UNIVERSITY OF PADUA

DEPARTMENT OF INDUSTRIAL ENGINEERING

MASTER DEGREE IN AEROSPACE ENGINEERING

Preliminary design tool of conventional/coaxial/tandem helicopters

Supervisors: *Prof. Ernesto Benini*

Prof. Filipe Szolnoky Ramos Pinto Cunha

Graduating: Anatol Cojocari

Registration number: 1084074

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I sincerely thank the people who have been next to me and believed in me all these years, especially my beloved parents and my beloved sister who supported and encouraged me in the most difficult moments of my life.

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A	Area of the rotor
Aov	Overlap area
Atr	Area of the tail rotor
c	Chord length
ctr	Chord length of the tail rotor
Cd	Drag coefficient
$Cd0$	Profile (viscous) drag coefficient
$Cd0tr$	Profile (viscous) drag coefficient of the tail rotor
Cl	Lift coefficient
$Cl\alpha$	Lift-curve-slope of the profile of the blades of the rotor
Cp	Coefficient of power
Cpi	Coefficient of induced power
Cp_{i0}	Sum of the induced and profile power coefficient integrals
$(Cp_{i0})_{coax}$	Sum of the induced and profile coaxial power coefficient integrals
$(Cp_{i0})_{TandemFront}$	Sum of the induced and profile tandem front rotor power coefficient integrals
$(Cpi0)_{TandemRear}$	Sum of the induced and profile tandem rear rotor power coefficient integrals
Cp_{fly}	Coefficient of power required for the level flight
$(Cp_{fly})_{Coax}$	Coefficient of power required for the level flight of the coaxial helicopter
$(Cp_{fly})_{Conv}$	Coefficient of power required for the level flight of the conventional helicopter
$(Cp_{fly})_{Tandem}$	Coefficient of power required for the level flight of the tandem helicopter
Cp_{max}	Coefficient of power installed on the helicopter
$Cpmr$	Coefficient of power of the main rotor of the conventional configuration

Cptr	Coefficient of power of the tail rotor
Cp0	Coefficient of profile power
Cq	Coefficient of torque
Ct	Coefficient of thrust
Cttr	Coefficient of thrust of the tail rotor
Cw	Coefficient of weight
d	Spacing between the two tandem rotor axes
$dist_{AC}$	Distance between the point A and the point C on the line s
$dist_{AB}$	Distance between the A and B points
dr	Step of integration of the adim. blade length in the blade element theory
dtr	Distance between the tail and main rotor axes
dxi	Step of integration in the blade element theory due to the azimuthal angle
dD	Drag per unit span on the blade
dFx,dFz	Forces per unit span on the blade
dL	Lift per unit span on the blade
D	Rotor diameter
Df	Fuselage drag
D_{tandem}	Tandem rotors diameter
Dtr	Tail rotor diameter
E	Endurance of the helicopter
$E_{(Mgtow-Mf/2)}$	Endurance of the helicopter considering the approximated helicopter mass *M*
f	Equivalent flat plate area of the fuselage
$f(\lambda)$, $f(\lambda i)$, $f(\mu)$	Function of the variable between the brackets
$f'(\lambda)$, $f'(\lambda i)$, $f'(\mu)$	Derivate of the funtion *f*
h	Altitude
k	Induced power factor
kint	Induced power interference factor between the coaxial rotors
kov	Induced power overlap factor
kx,ky	Deviation of the inflow from the value predicted with the momentum theory
K	Correction parameter

K_h	Adjustment coefficient in the regression expressions
m (eq.57)	Overlap of the tandem rotors
m (eq.175)	Angular coefficient of the straight line
$\dot{m}$	Mass flow
M	Mass of the helicopter
Mf	Mass of the fuel onboard
M_{gtow}	Gross mass at take-off
Nb	Number of blades
$Nbtr$	Number of blades of the tail rotor
P	Total power
Pc	Climb power
Pi	Induced power
$P_{installed}$	Power of the engines installed on the aircraft
$Pitr$	Induced power of the tail rotor
$(Pi)_{TandemFront}$	Tandem front rotor induced power
$(Pi)_{TandemRear}$	Tandem rear rotor induced power
P_I	Ideal induced power
P_{i0}	Sum of the induced and profile power integrals
$(P_{i0})_{Coax}$	Sum of the induced and profile coaxial power integrals
$(P_{i0})_{TandemFront}$	Sum of the induced and profile tandem front rotor power integrals
$(P_{i0})_{TandemRront}$	Sum of the induced and profile tandem rear rotor power integrals
Pp	Parasitic power
Ptr	Total tail rotor power
$P_{(Vmp)}$	Total power of the helicopter at the velocity for minimum power Vmp
$P_{(Vmr)}$	Total power of the helicopter at the velocity for maximum range Vmr
$P0$	Profile power
$P0tr$	Profile power of the tail rotor
$(P0)_{TandemFront}$	Tandem front rotor front power
$(P0)_{TandemRear}$	Tandem rear rotor induced power
P_1, P_2, P_{ov}	Induced power on each of the three areas described by the tandem rotors
Q	Torque

