the investment despite its positive net present value simply because it is unable to borrow.

Furthermore, the debtor is allowed to reject all "executory" contracts, such as leases and licensing agreements, which may hamper business operations. She can also sell assets through open auctions supervised by the court, which reduce risks and provide many advantages in terms of proceeds for the debtor as well as lien and encumbrances for the winner. Non-financial

¹⁶ ANDRADE, G., and KAPLAN, S., 1998. How Costly is Financial (Not Economic) Distress? Evidence from Highly Leveraged Transactions that Became Distressed. *The Journal of Finance*, vol. 53, issue 5, pp. 1443-1493.

¹⁷ ALTMAN, E. I., 1984. A further empirical investigation of the Bankruptcy cost question. *The Journal of Finance*, vol. 39, issue 4, pp. 1067-1089.

¹⁸ See Yost (2002), Moodarian (1994) and White (1994), in OUTEICHEVA N., 2007. *Corporate Financial Distress: An Empirical Analysis of Distress Risk*. Dissertation of the University of St. Gallen, Graduate School of Business Administration, Economics, Law and Social Sciences (HSG).

liabilities, such as those stemming from pensions and other post-employment benefit plans, may be managed in a more efficient way: the DIP can seek permission from the court to reject inconvenient agreements with unionized labour and in case of litigation, such that she can get a fresh start after recovery. A final advantage is the reduction of the tax liability faced by companies which incur in net operating losses and debt write-downs, which is only granted under Chapter 11 restructuring.

Many authors in the past argued that Chapter 11 was too costly and in the 80s Michael Jensen even theorized a “privatization of bankruptcy”, according to which the burdensome Chapter 11 proceedings would have been soon substituted by the more efficient out-of-court reorganization. However, times have changed since that. The opportunity to access “pre-packaged” plans, the eased access to financing and asset sales and the subsequent rise of private equity firms and hedge funds made the restructuring process much more efficient and fostered Chapter 11 proceedings, which nowadays seem to live a sort of revival.

Chapter II. The Valuation of distressed firms: adapting traditional approaches

1. Introduction

Distressed firms' economic and financial features make it particularly hard to apply traditional valuation methods, because their intrinsic assumptions could distort the results. As a consequence, it is necessary either to adapt traditional models to the peculiar features of those firms, privileging some methods to others, or to develop new models.

This chapter analyses first the issues which could cause distortions in the going-concern valuation and then addresses them when using traditional methods. However, their main task is to compute the enterprise value of the distressed firm, whereas what share- and debtholders of the company, as well as all other stakeholders, are mostly interested in are the consequences of default. The company could indeed liquidate or restructure its debt, causing a reduction in the value of debt, equity or both. Moreover, debt holders know that equity holders could even decide to strategically default in order to get as much value as possible, and they know this could result in a further reduction in debt value. Traditional income or market approaches cannot help to address these issues. It becomes then necessary to develop different methods to cope with them. This issue will be dealt with in the next Chapter, which describes the evolution of the Black, Scholes and Merton option pricing model and the following developments to make it as close to reality as possible.

2. The main issues in distressed firms' valuation

In general, the main approaches to obtain the going-concern value of a company are distinguished into asset, income and market approaches, as seen in Exhibit 2.1¹⁹. Valuing a company requires therefore first of all the choice of the most appropriate approach, which depends on the features of the company itself. Going-concern firms are typically valued using the market approach and/or the income approach. However, the asset approach may be more appropriate in some situations, such as in asset-intensive businesses with low profitability, where book value is not far from the enterprise value.

¹⁹ For reference, see e.g. Ratner, Stein and Weitnauer (2009).

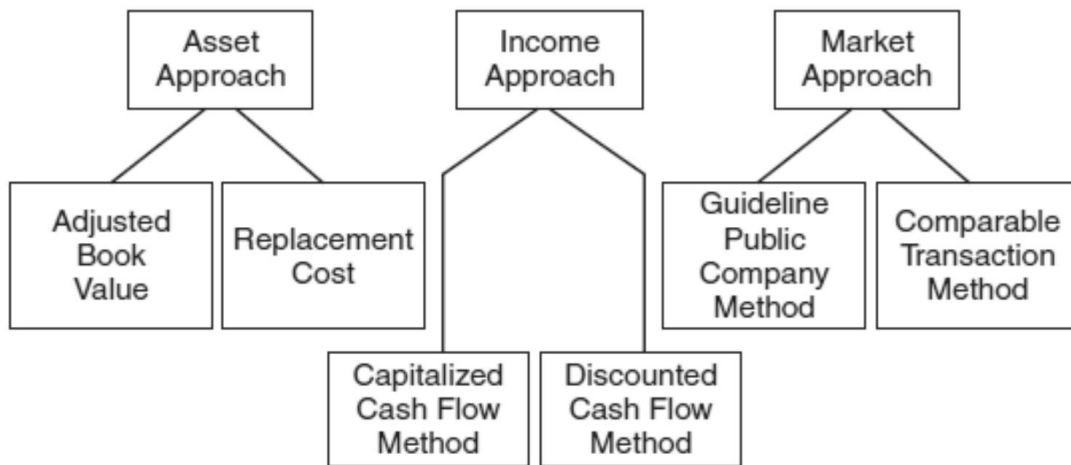


Exhibit 2.1: Overview of the business valuation approaches

Briefly introducing these approaches can lead to a better understanding of the difficulties incurred when considering distress. Asset approaches value companies starting from the value of assets and liabilities, which may be restated by using the fair market value to purchase or substitute the asset on the balance sheet. Income approaches, instead, are intrinsic valuation methods that value a company by discounting expected future cash flows by some discounting factor. This factor differs from model to model, in the main attempt to incorporate risk in the valuation. Income-based models usually require a detailed analysis of the balance sheet and they get expected cash flows by making some assumptions on growth and, in general, on the future outcome of the business. These assumptions allow to obtain expected NOPLAT (net operating profit less adjusted taxes) and to adjust it for noncash changes in accounts which were previously included in the computation of NOPLAT or for cash adjustments which were capitalized, to finally obtain cash flows for an explicit forecast period, which usually lasts 5 to 10 years, depending on when the company is expected to reach a steady state. As this happens, growth rate is expected to reduce in perpetuity. This allows to compute a continuing value, which represents the value of operations when the firm reaches a steady state and operates for a long period, and which is computed as in the last period of explicit forecast. After, the most appropriate discount rate to discount operating cash flows at each period is chosen, and summing discounted cash flows leads to the value of core business. Next, value per share is obtained by adding back to the business enterprise value nonoperating assets, subtracting debt, debt equivalents and equity equivalents, and finally dividing the value of equity by the undiluted number of shares outstanding.

Finally, the market approach relies upon the comparison with a peer group. According to these methods, for each competitor a ratio is computed, typically enterprise value over EBITA or NOPLAT, or, even if less appropriate, price-to-earnings, starting from its market current or

forward estimates. After that, the value of the firm is calculated by applying these multiples to the estimated performance of the analysed firm.

However, the application of these methods in the valuation of distressed firms is not straightforward. As explained by Damodaran (2010), the first issue refers to overoptimistic valuation assumptions, both in the explicit forecast and in terminal values. Decline and distress are usually associated with stagnant or shrinking revenues and margins, which are not always reflected in valuation. Most distressed firm valuations assume positive growth rates in the short term, simply because this is the usual approach in valuation, although there is no reason to be optimistic about future growth. On the other hand, long-term growth estimates (used for the computation of the terminal value) do not consider that the firm will usually not make it to a steady-state growth, but will instead fall bankrupt in many cases. Consequently, too high growth rates are usually assumed for valuation, causing an overestimation of free cash flows, and, eventually, of the value of the company.

A further problem is the computation of the discount rate when using Discounted Cash Flows (DCF) Valuation. This approach uses the so-called WACC (weighted average cost of capital), which is defined as:

$$WACC = \frac{D}{V}k_d(1 - T_m) + \frac{E}{V}k_e$$

where D/V and E/V represent the target market-value based levels of debt and equity to enterprise value respectively, k_d is the cost of debt, k_e is the cost of equity and T_m is the marginal tax rate on income.

However, several problems arise from this formula. First, dealing with a firm in distress usually means that the firm is overleveraged with respect to its competitors or that its leverage is anyway not sustainable in the long term. As a consequence, if we assume a target capital structure based on that of competitors, this will not be compatible with the current leverage of the firm. On the contrary, assuming that its current leverage will be maintained constant is not credible, because without restructuring a distressed firm will probably end up its operations and, as a consequence, expected cash flows will never realise. A solution to this could be to assume leverage changes from period to period, but this, besides being hard to credibly estimate, is also a cumbersome approach²⁰.

Second, using pre-tax cost of debt allows to account for benefits arising from the interest tax shields. However, interest tax benefits only accrue when operating income is sufficient to

²⁰ See e.g. PRATT S. P., GRABOWSKI R. J., 2010. *Cost of capital: applications and examples*. 4th Edition. New Jersey: John Wiley & Sons Inc., Chapter 6, for further insights on estimating the value of a company with changing capital structure.

completely cover interest expenses. If this is not the case, as it usually happens for distressed firms which are losing money and are usually expected to keep losing them, including it in the valuation could distort the cost of capital.

Finally, it is usually difficult to get a credible estimation for the cost of debt and equity, because book interest rates do not include expectations which came to light only as the company fell into distress, such that the currently existing probability of default is not included. On the other hand, market interest rates on bonds can result in very high costs of debt, which could even overcome the cost of equity. This results in a quandary because it leads to contrast between theory and practice, since equity should be riskier than debt.

Moreover, many problems refer to the estimated returns of assets. Even if profitable, they usually earn less than the cost of capital, leading DCF based on WACC to an enterprise value lower than invested capital. In this case, the best alternative is to divest them, but asset divestitures are hard to estimate in advance and cannot be easily included in valuation, although they increase cash flows and should be included in theory.

Furthermore, valuation is usually not only useful, but even required for distressed firms, for instance when they are going to file for bankruptcy. If this is the case, income-based valuation could become more an administrative than a market procedure (Gilson et al., 2000²¹). This substitution reduces the quantity and quality of information, because when investors cannot capitalize on superior information about future cash flows, for example through acquisitions or other open market transactions, they have “substantially less incentive to collect information about the bankrupt firm or to reality test management forecasts”. Moreover, in bankruptcy each claim holder will pursue her own interests. According to Crystal and Mokal (2006) and Gilson et al. (2000), who describe this issue as “strategic distortion of cash flows”, senior claimants have an incentive to underestimate cash flows in order to decrease the value of the bankrupt company and get it fully, thus maximising their share, whereas junior debt holders are willing to overestimate it, in order to get all that is left after senior payments are fulfilled. Having many opposing interests distorts enterprise value according to the bargaining power of the participants and leads eventually to higher valuation uncertainty.

Substituting an income with a market approach may lead to a more straightforward valuation, but causes other difficulties. Damodaran (2010) observes that market multiples can be applied both to historical and forward data. In the former case, however, declining firms which represent an outlier in a healthy sector will end up with a too low enterprise value, in which the chance to reorganize the company and go back to a healthy state is not included. On the

²¹ GILSON, S. C., HOTCHKISS, E. S., RUBACK, R. S., 2000. Valuation of bankrupt firms. *The review of financial studies*, vol. 13, issue 1, pp. 43-74.

contrary, the latter case does not include distress in valuation and shall be adjusted for the possibility of default.

Although all these arguments should compel valuation practitioners to include distress in valuation and adjust traditional techniques for these distorting issues, some analysts still argue there is no need to explicitly include it. In particular, they split up into two subgroups.

The more “purists” argue that distress shall never be considered in valuation. Indeed, according to them, only large caps and well-established firms need valuation, and they usually have a lower probability to go into distress. However, not only also small and high-growth firms need to be valued, whose uncertainty is much higher, but also large firms may declare bankruptcy, as it happened to Kmart²² or to large firms during the financial crisis. Furthermore, access to capital may be constrained by negative market conditions, leading to limits to access capital even for well-performing firms.

On the other hand, some analysts argue that it is enough to adapt either cash flows or WACC to a distress situation, starting from the assumptions in valuation. However, this hardly works in practice, since most practitioners continue to use overoptimistic assumptions. Moreover, every adjustment is just partial. Adjusting the WACC still leads to the formulation of a growth rate and a terminal value, which leads to implicitly assume that the firms will survive. On the contrary, cash flow adjustments should consider all possible scenarios: this is both very difficult and cumbersome to do in practice, because at each time step several situations shall be considered and several probabilities of default as well as liquidation values shall be computed. Consequently, adapting approaches to distress conditions is very difficult. In the following, we will introduce some ways to adapt cash flows and cost of capital in DCF valuation, and we will also deal with some income based approaches which simplify the issues related to DCF. Thereafter, two market approaches will also be presented.

3. Income approaches: Adapting DCF Valuation to distress

DCF valuation methods obtain the enterprise value of a firm by discounting its expected free cash flows by the WACC for an explicit forecast period and adding a terminal value which allows to include a long-term steady-state growth of the company. They start from some assumptions about the future performance of the firm. However, including uncertainties in this model is not straightforward. An easy way to deal with them is described by Damodaran (2010)

²² LEHAVY, R., UDPA, S., 2011. Kmart: Predicting Bankruptcy, Fresh Start Reporting, and Valuation of Distressed Securities. *Issues in Accounting Education*, vol. 26, n. 2, pp. 391-419.

and Koller, Goedhart and Wessels (2015)²³. They consider both discrete approaches, such as the scenario analysis or the binomial tree approach, and continuous methods, such as those which obtain volatility through a Montecarlo simulation and include it in valuation.

Bankruptcy, however, is not a simple source of uncertainty, since it could even lead to end up operations and liquidate the firm, thus strongly negatively affecting both debt and equity holders. Since the usual models for uncertainty did not cope well with these issues, they were adjusted or even substituted to more explicitly consider the possibility of filing for bankruptcy and include it in valuation.

The first two adjusted DCF models belong to the first group and they include bankruptcy as a form of uncertainty. The remnant ones, instead, deal with bankruptcy in a special way, the former by considering liquidation and the latter by modifying the WACC.

Montecarlo Simulation

Traditional DCF valuation relies upon some main assumptions which allow to estimate the future performance of the firm and thus to obtain FCFs. The main drawback refers to the computation of point estimates, which do not allow for uncertainty nor, specifically, for distress. Montecarlo simulations allow to start from assumptions related to the probability distribution of some (or all) inputs and to derive earnings and cash flows which include uncertainty. Although this can also be obtained through scenarios, Montecarlo simulation is more reliable and complete, because it addresses the consequences of continuous risk, and more flexible as well, because it can theoretically include the risk associated to each parameter of the valuation. Of course, assumptions can refer both to idiosyncratic and systematic risk. Specifically, it must be valued whether distress depends on the capabilities of the company. If this is the case, is it due to operational difficulties or lack of liquidity? Does the firm own the expertise and managerial skills to go out of distress? Is firm reputation good enough to get new sources of financing? Broad economic conditions or an industry crisis could also cause distress. In this case, uncertainty could be included in the market interest rates or inflation, and debt conditions could deteriorate despite a quite valid reputation. All the circumstances under which the firm will be pushed into distress must be defined and accounted for through changes in the variables associated to them. In a Montecarlo approach, all variables could follow a probabilistic distribution, at least in theory. However, this exponentially increases the number of simulations and subsequent computations which are necessary, thus leading to a too cumbersome procedure. As a consequence, just assuming a few key inputs change is usually considered enough.

²³ KOLLER, T., GOEDHART, M., WESSELS, D., 2015. *Valuation: Measuring and Managing the Value of Companies*. 6th Edition. New Jersey: McKinsey & Company.

Afterwards, probability distributions shall be defined. This is one of the key issues, which requires to estimate the type of distribution as well as the parameters associated to it. Furthermore, in most cases some boundaries could distort valuation and shall therefore be included. A probability distribution can be defined in several ways, the most common of which are hereafter presented:

- Analysing historical data: this approach is useful when dealing with data for which a long-time set is available, such as risk-free rates and market risk premia. This approach implicitly assumes the absence of structural changes from the past, and is applicable when considering both a constant and increasing confidence band over time²⁴.
- Relying on cross-sectional data: when the crisis hits the whole market, industry or sector, using data from competitors can be extremely useful. However, since the main assumption refers in this case to similar performance indicators as comparable firms, it is not convenient to rely on this kind of analysis when the firm behaves as an outlier.
- Using statistical distribution and parameters: in many cases the previous approaches could be imprecise or unreliable. If this is the case, it is possible to pick the statistical distribution which best fits the distribution of the variable on hand, and then choose the parameters. However, not only it is sometimes hard to find the best-fit distribution, but, also and most importantly, estimating parameters for it is not straightforward and, in some cases, requires relying upon times series or cross-sectional data. Furthermore, some input boundaries shall also be considered: as an example, Copeland and Antikarov (2001) observed that, since prices can never go negative, the most appropriate distribution is the lognormal. Including boundaries, however, is not always straightforward: according to Damodaran (2010), for instance, revenue growth cannot really be normally distributed, because the lowest value it can take on is -100% . Consequently, the choice of the statistical distribution will aim at not wreaking havoc on the whole valuation procedure.
- Using subjective estimates provided by management: when distress is firm-specific and has never been experienced before, none of the approaches can be used. The best alternative is thus to rely on credible assumptions on the future performance, for example those provided by the management, who often have in mind “subjective, non formal, non statistical estimates in their head” (Copeland and Antikarov, 2001). If this

²⁴ See for the first Damodaran (2010) and for the second: COPELAND, T., ANTIKAROV, V., 2001. *Real options: a practitioners' guide*. 1st edition. New York: W.W. Norton & co.

is the case, it is possible to deal with both a constant and an increasing band of values over time, as explained by Copeland and Antikarov (2001).

Before running the simulation, a further issue should be dealt with: correlation and autocorrelation. If two or more inputs are expected to be correlated with each other (e.g. prices and quantity, or interest rates and inflation rate), correlation shall also be included in simulation procedures, and it is also likely to improve the reliability and fitness to reality of the model. The same also works for correlation of times series. In general, positive autocorrelation increases the volatility of the valuation with respect to independence, whereas negative autocorrelation decreases it.

Thereafter, the simulation shall be run as many times as possible, in order to improve the precision of the model. Every random input is chosen by the statistical package according to the assumed distribution and parameters, so that in every simulation different random inputs are used to value cash flows and eventually the firm value. After defining a bankruptcy trigger for the company, which depends on its performance as well as on the overall market conditions, a comparison with the obtained firm value will determine whether it is possible to value it as a going concern, if the trigger is not breached, or if, otherwise, it is necessary to estimate its liquidation value. This can also allow to estimate the probability of distress and its effects on value. The final results allow to determine the whole probability distribution of the firm value, whose expected value can be considered the value of the firm.

Strengths and weaknesses of the Montecarlo approach

Montecarlo simulations provide practitioners with the main benefit of allowing to discuss implications of continuous distress risk, which is far more credible than discussing implications of a scenario analysis, where distress is considered as a discrete variable.

Moreover, modern statistical packages make it quite easy to implement. However, this requires time, since many simulations need to be implemented. As suggested by Damodaran (2009), indeed, (1) the larger the number of inputs to which a probability distribution is assigned, (2) the more different the statistical distributions chosen and (3) the larger the range of final outcomes (i.e. the volatility of the firm value), the larger is the number of simulations needed. Finally, the choice of the variable parameters and distribution may lead to valuation distortions, and having no clear method to make assumptions about it is not a help.

Modified DCF Valuation (Scenario Analysis)

As noted when introducing DCF valuation weaknesses, both cash flows and discount rates need adjustments for distress. The modified DCF considers the risk associated to distress discretely, by modelling different scenarios, and adjusts the discount rate accordingly.

Deriving Expected Cash Flows: A Scenario Analysis

Expected cash flows in modified DCF valuation must include the probability of distress explicitly. Rather than choosing the most likely scenario and set assumptions on it, without considering distress risk, this model attempts to consider all possible scenarios, or at least the best and worst ones, and to define assumptions for each of them. As in Montecarlo simulation, assumptions shall be somehow credible and shall be based on forecasts on the future performance of the firm, depending either on its specific conditions or on the state of the overall economy. Starting from them, it is then possible to compute the free cash flows associated to each scenario and at each time. This is similar to making simulations, but here the assumptions are made explicitly and not chosen randomly from a distribution. This renders this model more limited, since it only relies on discrete assumptions and allows to make less simulations, hampering the precision of the valuation. This approach computes explicitly free cash flows at each time step and then weighs them by the probability of each scenario, allowing to get an expected free cash flow per time step, which means as many expectations as the explicit forecast time period T. As a consequence, the formula will become (Damodaran, 2010):

$$E[FCF_t] = \sum_{i=1}^n \pi_{it} * Cash\ flow_{it}$$

where π_{it} is the probability of scenario i in period t and $Cash\ flow_{it}$ is the cash flow under that scenario and in that period. If it is complicated to compute many scenarios, it is sufficient to estimate the probability of distress and to compute:

$$E[FCF_t] = Cash\ flow_t(1 - Probability\ of\ Distress_t).$$

However, the main difficulty arises when computing probabilities for each time step. For example, after two periods the probability of survivorship is not simply the same as in the first period, but must include the chance to survive at each period, thus having to compute the probability of two events, and in the same way distress shall consider the cumulative probability of no cash flows in the future. For instance, if the probability of distress is 10% in year 1 and 2, there is now only an 81% = $(1-0.10)*(1-0.10)$ chance that the firm will have cash flows in year 3.

Adjusting discount rates: the cost of equity

Modifying DCF implicitly implies using WACC as a discount rate for the expected FCF estimated above, but, as noted above, WACC holds only in going-concern valuations and needs therefore some slight modification to include distress. To better understand the reason, the magnitude and the consequences of these changes, each component of the WACC is dealt with separately and the going concern approach is compared with the distress case.

The most difficult component to estimate is the *cost of equity*. This is already an issue when dealing with performing companies, but becomes even more complicated under distress. The usual approach is based on the CAPM, according to which risk is defined as the sensitivity to the market, such that the expected rate of return of any security equals the risk-free rate plus a measure of sensitivity, the beta, multiplied by the market risk premium:

$$E[R_i] = r_f + \beta_i(E[R_m] - r_f).$$

Alternatively, Fama and French proposed their Three-Factor Model, which regresses the stock's excess return on the market premium, the premium of small stocks over big ones, and the premium of value over growth stocks. This model was modified over years both by themselves and by other authors to include many more than only three factors, such as momentum, investments and so on. Nowadays a new approach seems to generalize the Fama - French Three-Factor Model by even choosing the factors which are significant and not only constitute noise. This approach is the Arbitrage Pricing Theory and, although it can better fit the regression than the previous approaches, it carries the risk to "go fishing for factors" that are secondary components in the computation of returns and that, thus, do not necessarily need to be included. Many discussions may be opened in asset pricing, but since it is not the main topic of this paragraph, we will assume that, having a solid economic theory about risk and return, the CAPM is the safer method for the estimation of returns²⁵.

Upon distress, a company requires higher returns (and, therefore, a lower price) to be traded, in order to incentivise investors to put money in it. Consequently, it is possible either to adapt the CAPM model to distress intrinsically or to add an additional premium which incorporates distress to the healthy-case CAPM formula.

In the former case, the cost of equity is obtained by estimating in a slightly different way the components of the original CAPM equation:

$$E[R_i] = r_f + \beta_i(E[R_m] - r_f).$$

The main components whose value needs to be estimated are:

- the risk-free rate: the long-term rate on government bonds (Treasury bonds) is usually chosen for this purpose. Its maturity is usually chosen to better match the cash flow stream being valued, although theoretically the rate should be different for each cash flow at every time;
- the market risk premium can be computed either by looking at the historical data or by computing the market implied cost of equity. In the first case, the market return is

²⁵ For further reference about asset pricing, see John Cochrane "Asset Pricing"; Claus Munk "Financial asset pricing theory"; Bali/Engle/Murray "Empirical Asset Pricing"

estimated from a long time set of market risk premia, whose median or average or an intermediate value is computed, and to which the current risk-free rate is added. In the second case, a simple formula, which is derived for instance in Koller et al. (2015), is obtained directly:

$$k_e = \left(\frac{1}{P/E}\right) \left(1 - \frac{g}{ROE}\right) + g$$

- Damodaran (2010) suggests estimating the beta of the company in a different way than in the going concern case. In particular, the usual process works as follows:
 - choose a peer group for each business in which the firm under valuation operates, pick the historical returns of each firm's stock and estimate the beta through a times series regression;
 - smooth the beta in order to make it closer to the mean of the companies by reducing the impact of specific features or events;
 - unlever the beta (finding β_u) of each company: indeed, beta is a function of the unlevered beta and a leverage factor, and accounts for both operating and financial risk:

$$\beta_e = \beta_u + \frac{D}{E} (\beta_u - \beta_d) - \frac{V_{txa}}{E} (\beta_u - \beta_{txa})$$

Unlevering, therefore, allows to strip out the effect of debt and make betas comparable across an industry. This is especially useful when dealing with distress, when financial risk is very large and leverage can distort betas with respect to competitors. It is usually assumed that $\beta_d = 0$ or 0.3 in investment grade firms and $\beta_{txa} = \beta_u$ under fixed capital structure, when tax benefits fluctuate with the value of operating assets, so that the formula simplifies at least to:

$$\beta_e = \beta_u + \frac{D}{E} (\beta_u - \beta_d).$$

- After removing the outliers, compute an industry beta for each business in which the firm operates by using a weighted average by the value of the firm, a simple average or a median. The last one is especially appropriate in case there are outliers in the sample. The latest two steps may also be inverted.
- Calculate the bottom-up unlevered beta of the company as a weighted average of the businesses of the company.
- Relever the bottom-up beta of the firm under valuation by using the formula above. In this case, it is necessary to remember the firm leverage and tax shields

must include distress, which are necessary to compute the cost of capital as well and thus will be treated later on.

The other choice to estimate the cost of equity is to use the beta which would be used in a healthy state in the CAPM equation, and then to adjust the equation for an additional premium to reflect distress:

$$E[R_i] = r_f + \beta_{i (healthy)}(E[R_m] - r_f) + \text{distress premium}.$$

Distress premium is obtained either by looking at historical data on returns earned by investing in the equity of distressed firms or by adding the difference between the company's own pre-tax cost of debt and the industry average cost of debt.

Adjusting discount rates: the debt to equity ratio

The target debt to equity ratio, the most used measure in the going-concern WACC, is not useful in distress. Indeed, as noted above, the firm is overleveraged with respect to a well performing firm in the same industry, and, if it wants to recover, it has to restructure its debt. According to Damodaran (2002)²⁶, bottom-up betas allow to use the current debt to equity ratio, which can be adjusted at any time. However, not only the firm is overleveraged, but also its capital structure is very complicated and restructuring might complicate it even further, for instance through the introduction of convertible bonds, debt-equity swaps and other forms of derivatives. Nonetheless, debt is expected to decrease for the firm to go back to a going-concern value. If this is the case, the book value of debt is never a good proxy of the debt value, on the contrary, it should be continuously updated, especially when a restructuring procedure is running. Moreover, it should be treated as a corporate bond, which is traded on the market, to estimate the interest rate depending on the rating of the company, i.e. its probability of default. A special issue refers to convertible bonds: in this situation, the option to convert must be stripped by debt and treated as equity. A simple way to do this is to value the convertible debt as if it were straight debt and consider the difference between the market value of the convertible debt and the straight debt portion as equity.

Adjusting discount rates: the interest tax shield

The interest tax shield cannot simply follow the statutory tax rate, since net operating losses, usually followed by tax loss carryforwards and tax loss carrybacks, investment tax credits and alternative minimum taxes may reduce the impact of tax benefits on income. It is convenient to follow the approach by Graham (1996)²⁷, tested by Graham and Mills (2009)²⁸, who determined

²⁶ DAMODARAN, A., 2002. *Investment valuation: Tools and Techniques for Determining the Value of Any Asset*. 2nd Edition. New Jersey: John Wiley & Sons Inc.

²⁷ GRAHAM, J. R., 1996. Debt and the marginal tax rate. *Journal of Financial Economics*, vol. 41, pp.41-73.

²⁸ GRAHAM, J. R., MILLS, L. F., 2009. Using tax return data to simulate corporate marginal tax rates. *Journal of Accounting and Economics*, vol. 46, pp. 366-388.

the “corporate marginal income tax rate”, i.e. the present value of current and expected future taxes paid on an additional dollar of income earned today. They first used firm historical data to compute the parameters of the distribution of the change in taxable income, which, according to Shevlin (1990)²⁹, follows a random walk with drift: $\Delta TI_{it} = \mu_i + \varepsilon_{it}$, where $\varepsilon_{it} \sim N(0; \sigma_{\Delta TI})$. After that, they drew randomly 18 realizations of ε_{it} and put them in the distribution above to consider the change in income for 18 years. The tax bill was then calculated using the entire corporate tax schedule and its present value for the 18-year period was obtained by discounting by the average corporate bond yield, gathered from Moody’s Bond Record; it is also computed for the 3 years before, in order to account for tax loss carrybacks, but in this case they are not discounted or grossed-up because, for all practical purposes, tax refunds are not paid with interest, and for the 3 years after, to adjust for tax loss carryforwards. Of course, 3 years is the period assigned by U.S. law, but it can be adapted to each country’s regulation. Next, a dollar was added to the income of the year under analysis and the present value of the tax bill was recalculated. The difference between the two tax bills represents the present value of taxes owed on an extra dollar of income earned by the firm i in year t (i.e., a single simulated estimate of marginal tax rate for the firm). The simulation was repeated 50 times per year and the average lead to the expected value and volatility of the marginal tax rate of firm i in year t , the MTR_{it} . This approach is far more credible than the traditional use of the statutory rate when dealing with a company with low or negative earnings, because it allows to compute a tax rate on a firm-by-firm basis and furthermore accounts for the fact that a distressed company does not fully benefit from the interest tax shield.

Adjusting discount rates: the cost of debt

The cost of debt is hard to obtain because expected yields do not match promised ones in below investment grade firms (Koller et al., 2015). In this case, the expected value of debt, and not its promised value, will determine the price and the yield to maturity will adjust accordingly.

An alternative to this is to adapt CAPM to distress. This relies on data which show that non-investment grade firms trade at a 0.1 higher beta than investment grade ones, so that it is only necessary to add 0.1 times the market risk premium to the value of a BBB rated bond.

Damodaran (2010), finally, proposed to add a default spread, based on the bond rating and therefore associated to the firm’s probability of default, to the risk-free rate. If the firm bonds are not traded, synthetic ratings can be simply estimated by looking at the main key value drivers of the company and adopting methods similar to the rating agencies’ ones. This

²⁹ SHEVLIN, T., 1990. Estimating corporate marginal tax rates with asymmetric tax treatment of gains and losses. *Journal of the American Taxation Association*, vol. 12, pp. 51-67 in GRAHAM, J. R., 1996. Debt and the marginal tax rate. *Journal of Financial Economics*, vol. 41, pp.41-73.

approach will still yield a high cost of debt, but probably more reliable than the estimation based on expected returns.

Going Concern DCF with adjustment for distress

An alternative to the adjustment in cash flows and discount rates is to measure the going-concern and the fire sale values of the firm, and then to compute the value of the firm as follows:

$$\text{Firm value} = \text{Going concern value} * (1 - \pi_{\text{distress}}) + \text{Distress sale value} * \pi_{\text{distress}}$$

where π_{distress} is the cumulative probability of distress over the valuation period.

The Going-Concern Value

In contrast with the Modified DCF, the valuation of the going-concern value will consider only scenarios in which the firm is expected to survive. The growth rate as well as the other assumptions for the firm assume it is going to recover and the discount rate is assumed to decrease over time, since the costs of equity and debt are decreasing and leverage is expected to shrink as the firm recovers. The firm also benefits from the interest tax shield, which will return to the statutory rate over time.

An even easier method, although less precise, is to assume the firm is in a going concern now. Cash flows are computed as if the firm were healthy, for instance by making assumptions which are appropriate for its competitors, whereas the cost of capital for the distressed firm can be set to the average target cost of capital for the industry.

The probability of distress

Several practitioners tried to estimate the probability of distress, using either statistical or rating based approaches. The most common approach, the *Z-Score approach*, was first developed by Altman in 1968 and is described by Altman and Hotchkiss (2006)³⁰ and Palepu, Healy and Peek (2013)³¹, but many similar models have been developed over time.

The assignment of appropriate default probabilities on corporate credit assets is a sequential three-step process which includes:

1. Credit scoring models.
2. Capital market risk equivalents—usually bond ratings.
3. Assignment of PDs (and possibly LGDs) on the credit portfolio.

The first step starts from evidence that some performance indicators differ between non-distressed and distressed firms, thus functioning as predictors of distress for the company. Afterwards, a multiple discriminant analysis allowed to develop the regression of the Z-score on a set of 22 potentially important indicators, which eventually reduced to five only. This

³⁰ ALTMAN, E. I., HOTCHKISS, E., 2006. *Corporate financial distress and bankruptcy: Predict and avoid bankruptcy, analyse and invest in distressed debt*. 3rd Edition. New Jersey: John Wiley & Sons Inc.

