



UNIVERSITA' DEGLI STUDI DI PADOVA
DIPARTIMENTO DI SCIENZE ECONOMICHE ED AZIENDALI
"M.FANNO"

CORSO DI LAUREA MAGISTRALE / SPECIALISTICA IN
ECONOMICS AND FINANCE

TESI DI LAUREA

"THE VALUATION OF FIRM DISTRESSED DEBT: MODELS AND
APPLICATIONS"

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ANNO ACCADEMICO 2015 – 2016

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Introduction

The valuation of distressed companies consists in a highly topical issue in modern business which has been developed by corporate finance literature over a long time. Despite its importance, few contributions allow taking into account correctly specific features of firms in financial troubles, such as low or even negative cash flows, declining growth rates, high levels of financial exposure, undeniable default probabilities. Models should appropriately consider that financial crisis can generate important consequences not only for the firm under valuation, in terms of liquidation or continuation as going-concern, but for the economy as a whole as well. According to Altman and Hotchkiss (2006), the unsuccessful business enterprise can be defined in different ways, by using terms very common in literature, such as *failure*, *default*, *insolvency* and *bankruptcy*. A brief definition of each of them let us clarifying the framework of valuation. In economic terms, *failure* means that the yield on invested capital is significantly and repeatedly lower with respect to rate of returns on similar investments for different reasons. Alternatively, failure occurs either when revenues are insufficient to cover the costs or the average return of investments is lower than the cost of capital. The economic interpretation of *failure* does not consider the implications of this sort of situation, such as firm's exit or continuation. *Default* can be technical and/or legal and always involve a relationship between creditors and debtors. *Technical default* occurs when a company (or, in general terms, a debtor) violates one or more conditions regulating the agreement with a creditor, such as the debt ratio. Usually these violations are signals of deteriorating firm performances. While, if a firm misses a scheduled loan or bond repayment, *legal default* can potentially occur. In some cases, firms' default can be avoided thanks to *distress restructuring* procedures agreed with creditors, avoiding bankruptcy declarations at least for some years. *Insolvency* describes a situation in which a lack of liquidity does not allow the reimbursement of current obligations. The condition of *technical insolvency* can be temporarily, even if it may turn into a formal bankruptcy declaration. More complex and dangerous is *insolvency in a bankruptcy sense*, which occur when firm's total liabilities exceed the value of total assets, generating overdue debts in balance sheets.

When a company suffers for financial distress, it has not enough proceeds to reimburse its obligations with the cash flows generated from operations and it is usually not able to obtain additional debt as a reasonable cost, given the high risk of insolvency. According to Jennergren (2013), in recent years many companies have faced severe operating (other than financial)

problems, among which low or even negative cash flows. All of them contribute to increase uncertainty surrounding future survival that is reflected in the market by a drop in equity prices (or, alternatively by an increase in the rate stocks' rate of return). This phenomenon cannot be ignored in valuation models; there are different opinions in literature about the way to deal with uncertainty when forecasting and actualising expected future cash flows generated by the distressed companies. While, for companies that are not listed, the valuation is more complicated and often based on past data or comparable companies. In this paper, we will provide some suggestions for valuing them as well.

It is in such framework that *credit analysis* intervenes in determining the creditworthiness of a firm, which is the ability to pay debt at the scheduled times (i.e. to respect covenants on debt). It involves a deep analysis of the business, financial and accounting conditions of the firm under valuation. The main task of *credit analysis* is assessing the probability that a firm will face financial distress, failing to repay its financial obligations. Once financial distress is recognised, we need to distinguish if such a situation is irreversible or can be turned around thanks to some insolvency procedures.

Consequently, the management should intervene and define the best strategies to overcome such a situation. The most efficient solutions are those that maximise firm value from the cash flow generated by the assets. We can identify almost three possible alternatives management can choose:

- Firm restructuring with the perspective of continuing operations as going-concern. In this case, the management stipulates an agreement with creditors, who usually accept a reduction and a deferment on their promised amounts in order to facilitate company's survival. If an entity's intrinsic or economic value is higher than its current liquidation value, it should try to reorganize itself and continue its operations;
- Change in firm's ownership (i.e. a change in management); even in this case, a continuity in operations is assumed;
- Firm's liquidation: assets are sold in the market when they cannot potentially produce additional value inside that particular context.

In this paper, we want to focus our attention mainly on the first alternative, remembering that firm's survival is subject to the possibility of exploiting all the existing assets, resources and skills in order to return to a sustainable financial equilibrium in the future. The going-concern hypothesis is usually included in the reorganization plan agreed with creditors. The latter also

involves assumptions about outputs and actions that will guarantee the success of the reorganization plan and will influence the computation of the going-concern capital value as well.

The valuation process can be considered as a step-by-step process that begins with an analysis of firm's historical financial results and balance sheets, which reflects past management decisions. Then, after a review of the main value drivers included in projection forecasts, estimates of the enterprise value and its variability should be performed. In doing so, we should not take into account only the assessments of management's strategic plans, but metrics of historical performances as well. In addition, a risk analysis conducted by changing forecast inputs (i.e. a *scenario analysis*) can be considered as a powerful instrument to address uncertainty in forecasts and to create a certain degree of confidence about expectations.

Summing up, in the first chapter we will provide an analysis of the methods developed in literature that allows obtaining estimates of the enterprise value. Therefore, we will distinguish cash-flow based methods (i.e. the Discounted Cash Flow, the Adjusted Present Value and the Capital Cash Flow approaches) from other methods, such as those referring to Option Pricing theory (i.e. Black-Scholes and Merton models) and Multiples as well. Then we will proceed by computing an estimate of the enterprise value by combining the DCF methodology and market multiples, taking into account that many distressed firms plan to reduce their leverage ratios to certain targets over time by devoting free cash flows to reimburse obligations, as Arzac (1996) suggests.

The valuation continues by computing Equity and Debt values. Indeed, option pricing theory allows considering equity as a *call option* on firm's value, assuming that the latter follows a continuous process with constant volatility. Thanks to Black-Scholes and Merton models, the value of the firm's risky debt can be decomposed in two components: a risk-free component and a *put option* on firm's value. In case of financial distress, companies show a market value of debt usually lower than its nominal value; therefore, we want to justify such discrepancy both in theory and in practice.

In chapter three, we distinguish between Out-of-Court and In-Court procedures as possible ways to overcome firms' financial troubles. Both have some associated advantages and disadvantages relative to the specific company under valuation and the surrounding framework as well. In literature, we find evidence about firms' propensity to adopt procedures that are independent from judicial intervention, given their relative convenience. So, Gilson et al. (1990)

sustain there is a large amount of firms in financial distress that avoid default through debt restructuring and thus stipulating some private agreements with creditors (i.e. reorganization plans), which usually involve some reductions in net transfers from debtors to creditors, other than payment delays. Creditors have a collective interest in unilaterally reduce their cash flow claims prior to liquidation, since the amount they can extract from the firm through a liquidation procedure would be even lower. Mella-Barral P. and Perraudin W. (1999) recognise that debtor's default rarely coincide with liquidation since debt is often reorganised through out-of-Court procedures. Moreover, Franks J. R. and Torous W. N. (1994) affirm that many firms adopt in-Court procedures only after attempting to resolve their financial difficulties informally and therefore as an alternative solution.

Finally, in the last chapter we want to apply the theoretical notions investigated in the previous sections specifically to a distressed firm. Therefore, we will provide a *case study* in which the company is valued as going-concern and it has been able to conclude private agreements with creditors in order to restore a sustainable financial equilibrium. The goal is to quantify the difference between the risk-free and risky components of debt as sized by the put value.

In the first case, we assume that Merton's assumptions hold: there is a unique tranche of a Zero Coupon Debt outstanding and therefore a single maturity date in the future. We will find out that, despite the limitations associated to such an approach, we are able to obtain closed-form solutions for capital components in a relatively simple and clear way. We also provide a *sensitivity analysis* that allows interpreting results in case of changes in input factors, such as enterprise values and asset volatilities.

In the second case, we take into account the introduction of two tranches of zero coupon debt: a short-term and a long-term one. In doing so, we increase the complexity of firm capital structure by introducing at least two maturity dates. Such a modification also allows dealing with situations in which the firm benefits from additional sources of finance (i.e. "new finance") which facilitate the continuation of operations and require a "super priority" reimbursement with respect to other pre-existing obligations. Referring to this aspect, we provide some adjustments to the original Merton's formula, which is considered as a linear combination of call options on firm assets.

CHAPTER ONE: The Enterprise Value computation

1.1 Different methods for valuing distressed companies: Cash Flow-based approaches

Analysts usually face some problems when valuing distressed firms due to their intrinsic characteristics, such as low or even negative cash flows, declining margins, high level of financial exposure and undeniable default probabilities. Traditional valuation models in literature therefore need to be “adjusted” in order to take into account the effect of financial distress in a consistent fashion. Among them, we can distinguish cash flow-based models, such as the Discounted Cash Flows (DCF), the Adjusted Present Value (APV) and the Capital Cash Flow (CCF) models. All these models involve the computation of the firm’s enterprise value from the expected cash flows generated from its assets, by assuming that the target will be able to continue its operations in the future (i.e. going-concern assumption). Such models are usually applied to firms in normal business circumstances, but they can be rearranged to value companies in financial distress as well. Concerning to this, Jennergren (2013) complains that uncertainty is not clearly visible in traditional valuation approaches, whereas highly uncertain cash consequences need to be discounted for valuing distressed firms correctly. The risk of firm’s default has a great impact on value and consequently it should be considered explicitly in valuations.

Gilson et al. (2010) recognise a significant dispersion error (i.e. from 20% to 250%) when comparing estimates from traditional valuation models based on cash flows forecasts and market values. We can identify some reasons that can justify such gap in values: the choice of the valuation model, the likelihood that firms will not survive in the future and the assumptions made about discount rates and future growth rates as well. Moreover, analysts often found their valuations on a set of limited information with respect to their amount and quality: they have a closer correspondence to market values than management forecasts, since they have some difficulties to access firms’ data, especially when companies are not listed. There is also a problem connected to “strategic valuations”: different incentives of the parties involved in debt negotiation can influence estimates of value. Indeed, junior and senior claim holders play a special role. They have different motivations in relation to cash flows estimates: senior claimants tend to underestimate firm’s value in order to increase their recovery in case of a bankruptcy procedure, while junior claimants have the opposite incentive, given that they are reimbursed at first in case of liquidation. As a matter of facts, we can conclude that relative

bargaining power of both claimants has an impact on firms' value estimates through cash flows models.

In the following paragraphs, we will provide a concise analysis of the advantages and limitations associated to each cash flow-based model. The same analysis will be extended to other approaches for valuing firms as well, such as those based on multiples and option pricing theory.

1.1.1 The Discounted Cash Flow (DCF) approach

This approach is the most widely used and common in corporate finance, since it is able to link firm's value to the ability of producing an adequate amount of cash flow in order to satisfy the expectations of investors. In normal circumstances, the value of a firm puts its basis on three main pillars: expected future cash flows forecasts, cost of capital estimates and terminal value computation. According to this approach, firm's value drivers - such as the expected annual sale growth and the EBITDA margin - play an important role. Projections need to be computed from historical data and market data (when available). Practically, there are some difficulties in obtaining precise estimates when the firm is actually in a financial distress situation and there is a positive likelihood of default in the future. In this case, it is better to extend the projection period until the end of debt restructuring and the achievement of a steady-state equilibrium. The DCF method is very applied in practice thanks to its flexibility, which allows valuing different types of firms. Anyway, it has some limitations: for instance, it assumes the firm has a potentially infinite life in the future and that it can continue to operate by overcoming almost all its financial problems. By the way, when there is a significant probability for a firm to default in the future, these assumptions seem not very realistic and reliable. This is only a small overview about the assumptions and the main advantages-disadvantages of the model; we will provide a deep analysis of such approach in the following paragraphs, since we will use it in our *case study* as well.

1.1.2 The Adjusted Present Value (APV) approach

Differently from the DCF approach, APV models consider explicitly the effects of obligations on firm's value: the value of a levered firm is computed by adding costs and benefits of debt to

its unlevered value. On one side, debt can generate positive value of tax shields, but on the other side, it can increase the probability of default. However, for distressed firms, negative effects tend to prevail with respect to the positive, leading to higher values of default probabilities with respect to healthy firms. Consequently, the model should be reformulated in order to take into account explicitly the increasing role of the costs associated to default, which are weighted by the probability of default. The total firm's levered value is therefore given by discounting the unlevered FCF, the value of the tax shield (that is a measure of debt benefits) and the adjusted value of bankruptcy costs. For instance, Damodaran (2006) suggests computing the cost of distress as the difference between the value of the firm as going-concern and its liquidation value, but in general, there are other computational problems when applying such an approach. Like all the other cash flow-based models, the APV approach requires a precise estimate of firm's default probability, but it also needs an analysis of operating firm's losses that compromises the possibility to benefit from tax shields. When firms face a deep financial crisis, tax benefits on debt are substantially reduced: either very low or negative EBIT values impede to take advantage from the deduction of interest expenses on taxable income, increasing the firm's leverage and probability of default. We can therefore conclude that this approach is not suitable for financially distressed firms with substantial operating problems, but it can be combined with other approaches to obtain more accurate estimates of value.

1.1.3 The Capital Cash Flow (CCF) approach

Ruback (2000) tries to overcome some traditional models limitations by introducing an alternative approach, known as Capital Cash Flow model, whose name derives from the fact that it includes all the cash available to capital providers. It follows directly from Free Cash Flow models, but, differently, it takes into account explicitly the value of debt tax shields. The APV approach takes into account tax shields on debt too but it usually uses a different rate: the cost of debt replaces the cost of assets used in CCF models. By attributing a higher value to the deductibility of interests, the enterprise value will be higher by applying the APV method than the CCF one. CCF and APV models differ for the assumptions made on leverage as well: the former assumes that debt is proportional to firm's value, while the second assumes that debt is fixed and independent on firm's value. We know that distressed firms' valuations usually rely on estimates about firms' future debt policy; when the latter involves a target debt-to-level ratio, the CCF approach seems less accurate than the APV. On the other side, if debt is likely to

increase as firm's value increases, then the proportional assumption seems more appropriate. CCF and APV approaches can be combined as well; an example is provided by Gilson et al. (2000) in the valuation of firms emerging from bankruptcy. Anyway, CCF or equivalent FCF methods are generally preferred to APV, since in most corporate circumstances debt levels tend to change as market values change.

1.2 Other approaches:

1.2.1 Market multiples

Differently from cash flow-based methodologies, we mention here the method of multiples, which is very used in practice since it is relatively easy to implement. Multiples are computed considering both market observations and data extracted from financial statements, such as enterprise values and equity values. Their popularity derives from the limits on the amount of underlying assumptions and the rapidity of calculation with respect to cash flow-based methods. They, furthermore, let a closer correspondence to market belief. Anyway, approaches based on multiples have some limitations as well, starting from the “simplicity” mentioned before, which can be a source of imprecise estimates. It is not easy to identify a homogeneous sample for distressed firms as the model suggests, since each firm shows specific characteristics that cannot be generalised to other companies. Moreover, multiples apply only on positive results; this can be a problem in case of very low or even negative expectations about annual growth rates on sales and expected cash flow generated by the firm as well. As Koller et al. (2010) underlines, we should take into account forward-looking multiples rather than multiples based on historical data in order to be consistent with the principles of valuation. Empirical evidence shows that forward-looking multiples are more accurate predictors of value than the historical-looking ones. Therefore, we need to make forecasts that are in line with the long-term prospects both of the firm and the business. As we will discuss later, multiples are commonly applied after assuming a steady-state equilibrium for distressed firms and therefore a certain stationarity in firms' capital structures. Multiples-based models can be combined with the DCF approach for computing the value of firms' capital structure components more precisely. Due to this, *asset side* multiples (such as EV/EBITDA margins) are generally preferred to *equity side* multiples because they are less dependent on firms' capital structure, which is likely to fluctuate frequently in distress situations.

1.2.2 The approach based on Options

Finally, we can involve another approach for valuing firms in trouble that relies on options. We can find the original version of such methodology in Black and Scholes (1973), and a modified version in Merton (1974). By considering option theory, we are able to determine a positive value for firm's equity even in situations where the value of firms' assets is lower than the value of firms' debt, as in case of distress frameworks. We can assess firms by taking into account the logic behind methods for valuing options: meaning that equity is considered as an option of firms' assets with a strike price equal to the nominal value of debt. In most companies, shareholders can decide in every moment to liquidate assets and use the proceeds to reimburse obligations, leaving the control on the hands of creditors. Moreover, shareholders usually have limited responsibilities, meaning that they cannot lose more than the capital invested in the firm. Equivalently, option pricing theory allows us considering debt as a put option on firm's assets. Among the merits of this model, we can mention the computation of a risk-neutral probability measure (which consists in the probability that the nominal value of debt will be higher than asset value at maturity) and the attribution of a certain value to uncertainty characterizing future firms' profitability. However, the limitations of such approach are mostly related to its assumptions, which are usually considered far from what happens in the real world. According to Koller et al. (2010) an important practical disadvantage is associated to the hypothesis that every source of uncertainty is considered as independent from the others, while in reality we can usually find some correlations between them. A crucial limitation is involved in the consideration of a single zero coupon bond as well. For its complexity and importance in distressed-firms' valuation, the option-based model merits a deep investigation, which will be provided in the second chapter of this paper.

1.3 A systematic solution for valuing distressed firms and some implications

The main goal of analysts and professionals is to determine the "correct" (or, better, the most adapted) model or combination of models that allows to deal with uncertainty, default probabilities, other than credible values for capital structure components. We propose to value highly distressed companies by considering the following steps one by one. Firstly, we assume that the firm will continue to operate even if it is temporarily in financial trouble. Indeed, we estimate firms' enterprise value by discounting expected cash flows from their assets, according

to the DCF approach. The latter relies on a set of assumptions that allows dealing with some important features of distressed companies, affording more precise estimates. When firms are assumed to restore sustainable financial stability, we can opt for methods based on multiples, which are easier and faster to compute. A steady-state framework is usually assumed in correspondence of firms' terminal values computation, assuming they will continue to grow at a constant rate in an infinite horizon.

The following paragraph is therefore assigned to the computation of the enterprise value. In usual circumstances, analysts obtain actual estimates of firm's value by discounting cash flows at the Weighted Average Cost of Capital (WACC); but, in case of distressed firms valuation, it seems not to be the best discount factor. Indeed, as we will discuss in the last paragraph of the first chapter, analysts and researchers have different opinions about the adjustments which need to be implemented in order to take into account fluctuations in firms' leverage and default risk as well. Anyway, we are allowed to employ the WACC for obtaining firms' terminal value without lots of practical implications, since capital structures are assumed to be stable over time.