r (*Fig.2.5*)	Dimentionless radius of the rotor
r (*eq.201*)	Name of the equation of the line that passes for the origin and the point A
R	Radius of the rotor
R (*eq.161*)	Range of the helicopter
Rtr	Radius of the tail rotor
$R_{(Mgtow-Mf/2)}$	Range of the helicopter considering the approximated helicopter mass M
s	Name of the equation of the line parallel to r and which passes through the point B
SFC	Specific fuel consumption of the engine(s)
T	Thrust
Tf, Tr	Thrust of the front and rear rotors of the tandem configuration
Ttr	Thrust required by the tail rotor
T_1, T_2	Thrusts of the two tandem rotors
U	Resultant of the velocity at the blade element
U_T, U_R, U_P	Components of the velocity U
vi	Induced speed
$vitr$	Induced speed of the tail rotor
vf, vr	Induced speed of the front and rear rotors of the tandem configuration
v_h	Induced speed for the hover regime
Vc	Climb velocity
$Vinf$	Free stream velocity
$Vmax$	Maximum speed of the helicopter
Vmp	Velocity for minimum power
Vmr	Velocity for maximum range
$Vtip$	Speed of the blade tip
$Vtipf, Vtipr$	Tip speed of the front and rear rotors of the tandem configuration
$Vtiptr$	Speed of the blade tip of the tail rotor
$Vvmp$	Vertical velocity for minimum power
w	“far” wake or thr slipstream velocity
W	Weight of the helicopter
y	Coordinate axis along the lenght of the blade

Ymp	*Z* multiplied per *k/(2μ)* and differentiated respect μ
Ymp'	Derivate of *Ymp*
Ymr	*Z* multiplied per $k/(2\mu^2)$ and differentiated respect μ
Ymr'	Derivate of *Ymr*
Z	Squared coefficient of power of the main rotor of the conventional helicopter *Cmpr*
α	Effective – aerodynamic angle of attack
$\alpha 0$	Zero-lift angle
γ	Angle between the AB vector and the line *r (eq.201)*
$\theta,\theta 0$	Pitch angle at the blade element
$\lambda,\lambda i$	Induced inflow ratio
λc	Climb case induced inflow ratio
λ_h	Hover case induced inflow ratio
λtr	Induced inflow ratio of the tail rotor
$\lambda 0$	Mean induced inflow ratio at the center of the rotor
μ	Forward airspeed ratio
μtr	Forward airspeed ratio of the tail rotor
$\mu x,\mu z$	Advance ratios, respectively, parallel and perpendicular to the disk of the rotor
ρ	Local air density
σ	Solidity of the rotor
σtr	Solidity of the rotor of the tail rotor
φ	Relative inflow angle or induced angle of attack
χ	Angle of the wake skew
ψ	Azimuthal angle
Ω	Angural velocity of the main rotor
Ωtr	Angular velocity of the tail rotor

ABSTRACT

The thesis work has the purpose of describing the theories and of applying them on the implementation of a tool useful for the preliminary design of a helicopter. This tool is a software developed in Matlab language and executes the calculations based on two different theories: momentum and blade element. With this tool the flight performances can be calculated, from hover to maximum forward speed, of three types of helicopter: conventional, coaxial and tandem. For the sizing of each rotor configuration, the power required for the flight, the necessary forward velocities, the range and the endurance can be easily calculated.

Keywords: Conventional, Coaxial, Tandem, Momentum Theory, Blade Element Theory, Hover, Level Flight, Power, Range, Endurance, Design, Configuration, Validation, Helicopter, Rotor

SOMMARIO

Lo scopo del lavoro svolto per la tesi è di descrivere le teorie e di applicarle per implementare uno strumento utile per la progettazione preliminare di un elicottero. Tale strumento è un software sviluppato in linguaggio Matlab ed esegue i calcoli basati su due differenti teorie: del momento e dell'elemento di pala. Con questo strumento è possibile calcolare le prestazioni in volo, dal sostentamento alla massima velocità di avanzamento, di tre tipologie di elicottero: convenzionale, coassiale e tandem. Per il dimensionamento di ciascuna configurazione di rotori, sono stati implementati i calcoli della potenza richiesta per il volo, le velocità di avanzamento necessarie, l'autonomia ed il tempo di autonomia.

Parole chiave: Convenzionale, Coassiale, Tandem, Teoria del Momento, Teoria dell'Elemento di Pala, Sostentamento, Volo Orizzontale, Potenza, Autonomia, Tempo di Autonomia, Progettazione, Configurazione, Validazione, Elicottero, Rotore

CHAPTER 1

INTRODUCTION

The helicopter is one of the most important and interesting aircraft created by the human being. This kind of aircraft is very useful because of its characteristics of hovering, forward flying and, of course, vertical take-off and landing. Almost all of the aircrafts can't hover and they need a landing strip. These characteristics of the helicopter allow them to be used in the most extreme situations by military, police, firefighters, medical services, emergency services. Also, they can be used for delicate works of transportation big and not standard cargos, working with big weights in faraway places, where other vehicles can't be used.

Modern helicopters are becoming more and more complex. They are characterized by many issues concerning with physical size, performance, safety standards, noise level etc. It is difficult to satisfy all of these issues, because of the conflicts that occur between each other.

It's easy to understand how much work and how much time the engineers need to design such an aircraft. The earlier projects took a lot of expensive and complicated *experiments, using wind tunnels and flight tests to be able to introduce new configurations and so to obtain some progress. With the invention of the computer, the engineers tried to create methods that are easy and fast to apply in order to have a preliminary design and, of course, to eliminate possible errors.*

On the other hand, design studies using the performance equation are very important. These simple analytical equations can provide the designer with an excellent method for doing first cuts in the preliminary helicopter sizing. The most important thing is the impressive reduction of time, of cost and of computational cost.

Helicopters were not able to evolve so rapidly as the fixed wing but this is not so important, as underlined by Dr.Alexander Klemin (1925) ([1] pag. 159):

"The future of the helicopter... therefore lies not in competition with the airplane, but in the ability to perform certain functions which the airplane cannot undertake."