³¹ PALEPU, K. G., HEALY, P. M., PEEK, E., 2013. *Business analysis and valuation – IFRS Edition*. 3rd Edition. Cengage Learning EMEA.

	Average Annual Number of Firms	Average Z-Score	Standard Deviation
AAA	66	6.2	2.06
AA	194	4.73	2.36
A	519	3.74	2.29
BBB	530	2.81	1.48
BB	538	2.38	1.85
B	390	1.8	1.91
CCC	10	0.33	1.16
D ^a	244	-0.20	n.a.

^aMedian, based on data from 2000 to 2004.

Source: Compustat data tapes, 1996–2001, author compilation.

Exhibit 2.2. The Altman Z-Score model

formula is presented in Exhibit 2.2³², although some adjustments may be required, for example for a sample in emerging markets, for privately held firms or for non-manufacturers.

One of the main reasons for building a credit scoring model is to estimate the probability of default given a certain level of risk estimation. To this purpose, it is helpful to associate credit scores with ratings estimated by rating agencies, which are certainly imperfect but still provide a proxy of expected and unexpected probabilities of default and perhaps even loss given default estimation. Exhibit 2.3³³ lists the bond rating equivalents for various Z-score intervals based on average Z-scores for bonds rated in their respective categories. Of course, this constitutes a valid alternative for non-rated firms, whereas rated firms, which have already been assigned a rating, do not require the Z-score estimation step.

Once the “synthetic” or real rating is assigned, it is possible to assign a marginal and cumulative probability of default to each credit score and rating, based on Exhibit 2.4³⁴. However, this approach is not free of limitations. Rating agencies could have assigned a wrong rating to a

$$Z = 1.2 X_1 + 1.4 X_2 + 3.3 X_3 + 0.6 X_4 + 1.0 X_5$$

X_1 = Working Capital/Total Assets
 X_2 = Retained Earnings/Total Assets
 X_3 = Earnings before Interest and Taxes/Total Assets
 X_4 = Market Value of Equity/Book Value of Total Liabilities
 X_5 = Sales/Total Assets
 Z = Overall Index or Score

Exhibit 2.3. Average Z Score by S&P Bond Rating

³² See Altman et al., 2006, Ch. 11.

³³ See Altman et al., 2006, Ch. 11.

³⁴ See Altman et al., 2006, Ch. 11.

firm, or its rating could change over the following years. Furthermore, in reality this reflects the bond default, and not the firm default, because the estimation is done from the perspective of investors, not of the firm. Default could also not lead to bankruptcy, for example in case of private restructuring or when the firms delays payments. Furthermore, the firm could continue operating after default, so that applying this value to weigh distress sale value could distort valuation.

An alternative model to determine the probability of default is the *KMV model*, which assumes the firm value follows a normal distribution. The portion of the distribution in which the asset value is lower than liabilities represents the default point, and its distance to the expected value of the firm is defined as the distance to default. Finally, a mapping is determined between a firm's distance to default and the default rate probability based on the historical default experience of companies with similar distance-to-default values, and the PD, or Expected Default Frequency (EDF), is computed.

Many other models attempt to estimate the probability of distress. One example is the *probit approach*. Starting from the same historical data as Altman (1968), a dummy variable was assigned to each company. It could take on the value 1 in bankruptcy and 0 otherwise. In a similar way as in Altman, a regression was developed and the following equation was obtained, which, when significant, directly computes the PD:

$$\begin{aligned} & \textit{Distress Dummy} \\ & = a + b (\textit{Debt to Capital}) + c (\textit{Cash Balance / Value}) \\ & + d (\textit{Operating Margin}) \end{aligned}$$

An application of this is the *dynamic logit approach*, which obtains the marginal PD over the next period by following a logistic distribution:

		1	2	3	4	5	6	7	8	9	10
AAA	Marginal	0.00%	0.00%	0.00%	0.00%	0.03%	0.00%	0.00%	0.00%	0.00%	0.00%
	Cumulative	0.00	0.00	0.00	0.00	0.03	0.03	0.03	0.03	0.03	0.03
AA	Marginal	0.00	0.00	0.32	0.16	0.03	0.03	0.00	0.00	0.03	0.02
	Cumulative	0.00	0.00	0.32	0.48	0.51	0.54	0.54	0.59	0.57	0.59
A	Marginal	0.01	0.10	0.02	0.09	0.06	0.11	0.06	0.21	0.11	0.06
	Cumulative	0.01	0.11	0.13	0.22	0.28	0.39	0.45	0.65	0.76	0.82
BBB	Marginal	0.36	3.22	1.43	1.28	0.77	0.45	0.20	0.20	0.14	0.40
	Cumulative	0.36	3.56	4.49	6.16	6.89	7.31	7.50	7.68	7.87	8.18
BB	Marginal	1.19	2.48	4.40	2.01	2.51	1.16	1.60	0.88	1.70	3.60
	Cumulative	1.19	3.64	7.88	9.74	12.00	12.93	14.36	15.07	16.52	19.60
B	Marginal	2.85	6.85	7.40	8.55	6.00	4.16	3.72	2.28	1.96	0.86
	Cumulative	2.85	9.51	16.20	23.37	27.94	30.96	33.46	34.97	36.25	36.80
CCC	Marginal	7.98	15.57	19.55	12.10	4.26	9.45	5.60	3.15	0.00	4.28
	Cumulative	7.98	22.31	37.50	45.06	47.37	52.35	55.01	56.43	56.43	58.30

*Rated by S&P at issuance.

Based on 1,796 defaulted issues.

Source: Standard & Poor's; Altman and Fanjul (2004).

Exhibit 2.4: All rated corporate bonds, Mortality Rates, 1971-2004.

$$PD(t-1, t) = Prob_{t-1}(Dummy_{it} = 1) = \frac{1}{1 + \exp(-a - bx_{i,t-1})}$$

where $x_{i,t-1}$ represents a vector of explanatory variables which is made up of bankruptcy indicators.

To use a more conventional approach, it is also possible to estimate PD by matching the *bond price* with its expected future payments multiplied by the PD, and then discounted at risk-free rate, so that risk is incorporated in the numerator and not, as usual, in the risk premium:

$$Bond\ price = \sum_{t=1}^T \frac{Coupon_t * (1 - PD)^t}{(1 + r_f)^t} + \frac{Face\ value * (1 - PD)^T}{(1 + r_f)^T}$$

Though this approach could seem straightforward at first glance, more complicated situations, such as convertible bonds or privately traded bonds, are not correctly estimated, and, in general, the distortions associated to the possible continuation of the business after default still hold.

Modified WACC in distress

As noted above, adapting DCF to distress implies to modify the expected cash flows, the discount rate or both. So far, we either simply chose the standard WACC and modified cash flows, or we adapted it while accounting for default in the cash flows as well. What follows, instead, proposes two models to incorporate distress in the WACC only.

Koziol (2013)³⁵ defined two further variables which need to be included in the WACC for distress, assuming that cash flows are given as if the firm were healthy: bankruptcy costs η and probability of survivorship p . The first ones follow Andrade and Kaplan (1998) estimation quoted above, include direct and indirect costs and are included in the WACC by increasing it if they incur, i.e. if default occurs. The probability of default is instead determined by the rating of the company, and its complementary value to 1 represents the probability to survive. If the firm survives, the interest tax shield may be applied, whereas, if it defaults, bankruptcy costs will arise. Furthermore, the interest tax shield is computed by using c , which represents the promised yield rather than the expected one. This works better than assuming $k_d = c$: the promised interest c is weighted by the probability of survivorship, which incorporates the uncertainty of the payment, such that only if the firm does not default it can fulfil its promised obligations and interest tax shield will arise. On the contrary, the cost of debt k_d already represents the expected yield and is therefore not weighted. The formula he derives works as follows:

$$WACC^K = \frac{E}{V} k_e + \frac{D}{V} k_d - p\tau \frac{D}{V} c + (1 - p)\eta$$

³⁵ KOZIOL, C., 2013. A simple correction of the WACC discount rate for default risk and bankruptcy costs. *Review of Quantitative Finance Accounting*, vol. 42, pp. 653-666.

In this formula, E, D and V represent the market values of equity, debt and assets respectively, k_e and k_d are the costs of equity and debt before incorporating distress. The so obtained pre-tax WACC is then summed to costs of distress η in case of default, which occurs with probability $(1-p)$, and reduced by the interest tax shields in a healthy state, where τ is the tax rate, c is the promised yield paid with uncertainty and p is the probability of survivorship.

As a natural result, an increase in the newly introduced factors, probability of default and bankruptcy costs, will lead to a larger gap from the non-corrected WACC, whereas if they are negligible the pricing error will be negligible as well.

Another approach was introduced by Saha and Malkiel (2015)³⁶, who used the continuing value formula to derive a rate adjusted by the probability of cessation of cash flows d , which may occur at any time in the future and is considered constant. The discount rate resulted:

$$WACC_{terminal\ value}^{SM} = \frac{d + WACC}{1 - d}$$

A further adjustment accounted not only for the probability of cessation, but also for a finite probability of reduction of a fraction f . The procedure was similar as before and resulted in:

$$WACC_{terminal\ value}^{SM} = \frac{df + WACC}{1 - df}$$

Hrdí and Šimek (2012) observe it may be worth applying two or three different costs of capital, allowing for distress and subsequent restructuring. The interest rate for the first phase would be the interest rate with a high average cost of capital on the very upper limit of the cost of equity and loan capital, usual for businesses which are in financial distress. In the second stage of transition, the cost of capital will be reflected where there is a decline in risk surcharge due to the stabilization of the company and in the third stage, i.e. stage of continuing value, the classical average capital cost of the company after recovery can be used.

Of course, many other similar approaches have been developed over time and just a brief overview was given here. However, most of them are theoretical and hardly work in practice, at least because the assumptions are not so reliable.

WACC, indeed, is not expected to be constant in practice: maintaining the peers' WACC could overestimate the value of the firm, whereas current values could underestimate it by neglecting the probability to survive. Corrections may involve the computation of PD, which is not always straightforward in practice, or the calculation of distress sale values or costs of bankruptcy, which are hard to be estimated, especially if the company is still quite distant to default, because they must rely on expectations of future values. In some cases, therefore, it is not worth valuing

³⁶ SAHA, A., MALKIEL, B., 2015. DCF Valuation with cash flow cessation risk. *Journal of Applied Finance*, vol. 22, issue 1.

a company by DCF valuation. Instead, it shall be substituted by other approaches, which are still income-based on one hand, but present less drawbacks on the other.

4. Income approaches: Adjusted Present Value

As mentioned above, the application of a constant WACC on a company which has necessarily to change capital structure can result in distortions, which could be only solved by adjusting the WACC on a yearly basis. However, this approach is cumbersome and imprecise. In order to deal with this issue, the Adjusted Present Value (APV) can be adopted as an alternative approach. This model starts from the main teachings by Modigliani and Miller that the value of a company does not depend on its capital structure unless market imperfections affect it, such as taxes, distress costs and agency costs.

As a result, the value of a firm under perfect market conditions is simply equal to the expected value of its future cash flows discounted by the unlevered cost of capital. After that, it must be adjusted by market imperfections and, specifically in distress, by the interest tax shield (ITS) and the bankruptcy costs. Indeed, as noted above, since interest is tax deductible (it is indeed subtracted to earnings before the computation of taxes), increasing debt leads to lower taxes and, thus, to a higher firm value. However, if the company gets too leveraged, its business will deteriorate since stakeholders will fear its ability to fulfil debt obligations, and the risk of bankruptcy will reduce the value of the company. Finally, as agency costs between stock- and stakeholders may also arise (as debt which is serviced strategically, as Fan and Sundaresan, 2000, and Anderson and Sundaresan, 1996,³⁷ proposed in their models), additional costs may reduce the firm value. However, they are usually not considered in APV approach.

Following the evolution of Modigliani and Miller's and following economists' studies, which are summarized in Exhibit 2.5³⁸, the levered value of the company will result in:

$$V_l = V_u + ITS - \text{Expected bankruptcy costs}$$

$$= \sum_{t=1}^T \frac{FCF_t}{(1 + k_u)^t} + \frac{CV_T}{(1 + k_u)^T} + \sum_{t=1}^T \frac{ITS_t}{(1 + k_d)^t} - PV(\text{Bankruptcy costs})$$

The main benefit to use this model under distress is that the consequences of distress on debt and its costs in case of bankruptcy are estimated separately, so that operational and financial distress can be analysed separately. This, of course, is accompanied by the clear advantage of neglecting assumptions on the future capital structure of the company.

³⁷ FAN, H., SUNDARESAN, S. M., 2000. Debt valuation, renegotiation and optimal dividend policy. *The Review of Financial Studies*, vol. 13, issue 4, pp. 1057-1099. ANDERSON, R. W., SUNDARESAN, S. M., 1996. Design and valuation of debt contracts. *The Review of Financial Studies*, vol. 9, issue 1, pp. 37-68.

³⁸ See Altman et al., 2006. Figure 6.5, Ch. 6

Estimating the unlevered value of the company

The procedure to evaluate the company is very similar to the DCF approach but differs in one main point: cash flows are discounted by the unlevered cost of capital k_u . Cash flows are the expected after-tax operating cash flows for the company, which are based on “reasonable” assumptions, as suggested by Damodaran (2010). Schueler (2007)³⁹ gives some additional indications about the set of assumptions which shall be chosen in distress. He argues that one might use historical ratios like the ratio of operating costs to revenues and the ratio of capital expenditures to revenues, although it must be noticed that in some cases the former values tend to increase, while the latter decrease. On the other hand, historical growth rates do not work well in distress. If distress hits the whole economy or sector, however, the growth rates on a consolidated basis can be used. If the company suffers from operational distress, it is convenient to use its proper growth rate, but, since the company is valued as if it were all-equity financed, and therefore the consequences of financial distress will be estimated separately, financial distress assumptions shall not be considered here. Furthermore, other assumptions based upon the growth of the European GDP, inflation rates and other macroeconomic indicators may be included.

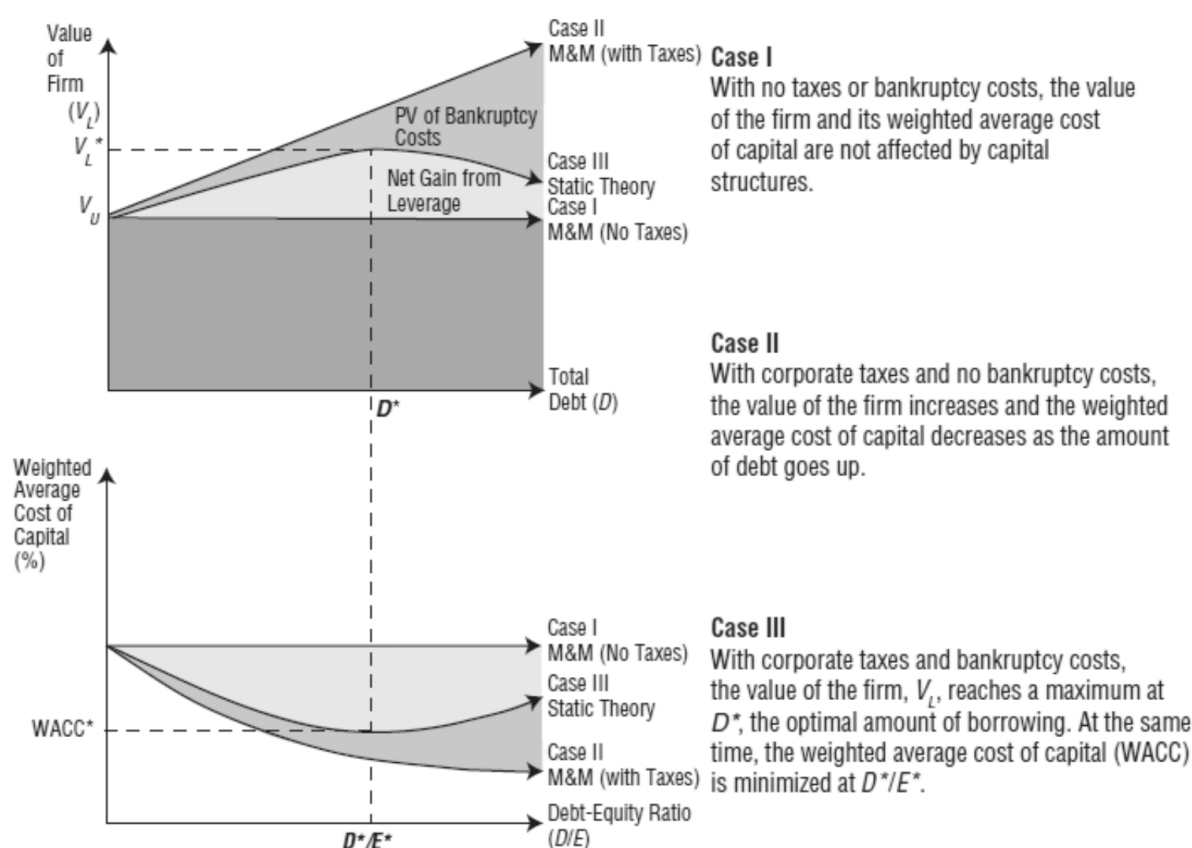


Exhibit 2.5: Net effects of leverage on firm value: M&M and Altman (1984)

³⁹ SCHUELER, A., 2007. Valuing companies in financial trouble – Eurotunnel. JASSA, issue 2, Winter 2007.

Once cash flow projection has been implemented, cash flows and continuing value shall be discounted by the unlevered cost of capital. The unlevered cost of capital is the cost which would be applied if the firm were all-equity financed, which means if there were no debt, and therefore it takes into account neither tax benefits on interest nor distress costs. The unlevered cost of capital k_u is obtained as:

$$k_u = \frac{D}{V}k_d + \frac{E}{V}k_e$$

This formula is equivalent to the WACC but for the absence of the marginal tax rate, since the firm is unlevered and thus does not profit from interest tax shields. The main advantage of using k_u is that its formula can simplify to:

$$k_u = r_f + \beta_u(r_m - r_f)$$

where $\beta_u = \frac{D}{V}\beta_d + \frac{E}{V}\beta_e$, r_f is the risk-free rate and r_m is the market return. As a result, k_u does not depend on the capital structure, does not modify as leverage changes and eliminates much of the complexity intrinsic in DCF valuation.

Computing the present value of interest tax shields

As far as interest tax shields are concerned, the most common method for companies whose leverage is at the target or that, however, only exploit benefits from interest payments is to estimate the expected interest payment by multiplying the prior year's net debt by the promised yield to maturity. These values are determined taking into account debt and debt equivalents. Next, it is multiplied by the marginal tax rate, using the statutory tax rate, and the resulting value represents the interest tax shields (Koller et al., 2015). To discount them, most practitioners use the cost of debt, so that if the tax shields are computed in perpetuity:

$$ITS = \frac{\tau D k_d}{k_d} = \tau D$$

However, too high leverage reduces the tax shield because it can lead to an exacerbation of economic conditions and therefore to losses. Using the statutory tax rate could overestimate future tax shields. Koller et al. (2015) suggest modelling expected tax shields, rather than promised ones, simply by reducing the promised tax shield by the cumulative probability of default. Alternatively, the MTR may conveniently substitute the corporate tax rate. Finally, Schueler (2007) proposes to deal with tax losses carryforwards by getting the difference between leverage and unleveraged valuation, which is due to interest payments and is thus associated to "real" tax shields.

Adjusting for leverage changes

As noted by Arzac (1996)⁴⁰, in distress “debt reduction is a function of cash flow realizations and, therefore, is uncertain. Leverage changes invalidate the use of a constant weighted-average cost of capital or a constant cost of equity. Furthermore, available formulas for adjusting the cost of equity as a function of leverage are not applicable to this case.” To deal with this issue, he introduced *recursive APV valuation*. The main assumption is that, when distress occurs, all cash flows will be used for debt reduction, thus avoiding one-time restructuring. As a consequence, the distinction between the unlevered value of the firm and the additional value coming from market imperfections will still hold, but the value of debt necessary to compute interest tax shields is computed in a more realistic way. Indeed, it is obtained by subtracting the present value of the past cash flows of the levered firm from the outstanding value of debt. After, the ITS obtained in this way is discounted by the cost of debt associated with that level of riskiness.

This approach is consistent with the idea of debt reduction through cash flows and includes uncertainty directly in the computation of interest tax shields. However, although it is for sure more realistic than the simple APV, where $PV(ITS) = \sum_{t=1}^T \frac{\tau D k_d}{(1+k_d)^t}$, it is also much more cumbersome in terms of calculations, because it requires all levered cash flows for all times in the past for the computation of each time “real” interest tax shield.

Introducing bankruptcy costs in the valuation

The final value which requires estimation is the present value of bankruptcy costs, which is usually estimated as the product of the present value of bankruptcy costs multiplied by the probability of default after the additional debt issuance.

If PD can be simply estimated using one of the methods described above, on the other hand there is no convergence of methods to compute bankruptcy costs. Damodaran (2010) simply proposes to compute them as the difference between the value of a firm as a going concern and the distress sale value.

Almeida and Philippon (2007)⁴¹ argued that costs of distress, as noted above, include not only litigation fees and other direct costs of distress, but also less quantifiable effects, such as reputational effects which imply, among other consequences, loss of customers, suppliers and financiers. As estimated by Andrade and Kaplan (1998), direct costs of distress only make up 3 to 5% of total firm value at the time of distress, but indirect costs can reach 10 to 23%. They tried to estimate these costs by first looking at the percentage costs of distress on the current

⁴⁰ ARZAC, E. R., 1996. Valuation of highly leveraged firms. *Financial Analysts Journal*, vol. 52, issue 4, p. 42.

⁴¹ ALMEIDA, H., PHILIPPON, T., 2007. The risk-adjusted cost of financial distress. *The Journal of Finance*, vol. 62, issue 6, pp. 2557-2586.

value, weighted by the probability they occur, i.e. the probability of distress. This is derived either from corporate bond spreads, akin to what described before, or risk-adjusted by using:

$$q = \frac{(k_d - r_f)}{(1 + k_d)(1 - \rho)}$$

where ρ is the recovery rate, which is obtained from historical recovery rates of firms with the same bond rating. The result is then discounted by the appropriate discount rate, which is the cost of the debt in the first case and the risk-free rate in the second.

5. Income approaches: Capital Cash Flows

The Capital Cash Flows model (CCF), presented by Ruback (2002)⁴², considers both the benefits of DCF and APV valuation. CCF valuation, indeed, strips out the effect of interest tax shields on valuation, and values them separately from the valuation of an all-equity financed firm discounted by the unlevered cost of capital, as APV approach does. It can be observed that, therefore, interest tax shields are included in the numerator of valuation formula, as in APV, and not in the denominator, as instead DCF does.

On the other hand, however, the discount rate for interest tax shields is not the cost of debt chosen depending on the bond rate or on other methods which allow to account for its intrinsic riskiness, as in APV approach. As argued by Ruback (2002), this implicitly assumes the risk associated to interest tax shields matches that of the debt, so that $k_{txa} = k_d$. This, however, roughly follows from the assumption that the amount of debt is fixed for perpetuity, so that the formula described above, $ITS = \frac{\tau D k_d}{k_{d,t}}$, where k_d (the fixed yield on debt) matches $k_{d,t}$ (the cost of debt, now constant), simply reduces to $ITS = \tau D$. The main idea here is that, therefore, the debt amount is kept constant and therefore that the interest tax shield is constant too and can be discounted by using a perpetuity formula.

When a firm is overleveraged, however, both its debt-to-value ratio and its amount of debt are expected to decrease for the firm to continue operations. If this is the case, the risk of interest tax shields will depend both on the riskiness of debt and on its changes in level, so that using k_d will underestimate the discount rate and overstate the firm value. According to Ruback (2002), interest tax shields shall therefore be discounted by the unlevered cost of capital, as free cash flows.

⁴² RUBACK, R. S., 2002. Capital Cash Flows: A simple approach to valuing risky cash flows. *Financial Management*, vol. 31, issue 2.

In this model, free cash flows are estimated either by starting from net income or from EBIT. Figure 2.6⁴³ gives a brief overview on this topic. In short, cash adjustments which are not included in the income statement as well as non-cash lines which were included in the balance sheet are considered to get FCF in the explicit projection period.

Furthermore, Gilson, Hotchkiss and Ruback (2000)⁴⁴ show a method to deal with continuing value for firms which present net operating loss carryforwards (NOLs) at the end of the explicit projection period. It is based on U.S. Bankruptcy Code: according to Section 382 Limitation, in case of change of ownership the pre-change NOLs shall be used to offset the taxable income after the change, unless they exceed each year fair market value of the company's equity times the long-term tax-exempt rate (provided by the Internal Revenue Service). To deal with this limitation, they estimated annual NOLs before the change in ownership, the projected Section 382 Limitation, if any, and the remaining balance of NOLs. Then, they multiplied this amount by the marginal tax rate, getting the tax shield due to NOLs, and they added it back to an extended forecast of cash flows as follows:

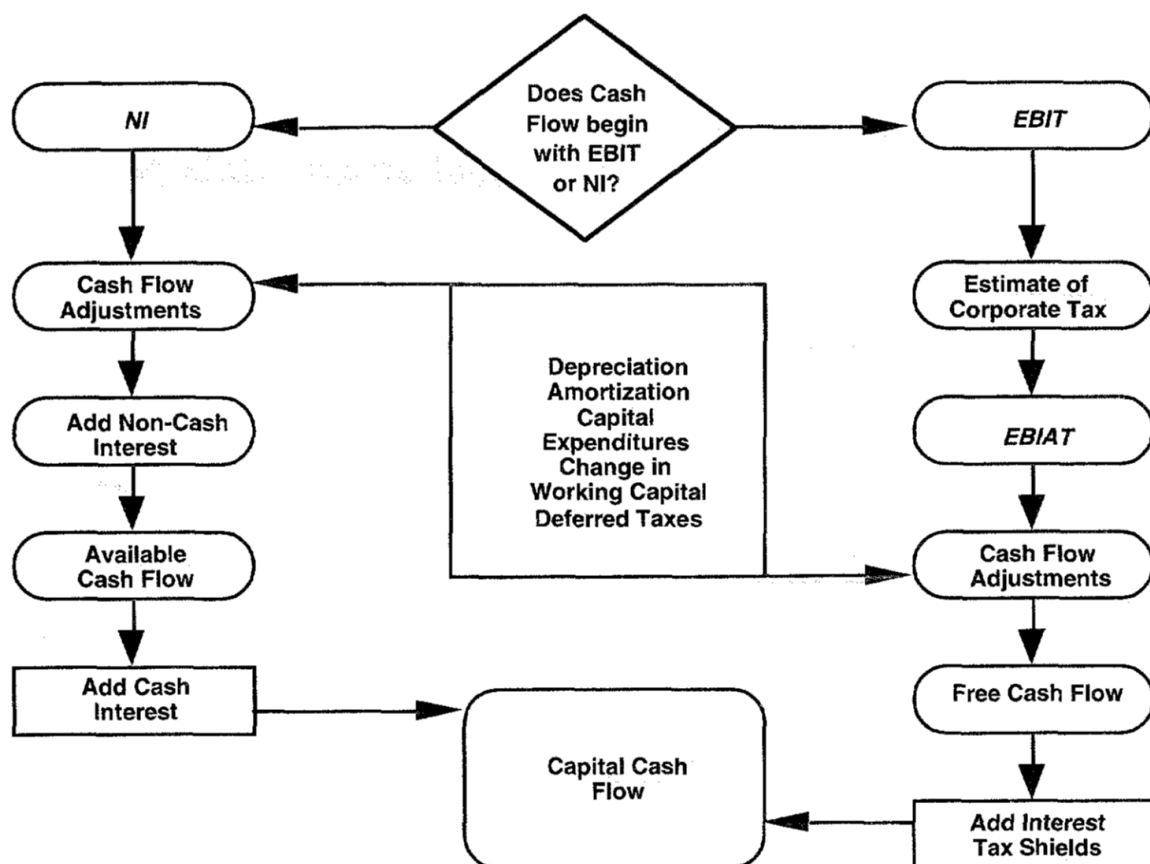


Exhibit 2.6. The computation of Capital Cash Flows following Net Income and EBIT approaches.

⁴³ See Ruback (2002)

⁴⁴ GILSON, S. C., HOTCHKISS, E. S., RUBACK, R. S., 2000. Valuation of bankrupt firms. *The Review of financial studies*, vol. 13, issue 1, pp. 43-74.

$$\begin{aligned}
CCF &= EBITA - \text{Operating taxes} + \text{Cash Flow Adjustment} + \text{Tax shield due to NOLs} \\
&= NI + \text{Cash Flow Adjustment} + \text{Cash and Noncash Interest} \\
&\quad + \text{Tax shield due to NOLs}
\end{aligned}$$

After extending the explicit forecast period, the value of the firm can be computed as a growing perpetuity after it starts experiencing earnings.

Afterwards, the discount rate k_u and ITS can be estimated as mentioned above in the discussion about APV. Again, beta cannot be estimated by simply looking at the single company's historical data, since stock returns of a bankrupt company are usually negative and this is not what is expected after restructuring. Instead, again, data provided by the whole industry are usually more significant, especially if the company is an outlier.

6. Comparing income approaches in distress

Although the income approaches described above are similar to each other, the subtle differences among them may make one or the other more convenient in some circumstances.

CCF is algebraically equivalent to DCF, as proved by Ruback (2002). In fact, the main difference is the incorporation of tax shields in the numerator in DCF valuation and in the denominator in CCF, but, if all the forecast assumptions match and if a constant WACC is applied, then the results are identical. However, a constant WACC is applicable only when the company follows a target capital structure, so that interest tax shields share the same risk as operations. In this case, dividing cash flows from interest tax shields is cumbersome and useless and DCF is a more convenient valuation method, although they coincide. If, instead, as in distress, complex financing plans are included, so that the capital structure is consistently changing over time, or if complex tax situations shall also be treated in valuation, then CCF allows to use a discount rate that both models explicitly the effect of deductible interest on taxes and discounts by a rate which does not depend on the capital structure.

As noted above, according to Ruback (2002), it is more correct to discount ITS by the cost of debt (and thus to use APV rather than CCF) only if their riskiness match, i.e. if debt is perpetual and fixed. However, unless there are regulatory restrictions which imply a fixed amount of debt, this is not very common in practice.

The best choice would be to model the debt policy and then apply the discount rate which best fits the policy, as suggested by the recursive APV model. However, in most cases insufficient information is available at the first stage of valuation to explicitly model cash flows and interest tax shields, so that CCF solves both for the problem of fixed debt level and of fixed capital structure. However, according to Arzac (1996), CCF is not consistent, because it does not consider that interest tax shields are negatively correlated to the unlevered value of the firm. As

a consequence, they cannot share the same risk, and this leads to an underestimation of the value of tax shield.

To summarize, finding a universally valid income approach is impossible, and several factors may lead to distortion in valuation. To this aim, the best solution is to make assumptions created by the analyst to value the company in a proper and theoretically sound way, having in mind that, since valuation assumptions are imperfect, a somehow distorted final value may result.

7. Market approach: Valuation using comparable company multiples

As described in the introduction made by Stein et al. (2009), the premise of the market approach is that the value of a company can be determined by comparing a company to current values assigned by the market for similar companies (*guideline company method*) or the prices paid in the past for similar companies (*transaction company method*). It is defined as a relative valuation since it assesses a company's value by calculating multiples on performance indicators, market values or prices of similar companies and by applying the resulting multiple to the corresponding financial metric of the subject company.

When dealing with distress, it is possible to consider it implicitly or to adjust the going-concern value obtained for distress explicitly.

Including Distress into Relative Valuation

When distress is considered implicitly in valuation, the company is valued through a comparison with other distressed companies in the same business. Therefore, the adjustment to the usual approach will apply directly to the initial assumptions, for instance when choosing the right multiple, the peer group and the forecast for the financial metric of the company (Damodaran, 2010). This approach is therefore most useful when distress hits the whole industry or sector.

First, the right multiple needs to be chosen, because earnings may be negative or low for companies in distress, and this could also not be due to operations. Consequently, it is necessary to dig into the balance sheet to find a positive, and thus comparable, value. This is usually EBITDA (Damodaran, 2010 and Gilson et al., 2000), although, according to Koller et al. (2005), in most cases EBITA is even more convenient because it includes all the operating costs, and also depreciation. Another alternative very convenient in these situation is to consider Revenues. The multiples which are applied are usually enterprise value to revenues, EBITDA or EBITA. Other multiples, instead, are not applicable in distress: for instance, Price-to-Earnings may lead to negative values in this case (Damodaran, 2010), and transaction multiples

are not useful in a liquidation scenario (Stein et al., 2005). Usually, the median of the peer group multiples will be chosen in order to strip out the outlier effect.