Our valuation continues in the second chapter with a description of option pricing theory and Merton's model (1974) as well. The closed-form option pricing formula derived by Merton (1974) allows us obtaining the value of firms' equity by considering estimates of enterprise value as underlying price and nominal value of debt as strike price. Then, by differentiating assets and equity values, we obtain estimates of market values of debt, which differ from nominal values of debt in distress circumstances. In this section, we will also provide some evidences about the computation of asset returns volatility, which is one of the main input entering the famous Merton's formula. Among different methodologies we can find in literature, we prefer computing asset returns volatility from equity returns volatility, by involving the Black and Scholes option pricing formula (1973). Once we have completed the theoretical valuation of distressed companies, we provide some proceeds that can be implemented in order to avoid default. In particular, we focus our attention on the debt restructuring through private agreements between creditors and distressed firm's manager. As we will discuss later, the parties involved can choose among different solutions in order to lead the company continuing its activities.

1.4 The Enterprise Value computation in DCF models

In order to determine the value of a company in financial distress, the starting point consists in estimating the Enterprise Value. The latter can be expressed as follows:

$$\text{Enterprise value} = \text{asset value} = \text{net debt} + \text{market value of equity}$$

$$\text{Where net debt} = \text{market value of debt} - \text{cash}$$

Firms' enterprise value is computed both considering expected financial results that firms will probably achieve in the future and information included in balance sheets as well. In doing so, we try to distinguish firms' capital components (i.e. enterprise value (EV), debt (D) and equity (E)) through the entire valuation, even if there are some interconnections between them. As Buttignon (2014) underlines, the realization of EV is subject to the approval of creditors, thus it depends on D. Creditors have a crucial position in distress firms: the adoption of the plan depends on their consent, but at the same time, by accepting the implementation of the financial remedies, they are granting added value to current and new shareholders. As we will discuss later, the value transferred from debt to equity holders after the plan's approval is measured by a put option on the firm's value.

We compute firms' enterprise value by applying both the DCF model and multiples. According to the first methodology, we have to forecast the proceeds the target will be able to achieve from its operations in order to reimburse its obligations, under a going-concern assumption. As Gilson et al. (2000) underline, such valuation is therefore performed through a set of negotiations that are involved in the reorganization plan.

The value of a firm can be expressed as a function of four main elements: the ability of the firm to generate cash flows from its assets in place, the expected growth rate of the cash flows, the length of time the firm needs to restore a sustainable financial equilibrium and, finally, the cost of capital. Thus, the firm's value is sensitive to a change in one or more of the previous variables. We can consider at first the value of a single asset: it is obtained by discounting the expected cash flows generated by such asset, at a rate that reflects its riskiness. This procedure allows computing the so-called "intrinsic value" of an asset:

$$\text{Asset value} = \sum_{t=1}^N \frac{E(\text{Cash Flows}_t)}{(1+r)^t}$$

Where the asset has a hypothetical life extending from one to N years, and r is the discount rate reflecting the riskiness of the cash flows. By considering the firm as a portfolio (i.e. a combination) of assets, it is possible to extend the previous equation to find the total firm's value. In this way, we consider all the expected cash flows generated by the assets as a whole.

The process is not so easy to implement, since a significant portion of the firm's value is influenced by expectations about future investments, other than assets that are already in place. In other words, we need an estimate of the cash flows the firm expects to realise, allowing the reimbursement of its obligations over time. According to Guthner (2012), the DCF model turns to be a forward-looking process that has to rely on a forecast of the general business conditions of the economy as a whole, including the unique factors driving the industry where the company operates and the context of the company's business strategy. Anyway, the model jointly implies an analysis of historical financial performances, operating and overall costs, operating profits and net income related to revenues as well. Thus, a more precise estimate of the company's future financial performances can be obtained by combining metrics of historical financial performances (i.e. past balance sheets and income statements), reorganization plans, and valuations of current and expected business conditions in the future.

An important assumption underlying the DCF model regards the life of the firm: we assume that the firm will continue to operate in the future, even if it is actually facing a financial distress situation. Therefore, the firm is valued as a going-concern, by looking at the cash flows it will achieve if it will follow a path back to financial health, exploiting existing assets, resources and skills in a potential infinite-life period. In order to compute the going-concern value of a firm it is useful to determine the projection period of the valuation. According to Buttignon (2014), it has at least the same duration of the reorganization plan proposed by the management (i.e. generally from three to five years). A second period is usually added in order to grant to the firm the possibility to restore an equilibrium condition. At the end of the reference period, we estimate the firm's Terminal Value or Continuing Value (CV), by assuming the cash flows growing at a constant rate for an infinite period. Thus, we can decompose the DCF value of a firm in this way:

$$\text{DCF value} = \text{Present value of cash flows during projection period} + \text{Present value of continuing value}$$

We proceed by differentiating the cash flows generated during the projection period from the calculation of the present value of the continuing value.

1.4.1 The Free Cash Flow during the Projection Period in distressed companies

The DCF methodology allows us reflecting the main effects of financial distress on firms' value determination, which should be incorporated in both firms' expected cash flows and discount rates. A deep valuation considers all the possible scenarios a firm can potentially face in the future (i.e. *scenario analysis*), from the most optimistic to the most pessimistic one. In particular, in a financial distress framework, it is interesting to study the firm's ability to service debt under challenges scenarios, especially those in which the probability of default is extremely high. To this purpose, we should consider not only the nominal amount of debt that the firm should repay over time, but the tenor of debt as well. According to Guthner (2012), we should not forget debt maturity when valuing the firm's financial sustainability: long-term debt may allow the company to survive at a reasonable cost with respect to short-term debt. The management may have time to decide and implement remedial actions, which contribute to the debt reimbursement.

The ability to service debt is necessarily linked to the expected cash flow generated in each scenario. More precisely, the expected cash flows generated by the firm during the reference period can be written as the sum of the probability-weighted estimates of the cash flows under all the scenarios considered. Analytically, the expected cash flow value can be derived as follow:

$$Expected\ cash\ flow = \sum_{j=1}^n \pi_{jt}(cash\ flow_{jt})$$

Where π_{jt} is the probability of scenario j in period t and $cash\ flow_{jt}$ is the cash flows generated under that scenario in that period. An important feature is that each input has to be estimated each year, since probabilities and cash flows may change over time. Adjustments are cumulative and have a greater impact in the later years. An approximation of the expected cash flow calculation would require estimates of only two possible scenarios: the going-concern and the distress one. Under the first scenario, we suppose that the financial distress is temporary, while, under the distress scenario, we assume that the firm will default and will be liquidated because of its insolvency situation. Thus, we should determine the cash flows generated from the liquidation procedure or the divestment of the entire firm. Precisely, the liquidation value of a firm derives from the difference between the market value of assets sold and the associated

liabilities in the balance sheet. In this simplified framework, the expected cash flows can be obtained as a weighted average of the two scenarios, weighted by the probability to face one of the two in the future:

$$\text{Expected cash flow}_t = (\text{Cash flow}_{\text{going-concern},t}) * (\pi_{\text{going concern},t}) + (\text{cash flow}_{\text{distress},t}) * (1 - \pi_{\text{going concern},t})$$

Where $\pi_{\text{going concern},t}$ is the cumulative probability that the firm will continue to operate as a going concern at time t .

In general, once we have an estimate of the expected cash flows generated each year in the projection period using the traditional DCF approach, we should discount them at the valuation date, using a proper discount factor. Thus, the value of the firm is analytically derived in the following way:

$$\text{Value of a firm} = \sum_{t=1}^N \frac{E(\text{Cash flows to firm}_t)}{(1 + \text{discount rate})^t}$$

The expected cash flow generated by the firm's assets are addressed to the firm's claimants, in particular bond and equity holders. According to Koller (2010), the DCF method employed to evaluate a firm is therefore built on the cash flows available to investors over time. Once we have an estimate of the firm's value during the projection period, we have to continue our valuation by computing its Continuing Value, which has a great impact on the total assessment.

1.4.2 The Continuing Value in distressed firms

The Continuing Value (CV) of a firm captures the value of the cash flows generated after the reference or projection period, in which we assume the firm will be able to restore an equilibrium condition, by reimbursing most of its financial obligations. Thus, the cash flows generated reflect almost "normalized" operations, which are expected to be sustainable over time. This assumption allows us to employ multiples in our valuation, taking into account comparable companies. Moreover, in the medium-long run firms in the same sectors tend to share the same growth rates, profitability and risk. Such reflections cannot be extended to the previous reference period, given the dynamic firm's capital structure and thus the conjectures included in the organization plan, which depend from firm to firm. Market multiples apply to

“normalised” financial results, such as earnings before interests, taxes (i.e. EBIT), depreciation and amortization (i.e. EBITDA) and/or EBIT and amortization. Other than select carefully a set of comparable companies, we should examine whether multiples registered today can be reasonably applied at the end of the forecasting period. If there are significant discrepancies between the company profile and the one of comparable companies, we should compute discounts to market multiples, since these differences are generally difficult to identify.

An alternative approach to compute the CV consists in applying the Growing Perpetuity formula to the expected cash flows generated in the subsequent period after the forecasting one. Assuming a “normalised” steady-state equilibrium, we can also consider the value of the underlying drivers (i.e. NOPLAT, RONIC and g) to determine the continuing value, as we can observe from the following equation:

$$CV_t = \frac{FCF_{t+1}}{WACC - g} = \frac{NOPLAT_{t+1} \left(1 - \frac{g}{RONIC}\right)}{WACC - g}$$

Since the perpetuity-based formula relies on parameters that never change, we should use them to find the CV of the firm when the firm has reached a certain steadiness with low but stable revenue growth (the value of the firm’s growth on its assets is expressed by g), and stable operating margins. The growth rate for distressed companies has to be potentially sustainable in an infinite horizon and cannot be higher than the growth rate of the economy (i.e. the risk-free rate). Once the CV is calculated, we have to update it by considering the discount factor employed in the last year of the explicit forecast period. Finally, the enterprise value is computed by summing the present value of the expected cash flows during the forecasting horizon and the actual value of the CV, which has a great impact in firms’ valuation and thus should be estimated carefully. Indeed, according to Damodaran (2009), some analysts are driven by “auto-pilot optimism” when valuing a firm with a history of financial health becoming in trouble. It means they have an excessive overconfidence in the firm’s ability to restore an equilibrium path. Optimism normally drive the determination of growth rates (i.e. positive a growth in the future), discount rates similar to those of healthy companies and high future profitability (i.e. margins and returns back to the pre-distressed framework). By assuming a going-concern framework, there is always a “happy ending” in which firms never default and their terminal value is large. While empirical observation suggest most firms do not survive in the long-term period and they are forced to exit the business; this usually happens when a firm is not able to reimburse its debts using the cash flows from operations. As consequence of the

financial failure, the company may have to liquidate its assets and use all the proceeds to repay debt.

In order to arrange traditional cash flow valuation for the risk of distress, we need to adjust the discount rate as well: in fact, riskier firms have higher cost of equity, debt and capital in comparison to safer firms. A higher risk implying a higher cost of capital leads to a reduction in the firm's expected value. A deep investigation of the possible solutions for the computation of the cost of capital is provided in the next paragraph.

1.5 The cost of capital for distressed firms

The determination of the cost of capital needed to discount expected future cash flows strictly depends on the company under valuation. Thus, the WACC is commonly applied since it ensures consistency between its components and free cash flow:

$$WACC = \frac{D}{D + E}(1 - \tau)r_D + \frac{E}{D + E}r_E$$

An accurate estimate of the discount factor is crucial for obtaining reliable firms' values. In order to compute the cost of capital, judgements of the cost of equity, the after-tax cost of debt and the company's target capital structure are required. Since each of the previous variable is not directly observable, we need to take into account different models, assumptions and approximations.

The cost of equity is determined by three factors: the risk-free rate of return, the market-wide risk premium and a risk-adjustment factor that reflects each company's riskiness. In order to estimate it, the CAPM approach is usually employed. The latter adjusts for the company *specific risk* thanks to *beta* (β), which measures stock co-movements with the market. Since it is not directly observable, we need to derive its value. The latter can be computed empirically through the market model, by regressing firm's return (i.e. r_i) against the market's return (i.e. r_m):

$$r_i = \alpha + \beta r_m + \varepsilon$$

With $\beta = \frac{Cov(r_i, r_m)}{Var(r_i)}$ represents the slope of the straight-line fitting stock and markets' rates of return, α is a constant value (i.e. the intercept) and ε the error term.

If the firm is not listed in the market, we need to follow a different approach for valuing *beta*; we should take into account *sector beta* or *beta* estimated for comparable firms (i.e. peers). Once identified, we should extrapolate the financial risk included in such a parameter, in order to obtain an indicator of the operating risk exclusively (i.e. *beta unlevered*). The following formula allows getting estimates of *unlevered beta* from *levered beta*, based on market observations:

$$\beta_u = \frac{\beta_l}{1 + \left(\frac{D}{E}\right) * (1 - \tau)}$$

From the *unlevered beta* of all firms belonging to a specific sector it is possible to find out the *sector unlevered beta*, known as Business Risk Index (BRI). From such an index, we can derive the *beta* of each firm that is not listed, considering its target financial structure:

$$\beta = BRI + BRI * (1 - \tau) * \frac{D}{E}$$

Where the ratio $\frac{D}{E}$ is computed from market multiples of comparable or companies in the same sector.

Beta estimates enter the CAPM formula for computing the cost of equity in the following way:

$$r_E = r_f + \beta(r_m - r_f)$$

Where r_f is the risk-free rate and r_m is the rate of return of the market portfolio. As a proxy for the latter, we can consider the S&P 500 index. The difference between the market and the risk-free rate consists of the extra yield of return asked by investors and it is known as *market* (i.e. *systematic*) *risk premium*. The latter can be computed by taking into account the historical average of the market excess return with respect to the risk-free rate, especially for firms that are not listed. Alternatively, given an analysis of future expected cash flows, it is possible to estimate the expected market return by determining the discount rate that is coherent with the actual value of the index. Such an approach based on fundamental usually requires the assumption of a constant growth in the future.

Alternative models to the CAPM for estimating the cost of equity include the Fama and French Three-Factor model and the Arbitrage Pricing Theory (APT) model as well. These models differ mainly on risk definition; the CAPM approach defines risk as stocks sensitivity to the market portfolio, while Fama and French (1992) measure stocks sensitivity with respect to three

portfolios: the market portfolio, a portfolio based on firm size, and a portfolio founded on book-to-market ratios¹.

In order to approximate the after tax cost of debt, Koller (2010) suggests employing the after-tax yield to maturity on companies' long-term debt. For companies with publicly traded debt, the *yield to maturity* can be calculated directly from the bond's price and promised cash flows. It is technically considered as a proxy for the expected return on company's debt, but it is actually a promised rate of return, which founds on the assumption that all coupon payments are made on time and debt is totally paid. Consequently, it is a valid proxy for the cost of debt only for companies with investment-grade debt rated BBB or better, in correspondence to low levels of default probabilities. For companies whose debt trade infrequently, it is better to take into account debt *rating* to estimate the yield to maturity. Professional rating agencies, such as Standard & Poor's (S&P) and Moody's provide such a rate, which is freely available to the public and can be downloaded from web sites. From firm's *rating* is therefore possible to derive a measure for the *default spread* that should be added to the risk-free rate to get an estimate of the cost of debt. If the firm is not listed, the *default spread* can be extrapolated from a *synthetic rating*. Damodaran (2014) suggests a *synthetic rating* based on firms' interest coverage ratio: EBIT/Interest Expenses. A more realistic approach would employ multiple ratios rather than a single one to set a score for estimating ratings. Anyway, the *synthetic rating* process would deliver reasonably close ratings for any firms. The model proposed is the following:

<i>EBIT/IE</i>		Rating	Spread
>	≤		
-100000	0,50	D	12,00%
0,5	0,80	C	10,50%
0,8	1,25	CC	9,50%
1,25	1,50	CCC	8,75%
1,5	2,00	B-	7,25%
2	2,50	B	6,50%
2,5	3,00	B+	5,50%
3	3,50	BB	4,00%
3,5	4,00	BB+	3,00%
4	4,50	BBB	2,00%
4,5	6,00	A-	1,30%
6	7,50	A	1,00%
7,5	9,50	A+	0,85%
9,5	12,50	AA	0,70%
12,5	100000,0	AAA	0,40%

Source: Damodaran (2014)

¹ For further information about this topic refer to Fama E. F. and French K. R. (1992)

For example, if the EBIT/IE ratio of the company is equal to 0,68 the associated rating according to the previous table is C and the spread amount to 10,5%. If the risk-free rate is equal to 0,5%, we will obtain a cost of debt equal to 11% (i.e. 10,5% + 0,5%).

According to Koller et al. (2010), the CAPM is the best approach we should follow if we want to estimate the cost of equity developing the WACC. Almost three years later, Jennergren (2013) recognises that even if poor, the CAPM is widely used for valuing companies and estimating the cost of equity capital. A possible explanation can be attributed to the fact that errors in cash flow estimates have probably a greater impact on firms' value than some discrepancies in the cost of capital. Moreover, it is an approach based on robust and few parameters. Unfortunately, we cannot conclude that the original version of such a methodology is the most appropriate to value distressed firms, since it tends to underestimate the risk of default. In addition, the WACC takes into account target weights (D/V and E/V), rather than current weights: if we expect the rebalancing will happen over a significant period (i.e. over the projection period in our case), we should employ a different cost of capital every year in order to be coherent with the capital structure each time. In practice, we should correctly model weights and changes in the cost of equity and debt. In case of extreme fluctuations in capital structures, the DCF approach with constant WACC as discount factor can lead to significant errors. Determining the correct discount factor for firms in financial troubles is surely a challenging task: different approaches to this problem have been proposed over time.

Among them, Damodaran (2006) supports the idea that distress event is often ignored in credit valuation, and the assumption about firm's ability to meet its financial obligations is often unrealistic. The author therefore proposes a different approach for estimating discount rates, which does not base on *regression beta*². Regression betas are usually involved for the cost of equity computation, while the cost of debt is derived by observing market interest rates or the interest rates on bonds already issued by the company itself. However, by considering *regression betas*, we take into account past prices over a long period (i.e. from two to five years) and distress events over short periods, underestimating the probability of distress. In other words, it can be the case that we estimate betas using two years of data, but the effects of distress on stock prices and debt-to-equity ratios are observable only in the last fraction of the regression period.

² We remember that *betas* in literature are considered as proxies for risk.

Hence, Damodaran (2006) identifies two alternatives to *regression beta* estimates: the CAPM beta adjusted for distress and the Distress Factor Model. The author aspires to value firms in financial troubles by means of approaches that are clear and relatively easy to implement. Indeed, they are appealing for analysts, even if they do not take into account the probability of distress in the discount factor correctly.