1.1 Objectives

This thesis focuses on the development of a software which delivers the general elements and features required for the preliminary design process of a helicopter. This program treats the three most common military/commercial configurations of helicopters: the conventional, the coaxial and the tandem.

The objectives are to create the script using two theories: the momentum theory and the blade element theory. So the three configurations of the helicopter can be studied from the both points of view.

This script calculates, for each configuration in dependence of the theory and the inputs inserted, the total power needed and it will plot the variation of it in function of the forward speed. The script will also calculate other important parameters like the endurance and the range of the helicopter chosen.

An objective is also to do the comparison between the results of the momentum theory and the blade element theory for each configuration. The next step is to compare the three configurations, giving the same values to the input variables of each configuration.

The preliminary design program is written in Matlab, simple and friendly to use for designers.

1.2 Helicopter configurations

The most common configurations of helicopters are the conventional, the coaxial and the tandem.

The conventional helicopter is one of the first designs in the history adopted to obtain a hover machine. In fact Sikorsky, the great russian engineer migrated to USA, used, in practice, this configurations for all his concepts and created flying machines that created the history of the helicopters. In a helicopter with a single main rotor, as the engine moves the rotor it's creating a torque effect that make the body of the helicopter to rotate in the opposite direction of the rotor. To counteract this effect, an anti-torque has to be applied. This allows the helicopter to maintain the fuselage steady and will ensure the yaw control. The most common control systems are the tail rotor, the NOTAR [2] and the Fenestron (or Fantail) [3].

The tail rotor is a smaller rotor mounted at the end of the tail at a precise distance from the center of gravity of the rotorcraft. The tail rotor provide the thrust in the opposite direction of the main rotor's rotation, in this way the torque effect created by the main rotor can be controlled. The pitch of the tail rotor blades is controlled by the pilot with the anti-torque pedals and this allows the rotation of the helicopter around its vertical axis. An example of a conventional helicopter is represented in the Figure 1.1:

Figure 1.1 A conventional helicopter: HX-1 (helicopter)/Sikorsky S-76 [4]

The coaxial configuration is more compact because the fuselage is entirely functional and doesn't need a long boom for a tail rotor for the rotor separation. This tipe of helicopter carries no rotor torque reactions because the rotors turn in opposite directions and therefore one cancels the torque of the other.

The control system is more effective and the configuration consists of less vulnerable area of critical components. Also the wetted area of the coaxial design, on the same payload basis, is much smaller than the tail rotor or the tandem configurations.

The absence of a tail rotor permits all of the engine power to be used by the rotor system for lifting. In comparison with a tail rotor configuration, the useful load of the coaxial design is higher [5]. The coaxial configuration, compared to an equivalent engine of a tail rotor configuration and to a similar tandem configuration, has lower weight and lower power losses in the transmission and in the shaft. In cross winds the handling of the coaxial helicopter is much simpler than any other type. The characteristic only of the coaxial design, due to symmetry of the coaxial rotor system, is the same aerodynamic efficiency and controllability in every direction of the flight [5].

Another advantage of the coaxial configuration is the reduced noise, that is associated with conventional helicopters where the noise derives, usually, from the interaction between the airflows of the main and tail rotors [6]. Another benefit is increased safety on the ground because of the absence of a tail rotor, so result less injuries and fatalities to ground crews. An example is exposed in the Figure 1.2:

Figure 1.2 A coaxial helicopter: the Kamov Ka-52 Alligator attack helicopter [7]

The tandem helicopters use counter-rotating rotors, like the coaxial one, with each canceling the other's torque. So all the power provided by the engines is used for lift. The two rotors are spaced and, usually have the same diameter. The two rotors are connected by a transmission that let the rotors be synchronized and don't hit each other if a failure occure.

The advantages of the tandem system are a larger range for the center of gravity position and good longitudinal stability. The tandem helicopters have the advantage to hold more weight with shorter blades and tend to have a lower disk loading than single rotor helicopters. Also the tandem rotor helicopters, typically, require less power to hover [8]. An example of a tandem helicopter is represented in the Figure 1.3:

Figure 1.3 A tandem helicopter: CH-47 Chinook [9]

1.3 State of art

There are several projects, research programs and professional tools for a preliminary tool design, but not all are easy to get or to work with. Below are some examples of them.

The Preliminary Helicopter Design Program Ver.1 (PHD-1) [10], typed in C++, is a program that performs the functions of helicopter design. In the PHD-1 program, the user have to insert as input the hover capability, the maximum payload, the range, the maximum level flight speed, the climb performance and other details. The PHD-1 will provide necessary data for the sizing of the rotor, the engine, for the determination of the weight of the helicopter and the power needed for every flight conditions. In the PHD-1 work, principally, the momentum theory was used, so it is good for a preliminary design process. This program can be applied to conventional type helicopters with a single main rotor, single tail rotor and and gas turbine engine.

An another tool is: RAPID / Rate: Rotorcraft Analysis for Preliminary Design / Rand Technologies & Engineering [11], it is still not freely accessible, but the results seem to be accurate and complete. It was developed and is maintained by Prof. Omri Rand Technion from Israel Institute of Technology.

In the paper “Determination of a Light Helicopter Flight Performance at the Preliminary Design Stage” [12], of the University of Belgrade, some of the calculation procedures used for the preliminary flight performance of the light conventional helicopter design are presented. In the work empirical relations, based on experimental data, are used with the momentum theory. It was developed at the Aeronautical Institute of the Belgrade Faculty of Mechanical Engineering.