Second, in this framework the choice of comparables will involve other distressed companies in the same business only. In order to have a sufficiently large sample, therefore, this approach only applies to distressed industries, sectors, or in case of a crisis. In the last case, business may also become a secondary choice and many distressed firms may be included in the sample independently on their business. However, this could also distort valuation, because firms in different businesses may have similar multiples but different severity of distress. Though in a more relaxed way, this risk also holds for firms in the same business: indeed, we could compare firms that are distressed to a different level or for differing reasons.

Finally, firms in distress usually have negative or very low present or even forward estimates, at least for the first explicit forecast years. Gilson et al. (2000) suggest using the first positive estimate and always annualizing it. However, they also argue this choice could understate the firm value, since its net operating earnings could be temporarily low with respect to its long-term growth after restructuring.

Relative Valuation weighted for Distress

In case distress is specific to a company, or if a sufficiently large group of distressed peers cannot be applied, the best choice is to correct explicitly for distress. The formula is equivalent to the going-concern DCF value with adjustment for distress:

$$Firm\ value = Going\ concern\ value * (1 - \pi_{distress}) + Distress\ sale\ value * \pi_{distress}$$

The distress sale value and the probability of distress are estimated exactly as in Section 3. The main difference refers to the going concern value, which is now computed according to the usual relative valuation. A broader set of industry or sector peers is chosen, including healthy firms too. The median multiple is then multiplied by the performance indicator of the company to get its value. However, in most cases this could be negatively affected by the overhang of distress, such that the adjustment will apply also directly in the going concern value, and the final firm value will be underestimated. To deal with this issue, it is possible to use a long-term forecast, such as 10 years from now, as suggested by Damodaran (2010), such that the firm is treated as healthy because it has recovered and its value will not include distress.

This valuation, however, is not very precise in most cases, since the forecast year used for comparables does not match that of the valued company, and because in some cases it would even lead to negative equity and zero value per share.

8. Conclusion

This Chapter analysed the main approaches to valuation and specifically adjusted them to incorporate distress. Some of them result in more realistic values than others, but this also depends on the assumptions and the circumstances of the valuation. Gilson et al. (2000) compared the results of CCF and relative valuation, and they observed the median valuation error from the market value was less than 1% through CCF and about 5% using multiples, but they also noticed a huge variation, higher in relative valuation. These results are substantially in line with those proposed by Kaplan and Ruback (1995) for highly leveraged transactions, showing that, in general, these valuation methods can resemble market valuation but, in some cases, also present huge errors.

As a consequence, using these approaches can be misleading, and this especially holds if we consider that equity and debt holders will be more interested in the value of their own claims than in the firm value. Indeed, they usually do not consider explicitly how to value debt and equity, and the usual methods based on book or market value could distort values in case of distress. If we try to deal with these issues, furthermore, as noted in the latest section, claims' value could even result in negative values if any assumption is wrong.

Consequently, new models which explicitly consider distress may be more convenient to value equity and debt values. In particular, the Merton model tries to value a firm based on option pricing theory and considering equity as a call option on the firm value. In the following Chapter, this approach and its evolution will be dealt with in detail.

Chapter III. Pricing Corporate Debt: An Option Pricing Approach

1. Introduction

In the conclusion of the previous chapter the limitations of the general approaches for the valuation of risky debt and stock were pointed out. To solve these issues, a special model has first been proposed by Merton (1974)⁴⁵, who used the option pricing theory proposed by Black and Scholes (1973)⁴⁶ and Merton (1973)⁴⁷. Merton observed that the value of equity behaves equivalently to a call option on the firm value V with strike price equal to the face value of debt F which can only be exercised at maturity T . Therefore, he modelled the distribution of V as a stochastic process and derived equity and debt values from Black and Scholes (1973) formula. This approach was far too simplistic and never applicable in practice, but strongly fostered further research. Over time, default started to be considered a consequence of other circumstances than “technical” default, such as illiquidity or strategic optimal default point for shareholders. Consequently, strategic default and bargaining games to optimally distribute bankruptcy proceeds were introduced.

Furthermore, assumptions became much more realistic. First, fixed and floating rate debt were distinguished and subordinated debt, debt covenants, convertible debt and other equity and debt equivalents were included in the valuation. Recently, reorganization has been considered even in its most complicated aspects, such as automatic stay of assets, grace period and arrears account.

On the other hand, other approaches were introduced, which considered stochastic interest rates or modelled EBIT rather than V , just to mention some of the most famous, or which assigned another distribution to V .

Option pricing valuation is a very broad topic, and analysing all its aspects is not possible in this limited framework. Although in the following only what is useful for the development of a simple but complete binomial model will be dealt with in detail, a brief overview of the main models which will be considered is given in the following table.

⁴⁵ MERTON, R. C., 1974. On the pricing of corporate debt: the risk structure of interest rates. *The Journal of Finance*, vol. 29, issue 2, pp. 449-470.

⁴⁶ BLACK, F., SCHOLES, M., 1973. The Pricing of Options and Corporate Liabilities. *Journal of Political Economy*, vol. 81, pp. 637-659.

⁴⁷ MERTON, R. C., 1973. Theory of Rational Option Pricing. *Bell Journal of Economics and Management Science*, vol. 4, pp. 141-183.

Topic	Research
Basic model: - default only occurs at maturity; - analysis of credit spreads.	Merton (1974)
More complicated capital structure	Black and Cox (1976) Morellec (2001) Leland (1994)
Breakdown of absolute priority rule	Longstaff and Schwartz (1995)
Binomial approach	Broadie and Kaya (2007)
Stochastic interest rates	Longstaff and Schwartz (1995) Mello and Parsons (1992) Kim, Ramaswamy and Sundaresan (1995)
Endogenous default threshold for liquidation	Leland (1994) Fan and Sundaresan (2000) Trigeorgis (2004) Morellec (2001)
Finite maturity	Leland and Toft (1996) Longstaff and Schwartz (1995) Kim, Ramaswamy and Sunderasan (1993) Broadie, Chernov and Sundaresan (2008) Paseka (2004)
Private workout	Fan and Sundaresan (2000) Mella-Barral and Perraudin (1997) Anderson and Sundaresan (1996)
Chapter 11 restructuring	Francois and Morellec (2004) Morellec (2001) Paseka (2004) Moreaux (2002) Broadie, Chernov and Sundaresan (2008) Broadie and Kaya (2007) Mella-Barral (1999)
Strategic default	Anderson and Sundaresan (1996) Mella-Barral and Perraudin (1997) Fan and Sundaresan (2000) Paseka (2004) Francois and Morellec (2004) Broadie, Chernov and Sundaresan (2008)
Dynamic model (change in leverage)	Morellec (2001) Goldstein, Ju and Leland (2001)
Illiquidity	Huang and Huang (2003) Trigeorgis (2004)
EBIT/Earnings based models	Mella-Barral and Perraudin (1997) Fan and Sundaresan (2000) Goldstein, Ju and Leland (2001) Broadie, Chernov and Sundaresan (2008)
Jump diffusion process	Kyprianou and Surya (2006)

2. Valuing Equity and Debt as options

Just after the introduction of the valuation method of options, a major application of the model was proposed by Merton (1974). He first assumed that a firm has an aggregate market value of assets V which has been financed by two types of securities, equity and debt. In particular, the firm has issued very standard zero-coupon bonds with aggregate value D , maturity T and face

value F . The class of bonds is homogenous and has no additional terms, such as bond covenants, and the absolute priority rule is always respected. This means that, until debt repayment, it cannot pay dividends, and, furthermore, default can only occur at maturity.

Indeed, upon maturity stockholders can:

- i. pay debt holders the promised amount F , if it is higher than the value of the company: they can get liquidity by selling their assets or issuing new debt, and they will finally keep control of the firm;
- ii. default on the promise to pay F to debt holders if, instead, they are not able to reach the face value of debt (i.e. $V < F$): shareholders get nothing, whereas the value and the control of the firm are transferred to debt holders.

As a result, equity can be considered as a call option on V with strike price F : if $V > F$ at maturity T , stock holders exercise the call and “buy”, or, better in this case, keep control of the firm by repaying the full amount promised to debt holders. If, instead, $V < F$ in T , they let creditors get control of assets by only repaying V , which is the full value of assets.

In the same way, it is also possible to value debt. If the option is exercised, debt holders get exactly the strike price, whereas they get V otherwise. As a result, they always get $\min[V; F]$, or, equivalently, $F - \max[F - V; 0]$. Debt can thus be considered as the difference between the face value and a put option with parameters equivalent to the call on equity.

The numerical approach adopted by Merton (1974)

The approach developed by Black and Scholes (1973) aims at obtaining the value of an option on a stock in continuous time, referring to the usual definitions of European call and put options. Merton (1974) developed a similar model, aimed not at pricing options on stocks, but instead at valuing firm’s claims as options on the firm value V , but following a procedure in line with Black and Scholes (1973). In order to implement this model, he elaborated the following assumptions:

1. The market is perfect, which means: lack of transactions costs, taxes or problems with indivisibilities of assets, presence of many investors who can sell as much of an asset as they want at the market price, borrowing and lending at the same rate and short sale permitted;
2. Trading in assets takes place continuously in time;
3. The Modigliani-Miller theorem holds;
4. The Term-Structure is "flat" and known with certainty;
5. The dynamics for the value of the firm, V , through time can be described by a diffusion-type stochastic process with stochastic differential equation:

$$dV_t = (aV_t - C) dt + \sigma V_t dW_t$$

where a is the instantaneous expected rate of return on the firm per unit time, C is the total dollar payouts by the firm per unit time (e.g., dividends or interest payments) if positive, and it is the net dollars received by the firm from new financing if negative, σ is the instantaneous volatility of the return on the firm per unit time and W_t is a standard Brownian motion. This allows price movements to be continuous in time and independent, so that price changes are unpredictable and efficient market hypotheses hold.

After, he supposed a security Y follows the diffusion process:

$$dY_t = (a_Y Y_t - C_Y) dt + \sigma_Y Y_t dW_t^Y$$

Applying Itô lemma on $Y=D(V,t)$ and rearranging, he obtained the second order partial differential equation (1):

$$\frac{1}{2} \sigma^2 V^2 D_{VV} + (rV - C) D_V - rD + D_t + C_Y = 0$$

which allows to get D as independent on the expected rate of return of the assets or the security as well as on the risk preferences of investors, so that the solution holds in general and the application of contingent claims valuation is consistent with risk neutrality. Moreover, in this case it is assumed that no chance for reorganization is allowed and that the value of assets plummets to V if the firm falls bankrupt.

In order to solve (1) for the value of risky bond with zero-coupon bond ($C_Y = 0$ and $\tau = T - t$, so that $D_t = -D_\tau$, which is the time left to maturity), two boundary conditions and an initial condition are needed. The boundary conditions stem from the provisions of the indenture and the limited liability of equity f :

$$D(0; \tau) = f(0; \tau) = 0$$

$$D(V; \tau) \leq V$$

The initial condition also follows from the fact that managers will act in the best interest of stakeholders, so that:

$$D(V; 0) = \min[V, F].$$

Alternatively, the same approach can be applied to equity, where, however, as noted above, f substitutes F in (1) and the initial condition becomes:

$$f(V; 0) = \max[V - F; 0].$$

As explained by B&S (1973) and by Merton (1973), this equation and boundary conditions are equivalent to the equations for a European call option on a non-dividend-paying common stock, so that it results exactly in the formulas given above. From this, debt can be derived as the

difference of the firm value from the value of the call option corresponding to equity and subsequent rearrangements for leverage lead to the computation of risk-premia:

$$f(V, \tau) = V\Phi(x_1) - F \exp(-r\tau) \Phi(x_2)$$

$$\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^x \exp\left(-\frac{1}{2}z^2\right) dz$$

$$x_1 = \frac{\ln\left(\frac{V}{F}\right) + \left(r + \frac{1}{2}\sigma^2\right)\tau}{\sigma\sqrt{\tau}}$$

$$x_2 = x_1 - \sigma\sqrt{\tau}$$

$$D(V; \tau) = V - f = V\Phi(-x_1) + F \exp(-r\tau) \Phi(x_2) = D_f\{L^{-1}\Phi(-x_1) + \Phi(x_2)\}$$

$$L = \frac{F \exp(-r\tau)}{V} = \frac{D_f}{V}$$

Adapting the model to real world: getting inputs

Although the model is theoretically consistent, adapting it to reality requires an estimation of the firm assets value and volatility, as well as further specifications about debt.

Estimating firm value

A main issue refers to the valuation of firm value, which, rather than an input, should be the result of any valuation approach. As a consequence, this model is much more convenient when dealing with claims valuation. According to option pricing idea, firm value should be a market value, so it has to be estimated as the value of the firm as if it were liquidated or sold now, rather the usual discounted value of future opportunities seen in the previous Chapter valuation approaches. Four chances to define V can be developed according to Damodaran (2002)⁴⁸.

The first is straightforward but inconsistent. Indeed, it proposes to sum up the market values of debt and equity and to reallocate the firm value between them. However, it results in completely different values for debt and equity than the market inputs.

Alternatively, if the firm has separable assets that are individually traded, such as in a real estate firm, cumulating the market values of the asset side (instead of the liability one) results in a more reliable firm value.

Multiples of revenues from healthy firms in the same business applied to the firm under valuation can also be assumed to be a liquidation estimate of the value of the firm. As observed in Chapter 1, however, there are more realistic models to compute the liquidation value.

The final approach is simply to use cash flows and discount them by a cost of capital, using either DCF or APV or CCF. According to Buttignon (2015)⁴⁹, the main parameters to be

⁴⁸ DAMODARAN, A., 2002. *Valuing Equity as an option*. Session 23. Slides available at NY Stern website: <http://people.stern.nyu.edu/adamodar/pdfiles/valonlineslides/session23.pdf>

⁴⁹ BUTTIGNON, F., 2015. *Distressed Firm Valuation: Reorganization Plan and Going-Concern Capital Value*.

assumed are the projection period and the terminal value. The former is usually computed as the length of the reorganization plan, to which a steady-state projection may also be added. However, this usually assumes too optimistic projections and may be conveniently substituted by a terminal value. This may be computed first by using the growing perpetuity formula, whose parameters are obtained as if the firm has reached a “normalized” steady state. Alternatively, a market multiple on the normalized forward financial results may be applied, where market multiples are estimated on the basis of comparable companies current values and require therefore a detailed analysis on whether they will be consistent with the values of the distressed company at the end of the explicit projection period.

In this case, however, we should only use estimates of operating income from current investments, neglecting future opportunities of the firm, since in theory the current value of the firm as if it were traded has to be estimated. However, this could lead to a too low firm value because financial distress may negatively affect operating income. Furthermore, in this case, option pricing valuation can be considered the natural step after traditional valuation rather than an alternative to them.

Estimating volatility

Volatility is estimated differently depending on whether the stock and bonds are traded.

If they are, the formula is simply the mean of the two volatilities σ_e and σ_d weighted by their market-value weights on the firm value w_e and w_d , squared to get the variance, where ρ is the correlation between equity and debt:

$$\sigma_V^2 = w_e^2 \sigma_e^2 + w_d^2 \sigma_d^2 + 2w_e w_d \sigma_e \sigma_d \rho$$

When a firm is distressed, however, this approach can be misleading. If this is the case, the inputs of the firm can be substituted by more reliable market values and weights of firms operating in the same industry and the average variance of these firms can be used.

If the firm is not traded, rather than using estimates of its values, values from bonds and stocks with a comparable rating as well as their correlation may also be used.

An alternative way to estimate firm volatility is to use a Montecarlo simulation, as proposed before and summarized by Copeland and Antikarov (2001). Rather than valuing firm value and volatility separately, they include uncertainty directly in the inputs for computation of cash flows, accounting for both idiosyncratic and general factors. Inputs’ expected value, their distribution and their volatilities are assumed exogenous, and autocorrelations among them or over time are set. After that, a Montecarlo simulation for each parameter is implemented, in order to get the distributions and parameters referring to returns, which correspond to firm values. These models can then be used to get the firm value on a DCF basis and the volatility

of the firm. Moreover, assumptions on constant or increasing volatility band may also slightly modify the model.

Debt values

The main inputs referring to debt are its maturity and its face value. These values seem straightforward at first glance, but the existence of more complicated capital structures makes it necessary to cope with this issue.

According to Damodaran (2010), the best method is to estimate the duration⁵⁰ of all debt outstanding and to determine the face-value weighted average, which will be used as an approximation of time to expiration in option pricing. If this is not possible, a simple face-value weighted average of the maturities of the several bonds converted into zero-coupon bonds in the option valuation will suffice.

On the other hand, the face value of debt seems to be clearly defined. However, this is not the case with coupon-paying bonds. Damodaran (2010) suggests using either simply the sum of the principal of all debt outstanding or to sum up principal with interests and coupon payments. In the former case, neglecting interests and coupon payments may underestimate the true value of debt payouts. Alternatively, it is possible to consider them as a percentage of the firm value which equals the dividend yield in the option pricing model. This approach is consistent because, as argued by Damodaran (2010), “each year that the firm remains in existence, we would expect to see the firm’s value decline by the expected payments on the debt”.

Black and Cox (1976)⁵¹ pointed out that this framework assumes a continuous flow of coupon payments, whereas when decisions, such as filing for bankruptcy, must be taken at discrete points, an optimal stopping problem may be more convenient. In this case, an American call option on a stock paying discrete dividend is modelled and a terminal condition at each stage is determined, such that in case of default the liquidation value is given, while, when default does not occur, the payoff is the value of the firm with one more running period, given that its current value is obtained by subtracting the coupon payment to the non-dividend paying option value. As a consequence, it is now possible to include coupon payments directly, although this only works as an approximation, being modelled in discrete time.

An analysis on risk premia

From expressions for D , D_f and L described previously, Merton (1974) also derived an expression for risk premia and analysed the factors which determine it. In particular, he observed that:

⁵⁰ For a definition of duration and convexity, see e.g. BODIE, Z., KANE, A., MARKUS, A., 2001. *Investments*. 5th Edition. McGraw Hill Companies.

⁵¹ BLACK, F., COX, J. C., 1976. Valuing corporate securities: some effects of bond indenture provisions. *The Journal of Finance*, vol. 31, n. 2, pp. 351-367.

$$R - r = -\ln\{L^{-1}\Phi(-x_1) + \Phi(x_2)\} / \tau$$

depends on the current leverage of the issuer, the volatility of assets and the time to maturity. As leverage and volatility increase, risk premia will clearly raise as well, since, being the firm riskier, investors are willing to invest only for a higher return. The term to maturity, instead, affects yield spreads in two ways, depending on leverage. If it is below one, which is the usual case in non-distressed situations, the spread is initially increasing and then declining on the time to maturity, because a longer term permits to amortize the risk-related price discount over a longer interval of time. If leverage is beyond unity, instead, the function has a negative slope. The ratio is that, if the firm current debt value overcomes assets value, then the likelihood of a maturity crisis is incorporated immediately, whereas with a longer maturity debt is expected to return to healthy levels.

This is already in line with risk premia predicted in bond rating, but this approach is far too simplistic. First, debt is not expected to be a zero-coupon and thus to default at T only. Second, single issuance and same priority for all debt is not a credible assumption. In order to include these issues in valuation, first-passage-time approach was introduced. This approach develops Merton (1974) in that it assumes the default event is triggered by a specified random process, most notably firm value, breaching some specified exogenous barrier or endogenous barrier process. In the following, these developments will be dealt with more in detail.

3. The introduction of a bankruptcy boundary to deal with liquidation

The introduction of more realistic boundaries

A first evolution of the original Merton (1974) model, aimed at including more realistic bond indenture features in the model, was already formulated in the optimal stopping problem mentioned above. Before, Cox and Ross (1976)⁵² defined four exogenous values which contribute to determine the value of the firm, i.e. its value at the maturity date if the firm is not reorganized before, its value if the firm is reorganized at the lower boundary, its value if the firm is reorganized at the upper boundary, and the value of the payouts it will potentially receive. The values are determined endogenously but they are generated as some exogenous (if determined by the contract) or endogenous (if determined by the problem) are breached. The inclusion of indenture provisions in the model already makes the approach more realistic and introduces some boundaries which are to be developed further later.

⁵² COX, J. C., ROSS, S. A., 1976. The valuation of options for alternative stochastic processes. *Journal of Financial Economics*, vol. 3, pp. 145-166.

Black and Cox (1976) also introduce valuation of debt with safety covenants and subordinated debt. Safety covenants allow debt holders to force bankruptcy as the firm hits a prespecified level, which may change over time and which is defined $K = C \exp(-\gamma(T - \tau)) = pP \exp(-\gamma(T - \tau))$ in this case. From this, theory about absorbing barriers is applied and the value of the firm is computed accordingly.

Subordinated debt follows instead the description of Exhibit 3.1⁵³. When different priorities are incorporated into the model, valuation can either occur separately or absorbing barriers may be used. The value of the senior bond (or stock), in fact, is the same as the corresponding security of an identical firm with a single bond issue having a promised payment of P (or (P+ Q)). It is possible to depict junior debt as the difference between a senior debt with face value P+Q and one with face value P, also including some safety covenant boundaries. Junior debt has some special features with respect to senior: first, it is initially convex, as senior debt, but then it becomes concave on V in the value V*:

$$V^* = \frac{1}{\sqrt{P(P+Q)}} \exp\left(-\left(r - a + \frac{1}{2}\sigma^2\right)(T - t)\right)$$

Second, it is increasing on volatility, since in this case a higher volatility leads to higher values for the junior bond, up to V*, after which it is declining. This stems from the idea that higher volatility may lead to more chances to get a higher value, since it is subordinated to a full payment.

Furthermore, it is possible for the junior debt to be worthless at maturity, and, if such a development is imminent, the junior bondholders find it in their interests to try to extend the maturity date of the entire bond issue. As a result, differently from senior debt, junior debt value increases with time to maturity.

Claim	$V < P$	$P < V < P + Q$	$V > P + Q$
Senior Bonds	V	P	P
Junior Bonds	0	$V - P$	Q
Stock	0	0	$V - P - Q$

Exhibit 3.1: Value of claims at maturity when dealing with subordinated debt.

⁵³ BLACK, F., COX, J. C., 1976. Valuing corporate securities: some effects of bond indenture provisions. *The Journal of Finance*, vol. 31, issue 2, p. 359.

These results may create a conflict of interests between senior and junior bond holders, with contrasting interests to be compared and protected.

In the following years, many other Merton (1974) assumptions were relaxed. First, Brennan and Schwartz (1978)⁵⁴ introduced uncertainty in interest rates, although this was not explicitly modelled as a stochastic process (as Longstaff and Schwartz, 1995⁵⁵ and Kim, Ramaswamy and Sundaresan, 1993⁵⁶ later proposed) and this volatility is not critical to the firm option pricing valuation. They also assumed that liquidation leads to some implicit costs, so that bond holders get the minimum between a fraction of the firm value in bankruptcy or the face value of debt, but the bankruptcy point was *de facto* determined exogenously and could only be hit at maturity, limiting the amount of bankruptcy-triggering events.

Longstaff and Schwartz (1995) modelled risky debt assuming the existence of a bankruptcy point and a passage time to bankruptcy, which allowed to model both fixed and floating-rate debt, and they also permitted for violation of the strict priority rule. However, Kim, Ramaswamy and Sundaresan (1993) innovated the literature by introducing a proper cash flow based bankruptcy point. In this model, bankruptcy could also be triggered at an intermediate point before maturity, when cash flows did not suffice to cover coupon obligations. Although the approach seems realistic at first glance, asset sales could increase liquidity and prevent from early default though not being incorporated in the model. The authors argued to this issue that upon distress managers are incentivised to sell in order to avoid default, but this is not in the desire of creditors because it reduces firm value, and therefore is restricted in most cases.

Leland (1994): Endogenous bankruptcy boundaries and infinite maturity

Endogenous bankruptcy was first introduced by Leland (1994)⁵⁷. His thrust was to derive closed-form solutions for debt and firm values in order to determine optimal leverage, taking into account interest tax benefits and the point at which they vanish due to the probability of default. As noted above, many other models, especially Brennan and Schwartz (1978), had analysed these aspects of firm value, but their approach, according to Leland (1994), was too simplistic. Furthermore, he pointed out that analysing leverage and probability of default may

⁵⁴ BRENNAN, M. J., SCHWARTZ, E. S., 1978. Finite difference methods and jump processes arising in the pricing of contingent claims: a synthesis. *Journal of Financial and Quantitative Analysis*, September 1978, pp. 461-474.

⁵⁵ LONGSTAFF, F. A., SCHWARTZ, E. S., 1995. A simple approach to valuing risky fixed and floating rate debt. *The Journal of Finance*, vol. 50, issue 3, pp. 789-819.

⁵⁶ KIM, J., RAMASWAMY, K., SUNDARESAN, S., 1993. Does default risk in coupons affect the valuation of risky bonds?: A contingent claims model. *Financial Management*, vol. 22, issue 3.

⁵⁷ LELAND, H. E., 1994. Corporate debt value, bond covenants and optimal capital structure. *Journal of Finance*, vol. 49, pp. 1213-1252.

lead to more consistent results than credit spreads derived from the model to analyse real world spreads (Broadie, Chernov and Sundaresan, 2007)⁵⁸.

Leland (1994), indeed, assumed that the firm follows a diffusion process with constant volatility:

$$\frac{dV}{V} = \mu(V, t)dt + \sigma dW$$

where W is a standard Brownian motion. The firm has issued infinite maturity debt with coupon C . Consequently, debt, interest tax shields and bankruptcy costs are derived as solutions of time-independent second order differential equations with consistent boundary conditions. The Modigliani-Miller value V is then added both ITS and BC, and equity is derived as:

$$E(V) = v(V) - D(V) = V - \frac{(1 - \tau)C}{R} + \left[\frac{(1 - \tau)C}{R} - V_B \right] \left(\frac{V}{V_B} \right)^{-X}$$

where $X = \frac{2r}{\sigma^2}$ and $\left(\frac{V}{V_B} \right)^{-X}$ is the Arrow-Debreu price of one unit contingent on future bankruptcy, which is a proxy of the probability of default. Equity is therefore a convex function of V if $\frac{(1-\tau)C}{R} - V_B > 0$, which is consistent with the option-like nature of equity.

From this, he derived an endogenous bankruptcy point which accounted for market imperfections as well as for the fact that managers will act in the interest of shareholders. Clearly, they will not file for bankruptcy unless the firm cannot fulfil its periodic coupon obligation. This cannot only be covered by current cash flows, as in Kim et al. (1993), but also issuing additional equity or diluting it up to the point at which equity reaches the zero level. This idea follows from the absolute priority rule. However, managers may default at the bankruptcy boundary level which is optimal for the firm value or for the equity value. If they maximised the firm value v or, equivalently, equity value without limited liability, the optimal V_B would simply be as low as possible because these functions are decreasing on V_B . This is defined the *first-best* solution. However, this is not the case in the analysis on hand: limited liability prevents managers from choosing an arbitrarily small V_B because, to satisfy shareholders, they must choose it in order to get a positive equity value for all $V > V_B$ (*second-best* solution). In other words, they will choose the first value of the firm V_B at which equity is zero. Because of the convexity mentioned above, the value which maximises equity has to satisfy the smooth-pasting condition (2) at $V = V_B$:

$$\frac{\partial E(V)}{\partial V} \Big|_{V=V_B} = 0$$

⁵⁸ BROADIE, M., CHERNOV, M., SUNDARESAN, S., 2007. Optimal Debt and Equity Values in the Presence of Chapter 7 and Chapter 11. *The Journal of Finance*, vol. 62, issue 3, pp. 1341-1377.

As pointed out by Dixit and Pindyck (1994)⁵⁹, the smooth pasting condition is necessary when the threshold must be imposed endogenously. In this case, indeed, the value must be such that not only continuity but also continuity of the slopes are imposed along the boundary. The conditions, however, depend on considerations which hold specifically for the problem.

Furthermore, (2) coincides with $\frac{\partial E(V)}{\partial V_B} = 0$. This implies that V_B is incentive compatible not only *ex ante*, satisfying the smooth-pasting condition and thus being set before debt issuance as the value which maximizes firm value upon limited liability, but also *ex post*, preventing from the incentive to set a new value since V_B also maximises equity value for shareholders (Merton, 1973)⁶⁰. The closed-form solution for V_B is:

$$V_B = \frac{(1 - \tau)C}{r} \frac{X}{1 + X}$$

Plotting it into debt and equity equations, it is observable that debt value is decreased by bankruptcy costs but increased by the corporate tax rate through the decrease of the bankruptcy boundary. Furthermore, when the firm is distressed, i.e. $V \approx V_B$, higher coupons, lower volatility and lower risk-free rate decrease debt value. This suggests a different behaviour of junk bonds with respect to investment grade firms, whose debt value is increasing in coupon rate and does not depend on volatility.

Total distressed firm value presents a perverse behaviour with respect to the healthy state too: in an imperfect market, riskiness as well as higher risk-free rate are likely to increase its value, whereas coupon rate decreases it. Equity value, instead, is not that affected by the existence of imperfections: indeed, bankruptcy costs are mostly born by debt holders because of the residual claim feature of equity.

Leland and Toft (1996): Endogenous boundary in finite maturity debt

Subsequently, Leland and Toft (1996)⁶¹ develop a similar model for finite maturity debt. In some way, however, they maintain a constant capital structure by assuming the firm issues a debt equivalent to a constant fraction P/T of its value at each time, such that it has a constant defined debt amount. Although the previously treated models by Longstaff and Schwartz (1995) and Kim et al. (1993) already dealt with finite maturity debt, none of them approached the bankruptcy trigger as an optimal decision by equity holders. On the contrary, Leland and Toft

⁵⁹ DIXIT, A. K., PINDYCK, R. S., 1994. *Investment under uncertainty*. 1st Edition. Princeton, New Jersey: Princeton University Press. pp. 108-132.

⁶⁰ see Merton (1973), footnote 60, in LELAND, H. E., 1994. Corporate debt value, bond covenants and optimal capital structure. *Journal of Finance*, vol. 49, pp. 1213-1252.

⁶¹ LELAND, H. E., TOFT, K. B., 1996. Optimal Capital Structure, Endogenous Bankruptcy, and the Term Structure of Credit Spreads. *The Journal of Finance*, vol. 51, n. 3, pp. 987-1019.

(1996) use the same “strategic” procedure as Leland (1994), but including finite maturity. Therefore, they model debt as the solution to the partial differential equation:

$$\frac{1}{2}\sigma^2V^2d_{VV} + (r - \delta)Vd_V - rd + d_t + \frac{C}{T} = 0$$

where δ is the payout rate and α is the proportion of bankruptcy costs, s.t.:

$$d(\infty, t) < \infty \quad \forall t > 0$$

$$d(V_B, t) = D_B = (1 - \alpha)V_B \quad \forall t > 0$$

$$d(V, T) = P/T$$

which results in $d(V, t) = \int_0^t \frac{e^{-rsC}}{T} 1_{s < \tau_B} ds + D_B e^{-r\tau_B} 1_{t > \tau_B} + \frac{P}{T} e^{-rt} 1_{t \leq \tau_B}$, where τ_B is the first passage time of the process V to V_B and $1_{t > \tau_B}$ represents its probability distribution. Exploiting a corollary of the Girsanov theorem, the probability distribution can be derived as in e.g. Harrison (1990)⁶² or Karatzas and Shreve (1991)⁶³:

$$\mathbb{E}[1_{t > \tau_B}] = \Phi(h_1(t)) + \exp(-2ab)\Phi(h_2(t))$$

where $\Phi(\cdot)$ is the standard cumulative normal distribution and:

$$h_1(t) = \frac{-b - a\sigma^2 t}{\sigma\sqrt{t}}; \quad h_2(t) = \frac{-b + a\sigma^2 t}{\sigma\sqrt{t}}$$

$$a = \frac{r - \delta - \frac{\sigma^2}{2}}{\sigma^2}; \quad b = \ln\left(\frac{V}{V_B}\right)$$

From this the following expressions are also derived:

$$\mathbb{E}[1_{t > \tau_B}]e^{-r\tau_B} = \left(\frac{V}{V_B}\right)^{-a+z}\Phi(q_1(t)) + \left(\frac{V}{V_B}\right)^{-a-z}\Phi(q_2(t))$$

where

$$q_1(t) = \frac{-b - z\sigma^2 t}{\sigma\sqrt{t}}; \quad q_2(t) = \frac{-b + z\sigma^2 t}{\sigma\sqrt{t}}; \quad z = \frac{\sqrt{(a\sigma^2)^2 + 2r\sigma^2}}{\sigma^2}$$

and

$$\mathbb{E}[1_{t < \tau_B}] = 1 - \mathbb{E}[1_{t > \tau_B}].$$

Integrating debt value and using:

$$I(T) = \left(\frac{1}{T}\right) \int_0^T e^{-rt} \mathbb{E}[1_{t > \tau_B}] dt = \frac{1}{rT} (\mathbb{E}[1_{t > \tau_B} e^{-r\tau_B}] - \mathbb{E}[1_{t > \tau_B}] e^{-rt})$$

$$J(T) = \frac{1}{T} \int_0^T G(t) dt = \mathbb{E}[1_{T > \tau_B} e^{-r\tau_B}] - \frac{1}{T} \mathbb{E}[\tau_B 1_{T > \tau_B} e^{-r\tau_B}]$$

debt value is defined as:

⁶² HARRISON, J. M., 1990. *Brownian motion and stochastic flow systems*. 1st Edition. Malabar, Florida: Robert E. Krieger Publishing Company.