Almeida and Philippon (2007) complain that the “correct” discount factor incorporating true risk and uncertainty about firm’s future framework is very complicated to compute. They deviate the problem by adjusting the probability that distress actually occurs. They propose a way to incorporate systematic risk into default probability by using risk premia that are implicit in corporate yield spreads. By assuming that financial distress costs are more likely to occur in states of nature where bonds default, corporate bond prices can be used to estimate the distress-risk adjustment. It is possible to observe that actual spreads are greater than if they, conversely, would have been implied by historical default rates and that almost part of the additional premia reflects a systematic risk premium. From such spreads, we can derive a risk-neutral market implied probability of default that can be employed to estimate financial distress costs. In doing so, a tree valuation can be useful to illustrate the payoff to bond holders, considering a certain probability that a firm will default each year. In such a manner, we want to value the distress costs for a firm that has issued an annual-coupon bond maturing in one year, which is priced at par and promises a yield equal to y . The recovery rate is certain today and it is expressed by ρ . Therefore, in case of default on bond, creditors can recover an amount equal to $\rho(1+y)$. The actual value of the bond is determined by discounting the expected future cash flows generated, adjusted for the systemic risk of default. If q is the risk-adjusted probability of default in one year, the bond value can be defined as follows:

$$1 = \frac{(1 - q)(1 + y) + q\rho(1 + y)}{1 + r_f}$$

Where r_f is the risk-free rate. The default risk premium is therefore included in the probability of default q and it can be represented by the difference between the promised yield on bond and the risk-free rate. The risk-adjusted probability of default is certainly higher than the risk neutral one and it can be defined in this manner:

$$q = \frac{y - r_f}{(1 + y) * (1 - \rho)}$$

It is possible to perform a tree analysis for the cost of distress as well. In case of default, the loss in value is included in φ and it is assumed certain at the time of valuation. Obviously, if the firm is able to avoid default on its obligations, the loss in value is 0. Φ is the actual value of financial distress costs and it is given by:

$$\Phi = \frac{q\varphi + (1 - q)0}{1 + r_f}$$

Following such an approach, we are able to derive risk-adjusted probabilities of financial distress from yield spreads and recovery rates, without taking into account historical default probabilities, which usually have few predictive power in our valuation.

As a company comes under distress, its bonds fluctuate down the credit spectrum and consequently the expected default frequencies begin to rise. At some point, the risk is too high that default is taken as given and the price of the loan is associated to uncertainty in recovery values and loss given default. Such a change in the risk profile has a great influence on the price of credit-risky debt: the probability of default is driven primarily by the value of firms' equity, since it represents the difference between enterprise values³ and debt values, which in turns determines the distance to default. Therefore, by taking into account the relationship between equity and debt values, we can employ the price of common stocks to estimate the price of corporate debt obligations. Changes in share prices can be useful for estimating how bond prices should change as the operating environment fluctuates. In some circumstances, it is better to estimate the price of debt obligations directly from the price of common stocks, since most markets do not trade in public markets with the same frequency of stocks or they are not traded at all. Therefore, by involving a certain interrelation between companies' debt and equity securities, it is possible to value loans and securities in real time.

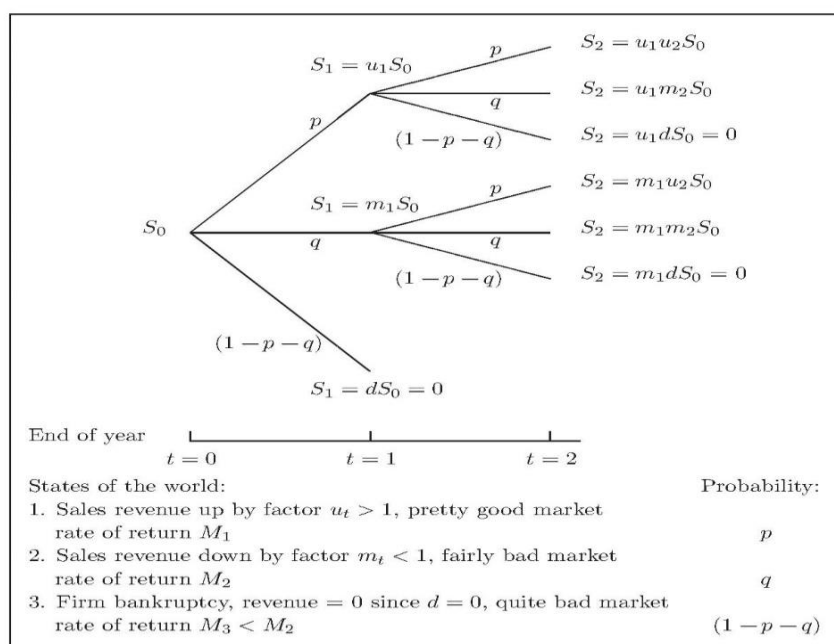
At this proposal, Guthner (2012) considers a typical industrial/manufacturing company with only one 10-year tranche of debt outstanding. The goal is to compute the price of such a loan by taking into account company's financial statements and information extracted from equity market as well. For example, based on a stock price of \$36.50 and an implied volatility extracted from option prices of 40%, the resulting bond price is \$98.618 and the yield to maturity is 5.18%. The estimate probability of default over 10 years is 6.77%, which corresponds to an

³ The enterprise value is obtained by summing the value of operations (core business) and non-operating assets, such as excess cash. Once we obtain such value, we can subtract the value of debt to compute indirectly equity value.

implied credit rating Baa3/BBB-. By observing the behaviour of stock and bond prices over time, we can make some interesting observations. Bonds issued by the company that enjoys very high rating (i.e. from Aaa/AAA to A3/A-) show low sensitivity to share price. This behaviour is related to the fact that companies whose asset value is largely greater than debt value have lower potential to default over a business cycle. Therefore, moderate changes in equity values do not have a large impact on expected default frequencies, causing a slightly variation of debt value. As the credit rating falls from B to BBB range, debt valuation is very responsive to changes in share prices. Companies have a higher amount of debt outstanding with respect to those in the high end of the investment-grade arena. Consequently, a change in the share price has a greater impact on firm's capital structure on a marked-value basis. If equity value tends to decline over time, the "cushion" that is a protection for creditors against the erosion in their collateral values reduces as well, causing an increase in the probability of default. Consequently, company's debt begins to assume some equity characteristics. If we consider levels of credit rating lower than CCC, financial distress and expected default frequencies are very high. As default becomes certain and the value of firm's debt drops significantly, the latter begins to assume the characteristics of a defaulted security. Debt values translates to loan estimated recovery values in a default scenario. Therefore, changes in equity prices may influence the probability of default but do not affect the value of distressed debt. At default, loss given default and uncertainty related to its estimates drive the value of firm's debt, which trades at a discount with respect to its expected amount. The previous analysis can be further extended to include the possibility that default may occur not only at the end of the first year, but rather in future years as well, assuming that the risk-adjusted probability of default and the risk-free rate do not change over time even if this is counterfactual and not met in reality.

A different approach for valuing distressed firms has been proposed by Jennergren (2013). Sales revenues are the driving variable, bankruptcy risk and other sources of uncertainty are considered explicitly. The object of valuation is an unlevered firm whose revenues over time are represented by a tree diagram. A node defines each year of valuation. At the end of each year, it is possible to distinguish among at least three states of nature. The first state is characterised by a jump in sales revenues that is captured by a multiplicative up factor $u_t > 1$ and a pretty good market rate of return M_1 . A probability p is associated to such state of nature and it is supposed constant over time. The second state of nature is characterised by a moderate jump in sale revenues $m_t < 1$ and a fairly bad market rate of return M_2 . Therefore, with a probability q , the value of firm's sales after a year could be $S_1 = m_1 * S_0$, where S_0 is the actual

value of firm sales. To the third state of nature, the associate probability is given by difference (i.e. $1 - p - q$) and it is the worst one for the firm and economy as a whole: there is a bankruptcy down jump $d = 0$, leading to revenues closed to 0. Moreover, the market rate of return is quite bad and even lower with respect to the intermediate step (i.e. $M_3 < M_2$). It is discounting back all the nodes in the tree that we can determine the value of the unlevered target. By considering this approach, as we following figure shows, cash flows in each node depend on three main factors: period of valuation, sales revenue and type of jump in relation to the state of nature where the company moves. Once we obtain an estimate of the unlevered firm value, the value of the levered firm can be therefore obtained in three methods: as the sum of the unlevered value and the tax shields, as the sum of debt and equity values or discounting cash flow by levered WACC.



Source: Jennergren (2013)

Discount rates in this model are derived from the CAPM taking into account some basic parameters: the risk-free rate, the jump probabilities p and q , the market rate of return characterising each state of nature, other than jump factors.

According to Damodaran (2010), there seems to be at least two excuses for not considering the event of distress explicitly. Some analysts believe that discounted cash flow valuations already

include the effect of distress both in discount rates and in cash flows determination, but such inclusion is hidden somewhere, it is not explicit. When we use expected cash flows that already incorporate the probability of distress and discount rates adjusted for a high risk of distress, there is not a material impact on value with respect to approaches that consider the distress event explicitly. The main point is therefore computing financial forecasts in an accurate fashion. The valuation model proposed earlier can be criticised for its simplicity: three possible states of nature for every node are not enough to describe all the possible events that can happen in the real world. Moreover, it simplifies the process that leads into bankruptcy, by assuming that such event occurs with a fixed probability every year. This means that, bankruptcy is not more likely to occur after a number of previous bad years as it usually happens in reality; it is reasonable to let bankruptcy depending on preceding firms past history.

Finally, Meitner and Streitferdt (2014) propose a recent valuation method for distressed companies where the economic landscape is characterised by a deep world recession and severe operating problems, such as low or negative cash flows. As other practitioners, they find some difficulties in determining the right *beta* to include in the determination of risk-adjusted discount rate according to the CAPM. Historical data are not very useful since they usually refer to normal business circumstances and consequently generate significant valuation errors. In order to value a company in financial troubles, we can involve two approaches that allow taking into account for uncertainty explicitly: the *risk-adjusted discount rate* and the *certainty equivalent* approach. According to the former, risk is considered in the denominator of the valuation formula: the actual firm value is computed by discounting expected cash flows during and after the projection period at the risk-free rate plus a risk-adjustment factor that takes into account the specific firm's financial conditions and the situation of the economy as a whole. On the other side, the *certainty equivalent* approach takes into account risk in the numerator of the valuation formula. With respect to the CAPM environment, both approaches can be applied when dealing with distressed firms. For simplicity, we will consider two time intervals: $t = 0$, that represents the actual valuation date and $t = 1$ that stands for a certain period in the future. According to the DCF approach, we need an estimate of the expected cash flows generated by the firm during the projection period and an estimate of its continuing value as well; they will determine the numerator of our valuation formula.

Following the *risk-adjusted discount rate* approach, the risk-free rate is adjusted by multiplying *beta* (as a relevant measure of risk) for the market risk premium. Analytically, we can write the expression of firm's value as follows:

$$V_0 = \frac{E(CF_1) + E(CV_1)}{1 + r_f + z}$$

Where z is the risk-adjustment to the discount rate. The latter can be expressed as a function of firm's *beta* (β_i) and the market risk premium ($E(r_m) - r_f$):

$$V_0 = \frac{E(CF_1) + E(CV_1)}{1 + r_f + \beta_i * (E(r_m) - r_f)}$$

The previous formula can be reformulated by taking into account the traditional CAPM *beta* definition.

Therefore, we obtain:

$$V_0 = \frac{E(CF_1) + E(CV_1)}{1 + r_f + \left[\frac{Cov(r_i; r_m)}{\sigma_{r_m}^2} \right] * (E(r_m) - r_f)}$$

Then, we use λ as expression of the market price of risk, i.e. $\lambda = \frac{(E(r_m) - r_f)}{\sigma_{r_m}^2}$:

$$V_0 = \frac{E(CF_1) + E(CV_1)}{1 + r_f + \lambda * Cov(r_i; r_m)}$$

With respect to the *certainty equivalent* approach, we need to adjust the numerator of the formula. The certainty equivalent represents the amount of money that - if received with certainty - would make the decision-maker indifferent between receiving that amount or the expected but uncertain one. According to such an approach, the valuation formula becomes the following:

$$V_0 = \frac{CE(CF_1 + CV_1)}{1 + r_f} = \frac{E(CF_1 + CV_1) - \lambda * Cov(CF_1 + CV_1; r_m)}{1 + r_f}$$

And:

$$V_0 = \frac{CE(CF_1) + CE(CV_1)}{1 + r_f} = \frac{E(CF_1) - \lambda * Cov(CF_1; r_m) + E(CV_1) - \lambda * Cov(CV_1; r_m)}{1 + r_f}$$

By imposing some assumptions on the difference between healthy-distressed firms and involving the findings by Modigliani and Miller (1958), we can try to find out a relationship

between their respective discount rates. In an ideal world where certainty reigns, the interest rate on bonds can be considered as a proxy for the cost of capital, regardless to the firms' capital composition. While, when dealing with uncertainty, some “risk discounts” need to be subtracted to the expected yield, or a rather risk premium to the market rate of return should be incorporated.

In identifying such a relationship, we assume that the distressed firm generates a low amount of cash flows; more precisely, it is computed by subtracting to the cash flows of the healthy firm a constant amount every year. Moreover, we assume that the cash flow reduction does not add relevant risk in our valuation, meaning that the CAPM-relevant risk position does not change only because we add financial obligations in the company. Despite the fundamental cash flow risk is the same for both companies, there are risk-adjusted discount rate differentials between distressed and healthy companies:

$$r_D = r_H + (r_H - r_f) * \frac{D_0^{D,virtual}}{V_0^D}$$

Where $D_0^{D,virtual}$ represents the market value of “virtual risk-free debt” and V_0^D the value of the distressed company in $t = 0$. Consequently, it seems possible determining the actual value of a distressed firm from both the actual value of a healthy firm and the market value of “virtual risk-free debt” in the following way:

$$V_0^D = V_0^H - D_0^{D,virtual}$$

Anyway, in reality we know that distressed companies exhibit quite different cash flows risk structures with respect to healthy entities. Therefore, the difference between values of healthy firms and distressed firms is not risk-free, but there is a rather certain amount of risk that can be represented in mathematical terms by a non-negative correlation between the cash flow difference and the market rate of return. Consequently, we need to adjust the relationship between discount rates for distressed and healthy firms in this way:

$$r_D = r_H + (r_H - r_{virtual\ debt}) * \frac{D_0^{D,virtual}}{V_0^D}$$

Where $r_{virtual\ debt}$ is the CAPM-relevant cost of the “virtual debt” in the distressed company. After recognising that distressed and healthy companies show different cash flow risk, we face a computational problem: differently from financial debt, the “virtual cost” of debt cannot be

computed directly since it is not observable. A different approach for discount rates determination should therefore be pursued by analysts when valuing firms with financial troubles. A good practice involves a deep analysis of the risk profile of the company, reminding that distressed companies are normally characterized by low or even negative cash flows. According to this, Meitner and Streitferdt (2014) provide some examples, but almost all of them assume the same fundamental cash flow risk for distressed and healthy companies, determining the rate of return on virtual debt directly.

CHAPTER TWO: Option Pricing Theory and Merton's model

2.1 The assumptions of the model

The Merton's model tackles the problem of pricing and hedging a European option (call or put) on a non-dividend paying stock⁴. It forms the benchmark model for pricing options on a variety of underlying assets including equities, equity indices, currencies, and futures. The assumptions of the model follow directly those made by Black and Scholes (1973). They are in a certain sense "ideal" and simplistic, starting from the assumption that asset prices follow a Geometric Brownian motion with known mean and variance. Thus, the evolution of asset prices over time can be described by a *normal* distribution⁵. In this pricing model, the Modigliani and Miller theorem is valid; thus, the value of the firm is independent from its capital structure⁶, the interest rate is constant and it is risk-free. There is a sufficient number of investors with more or less the same wealth, so they can buy or sell the desired quantity of assets. Investors can borrow and lend at the same interest rates and they do not have any penalties in practicing the short-selling procedure. Trading takes place continuously and there are not arbitrage opportunities. We consider firms with a simple capital structure; it is composed of equity and a single issue of a zero coupon bond (i.e. a risk-free debt with a single maturity date T). We know that in reality firms have very complicated capital structures made up by different bond issues (i.e. bonds with different maturity, seniority, remuneration methods), convertible stocks, privileged shares and so on. As with all "structural models", Merton's model specifies how the value of a firm evolves over time. In fact, the economic value of firm's assets is modelled by mean of a stochastic process, which allows taking into account that future assets values are uncertain:

$$dV_t = rV_t + \sigma V_t dW_t$$

Where r is the drift of the firm's value and σ its volatility; they are both constant. dW_t is a Geometric Brownian motion (GBM). By assuming a *normal* (or *lognormal*) diffusion process for asset values and constant volatilities, the value of the firm's assets at a generic time t can be obtained from the following formula:

⁴ Afterwards, the valuation has been extended to include a stock paying dividends, but it is not the purpose of this elaboration.

⁵ This is a strong assumption with important implications that will be discussed later in our valuation. In reality, by observing the market we can state that asset prices usually jump and do not evolve continuously.

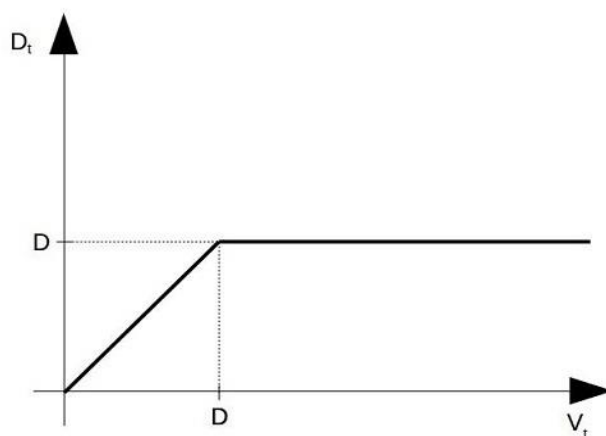
⁶ In particular, the value of the firm's assets equal the sum of debt and equity values. For further information see Miller M. and Modigliani F. (1958).

$$V_t = V_0 * \exp \left\{ \left(r - \frac{\sigma^2}{2} \right) t + \sigma \sqrt{t} W_t \right\}$$

Where V_0 is the initial value of the assets at time 0.

2.2 The payoffs to Debt and Equity holders

According to Merton (1974), there are no covenants triggering default before the maturity date on debt, whose face value is D . Thus, if at maturity T the firm's asset value exceeds the promised payment D , lenders will receive the promised amount, leaving the residual asset value to shareholders. While, if the asset value at maturity is lower than the promised payment, lenders will receive a payment equal to the residual asset value and shareholders will get nothing (Hull et al. 2004). Thus, we can distinguish the payoffs to equity and debt holders in the following way: on date T , there are two possibilities: if $V_T \geq D$, then debt holders get the promised payment D ; otherwise, they get the firm value they can extract at maturity T . Thanks to Merton's paper (1974), we can rewrite the payoff in terms of option notations. Debt holders receive the $\min \{V_T, D\}$ or, in other terms, they receive: $D_T = D - \max \{D - V_T, 0\}$. In this way, we recognise that debt holders have a long position on a default-free bond paying D at maturity and a short position on a put option on the firm's value with strike price equal to the face value of debt at maturity. The riskiness of the zero coupon bond located in the firm's capital structure is linked to the possibility that the firm's value at maturity will be lower than the promised payment to debt holders. The following figure shows the payoff of debt related to asset value:



Source: adapted representation of a put option

By involving the *Put-Call parity* and Sundaram (2010), the value of a firm's risky debt (i.e. the market value of debt) is equal to the value of a risk-free discount bond minus the value of a put option. In fact, for low values of the firm, we consider the possibility to conclude an agreement (i.e. a reorganization or restructuring plan) between creditors and management; in other words, when $V_T < D$, debt holders are willing to accept a discount and eventually a delay on their promised payments, rather than declaring the firm's bankruptcy and obtaining the liquidation value. They are disposed to accept such agreement if the estimated going-concern value of the firm is larger than the value reachable from plausible alternatives (i.e. change in ownership or combination with other entities and firm's liquidation). The main point we want to stress through this valuation is that, in a distress situation, the nominal and economic value of debt are usually different, while these values tend to converge in normal circumstances. By accepting a discount on their credits, debt holders are implicitly increasing the equity value of a firm. Indeed, the difference between the nominal and economic value of debt represents the price of a put option on debt, which in turns is a proxy of the benefits derived by shareholders thanks to the reorganization plan. Analytically, we can derive the following relationship:

$$D_{mkt} = D_{rf} - (Put\ Option)$$

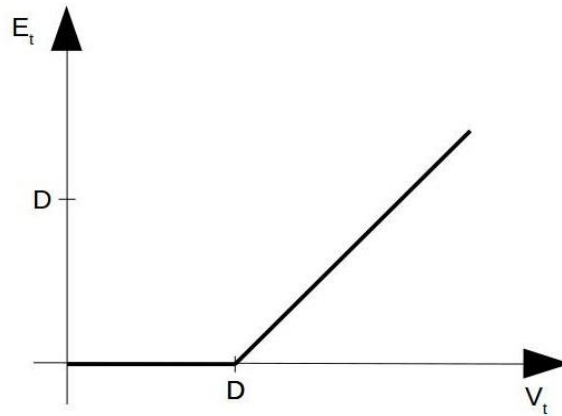
Where: $D_{rf} = D * e^{-r_f(T-t)}$.