Another aeromechanical analysis tool of helicopters and rotorcraft is CAMRAD II [13]. This very sofisticated program is used for the design, testing, and evaluation of all the steps of the project: the conceptual design, the detailed design, and the development of the project. The software uses multibody dynamics, non linear finite elements, the computational fluid dynamics, the rotorcraft aerodynamics and wakes (Momentum Theory). This program can be used to design the conventional, tandem, coaxial and tilting proprotor aircrafts. CAMRAD II results very efficient and complete, but it's still expensive and too complicated for a preliminary design.

The power estimation algorithm in HCDO (Helicopter Conceptual Design Optimization), upgraded in the reference paper [14], consider the compound rotorcraft with a lift offset. The HCDO program predicts the required power of a rotorcraft. The performance analysis in this design program are based on the simple momentum theory with empirical corrections. To improve the power estimation accuracy, the blade element rotor aerodynamics and the trim analysis were integrated. The HCDO program provides design capabilities for a conventional, coaxial or a tandem rotor helicopter.

In the thesis work of Roman Vasyliovych Rutskyy of Instituto Superior Técnico of Lisbon [15], a preliminary design tool for conventional helicopters was developed. Using statistics a great amount of available data of conventional helicopter were analyzed to describe the main parameters. Empirical formulas to estimate the main and the tail rotor diameters, the tip speed, the rotors chord were applied. In this work two different theories are applied in order to obtain the total power required by the helicopter: the linear momentum theory and the blade element momentum theory.

1.4 Structure of the thesis

The thesis is made of the exposition of the theory used to describe the performances of a helicopter like: the power needed to the various conditions of flight, the specific velocities, the efficiency in terms of the endurance and the range of the helicopter etc (Chapter 2). The program code, that is the aim of this thesis, was developed using certain assumptions and simplifications of the theory (Chapter 3). The three most important configurations of helicopters: the conventional, the coaxial and the tandem are presented and discussed.

Two theories are presented, the Momentum Theory and the Blade Element Theory (Chapters 2.1, 2.2). These are the most simple ones used to describe a rotor. The two theories were used separately to be able after to compare the results and to verify how different are the two approaches.

The program was made applying the various equations and relations from the two theories to calculate, with certain precautions, the numerical behavior of the each configuration of the helicopter. To have a preliminary design tool there have to be made certain choices in the both theories, that makes the program be simpler and faster.

A very important step is the validation of the results (Chapter 3.5). This means to compare the results obtained from the program with the experimental data collected in the wind tunnel of real models.

An another point of the thesis is to choose certain input values, the same to all the configurations of the helicopter, and to analyze the progress of the output results in comparison to each other. In fact this step is the creation of a preliminary design of a new helicopter. So in this phase can be compared three different configurations using two different theories (Chapter 4).

CHAPTER 2

THEORETICAL ANALISIS

2.1 The Momentum Theory

The Momentum Theory is used to analize the rotor flow field problem using the three basic conservation laws of fluid mechanics: mass, momentum and energy. This method was proposed first time by Rankine in 1865 to study the marine propellers, the following progress in this direction was made by Froude and Betz, and further formally generalized by Glauert in 1935 [16].

The most simple mathematical model of a rotor is an actuator disk. The rotor is approximated by an infinitesimally thin disk, over which pressure differences exist, producing an instantaneous change in the momentum of the flow. The application of the fluid conservation laws on a control volume results, in this case, a solution for the uniform induced velocity at the uniformly loaded rotor disk. The control volume, as illustrated in Figure 2.1 [17], is the surrounding area of the thrust carrying disk and it's wake.

This one-dimensional method provides only a first order global estimation of the rotor thrust and power. It doesn't include load distributions and non-linearities in the flow environment (for example: tip losses).

2.1.1 The conventional helicopter

The power required by the helicopter is given by the following expression:

$$P = Pi + P0 + Pp + Ptr + Pc, \tag{1}$$

where P is the total power, *Pi* is the induced power, *P0* is the profile power due to the trawling in the main rotor blades force, *Pp* is the power resulting from the fuselage drag force due to the horizontal movement of the helicopter, *Ptr* is the power required by the tail rotor and *Pc* is the power consumed by the helicopter to climb up.

2.1.1.1 Hover Regime

Pi is described by the expression (2), where P_I is the power ideally consumed by the helicopter and *k* (typical value being about 1.15) is the induced power factor which is introduced to consider the non-ideal physical effects (for example: finite number of blades, tip losses, wake swirl, nonuniform inflow):

$$Pi = k \cdot P_I, \tag{2}$$

$$P_I = \frac{W^{\left(\frac{3}{2}\right)}}{\sqrt{(2 \cdot \rho \cdot A)}} = \frac{T^{\left(\frac{3}{2}\right)}}{\sqrt{(2 \cdot \rho \cdot A)}} \tag{3}$$

where the weight of the entire helicopter was assumed to be equal to the thrust required by the main rotor to allow the hover condition, ρ is the local air density and A is the area described by the blades of the main rotor.

The expression (3) can be deduced, considering the Figure 2.1, in the following way. The applied force equals to the variation of the linear momentum:

$$T = \dot{m} \cdot w - 0, \tag{4}$$

where $\dot{m}$ is the mass flow and w is the “far” wake or thr slipstream velocity (plan ∞).

Here is considered $\dot{m}_0 = \dot{m}_\infty = \dot{m}_1 = \dot{m}_2 = \dot{m}$ (the mass conservation) and $A_1 = A_2$. From this results that $vi_1 = vi_2 = vi$. Therefore, there is no speed variation through the rotor disc.