⁶³ KARATZAS, I., SHREVE, S. E., 1991. *Brownian motion and stochastic calculus*. 2nd Edition. New York: Springer Verlag.

$$D(V, V_B, T) = \frac{C}{r} + \left(P - \frac{C}{r}\right) \left(\frac{1 - e^{-rT}}{rT} - I(T)\right) + \left((1 - \alpha)V_B - \frac{C}{r}\right) J(T)$$

which can be seen as the sum of the coupon and the principal up to maturity if bankruptcy is avoided summed to the difference between the liquidation value and the value lost in bankruptcy (future coupons) if bankruptcy instead occurs.

Following Leland (1994) step by step, finite-maturity V_B was also derived from $\partial E|_{V=V_B} = 0$.

The closed form solution for finite maturity bankruptcy boundary is:

$$V_B = \frac{\frac{C}{r} \left(\frac{A}{rT} - B\right) - \frac{AP}{rT} - \frac{\tau Cx}{r}}{1 + \alpha x - (1 - \alpha)b}$$

where $x = a + z$ and, defining $n(\cdot)$ as the standard normal density function,:

$$A = 2ae^{-rT} \Phi(a\sigma\sqrt{T}) - 2z\Phi(z\sigma\sqrt{T}) - \frac{2}{\sigma\sqrt{T}} n(z\sigma\sqrt{T}) + \frac{2e^{-rT}}{\sigma\sqrt{T}} n(a\sigma\sqrt{T}) + (z - a)$$

$$B = -(2z + \frac{2}{(z\sigma^2\sqrt{T})} \Phi(z\sigma\sqrt{T}) - \frac{2}{(\sigma\sqrt{T})} n(z\sigma\sqrt{T}) + (z - a) + \frac{1}{(z\sigma^2T)})$$

V_B is independent of time, indicating that the boundary is constant over time also for finite maturity bonds. Furthermore, applying Itô lemma, the equity appreciation around V_B was computed:

$$\partial E|_{V=V_B} = \left((1 - \tau)C + \frac{P}{T} \right) dt - (d(V_B; V_B; T) + \delta V_B) dt$$

Since it corresponds to the costs of debt service less the sum of the revenues from selling bonds at their market price for an amount equal to the principal and the payout available from the firm's activities, this is the additional cash flow which is needed to service debt after issuing bonds and using proper cash flows and therefore can be interpreted as the change in equity needed for debt service. As in Kim et al. (1993), therefore, this is a flow condition: bankruptcy will occur when the change in equity is just enough to service debt. However, for short maturities and positive bankruptcy costs this can occur although the firm value overcomes debt, because the default triggering condition relates to cash flows in this framework and not to firm value. This is in contrast with longer term debt structures, where it is necessary for the value to be lower than the debt principal amount.

Analysing the results empirically, the results by Leland (1994) are mostly confirmed as far as the effect of volatility and risk-free rates in the valuation of distressed short-term debt as well, which seem in contrast with both healthy debt and distressed debt with exogenous boundary. Since formulas for an endogenous boundary are decreasing on these parameters, bankruptcy is expected to occur after and, being debt holders the most damaged by this decision, larger room before it will drive up the value of debt.

Maturity also affects bond prices and yield spreads: debt is usually sold at par at the time of issuance and maturity, and above par otherwise, but if the firm is highly leveraged and risk of distress occurs, this humped figure is not followed, and bonds with short left maturity sell above par, and those with long left maturity sell even below it. In general, however, all figures are more pronounced for higher initial maturity. Finally, yield spreads also follow Merton (1974).

Charitou and Trigeorgis (2004): Intermediate default triggers and illiquidity

Further detail has been provided by Charitou and Trigeorgis (2004)⁶⁴, who proposed a model accounting for intermediate default driven explicitly by illiquidity and modelled as a compound option, which is referred to as an option on an option, or, when dealing with the valuation of investment projects, as an option to complete a sequential investment, where each stage completed gives the option to complete the following stage. In this option, the strike price I is represented at each time by the interest and debt repayment necessary to the stockholders to service the debt and, thus, to proceed to the following stage. This value I corresponds to the cut-off value of equity $E(V_B, T')$, which following Merton (1974) is an option on the value of the firm, but now computed in an intermediate time T' . From this, V_B can be obtained and compared to the value of the firm at that time V .

Beyond this basic framework, Charitou and Trigeorgis (2004) also considered illiquidity explicitly as a source of bankruptcy, because if the firm has limited liquid assets or cash flows, it is not able to service debt. They modelled the cash flow coverage (CFC), aimed at comparing available cash flows from operations with the difference between debt to be serviced and already available cash and cash equivalents. If the ratio is lower than 1, then illiquidity issues will arise and the firm will be forced by bond holders to file for bankruptcy in an intermediate stage.

In order to test the consistency of the model, empirical results based on real Compustat data were compared to Leland (2004) simulation. They proved his theoretical work by observing that default probability increases with leverage and bankruptcy costs, whereas it declines for longer maturity. Charitou and Trigeorgis (2004) also substantially confirm that higher leverage and risk lead to higher probability of default, but also consider CFC, which instead reduces it.

Further parallel developments

In this complicated framework, endogenous “strategic” default was not the only field developed. Other studies focused on the effects of the incorporation in the model of market imperfections, such as taxes and their benefits or bankruptcy costs. For example, Mello and

⁶⁴ CHARITOU, A., TRIGEORGIS, L., 2004. Explaining bankruptcy using option theory.

Parsons (1992)⁶⁵ introduced agency costs, which resulted a preparatory model for much of the following literature, including the models by Leland (1994) and Leland and Toft (1996) analysed above. Agency costs are referred to as the divergence between the firm value with a healthy firm's operating policy and that following distress. They arise in case of distress, when firm leverage is so high that the firm cannot fully exploit the benefits of debt incorporated in interest tax shields. In particular, they observe that, the higher leverage, the more will these costs, modifying the financial structure, change operating policy to maximise equity value. This results in a second-best firm value, different from the healthy firm case.

Personal taxes and endogenous dividend policy also allowed to alter the distribution of cash flows. This also lead to the introduction of dynamic capital structure, where leverage is allowed to change over time in accordance to an optimal capital structure. An example to this is Goldstein, Ju and Leland (2001)⁶⁶. We can also notice that optimal capital structure is one of the main issues in these models, although this work will not focus on it in the following.

Asymmetric information and some first forms of strategic behaviour, which will be better analysed later, were also included in the model.

Others considered assets as a Levy jump diffusion process (see e.g. Kyprianou and Surya, 2006⁶⁷) or, instead of modelling firm value, determined a stochastic process for operating earnings as a proxy of cash flows (see e.g. Huang and Huang, 2003⁶⁸ and Broadie, Chernov and Sundaresan, 2007).

A huge literature was developed over time and found evidence in many cases. The following will only follow in the main points necessary to develop a binomial approach.

4. Modelling bargaining games for debt service

The main thrust of debt valuation theory was to apply the insights of option theory for debt valuation purposes. However, at least initially, most trigger points were exogenous and liquidation was an automatic consequence in case the lower boundary was breached. The subsequent models included an endogenous liquidation boundary which was set at the level at which equity value was maximised. However, they did not model the behaviour of players explicitly, but were only limited to an action undertaken by managers in the interest of

⁶⁵ MELLO, A. S., PARSONS, J. E., 1992. Measuring the agency cost of debt. *The Journal of Finance*, vol. 47, issue5, pp. 1887-1904.

⁶⁶ GOLDSTEIN, R., JU, N., LELAND, H. E., 2001. An EBIT-based model of dynamic capital structure. *The Journal of Business*, vol. 74, issue 4, pp. 483-512.

⁶⁷ KYPRIANOU, A. E., SURYA, B. A., 2006. Principles of smooth and continuous fit in the determination of endogenous bankruptcy levels. Springer Verlag. Available on: <https://link.springer.com/article/10.1007/s00780-006-0028-y>.

⁶⁸ HUANG, M., HUANG, J., 2002. How much of the corporate-Treasury yield spread is due to credit risk?. Working paper, Stanford University.

shareholders, without any choice left to other players, and they were immediately followed by liquidation, which is the case in real world only if the parties agree on it. This, therefore, implies the need for an extensive game form representation which accounts for these issues.

Discrete time games

The first game (Anderson and Sundaresan, 1996⁶⁹) assumed that the amount of debt service is chosen by the owners at each time and is defined $S_t \in [0; f_t]$, where f_t is the cash flow available at that time. If it is higher than the promised debt payment CS_t , the game will proceed to the next step. Otherwise, debt holders can decide whether to initiate a legal action, which rejects debt service but gives the right to get the liquidation value incurring in a fixed cost K , or to simply accept the service, in which case the game continues to the following date. It can be already pointed out that the model differs from the usual approaches in that bankruptcy does not imply liquidation, but instead allows for a first form of debt restructuring without the court intervention, as in the private workout. Furthermore, the model focuses on a choice of debt service which is made strategically by equity holders and, thus, violates the absolute priority rule.

Being designed in discrete time, the binomial model can be solved recursively. First, focusing on each time T subgame, the creditor will obtain S_T if she accepts debt service and $\max[V_T - K; 0]$ otherwise, where K are the lumpy liquidation costs. On the other hand, the shareholder only obtains $V_T - S_T$ if debt service is accepted, and will therefore choose S_T that triggers this solution but at the same time maximises her payoff. As a consequence, she sets $S_T = \max[V_T - K; 0]$, which leaves the debt holder indifferent between accepting and rejecting and at the same time leaves more value to equity. Including also the case in which debt service is equal to the contracted payments, the claims result:

$$B(V_T) = \min[CS_T; \max[V_T - K; 0]]; E(V_T) = V_T - B(V_T)$$

The same procedure also holds for previous periods, but for the fact that the risk-neutral value of debt must be added to debt. At each time, the optimal debt service, debt and equity values are respectively:

$$S(V_t) = \min \left[CS_t; \max \left[0; \max[V_t - K; 0] - \frac{pB(V_u) + (1-p)B(V_d)}{1+r_f} \right] \right]$$

$$B(V_t) = S(V_t) + \frac{pB(V_u) + (1-p)B(V_d)}{1+r_f}$$

$$E(V_t) = f_t - S(V_t) + \frac{pB(V_u) + (1-p)B(V_d)}{1+r_f}$$

⁶⁹ ANDERSON, R. W., SUNDARESAN, S., 1996. Design and valuation of debt contracts. *The Review of Financial Studies*, vol. 9, issue 1, pp. 37-68.

So far, the model considered the case of voluntary bankruptcy by shareholders and subsequent optimal decision by debt holders and could therefore be considered a form of private restructuring or workout. However, if the choice of debt service is higher than the cash flow available, liquidation may be forced by creditors and will result in:

$$B(V_t) = \max[0; \min[V_T - K; CS_t + P]]; E(V_t) = V_t - K - B(V_t)$$

The results are mainly similar to the traditional option approaches, knowing that a decrease in the yield spread will usually be fostered by the reduction of the opportunities to strategically service debt and, thus, by immediate liquidation, which increases the value available for creditors. As before, bankruptcy costs, volatility and maturity increase the yield. This holds both for discount and coupon paying debt. On the contrary, the effect of cash payouts and coupon is ambiguous. The latter, specifically, increases the chance to choose a service lower than the promised one, and thus increases strategic behaviour at odds of creditors, who find it more convenient to accept anyway in most cases. However, it can also result in illiquidity and thus forced liquidation, which always favours debt holders and increases debt value.

This model has two main drawbacks: first, it is far too simplistic being in discrete time; second, it excludes asset sales and issuance of new equity to get liquidity, and only considers cash flows from operations. As a consequence, some improvements resulted necessary.

Continuous time games

The model developed by Mella-Barral and Perraudin (1997)⁷⁰ was in some sense driven by Anderson and Sundaresan (1996), but for that it was developed in continuous time and with agency costs. Two boundaries are involved in the discussion. They are defined in terms of the state variable which refers to the value of the firm, but, in reality, they are derived at each time from piecewise right continuous functions of the debt service, exactly as Anderson and Sundaresan (1996) did. The first boundary refers to the trigger price at which equity holders are incentivised to wind up firm operations and the firm is therefore eventually liquidated. In this case, the strategic choice of the optimal debt service is simply given by the liquidation value. As the price raises, if the firm debt still overcomes its assets, the shareholders can extract a surplus by offering debtholders a service flow less than the promised amount, which is optimally defined as the value of the option to reject the offer and initiate a legal action. Bondholders will not wish to declare bankruptcy, since the value of the firm's assets will be less than the current value of the debt. As the price increases so much that it overcomes debt, equity holders will eventually have an incentive to pay the full, initially contracted service flow, since

⁷⁰ MELLA-BARRAL, P., PERRAUDIN, W., 1997. Strategic debt service. *The Journal of Finance*, vol. 52, issue 2, pp. 531-556.

otherwise the liquidation value will be so large that debtholders will definitely wish to close the firm if given the opportunity. The second trigger is thus the level at which the debt service equals the contracted coupon payment.

Except for the continuous time framework, the model follows the rationale of the strategic game analysed above, depending on the payoff of the shareholders. Instead of proceeding recursively, however, the solutions to the partial differential equations with related boundaries for the values of equity and debt and finally the boundaries described above were computed. The results allow to point out that renegotiation increases the efficiency of the model. Indeed, liquidation is still triggered by maximising the value of a pure equity firm, so that this value is not affected by leverage. The existence of renegotiation, however, allows shareholders to extract any surplus in the value of the bonds with respect to the optimally chosen level of debt service, which is the value of the exit option for debt holders. This permits to recover efficiency in the decision to go bankrupt, since equity holders are incentivised to renegotiate rather than liquidate, and will act in the interest of the firm rather than in their own one.

Bargaining games

So far, games were designed imagining that the whole bargaining power belonged to equity holders and that they could bargain indifferently on the value of assets or on the value of the firm as a whole. However, debt holders may also be ensured some power (in past papers, this was only included, for example, in the form of bond covenants) and corporate taxes may create a divergence between assets and firm value. Fan and Sundaresan (2000)⁷¹ gave insight to these issues by assuming the usual lognormal diffusion process for the value of the firm and a basic capital structure made up of equity and a single issue of perpetual (then finite maturity is also treated) debt paying a coupon. Asset sales are not permitted and liquidation costs must be faced by the firm, including both fixed and proportional costs, indicated respectively as K and α . Taxes are included in the model and create an interest tax shield in healthy states.

They observe that debt renegotiation can occur through debt-equity swap, where bargaining refers to the value of assets, and strategic debt service, where instead the players bargain over the value of the firm.

In the first case, reorganization is triggered at an endogenously specified boundary and it leads to the conversion of debt to equity and the creation of an all-equity firm, where, as the firm is bankrupt, no tax benefit arises. In this case, the equity holders' bargaining power $0 \leq \eta \leq 1$ affects the optimal sharing rule, θ^* , which is obtained by maximising the Nash equilibrium

⁷¹ FAN, H., SUNDARESAN, S. M., 2000. Debt valuation, renegotiation, and optimal dividend policy. *The Review of Financial Studies*, vol. 13, issue 4, pp. 1057-1099.

solution. Since the incremental value available for equity is $(\theta V_B) - 0$ and for debt $(1 - \theta)V_B - \max[\alpha V_B - K; 0]$, then:

$$\theta^* = \operatorname{argmax} \left\{ (\theta V_B)^\eta \left((1 - \theta)V_B - \max[(1 - \alpha)V_B - K; 0] \right)^{1-\eta} \right\} = \eta \min \left\{ \frac{\alpha V_B + K}{V_B}; 1 \right\}$$

This formula reduces to the Anderson and Sundaresan (1996) if all the bargaining power is granted to shareholders, and to Leland (1994) if no room for renegotiation is left.

Alternatively, when the firm reaches an endogenously determined lower boundary on its value, equity holders can negotiate to reduce the debt service but continue operating the firm. As mentioned above, claim holders negotiate on the value of the firm rather than on the assets value in this framework. The difference stems from the chance to continue operating and, therefore, to get tax benefits from interest. It is thus necessary to define $v(V_t)$, which refers to the firm value on the whole and equals the sum of assets, tax benefits and bankruptcy costs weighted by the probability to default designed by Arrow-Debreu. The incremental value is now defined as $\tilde{\theta}v(V) - 0$ and $(1 - \tilde{\theta})v(V) - \max[(1 - \alpha)V - K; 0]$ for equity and debt respectively, and:

$$\begin{aligned} \theta^* &= \operatorname{argmax} \left\{ \left(\tilde{\theta}v(V) \right)^\eta \left((1 - \tilde{\theta})v(V) - \max[(1 - \alpha)V - K; 0] \right)^{1-\eta} \right\} \\ &= \eta \min \left\{ 1 - \frac{(1 - \alpha)V - K}{v(V)}; 1 \right\} \end{aligned}$$

Again, by modelling the respective PDE for equity and debt, their values are obtained and the strategic debt service optimal value is defined as well. It is worth noting that, although debt holders get less power, both claims are eventually worth more. Furthermore, the trigger is also breached before, allowing for a more valuable renegotiation.

Introducing strategic behaviour leads to some modifications on debt valuation. In general, debt value increases with higher cash payout ratio, lower volatility and higher statutory tax rate.

If leverage is too high upon debt-equity swap, these results reverse. Indeed, the higher the dividend payout ratio, the lower is the bankruptcy trigger and the lower will be the value left to debtholders. Higher volatility may instead increase hopes in case of default and thus increase debt value, whereas tax benefits disappear.

Upon strategic debt service and in finite maturity, instead, the results are substantially confirmed. However, risk premia are more pronounced than in Merton (1994), although the idea that upon distress firms pay a substantially higher premium in the short term than in the longer maturity is confirmed. In general, it produces higher premia than Leland (1994), even if the evidence is mixed, depending both on the default probability and on the recovery rate, and lower premia than Anderson and Sundaresan (1996). Nevertheless, they are more realistic than the Merton (1974) risk premia.

Bargaining games in finite maturity and after filing for bankruptcy

The models described above mostly focus on strategic debt service and, as a result, only include private workout and result in values that are reliable only before entering the bankruptcy period, without considering the impact of Chapter 11 reorganization and all its legal implications. One of these features is the so-called “cram-down provision” described above in Chapter 1, which implies that the plan is not approved unless all the impaired classes of debt holders are satisfied. The possibility of cramming down reduces the spreads by enhancing the value of the outside option for creditors, but at the same time assumes only equity holders can propose the plan. This assumption is consistent with the empirically studied majority of reorganization proposals⁷².

Furthermore, the models did not account explicitly for time and they did not define the asset value diffusion process. They were finally derived for infinite maturity debt.

The bargaining process is almost equivalent to the Anderson and Sundaresan (1996) discrete time game, but in that the judge can end the game automatically as soon as an exogenous threshold $V_L = \zeta V_B$ is breached and liquidate the firm according to the re-established absolute priority rule.

Perfect information holds for all the players but the court, which cannot observe whether the offer made by the shareholder is lower than the liquidation value. If this occurs, the cram-down provision would enforce liquidation, which is always less convenient to the shareholder. However, since the underlying asset value is not observable to the court, neither ex-post, an equilibrium may be achieved although liquidation should be theoretically enforced when unsatisfied creditors do not object.

The Nash equilibrium is obtained by offering slightly more than the lowest possible outcome to the bondholder upon reorganization:

$$D(V) = (1 - \alpha)\zeta V_B \left(\frac{V}{V_B}\right)^{-x}$$

where $x = -\frac{r - \delta - \frac{1}{2}\sigma^2}{\sigma^2} - \sqrt{\frac{r - \delta - \frac{1}{2}\sigma^2}{\sigma^2} + \frac{2r}{\sigma^2}}$ as before. Bond holders have no other choice but accepting, since objecting does not lead to immediate liquidation, but rather to a new reorganization plan, and the creditor knows she will never get more from the debtor. Therefore, although the value to bond holders is lower than the liquidation value, bond holders will never

⁷² See Weiss (1991), Betker (1995) and Eraslan (2001), mentioned in PASEKA, A. I., 2004. Debt valuation with endogenous default and Chapter 11 reorganization. Dissertation for Degree in Philosophy with Major in Management, The University of Arizona.

object and the court intervention will be avoided. The model on hand will be dealt with more in detail in Section 5.

5. Models involving the choice of Chapter 11 restructuring

Any firm has a third choice when it falls bankrupt, alternatively to liquidate under Chapter 7, as described by Leland (1994) or experience private workout, which was dealt with when referring to strategic debt service. The third alternative is to file for a court-supervised debt renegotiation, according to Chapter 11. Key differences among these models refer to the treatment of claimholders. Upon Chapter 11 restructuring, the court intervenes and grants the firm a grace (or observation or exclusivity) period d , during which the firm continues operating. Furthermore, automatic stay of assets is granted and thus no legal action on firm assets by creditors can be enforced. At the end of this period, whose maximum length is predetermined, the firm has to present a reorganization plan or it will be liquidated. The validity of the plan needs to be evaluated by the court, which determines whether the firm will be able to continue as a going concern.

As in the previous models, two boundaries are required. The first refers to the point at which the firm files for bankruptcy and is the starting time of the grace period. At this level the firm is granted a limited period of protection from legal actions by the court and can continue operating. However, at the end of this period the firm has to present a plan to reorganize its capital structure, otherwise it will be forced to liquidate. This second time represents the second boundary.

In the following, Chapter 11 reorganization was involved in the contingent claims analysis in several ways, both as an alternative to liquidation or as a process leading in some cases to it.

Modelling endogenously Chapter 11 procedures

Francois and Morellec (2004)⁷³ first introduced Chapter 11 reorganization as a main component of option pricing valuation. Restructuring was explicitly introduced in the value of the firm and following the strategic debt service model by Fan and Sundaresan (2000). As far as the latter is concerned, the incremental value of equity and debt upon restructuring found above is summed up in order to obtain the total surplus, and this amount is plotted into the optimal sharing rule:

$$E(V_B) = \theta^* v(V_B) = \eta(v(V_B) - (1 - \alpha)V_B)$$

$$D(V_B) = (1 - \theta^*)v(V_B) = (1 - \eta)(v(V_B) - (1 - \alpha)V_B) + (1 - \alpha)V_B$$

⁷³ FRANCOIS, P., MORELLEC, E., 2004. Capital structure and Asset prices: some effects of bankruptcy procedures. *Journal of Business*, vol. 77, issue 2, pt. 1, pp. 387-411.

These values are thereafter equalled to the values for equity and debt derived as solutions of the usual partial differential equations and designed as the sum of the payoffs which will arise at each time period, weighted by the probability they arise: debt is defined as the sum of contracted and realized debt payments plus the firm value upon liquidation, while equity is made up of dividend payments less the fulfilled obligations amount less the costs of bankruptcy. Again, equity is finally maximised to obtain the endogenous bankruptcy boundary. The values are found in the following table.

	Leland (1994) $d = 0; \eta = 0$	Fan and Sundaresan (2000) $d \rightarrow \infty; \varphi = 0$	Francois and Morellec (2004)⁷⁴
D(V)	$\frac{C}{r}$ $+ \left((1 - \alpha)V_B - \frac{C}{r} \right) \left(\frac{V}{V_B} \right)^{-X}$	$\frac{C}{r} \left(1 - \left(\frac{V}{V_B} \right)^{-X} \right) + (1 - \eta\alpha)V_B \left(\frac{V}{V_B} \right)^{-X}$	$\frac{C}{r} \left(1 - \left(\frac{V}{V_B} \right)^{-X} \right) + (1 - \alpha)V_B \left(\frac{V}{V_B} \right)^{-X} + (1 - \eta)R(d) \left(\frac{V}{V_B} \right)^{-X}$
E(V)	$V - \frac{(1 - \tau)C}{r} \left(1 - \left(\frac{V}{V_B} \right)^{-X} \right) - V_B \left(\frac{V}{V_B} \right)^{-X}$	$V - \frac{(1 - \tau)C}{r} \left(1 - \left(\frac{V}{V_B} \right)^{-X} \right) + (\eta\alpha - 1)V_B \left(\frac{V}{V_B} \right)^{-X}$	$V - \frac{(1 - \tau)C}{r} \left(1 - \left(\frac{V}{V_B} \right)^{-X} \right) + \eta R(d) \left(\frac{V}{V_B} \right)^{-X}$
V_B	$\frac{(1 - \tau)C}{r} \frac{X}{1 + X}$	$\frac{(1 - \tau + \eta\tau)C}{r(1 - \eta\alpha)} \frac{X}{1 + X}$	$\frac{X}{1 + X} \frac{C(1 - \tau + \eta\tau(1 - B(d)))}{r - r\eta \left(\alpha(1 - C(d)) - \frac{\varphi}{\delta}(\delta A(d) - C(d)) \right)}$

In general, it can be pointed out that this model extends the expressions found in past papers to account for the possibility of Chapter 11 restructuring within a positive but finite grace period. In particular, the bankruptcy boundary reduces to the amount found by Leland (1994) setting $d=0$ and to that by Fan and Sundaresan (2000) if bankruptcy determines private workout and

⁷⁴ In the paper, $R(d)$ refers to the renegotiation surplus at the time of default and is defined as follows:

$$R(d) = \alpha V_B (1 - C(d)) - \frac{\varphi}{\delta} (\delta A(d) - C(d)) V_B + \frac{\tau C}{r} (1 - B(d))$$

where, being $b = \frac{1}{\sigma} (r - \delta - \frac{\sigma^2}{2})$, $\lambda = \sqrt{2r + b^2}$, $X = \frac{b + \lambda}{\sigma}$:

$$A(d) = \frac{1}{\lambda} \left(\frac{1}{\lambda + b + \sigma} + \frac{1}{\lambda - b - \sigma} \frac{\Phi(-\lambda\sqrt{d})}{\Phi(\lambda\sqrt{d})} \right)$$

$$B(d) = \frac{\lambda - b}{2\lambda} + \frac{\lambda + b}{2\lambda} \frac{\Phi(-\lambda\sqrt{d})}{\Phi(\lambda\sqrt{d})}$$

$$C(d) = \frac{\Phi(-(\sigma + b)\sqrt{d})}{\Phi(\lambda\sqrt{d})}$$

where α =liquidation costs, d =grace period and φ =renegotiation costs. Fan and Sundaresan (2000) fixed bankruptcy costs are omitted.

liquidation never occurs, and if no renegotiation costs are faced. As a result, the value discussed in this Section entails a bankruptcy boundary which is higher than the liquidation boundary proposed by Leland (1994), i.e. it occurs before in time, while being lower than the boundary which would be settled if liquidation were never triggered (Fan and Sundaresan, 2000). Restructuring allows to continue operations and to extract strategically the surplus conceded by bond holders. This incentivises shareholders to default earlier. Specifically, a longer grace period drives down the probability of final liquidation and therefore increases the incentive to default earlier to extract more surplus. As a result, a longer grace period leads to a higher V_B . On the contrary, immediate liquidation is irreversible and equity holders are treated as residual claimants, so they will file for it only when equity is worthless. Indeed, having no strategic opportunity, this choice is optimal to them.

The length of the exclusivity period also affects credit spreads through the reduction of the probability of liquidation, which decreases the spread, and through the introduction of strategic debt service, which increases the spread and usually dominates the first effect. The shareholders' bargaining power has an ambiguous effect on the spread, because it decreases the boundary but increases the cost of debt.

Again, a higher risk-free rate decreases credit spreads and the same do higher liquidation costs, because, reducing the optimal coupon, they also reduce the threshold and thus the probability of liquidation.

A structural model including two periods and two thresholds

This first form of inclusion of Chapter 11 filings is very detailed in considering the legal implications in the model, but presents some drawbacks in that it derives infinite maturity closed form solutions only and in that it just applies the model by Fan and Sundaresan (2000) to determine strategic reorganization, which was designed for private workouts and no court intervention. Moreover, it does not consider all the choices available to debt holders in case of bankruptcy. In general, indeed, bankruptcy occurs as debt payments are stopped in an optimal stopping problem (which results in an optimal endogenous bankruptcy boundary V_B) and it can be avoided if the firm can pay its coupons, also by diluting the equity value. This results in the firm's entry in the bankruptcy period. Upon Chapter 11 bankruptcy, the firm is granted a grace period in which it is protected by the court and payments cannot be enforced by creditors. However, Francois and Morellec (2004) implicitly assumed that the grace period has a fixed length. In this case, at the expiration of this period the firm can either present a reorganization plan, to be approved by the court if fair, or be liquidated. Therefore, liquidation may only be triggered at the end of the grace period, if no plan is presented or it is not approved.

On the other hand, Paseka (2004)⁷⁵ argued that this period can end up in liquidation not only if at the end of the grace period no valid reorganization plan is presented, but also if the value of the firm drops to an exogenously specified lower threshold V_L even before the end of the grace period. Otherwise, it ends at the upper threshold V_R , determined endogenously to maximise equity value, in which a reorganization plan is proposed by the debtor, as shown in Exhibit 3.2⁷⁶.

The model proposed by Paseka (2004) therefore endogenizes not only the bankruptcy boundary, as all the models for immediate liquidation and the previously described Francois and Morellec (2004), but also the threshold which determines the exit to the bankruptcy period and, differently from before, does not fix the grace period length.

In order to determine these thresholds, the model was split up to a pre-bankruptcy and a bankruptcy period.

The pre-bankruptcy period was modelled exactly as in the finite maturity immediate liquidation case by Leland and Toft (1996). Indeed, equity holders decide to default at the value where equity is maximised, although the firm is liquid, because it is less convenient for them to raise new equity to meet debt obligations than to get the value which results from optimal stopping. As developed above, in fact, the debtor is willing to tolerate equity dilution only until the risk-neutral expected equity appreciation exceeds the cash flows she must contribute to avoid

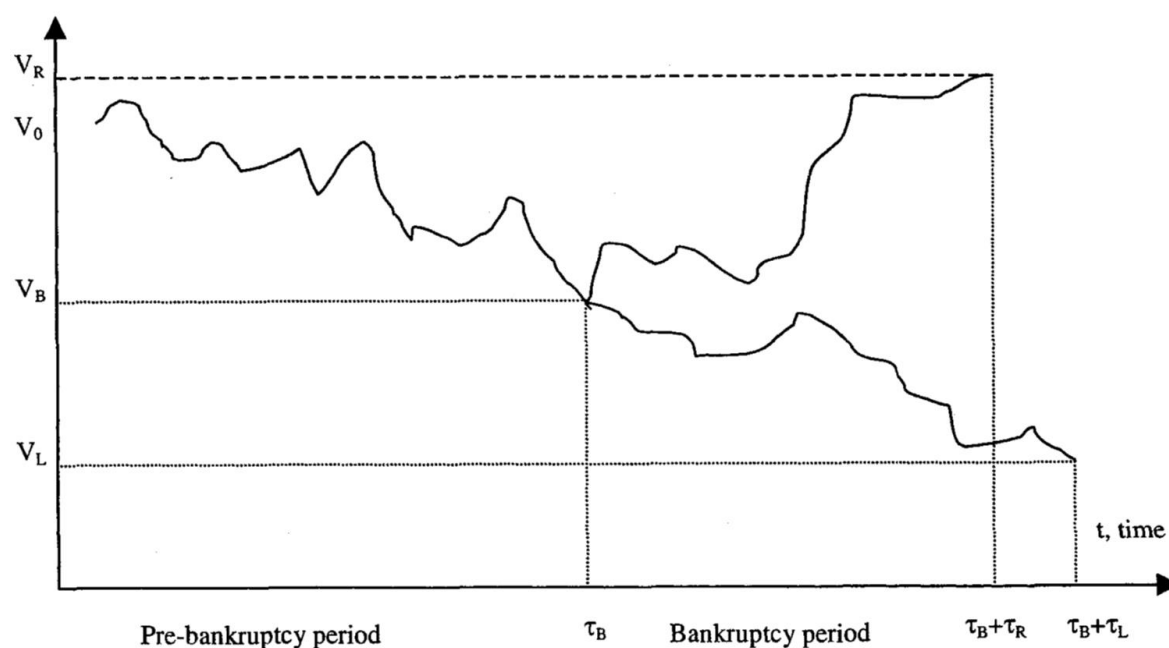


Exhibit 3.2. The time line of the model

⁷⁵ PASEKA, A. I., 2004. Debt valuation with endogenous default and Chapter 11 reorganization. Dissertation for Degree in Philosophy with Major in Management, The University of Arizona.