D_{rf} is the actual value of the sum of the cash flows expected to debt holders, as designed in the reorganization plan, and r_f is the risk-free rate.

We can now determine the payoff to equity holders according to real option theory as well. If at maturity of firm's obligations, the enterprise value is larger than the face value of debt (i.e. $V_T > D$), then shareholders will get the residual amount ($V_T - D$), in line with the Absolute Priority Rule⁷. In the worst scenario, the firms do not have enough proceeds to reimburse the promised amount debt holders, leaving equity holders without any money in their pocket. According to real option theory, the payoff to equity holders can be therefore replicated by holding a long call position on the company's assets, with a strike price equal to the face value of debt and maturity T : $E_T = \max \{0, V_T - D\}$. For equity holders, it is optimal to exercise the

⁷ The Rule of Absolute Priority stipulates an order of payment -creditors before shareholders- that we assume being valid even before the liquidation event. By accepting the reorganization plan, creditors pretend the reimbursement prior to all the other claimants.

option as V_T gets larger than D ; as a matter of fact, they have enough proceeds to repay debt holders, keeping the residual for themselves. The payoff distribution as explained before is in line with the common attribution of the firm's ownership and control: it is usual to attribute the ownership of the firm to creditors, while shareholders hold a call option of the firm's assets.



Source: adapted representation of a call option

2.3 Equity as a Call Option on firm assets

By recognising the presence of real options in the payoff to equity and debt holders, we are allowed to employ the Black-Scholes and Merton's model for the computation of the firm value. At the beginning of his pricing valuation, Merton (1974) focuses his attention on the Black and Scholes formula:

$$C = Se^{-y(T-t)}N(d_1) - Ke^{-r_f(T-t)}N(d_2)$$

With the boundary condition: $C = \max(0, S - K)$ at expiration and

$$d_1 = \frac{\left\{ \log\left[\frac{S}{K}\right] + \left(r_f - y + \frac{1}{2}\sigma_s^2\right)(T-t) \right\}}{\sigma_s\sqrt{(T-t)}} \quad \text{and} \quad d_2 = d_1 - \sigma_s\sqrt{(T-t)}.$$

Where C is the price of a call option on company stock, S is the price of the stock, K is the strike price, y is the dividend yield⁸, r_f is the risk-free rate, σ_s is the standard deviation of stock returns and $N(\bullet)$ the cumulative standard normal distribution function⁹.

In his paper, Merton (1974) proposes an adjusted version of the previous formula, as a function of the firm's value (i.e. V) and the face value of debt:

$$E = Ve^{-y(T-t)}N(d_1) - De^{-r_f(T-t)}N(d_2)$$

Where

$$d_1 = \frac{\{\log\frac{V}{D} + (r_f - y + \frac{1}{2}\sigma_V^2)(T-t)\}}{\sigma_V\sqrt{(T-t)}} \text{ and } d_2 = d_1 - \sigma_V\sqrt{(T-t)}.$$

The boundary condition is still the same (i.e. we are still considering a call option), E is the equity value of the firm, V the value of the firm's assets, D the face value of debt, y the dividend yield and σ_V in the standard deviation of the enterprise value.

We can observe the differences from the original Black and Scholes formula: the underlying price consists in the value of the firm's assets, the strike price is given by the face value of debt at maturity¹⁰ and the standard deviation of the asset returns (i.e. σ_V) replaces the standard deviation on equity returns. In fact, as Guthner (2012) points out in his valuation, option-pricing models referring to the value of the firm's assets as state variable should no longer take into account the volatility of stock return but rather the volatility of asset returns.

2.3.1 From the volatility of Equity returns to the volatility of Asset returns

The closed-form option pricing formula derived by Merton (1974) offers a new theoretical approach for valuing risky debt, which combines option pricing theory and corporate finance theorems. Such an approach is appealing for valuing risky assets with only a small set of variables. Almost all the parameters involved in the valuation can be observed from the market (such as the risk-free rate) or firm's financial statements (such as the face value of debt). The

⁸ We assume that the dividend rate is 0, but the valuation can be extended by assuming a positive dividend yield.

⁹ $N(\bullet) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\bullet} \exp\left[-\frac{1}{2}z^2\right] dz,$

¹⁰ We remember that Merton does not involve the possibility of default prior the maturity of the bond.

two unknown variables are the market value of the firm's assets and the value of assets' standard deviation; the latter is included in d_1 and d_2 of Merton's formula. For the first unknown we have already proposed a solution: thanks to the DCF model, we compute the actual value of the firm's assets as the sum of the present value of the expected cash flows generated during the projection period and the CV, using proper discount rates. The major problem consists in estimating assets' standard deviation: the enterprise value is not quoted in the market, differently from the equity, impeding the computation of σ_V in a standard fashion. A feasible solution for a listed firm is an estimate of standard deviation (or volatility) of asset returns as a function of standard deviation of equity returns, by solving the preceding Black and Scholes formula in reverse. In other words, if we assume that the price of equity and thus the price of the call option are correct, we can compute an estimate of implied equity volatility from those prices. Once we have an estimate of the implied equity volatility, we can compute an option price and compare it with the market price of such option. If the resulting price is above the market price, we should obtain a second estimate of the implied volatility that is lower than the first one. Conversely, if the price obtained by applying the model is lower than the market price, we try with a higher estimate of volatility. The process continues until we obtain a price for the call option equals to the market price of such option. Implied volatility determined in this method allows having a forward-looking estimate of future equity and asset volatility, which can be different from realized historical volatility. In case of non-listed firm, this approach can be applied with reference to some (at least one) comparable listed firm.

We should explain the relationship between equity and asset volatility before computing the volatility of asset returns; in doing so, we involve option pricing theory and Black-Scholes and Merton's model as well. According to the previous literature, equity value can be considered as a function of the firm's value and time¹¹, given that it behaves like a call option on the enterprise value: $E = f(V, t)$. Then, by applying Ito's formula, we can derive the dynamics of equity through time in the following way:

$$dE = \frac{\partial E}{\partial t} dt + \frac{\partial E}{\partial V} dV + \frac{1}{2} \frac{\partial^2 E}{\partial V^2} d < V >$$

$$= \left[\frac{\partial f(V, t)}{\partial t} + \frac{\partial f(V, t)}{\partial V} * V * r + \frac{1}{2} \frac{\partial^2 f(V, t)}{\partial V^2} (V * \sigma_V)^2 \right] dt$$

¹¹ As explained earlier, equity behaviour can be approximated by a call option on the enterprise value and debt behaviour like a put option on the same underlying asset.

$$+ \frac{\partial f(V, t)}{\partial V} * V * \sigma_V * dW_t$$

We are interested in the stochastic components of equity dynamics: $\frac{\partial f(V, t)}{\partial V} * V * \sigma_V * dW_t$, that allows obtaining a closed-form expression for equity standard deviation (or volatility). Thus, we can set $dt = 0$ and divide both side of the previous equation by E :

$$\frac{dE}{E} = \frac{\partial f(V, t)}{\partial V} * \frac{V}{E} * \sigma_V * dW_t$$

Then, by substituting $\frac{dE}{E}$ with $\sigma_E * dW_t^{12}$, we can simplify the equation and obtain the final result:

$$\sigma_E = \frac{\partial f(V, t)}{\partial V} * \frac{V}{E} * \sigma_V.$$

Professionals recognise that $\frac{\partial f(V, t)}{\partial V} = \frac{\partial E}{\partial V}$ is the sensitivity of firm's equity with respect to changes in firm's value; the ratio is known as the *delta* of an option, which in turns is equal to $N(d_1)$, we have found in deriving the Black and Scholes equation do far. We can therefore rewrite the previous formula in this way:

$$\sigma_E = N(d_1) * \frac{V}{E} * \sigma_V$$

We can observe that standard deviation of equity returns depends linearly on standard deviation of the asset returns, firm's leverage and sensitivity of equity value to changes in asset value (in a percentage term). It depends on time to maturity as well, since it is included in d_1 formulation. The previous equation is useful to obtain both implied and historical asset volatility; in fact, it can be rearranged to express the value of the asset standard deviation as follows:

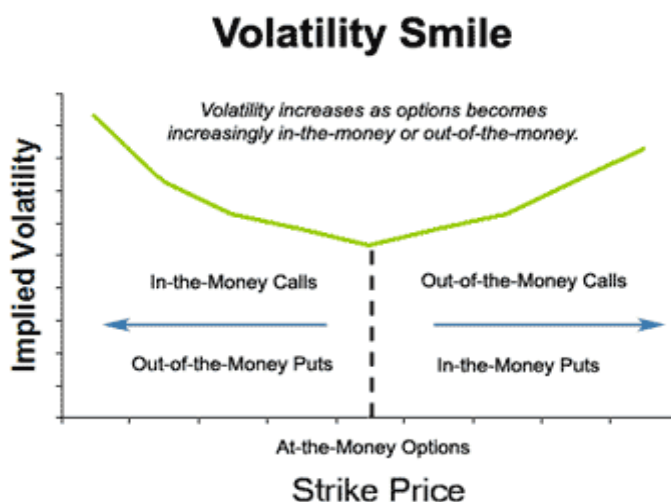
$$\sigma_V = \sigma_E * \frac{E}{V} * \frac{1}{N(d_1)}$$

The main difference between implied and historical asset volatility is that the latter is computed from historical equity volatility, and thus it is a function of past data.

¹² Equity is a call option on the firm's assets, which follows a stochastic process driven by a GBM, thus we can rewrite the dynamics of E in a similar way, assuming $dt = 0$.

2.3.2 Black-Scholes-Merton's volatility limitations and possible solutions

Merton's volatility involves some important limitations that compromise our estimates. The strictest assumption characterising Black-Scholes and Merton's model is probably the one associated to constant volatilities. If we assume the option pricing correct, n call options with the same underlying assets but different strike prices are likely to share the same value of implied volatility, which is employed in the valuation formula. Nevertheless, we can observe from the market that options on the same underlying securities but with different strike values and expiration times yield different values of implied volatility. This finding is not achieved with traditional valuation techniques, since implied volatilities can be represented graphically as flat lines, independently from the value of strike prices. In reality, representations of implied volatility show some skewness¹³, which is especially justified by the presence of the firm's leverage. Another volatility shape that differs from the original pricing assumption is termed "volatility smile": a symmetric distribution similar to a smile as shown in the figure below, which suggests that implied volatility has a positive concavity with respect to the strike price. Consequently, estimates of implied volatilities employed in Merton model tend to be biased and distorted with respect to market experience.



Source: www.theoptionguide.com

¹³ According to Corrado and Su (1997) a volatility skew is a pattern that results from calculating implied volatilities across a range of strike prices for a certain class of options (i.e. put or call options).

By observing the market, it is clear that asset volatility fluctuates over time: there are periods with very low or rather very high volatility and it is difficult to understand if the firm faces the first or the second scenario. In order to improve the option-pricing model developed by Merton (1974), we should incorporate the likelihood of volatility fluctuations over time. Thus, implied volatility should not be considered as a deterministic (i.e. constant) number, but rather as a random variable that changes with time. Stochastic asset volatilities can be obtained by introducing a stochastic volatility model in which the value of a contingent claim depends mainly on the randomness characterizing future values of firm's assets and its volatility. A popular model dealing with stochastic volatility is the Heston one (1993). Heston develops Black-Scholes and Merton patterns by taking into account stochastic volatilities and a different price distribution. He exploits the correlation between volatility and spot asset returns, which seems relevant in explaining return skewness and strike price bias in the Black-Scholes and Merton model. Following Gatheral and Lynch opinion (2004), stochastic volatilities models allow to explain consistently why stocks with different strike prices and maturities have different implied volatilities. By looking at the distribution of stock price returns, we can observe that it is highly peaked and fat tailed relative to a normal distribution. Effectively, the market usually attributes higher probabilities to extreme events with respect to a *normal* distribution, leading to higher probabilities of the default for firms. Empirical studies confirm that even assets' log-return distribution is not *normal*, but rather shows heavy tails and high peaks (leptokurtic); such behaviour justifies the relation between equity and asset returns, which in turns drives equity and asset volatilities. However, if we abandon the *normality* assumption of asset distribution over time, we cannot derive a formula for asset return volatility as a function of equity volatility and firm's leverage. Moreover, we cannot obtain a closed-form solution for option prices anymore. We can therefore conclude that a stochastic volatility model can solve the problem associated to constant volatilities but, on the other side, it complicates our estimates of asset returns volatilities. Rather than considering stochastic volatilities, we can focus our attention on historical estimates of volatility. Indeed, we need to take into account the returns achieved by debt and equity in the past. With respect to debt returns, we refer to time-series data of prices and accrual interests on debt, while we consider data of prices and dividends for equity returns. From these data, we calculate the standard deviation of each financial asset, taking into account the correlation of returns: in fact, we expect to have a positive correlation between debt and equity securities as their price tend to rise with the enterprise value. In practice, the correlation between debt and equity is less than perfect (i.e. lower than 1.0),

especially in case of operational and financial restructuring¹⁴. Given the estimates of debt and equity volatilities, we can compute the volatility of the enterprise value as a weighted average, where the weights consist in the fraction of debt and equity outstanding on a market-value basis:

$$\sigma_V^2 = x_E^2 \sigma_E^2 + (1 - x_E)^2 \sigma_D^2 + 2\rho_{E,D} \sigma_E \sigma_D x_E (1 - x_E)$$

x_E is the fraction of the firm capital structure represented by equity (i.e. $x_E = E / E+D$) and $(1 - x_E)$ by debt, σ_E and σ_D are respectively the standard deviations of debt and equity returns. The historical correlation of debt and equity returns is thus represented by $\rho_{E,D}$. Such a solution has some restrictions as well: historical values of debt and equity returns lead only to a backward-looking valuation, rather than a forward-looking one. Furthermore, it is difficult to determine exactly a value for the correlation between debt and equity returns. On the other side, it allows us abandoning the hypothesis of *normality* of equity and asset returns.

2.3.3 Final considerations about volatility estimates

After a deep investigation about the methods for the computation of firms' volatility, we can conclude that a more precise estimate - following the findings of Correia et al. (2015) - derives from a combination of:

- a) Historical volatility of returns;
- b) Implied volatility from equity options
- c) Firm's financial statements.

It can be shown that a mixture of the three methodologies improves the explanatory power of corporate bankruptcy models, when dealing with publicly traded companies. If companies are not listed, valuation procedures cannot take into account quoted equity prices but rather only historical financial performances or comparable companies. From such data, analysts try to compute an estimate of firm's asset volatility by distinguished among different industries, sectors. They take into account a set of drivers influencing asset volatility, which includes systemic and company specific factors as well. Among the systemic factors we can identify the

¹⁴ Restructuring can involve transferring value from shareholders to bondholders or vice versa. While, in case of takeovers, shareholders tend to benefit more than bondholders. For further information, see Guthner M. W. 2012.

business in which the firm operates and the economic cycle. While, among the company specific factors there are the stability and the quality of the management, the relationship between labours and management, changes in business strategy and so on.

In credit markets, total volatility plays a central role: despite the source of volatility can be systematic or idiosyncratic, it is critical to measure both in order to investigate whether future asset values will fall below a certain default threshold. Consequently, if we limit measures of asset volatility only to systematic sources, we will generate inferior estimates of default probabilities. Using a large sample of firms belonging to different sectors with liquid corporate bond data, the mixing of information about volatility from market and accounting based sources allows improving estimates of corporate bankruptcy with respect to situations in which only market based values are taken into account. A naïf measure of historical asset volatility is obtained as a function of historical equity volatility (σ_E) multiplied by the ratio of market value of firm's equity to the book value of debt (ω), which allows de-leveraging historical equity volatility:

$$\sigma_V^{NAIVE} = \sigma_E * \omega$$

A second estimate of historical asset volatility combines historical credit and equity market data and it is obtained according the formula shown in the previous paragraph:

$$\sigma_V^\omega = \sqrt{\omega^2 \sigma_E^2 + (1 - \omega)^2 \sigma_D^2 + 2\rho_{E,D} \sigma_E \sigma_D \omega(1 - \omega)}$$

Where ω is defined as the fraction of asset value attributable to equity, σ_D is the annualised standard deviation of total monthly bond returns and $\rho_{E,D}$ is an estimate of the historical correlation between equity and debt returns. Implied volatility estimates (i.e. forward looking estimates of volatilities) are computed by considering Black-Scholes and Merton's models and therefore at-the-money call and put options. Even in this case, we can compute two asset volatility estimates based on implied volatilities following the approaches described earlier. With respect to the accountable approach for measuring asset volatility, an average of the returns on net operating assets are taken into account, since they are considered as a measure of enterprise profitability. By regressing the probability of firm's default on those different estimates of asset volatility, it is possible to observe that each of them is significantly positively correlated to the probability of bankruptcy: if market or fundamental based volatility increases, credit spread increases too, after controlling for leverage. In addition, Correia et al. (2015) provide models that allow to examine the combination of different component measures of asset

volatility, such as equity volatility or implied volatility (given the fact they are correlated they should not be considered in pairs in order to avoid correlation problems), volatility from credit markets and fundamental volatility. When models take into account all the sources of asset volatility for determining the likelihood of bankruptcy, the resulting goodness of fit (i.e. R^2) is higher with respect to the case in which only equity volatility is taken into account. Therefore, it turns out that a combination of market and accounting based measures of volatility provides superior results with respect to each source considered alone.