The power, at the same time, is equal to the force (the thrust) multiplied per the velocity and to the energy variation rate:

$$T \cdot vi = \frac{1}{2} \cdot \dot{m} \cdot w^2 - 0 \quad (5)$$

From the relations (4) and (5) results:

$$w = 2 \cdot v_i = 2 \cdot v_h \quad (6)$$

where v_h is the induced speed for the hover regime.
So results:

$$\dot{m} = \dot{m}_1 = \dot{m}_2 = \rho \cdot v_h \cdot A \quad (7)$$

from the next combination of the relations:

$$T = \dot{m} \cdot w = \dot{m} \cdot (2 \cdot v_h) = \rho \cdot v_h \cdot A \cdot (2 \cdot v_h) = 2 \cdot \rho \cdot A \cdot v_h^2 \quad (8)$$

we obtain the relation of v_h:

$$v_h = \sqrt{\left(\frac{T}{(2 \cdot \rho \cdot A)}\right)} \quad (9)$$

From the relations (5) and (9) result the following expression for the ideal induced power:

$$P_I = T \cdot v_h = T \cdot \sqrt{\left(\frac{T}{(2 \cdot \rho \cdot A)}\right)} \quad (10)$$

So the induced power of the helicopter results:

$$Pi = P_I \cdot k = k \cdot \frac{T^{\left(\frac{3}{2}\right)}}{\sqrt{(2 \cdot \rho \cdot A)}} \quad (11)$$

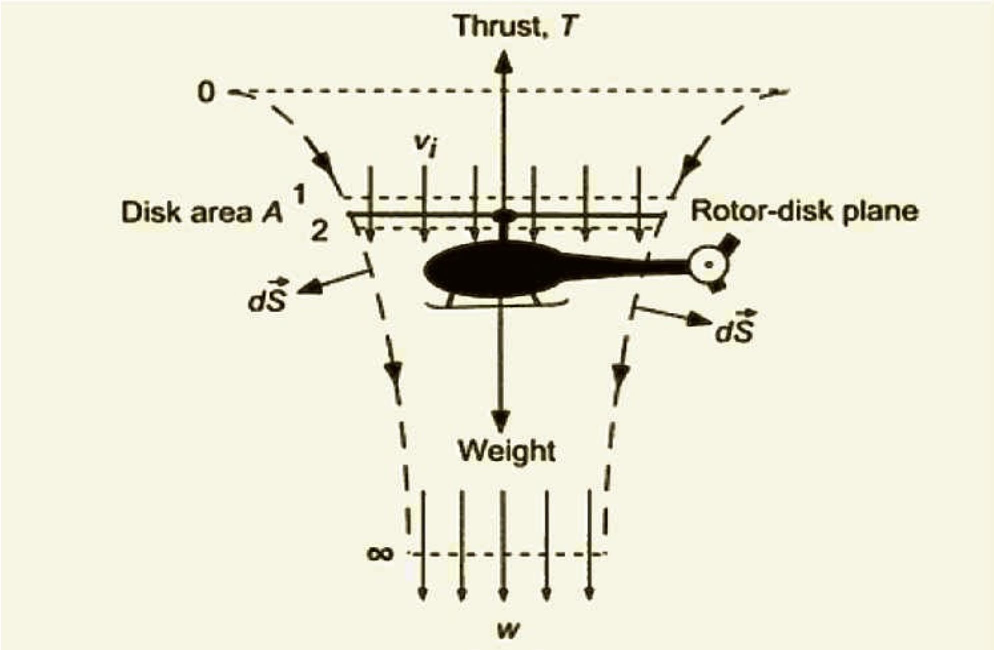


Figure 2.1 Flow Model for momentum theory analysis of a rotor in hovering flight, taken from Leishman [17]

The profile power, that derives from the blade element theory (see relations (109) – (112)), has the following expression:

$$P0 = \frac{1}{8} \cdot \sigma \cdot Cd0 \cdot \left(\rho \cdot A \cdot Vtip^3 \right) \tag{12}$$

where σ is the solidity of the main rotor, which is the ratio between the area of the blades of the main rotor and the area of the entire main rotor, *Cd0* is the aerodynamic drag coefficient of the main rotor blade profile with zero lift and *Vtip* is the speed of the blade tip.

The local solidity σ is expressed by ([1] pag. 81):

$$\sigma = \frac{(Blade\ area)}{(Disk\ area)} = \frac{Ab}{A} = \frac{(Nb \cdot c \cdot R)}{(\pi \cdot R^2)} = \frac{(Nb \cdot c)}{(\pi \cdot R)} \tag{13}$$

where *Nb* is the number of blades of the main rotor, *c* is the chord of the blade when it is rectangular, if not, is chosen an average value of the chord. *R* is the radius of the main rotor.

Pp is the power required to overcome the drag force on the fuselage, and in the case of hovering is null:

$$Pp = 0 \tag{14}$$

Pc represents the part of the power needed to climb. If there is no climb velocity:

$$Pc = 0 \tag{15}$$

Ptr, the power consumed by the tail rotor, for the first steps is considered around 10% of the total power required for the main rotor.