⁷⁶ PASEKA, A. I., 2004. Debt valuation with endogenous default and Chapter 11 reorganization. Dissertation for Degree in Philosophy with Major in Management, The University of Arizona, p. 63.

bankruptcy. It is noteworthy that the same outcome is also derived by Moraux (2004)⁷⁷, who elaborated a signalling approach according to which some factors may be able to predict the default probability, and Broadie, Chernov and Sundaresan (2007), which will be further analysed later.

Upon bankruptcy, instead, the owner can service debt strategically, because during the grace period no action can be enforced against her by debt holders and protection is granted by the court. This period lasts until $\min[\tau_R; \tau_L]$, causing firm reorganization in the first case and liquidation in the second.

As in Francois and Morellec (2004), upon bankruptcy shareholders can play strategically and thus determine the debt value which least deteriorates the equity value but at the same time forces creditors to accept. In this case, however, they are threatened by the exclusivity or grace period, which grants control of operations to shareholders and therefore does not pose liquidation as the alternative. The game follows the strategy described above by Paseka (2004) himself and results in the value of debt in the bankruptcy period.

The value of equity is determined as a perpetual down-and-out American call option with exercise value endogenously depending by the other parameters. In this case, the diffusion process for the assets value is determined by stopping cash flows δ and including a continuous flow of bankruptcy costs b , as well as by changing the instantaneous volatility of the assets value to ω . The resulting value of equity in bankruptcy is:

$$E_+(V) = \left(V_R - K - (1 - \alpha)V_L \left(\frac{V_R}{V_L} \right)^{\frac{r-b-\frac{1}{2}\omega^2}{\omega^2} - \sqrt{\left(\frac{r-b-\frac{1}{2}\omega^2}{\omega^2} \right)^2 + \frac{2r}{\omega^2}}} \right) \xi(V)$$

$$\text{where } \xi(V) = \frac{\frac{V}{V_L} \left(\exp \left(-\frac{r-b-\frac{1}{2}\omega^2}{\omega^2} + \sqrt{\left(\frac{r-b-\frac{1}{2}\omega^2}{\omega^2} \right)^2 + \frac{2r}{\omega^2}} \right) - \exp \left(-\frac{r-b-\frac{1}{2}\omega^2}{\omega^2} - \sqrt{\left(\frac{r-b-\frac{1}{2}\omega^2}{\omega^2} \right)^2 + \frac{2r}{\omega^2}} \right) \right)}{\frac{V}{V_R} \left(\exp \left(-\frac{r-b-\frac{1}{2}\omega^2}{\omega^2} + \sqrt{\left(\frac{r-b-\frac{1}{2}\omega^2}{\omega^2} \right)^2 + \frac{2r}{\omega^2}} \right) - \exp \left(-\frac{r-b-\frac{1}{2}\omega^2}{\omega^2} - \sqrt{\left(\frac{r-b-\frac{1}{2}\omega^2}{\omega^2} \right)^2 + \frac{2r}{\omega^2}} \right) \right)}$$

K correspond to the fixed reorganization costs, whereas α are the proportional liquidation costs. Different values than Leland and Toft (1996) for both V_B and V_R are determined in this case, since the bankruptcy boundary will also be affected by the choice, timing and allocation of the values in case of reorganization. In this case, since the real option is sequential, a joint optimal stopping problem shall be solved. The way to do this is to invoke a high contact condition,

⁷⁷ MORAUX, F., 2004. A closed form solution for pricing defaultable bonds. *Finance Research Letters*, vol. 1, issue 2, pp. 135-142

which is essentially the first order condition for optimal stopping problems and turns out to be a sufficient condition for optimal stopping, as suggested by Brekke and Oksendal (1990)⁷⁸.

Unlike Leland and Toft (1996), the high contact condition requires equity value to be continuously differentiable across the bankruptcy boundary, such that $\partial E_- / \partial V|_{V=V_B} = \partial E_+(V_B) / \partial V_B$. Entering bankruptcy allows equity to maintain a positive equity value even in default if liquidation is not immediately triggered, such that the right-hand side differs from 0. In the following this insight will be considered more in detail.

During bankruptcy, the shareholder must weigh the possibility to get nothing upon liquidation and reorganize the firm. In this case, the high contact condition is $\partial E_+ / \partial V|_{V=V_R} = \partial E_+(V_R) / \partial V_R$.

On the contrary, the liquidation value is simply a fraction of the boundary which is given exogenously, although, as a trigger, it may be more senseful to define it endogenously as well. The comparative statics of the model follow the results found above, at least on the whole, but also allows for an analysis of the time spent in bankruptcy. Liquidation costs reduce the value of the outside option of bond holders such that, anticipating this, the debtor will file for bankruptcy earlier and will wrap up this phase earlier as well. Volatility makes the owner default later because of limited liability. However, the grace period length will increase as well and a lower value after reorganization is expected by debt holders, who therefore ask for a higher credit spread. The same reasoning also applies to maturity. As far as bankruptcy volatility is concerned, the main issue is that it can trigger liquidation more often, thus leaving no value to the debtor. Acting strategically, she will delay the default point and spend more time in bankruptcy. However, the spread is decreased in this case, since the outside option is more valuable.

Endogenizing the liquidation boundary: a more realistic approach

The recognition of the existence of two endogenous thresholds is clearly an enhancement of the model to include Chapter 11 restructuring.

As noted above, the bankruptcy boundary V_B is derived differently from Leland (1994) and Leland and Toft (1996). In those frameworks, indeed, coupon payments are to be met both with cash flows and equity dilution or additional equity issuance and, due to limited liability, they just stop as equity value breaches the zero level. This is because liquidation will lead anyway to zero value for shareholders and they have no interest to default earlier. Therefore, this *ex post*

⁷⁸ BREKKE, K. A., OKSENDAL, B, 1990. The high-contact condition as a sufficient condition for optimal stopping. In reality, this only holds for linear reward, but the paper by Paseka (2004) proves this is the case (page 37).

result is consistent with the *ex ante* equity maximization procedure. Furthermore, equity maximization coincides with firm value maximization subject to limited liability constraint. On the other hand, the introduction of reorganization leaves room for strategic behaviour by stock holders, because they can extract the surplus deriving from reorganization (such as the delay of payments or debt forgiveness) and are thus incentivised to default as the equity value is still positive. This leads to an earlier bankruptcy threshold, which impairs debt holders by reducing the debt obligation flows and also reduces the firm value. Earlier bankruptcy with respect to the immediate liquidation framework leads to a divergence of the boundary derived in an equity and in a firm maximization procedure, and entails the creation of a first-best outcome coming from firm (or, almost equivalently, debt) maximization, and a second-best outcome, where equity maximization is treated.

On the other hand, a second threshold was explicitly derived as the assets value when the firm reaches either the liquidation or the reorganization time. However, in Francois and Morellec (2004) this time is exogenously defined as the grace period, whereas Paseka (2004) suggests liquidation being exogenous and reorganization being triggered endogenously as to allow for strategic behaviour by shareholders. Furthermore, other models were also proposed in the literature. Moraux (2002) suggests adopting a cumulative approach in which the firm cannot exit from bankruptcy once it enters, while Galai et al. (2003)⁷⁹, though recognizing Moraux's improvement, define liquidation as the state in which the weighted average of the distance between V and an exogenous time dependent boundary exceeds a certain value. Nevertheless, none of these models recognized the existence of two endogenously specified (i.e. strategically determined) thresholds V_B and V_L .

Broadie, Chernov and Sundaresan (2007) and Broadie and Kaya (2007)⁸⁰ explicitly model Chapter 11 reorganization in continuous and discrete time respectively. They assume that the firm can liquidate either at the end of a prespecified grace period, whose length is eventually efficiently determined by creditors in order to reduce the gap between the first- and second-best outcomes but is all in all exogenous, or as soon as the firm value reaches a specified endogenous lower threshold V_L . As a result, the liquidation boundary is now endogenous, whereas reorganization can only be triggered as in Francois and Morellec (2004) at the end of the grace period if the plan is accepted. Otherwise, liquidation can occur at the end of the grace period as well.

⁷⁹ GALAI, D., RAVIV, A., WIENER, Z., 2003. Liquidation triggers and the valuation of equity and debt. Working paper, Hebrew University of Jerusalem.

⁸⁰ BROADIE, M., KAYA, O., 2007. A binomial lattice method for pricing Corporate Debt and modelling Chapter 11 proceedings. *Journal of financial and quantitative analysis*, vol. 42, issue 2.

Unlike previous formulations, the primitive variable is now assumed to be the operating cash flows or earnings before interest and taxes δ_t , whose process in a filtered probability space and under the risk-neutral probability is:

$$\frac{d\delta_t}{\delta_t} = \mu dt + \sigma dW_t$$

which results in the firm value:

$$V_t = \frac{\delta_t}{r - \mu}$$

Since μ is constant, then the process for V_t results in the same drift and instantaneous volatility as δ_t :

$$\frac{dV_t}{V_t} = \mu dt + \sigma dW_t$$

The firm pays a fixed coupon stream C and has issued infinite maturity debt. If cash flows are enough to pay the coupon, then the firm is liquid and remains in the healthy state. If instead cash flows are not sufficient, the firm should dilute or issue equity until it breaches the zero equity level according to Leland (1994). However, bankruptcy is not triggered by zero equity value in this case. Similarly to Paseka (2004), the owners may decide to default prior to destroying the equity value completely, in order to exploit the rents arising from Chapter 11 bankruptcy. Therefore, they will default at a level at which cash flows alone are not sufficient to repay the debt obligations, but equity is still not completely diluted in value and could still increase liquidity. One consequence to this is that distress will be purely financial and will not cause the diffusion process parameters to change, unlike suggested by Paseka (2004). A continuous flow of bankruptcy costs ω proportional to the assets value is also forecasted in continuous time, though without affecting the cash flows value. In discrete time, Broadie and Kaya (2007), instead, suggest subtracting bankruptcy costs ω to the cash flow drift (unlike Paseka, 2004, who subtracts them to the assets value drift), such that the distress cost adjusted operating cash flow $\bar{\delta}_t$ is modelled.

Once the company files for bankruptcy, it is granted an exclusivity or grace period, during which interest payments on all unsecured debt are stopped and all debt obligations are recorded in an arrears account A_t . Furthermore, cash flow or EBIT is accumulated in a separate account, S_t . Arrears will be repaid if the firm returns to a healthy state, but are reduced by some forms of debt forgiveness, which result from equity holders bargaining power θ , similarly to the optimal sharing rule proposed by Fan and Sundaresan (2000). Therefore, if V exceeds V_B and the firm returns to a healthy state at some point in time T before the expiration period, the separate account amount is used to repay the impaired arrears. Nonetheless, if it does not

suffice, equity is diluted up to the zero level. If the grace period expires before the firm is able to recover or if the value of unlevered assets reaches V_L , then the firm bears some liquidation costs α and has to liquidate.

Equity, debt and firm values are computed following the usual approach, but for the fact that they include all the possible outcomes from bankruptcy. Therefore, equity is the weighted algebraic sum of the dividend payouts in a healthy state, the distress costs and the residual amount available if the firm recovers before the expiration of the grace period. Debt value, instead, is made up of the coupon flow if the firm is healthy, the impaired arrears if the firm recovers, and the liquidation value if the firm falls bankrupt. These values can thereafter be used to determine V_B as the value which maximises either the firm value subject to limited liability in a first-best environment, or the equity or debt value in a second-best framework.

First-best bankruptcy threshold

When the debtor files for bankruptcy under Chapter 11 but maximises the firm value subject to limited liability, the illiquidity problem is already sorted out sooner than forecasted by Leland (1994), such that V_B will be higher (i.e., will be reached earlier) when a longer grace period is granted (this minimizes the probability of liquidation), when distress costs decrease (increasing

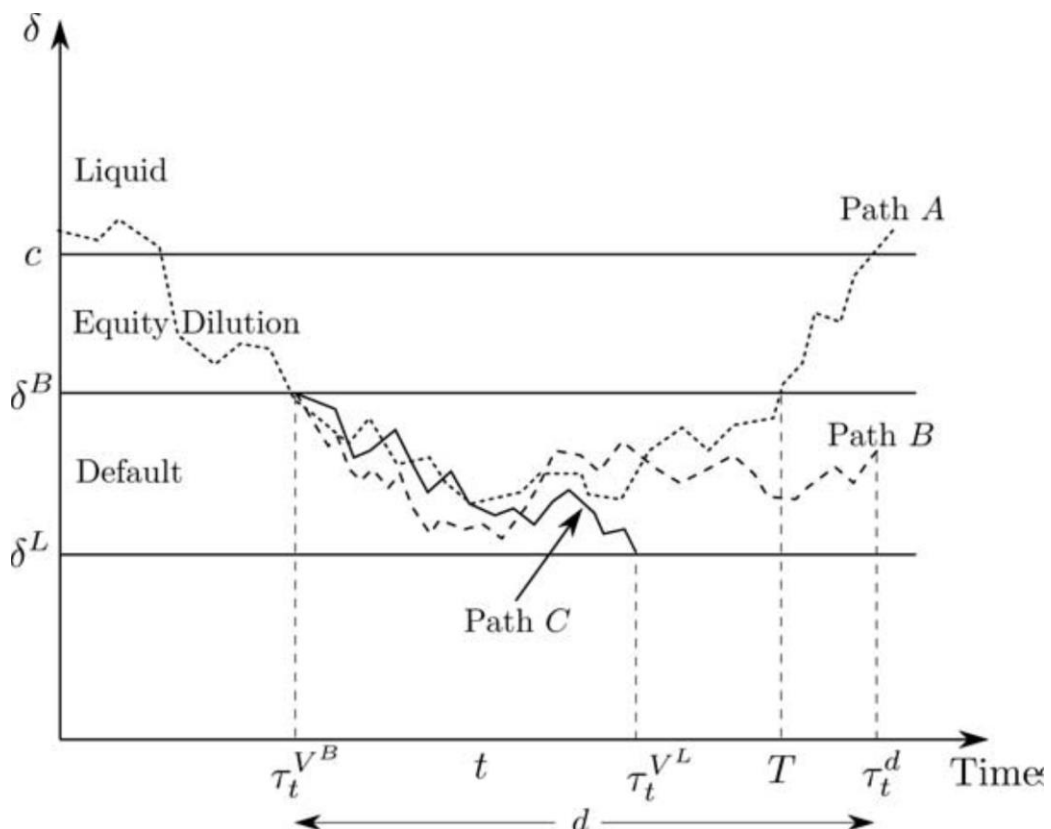


Exhibit 3.3: The time path of the model by Broadie et al. (2007). Bankruptcy occurs as the firm value goes below the bankruptcy boundary. Thereafter, the firm can (A) reorganize and recover; (B) arrive to the end of the grace period and, if it does not recover, it is liquidated; (C) reach the liquidation boundary and be liquidated.

the firm value upon bankruptcy) and when some of the debt is forgiven (decreasing the probability of being liquidated).

Decreasing bankruptcy costs and increasing debt forgiveness will also benefit firm, equity and especially debt, such that debt holders will be incentivized to return to the first-best outcome. They could achieve doing this by getting control of bankruptcy procedures, as demonstrated by the fact that debt value maximization leads to substantially similar results as firm value maximization. However, this is not applicable in practice, since equity holders keep control of these procedures. An alternative proposed by Broadie et al. (2007) is to allow them to determine the length of the grace period, in order to offset the incentive to anticipate bankruptcy by shareholders to get debt relief and other benefits with the attempt to avoid liquidation they have, especially if liquidation is costly.

Liquidation costs also make the values diverge between Chapter 7 and Chapter 11 bankruptcy, since they decrease firm value: the higher liquidation costs, the more beneficial is renegotiation, since the probability to incur in them is lower than in the immediate liquidation case.

Second-best bankruptcy threshold

As noted earlier, equity holders may have the incentive to file for bankruptcy before the first-best bankruptcy threshold, at a level which is optimal for their own equity value and where they can exploit the advantages of restructuring although being liquid. The results for this optimization problems are shown in Exhibit 3.4 and 3.5⁸¹. The firm and debt value decrease both relative to the liquidation only case and to the first-best situation. Furthermore, the presence of debt forgiveness and the absence of bankruptcy costs incentivize the divergence from the first-best outcome and a longer grace period associated with low bankruptcy costs can trigger early bankruptcy. In order to reduce the gap, the most effective solution is thus to reduce the grace period. Since this is not in the interest of shareholders but benefits debtholders, it is optimal to assign them this decision.

⁸¹ BROADIE, M., CHERNOV, M., SUNDARESAN, S., 2007. Optimal Debt and Equity Values in the Presence of Chapter 7 and Chapter 11. *The Journal of Finance*, vol. 62, issue 3, pp. 1356 and 1359.

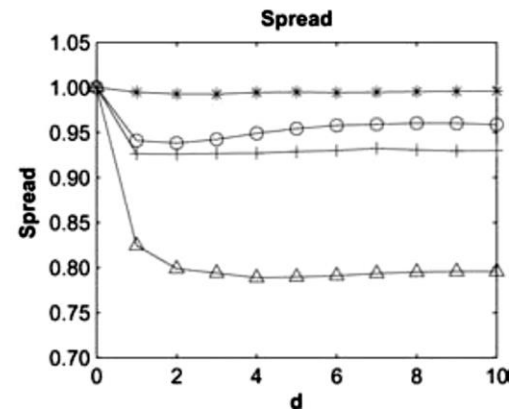
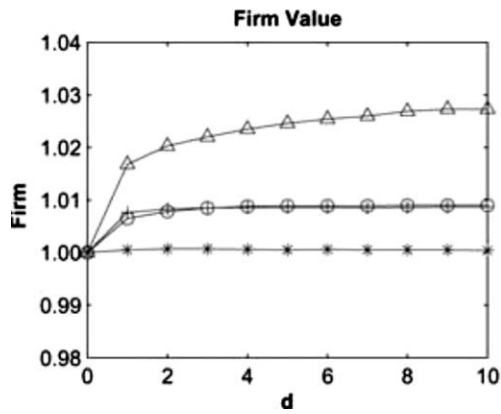
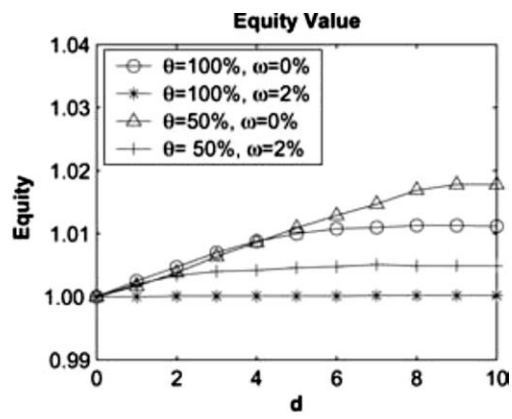
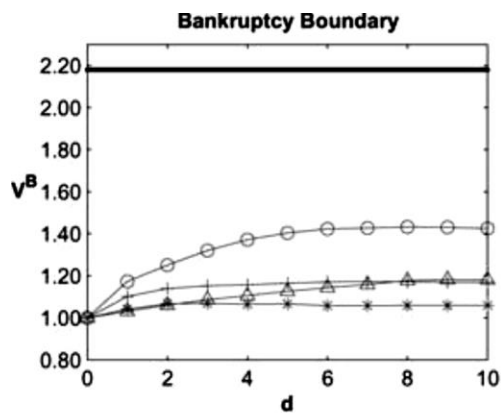


Exhibit 3.4: Firm value maximization effects as a percentage of Leland (1994)

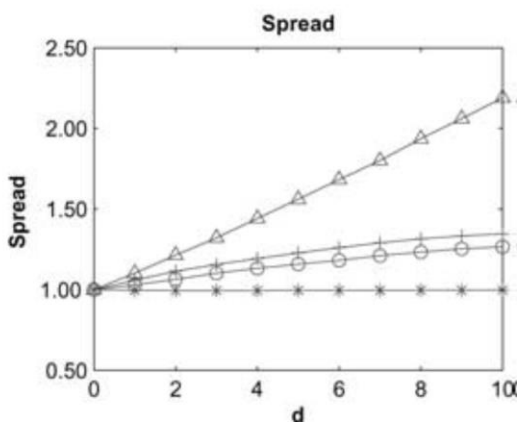
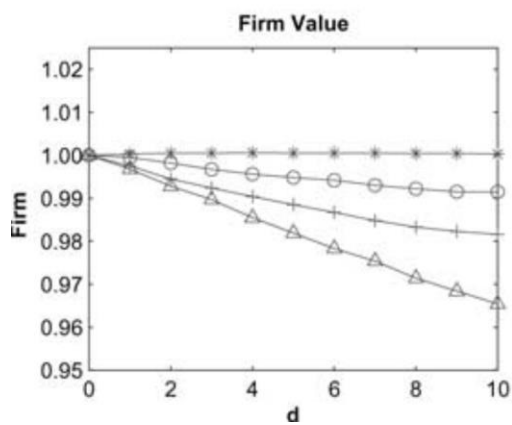
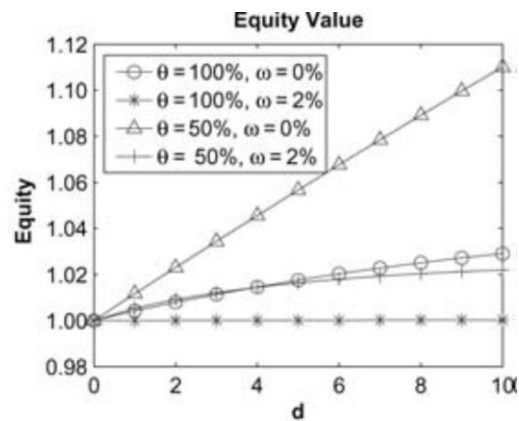
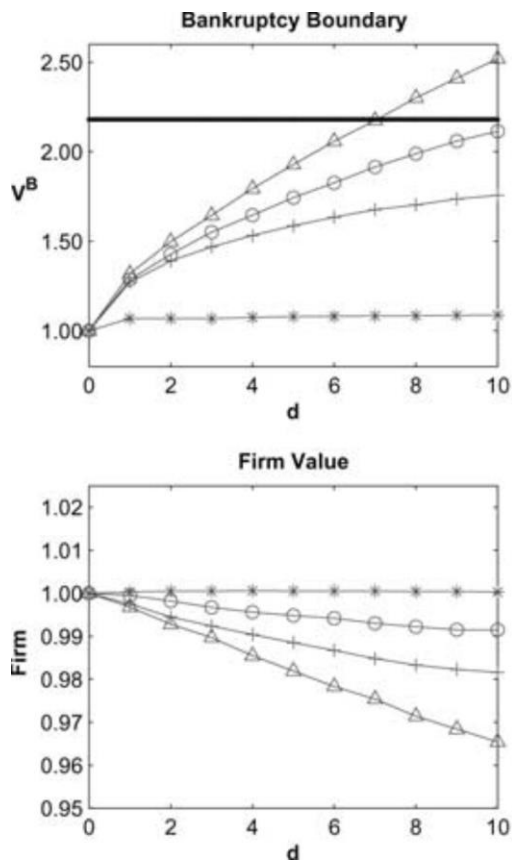


Exhibit 3.5: Equity value maximization effects as a percentage of Leland (1994)

Liquidation value

V_L , on the other hand, follows the optimization process in the liquidation only environment, since intuitively equity holders will behave exactly as in this model. Therefore, as upon liquidation, firm value maximization leads to the same liquidation threshold as equity maximization. The main difference from Leland (1994) et ss. is that the boundary now depends upon d and S_T and that it is modelled as the time dependent optimal strike price of an American option.

In order to determine it, Broadie and Kaya (2007) followed a discrete time approach, assuming a fixed coupon paid out in discrete time interval and focusing on finite maturity debt. They observed that the liquidation boundary may simply be hit in case of illiquidity, which occurs as equity (or firm value at maturity) and cash flows are not sufficient to repay coupon (and principal). The model is obtained recursively and will be further analysed in all its implications in the following Chapter. In it, it will also be proved in a binomial model that the liquidation boundary maximizes firm value subject to the limited liability constraint as well as equity value.

Chapter 11 vs. Chapter 7

As pointed out by Broadie et al. (2007), Chapter 11 entails a higher bankruptcy boundary and therefore early bankruptcy than in the immediate liquidation framework. On the whole, therefore, the probability of default increases with respect to immediate liquidation. The main exceptions when no debt forgiveness is granted and distress costs are high.

By looking at the probabilities of liquidation, on the other hand, it emerges that restructuring opportunity decreases the probability of liquidation for a fixed grace period. This is a clear consequence of the fact that the presence of Chapter 11 helps firms avoid unnecessary liquidation. A longer grace period d would be helpful in reducing these probabilities even further.

6. Conclusion

In this Chapter, the main developments of option pricing theory applied to debt and equity valuation have been introduced. Of course, this is just a limited overview and does not consider all the shades which have been analysed in the literature. However, it is enough to develop a binomial option pricing model through which an analysis and comparison of the main models can be implemented. In the next chapter, the implementation of the model will be split in a theoretical description and an application to a hypothetical company. The results are of interest for the analysis in the following.

Chapter IV. A binomial option pricing perspective

1. Introduction

The previous Chapter described the main option models available in the literature to value equity and debt. However, these approaches are very theoretical and the assumptions and structure they are based upon are not always credibly applicable in the real world. For example, most models use simplifications to avoid time and path dependencies. One example over all refers to the assumption of infinite maturity debt, which excludes time dependencies but never occurs in practice. Furthermore, modelling grace periods, automatic stay provisions and arrears payments is not straightforward in continuous time. The implementation of a partial differential equation, indeed, does not always lead to closed-form solutions, and the determination of the right formula may itself be not straightforward at all.

In order to cope with these issues, Broadie and Kaya (2007) modelled a binomial option pricing method. They considered the firm's assets value as primitive variable and they modelled it in discrete time. Afterwards, the values of equity and debt were obtained by working recursively, taking into account explicitly some features of equity and debt.

Indeed, this model proves to be much more reliable because it considers explicitly all the complications related to the limited liability of equity and to the debt payments. Furthermore, it also allows to consider more complicated regulatory frameworks, such as those associated to Chapter 11, and prevents from introducing strategic restructuring, which is theoretically consistent but unrealistic in many cases.

The paper can be split off into three main models, which incorporate three different regulatory frameworks and are associated to three different continuous time models:

- The first models in discrete time the immediate liquidation case, following Leland (1994), which assumed the existence of a consol bond, and Leland and Toft (1996), which allowed for a debt which is continuously rolled over, in order to keep constant leverage;
- The second includes the chance for Chapter 11 renegotiation by Francois and Morellec (2004), which assumed infinite maturity debt;
- The third allows for Chapter 11 renegotiation, but also includes some technicalities of Chapter 11 in a discrete lattice: it records the arrears payments and the automatic stay. This model is adapted from Broadie, Chernov and Sundaresan (2007).

In the following, these models will be analysed for a toys company example and the results will be compared with the theoretical models' empirical studies.

2. An illustrative example

As specified by Broadie and Kaya (2007), the analysis of a binomial lattice valuation cannot only rely upon theoretical formulas, but is much more complete when considering a numerical example, although this may hamper the generality of the model.

The main inputs of the numerical model were designed according to Buttignon (2015) and refer to Happy Toys, an imaginary financially distressed firm operating in the sector of games and entertainment.

In order to define the value of its claims, the first step is the reorganization and analysis of the historical financial statements and the projection of future cash flows. Since this analysis is only preparatory to option pricing, which will effectively include volatility, only a base-case scenario is proposed. The key value drivers are defined as described above when analysing the determinants of firm's value, i.e. by considering only existing investments and by incorporating the likely negative effects of financial distress on operating income. In the model on hand, the explicit projection period is assumed to be 3 years⁸² and the assumptions of the model are described in Exhibit 4.1⁸³. At least in the last balance-sheet year and in the first year of projection, the firm is expected to incur in negative revenue growth and reduced margins, which will themselves decrease the operating working capital as a consequence of the indirect costs of distress.

	-1	1	2	3
Sales annual growth (g)	-10%	-5%	2%	3%
EBITDA margin	0%	3%	5%	8%
NWC/Sales	16%	16%	15%	15%
CAPEX/Sales	2,0%	3,0%	3,0%	2,5%

Exhibit 4.1. HAPPY TOYS: Operating financial projections (Value Drivers)

⁸² The choice of a so short explicit projection period was due to complications of the model arising in the following. Of course, a DCF analysis usually requires an explicit projection period sufficiently long for the firm to reach the steady state. See f.e. Koller, Goedhart and Wessels (2005) to deal with this topic.

⁸³ Buttignon (2015), Exhibit 4.

Using these assumptions and the usual DCF procedure, the main financial projections and the business enterprise value were computed, as shown in Exhibit 4.2 and 4.3 respectively. Apart from those directly derived from key value drivers, the other formulas, such as depreciation or net fixed assets, use the most adequate forecast ratios. Interest is computed by multiplying the promised interest rate 4% to the value of principal (350 million €), and the tax rate is assumed $\tau=25\%$. The resulting business enterprise value is $V_0=260$ million € in the DCF scenario. It coincides with the enterprise value because non-operating assets were omitted for the sake of

	-1	1	2	3
Sales (S)	750	713	727	749
EBITDA	0	21	36	60
Depreciation	-25	-18	-18	-19
EBITA	-25	3	18	41
Interest expenses	-15	-14	-14	-14
EBT	-40	-11	4	27
<i>Income tax rate</i>	25%	25%	25%	25%
Income taxes	0	0	-1	-7
Net earnings	-40	-11	3	20
Net working capital	120	114	109	112
Net fixed assets	180	183	187	187
Invested capital	300	297	296	299
Senior debt	350	350	350	0
Junior debt	0	0	0	0
(Cash) debt (plug)		8	3	336
Net financial position	350	358	353	336
Shareholders' Equity	-50	-61	-58	-37

Exhibit 4.2. HAPPY TOYS: Financial Projections.

	-1	1	2	3	CV
EBITDA	0	21	36	60	61
EBITA	-25	3	18	41	42
Operating taxes	6	-1	-5	-10	-11
<i>Operating tax rate</i>	25%	25%	25%	25%	25%
NOPLAT	-19	3	14	31	32
Net working capital variation		6	5	-3	-2
Net capex		-3	-3	0	-4
Free cash flow (FCF)		5	15	28	26
Cost of capital		10,5%	10,5%	10,5%	10,5%
Growth rate (g)					2,0%
<i>Discount factor</i>		0,90	0,82	0,74	
Present value of FCF		5	12	20	
Present value of FCF	37				
Continuing value	222			300	
Enterprise value (EV)	260	282	297	300	
EV dynamics	260	282	297	300	
Return on capital		10,5%	10,5%	10,5%	

Exhibit 4.3. HAPPY TOYS: EV calculation.

simplicity. V_0 is thereafter used as the primitive variable for the introduction of a structural model. Following the models described previously, V_t follows a diffusion process with constant volatility of return σ :

$$dV_t = \mu(V_t, t)V_t dt + \sigma V_t dW_t$$

where W_t is a standard Brownian motion under the risk-neutral probability Q and $\mu(V_t, t)$ is the total expected rate of return, which also considers the payout ratio δ , whose behaviour is specified in the following. The risk-free rate is assumed to be constant and equal to 3% and, following Merton (1974), there are no frictions to lending and borrowing. The volatility is exogenous and equals 20%. The firm has initially issued senior debt only with a principal $P=350$ and interest rate $r_D=4\%$. The principal will be fully repaid at maturity, whereas interests are paid out continuously unless default occurs and are discretized as $C\Delta t$.

The firm is also expected to produce a free cash flow (FCF), which is assumed to be generated instantaneously as a fixed and known fraction δ of the firm's assets value, which is computed as the mean of the cash flows obtained in the DCF valuation for the explicit projection period and results $\delta=5\%$. This feature is modelled explicitly in this model, in order to estimate future free cash flows, which represent the basic source of newly generated liquidity necessary to satisfy debt obligations. That is why δ is defined by Broadie and Kaya (2007) as the cash flow or payout ratio. At each infinitesimal time interval, the free cash flow is computed as follows:

$$FCF_t = \delta V_t$$

However, for more accuracy, the payout is assumed here to be paid discretely and thus (4.2):

$$FCF_t = V_t e^{\delta\Delta t} - V_t.$$

As a consequence, V_t can be treated as a dividend paying stock with known dividend yield, whose dividend is paid at each time step from time 1⁸⁴. This slightly differs from the assumptions of most literature, in which the dividend yield is already incorporated in the diffusion process. These models, therefore, directly derived the value of firm's assets after payout. However, since the model requires the explicit computation of both V_t+FCF_t and of FCF_t , it is more convenient to model them separately, as the final model does not differ.

As suggested by Hull (2012), this resembles an option on a dividend-paying stock, where the dividend accrues continuously but is paid out discretely. Consequently, the parameters to construct the option can be computed as though no dividends were expected, according to the Cox, Ross and Rubinstein (1979) binomial model formulas for option pricing:

$$u = e^{\sigma\sqrt{\Delta t}}; d = e^{-\sigma\sqrt{\Delta t}}$$

⁸⁴ HULL, J. H., 2012. Options, futures, and other derivatives. 8th Edition. Harlow, England: Pearson Education Ltd., pp. 437-438.