CHAPTER THREE: The Reorganization Plan

3.1 Introduction

As more and more firms defaulted on their debts or filed for bankruptcy in the recent recession, there is an increasing interest in understanding how firms can deal with distressed situations. Different mechanisms have been introduced over time in order to overcome firms' financial difficulties. The academic literature has identified different possible solutions and at the same time, according to Kose (2001), investors have become increasingly concerned about default risk and valuation of distressed securities. Anyway, in his opinion, there is already an extensive literature related to financial distress management, whereas valuations of distressed securities in troubled reorganization have not been developed completely until that time. In general terms, all the possible solutions to financial crisis can be classified as private (i.e. informal) or Court-supervised (i.e. formal) insolvency procedures. Each of them has associated advantages and disadvantages that will be discussed in the following paragraph.

Previous researchers in this field provide different suggestions to financial distressed companies to join a resolution. Referring to this, Asquith and al. (1994) take into account the restructuring of firms' assets and liabilities, asset sales, mergers, capital expenditure reductions and layoffs on the asset side. Debt reorganization in a process letting distressed firms to modify outstanding debt contracts (with the consent of creditors) in order to reduce their exposure and improve capital conditions; it can require a reduction of promised interests or principal payments, maturity extensions or the placement of new equity securities among creditors. With respect to partial sell-offs of existing assets for distress resolution, managers can benefit from the acceleration of future cash flows originated by those assets and the reduction of outstanding firms' obligations through the proceeds obtained as well. Anyway, by combining poor financial conditions and urgent need of liquidity, the bargaining power of debtors is likely to be very low and therefore the price they can obtain for selling the asset is moderate. In addition to the previous quoted alternatives, Lemma et al. (2012) consider the injection of "new money" in distressed firms, even if they are aware that raising money can be very complicated because of the high risk involved. Successful restructurings usually require debtors to access additional finance that allows continuing operations.

As we will discuss later, in some circumstances there are some potential impediments to informal workouts; the Court intervention seems an alternative explanation for limiting the negative effects of distress. Even in this framework, the goal is to develop a consensual

restructuring with creditors while preserving firms' value. Firms can continue to operate, debt payments persist, secured creditors cannot take possession of collaterals and new borrowings have the priority with respect to pre-bankruptcy claims. The debtor has the right to propose a reorganisation plan whose exclusivity period can be extended only after the Court approval. The Court oversees all firms' operations as well. Almost all insolvency laws in different countries consider liquidation procedures where the control shifts to creditors, some assets are sold and firms continue to operate as going-concern. However, as Hotchkiss and al. (2008) highlight, there some differences in the provisions for Court-supervised reorganisations: for example, some countries offer few alternatives to sales of distressed firms' assets, whereas others guarantee larger protections to incumbent managers and equity holders in order to facilitate the continuation of operations.

When the financial situation is deeply depressed, firms usually incur into liquidation procedures, which have the goal to ensure that all companies' affairs have been dealt with and all assets being realised. In this legal process, a liquidator is appointed to conclude the affairs and at the end, the firm ceases to exist. Sometimes it can be argued that firms that are not able to compete in the market, because of the inability of meeting obligations at maturity, as debt value is higher than asset value, should be removed from the marketplace.

3.2 Advantages and disadvantages

In this chapter and in our *case study* too, we will consider a firm in financial trouble that has been able to conclude a private agreement with its creditors (i.e. reorganization plan), avoiding the Court intervention to solve problems. When a debtor is not able to repay its debt as it comes due, most legal systems provide a legal mechanism to satisfy the outstanding claim holders on assets. Moreover, most legal mechanisms contain rules with various types of proceedings that allow firms to solve their financial difficulties. Among them, we can distinguish between formal and informal insolvency proceedings, even if they share the same restructuring goals. Insolvency laws govern the first proceedings, while the second ones generally involve voluntary negotiations between debtors and creditors. Even if they are not regulated by the law, their effectiveness depends on an insolvency law, which can provide incentives to the reorganization. In some cases, informal procedures can be complementary to formal procedures. The restructuring of firm's debt through out-of-Court procedures can involve measures that reorganise debtor's business (*operational restructuring*) other than debtor's finance (*financial*

restructuring). They perform an important role in all insolvency systems, providing a tool for creditors and firms to protect their respective interests in an efficient fashion. In order to decide whether out-of-Court procedures are better than formal procedures, we need to consider relative advantages and disadvantages. According to Garrido (2012), one of the main advantage connected to informal procedures is their flexibility¹⁵ and ease of adoption with respect to the specific needs of debtor's business. Under numerous insolvency laws, limits to provisions in reorganization plans can be very strict, especially if they involve a minimum credit recovery or a payment delay. In addition, negotiations of new securities and provisions of additional money are also easier to obtain through private agreements. In a framework where corporate financial difficulties dominate, workouts provide a quick time-response: the process is typically shorter and there are not delays caused by the intervention the judicial system. Consequently, it seems easier for the debtor to continue operations through private agreements than formal procedures, which are more expensive in terms of time, money and reputation.

Following Fischer and Wahrenburg (2012) opinion, firms with viable going-concern values should try to pursue out-of-Court restructurings, since they cause less distortion to the business where they operate other than provide quicker responses to distress.

Anyway, there are some disadvantages associated to out-of-Court procedures as well. The first one is associated to the difficulty in analysing the debtor's financial and economic situation in a relatively short time framework. In addition, the contractual nature of the workout requires the consent of the target firm and therefore the shareholder's vote, while formal procedures allow the liquidation of distressed firms without their consent. With respect to the target, it is sometimes difficult to deal with a great number of creditors and find a common solution through informal procedures. Empirical studies about publicly traded company in U.S. demonstrate that a feasible workout depends on the concentration of debt structure: firms with dispersed debt should adopt formal insolvency procedures, while firms with concentrated debt should prefer out-of-Court procedures. On this subject, Gertner and Scharfstein (1991) recognise that coordination problems can generate important sources of inefficiency: the target might have problems of under investments (i.e. it might forgo investments with positive net present value) or over investments (i.e. shareholders may want to increase the riskiness of cash flows). High indebtedness and poor financial performance are crucial features for pushing a company into distress. We expect highly leveraged firms with a large portion of debt due in the short-run

¹⁵ An example of the mentioned flexibility is related to the Absolute Priority Rule that does not apply in out-of-Court insolvency procedures.

facing some time constraints in attempting to reorganize their debt through out-of-Court procedures. Debt structure (in terms of composition and seniority) is therefore crucial for determining the choice between in and out of Court procedures in the form of both debt seniority and composition.

3.3 The contents of the Reorganization Plan

A restructuring plan or workout should incorporate changes in the financial distressed business, other than debt and financial structures. According to Garrido (2012), it consists in an opportunity for the target to restructure its activities and adjust cash flows to the maturity of debt. Sometimes it can benefit from additional funds to overcome liquidity problems as well¹⁶. The agreement between managers and creditors can include different suggestions, such as the transfer of firms' assets without changes in ownership¹⁷ or the sale of those assets with the goal of obtaining liquidity for the continuation of the business. Provisions regarding debt restructuring may differ according to the characteristics of the distressed company. They can be adapted to different creditors' perspectives, by distinguishing creditors who prefer a partial payment as soon as possible from those who are more confidential about future firm's profitability and may opt for a full payment in the future. In this regard, provisions can involve rescheduling of payments (i.e. payment deferment), debt roll-overs (i.e. changes in maturity date) and alterations of interest rates as well. Debts associated to high interest rates are one of the most common reasons of financial distress, since the firm is not able to generate enough cash flows to reimburse them. Reorganization plans normally include some partial or total debt write-offs: creditors may agree to reduce the outstanding debt of the distressed companies through "hair-cuts". In order to preserve firm survival, creditors may accept some covenant violations and/or alterations as well. On the other side, they can ask for the inclusion of additional guarantees that facilitate the reimbursement of their obligations. Therefore, firm's management and creditors can agree some debt/equity swaps, debt/debt swaps and equity/equity swaps, allowing claim holders to obtain new debt or equity instruments. Generally, such agreements based on going-concern assumptions: after the accord has been executed, there is the possibility for the firm to continue its operations when it represents the best solution for the firm itself and creditors as well. Participants in insolvency procedures

¹⁶ We will consider the valuation of a financial distressed firm receiving injections of financial resources needed to continue its operations in the following *case study*.

¹⁷ These assets are usually addressed to liquidation procedures.

should have strong incentives to achieve the maximum value from firms' assets in order to guarantee maximal distributions to lenders and reduce the burden of insolvency.

As a matter of fact, the plan proposed by the management has to include all the activities the firm intends to pursue in order to reduce its level of financial exposure. The plan is subject to creditors' approval, which is essential for its implementation. Creditors are usually divided in homogeneous classes with respect to their economic interest and judicial position, allowing a sort of hierarchy in credits' reimbursement. Indeed, creditors with similar legal rights should be treated fairly, receiving distributions that are proportional to their relative ranking and interests. Creditors with similar legal rights are threaten approximately the same way: for example, the principle of equitable treatment can be modified by the introduction of some priorities in repayments. When the management proposes the plan to creditors, all information suitable to influence the reimbursement needs to be exposed. Such information mainly involves the disclosure of financial statements with a focus on the activities that the firm can realise in the following years. By representing the previous experience of the distressed entity, financial statements provide a tool for better explaining current financial problems. The comparisons of annual ratios provide an analysis of firms' economic and financial evolution over time and therefore allows developing and understanding forecasts about future profitability, which are crucial for plan feasibility. Other than focus the attention on of the distressed firm under valuation, the management needs to consider general market conditions as well. In this way, it is possible to identify the best strategies to implement, which enable the distressed entities to overcome internal and external pressures. We will take into account all this information in the following analysis, which is a prerequisite to our *case study*.

3.4 The Value Drivers in the *Scenario Analysis*

A *scenario analysis* consists in a powerful tool for developing alternative visions of the future in which decisions may be played out. In this specific analysis, we consider a projection period of five years and a subsequent period in which we suppose the firm will join a steady-state equilibrium where financial problems are solved. The period correspondent to $t = 0$ takes into account actual firm's values and therefore they are the same in all scenarios. It is also considered the valuation date.

A central role in such a valuation procedure is played by the FCF that the firm will be able to generate as going-concern during the reference period. In case of distress, a portion of the FCF

produced is addressed each year to the gradual reimbursement of financial obligations (i.e. FCD), in order to reduce, over time, the probability of default. Both FCF and FCD depend on estimates about some firm's actual and forecasted value drivers, such annual growth rates on sales and EBITDA margins, which are driven by managers' expectations and therefore subject to a certain degree of uncertainty. A way to reduce the latter is performing a *scenario analysis* that allows taking into account different future frameworks. The following example is extracted from Buttignon (2014) and it will help us performing our *case study*. By looking at the financial projections, we can observe that the firm is planning to reduce the value of its *Net financial position* over time: thanks to a raise in EBITDA margin, the firm is able to allocate almost an increasing portion of the cash flow generated to debt holders, reducing in such manner the value of its indebtedness. Anyway, final estimates of firm's enterprise value obtained from forecasted FCF depend on the scenario we are considering. That is why we take into account three main scenarios: the *best scenario*, the *base scenario* and the *worst one*.

The *base scenario* is the most prudential one: the management assumes to realise a consistent portion of firm's enterprise value in order to reduce most of outstanding obligations. The EBITDA margin – which is a good measure of company's profitability - starts to increase gradually (i.e. by almost 1% every year during the projection period) and the annual sale growth starts to improve as well. In this intermediate scenario, we suppose there will be an increase in equity prices since the firm will reduce its exposure and reach a more stable financial path. Actual and expected values involved in such a scenario are represented in the following figure:

Base case	t = 0A	t = 1P	t = 2P	t = 3P	t = 4P	t = 5P	CV
Sales annual growth (g)	-10%	-5%	0%	3%	5%	5%	1,50%
EBITDA margin	3%	5%	6%	7%	8%	9%	10%
NWC/Sales	23%	24%	25%	25%	25%	25%	25%
CAPEX/Sales	2,00%	2,00%	2,00%	2,00%	2,00%	2,00%	2,00%

In the *downside* scenario, forecasts about future profitability are more pessimistic and more prudential in a certain sense. The value that managers expect to extract from continuing operations is very low and the EBITDA margin tends to be stable and moderate over time. Such a scenario is the most interesting for creditors, who can be asked to convert part of their credit into equity, sharing part of equity risk with current shareholders, as agreed in one of the feasible proceeds included in the reorganization plan. In such extreme events, Guthner (2012) recognises that credit risk can be considered as equity risk in disguise. Given the relationship

between capital components, especially as a consequence of a drop in firm's value, debt and equity tend to support the same risk; the higher the probability of default, the higher the equity or credit risk faced by lenders. In the following figure, we can observe the evolution of value drivers over time:

Downside scenario	t = 0A	t = 1P	t = 2P	t = 3P	t = 4P	t = 5P	CV
Sales annual growth (g)	-10%	-5%	-5%	-5%	-4%	-3%	0,50%
EBITDA margin	3%	3%	3%	4%	5%	5%	5%
NWC/Sales	23%	25%	26%	27%	27%	27%	27%
CAPEX/Sales	2,00%	2,00%	2,00%	2,00%	2,00%	2,00%	2,00%

Finally, the *best scenario* is the most challenging one: the management forecasts that firm's value will potentially join very high levels. If we consider that firm's value over time can be represented as a *normal* distribution, we need to focus our attention on the right tail, where extremely positive values concentrate. The annual sale on growth is expected to reach a very high level in few years (i.e. from -10% to 6% in four years); the EBITDA margin is forecasted to increase fast as well (i.e. from 3% to 9% in four years). It is clear that such results are difficult to achieve if the firm faces heavy financial and operating problems at the time of valuation. The figure below shows the expected trend through time:

Best scenario	t = 0A	t = 1P	t = 2P	t = 3P	t = 4P	t = 5P	CV
Sales annual growth (g)	-10%	0%	3%	5%	6%	7%	2,00%
EBITDA margin	3%	6%	7%	8%	9%	10%	10%
NWC/Sales	23%	24%	25%	25%	25%	25%	25%
CAPEX/Sales	2,00%	2,00%	2,00%	2,00%	2,00%	2,00%	2,00%

This *scenario analysis* will help us in performing our *case study*, where we will consider the *base scenario* as our benchmark. We will involve values obtained in the other scenarios as well, in order to make some comparisons.

CHAPTER FOUR: The *Case Study*

4.1 Introduction

Thanks to option pricing theory and the Contingent Claim Analysis (i.e. CCA), the enterprise value of any firms can be obtained by summing up the values of the securities included in the capital structure. As mentioned in the previous chapters, we suppose that asset value is driven by a random component to which a certain volatility measure is associated. If the latter is very high, the probability that asset will fall below the level necessary to meet senior debt payments over the horizon period increases as well. According to the CCA (which can be considered as an extension of the option pricing theory), debt represents a senior claim on asset value, while equity a junior or residual claim. In other words, debt holders benefit from reimbursement priority with respect to equity holders. In distressed companies, debt is risky because asset value may not be sufficient to reimburse promised payments in the future. In presence of default risk, the economic value of debt is lower than its nominal amount. In other words, the value of risky debt can be decomposed in a risk-free component and a put option written on firm's assets, which can be considered as a proxy for the expected loss in case of depressed levels of enterprise value are reached. The "price" of such a put option causes a reduction in the amount of risky debt, as we can observe from the following relationship:

$$D_{risky} = D_{rf} - Put\ Option$$

As we will demonstrate analytically, an increase of the value of the put option can be derived by an increase in asset volatility as well.

In our *case study*, we want to estimate the difference between the risk-free and the risky component of debt, which is captured by the value of the put option written on the enterprise value (i.e. $Put\ Option = D_{rf} - D_{risky}$), by following Merton's model (1974). In doing so, a previous computational analysis and interpretation of the enterprise value of the target is required. The latter mainly depends on estimates of the expected future cash flows that would be extracted from firm's assets, according to the value drivers included in the previous *scenario analysis*.

Such a *case study* is in turn decomposed in two cases. In the first case, we follow directly the recommendations implicit in Merton's model (1974): we assume that all the firm's outstanding obligations can be pooled together in a single tranche of a zero coupon debt with a unique maturity and therefore reimbursement in the future, regardless to the fact that companies usually

have more complicated capital structures. In addition, a *sensitivity analysis* is provided in order to verify what happens to the value of the put option of debt as some inputs of the Merton's formula (i.e. the enterprise value and asset volatility) change. In addition, such an analysis allows to test if the model is coherent with our expectations.

In the second case of our analysis, we introduce a second tranche of zero coupon bond and therefore a second maturity date. In this way, we are able to increase the complexity of the capital structure, by distinguishing short-term from long-term obligations. This lets us suppose the company will benefit from “new finance”, i.e. the injection of new sources of liquidity with a reimbursement priority with respect to other existing loans. Creditors who provide liquidity in such emergency conditions have to be satisfied before other existing creditors. Anyway, such an approach can be extended for valuing firms with privileged debt as well.

4.2 The Enterprise Value of the distressed company

When assessing firms in financial distress, value drivers characterising each scenario play an important role; when they are associated to the initial balance sheet, they provide consistent results. We know that the going-concern value of the target has been computed by summing up the actual value of the expected FCF and CV as well. The former is derived by applying the DCF model (that has to be adjusted for the probability of distress), while for the computation of the CV, we can distinguish two possible solutions:

1. we can apply the growing-perpetuity formula to FCF projected in the first period after the forecasting one or, alternatively, we consider the underlying value drivers (NOPLAT, RONIC, g), assuming the firm will be able to join a steady-state equilibrium after the reference period;
2. the assumption of a “normalised” equilibrium after the forecasting one enables us applying market multiples to forecasted financial results, such as EBIT and EBITDA margins. By following this alternative, we need to select carefully comparable companies. If there are significant differences between the target and comparable companies, it is difficult to quantify discounts on market multiples capturing these discrepancies.

The succeeding table shows the entire procedure followed for the computation of the firm's enterprise value, where $t = 0$ represents the actual valuation period. The succeeding years include values derived from projections, in line with the value drivers estimated for the *base scenario*.