A good approximation of the density of the air that changes in function of the altitude is given by ([15] eq.16):

$$\rho = 1.225 \cdot \left(1 - 0.0065 \cdot \frac{h}{288.15}\right)^{4.25588} \tag{16}$$

2.1.1.2 Horizontal flight regime with a given altitude

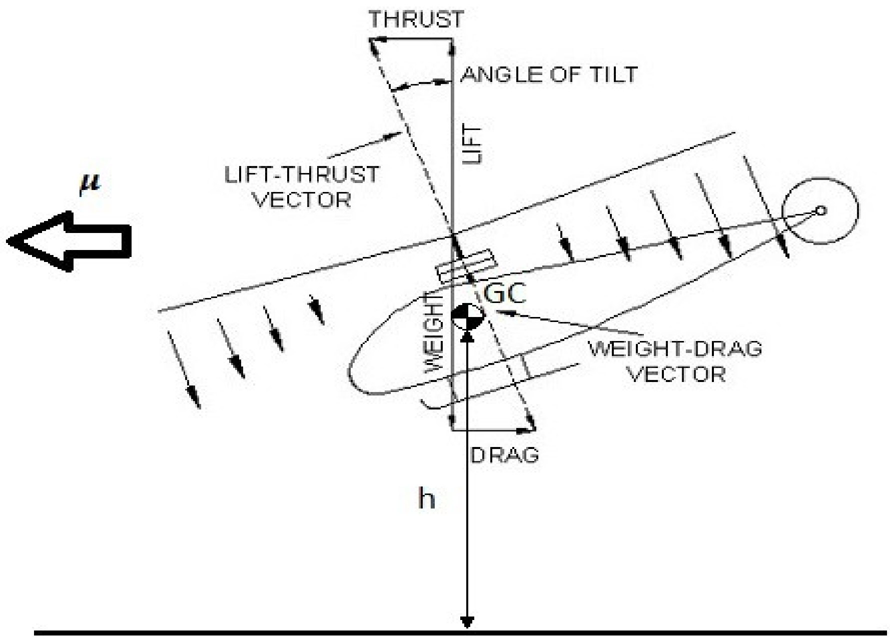


Figure 2.2 Illustrates the horizontal flight regime of a helicopter, with a certain speed at a given altitude [18]

The expression (1) is also used in forward flight regime. In this case there is no climb velocity so *Pc* equals zero too. The power required from the tail rotor, *Ptr*, as in the hover case, can be consider to be around 10% of the power needed for the main rotor and it will be exposed below.

So the expression (1) reduces to:

$$P = (Pi + P0 + Pp + Ptr) \tag{17}$$

Introducing μ as the ratio between the forward speed (free stream velocity) and the blade tip speed, or called the dimensionless forward speed of the helicopter, for very small α results:

$$\mu = \frac{(Vinf \cdot \cos(\alpha))}{Vtip} = \frac{Vinf}{Vtip} \tag{18}$$

The expression of the induced power *Pi*, valid for any value of *vi* (can be simplified for μ >0.15) results:

$$Pi = k \cdot T \cdot vi \tag{19}$$

The induced velocity *vi*, is given by ([1] pag. 65):

$$vi = \lambda \cdot (\Omega \cdot R) - Vinf \cdot sin\alpha = \lambda \cdot (\Omega \cdot R) \tag{20}$$

The solution for the inflow ratio λ can be obtained from the relation([1] pag. 65 eq. 2.102 and eq. 2.106):

$$\lambda = \frac{(Vinf \cdot sin\alpha)}{(\Omega \cdot R)} + \frac{vi}{(\Omega \cdot R)} = \mu \cdot tan\alpha + \frac{(\lambda_h)^2}{(\sqrt{(\mu^2+\lambda^2)})} = \frac{(\lambda_h)^2}{(\sqrt{(\mu^2+\lambda^2)})} \tag{21}$$

where λ_h is the inflow ratio from the hover case and is given by the equation ([1] pag. 65 eq. 2.104):

$$\lambda_h = \sqrt{(\frac{Ct}{2})} \tag{22}$$

Note that λ appears on both sides of the expression (21), so, for the generical case, has to be applied a numerical procedure to solve for λ, for example, the Newton-Raphson method.

The coefficient of induced power *Cpi* (valid for μ>0.15) is given by the expression ([1] pag. 164, eq. 5.18):

$$Cpi = k \cdot \frac{Ct^2}{(2 \cdot \mu)} \tag{23}$$

where *Ct* is the coefficient of thrust of the helicopter. So the induced power results:

$$Pi = k \cdot \frac{T^2}{(2 \cdot \rho \cdot A \cdot Vinf)} = k \cdot T \cdot vi \tag{24}$$

where *vi* is the induced velocity:

$$vi = \frac{T}{(2 \cdot \rho \cdot A \cdot Vinf)} \tag{25}$$

For the calculation of the profile power *P0* the formula is (see [19] pag. 219 eqs. 5.26 - 5.29):

$$P0 = \frac{(\sigma \cdot Cd0)}{8} \cdot (1 + K \cdot \mu^2) \cdot (\rho \cdot A \cdot Vtip^3) \tag{26}$$

where *K* is a parameter that varies, in base of the various assumptions and approximations made, from 4.5 in hover to 5 at $\mu = 0.5$. Stepniewski (1973) suggests to use an average value of $K = 4.7$ ([1] pag. 165).

The parasitic power *Pp* can be given by the expression ([1] p. 166, eq. 5.24):

$$Pp = \frac{1}{2} \cdot (\frac{f}{A}) \cdot \mu^3 \cdot (\rho \cdot A \cdot Vtip^3) \tag{27}$$

where *f* is the equivalent flat plate area of the fuselage. The value of *f* varies from about $0.93m^2$ for a smaller helicopter to $4.65m^2$ for a large utility helicopter design ([1] pag 166). *f* is defined by the formula:

$$f = \frac{Df}{(\frac{1}{2} \cdot \rho \cdot Vinf^2)} \tag{28}$$

where *Df* is the fuselage drag.