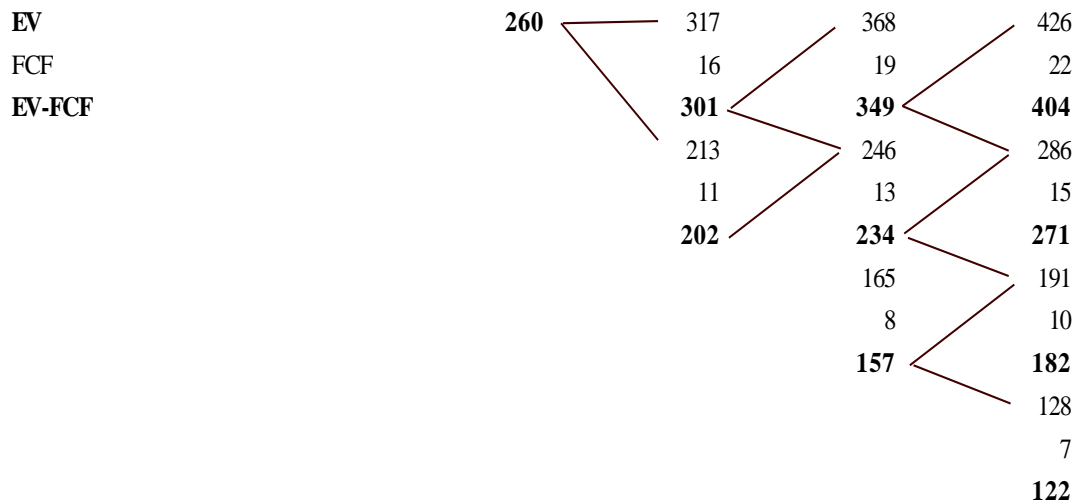


Exhibit 4.4. Evolution of EV and FCF. EV is computed using Cox, Ross and Rubinstein (1979) binomial model formulas for option pricing at each up or down node. FCF are computed according to formula (4.2) since they are paid at discrete time points. Then they are subtracted to the EV.

In these formulas, u and d are multiplied by the value of assets V_t in order to obtain the value of assets at the following node for an up-move or a down-move respectively. The cash flow discretized in that period, obtained in (4.2), is thereafter subtracted to the obtained assets value for each move. The result so obtained is then used as input for the computation of the moves in the following nodes. The results are shown in Exhibit 4.4. Notice that this method coincides with the construction of a binomial tree for a stock paying a known dividend yield, where, after the stock goes ex dividend, the nodes correspond to the stock prices: $V_0(1 - \delta)u^j d^{i-j}, j = 1, \dots, i$. The risk-neutral probability of an up-move is computed as if no dividends were expected as well:

$$p = \frac{e^{r\Delta t} - d}{u - d}$$

Before proceeding with the analysis of each single model, it is necessary to notice how the construction of a binomial lattice can enhance the analysis of financially distressed firms. This will be treated in the next Subsection. Binomial models also require that limited liability and equity dilution issues are dealt with explicitly. This is what the last Subsection refer to.

Enhancing the analysis through binomial models

The binomial lattice methods described in this model rely on the idea initially theorised in the Merton (1974) model, aimed at valuing equity and debt as options. However, these models are developed in discrete time and relax some of the tightening assumptions of the original model. Furthermore, some adaptations are introduced to comply with real world features. First, the Merton (1974) model assumes that the firm has issued a risk-free zero-coupon bond, which may default only at maturity if the face value is higher than the assets value. In the models analysed now, instead, the firm issues debt which will have interest accruing at any discrete time step

(similarly to a coupon bond) and may therefore declare bankruptcy at any time. If this is the case, it is necessary to determine which bankruptcy-triggering assets value shall be used at each time. This choice also depends on whether bankruptcy entails immediate liquidation, as in Leland (1994) and Leland and Toft (1996), or whether some restructuring is included in the model, for example as in Francois and Morellec (2004). In particular, the binomial approach also allows to deal with real Chapter 11 restructuring features, especially the introduction of a grace period, automatic stay of assets and arrears account, following the approach by Broadie, Chernov and Sundaresan (2007). Furthermore, the model also included imperfections, such as taxes τ , whose rate is assumed to be 25% and which produce a benefit up to the bankruptcy point, and bankruptcy costs, which are split up to liquidation and restructuring costs, respectively α and ω . In the model, α is derived as the ratio between the value of assets as if they were liquidated today and their book value, thus accounting for the fire sales effect and the reduction in the expected cash flows from them as they are sold, minus the direct costs of liquidation. It is assumed that $\alpha=30\%$ and $\omega=0.5\%$.

Another adaptation with respect to Leland (1994) and Francois and Morellec (2004), but also to most authors, is the introduction of the more realistic finite maturity debt. Indeed, continuous time models make use either of consol bonds or of finite maturity debt that continuously rolls over at a fixed interest rate or premium to the risk-free rate. This is aimed at getting time independent variables. However, using the binomial lattice method avoids this problem and makes it possible to extend these models for finite maturity debt.

Equity dilution and limited liability

One of the main advantage of the binomial model is that it allows to incorporate the limited liability requirement explicitly into the model, i.e. the requirement that debt obligations can only be satisfied through available cash or by issuing or diluting equity, but equity can never go negative and therefore the management has to file for bankruptcy before or at the latest in the moment that equity reaches zero level.

In this Subsection, it is anticipated how the following models will incorporate the limited liability requirement and equity dilution. This requires knowing the value of equity as a call option at maturity T , which will be computed differently from model to model, depending on whether only liquidation or also restructuring are considered, and which, therefore, will be analysed step by step when the models will be described in the following Sections. For now, the values of E_u and E_d are supposed to be known and to represent the values of equity in the next step for an up and down move respectively.

Being equity a call option on the value of the firm, its value ignoring interest payments and cash flows is the usual value computed at each node for a European call option, obtained recursively as the expected value of equity under the risk neutral probability (4.3):

$$\tilde{E} = e^{-r\Delta t}(pE_u + (1 - p)E_d)$$

The equity value obtained recursively is not sufficient to obtain the initial value, because some changes modify it at each step. Indeed, following Leland (1994), in this model debt obligations are fulfilled not only through available cash flows but also through equity dilution or new equity issuance⁸⁵. As a result, at each time net coupon payments are defined as the difference between the promised coupon and the available cash ($\bar{C} = C - \delta$). If \bar{C} is negative, cash flows are sufficient to cover debt payments and therefore no equity intervention is needed. Equity is consequently increased by the value in advance, and $E = \tilde{E} - \bar{C}$. If \bar{C} is positive, equity may be diluted if money raised by this intervention is sufficient to make the payment and equity holders will still get a positive equity value. The amount of newly issued equity shares x is such that their value is able to satisfy debt obligations at that time, knowing that this issuance will reduce the value of shares and, therefore, the new share is worth total equity divided by the after-dilution number of shares outstanding⁸⁶:

$$\frac{x}{1+x}\tilde{E} = \bar{C} \Leftrightarrow x = \frac{\bar{C}}{\tilde{E} - \bar{C}}$$

and equity is the original value without considering net payments divided by the current amount of shares, which is the amount which allows to repay the bond. Easy simplifications lead again to $E = \tilde{E} - \bar{C}$.

If instead, despite a still positive \bar{C} , equity is not sufficient, liquidation occurs and equity value is zero.

This reasoning leads to a modified equity value than the recursively obtained one, which incorporates explicitly limited liability and equity change:

$$E = \begin{cases} \tilde{E} - \bar{C} & \text{if } \tilde{E} \geq \bar{C} \\ 0 & \text{if } \tilde{E} < \bar{C} \end{cases}$$

This approach will be very useful to define option pricing at each node in the models which follow.

⁸⁵ The asset value based model differs in this from most cash flow based approaches, where illiquidity arises as cash flows do not suffice to make the payments, as in Kim et al. (1993).

⁸⁶ See Broadie and Kaya (2007). They assume initially one share only is outstanding.

3. Bankruptcy with immediate liquidation

This Section is concerned with the definition of a binomial model which follows Chapter 11 bankruptcy, according to the model by Leland (1994) and specifically by Leland and Toft (1996), where finite maturity debt is considered. As mentioned in the previous theoretical description, the bankruptcy-triggering assets value V_B is derived from the equity maximization procedure and, due to convexity of equity, results in the first value of V which triggers zero equity value, because otherwise shareholders will never prefer liquidation-triggered zero equity value to a positive equity value in continuation. When incorporating limited liability and equity dilution, this value can be obtained comparing cash flows available at each node with the coupon due payment. If the firm does not dispose of sufficient liquidity to meet its obligations, neither by using available cash flows, nor by diluting equity, the management will file for bankruptcy under Chapter 7 and the firm will be immediately liquidated. If this is the case, all the residual enterprise value and the accrued cash flows will be paid to debt holders, net of liquidation costs, which are assumed to be a proportion α of this value. On the other hand, since liquidation occurs exactly as shareholders reach zero equity value, their residual claim will be worth zero.

Otherwise, the firm will continue its operations and it will obtain the value of equity as if nothing happened at that node summed to the net debt obligations payment. Therefore, the value of equity increases if the cash flow is sufficient to repay debt net of interest tax shields, whereas it decreases if equity dilution is necessary.

It is noticeable that equity behaves as a call option on the firm assets value with strike price equal to the due obligations, while debt corresponds to its face value net of a corresponding put option on V . This is easier to observe at maturity, but the option can be exercised at each time, in contrast with the original theoretical model.

At time T (which coincides with maturity), equity is valued as a call option and debt coincides with the minimum between its principal and interest value and the liquidation value, which is a fraction $(1-\alpha)$ of the enterprise value used as primitive variable summed with the explicitly derived FCF_t .

Furthermore, an additional value was derived following this procedure and was named F . It corresponds to the firm value, being computed as the sum of equity and debt. It is necessary to distinguish this value from the primitive variable V , which was already defined “enterprise value” (more correctly, “business enterprise value”, but they coincide if the firm has no non-operating assets). Indeed, the latter is computed as the sum of cash flows from currently existing assets and it neglects the opportunities available to the firm, since the firm is valued as if it were

to be traded immediately. Indeed, as pointed out in the previous Chapter, other models than DCF may be used to derive this value. On the other hand, F also incorporates the additional value of flexibility for the firm, and therefore includes the additional value of a firm being able to continue its operations. Indeed, it is remarkable that the value is obtained including the two options referring to equity and debt.

At maturity T, therefore, the values are obtained as follows and are summarised in the first table of Exhibit 4.5:

$$\begin{aligned} \text{If } V_T + FCF_T \geq (1 - \tau)C_T + P_T: & \begin{cases} E = V_T + FCF_T - ((1 - \tau)C_T + P_T) \\ D = C_T + P_T \\ F = V_T + FCF_T + \tau C_T \end{cases} \\ \text{If } (1 - \tau)C_T + P_T < V_T + FCF_T: & \begin{cases} E = 0 \\ D = (1 - \alpha)(V_T + FCF_T) \\ F = (1 - \alpha)(V_T + FCF_T) \end{cases} \end{aligned}$$

In this formula, τ is the marginal tax rate, C_t and P_t are the values of interest accruing at t and of principal to be repayed at t . This make up the debt obligations net of the interest tax shields. At any time t before maturity, equity is derived similarly to an American option where equity features need to be incorporated. Consequently, it is not only the recursively computed amount under the risk neutral probability derived in (4.3), but also includes changes in equity necessary to avoid liquidation, as discussed in Section 2. If equity holders cannot meet their obligations at any node, they will hand over the firm to bond holders and equity will end up worthless. In the same way, debt is obtained not only from the result of recursive computation, but also by adding at each time the coupon payment gross of taxes, while it corresponds to the liquidation value if debt payments are not fulfilled. It can be priced as a knock-out barrier option⁸⁷, where the barrier coincides with the debt payments (the strike price for equity-call option) below which debt value coincides with the liquidation value. Finally, the firm value F is, as usual, the sum of equity and debt:

$$\begin{aligned} \text{If } \tilde{E} + FCF_t \geq (1 - \tau)C_t: & \begin{cases} E = \tilde{E} + FCF_t - (1 - \tau)C_t \\ D = C_t + e^{-r\Delta t}(pD_u + (1 - p)D_d) \\ F = FCF_t + \tau C_t + e^{-r\Delta t}(pF_u + (1 - p)F_d) \end{cases} \\ \text{If } (1 - \tau)C_t > \tilde{E} + FCF_t: & \begin{cases} E = 0 \\ D = (1 - \alpha)(V_t + FCF_t) \\ F = (1 - \alpha)(V_t + FCF_t) \end{cases} \end{aligned}$$

⁸⁷ A barrier option is a type of exotic option whose payoff depends on whether or not the underlying asset has reached or exceeded a predetermined price. Specifically, knock-out barrier options cease to exist if the underlying asset reaches a barrier during the life of the option. In the case on hand, liquidation is triggered as the barrier is breached.

These steps are described in the second and third tables of Exhibit 4.5: working backward, equity, debt and firm value can be found throughout the lattice. The resulting values are E=15.07, D=197.45, with a 44% discount on its nominal value, and F=212.53.

At T

$V_T + \Delta T$	$(1 - \text{tax}) C + P$		E	D_s	F
426	360,50	TRUE	0,00	364,00	298,18
286	360,50	FALSE	0,00	199,88	199,88
191	360,50	FALSE	0,00	133,98	133,98
128	360,50	FALSE	0,00	89,81	89,81

at T2

$\tilde{E} + FCF_t$	$(1 - \text{tax}) C$		E	D	F
52,31	10,50	TRUE	0,00	291,71	257,36
12,67	10,50	TRUE	0,00	177,65	172,51
8,49	10,50	FALSE	0,00	115,64	115,64

at T1

$\tilde{E} + FCF_t$	$(1 - \text{tax}) C$		E	D	F
16,31	10,50	TRUE	0,00	244,60	222,13
10,94	10,50	TRUE	0,00	157,86	148,90

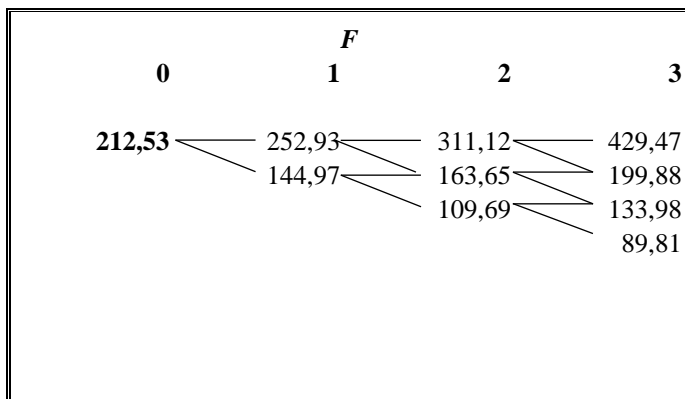
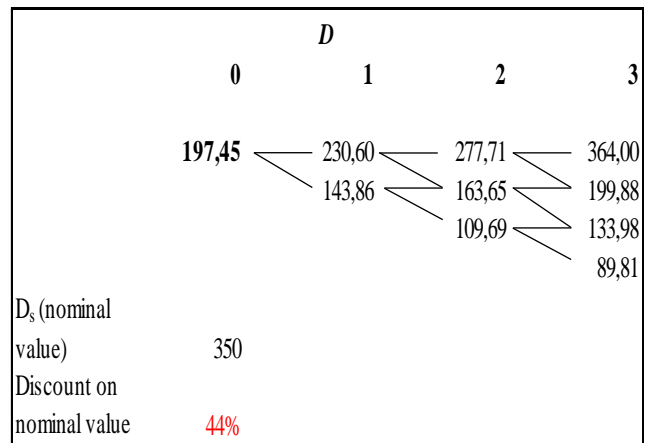
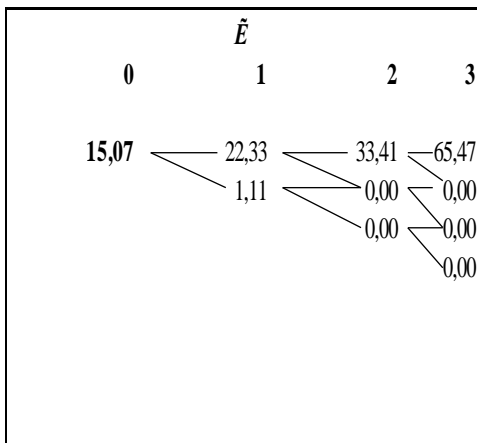


Exhibit 4.5. Immediate liquidation binomial option pricing results.

Notice that backward computations do not make use of the values written in the next node as input, but instead on the values obtained for that node after considering limited liability and equity dilution. For instance, the up move at time 1 derives 22.33 as $\tilde{E} = 22.33 = e^{-3\% \cdot 1}(0.526 \cdot 41.81 + 0.474 \cdot 2.17) \neq e^{-3\% \cdot 1}(0.526 \cdot 33.41 + 0.474 \cdot 0)$. In other words, limited liability requirements and equity dilution alter the input for the computation of the next node from the recursively obtained one. The same procedure is adopted for all the time steps and all the node but the initial value, where no decision can be made and therefore the value is simply the obtained recursively.

Analysing the bankruptcy boundary

Leland (1994)'s bankruptcy boundary was obtained in continuous time by applying the smooth pasting condition, and the resulting closed-form solution is proportional to the coupon and decreasing in the corporate tax rate, risk-free rate and volatility.

The discrete modelling proposed by Broadie and Kaya (2007), instead, entails explicit dependencies on the corporate tax rate and the interest rate only. Nevertheless, a higher volatility may also lead to higher assets value in up-steps, while still satisfying positivity of assets value for the lower bound, and could implicitly lead to a reduction of cases when the threshold is breached. In the discrete model, higher coupons will increase the boundary, whereas a higher tax rate will decrease it. At first glance, it seems straightforward that a higher V_B will entail early liquidation and thus will decrease the value of equity, which will drop to 0 more frequently, as well as debt, since the full payment is not fulfilled and, on the other hand, liquidation will become more imminent. Liquidation, of course, is expected to reduce debt value though still letting it be higher than 0, because it is invoked only when due payments cannot be made by using the current value of assets and cash flows available, and furthermore it also suffers from liquidation costs.

However, this analysis is only partial: not only is V_B only one of the determinants of equity and debt, but also in some cases other effects are dominant on the determination of claims' value, making a complete analysis much more complicated.

Comparative statics of Debt and Equity

Leland (1994) and Leland and Toft (1996) pointed out that the value of debt was also directly affected by market imperfections, i.e. bankruptcy costs and corporate tax rate, which decreased and increased it respectively. Discrete option pricing only partially confirms these results. If on one hand α clearly reduces the liquidation value and therefore implies a reduction in the debt value, while not affecting equity, on the other hand tax rate increases show to make debt value decline. This result seems counterintuitive at first glance, because interest tax shield should

make debt more convenient and therefore should make debt and firm value raise. However, the discrete model starts from an initial DCF analysis, causing assets value to be itself affected by the tax rate. Since higher tax rates are also expected to affect NOPLAT, they will decrease the primitive variable V_t : this triggers an increase of bankruptcy nodes and therefore a decrease in debt and firm value. This effect is also proved by fixing exogenously the assets value, which leads back to an increase in debt as tax rate increases.

Other determinants of the debt value are the interest rate, the risk-free rate and the riskiness of the firm. In investment-grade companies, increases in volatility and coupon rate are expected to increase debt value, whereas risk-free rate makes it decline. The behaviour in this model is reversed when specific levels of some of these parameters are reached. Above the value where equity reaches the zero level, higher interest rates lead to a lower debt value and then stop affecting it. The resulting trends of debt and firm values, shown in the second graph of Exhibit 4.6, are increasing up to a certain level of the interest rate, then they plummet and they stop at this final level.

Volatility does not determine the debt slope univocally as well. If for sufficiently low levels debt is decreasing in it, when it exceeds a certain level, 43.2% in the case analysed in this work, it starts raising on increases in riskiness, and, thereafter, it returns to a declining trend. This is determined by whether bankruptcy will occur at certain nodes, especially at final nodes.

Although this seems counterintuitive, Leland (1994) suggests how to explain this effect. He argues that, for V being sufficiently close to V_B , higher asset volatility and risk-free rate as well as lower coupon rates also reduce the value of V_B , thus reducing the probability of filing for bankruptcy and increasing debt value. In the model on hand, indeed, changes in behaviour occur as the threshold is breached at a certain node, as it is the case for 43.2%, which satisfies

$$V_0 u^2 d = V_B \Leftrightarrow V_0 e^{2\sigma\sqrt{\Delta t}} e^{-\sigma\sqrt{\Delta t}} = V_B \Leftrightarrow \sigma = \frac{\ln\left(\frac{V_B}{V_0 d_{t-1}}\right)}{\Delta t}.$$

This effect is clear when looking at the trend of the debt value with respect to volatility in Exhibit 4.6.

Furthermore, in the discrete time model, risk-free rates do not affect the boundary nor the tree, but for the fact they affect the WACC for the initial business enterprise value computation and the values obtained recursively, which do not constitute explicit inputs in the model. Consequently, the reversion of the relationship does not occur in this case.

As for equity and firm values, Exhibit 4.6 shows the effect of volatility and interest rate. Higher volatility may also increase the value of the firm, whereas the other parameters do not cause changes in behaviour of junk bonds with respect to investment-grade ones. Equity does not show reversals of comparative statics results as V reaches V_B , nor it is affected by bankruptcy

costs. Indeed, they only arise when liquidation is triggered. Therefore, they only decrease debt value, whereas equity as a residual claim, i.e. as a claim with zero value upon liquidation, will not be affected.

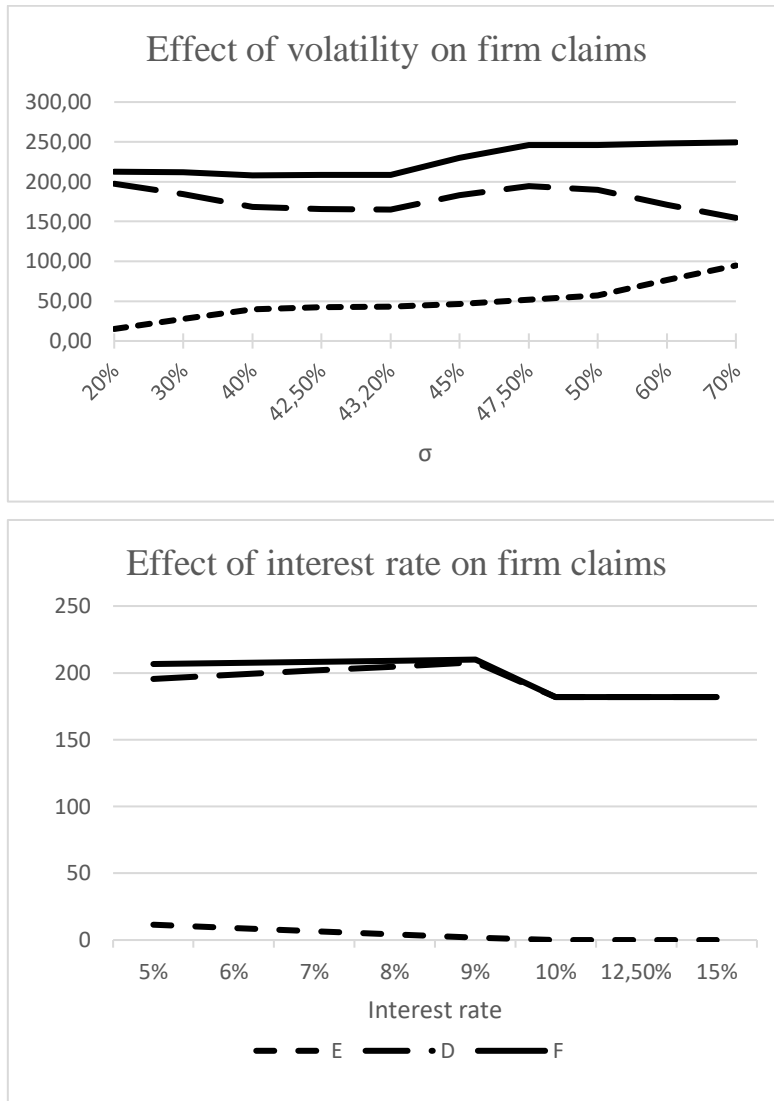


Exhibit 4.6. The effect of volatility and interest rate on debt, equity and firm value.

4. Bankruptcy with grace period and bargaining

The option pricing method implemented in the previous Section assumed that filing for bankruptcy leads to immediate liquidation and the models dealing with strategic debt service analysed the values of claims which are agreed by shareholders and creditors in a strategic non-cooperative game. However, this result was only limited to private workout. Francois and Morellec (2004) proposed a model which explicitly deals with Chapter 11 debt restructuring. Upon it, bankruptcy leads to a grace period, in which the debtor is protected by the court and debt obligations cannot be enforced. The debtor can therefore stop making the contractual debt payment and start servicing debt strategically unless the bankruptcy threshold is breached again from below and the firm goes back to a healthy state (Fan and Sundaresan, 2000). In the

meantime, she can continue her operations but has to prepare a restructuring plan, which needs to be presented at the end of the grace period, set by the court. The court finally evaluates the plan and determines whether the firm shall end up in liquidation or may continue as a going concern.

The various developments which may be triggered by declaration of bankruptcy are better off be treated explicitly in a binomial model. Indeed, in this case, at each node the option values may be triggered by several decisions:

- The firm may be in a healthy state of file for bankruptcy and thus enter the bankruptcy grace period, assumed $\bar{g}=2$. The latter is usually decided by managers on behalf (and normally in favour) of shareholders and satisfies the smooth-pasting condition applied to an equity value which also considers renegotiation. How do determine this value will be dealt with later in this Section;
- During the grace period, the firm may liquidate at any time when hitting the liquidation boundary, which coincides with the case of lack of liquidity despite equity dilution as in the previous model;
- Otherwise, during the grace period, the firm may service debt strategically according to its bargaining power $\eta=60\%$ and will also have to face a reduction of cash flows which is determined as reduction of δ by the amount of bankruptcy costs ω , such that during the grace period as well as at the first step after recovery $\overline{FCF}_t = V_t e^{(\delta-\omega)\Delta t} - V_t$;
- At the end of the grace period, if the firm is able to recover then it will go back to a healthy state, while if not it will be liquidated.

In this framework, therefore, two boundaries were determined. The first is the bankruptcy boundary, which may be obtained exogenously or endogenously. This boundary represents the value of assets which satisfies the smooth-pasting condition, i.e. which is most convenient to shareholders as a bankruptcy trigger because all equity values are positive for any V higher than this boundary, and is defined as V_B . Despite the same notation, this value does not coincide with the bankruptcy boundary by Leland (1994), which now corresponds to a liquidation boundary and is defined as in the previous Section. Initially, V_B is defined exogenously, but this issue will be discussed later more in detail.

Determining the values at the final nodes

In order to proceed with the analysis, it is necessary to structure the grace period. This concession is initiated as soon as the firm falls bankrupt, which means $V_t < V_B \wedge V_{t-1} > V_B$ in the case on hand. In this situation debt is served strategically and its formula follows Fan and Sundaresan (2000). Afterwards, the firm might either recover or spend up to a maximum of \bar{g} periods in bankruptcy. In the former case, $V_t > V_B \wedge V_{t-1} < V_B$, g will go back to zero and

bankruptcy costs may also accrue. In the latter case, instead, the firm reaches $g = \bar{g}$: if its assets value exceeds the liquidation boundary, then the firm will return to a healthy state; otherwise, it will be liquidated anyway.

At the final node, the firm may either liquidate or restructure. Liquidation may occur if, though the firm is healthy, the liquidation threshold is reached anyway (i.e. the firm is unable to meet its debt obligations) or if the firm had previously filed for bankruptcy. The latter situation is not modelled explicitly by Broadie and Kaya (2007), who assumed that as the firm goes bankrupt debt and equity are serviced strategically and during the grace period their value is just equal to the strategic value provided by Fan and Sundaresan (2000) and can be omitted. Therefore, if at maturity $V_T < V_B$, they simply assume liquidation. To better understand whether restructuring determines some benefit to the firm claims values, the model assumes that, if the expiration of the grace period coincides with the debt maturity, the firm may decide either to restructure or to liquidate. As a consequence, at $g = \bar{g}$ the model slightly differs from the binomial option pricing proposed in the paper on hand in order to be closer to the original Francois and Morellec (2004) framework.

If $V_T > V_B \wedge V_{T-1} > V_B$ ($g = 0$), the firm is in a healthy state and may eventually file for bankruptcy under Chapter 7 if the liquidation boundary is reached.

$$\text{If } V_T + FCF_T \geq (1 - \tau)C_T + P_T: \begin{cases} E = V_T + FCF_T - ((1 - \tau)C_T + P_T) \\ D = C_T + P_T \\ F = V_T + FCF_T + \tau C_T \end{cases}$$

$$\text{If } (1 - \tau)C_T + P_T < V_T + FCF_T: \begin{cases} E = 0 \\ D = (1 - \alpha)(V_T + FCF_T) \\ F = (1 - \alpha)(V_T + FCF_T) \end{cases}$$

Similarly, if the firm is in the first bankruptcy state or it has just recovered in any final node, then it will have to decide whether to liquidate. Therefore, if $V_T < V_B \wedge V_{T-1} > V_B$ ($g = 1$ and $g < \bar{g}$) or if $V_T > V_B \wedge V_{T-1} < V_B$ ($g = 0$ and $g - 1 > 0$), the model will be similar as the case where the firm is healthy, but for the presence of the firm value $F[i]$ when the firm is in the grace period, where the usual cash flow is substituted by the cash flow available upon bankruptcy $\overline{FCF_T}$.

Therefore, $F[0]$ corresponds to the firm value at the bankruptcy boundary without the firm having been in bankruptcy before and is used as input for the nodes reaching V_B from above. On the other hand, $F[i]$ represent the values of the firm in bankruptcy or just coming out from it and constitute the inputs for the nodes reaching V_B from below.

Notice that $F[i]$ also represents the firm value during bankruptcy and, therefore, during the grace period. If at the final node the firm is in the grace period, it will be forced to liquidate:

$$F[i] = (1 - \alpha)(V_T + \overline{FCF_T}).$$

Since debt is served strategically and its value is computed at the beginning of the grace period, it is not needed to keep track of the equity and debt value. Their values, in fact, are determined at the beginning of the grace period, in the previous nodes.

Recursive analysis

The next step is to determine the value of the claims by working backward. As in a normal option pricing model, the value of equity, i.e. the call option, is initially determined recursively as:

$$\tilde{E} = e^{-r\Delta t}(pE_u + (1-p)E_d)$$

Due to limited liability requirement and equity dilution assumption, the value is again compared with the due payments to ensure the firm does not need to liquidate its assets. This would entail immediate liquidation although the firm is healthy ($g = 0$ and $g - 1 = 0$):

$$\text{If } \tilde{E} + FCF_t \geq (1-\tau)C_t: \begin{cases} E = \tilde{E} + FCF_t - (1-\tau)C_t \\ D = C_t + e^{-r\Delta t}(pD_u + (1-p)D_d) \\ F = FCF_t + \tau C_t + e^{-r\Delta t}(pF_u + (1-p)F_d) \end{cases}$$

$$\text{If } (1-\tau)C_t > \tilde{E} + FCF_t: \begin{cases} E = 0 \\ D = (1-\alpha)(V_t + FCF_t) \\ F = (1-\alpha)(V_t + FCF_t) \end{cases}$$

The procedure, which follows exactly the immediate liquidation formula, is consistent until the firm reaches V_B , when shareholders are incentivised to file for bankruptcy. If this occurs, Chapter 11 ensures protection against enforcements and stops scheduled debt payments in favour of strategic debt service. Therefore, as the firm files for bankruptcy ($g = 1$), the values are:

$$\begin{cases} F[0] = FCF_t + e^{-r\Delta t}(pF_u + (1-p)F_d[1]) \\ E = \eta(F[0] - (1-\alpha)(V_t + FCF_t)) \\ D = (1-\eta)(F[0] - (1-\alpha)(V_t + FCF_t)) + (1-\alpha)(V_t + FCF_t) \end{cases}$$

Equity and debt follow Fan and Sundaresan (2000): they are the amounts served strategically depending on the bargaining power of equity holders η and are constructed as a sort of barrier option triggered by V_B , whereas the firm value is obtained as in the immediate liquidation case, noting that it coincides with the sum of equity and debt by construction. These values are not recomputed at any node where the firm is in the grace period. Indeed, they represent total values though they have no clear link with current payments at any time. Therefore, the choice of computing them as the result of a bargaining game is very convenient, because, as pointed out by Broadie and Kaya (2007), this only requires the firm value at the bankruptcy point.