€ million	t = 0A	t = 1P	t = 2P	t = 3P	t = 4P	t = 5P	CV
EBIT		29,50	38,90	50,31	63,87	78,54	90,65
Operating taxes		-10,33	-13,62	-17,61	-22,35	-27,49	-31,73
Tax rate		35,00%	35,00%	35,00%	35,00%	35,00%	35,00%
NOPLAT		19,18	25,29	32,70	41,51	51,05	58,92
Net working capital variation		2,00	-9,50	-7,13	-12,23	-12,84	-4,05
Depreciation		18,00	-18,10	-18,19	-18,33	-18,55	-18,85
Gross capex		-19,00	-19,00	-19,58	-20,54	-21,58	-21,90
Operating invested capital variation		1,00	-10,40	-8,51	-14,45	-15,87	-7,09
Operating free cash flows		20,18	14,89	24,19	27,06	35,18	51,83
Cost of capital		12,00%	12,00%	12,00%	12,00%	12,00%	10,03%
Growth rate (g)							1,50%
Discount factor		0,89	0,80	0,71	0,64	0,57	
Present value of (annual) FCF		18,01	11,87	17,22	17,20	19,96	
Present value of FCF in the forecasting period	84,26						
Continuing value (CV)	297,69					524,64	
BEV before debt tax shield and other items	381,96	408	442	470	500	525	
Tax shield on debt*	9,27						
Non-recurrent extraordinary asset value**	-13,21						
BEV	378,02						
BEV/EBITDA(t+1)	7,96						
BEV/EBIT(t+1)	12,81						
Non-operating assets (NOA) value	26,00						
Enterprise Value (EV)	404,02						

*Debt tax shield value						
Interest expenses		-12,00	-12,00	-10,00	-9,00	-7,00
Tax shield rate		25,00%	25,00%	25,00%	25,00%	25,00%
Tax shield flow		3,00	3,00	2,50	2,25	1,75
Cost of capital	12,00%					
Debt tax shield value	9,27					
**Non-recurrent extraordinary asset value						
Non-recurrent and extraordinary items		-10,00	-8,00	-5,00	0,00	0,00
Tax rate		30,00%	30,00%	30,00%	30,00%	30,00%
Net cash flow		-7	-5,6	-3,5	0	0
Cost of capital	12,00%					
Non-recurrent and extraordinary items value	-13,21					

Table 1: the EV computation through DCF method

Estimates of future EBIT values are sensitive to the increasing trend in annual growth rate forecasted for the *base scenario*, as reported in the previous chapter. They are influenced by the expectations about the evolution of the EBITDA margin as well. Assumptions about net

working capital and gross capex depend on the value drivers (NWC/Sales and CAPEX/Sales) reported in the scenario analysis and determine the operating invested capital variation over time. Then forecasts of FCF are obtained each year by summing the expected variation in the operating invested capital and NOPLAT value. A crucial step consists in the determination of the cost of capital. For the projection period, it is estimated by considering the *unlevered* value of the firm. Therefore, the cost of capital employed for discounting FCF is simply given by the cost of equity, which is obtained by taking into account the risk-free rate (i.e. we consider the interest rate of a 10-year Treasury Bill as a proxy) and the sum of equity market premium and company specific risk premium, which is weighted by *beta equity*. The latter is equal to the value attributed to the *unlevered beta*, given that we assume the firm is funded only by equity. For the CV, the cost of capital is obtained by applying the WACC, considering that the firm is financed by both equity and debt. The cost of capital employed is then adjusted by using a discount factor that takes into account the likelihood of distress. Finally, we obtain the BEV as the sum of the present value of the FCF during the forecasting period and the continuing value, adjusted for the value of the tax shield on debt and the value of non-operating assets. A similar evaluation can be done by considering the value drivers characterizing the *best* and *worst scenarios* as well; in the first case we will obtain a higher estimate of the enterprise value (i.e. it will be almost equal to € 157 million) while in the second case it will be lower and closed to € 158 million. Such values will be employed in performing our *sensitivity analysis* in the following paragraph.

4.3 CASE ONE: a unique debt reimbursement in the future

In this first case, we assume having all the ideal conditions for applying Merton's model (1974) shown earlier. According to such an approach, the cost and the value of debt are quantified at the overall level, therefore different creditors are pooled in a single category, regardless to the presence of differences in remuneration and reimbursement methods. The starting point is the enterprise value of the firm that we have obtained through the DCF approach. Furthermore, we need an estimate of the risk-free value of debt, which is obtained by discounting the cash flow projected to creditors at the risk-free rate, adjusted for the risk of default, considering the specific features of the firm under valuation and the overall economic conditions. The risk-free debt value obtained before is then taken into account for the computation of the nominal debt value at maturity, which will be our strike price in the BSM model. A proxy for debt duration

is obtained by computing the ratio of the total FCF per time and the actual value of risk-free debt.

€ million	t = 0A	t = 1P	t = 2P	t = 3P	t = 4P	t = 5P
Risk-free debt value and duration						
a. Year from 0 (time)		1	2	3	4	5
b. Free cash flow to debt (FCD)		23,14	53,24	49,02	68,54	256,89
c. Risk-free rate		0,50%	0,70%	0,90%	1,10%	1,30%
d. Discount factor		1	0,99	0,97	0,96	0,94
e. PV(FCD)		23,02	52,5	47,72	65,6	240,83
f. Risk-free value of debt	429,67					
g. FCF per time		23,02	105,01	143,16	262,41	1.204,13
h. Sum FCF per time	1.737,73					
i. Duration (h/f)	4					

Table 2: The risk-free debt value and duration

Another indispensable input of the model consists in the EV volatility estimate, which we suppose being equal to 35%. This means that the enterprise value is expected to fluctuate up and down, within the range of 35% the initial amount. With all this information in hands, we are therefore able to derive the BSM formula and consequently the value of the firm's equity, as the value of a call option of firm's assets:

$$E = C = Ve^{-y(T-t)}N(d_1) - De^{-r_f(T-t)}N(d_2)$$

€ million	t = 0A
a. Enterprise value	404,02
b. Risk-free debt value	429,67
c. Debt duration (years)	4
d. Risk-free rate	1,10%
e. Debt nominal value at maturity	449,22
f. EV volatility (sigma)	35%
g. d1	0,31
h. d2	-0,39
i. N(d1)	62,17%
j. N(d2)	34,83%
k. Equity value (BSM formula)	101,53
l. Debt value	302,5
m. Put value	127,18

Table 3: The computation of debt and equity values with Merton's formula

V of the previous formula represents the estimated enterprise value of the firm under consideration, whereas the risk-free debt value and duration has been derived earlier in Table 2. The strike price of our call option is the nominal debt value at maturity, which has been calculated as a function of the actual value of the risk-free debt, debt duration and risk-free rate, according to this formula: $e = b \exp(cd)$. By applying Merton's formula, the resulting equity value is € 101,53 million. Such a value is then subtracted from the enterprise value in order to obtain the risky value of debt, which is equal € 302,50 million. Finally, the worth of the put option of debt is obtained as the difference between actual risk-free and risky debt. It represents at the same time the amount of promised payments that creditors are asked to "sacrifice" to facilitate business continuity and the shareholders value of debt reorganization plan. The inequality between the risk-free and the market value of debt (i.e. $l = b - k$) is therefore captured by the put value of debt.

The situation before and after debt reorganization can be outlined as follows:

BEFORE DEBT REORG.		AFTER DEBT REORG.		
EV	D rf	EV	D mkt	
404,02	429,67	404,02	302,50	➡ 429,67 - Put value
	"E"		E	
	-25,65		101,53	➡ -25,65 + Put value

A quicker approach for obtaining the difference between the riskless and the risky debt components consists in applying directly the Merton's formula for pricing put options:

$$P = D e^{-r_f(T-t)} N(-d_2) - V e^{-y(T-t)} N(-d_1)$$

Where D is still the nominal debt value at maturity and V is the enterprise value as estimated from future expected cash flows.

€ million	t = 0A
a. Enterprise value	404,02
b. Risk-free debt value	429,67
c. Debt duration (years)	4
d. Risk-free rate	1,10%
e. Debt nominal value at maturity	449,22
f. EV volatility (sigma)	35%
g. -d1	-0,31
h. -d2	0,39
i. N(-d1)	37,83%
j. N(-d2)	65,17%
k. Put value	127,18
l. Debt value	302,5
m. Equity value	101,53

Table 4: The direct computation of the put value

As before, we assume that the dividend yield y is equal to 0. From the resulting value for the put option, we compute the market value of debt by subtracting the actual risk-free debt value. Finally, the equity value is derived as the difference between the enterprise value and the market value of firm's obligations.

4.3.1 The *Sensitivity Analysis*

At this point, it is interesting to perform a *sensitivity analysis* in order to highlight the effects on the put option of debt as some key parameters change. We know that almost all the inputs involved in the valuation can be observed from the market, except for the current value of firm's assets and asset volatility. Their value is the result of some estimates and therefore is subject to a certain degree of uncertainty. We want, therefore, to study the effects on debt derived from:

- a) Changes in *enterprise value* expectations;
- b) Changes in *asset volatility*.

In case of changes in EV expectations, holding the volatility constant, we can observe the following results:

€ million	WORST SCENARIO	BASE SCENARIO	BEST SCENARIO
ENTERPRISE VALUE	158,00	404,02	517,36
EV volatility	35%	35%	35%
Risk-free debt value	429,67	429,67	429,67
Debt duration (years)	4	0,011	4
Risk-free rate	1,10%	1,10%	1,10%
Debt nominal value at maturity	449,22	449,22	449,22
d1	- 0,63	0,31	0,55
d2	- 1,33	- 0,39	- 0,15
N(d1)	26,76%	62,17%	70,88%
N(d2)	9,34%	34,83%	44,04%
Equity value	2,13	101,53	177,38
Debt value	155,87	302,50	339,98
Put value	273,80	127,18	89,70

Table 5: *Sensitivity analysis* with respect to changes in enterprise values

For the value of firm's assets, we replace the results obtained from the valuation using the value drivers characterising the *worst scenario* and the *best scenario*, in comparison with our benchmark (i.e. the *base scenario*). Assuming all the other parameters being equal, we can observe the huge value of the put option of debt for very low expected values of firm's assets. This means that, creditors are asked to accept a larger discount on their credits in order to let the firm continues its operations; in the *worst scenario*, the probability of default joins high levels. The higher is the initial value of the firm's assets with respect to the debt amount and the lower is the probability of default. In the *best scenario*, therefore, the value of the put option is lower than in the *base case*, so that the difference between the risk-free and risky (i.e. market) value of debt is shrinking. We can conclude that these results are in line with our expectations about the relationship between uncertainty and values of capital components. The following graph captures the *sensitivity* of put values with respect to changes in the underlying assets, considering the results obtained in the previous table. It is clear the negative relationship between the enterprise value and put price: as the former increases, the latter declines with the probability of default.

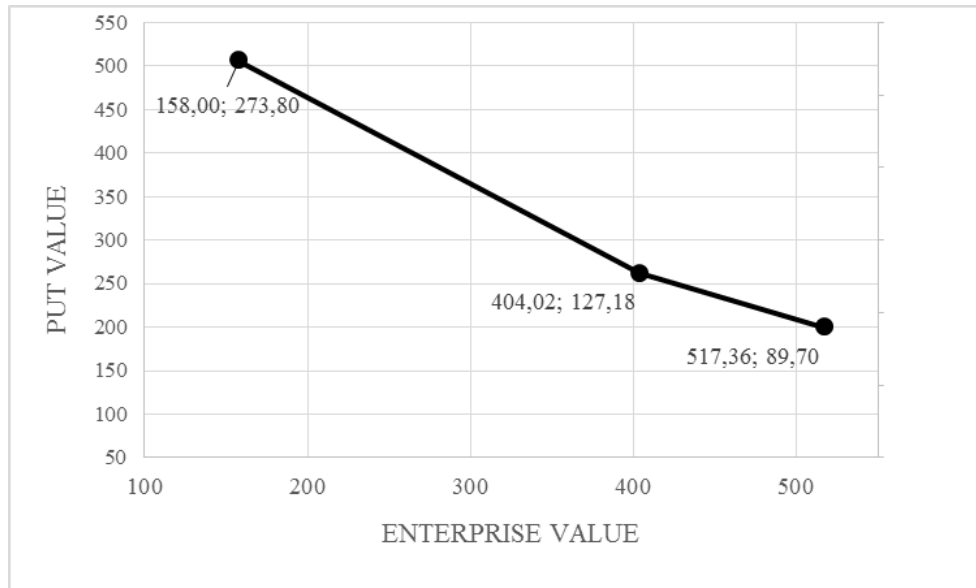


Table 6: Graphical representation of the put option *sensitivity* with respect to changes in enterprise value estimates

We can now perform a *sensitivity analysis* with respect to firm's asset volatility, by capturing the effects of a change in such a value on the put option. Understanding the changes in asset volatility through time is a fundamental issue in corporate finance. Asset volatility is a proxy for the riskiness associated to the realisation of future cash flow, which is the main driver of debt reimbursement. We will present such an analysis by focusing on the *base scenario* (correspondent to a volatility of 35%), with the goal of checking whether the model is able to account correctly changes in asset volatility:

€ million	t = 0A			
ENTERPRISE VALUE	404,02	404,02	404,02	404,02
EV volatility	25%	35%	42%	50%
Risk-free debt value	429,67	429,67	429,67	429,67
Debt duration (years)	4,00	0,01	4,00	4,00
Risk-free rate	1,10%	1,10%	1,10%	1,10%
Debt nominal value at maturity	449,22	449,22	449,22	449,22
d1	0,23	0,31	0,37	0,44
d2	-0,27	-0,39	-0,47	-0,56
N(d1)	59,10%	62,17%	64,43%	67,00%
N(d2)	39,36%	34,83%	31,92%	28,77%
Equity value	69,57	101,53	123,09	147,02
Debt value	334,45	302,50	280,93	257,00
Put value	95,23	127,18	148,74	172,67

Table 7: Sensitivity analysis with respect to changes in asset volatility

If the value of the assets is expected to fluctuate widely up or down, the likelihood of default on debt is considerable and therefore the value of the put option tends to rise. This is due to the fact that the initial enterprise value can potentially reach very low levels as the volatility is high. From the previous table, we can state that as uncertainty associated to future values of firm's assets increases, the value of the put option of debt increases as well, causing a rise in the difference between the nominal and the market value of debt. The price of the put option on firm's asset is therefore monotonically increasing in the volatility of the assets, as we can observe from the following graph, by substituting the results obtained in the previous table:

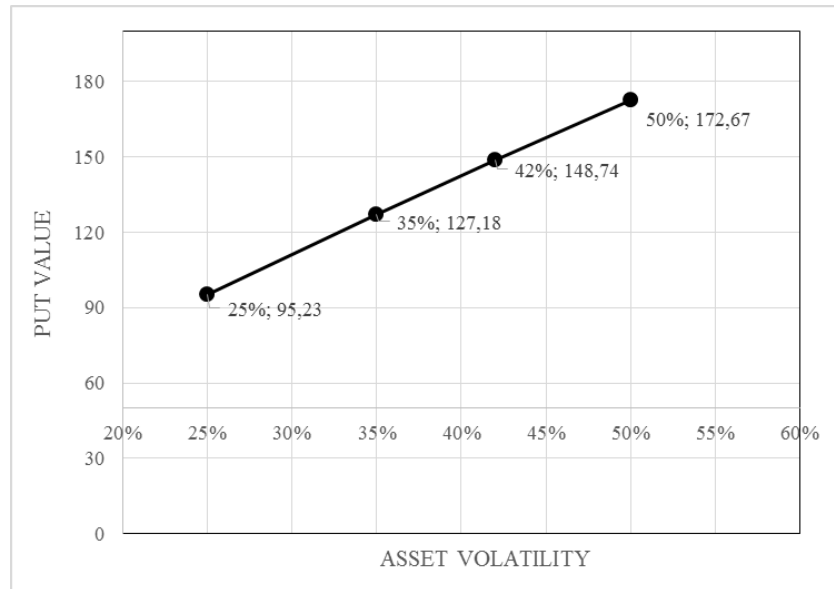


Table 8: Graphical representation of the put option *sensitivity* with respect to changes in volatility estimates

4.4 CASE TWO: How to account for different maturity dates: Merton's model adjustments

In this framework, we would like to increase the complexity of the target capital structure, by considering at least two tranches of zero coupon bonds rather than a single one, with different maturity dates. In doing so, we remember Merton's formula deriving equity value as a call option:

$$E = C = Ve^{-y(T-t)}N(d_1) - De^{-r_f(T-t)}N(d_2)$$

Where *delta*, representing the sensitivity of the call option with respect to changes in the underlying assets, is derived as follows:

$$\Delta = \frac{\partial E}{\partial V} = N(d_1)$$

Whereas, $N(d_2)$ of the previous formula represents the probability that the call option will finish in-the-money¹⁸ on a risk-neutral basis and consequently $(1 - N(d_2))$ is the associated probability of default. Indeed, d_2 stands for the number of standard deviations the assets value must fall to reach the default point. In other words, d_2 symbolises the Distance to Default in this way:

$$d_2 = DD_t = \frac{\ln\left(\frac{V}{DP}\right) + (r_f - y - \frac{\sigma_V^2}{2})(T - t)}{\sigma_V \sqrt{(T - t)}}$$

DP is the Default Point, which in Merton's model consists in the amount of the risk-free debt at maturity. A different interpretation of the default point has been developed in the 1980s by Moody's KMV model, after the introduction of a more sophisticated capital structure. Such a default threshold is obtained by setting the face value of the zero coupon bond equal to the sum of face values of short-term liabilities and a fraction of the value of long-term liabilities (i.e. usually a half of them).

The first half of the numerator represents the distance between the enterprise value and the default point at the evaluation date. The higher the enterprise value with respect to the default point, the higher would be the probability of the call option to finish in-the-money. The second term in the numerator is a *drift term* and indicates how the enterprise value is likely to evolve from the valuation date until the expiration of the option. The higher the risk-free rate (considered as a proxy for the risk-neutral return of firm assets), the higher is the probability of the call option to finish in-the-money. Whereas, dividends and asset volatility both reduce that probability: an increase in asset volatility or dividend yield determines a reduction in firms' compound returns. The denominator normalises the numerator in order to establish the DD in terms of standard deviations.

¹⁸ A call option is said to be in-the-money as the market value of the underlying asset (i.e. the enterprise value) is higher than the strike price (i.e. the value of risk-free debt at maturity).

As mentioned in the previous chapters, one of the main limitations of Merton's model consists in assuming too simplistic capital compositions, ignoring the presence of many issues of loan outstanding, with different coupons, maturities and subordinated structures. In order to overcome such a restriction, we can attempt to extend the theoretical structure of the model, increasing therefore its complexity. A first attempt has been made by Geske (1979), who generalises Merton's model by considering equity not as a simple call option on firm's value, but rather as a compound option. He also increases the complexity of the balance sheet structure by introducing short and long-term debt, junior or subordinated debt, amortising debt, safety covenants and other possible promised or restricted payments. We want to focus our attention on the presence of both short and long-term debt. By considering two points in time t_1 and t_2 , with $t_1 < t_2$, we can state that the firm is solvent in t_1 whereas its value V_{t_1} is higher than the value of the short-term debt D_1 plus the expected value of the long-term debt. In this regard, a refinancing assumption is usually involved, since the firm is unlikely to pay off its short-term obligations without shrinking the value of the firm and increasing the probability of default on the remaining ones. Since there are two maturities of debt, we can firstly identify two probabilities of default: a short-term probability in t_1 , associated to the short-term debt and a conditional long-term probability in t_2 , which is known as "forward" probability. The latter is conditional on not defaulting on the short-term debt. In addition to the short and long-term probabilities, we can mention a total or jointly probability of default in either t_1 or t_2 as well.

According to Delianedis and Geske (1998), by considering equity as a compound option, there is more than one *option to default* on firm obligations: in correspondence of each maturity, equity holders can chose between reimburse the current obligation, maintaining the control over the company, or rather default, transferring the control to creditors. Each payment gives them the right to proceed to the next reimbursement, so it is like an option on the option. In other words, if equity holders exercise the option in t_1 (i.e. they repay short-term obligations), then in t_2 they will have another option: debt reimbursement or default. Whereas, the option in t_2 expires as the firm has defaulted on its debt in t_1 . We can formally explain such a framework in the following way; at the first maturity date, the firm is solvent if:

$$V_{t_1} > D_1 + PV_{t_1}(D_2)$$

The enterprise value at that date is higher than the nominal amount of short-term debt and the expected actual value of long-term debt.