2.1.1.3 Tail rotor power of a conventional helicopter

The tail rotor is needed in a convetional helicopter to create a thrust that has to balance the main rotor torque reaction on the fuselage. The thrust required by the tail rotor *Ttr* is ([1] pag. 167 eq. 5.28):

$$Ttr = \frac{(Pi + P0 + Pp)}{(\Omega \cdot dtr)} \tag{29}$$

where *dtr*, theoretically, is the distance from the tail rotor shaft to the center of mass of the helicopter, but was assumed that the position of the center of mass is on the main rotor shaft axis. *Ω* is the angular velocity of the main rotor. The interference between the main rotor and the tail rotor is usually neglected in the preliminary design analysis.

The power of the tail rotor is calculated in a similar way to the main rotor:

$$Ptr = Pitr + P0tr \tag{30}$$

where *Pitr* is the induced power of the tail rotor and *P0tr* is the profile power of the tail rotor.

The induced power of the tail rotor is calculated, as in the previous relation (19) for the main rotor, for the thrust *Ttr* (29):

$$Pitr = k \cdot Ttr \cdot vitr \tag{31}$$

where *vitr* is the induced speed of the tail rotor (for small α, as for the main rotor):

$$vitr = \lambda tr \cdot (\Omega tr \cdot Rtr) - Vinf \cdot sin\alpha = \lambda tr \cdot (\Omega tr \cdot Rtr) \tag{32}$$

where *Rtr*, Ω*tr* and *λtr* are respectively the radius, the angular velocity and the inflow ratio of the tail rotor. The inflow ratio of the tail rotor *λtr*, for the level flight, is calculated as in the expression (21) for the main rotor:

$$\lambda tr = \frac{(Vinf \cdot sin\alpha)}{(\Omega tr \cdot Rtr)} + \frac{vitr}{(\Omega tr \cdot Rtr)} = \frac{Cttr}{\left(2 \cdot \sqrt{(\mu tr^2 + \lambda tr^2)}\right)} \tag{33}$$

For the hover case the inflow ratio of the tail rotor is found with the relation (22), where the coefficient of thrust of the tail rotor $Cttr$ is calculated by the formula:

$$Cttr = \frac{Ttr}{\left(\rho \cdot Atr \cdot Vtiptr^2\right)} \tag{34}$$

where $Atr = \pi \cdot Rtr^2$ is the area of the tail rotor, $Vtiptr$ is the blades tip speed of the tail rotor and the dimensionless forward speed of the tail rotor is given by:

$$\mu tr = \frac{Vinf}{Vtiptr} = \mu \cdot \frac{Vtip}{Vtiptr} \tag{35}$$

To semplify the calculations we considered $Vtiptr = Vtip$.

λtr appears on both sides of the expression (33), so a numerical solution can be the Newton-Raphson method, as in the case of the main rotor.

The induced power of the tail rotor is calculated, as in the relation (24) for the main rotor, with the thrust Ttr (see relation (29)):

$$Pitr = k \cdot \frac{Ttr^2}{\left(2 \cdot \rho \cdot Atr \cdot Vinf\right)} = k \cdot Ttr \cdot vitr \tag{36}$$

The induced velocity of the tale rotor results in this case:

$$vitr = \frac{Ttr}{\left(2 \cdot \rho \cdot Atr \cdot Vinf\right)} \tag{37}$$

For the calculation of the profile power of the tail rotor, $P0tr$, the expression is the same for the main rotor (26):

$$P0tr = \frac{\left(\sigma tr \cdot Cd0tr\right)}{8} \cdot \left(1 + K \cdot \mu tr^2\right) \cdot \left(\rho \cdot Atr \cdot Vtiptr^3\right) \tag{38}$$

where $Cd0tr$ is the aerodynamic drag coefficient of the tail rotor blade profile with zero lift. As for the main rotor (13), the solidity of the tail rotor is given by the expression:

$$\sigma tr = \frac{\left(Nbtr \cdot ctr\right)}{\left(\pi \cdot Rtr\right)} \tag{39}$$

where $Nbtr$ is the number of blades of the tail rotor and ctr is the chord length of the tail rotor blades (rectangular).

2.1.2 The coaxial rotor

The coaxial rotor configuration has some advantages, for example, there is no tail rotor required for the anti-torque purposes, the net size of the rotors is reduced (for a given rotorcraft weight) because each rotor creates thrust. But one of the problems is that the lower rotor is in the wake of the upper rotor and this generates a much more complicated flow field than for a single rotor design. This can be simplified by assuming that the interference is simulated with a loss of power.

2.1.2.1 Hover regime

The total power, in hover, of the coaxial helicopter is:

$$P = Pi + P0 \tag{40}$$

because the parasitic power *Pp* and the climb power *Pc* are null in hover.

If the rotor plans are assumed sufficiently close and each rotor creates a thrust equal to *T/2*, the total thrust generated by the helicopter results *T*. The effective induced velocity for the hover, in this case, will be ([1] pag.69 eq.2.124):

$$vi = \sqrt{\left(\frac{\left(2 \cdot \left(\frac{T}{2}\right)\right)}{\left(2 \cdot \rho \cdot A\right)}\right)} \tag{41}$$

So the ideal induced power for the coaxial design, P_I, in hover condition, results ([1] pag.69 eq.2.125):

$$\left(P_I\right)_{tot} = \frac{\left(2 \cdot \left(\frac{T}{2}\right)\right)^{\left(\frac{3}{2}\right)}}{\sqrt{\left(2 \cdot \rho \cdot A\right)}} \tag{42}$$