If instead the firm reaches the bankruptcy boundary from below, the firm recovers before the end of the grace period by presenting a restructuring plan. The restructuring plan, again, is presented by the shareholders and therefore mostly benefits them. As a consequence, in order

not to need any specific information about it, the bargaining game's values for equity and debt are considered again. The main difference is that the firm value chosen as an input has to incorporate the distress suffered from filing for bankruptcy and being in the grace period:

$$\begin{cases} F[i] = \overline{FCF}_t + e^{-r\Delta t}(pF_u + (1-p)F_d[i+1]) \\ E = \eta(F[i] - (1-\alpha)(V_t + \overline{FCF}_t)) \\ D = (1-\eta)(F[i] - (1-\alpha)(V_t + \overline{FCF}_t)) + (1-\alpha)(V_t + \overline{FCF}_t) \end{cases}$$

If the firm is inside the grace period but before the expiration date, equity and debt values correspond to the values derived according to the bargaining game, because this allows to conveniently get a total value which does not consider cash flows at any time, which are not predictable. The only value which must be kept track of is the firm value, which evolves as an option obtained recursively:

$$F[i] = \overline{FCF}_t + e^{-r\Delta t}(pF_u[i+1] + (1-p)F_d[i+1]) \text{ for } i = 2, \dots, \bar{g} - 1$$

At the expiration of the grace period, the model by Francois and Morellec (2004) suggests that if the firm does not come out of bankruptcy and is not able to honour its debt obligations, then it will be liquidated:

$$\begin{cases} E = \max\{\widetilde{E}_t + \overline{FCF}_t - (1-\tau)C_t; 0\} \\ D = \min\{C_t + e^{-r\Delta t}(pD_u + (1-p)D_d); (1-\alpha)(V_t + \overline{FCF}_t)\} \\ F = \min\{\overline{FCF}_t + \tau C_t + e^{-r\Delta t}(pF_u[i+1] + (1-p)F_d[i+1]); (1-\alpha)(V_t + \overline{FCF}_t)\} \end{cases}$$

The bankruptcy boundary

The procedure proposed in all models to cope with the bankruptcy boundary, which represents the endogenous barrier to determine whether the firm files for Chapter 11 bankruptcy, is very similar. Being this value determined by shareholders, it is expected to maximise the value of equity and, due to limited liability, this does not coincide with the firm value maximisation, which would set V_B as low as possible, in order to delay bankruptcy as much as possible.

In the Leland (1994) model, equity maximisation is determined by the smooth-pasting condition and returns a bankruptcy boundary above which the equity value is always positive. Indeed, as noted above, the debtor had better be solvent until debt obligations fulfilment ensures a positive equity value, not a positive firm value. Otherwise, she is better off liquidating the firm, because she will get zero value anyway.

This reasoning does not hold when restructuring is taken into account: as explained by Broadie, Chernov and Sundaresan (2007), the firm may choose to default prior to the complete disruption of equity value: "When debtors are given the right to decide when to file for Chapter 11, they attempt to capture the rents associated with the additional option, such as debt forgiveness and suspension of contractual payments, by filing too early". This is because, upon Chapter 11 bankruptcy, equity holders expect no longer to get nothing, as in the immediate liquidation case

(in which they delay bankruptcy as much as possible because they get 0 for sure upon it), but instead they know that they can still get something before liquidation or even avoid it thank to their bargaining power, thus getting probably some more benefit than by not filing and being forced to liquidate immediately if the firm is not able to satisfy its debt obligations. In other words, Chapter 11 reduces the probability of liquidation or delays over time, in favour of equity holders.

The consequences of early default, however, mostly benefit equity holders. Broadie et al. (2007) argue that “early default leads to a decline in overall firm value relative to the first–best scenario in both the Leland model and our model. Debt forgiveness may also be in the interest of the lenders and the firm as a whole because costly liquidation could be avoided. Nonetheless, total firm value maximization generally requires filing for Chapter 11 later (relative to equity value maximization), because this extends the period of complete contractual payments by the debtors. The divergence between firm value maximization and equity value maximization raises some important issues that are absent in the benchmark model of Leland (1994) and, more generally, in the corporate debt literature”.

In the explanation of the restructuring model, the barrier was assumed exogenous. However, determining an endogenous barrier may be beneficial in terms of the following sensitivity analysis of the parameters. Since binomial trees do not appear much in corporate claims literature, it is complex to determine a discrete exogenous barrier. As a consequence, two main procedures were followed.

The first procedure refers to the choice of a continuous time barrier identified in the papers analysed above. Specifically, the determinants of the boundary in Leland and Toft (1996) were modified to consider the renegotiation value. As observed by Broadie et al. (2007), the probability of liquidation differs from the probability of bankruptcy. Although they can both be computed as the probability of hitting one absorbing barrier, the latter differs and can be either V_B or V_L . Adapting Leland and Toft (1996), the debt value is determined as:

$$d(V, t) = \int_0^t \frac{e^{-rs}C}{T} 1_{s < \tau_B} ds + (1 - \alpha)V_L e^{-r\tau_L} 1_{t > \tau_L} + \int_0^t (1 - \theta^*)V_B 1_{\tau_B < s < \tau_L} ds + \frac{P}{T} e^{-rt} 1_{t \leq \tau_B}$$

According to Leland and Toft (1996), furthermore, tax benefits and bankruptcy costs are still determined as in the infinite maturity framework, and summing equity is obtained as:

$$E = v(V) - D = V + ITS - BC - D$$

Maximising this value should lead to the second-best or equity maximising boundary in continuous time. The results in the analysed model were:

- $V_B=V_L=128.47$ million € according to the Leland (1994) infinite maturity analysis;
- $V_B=V_L=183.85$ million € according to the Leland and Toft (1996) immediate liquidation and finite maturity boundary;
- $V_B=205.55$ million € when following Francois and Morellec (2004) but including finite maturity;
- $V_B=208.28$ million € when relying on the formulas shown just above.

Especially this last model, however, includes many imprecisions due to the difficulties in defining the inputs and the behaviours of the claims, boiling down to a value which is probably hardly reliable as a point estimate.

In general, furthermore, a generalisation to the discrete time would be imprecise and may result inappropriate or even inconsistent for many reasons. First of all, this would assume that bankruptcy and liquidation triggers can be breached not only at prespecified time steps, but at any infinitesimal time interval, leading immediately to the flexibility option. The same reasoning also holds for the underlying value V , which, though following a diffusion process as in continuous time, is only valued at each node, and never at the time intervals between the nodes themselves. The options of this value, therefore, are determined discretely as well. Furthermore, the idea of applying a smooth-pasting condition is not usually applied for discrete time options, but instead constitutes a continuous time models' feature. Therefore, extending its applicability can be in contrast with the theory.

In order to overcome these issues, the continuous-time resulting boundary is only used as a benchmark to observe the behaviour of the boundary by changing the parameters and therefore analyse the combined effect of changing the boundary on the value of the claims and on the timing of default, but is not directly applied to the model.

Instead, Broadie and Kaya (2007) proposed to apply a numerical optimization of equity (and possibly of other claims). They first identified a minimum bankruptcy boundary in the model by Leland (1994) since, as mentioned above, all bankruptcy threshold with restructuring will occur earlier than this, and will therefore be higher (because the boundary, of course, is breached from above). Then they obtained the optimal V_B by increasing the boundary and repricing equity and thereafter extrapolating the equity maximising boundary from the curve approximation.

A similar procedure was followed in this analysis and the results are in line with those described by Broadie et al. (2007), as shown in Exhibit 4.7. The points at which the values change correspond to the values of V at each node, and the minimum was set at $V_B=128.47$, according

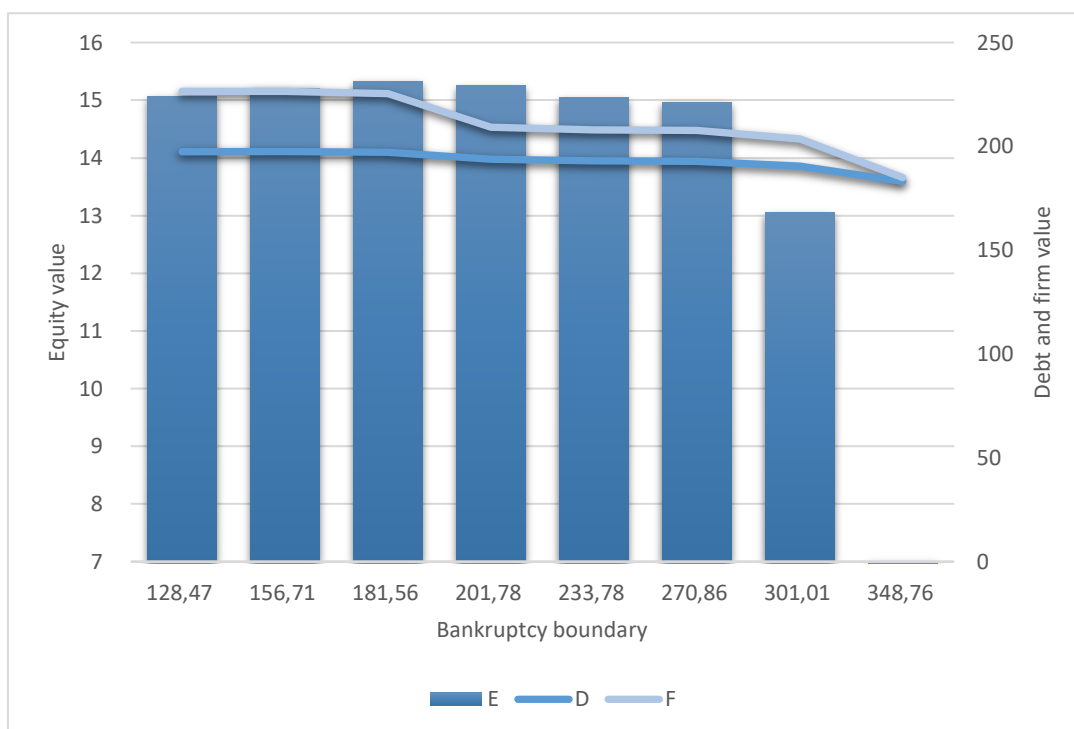


Exhibit 4.7. The effect of V_B on equity, debt and firm value.

to Leland (1994). Then, each point in the horizontal axis corresponds to the minimum V_B that triggers the represented values of equity, debt and firm value.

It is noticeable that upon Chapter 11 the bankruptcy boundary which maximises the equity value is $181.56 \leq V_B < 201.78$, whereas debt and firm value maximisation would imply a lower V_B ($V_B < 181.56$). It turns out that the first boundary is breached before, so that equity holders are incentivised to default earlier than it would be optimal for creditors and for overall welfare. Indeed, by filing for bankruptcy before, equity holders can exploit the probability to obtain something more, whereas debt holders stop receiving the contractual payments and are therefore damaged by bankruptcy. A reduction of cash flows coming from bankruptcy costs continuously arising also decreases the firm value.

Moreover, restructuring might also be expected to increase the value of all claims with respect to immediate liquidation. Before proceeding with the discussion, it is interesting to notice that the immediate liquidation values coincide with those which would be triggered upon restructuring by choosing the lowest possible V_B . Indeed, if equity holders do not choose to file for bankruptcy before equity value reaches zero level, then they will not be advantaged by restructuring. Moreover, this is also in line with the Leland (1994) and Leland and Toft (1996) formulas.

At time T:

V	V _B	V+FCF	V _L	E	D	F[0]	F[i]	g
404,07	200,00	425,97	360,50	65,47	364,00	FALSO	429,47	0,00
270,86	200,00	285,54	360,50	0,00	199,88	FALSO	199,88	0,00
270,86	200,00	285,54	360,50	0,00	199,88	FALSO	199,88	0,00
270,86	200,00	280,70	360,50	0,00	196,49	FALSO	196,49	0,00
181,56	200,00	190,40	360,50	0,00	133,28	133,28	133,28	1,00
181,56	200,00	190,40	360,50	0,00	133,28	133,28	133,28	1,00
181,56	200,00	190,40	360,50	0,00	133,28	FALSO	133,28	2,00
121,71	200,00	127,63	360,50	0,00	89,34	FALSO	89,34	2,00

At t=2

V	V _B	\tilde{E} +FCF	V _L	E	D	F[0]	F[i]	g
348,76	200,00	52,31	10,50	41,81	291,71	FALSO	333,52	0,00
233,78	200,00	12,67	10,50	2,17	177,32	FALSO	179,49	0,00
233,78	200,00	12,67	10,50	2,17	175,59	FALSO	177,76	0,00
156,71	200,00	7,63	10,50	1,18	116,43	117,61	#N/D	1,00

At t=1

V	V _B	\tilde{E} +FCF	V _L	E	D	F[0]	F[i]	g
301,01	200,00	38,65	10,50	28,15	244,45	FALSO	272,60	0,00
201,78	200,00	12,59	10,50	2,09	157,18	FALSO	187,83	0,00

15,32	197,06	225,53
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Exhibit 4.8. The value of claims upon restructuring.

Instead, looking at the claims/options values triggered by the equity maximising V_B, described in Exhibit 4.8, E=15.32, which is effectively higher than in the immediate liquidation case, where E=15.07. On the other hand, debt value (D=197.06) is now lower than in the previous model (D=197.4). Again, when the decision to file for bankruptcy is determined by shareholders, debt holders stop receiving their contractual payments, cannot enforce liquidation for the whole grace period and will only get some debt service determined strategically. Furthermore, they eventually get a liquidation value at the end of the grace period, which may even be lower than the immediate liquidation case due to bankruptcy costs and to the probability that the firm value declines even more.

Comparative statics of Debt and Equity upon Restructuring

Once the behaviour of the bankruptcy boundary is determined, it is also possible to test how other parameters may affect the value of equity and debt.

The first parameter to be analysed is again the riskiness of the firm, represented in Exhibit 4.9. In this model, it can be observed that an increase in volatility has both an effect on the optimal boundary and on the claims. As observed by Paseka (2004), higher volatility reduces the optimal default boundary and incentivises equity holders to default later, which corresponds to points where the firm is less solvent. This is also confirmed in the model in discrete time, where

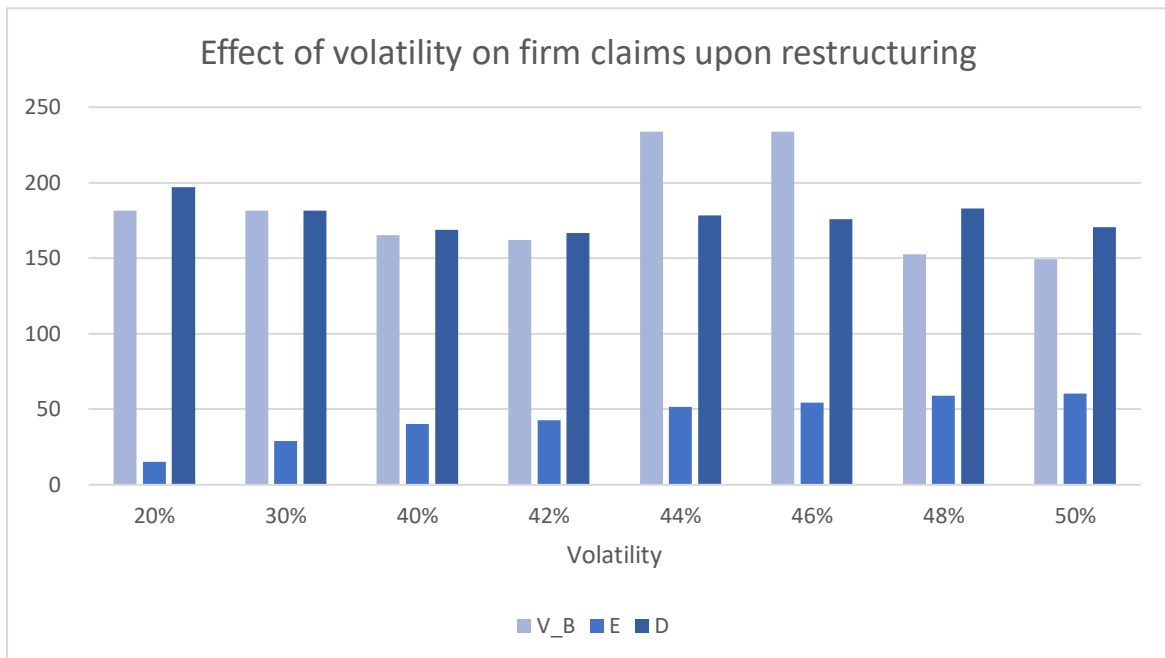


Exhibit 4.9. The effect of volatility on V_B , E and D upon restructuring.

the minimum point of the optimal bankruptcy interval progressively drops as volatility raises. This is because higher firm value uncertainty allows the debtor to exploit limited liability of equity, which imposes a lower but not an upper boundary, and to act strategically at odds of debt holders, who suffer from asymmetries of information and cannot impose any decisions. However, this results both in lower chances that bankruptcy ends up in reorganization and, in case this does not occur, in a lower firm liquidation value. Equity holders benefit from the higher firm values associated with renegotiation but are not damaged by lower liquidation levels, since the value they obtain is at least 0. On the contrary, higher volatility mostly comes at the expenses of debt holders, who suffer both effects. Consequently, equity value increases as volatility increases, whereas debt value tends to decrease. Interestingly, however, the behaviour of debt is similar to the immediate liquidation case. As volatility reaches the boundary mentioned before ($\sigma=43.2\%$), the debt value starts to increase for the same interval as in the model mentioned above, to finally decrease again as V “gets further” from V_B . This feature was pointed out by Leland (1994) for long-term debt and subsequently confirmed by Leland and Toft (1996) for finite maturity debt. However, the explanation given in these models referred to coinciding bankruptcy and liquidation boundaries: according to these works, volatility reduces the optimal liquidation boundary, ensuring more room before bankruptcy, which is especially beneficial for debt holders when they own junk bonds because, in presence of liquidation costs, delaying bankruptcy can also delay these costs. If Chapter 11 is included in the model, however, liquidation costs are no more delayed by decreasing the boundary which leads to immediate liquidation. On the contrary, liquidation costs will become less imminent by filing earlier for bankruptcy, i.e. when equity holders are more solvent, for a higher

boundary. Therefore, especially when V is close to V_L and therefore liquidation constitutes a serious threat, filing before for bankruptcy decreases consistently the probability of liquidation. If this happens, the positive effect of more rents for equity holders dominates on the advantages of behaving opportunistically because this decreases the chance to get the lowest possible level zero, which is a very likely scenario in this framework. Furthermore, debt will benefit from the decreased probability of getting liquidation value only and will consequently raise.

Francois and Morellec (2004) also predicted the effect of the debtor's bargaining power η , which determines the optimal sharing rule. They argued η does not follow a univocal trend, due to the presence of two contrasting effects: on one hand, a higher η triggers a decrease in the optimal V_B , which benefits debt holders; on the other hand, it increases the cost of debt and makes it less attractive. These two effects offset with each other, leaving the final debt value roughly unchanged.

The discrete time model, however, does not explicitly compute an optimal boundary, and furthermore it does not consider some of the main features of the theoretical model by Francois and Morellec (2004), such as the assumption of being at the optimal leverage and optimal coupon. Therefore, it was not straightforward to observe the first effect. On the contrary, by moving the assumed η and determining the optimal V_B interval as that interval where equity is maximised, V_B seemed even to slightly increase, in contrast with the predicted result. This may be because the theoretical V_B obtained in continuous time depends itself by η , while obtaining it as the exogenous value which maximises equity may be misleading. However, the formula defined by Francois and Morellec (2004) for V_B determines a value which is clearly increasing in η . As they notice themselves, the behaviour they describe may also be due to their change of the parameters.

On the contrary, the second relationship between η and D , which can be defined as the fact that higher shareholders' bargaining power decreases the value of debt, is clear by fixing exogenously the boundary. Indeed, although the total firm value remains the same, the distribution between equity and debt changes: as expected, higher bargaining power benefits shareholders at odds of debt holders.

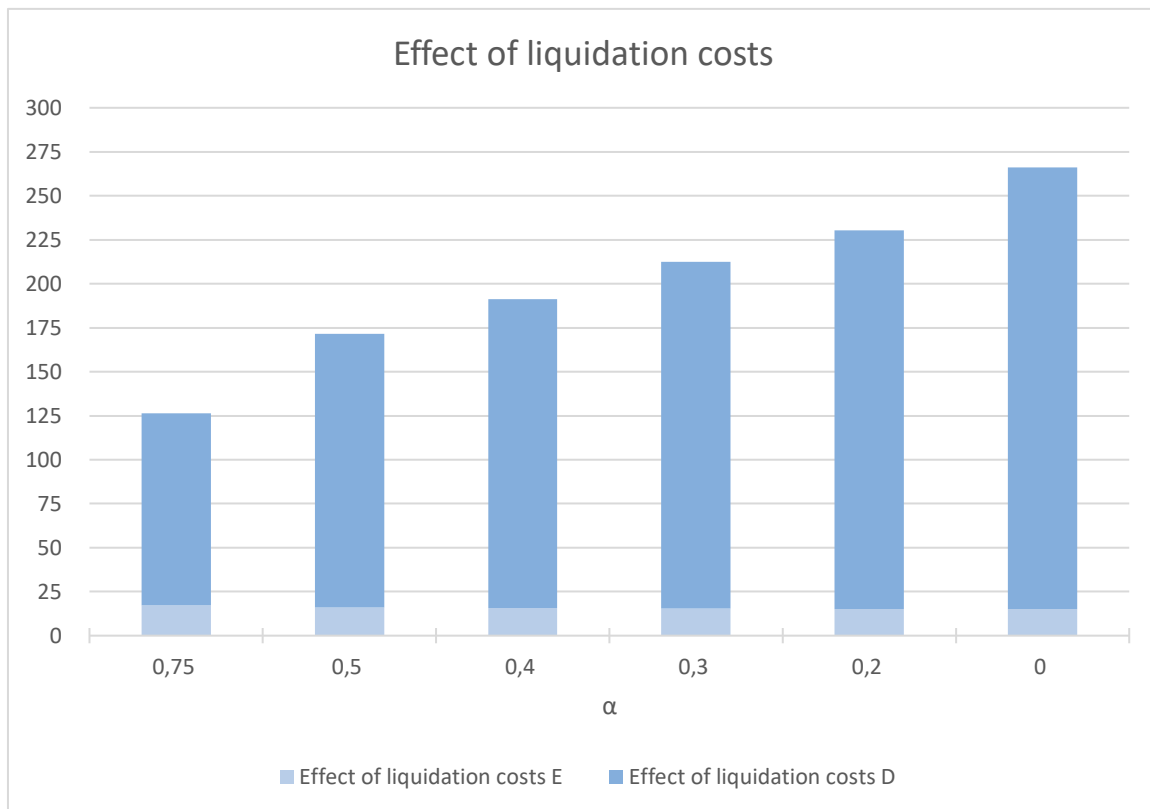


Exhibit 4.10. The effect of liquidation on firm claims.

As shown in Exhibit 4.10, higher liquidation costs α at the optimal boundary increase the equity value but decrease more consistently debt value, leading to a drop in the firm value. Indeed, higher liquidation costs lead to a decline in the liquidation allocation for debt holders. The value they lose is lost for the total welfare. However, anticipating this, equity holders are incentivised to default earlier, as proved in the model, where higher α implies higher V_B . This leaves more room for restructuring and benefits equity holders, differently from the immediate liquidation case, where equity value is not affected by these costs. On the contrary, the effect is not univocal for lenders. Specifically, if these costs are predominant to the restructuring probability, the final debt value decreases. Paseka (2004) also specifies that debt value may increase if higher α decreases the length of the grace period. Being the discrete time model too short to explicitly observe a claim value trend on g , this parameter is not studied.

As expected, bankruptcy costs ω lead to dropping equity, debt and firm values. Furthermore, since they arise only when the firm files for bankruptcy and they affect the debtor's value, they are expected to lead to a lower optimal boundary and therefore to late bankruptcy. This both prevents them from arising but increases the liquidation probability. This explains why no benefit is expected from them.

5. Bankruptcy with grace period, automatic stay and arrears account

As explained above, during bankruptcy the firm is not forced to fulfil its debt obligations and debt holders are prevented from liquidating the firm. Until debt restructuring or expiration of the grace period, which may also coincide with the firm reaching the final node in a finite maturity framework, the debt obligation does not need to be paid in full and no legal action can be undertaken by bond holders. However, the previous model, although allowing for a grace period, simply divided equity and debt strategically following Fan and Sundaresan (2000), but did not account for the cash flows at each time step. This approach is far more convenient because it avoids explicit cash flows estimation at any node, but is not realistic in practice. Therefore, Broadie, Chernov and Sundaresan (2007) aim at developing a more realistic model, which introduces two accounts which accumulate cash flows when payments are stopped during the grace period.

A_t is the arrears account, which accumulates all the coupon payments which are not made after filing for bankruptcy. S_t is instead a separate account where all cash flows of the company are accumulated instead of being paid out for reimbursement or distribution of dividends. Since they are retained, at each time in the grace period this is obtained by summing up the cash flows attained after the declaration of bankruptcy, which also include continuously accruing bankruptcy costs.

Before proceeding, it is worth noticing that Broadie and Kaya (2007), in the adoption of the model by Broadie, Chernov and Sundaresan (2007), substitute the primitive variable for the sake of coherence. Indeed, the continuous time model by Broadie, Chernov and Sundaresan (2007) adopts $EBIT_t$ as primitive variable, as it can be considered as an approximation of operating cash flows, and assumes it follows a diffusion process. However, this implies⁸⁸ that also V_t follows a diffusion process if q is constant, and therefore the unlevered value V_t based model used so far still holds. The main difference relates to the treatment of taxes, which is not so relevant since they are not considered upon bankruptcy.

The determination of the arrears account A and of the automatic stay payoffs S rely upon the continuous time models suggested by Broadie et al. (2007), although the authors attempted at discretising them.

To keep track of the automatic stay payoffs S , the starting point is the maximum value of S at each time:

⁸⁸ Broadie, Chernov and Sundaresan (2007), page 1344.

$$\bar{S} = V_B(e^{q\Delta t} - 1)G.$$

G denotes the maximum grace period granted by the court, which is set at G=3 in the model in discrete time, in line with the number of nodes. Afterwards, it is possible to determine the value of S at each time of the grace period by computing the values in a discretized grid and then using linear interpolation. By using M=10 values in the discretized grid, where M can be chosen exogenously since it does not affect much the value of the claims, $S_j = j\bar{S}/M$, where $j=[0, 1, \dots, M]$. For any node, both g and S should be assigned any value from 1 to G and from S_0 to S_M respectively, and then equity and debt should be computed for each of them by using linear interpolation. However, for the purposes of this analysis, only $g=i$, S_j and S_{j+1} were computed when needed to proceed with linear interpolation. Then, it is easy to proceed as in Broadie and Kaya (2007) for the computation of S_u , S_d and the interpolated values for E and D. However, this is only possible some steps before maturity. Broadie and Kaya (2007) give no indication on how to deal with finite maturity debt, as it is in the case on hand. Thus, it was necessary to adapt the previous models to the accumulated FCF and to the arrears account.

If $g = \bar{g} \vee 1 < g \leq \bar{g}$:

$$\text{If } V_T + S_j \geq A_j + P: \begin{cases} E = V_T + S_j - P - A_j \\ D = A_j + P \end{cases}$$

$$\text{If } A_j + P > V_T + S_j: \begin{cases} E = 0 \\ D = (1 - \alpha)(V_T + S_j) \end{cases}$$

If $g=0$ (and no bankruptcy has occurred in previous steps) or $g=1$ (since no arrears and separate accounts have accrued yet), the values are the ones which would be computed with no chance for restructuring and immediate liquidation.

From $T - 1$, it is thus possible to compute up and down moves for S by compounding the current value of S and adding FCF of next time step according to that up or down move:

$$S_u = S_j e^{r\Delta t} + FCF_u$$

$$S_d = S_j e^{r\Delta t} + FCF_d.$$

After this, interpolation is finally easy to derive by associating respectively S_j with $E_u[i+1; j]$, S_{j+1} with $E_u[i+1; j+1]$, and S_u with $\tilde{E}_u[i+1; j]$, and the same can be done for E_d and the values of D_s and D_j .

$$\tilde{E}_u[i+1; j] = E_u[i+1; j] + \frac{S_u - S_j}{S_{j+1} - S_j} (E_u[i+1; j+1] - E_u[i+1; j])$$

$$\tilde{E}_d[i+1; j] = E_d[i+1; j] + \frac{S_d - S_j}{S_{j+1} - S_j} (E_d[i+1; j+1] - E_d[i+1; j])$$

Next, the present value of equity (or debt with the same procedure) without anything happening at the current node can be computed as the present value of an option obtained by working backward:

$$\tilde{E}[i; j] = e^{-r\Delta t} (p\tilde{E}_u[i + 1; j] + (1 - p)\tilde{E}_d[i + 1; j]).$$

$\tilde{E}[i; j]$ resembles \tilde{E} obtained for the bankruptcy models which do not consider explicitly the accounts which are opened upon Chapter 11 restructuring. However, in this case, it is still incomplete, not because it ignores the current coupon payment, which is suspended, but because of the risk of liquidation, which may arise because of the continuously accruing distress costs ω , which at any time step result in the following amount:

$$BC_t = V_t(e^{\omega\Delta t} - 1) \approx \omega V_t \Delta t$$

As a result, equity dilution may be necessary despite the stopping of owed payments if shareholders must face bankruptcy costs. Following the same rationale applied for liquidation risk arising from coupon payments, the issue related to bankruptcy costs can also be incorporated:

$$\begin{aligned} \text{If } \tilde{E}[i; j] \geq \omega V_t \Delta t: & \begin{cases} E[i; j] = \tilde{E}[i; j] - \omega V_t \Delta t \\ D[i; j] = e^{-r\Delta t} (p\tilde{D}_u[i + 1; j] + (1 - p)\tilde{D}_d[i + 1; j]) \end{cases} \\ \text{If } \tilde{E}[i; j] < \omega V_t \Delta t: & \begin{cases} E[i; j] = 0 \\ D[i; j] = (1 - \alpha)(V_t + S_j) \end{cases} \end{aligned}$$

Of course, this procedure can only be applied once the firm has filed for bankruptcy, when both i and j become positive values. If the firm is in a healthy state, as suggested before, the model is identical to the case of immediate liquidation, where liquidation occurs only if equity and available cash flows do not suffice for debt payments. If the firm is instead in the transition state, i.e. it is either filing for or recovering from bankruptcy, then the computation of the present value of \tilde{E} is based on E_u , in which the firm is not bankrupt and which is therefore computed as if the firm were healthy, and $\tilde{E}_d[1; 0]$, which is the first unhealthy state obtained through interpolation.

If the firm comes from a healthy state, which means $g_t = 1$ and $g_{t-1} = 0$, then there is no arrears account and no cumulated payoffs account. The value of equity and debts are therefore computed as if the firm was in the liquidation state, but the present value of debt uses the healthy state value as an input for the risk-neutral upward step and the unhealthy state for the downward one.

If instead the firm comes out of bankruptcy, then the arrears shall be cleared and used for the previously stopped payments in case they are sufficiently high, otherwise the firm will be liquidated (because $V < V_B$, as before, does not imply liquidity is sufficient to immediately repay). As a consequence:

$$\begin{aligned}
\text{If } \tilde{E} + S_j \geq A_i + P_i: & \begin{cases} E = \tilde{E} + S_j - P_i - A_i \\ D = A_i + P_i + e^{-r\Delta t}(pD_u + (1-p)\tilde{D}_d[1;0]) \end{cases} \\
\text{If } A_i + P_i > \tilde{E} + S_j: & \begin{cases} E = 0 \\ D_s = (1-\alpha)(V_t + S_j) \end{cases}
\end{aligned}$$

By proceeding recursively, the values obtained upon maximisation of equity with limited liability were E=20.05, D=187.23, F=207.29.

Before proceeding with the analysis, it must be observed that the model is aimed at being as realistic as possible. Although it starts from a double-barrier option, where the barriers should be endogenous, at least in theory, and although it obtains the final value of the option and afterwards proceeds recursively and add the specificities of the model to get the result, the model is not completely credible from a theoretical perspective. Indeed, it assumes too many conditions and results in an artificial and in some cases unreliable option modelling. A clear example to this is the presence of linear interpolation, which can be hardly implemented in practice, or the artificial way used to determine the automatic stay account, led by the difficulty to estimate cash flows upon restructuring. Furthermore, empirical results are somehow driven by simplifications and cause deviations from the correct point estimates. This can be observed when trying to determine the trend of claims' values. The analysis of the model is yet implemented, but of course its limits deserve to be reminded as well.

The bankruptcy boundary

On the whole, empirical evidence shows to confirm both theoretical results and empirical observations of the previous models, in spite of some specificities.