From the previous formula we obtain a certain threshold for firm's value in t_1 , below which the default is triggered:

$$V_{t_1}^* = D_1 + PV_{t_1}(D_2)$$

$V_{t_1}^*$ is exactly the strike price of the first option in a compound option context. Whereas, the strike price of the call option expiring in t_2 is just the face value of the second debt tranche, i.e. D_2 . In general terms, V^* - known as "cut-off value" - is an additional variable to be solved for in the problem. The idea of introducing a certain threshold for asset value determining default is not unused in literature. For example, Leland (1994) takes into account two possible bankruptcy determinants: the first is endogenously determined¹⁹, while the second involves a positive net-worth covenant²⁰. In both cases, they have an important role in setting the values of debt and equity. When the bankruptcy threshold is chosen by the firm, rather than being imposed by a covenant (such as a positive net-worth requirement), its value will be as low as possible in order to maximise firm's enterprise value. Whereas, in other cases, the choice of the bankruptcy level can be functional to equity value maximisation for any level of firm's enterprise value. The asset value at which bankruptcy occurs is usually independent from firm's current asset value; it decreases as the risk-free rate or asset volatility increases as well.

Some years later, Leland and Toft (1998) assume either that bankruptcy can be triggered at exogenously specified asset values, such as debt principal value, or when cash flows are not enough to cover interest payments on debt. According to this theory, in case of a single tranche of debt outstanding, before its maturity, asset value may be low but still sufficient to pay debt holders and avoid default. Only at maturity, the firm will need substantial assets to avoid bankruptcy. These findings are in line with Merton's assumption about bankruptcy occurring exclusively at maturity, since for a single zero coupon bond there is never an endogenous reason to default before expiration. Anyway, if the firm has more than one tranche of debt outstanding, the bankruptcy trigger is better determined in an endogenous way, depending on the maturity and the amount of debt as well. In such a framework, bankruptcy can occur at asset values that may be either lower or higher than the principal value of debt. As supported by many researchers in this field, the choice between endogenous and exogenous bankruptcy threshold

¹⁹ The endogenous bankruptcy threshold depends on some parameters, such as the risk-free interest rate, the corporate tax rate and asset volatility.

²⁰ The net worth covenant involves computing the difference between the market value of assets and firm's default point (i.e. the asset value at which default will be triggered, which generally lies between total and current liabilities).

leads to substantial valuation differences, mainly for “junk” bonds. Indeed, with respect to the bankruptcy boundary chosen, we will obtain different results for equity and debt value as well.

Delianedis and Geske (1998) take into account the asset value thresholds as identified earlier in order to obtain a closed-form solution for equity as a compound option on firm assets:

$$E = VN_2[d_1 + \sigma\sqrt{T-t_1}, d_2 + \sigma\sqrt{T-t_2}; \rho] - D_2e^{-r_f(T-t_2)}N_2[d_1, d_2; \rho] - D_1e^{-r_f(T-t_1)}N(d_1)$$

Where:

$$\rho = \sqrt{\frac{T-t_1}{T-t_2}}, d_1 = \frac{\ln\left(\frac{V}{V^*}\right) + \left(r_f + \frac{\sigma_V^2}{2}\right)(T-t_1)}{\sigma_V\sqrt{T-t_1}} \text{ and } d_2 = \frac{\ln\left(\frac{V}{D_2}\right) + \left(r_f + \frac{\sigma_V^2}{2}\right)(T-t_2)}{\sigma_V\sqrt{T-t_2}}. N_2(\bullet) \text{ is the cumulative}$$

bivariate normal standard distribution with correlation coefficient ρ . With respect to Merton’s formula, the *bivariate normal* standard distribution replaces the simple *normal* one. The strike prices – as we can observe from d_1 and d_2 formulas - correspond to the bankruptcy boundaries in t_1 and t_2 , respectively. The evident complexity of the solution for equity values with respect to the original Merton’s equation creates some implementation problems: firstly, the model requires precise and complete information about firm’s actual capital structure. Secondly, the process involved for transforming equity volatility into asset volatility is more complicated, since an additional variable needs to be computed (i.e. V^*).

At this point, we want to propose an alternative version of the original Merton’s formula, which allows taking into account at least two tranches of zero coupon bonds with different maturity dates. Rather than considering equity as a compound option on firm value, we want to model equity as a combination of call options, i.e. $E = E_{t_1} + E_{t_2}$, where E_{t_1} is call option with expiration date t_1 and E_{t_2} is a call option as well but with maturity date t_2 , with $t_1 < t_2$. The option-pricing formula for equity value we can derive from the previous assumption is the following:

$$E = C = V_1e^{-y(T-t_1)}N(d_1^{t_1}) - D_1e^{-r_f(T-t_1)}N(d_2^{t_1}) + (V - V_1)e^{-y(T-t_2)}N(d_1^{t_2}) - D_2e^{-r_f(T-t_2)}N(d_2^{t_2})$$

$$\text{Where } d_1^{t_1} = \frac{\ln\left(\frac{V_1}{D_1}\right) + \left(r_f + \frac{\sigma_V^2}{2}\right)(T-t_1)}{\sigma_V\sqrt{T-t_1}} \text{ and } d_1^{t_2} = \frac{\ln\left(\frac{V-V_1}{D_2}\right) + \left(r_f + \frac{\sigma_V^2}{2}\right)(T-t_2)}{\sigma_V\sqrt{T-t_2}}.$$

The total enterprise value is still the underlying asset, as derived in *case one*, but it is portioned between the two call options in accordance to the cash flow expected to be generated until maturity, including the proceeds obtained from possible final divestitures and reorganizations. More precisely, in order to obtain a positive value for the put option, we assume that the enterprise value we can derive is almost proportional to the nominal debt that has to be reimbursed at maturity. D_1 and D_2 are respectively the strike prices of the first and the second call option on firm assets. They can be considered as proxies for the short-term debt (i.e. D_1 reimbursed in t_1) and the long-term debt (i.e. D_2 is reimbursed in t_2). The idea of separating in such a way the total value of nominal debt follows directly from Moody's KMV experience. The introduction in the model of two tranches of debt and therefore two expiration dates allows setting the barycentre of our analysis in more than a single point in the future, since the valuation approach proposed by Merton focuses exclusively on bonds maturity. In addition, such a modified version enables us dealing with situations in which firms in financial troubles benefit from the injections of liquidity, facilitating the continuation of operations. As mentioned in the previous chapter, agreements between debtors and creditors usually based on going-concern assumptions involve the introduction of "new finance" that in most circumstances contributes to avoid firm default, increasing the probability of success associated to the attempt of target reconditioning. While most of pre-bankruptcy liabilities are frozen, the target needs liquidity to face the up-front costs for stabilising the business. It might seem counterintuitive that banks and other financiers would provide loans to companies in financial distress; anyway, credit extensions may be necessary to "protect" existing obligations. All types of lenders (banks, financial intermediaries...) benefit from pre-deductibility. As in case of privileged debts, funds obtained in such circumstances have shorter maturity and they are "senior" with respect to other loans. If we assume that the debt that has to be reimbursed in the short-term run consists in pre-deductible or privileged debt, we are able to deal with different debt seniorities as well.

As in Delianedis and Geske (1998), the short-term debt needs to be refunded before the long-term debt, whose reimbursement is conditional to the preceding one. Therefore, the second tranche of debt will be paid if and only if the first tranche of debt has been completely reimbursed in t_1 . After assuming a certain "seniority" among obligations, we can represent graphically the value of firm's securities at maturity in the following fashion:

Claim	$V < D1$	$D1 < V < (D1 + D2)$	$V > (D1 + D2)$
Senior bonds	V	$D1$	$D1$
Junior bond	0	$V - D1$	$D2$

Our analysis will continue by applying the modified version of Merton's formula (in which equity is considered as the sum of two call options on firm's assets), considering the same estimates of enterprise value, risk-free debt value, asset volatility and risk free rate characterising *case one*. The goal is to test if our model is consistent with our expectations after the inclusion of two maturity dates for firm's obligations. We will proceed by performing a *sensitivity analysis* to check the impact on the put value of debt due to changes in debt structure:

- A. 70% of the estimated risk-free debt has to be reimbursed in t_1 and the remaining amount in t_2 ;
- B. 30% of the estimated risk-free debt has to be reimbursed in t_1 ;
- C. 50% of the estimated risk-free debt has to be reimbursed in t_1 ;

4.4.1 Put values and put *sensitivity* with two tranches of debt

- A. 70% of the estimated risk-free debt reimbursement in t_1

We consider at first the framework in which the firm plans to reimburse 70% of the actual value of the risk-free debt in the short-term run (i.e. t_1) and the remaining 30% in the medium-long run (i.e. t_2). The probability of default should decline over time, or in other terms, the value of the put option should be lower in correspondence to the second debt maturity with respect to the first one, since the promised amount to reimburse is smaller. By considering the data of the previous case, we obtain the following results:

€ million	t = 0A	
a. Enterprise value	282,82	121,21
b. Risk-free debt value	300,77	128,90
c. Debt duration (years)	2,20	4,79
d. Risk-free rate	1,10%	1,10%
e. Debt nominal value at maturity	308,14	135,87
f. EV volatility (sigma)	35,00%	35,00%
g. d1	0,22	0,34
h. d2	-0,30	-0,43
i. N(d1)	58,71%	63,31%
j. N(d2)	38,21%	33,36%
k. Equity value (BSM formula)	51,12	33,73
l. Debt value	231,70	87,47
m. Put value of debt	69,07	41,43
n. Total Enterprise value		404,02
o. Total Equity value		84,85
p. Total Debt value		319,17
q. Total Put value		110,50

Table 9: 70% debt reimbursement in t_1

The total enterprise value is still the same (i.e. € 404,02 million), as the sum of the risk-free debt value (i.e. € 429,67 million), which is taken into account for the computation of the strike prices of each call option. The maturity dates i.e. $t_1 = 2,20$ and $t_2 = 4,79$ years have been computed according to formula applied in *case one*. As expected, the value of the put option tends to decline from the first to the second expiration date. Almost 62,51% of the total put amount is explained by the price of the put option in correspondence to t_1 , where creditors are asked to accept a higher cut on their promised payments to let the firm continuing its operations. Once the first tranche of debt has been reimbursed in t_1 , the value of the second put option is lower, since lower is the promised amount to creditors. The model seems therefore in line with our expectations about a declining default probability over time. If we compare the resulting total put option (obtained by summing up the put options in t_1 and t_2) with that obtained in *case one*, we can see that it is slightly lower:

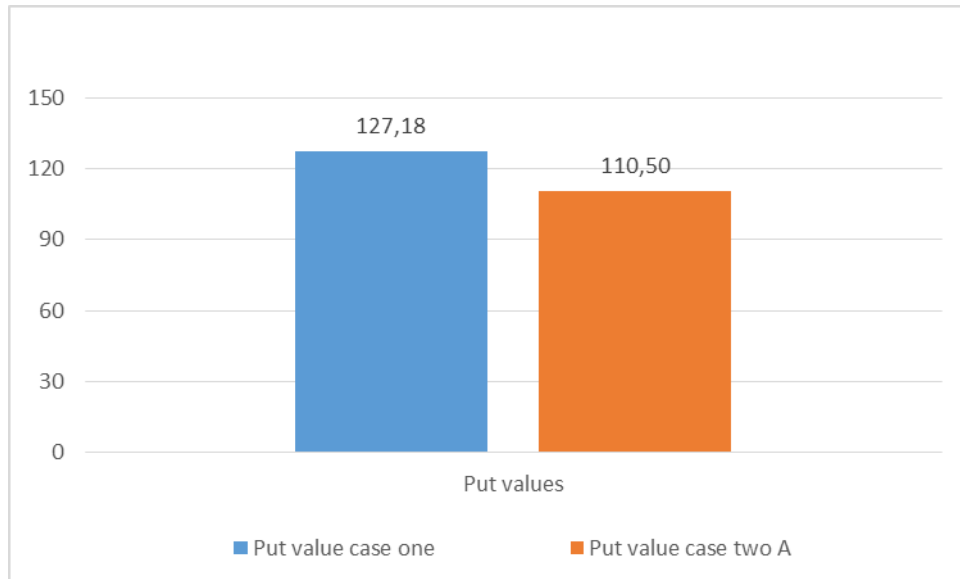


Table 10: Graphical representation of put values *sensitivity A*

This result is probably due to the fact that the risk of default is better proportioned between the maturity dates, especially if we assume the firm benefits from the injection of “new finance” that allows reimbursing the big portion of debt in the short-term run.

B. 30% of the risk-free debt reimbursement in t_1

We now take into account the reverse situation in which the firm plans to reimburse 30% of the actual value of the risk-free debt in the short-run with the proceeds obtained until that maturity, facing a bigger portion to pay back in the following years. In this framework, we expect to obtain an opposite trend with respect to the previous one and therefore a larger value for the put in correspondence to the second maturity date, leading to an increase in default probability over time. The firm in this case may face some difficulties in reimbursing the second tranche of debt if, after the payment of the first tranche, its value starts to follow a downward path and there are not significant divestitures or additional liquidity provided. On the other hand, under an *optimistic scenario*, we can suppose the company will have enough time to collect proceeds for ensuring the reimbursement of the long-term obligation as well. The results obtained after applying the formula are reported below:

€ million	t = 0A	
a. Enterprise value	121,21	282,82
b. Risk-free debt value	128,90	300,77
c. Debt duration (years)	2,20	4,79
d. Risk-free rate	1,10%	1,10%
e. Debt nominal value at maturity	132,06	317,03
f. EV volatility (sigma)	35,00%	35,00%
g. d1	0,22	0,34
h. d2	-0,30	-0,43
i. N(d1)	58,71%	66,31%
j. N(d2)	38,21%	33,36%
k. Equity value (BSM formula)	21,91	87,20
l. Debt value	99,30	195,62
m. Put value of debt	29,60	105,15
n. Total Enterprise value		404,02
o. Total Equity value		109,10
p. Total Debt value		294,92
q. Total Put value		134,76

Table 11: 30% debt reimbursement in t_1

By considering the previous expiration dates, but a different capital structure composition, we can confirm a different trend surrounding the value of the put option and default probability. The risk the firm will not be able to meet the second term obligation is higher with respect to the previous framework and should not be ignored. As mentioned before, one of the main limitation of Merton's model is that it does not consider what happens to the firm enterprise value over time before debt maturity. It can fluctuate both up and down, influencing debt reimbursement. If the proceeds obtained from the assets at a certain point in time were lower than the promised amount to bond holders, it would be reasonable to declare firm's default on current and future obligations as well. The model does not take into account that unpaid obligations cannot be cancelled out, but rather they increase the following promised tranche of debt to reimburse. In other words, Merton's model helps investigating firm's financial situation only at maturity, ignoring intermediate default. As mentioned earlier, a possible solution to such a limitation should be introducing a certain barrier to the asset value determining the default point. Anyway, by considering the resulting value of the risky debt sensitive to the chosen barrier²¹, there is not a unique response.

²¹ For further information about this topic, see Leland and Toft (1996).

Even in this scenario, we want to compare the value of the put option with the results obtained in the previous cases:

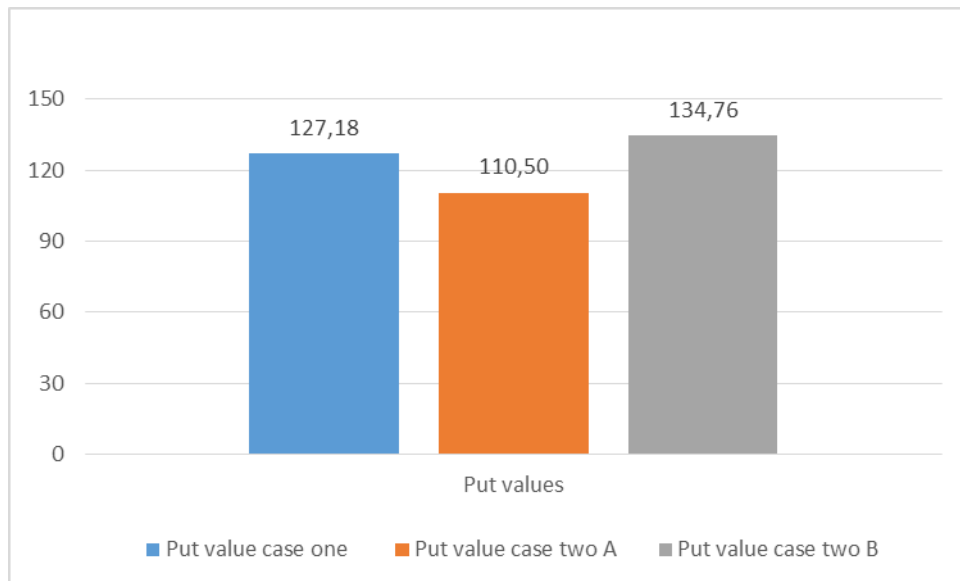


Table 12: Graphical representation of put values *sensitivity A and B*

From the previous table, we can observe that the amount of the put option obtained in correspondence to *case two B* is to some extent higher than that obtained both in *case one* and *case two A* as well. This result is probably due to the effect of a higher probability of default associated to the second debt tranche, which contributes in determining almost 78% of the total put value.

C. 50% Risk-free debt reimbursement in t_1

An intermediate framework with respect to those presented earlier consists in assuming a more balanced capital structure where half of the debt should be reimbursed in t_1 and the remaining amount in t_2 . The following results are obtained by appropriately modifying the inputs of Merton's formula:

€ million	t = 0A	
a. Enterprise value	202,01	202,01
b. Risk-free debt value	214,84	214,84
c. Debt duration (years)	2,20	4,79
d. Risk-free rate	1,10%	1,10%
e. Debt nominal value at maturity	220,10	226,45
f. EV volatility (sigma)	35,00%	35,00%
g. d1	0,22	0,34
h. d2	-0,30	-0,43
i. N(d1)	58,71%	63,31%
j. N(d2)	38,21%	33,36%
k. Equity value (BSM formula)	36,51	56,22
l. Debt value	165,50	145,79
m. Put value of debt	49,34	69,05
n. Total Enterprise value		404,02
o. Total Equity value		92,74
p. Total Debt value		311,29
q. Total Put value		118,39

Table 13: 50% debt reimbursement in t_1

From the previous table, we observe that the main value influencing the amount of the total put value is debt duration. As the latter increases, the difference between riskless and risky debt increases as well, rising the probability of firm's default. If we would try to plot a graph relating bond's maturity to the spread between riskless and risky debt over time, we will observe a humped shape: spreads are very low at short maturities, they increase with the latter and then decline again. This behaviour is in line with Merton's findings, as $V_t > D$. Intuitively, for shorter maturities, the obligation is not likely to default and therefore spreads are low. While, if maturity lengthens, there is sufficient time for the bond to default as there are more possibilities for the enterprise value to drop below the face value of debt, increasing spreads. For much longer maturities, conditionally on the absence of default in the past, the likelihood of default on average is declining, since firms have enough time to find resources for granting survival. In general, we think about two possible trajectories for the firm's value, the former is increasing and the latter is declining with time. For shorter maturities, there may be not sufficient time to default on bond, even if the trajectory is low. Whereas, in correspondence of medium maturities, the declining path of firm value would have enough time to reach the default, leading to higher spreads with respect to short-term maturities. Finally, if the firm is solvent after a long period, it is much likely to have experienced increasing trajectories,

obtaining lower credit spreads. By taking into account the option theory context, the reasoning is almost the same. The extent of the spread between riskless and risky debt values depends on the value of a European option on V , with strike D and time to maturity $(T - t)$. Provided $V_t > D$, the firm is likely to reimburse its obligations, so that the put value is low. When maturity increases, the put value tends to rise as well, reflecting a higher chance of default. Whereas for long-term maturities, the value of the put option declines again. As mentioned in the previous chapters, Merton's model is based on the assumption of a Brownian Motion driving asset value over time. Since the latter involves a continuous path, for very short maturities and $V_t > D$, the possibility for the firm value to “jump” down is very strict. If the process would include some discontinuities, then even in the short-term horizon, higher spreads would be generated since a “jump” to default can no more be excluded. According to Zhou (1997), a jump-diffusion model would better match the size of credit spreads on corporate bonds, generating various shapes of spread curve with respect to the diffusion approach adopted by Black, Scholes and Merton. The inclusion of a jump-diffusion process for valuing risky debt provides another way to increase the complexity of the original model, leading to implementation problems that we try to avoid in our analysis.