First, when the bankruptcy boundary is chosen by the debtor, it does not correspond to the first-best outcome, which might be obtained by optimizing the firm value. Specifically, debtors tend to choose to default earlier (which implies a higher V_B) than it would be optimal for the firm, in order to exploit the rents against debt holders and all the benefits accruing from restructuring. The probability of liquidation, which mostly affects shareholders, in fact, wanes by increasing the boundary. That is why they are incentivised to choose early default. This is also confirmed in this model, where explicit modelling of specific accounts is done, avoiding the need for the simple but unrealistic Fan and Sundaresan (2000) study.

As shown in Exhibit 4.11, the optimal V_B chosen by equity holders is at $V_B = 232$. This value is obtained again by setting $V_{B \min}=183.85$ (Leland and Toft, 1996 boundary) and by working similarly to the continuous repricing of equity described above and suggested by Brodie and Kaya (2007). However, the boundary so obtained is not in line with the first-best outcome, which would imply a lower boundary, just higher than $V_{B \min}$. Again, filing too early reduces

the period in which debt payments are made in full and damages debt holders. Indeed, the optimal boundary which they would choose restores or comes closer to the first-best outcome. Theoretical works argue that this issue exacerbates the longer the grace period. Although a longer g delays liquidation and therefore reduces the probability to incur in liquidation costs, thus increasing equity, debt and firm values, it is also usually associated to a higher optimal V_B for shareholders. Indeed, they may extract more surplus from the suspension of the contractual payments, increase the probability of successful restructuring and reduce the chances to have to liquidate. The empirical analysis of this result is very limited in this case, due to the choice of $T=3$, which allowed the grace period to be chosen only between $g=2$ and $g=3$. Nevertheless, the results seem to confirm the theoretical predictions: at $g=3$, $V_B=300$, $E=20.43$ and $D=193.86$. Though the claims' values are higher than in the case of $g=2$ specified above, second-best debt value is lower than the maximum one, which can also result by firm value maximisation ($D_{\text{first-best}}=196.31$).

Broadie et al. (2007), following this result, proposed to let debt holders choose the grace period as a measure to limit shareholders' opportunistic behaviour: they argue that, reducing the grace period, equity holders are willing to reduce the optimal boundary, leading to a situation which is closer to the first-best. This is yet true also in this model, but it must be pointed out that it would not be convenient for debt holders to choose $g=2$, since the reaction of shareholders will be to reduce the boundary at their second-best optimal one, but this would lead to both a lower debt and firm value. As a consequence, debt holders will choose $g=3$ and the result will be far from the first-best. The attempt to restore the first-best by letting the creditor choose the optimal grace period fails. Indeed, also theoretical results suggest that this choice reduces total value

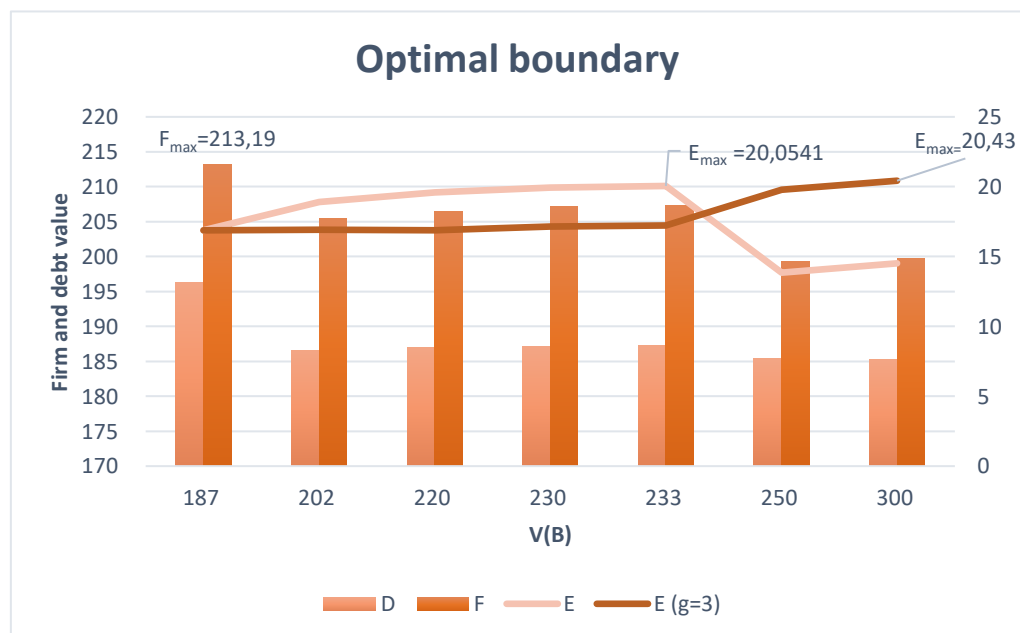


Exhibit 4.11. First and second best optimal boundaries

and it would be more convenient if creditors took the reins of the bankrupt firm or decided directly whether and when to force bankruptcy.

Comparative statics of Debt and Equity in presence of arrears account and automatic stay

A point which deserves to be pointed out refers to the bankruptcy costs. Broadie et al. (2007) observed that zero distress cost and high debt forgiveness lead to early default in continuous time, because they both reduce the probability of final liquidation and raise the value available to shareholders. According to their findings, this is especially true for longer g , which further wanes liquidation.

The effect of debt forgiveness cannot be studied in the discrete time model. In fact, Broadie, Chernov and Sundaresan (2007) suggested that this leads to a reduction of the due arrears during and at the end of the grace period. On the contrary, the discrete time model does not propose a similar reduction and assumes all the bargaining power belongs to the debt holders. However, the results should be similar to the model explained in Section 4, where debt forgiveness was modelled as a consequence of the bargaining power of shareholders η . Higher η increased the equity value, thus leading to higher incentive to default earlier.

Bankruptcy costs seem to confirm these observations. Although the effect is taken to the extremes by setting unlikely high ω , it can be observed that, especially if $g=3$, the optimal boundary shows a downward trend with respect to bankruptcy costs. Furthermore, this leads to a decrease of all claims values. Indeed, higher bankruptcy costs lead to a continuously accruing loss in the grace period and therefore trigger delayed bankruptcy. As this happens, however, the value available to all claim holders is reduced by the reduction of available cash flows. Again, a longer grace period leads to early default, but high bankruptcy costs also imply an overall decline of the claims values. Therefore, if on one hand it is true that zero or sufficiently low distress costs are mostly beneficial for longer grace periods, therefore implying higher equity and debt values, on the other hand if they are too high a longer grace period would go in disfavour of the claims' values: in fact, the loss owed to these costs may dominate the positive effect triggered by the prolongment of the grace period, which is the decrease of the probability of liquidation and therefore to incur in its costs. Summarising, low distress costs result in higher optimal boundary and equity and debt values for a longer grace period, whereas a sufficient increase in these costs leads to the opposite effect: they reduce the outcomes more for $g=3$, such that shorter grace periods may even become more beneficial than longer ones. These findings are shown in the following table.

Ω	G=2			G=3		
	V_B	E	D	V_B	E	D
0,05	230	20,05	187,23	300	20,43	193,86
0,02	230	19,81	186,94	300	19,77	193,20
0,04	230	19,63	186,64	300	18,93	192,34
0,06	230	19,54	186,35	250	18,23	191,50
0,08	230	19,54	186,07	250	17,70	190,67
0,1	230	19,54	185,79	250	17,17	189,86
0,2	230	19,54	184,48	230	16,74	182,71

The final point which deserves to be mentioned is the effect of liquidation costs, shown in Exhibit 4.12. As observed so far, avoiding liquidation is the main aim which is implicitly pursued when equity holders increase the optimal boundary, because liquidation enforces them to give up the rents associated with restructuring. However, it is not straightforward that an increase in liquidation costs leads to a decrease in equity value, since who suffers from liquidation costs are debt holders, at least at first glance. Evidence from the previous models showed two contrasting results: upon immediate liquidation, α only affects debt holders, while the chance of restructuring leads to consequences of liquidation costs for shareholders as well. Also in this framework, higher liquidation costs affect both debt and equity value, though, naturally, debt holders are much more affected in terms of their value. In the case on hand, no clear evidence of changes in the optimal boundary was observed, and, as a consequence, equity showed to be slightly decreasing rather than increasing, in contrast with the restructuring model handled before.

An explanation to this is therefore not univocal. The slight decrease of optimal equity value may lead discrete-time boundary intervals not to change, while a more precise point estimation, only feasible in continuous time, may lead to a result similar to the one described above.

On the other hand, Paseka (2004) expected that increasing liquidations costs trigger a decrease in the optimal grace period, which should turn to a raise in the debt. The relationship between liquidation costs and the choice of the grace period is hard to test and describe, because the latter is chosen exogenously. On one hand, a longer grace period is expected to anticipate bankruptcy, making liquidation less likely and increasing equity value. This is proved empirically only when liquidation costs are sufficiently low: in this case, the positive effect of a longer period of debt extraction dominates on the increase in the costs of the process. If these

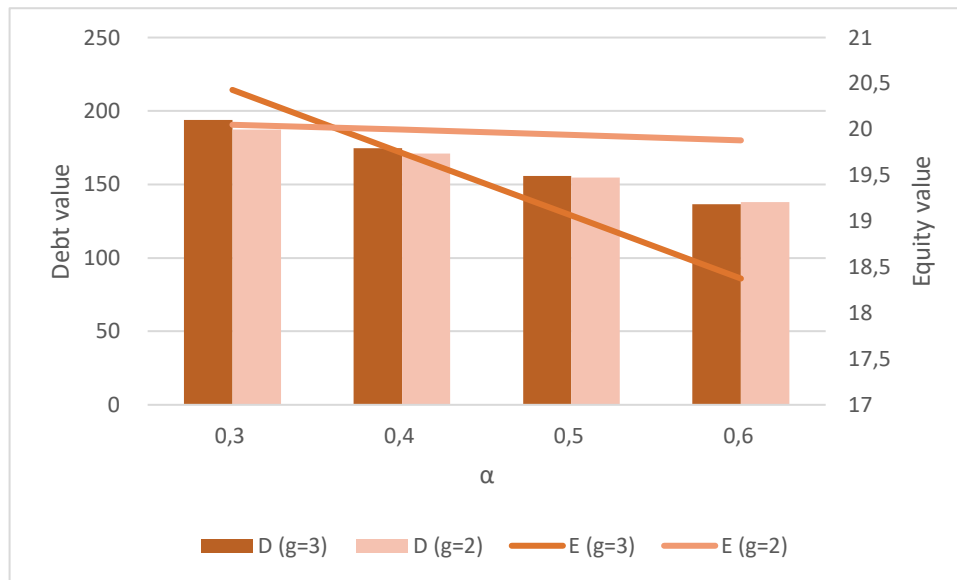


Exhibit 4.12. Effect of liquidation costs on debt (columns, primary vertical axis) and equity (lines, secondary vertical axis) depending on the grace period chosen.

costs are too high, instead, the loss seems to be dominant on the anticipation of the decision to file for bankruptcy. On the other hand, debt value is more volatile for longer g . Indeed, if liquidation costs are not very high, a longer grace period may benefit debt holders by decreasing the likelihood of liquidation. If, instead, these costs are high, they dominate on the lower probability of coping with them and the resulting outcome is that a shorter grace period increases debt value. Empirical evidence confirms what expected: for low liquidation costs, a longer grace period is beneficial to debt holders, whereas high liquidation costs dominate on the benefits of a longer grace period, which therefore damages debt holders.

6. Introducing changing Free Cash Flows over time

One of the main benefits of relying on a binomial model for a stock paying a dividend which accrues continuously but is paid out discretely is the possibility to assume several known payout yields rather than simply considering the average over time, $\delta=5\%$. Although it was chosen to follow strictly the model by Broadie and Kaya (2007), it is more credible to assume that the firm's free cash flows raise over time, since the key value drivers showed a recover in years 2 and 3. Exhibit 4.13 shows how the free cash flows yields were derived and the consequences on the modelling of the tree.

On the whole, increasing cash flows over time leads to an overall decrease of the claims' value. This happens because at time 1 δ is so low that cash flows are not sufficient for the firm to meet its debt obligations. Consequently, the firm files for bankruptcy more often and before.

	0	1	2	3
FCF		5	15	28
EV+FCF	260	287	312	328
FCF/(EV+FCF)		1,8%	4,8%	8,4%
EV	260	317	381	442
FCF		6	19	39
EV-FCF		312	362	403
		213	255	296
		4	13	26
		209	242	270
			171	199
			8	17
			163	181
				133
				12
				121

Exhibit 4.13. The effect of changing free cash flows on the assets value binomial tree

With respect to constant free cash flows, which result in $E=15.07$, $D=197.45$ and $F=212.53$, the values in the changing FCF yields are $E=11.04$, $D=195.66$ and $F=206.69$. Furthermore, the main results seem to confirm the effects of the main parameters on the claims' values, as described in Section 3.

A similar result also holds when considering restructuring according to Francois and Morellec (2004). With respect to the equity maximising values upon constant δ ($181.56 \leq V_B < 201.78$, $E = 15.32$, $D = 197.07$, $F = 225.52$), the optimal bankruptcy boundary is now $V_B < 242.50$, but equity, debt and firm values are now lower and they correspond to $E=11.04$, $D=194.95$ and $F=205.87$. It must be highlighted that equity holders choose to default early in this case as well, but the higher probability to liquidate reduces the rents they can extract during the grace period. Consequently, the equity value corresponds to the optimal value upon liquidation. On the other hand, debt holders are damaged by anticipated bankruptcy. If the optimal bankruptcy boundary was chosen to maximise the overall firm value, indeed, they would obtain $D=195.03$. This would occur by filing for bankruptcy at a lower boundary, $V_B < 181.09$, which maximises both debt and firm value. At this level, equity is still equal to the immediate liquidation case, which was shown to be value maximising also in the constant- δ model. However, in this case, $E=11.04$ also coincides with the second-best outcome. Therefore, debt and equity holders may agree to delay bankruptcy, and equity holders will obtain the same amount, whereas debt holders may benefit from a more prolonged period in which debt obligations are met, thus increasing debt value.

However, this result may also be due to simplifications assumed in the model by Francois and Morellec (2004). Indeed, benefits exploited by equity holders during the grace period are only modelled as a strategic behaviour in that model. Moreover, they are associated more to the bargaining power η rather than to a complete model of rents extraction.

On the contrary, modelling explicitly not only the grace period, but also arrears account and automatic stay, preserves the different results obtained from the first and second-best optimal boundaries. For $g=2$, the optimal boundary for shareholders is $V_B=242.5$, leading to $E=13.81$ and $D=190.66$. The equity value is now higher than both equity upon immediate liquidation $E_{\text{immediate liquidation}}=11.04$ and the first-best equity outcome upon reorganization (which implies a lower V_B) $E_{\text{first-best}}=11.45$. The contrary holds for debt which is lower than the Leland case, where $D_{\text{immediate liquidation}}=194.95$, and than $D_{\text{first-best}}=195.03$. These values are also lower than the constant- δ correspondents. Furthermore, if $g=3$ was assumed, higher chances to extract rents and lower probability of liquidation lead to anticipated default, higher equity and debt values, when the base-case scenario costs $\alpha=30\%$ and $\omega=0.5\%$ are chosen as inputs. This is in line with empirical evidence of the constant payout yields as well.

A broader analysis of the model allows to confirm what was shown in the restructuring case. In particular, the effect of liquidation costs is shown in Exhibit 4.14, which mirrors Exhibit 4.12. The effect on debt is similar in the two models: for low liquidation costs, a longer grace period is beneficial to debt holders, as it would be expected. Instead, high liquidation costs dominate on the benefits of a longer grace period, thus reducing debt value. On the other hand, no reversal is shown for equity. For any liquidation costs observed, a longer grace period is beneficial. This may be explained as follows: lower cash flows may lead to anticipate bankruptcy and, if a longer grace period can be chosen, the rents extraction may dominate on

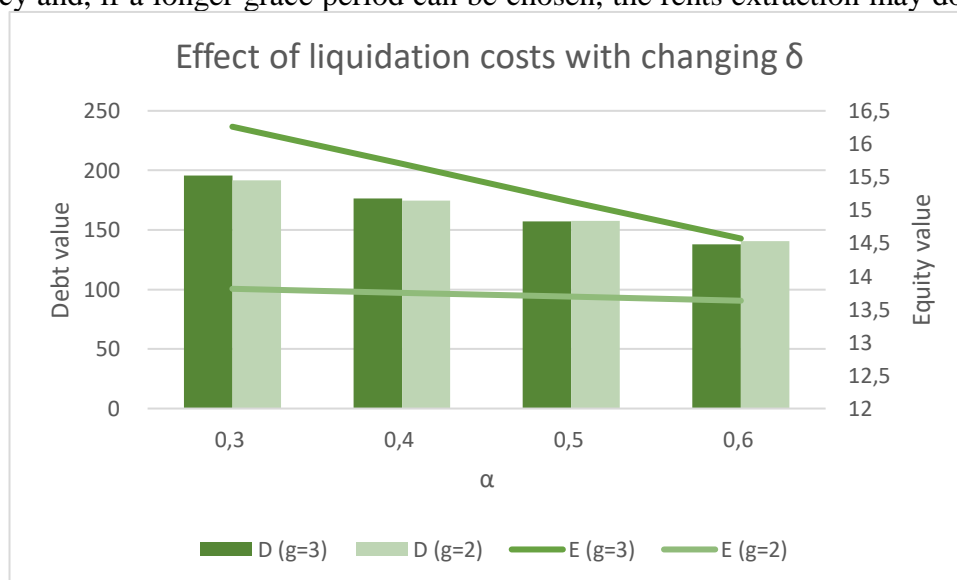


Exhibit 4.14. Effect of liquidation costs upon restructuring, with different grace periods g , and with changing payout yields over time.

the risk of liquidation even for higher liquidation costs. The lines shown in the histogram, however, are not tendency lines, but connect the exact point estimates. If liquidation costs were to be increased even more, the two lines would probably intersect and, even if for a higher level of α , the reversal of behaviour would occur as well. As a result, too high liquidation costs may dominate on the rents extraction and make it optimal to reduce g for equity holders too.

The effect of increasing bankruptcy costs is mostly interesting when comparing it to the effect of liquidation costs. Liquidation costs showed an analogous behaviour of debt value for constant and changing δ , while the effect on equity was delayed: the fear for liquidation seems to be delayed through a longer grace period. Distress costs, instead, are mainly suffered by equity holders. Therefore, higher costs will first affect them: though their increase makes both claims' values drop for $g=2$ and $g=3$, the decrease in equity is much steeper for a longer grace period, such that low ω leads to a higher equity value for a longer grace period, but then a reverse behaviour occurs, and for high ω equity holders will prefer to decrease the optimal grace period and, if the grace period is given, delay bankruptcy as much as possible.

On the other hand, no reversal of behaviour is shown for debt in this case, since bankruptcy costs mainly affect equity holders. This holds especially when cash flows are low and floating, since equity represents the residual claim.

Finally, volatility follows the same behaviour which was observed in the previous models, described in Exhibit 4.15. Debt is decreasing in volatility, but as V approaches V_B , where it is increasing. Also in this case, the optimal boundary does not reflect the behaviour theorized by

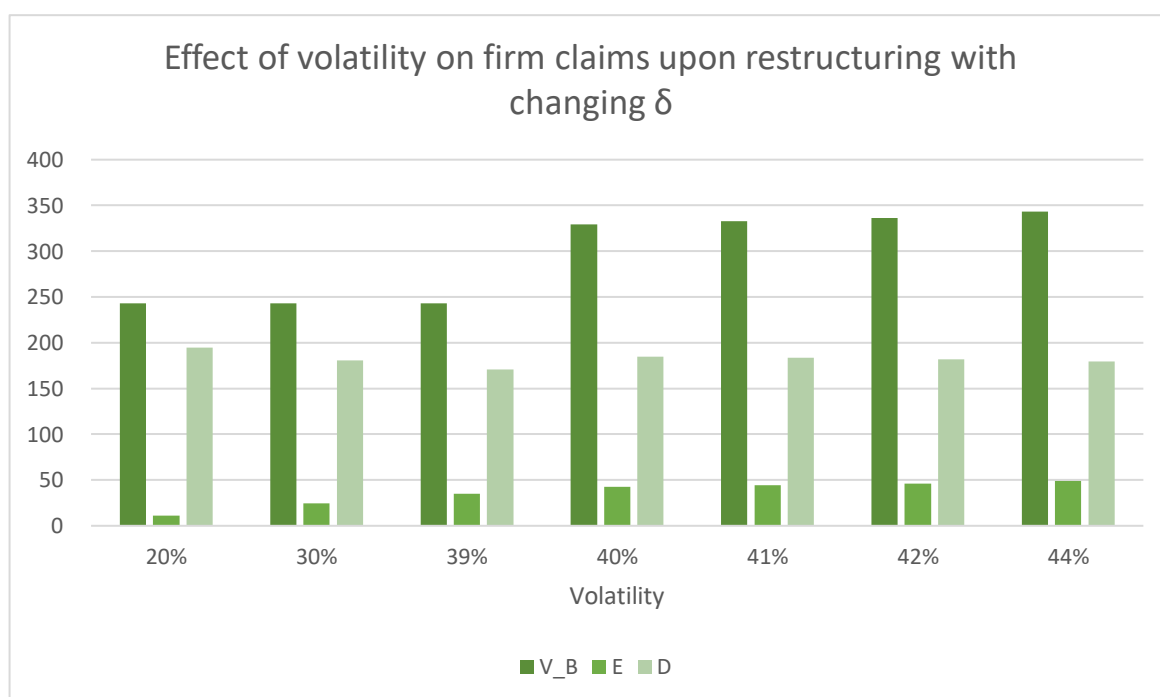


Exhibit 4.15. Effect of volatility with changing payout yields.

Leland (1994). On the contrary, it is decreasing up to $\sigma^* = \ln\left(\frac{C+P}{V_0(1-\delta_1)(1-\delta_2)ud}\right)$, thereafter it jumps upwards and has an increasing trend for a short interval only. It is also relevant to point out that, since the recombination of the tree is no more at $V = V_0ud(1-\delta)^2 = 233.78$, but at $V = V_0ud(1-\delta_1)(1-\delta_2) = 242.5$. Therefore, the resulting optimal $\sigma^* = 39.2\%$.

7. An overall overview of the models

This Section shortly deals with the comparative analysis of the three models, which, in spite of some specificities, seem to be generally in line with each other and with theoretical predictions. In particular, liquidation is triggered as equity value subject to limited liability is maximised, which corresponds to the first value of equity which can be hit from above such that all the values of the assets above it are higher than the boundary when limited liability and equity dilution are also considered. This results in a very low boundary, because immediate liquidation prevents shareholders from implementing any strategic behaviour. On the other hand, restructuring gives more chances to equity holders to appropriate the rents ex post though they should be destined to debt holders, thus increasing their optimal boundary and equity value. Specifically, Chapter 11 modelling leads to a much higher result for equity value, suggesting that the presence of separate accounts is mostly beneficial to the debtor. On the other hand, debt holders are expected to suffer restructuring the most, especially when bargaining power of equity holders is high. Indeed, in general, restructuring leads to early default, which comes at the expenses of creditors, and may end up in liquidation, which again mostly damages this category.

However, this does not result in immediate liquidation being always the most convenient choice. Indeed, both strategic and economic parameters affect the binomial option pricing valuation.

First, strategic behaviour is triggered by higher volatility and bargaining power. In these cases, in fact, the debtor exploits limited liability the most. Anticipating that she has a lower boundary (the zero equity value), she is more incentivised to go bankrupt earlier in order to exploit these benefits at odds of debt holders. To restore the first-best outcome, therefore, it may be convenient to reduce the chances of acting opportunistically, by fixing the due payments to debt holders, at least partially, as separate accounts implicitly suggest.

Second, bankruptcy and liquidation costs affect both equity and debt holders. Upon immediate liquidation, liquidation costs reduce only the debt value, whereas their effect is mixed when restructuring is filed for. In general, by letting shareholders default earlier, equity value raises and debt value declines. However, reducing the grace period may also increase the debt value

if these costs are sufficiently high. This is a consequence of the drop in the optimal V_B , which in turn reduces the probability of default and ensures that debt obligations are satisfied for a longer period. However, if costs are sufficiently high, a shorter grace period may damage shareholders by decreasing the period in which rents are extracted, preventing them from choosing a value which, instead, would be optimal to debt holders. Therefore, according to the estimated liquidation costs, it is possible to model the optimal grace period which best satisfies all claim holders interests.

The same rationale also applies to bankruptcy costs: they always decrease the value of both equity and debt, but in a different way. As before, unless they are sufficiently low, they tend to reduce the optimal grace period from an equity holder perspective. Furthermore, the equity value is lower if these costs are higher, because bankruptcy is delayed at a lower V_B , because it causes the rise of these costs. Debt value is instead maximised by reducing the grace period (and therefore the optimal boundary) if the costs are high, and by increasing it if they are low. Therefore, on the whole, modelling the grace period according to the level of the costs for the firm may result in a more convenient result for the parties. For example, if these costs are high, lowering the grace period may reduce the equity maximising V_B and let the debt and firm value come closer to the first-best outcome. If they are low, instead, a longer grace period may be more convenient.

Finally, Broadie, Chernov and Sundaresan (2007) also pointed out that immediate liquidation is not the best possible outcome in spite of the results. Indeed, though the probability of default is higher upon restructuring, the final probability of liquidation is lower than in the Leland (1994) model. Therefore, restructuring may help avoiding unnecessary liquidation. This holds especially by setting the grace period: everything else being equal a longer grace period reduces the probability of final liquidation. An optimal choice of it depending on the parameters may also induce a result closer to the first-best and value maximising in many more cases.

Conclusions

The option pricing method offers an alternative approach to value companies in financial distress, which proves to be more reliable than the adaptation of traditional models in many cases. Indeed, it allows to explicitly incorporate the features of companies in financial distress in the model, instead of making unrealistic assumptions or, even worse, of “forcing” traditional models to make valuation consistent, although they were initially implemented by assuming fairly different behaviours.

However, the first attempt to use options to value firms’ claims was neither realistic nor simple to implement in the real world. As a consequence, it was subject to further improvements over time. They not only made it more credible and applicable, but also developed the call option available to shareholders even more, which initially simply consisted of keeping control of the firm (by exercising the option) or letting control to debt holders by filing for bankruptcy (and not exercising the call). New scenarios introduced the chance to restructure the firm, both from a strategic and a regulatory perspective.

Three models, respectively for the immediate liquidation case, a simplified restructuring framework as well as a more detailed analysis of restructuring following Chapter 11 regulation were thereafter implemented in a discrete time framework, which allowed to make use of a binomial lattice to implement, analyse and compare their results.

First, the primitive variable was derived. It was selected to be V , corresponding to the enterprise value obtained from a DCF valuation adapted to only consider return on current investments. Therefore, it is derived as if the company were to be sold immediately, but without including future opportunities. Second, the binomial tree for V and FCF were derived. Thereafter, the economic values of equity, debt and of the firm were derived as traditional options on the firm value in the first model, where the option may be exercised depending on whether the firm is solvent. In the other models, a more complex double barrier option is implemented. Specifically, the boundary which triggers bankruptcy corresponds to the value which maximises equity. This value differs from the (first-best) firm value-maximising outcome in that it anticipates bankruptcy to permit shareholders to take advantage of the grace period and of restructuring. The liquidation boundary, instead, is again determined as the lowest value at which the firm is able to meet its obligations, and the option is exercisable at each node.

In the analysed example, equity, debt and the firm value are discounted with respect to their nominal value for all the three implemented situations. However, restructuring reduces the equity value discount both in the second and first-best solutions, but it is also beneficial for debt

and firm value in the first-best outcome. This is because, if the company is not forced to liquidate immediately, it will benefit from additional cash flows if the firm enters the grace period, or may even return to a healthy state and ensure continuation of the business, thus reducing the most damaging outcome, i.e. liquidation.

However, as mentioned, what really stands out is the fact that restructuring seems to mostly benefit shareholders at odds of creditors if bankruptcy is filed for by the first category. With respect to immediate liquidation, the restructuring value of equity raises, while the value of corporate debt declines, and this holds in both restructuring models. This result can be explained by the incorporation of strategic behaviour by equity holders: if they can choose when to file for bankruptcy, the chance to extract some rents before liquidation, available if they have some bargaining power, will incentivise them to anticipate default. This, in turn, damages debt holders, whose due obligations are no more to be satisfied and who, upon restructuring, cannot enforce any payments and therefore mostly suffer the situation.

This result would lead regulators to reflect about the validity of the restructuring procedure: if it incentivises strategic behaviours and if it damages debt holders even more than liquidation, then regulators should probably prohibit or simply reduce the scope for this procedure. Although it is true that debt holders may probably prefer liquidation in this framework, this conclusion is probably rushed.

The described result is driven by an endogenous decision made by shareholders, without any limitations in their actions by the regulator or by debt holders themselves. If they are permitted to choose the bankruptcy point as well as the possibly endogenous parameters of the model, such as the grace period, they will of course choose them to maximise their value. In general, the longer the grace period, the more anticipated is the bankruptcy-triggering threshold, and the higher is the fraction of rents which can be extracted by equity holders. This holds especially if bankruptcy and liquidation costs, which only arise as the two boundaries are breached, are sufficiently low. Otherwise, these two decisions will be delayed as much as possible to avoid the costs associated with them, and the grace period is optimally reduced from the point of view of both parties.

Summarising, as argued by Broadie, Chernov and Sundaresan (2007), the issue referring to early default, which represents the main deviation from the first best outcome, may be limited by reducing the grace period, reducing the bargaining power of shareholders (or debt relief, which was not explicitly considered in the discrete time model) or transferring control of the firm to debt holders.

The first two solutions proved to be beneficial by limiting the opportunistic behaviour by equity holders. When costs are low, these decision is in contrast with the desire of equity holders, who

would like to prolong the grace period to exploit more benefits. When they are sufficiently high, it could be in their same interest to reduce the optimal boundary and delay these costs. In any case, imposing a sufficiently short grace period gets closer to the first best outcome, but is not always total value maximising. This is not only forecasted by theoretical approaches, but was also observed in the numerical approach and showed to be especially damaging in the base-case scenario. In this case, the reduction of the grace period allowed to reduce the optimal boundary but was not able to achieve the first best. Due to the fact that a lower grace period limits the time before the potential final liquidation, indeed, the negative effect dominates the positive effect of being closer to total firm value maximisation, and this leads to an overall firm value decrease.

The other suggestion is to let the lender take an active role in either deciding when the firm should file for Chapter 11 or taking the reins of the firm once the borrowers decide when to file for Chapter 11. In the first case, debt value usually follows the same outgoing as total firm value, as mentioned in the previous Chapters. As a consequence, their maximisation usually leads to a similar boundary. Letting debt holders impose bankruptcy, therefore, may be better off for the whole firm. Though reducing equity value, it still leads to a higher value than the immediate liquidation case for all claim holders, everything else being equal. Therefore, restructuring may lead to a more beneficial outcome than direct liquidation.

On the other hand, letting debt holders take control of the firm in the restructuring procedure, by proposing their reorganization plan or being granted the veto right on the reorganization plan proposed by managers, ensures that they are not excessively damaged by the equity favourable behaviour and that, instead, they can gain more from the firm restructuring than upon liquidation.

This is especially true because restructuring is aimed at granting a future to the firm. A three-year model is too limited in this sense, since it does not allow to explicitly capture the benefits of a firm returning to a healthy state. From this point of view, it could be beneficial to use a longer period, to include in the model a sort of continuing value which assumes the firm may be healthy again. Indeed, this model is limited by the assumption that, at maturity, something has to happen, although this is not explicitly modelled.

An interesting development may include a more realistic modelling of the reorganization plan. In the models analysed so far, the grace period implicitly allowed for a reorganization which simply came from playing a bargaining game or, in the third model, by delaying debt obligations (and reducing them by a fraction, factor which was not considered explicitly by the authors of the discrete time model themselves). However, in the real world, reorganization is a well-organized plan, where new seniorities, changes of maturities and most of all changes of

the amount and timing of due payments are introduced. If this plan is accepted, of course, it will allow for further developments of the option to liquidate in case the liquidation barrier, which refers to the new due payments, is now hit.

This development would impose an exogenous reorganization plan, different from those implied by the models and used in the model implemented by Broadie and Kaya (2007). Since the model is exogenous, including it would be cumbersome and useless if it is not yet available. If, instead, it is possible to define it, its incorporation may even lead to more realistic results and may also allow binomial models to be useful for practitioners.

Option pricing has huge opportunities which may be exploited not only for theoretical purposes, Instead, it may also be beneficial for single firm valuation and for the valuation of their future potential, in spite of their current phase of financial distress. Evolving it to make it even more realistic, reliable and applicable in the real world may lead to improvements in many fields, first of all in regulation and advisory. Although the binomial option presented above is far more realistic than many other option-based models, non-credible assumptions are still many. Relaxing these assumptions and introducing less simplistic and at the same time more realistic forecasts and behaviours for future developments may lead to huge improvements of the model and of its applicability in practice.

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