As before, we compare the results obtained for the put values:

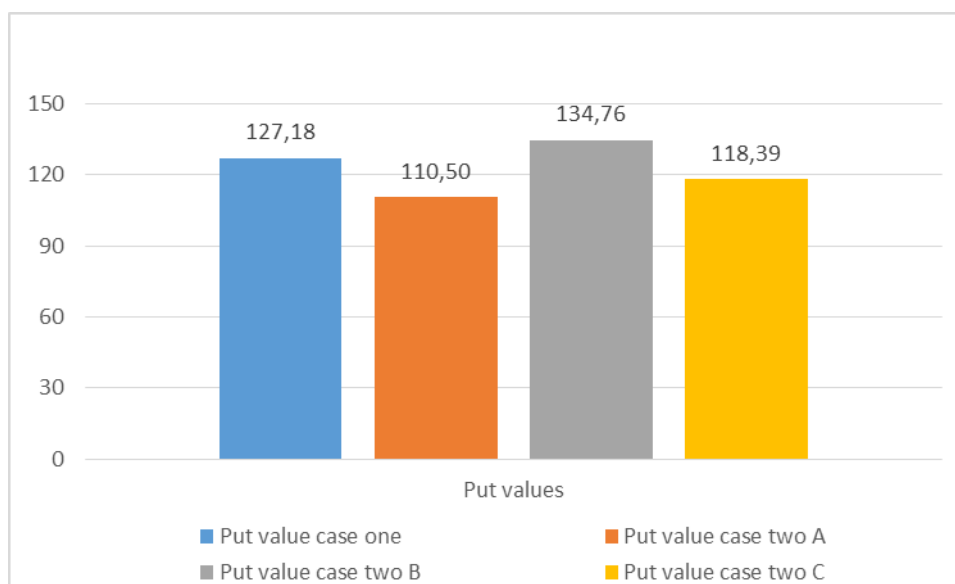


Table 14: Graphical representation of put value *sensitivity* with respect to all debt structures

Even if we assume that the firm is able to reimburse half of its obligations in the short-term and the remaining amount in the long-run, the amount of the put option associated to each maturity

date is not the same. Put values computed in correspondence to longer maturity dates are worth more than put options with shorter maturities, because of the potential risk for the firm to follow declining paths in value when expiration dates are not excessively too far in the future. Whereas the firm would be able to decide both the portion of the expected cash flow generated from the assets for debt reimbursement at each maturity and the amount of debt to repay as well, the combination enterprise value – risk-free debt that would ensure almost a flat probability of default over time is the following one:

€ million	t = 0A	
a. Enterprise value	210,09	193,93
b. Risk-free debt value	234,17	195,50
c. Debt duration (years)	2,20	4,79
d. Risk-free rate	1,10%	1,10%
e. Debt nominal value at maturity	239,91	206,07
f. EV volatility (sigma)	35,00%	35,00%
g. d1	0,17	0,39
h. d2	-0,35	-0,37
i. N(d1)	56,75%	65,17%
j. N(d2)	36,32%	35,57%
k. Equity value (BSM formula)	34,18	56,84
l. Debt value	175,92	137,09
m. Put value of debt	58,26	58,42
n. Total Enterprise value		404,02
o. Total Equity value		91,02
p. Total Debt value		313,00
q. Total Put value		116,67

Table 15: Flat put values computation with two tranches of debt

The underlying value of the first call option is equal to 52% times the total expected enterprise value at the moment of valuation (i.e. $52\% * € 404,02 \text{ million} = € 210,09 \text{ million}$). While the underlying value of the second call option is computed by taking the difference between € 404,02 million and the value obtained earlier. 54,5% of total risk-free debt (i.e. € 429,67 million) should be reimbursed in correspondence to the first expiration date (i.e. t_1) and the remaining 45,5% in the second one. Therefore, by considering the riskless interest rate, debt duration and asset volatility unchanged with respect to the previous cases, we obtain two values for the put option that are almost the same; meaning that the probabilities of default both in t_1 and t_2 are quite smoothed. The intuition for obtaining such results mainly derives from the

previous assumptions about loan reimbursement over time. We remember that in *case two A* the probability of default is focused mainly in the short-term horizon and its declining over time, whereas in *case two B* the probability of default is increasing, due to the large amount that needs to be refund in the long-term horizon and the “maturity effect” as well. The latter is evident in *second case C*, where we assume a balanced debt reimbursement.

4.4.2 Put values and put sensitivity with three tranches of debt

After considering only two tranches of debt, we want to verify if our analysis can be extended to three tranches of risk-free debt as well, by adapting the original Merton’s formula. As a proxy for their maturities, we consider $t_1 = 1$ years, $t_2 = 3$ years and $t_3 = 5$ years. The inputs of our analysis, such as the actual enterprise value, the actual risk-free debt value, asset volatility and risk-free rate are still the same, allowing some comparisons among the results. In such a framework, we consider that 20% of obligations are reimbursed in $t_1 = 1$ years, 50% in $t_2 = 3$ years and the remaining amount in $t_3 = 5$ years. As before, we expect a higher amount for the put option in correspondence to the second maturity date, where half of the reimbursement will take place. The results obtained are the following:

€ million	t = 0A		
	a. Enterprise value	80,80	202,01
b. Risk-free debt value	85,93	214,84	128,90
c. Debt duration (years)	1,00	3,00	5,00
d. Risk-free rate	1,10%	1,10%	1,10%
e. Debt nominal value at maturity	86,89	222,04	136,19
f. EV volatility (sigma)	35,00%	35,00%	35,00%
g. d1	0,12	0,02	0,35
h. d2	-0,23	-0,58	-0,44
i. N(d1)	54,78%	50,80%	63,68%
j. N(d2)	40,90%	28,10%	33,00%
k. Equity value (BSM formula)	9,12	42,25	34,65
l. Debt value	71,69	159,76	86,56
m. Put value of debt	14,25	55,08	42,34
n. Total Enterprise value			404,02
o. Total Equity value			86,02
p. Total Debt value			318,01
q. Total Put value			111,67

Table 16: Put value sensitivity with respect to three tranches of ZCB

Our expectations are confirmed in this case as well: the value of the put option tends to increase as the nominal value of debt increases, rising the associated probability of default. From the succeeding figure, we can observe the tendency of the spread between riskless and risky debt over the time horizon and the humped shape mentioned before as well:

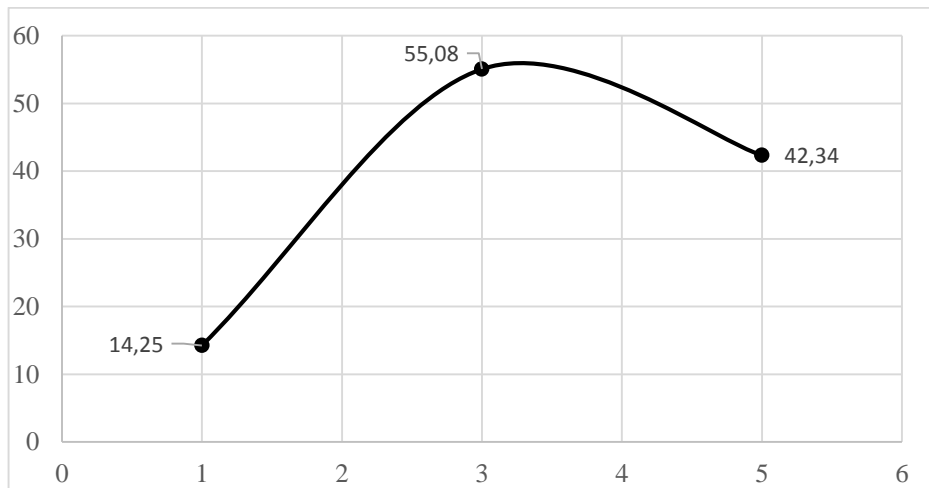


Table 17: Put value evolution over time

The probability of default in correspondence to very short maturities is very low, while it tends to increase in correspondence to medium-term maturities and decline again for longer maturities.

Even after the introduction of three tranches of debt, we can consider a balanced debt structure, as we did in case of two tranches. The total actual enterprise value and risk-free debt are distributed in equal amount each year. The resulting amounts for the put options are therefore influenced by the presence of different maturity dates. We expect to obtain increasing put values from the first to the last expiration dates for the same reasons explained when dealing with two tranches of debt and an equilibrated bond composition. The following results demonstrate the validity of our assumptions:

€ million	t = 0A		
	a. Enterprise value	134,67	134,67
b. Risk-free debt value	143,22	143,22	143,22
c. Debt duration (years)	1,00	3,00	5,00
d. Risk-free rate	1,10%	1,10%	1,10%
e. Debt nominal value at maturity	144,81	148,03	151,32
f. EV volatility (sigma)	35,00%	35,00%	35,00%
g. d1	0,13	0,26	0,40
h. d2	-0,22	-0,34	-0,38
i. N(d1)	60,26%	60,26%	65,54%
j. N(d2)	41,29%	36,69%	35,20%
k. Equity value (BSM formula)	22,02	28,61	37,85
l. Debt value	112,66	106,07	96,82
m. Put value of debt	30,57	37,16	46,40
n. Total Enterprise value			404,02
o. Total Equity value			88,47
p. Total Debt value			315,55
q. Total Put value			114,12

Table 18: Put value *sensitivity* with a balanced debt structure

Differently from the situation in which the debt structure is not balanced, we do not obtain a humped shape for the distribution of the put value over time, since it is increasing with the expiration date (i.e. there is a “maturity” effect), as we can observe from this graph:

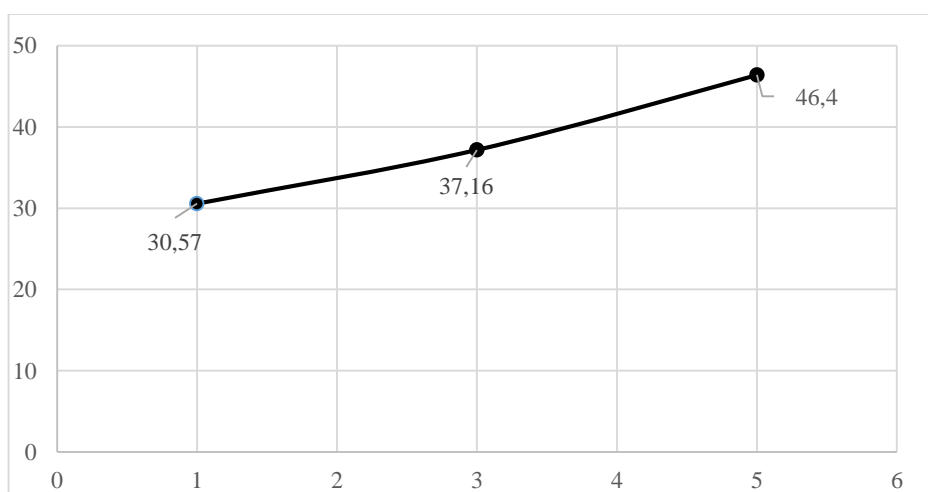


Table 19: Put value evolution over time with a balanced capital structure

In a framework characterised by three tranches of debt, determining the combination of enterprise value and risk-free debt that ensures a flat probability of default over time can be more ambitious than in case of two tranches, even if the underlying reasoning is almost same. The following spreadsheet shows the necessary steps for obtaining almost equal values for the put option at each maturity:

€ million	t = 0A		
a. Enterprise value	204,03	117,17	82,82
b. Risk-free debt value	210,54	124,61	94,53
c. Debt duration (years)	1,00	3,00	5,00
d. Risk-free rate	1,10%	1,10%	1,10%
e. Debt nominal value at maturity	212,87	128,79	99,87
f. EV volatility (sigma)	35,00%	35,00%	35,00%
g. d1	0,16	0,26	0,33
h. d2	-0,19	-0,34	-0,45
i. N(d1)	56,36%	60,26%	62,93%
j. N(d2)	42,47%	36,69%	32,64%
k. Equity value (BSM formula)	25,58	24,89	21,27
l. Debt value	178,46	92,28	61,56
m. Put value of debt	32,08	32,33	32,97
n. Total Enterprise value			404,02
o. Total Equity value			71,73
p. Total Debt value			332,29
q. Total Put value			97,38

Table 20: Flat put value computation with three tranches of debt

The combination of the enterprise value that should be addressed to debt reimbursement over time is 50,5% * € 404,02 million in t_1 , 29% * € 404,02 million in t_2 , and therefore 20,5% * € 404,02 million in t_3 . Moreover, in order to obtain a smoothed probability of default over time, the company should reimburse 49% times the total amount of risk-free debt in t_1 , 29% in the t_2 and the remaining 22% in t_3 , with $t_1 < t_2 < t_3$. Although the resulting values for the put options are almost equivalent, the increasing trend is not completely eliminated, as the succeeding representation suggests:

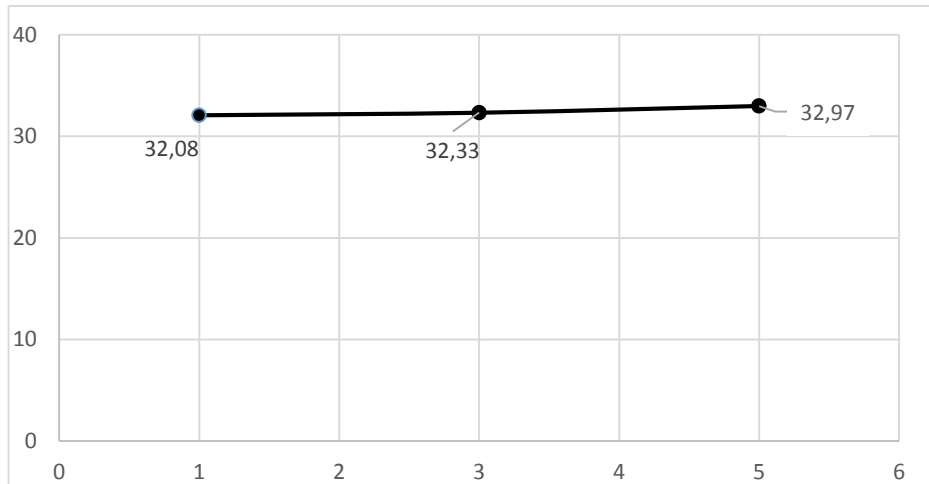


Table 21: Flat values for the put option with three tranches of debt

When dealing with two or more tranches of bonds, it is necessary to assume that the firm has enough resources to meet all its intermediate obligations. Once the firm is not able to refund one of its promised payment, default on current and future obligations should be declared, unless another agreement with creditors is not achieved. Unfortunately, such a modified version of Merton's model (as the original one) does not take into account explicitly the possibility of intermediate default, once the firm value has started to follow a downward path. On the other hand, the introduction of more than a single loan reimbursement may allow investigating the tendency of firm to default over time, which is measured by the value shifted from creditors to shareholders in light of debt reorganisation.

Conclusion

The valuation of distressed firms is a very complicated topic, since there are not “ad hoc” models that take into account all their specific features, such as low or negative cash flows, declining margins and decreasing growth rates as well. Actual academic literature provides a large spectrum of traditional models usually applied for valuing companies in normal business circumstances. Among them, there are cash flow-based models and other approaches based on market multiples and option theory as well. They need some “adjustments” before being applied for valuing firms in financial troubles, since their original version tends to underestimate default probabilities. The systematic solution proposed in this writing starts with the assessment of firm enterprise value, which is provided by considering the discounted cash flow approach. In this regard, negotiations among management, creditors and shareholders play an important role. Assuming the company will be able to continue operating as a going-concern, the realisation of future free cash flows depends on the approval of a reorganization plan by creditors, who usually accept a discount on their promised payments in order to avoid a bankruptcy situation. When discounting the expected proceeds achievable from the existing assets, a particular attention should be focused on the cost of capital determination, since the commonly applied WACC approach does not admit dynamic capital structures and therefore decreasing levels of firm exposure, with the goal of reaching a more sustainable path. Once we obtain an estimate for the enterprise value, the valuation continues by implementing Black, Scholes and Merton (BSM) model, which is useful for the determination of debt and equity values. Distressed companies usually show a market (i.e. risky) value of debt lower than the nominal (i.e. riskless) amount due to the presence of a substantial probability of default. The spread between them quantifies the benefits of debt restructuring for equity holders. In the BSM model, such a difference between riskless and risky debt is quantified by the price of a put option written on firm assets. In our *case study*, we show the entire valuation process, starting from the simplified framework proposed by Merton, which involves a single tranche of zero coupon bond outstanding. The results are strictly sensitive to variables that cannot be observed directly from the market, such as the enterprise value and asset volatility, which needs to be estimated somehow. We therefore propose a *sensitivity analysis* to verify the effects on put values as one of the previous input changes. Moreover, firm capital structure can be progressively enriched by introducing two tranches of zero coupon bond with different maturity dates. In this way, we

try to increase the original model complexity in order to attenuate the gap between theory and real firms' experience. The admittance of short and long-term obligations requires some arrangements to the traditional equity pricing formula: equity is no longer considered as a single call option, but rather as a linear combination of call options written on firm assets with different strike prices, depending on the promised amount to creditors. In this framework, we mainly examine the effects of debt maturity and debt amount on put values determination. We consequently study the evolution of firm default probability over time as debt structure changes. Finally, we suppose the existence of three tranches of debt (and three maturity dates). An interesting judgement involves finding the combination of enterprise value and risk-free debt amounts that allows the firm having almost a smoothed probability of default over time. In general, suggested modifications to the original Merton's model let focusing the assessment on more than one barycentre, since the likelihood of firm's default is perceived only at maturity, when the enterprise value and the promised amount to debt holders are compared. We provide in this way a valid attempt for upgrading traditional valuation models that are usually far from the real word experience and complexity.

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