If each rotor is considered separately, the induced power per single rotor results *(T/2)· vi*. So the ideal induced power of the coaxial configuration is given by:

$$P_I = 2 \cdot \frac{\left(\frac{T}{2}\right)^{\left(\frac{3}{2}\right)}}{\sqrt{(2 \cdot \rho \cdot A)}} \tag{43}$$

The induced power interference factor between the rotors, *kint*, can be defined from the ratio between the relations (42) and (43) ([1] pag.70 eq.2.127):

$$kint = \frac{((P_I)_{tot})}{P_I} = \frac{\left(\frac{\left(2 \cdot \left(\frac{T}{2}\right)\right)^{\left(\frac{3}{2}\right)}}{\sqrt{(2 \cdot \rho \cdot A)}}\right)}{\left(2 \cdot \frac{\left(\frac{T}{2}\right)^{\left(\frac{3}{2}\right)}}{\sqrt{(2 \cdot \rho \cdot A)}}\right)} = \sqrt{2} \tag{44}$$

From the relation (44) results that there is an increase of 41% in the induced power needed to operate the coaxial configuration respect to the configuration of the two isolated rotors. Usually the two rotors of the coaxial configuration are spaced enough to make the lower rotor work in a fully developed slipstream of the upper rotor.

So with the momentum theory, *kint = 1.41* calculted assuming the rotors not separated, is changed to *kint = 1.28* found assuming the rotors enough vertically separated (see [19] pag. 103). From the experiments, for example Dingledein (1954), was deduced a *kint = 1.16* (used in all this thesis), that still overpredicts the value ([1], pag. 70).

The induced power of the coaxial helicopter in hover results:

$$Pi = (P_I)_{tot} \cdot k \cdot kint = kint \cdot k \cdot \frac{\left(2 \cdot \left(\frac{T}{2}\right)\right)^{\left(\frac{3}{2}\right)}}{\sqrt{(2 \cdot \rho \cdot A)}} \tag{45}$$

where *k* is assumed the same for each rotor.

The profile power of the coaxial helicopter in hover has the expression:

$$P0 = \frac{1}{8} \cdot (2 \cdot \sigma) \cdot Cd0 \cdot (\rho \cdot A \cdot Vtip^3) \tag{46}$$

There is $2 \cdot \sigma$ because the profile power considers both rotors of the coaxial configuration. The experimental data and the momentum theory are in a good agreement and suggest that the coaxial rotor can be considered as a system of two isolated rotors with an induced interference between them ([1] pag. 71).

2.1.2.2 Horizontal flight regime with a given altitude

The following expression is for the induced power *Pi* (valid for any value of *vi*):

$$Pi = kint \cdot k \cdot \left(2 \cdot \left(\frac{T}{2}\right)\right) \cdot vi \tag{47}$$

where *vi* is the induced velocity ([1] pag. 65):

$$vi = \lambda \cdot (\Omega \cdot R) \tag{48}$$

The solution for the inflow ratio, λ is:

$$\lambda = \frac{(\lambda_h)^2}{\left(\sqrt{(\mu^2 + \lambda^2)}\right)} \tag{49}$$

As for the conventional helicopter (21), to solve for λ can be used, for example, the Newton-Raphson method.

The inflow ratio for the hover case λ_h results:

$$\lambda_h = \sqrt{\left(\frac{Ct}{2}\right)} \tag{50}$$

The coefficient of thrust for the coaxial design is:

$$Ct = \frac{\left(\frac{T}{2}\right)}{\left(\rho \cdot A \cdot Vtip^2\right)} \tag{51}$$

where *A* and *Vtip* are the area of each rotor and the blade tip speed of the coaxial helicopter.

The coefficient of induced power *Cpi* of the coaxial system (valid for μ>0.15) results (see the relation for the conventional helicopter (23)):

$$Cpi = 2 \cdot kint \cdot k \cdot \frac{Ct^2}{(2 \cdot \mu)} \tag{52}$$

So the induced power is given by the expression:

$$Pi = Cpi \cdot \rho \cdot A \cdot Vtip^3 = kint \cdot k \cdot 2 \cdot \frac{(\frac{T}{2})^2}{(2 \cdot \rho \cdot A \cdot Vinf)} = kint \cdot k \cdot 2 \cdot (\frac{T}{2}) \cdot vi \tag{53}$$

where the last equality is the expression (47). So the induced velocity *vi* can be expressed:

$$vi = \frac{(\frac{T}{2})}{(2 \cdot \rho \cdot A \cdot Vinf)} \tag{54}$$

For the calculation of the profile power *P0* for the coaxial design the formula is:

$$P0 = \frac{(2 \cdot \sigma \cdot Cd0)}{8} \cdot (1 + K \cdot \mu^2) \cdot (\rho \cdot A \cdot Vtip^3) \tag{55}$$

The expression of the parasitic power *Pp* is the same as for the coventional configuration given by the relation (27).

2.1.3 The tandem rotor

The momentum theory can be applied also to an overlapping tandem rotor. The tandem configuration, as the coaxial, doesn't need a tail rotor, so all of the rotor power is used to create lift. But the induced power of the partly overlapping tandem results higher respect to the two isolated rotors, like for the coaxial design, because one of the rotors has to work in the slipstream of the other one. This problems were studied by Stepniewsky & Keys (1984) ([1] pag. 71).

2.1.3.1 Hover Regime

As Payne (1959) suggests ([1] pag. 71), the overlapping rotors are studied with the momentum theory using the "overlapping areas".

$$Aov = m \cdot A \tag{56}$$

where *Aov*, described in the Figure 1.6, is the overlap area.

As Payne (1959) suggests, the overlapping rotors are studied with the momentum theory using the "overlapping areas" and the two rotors are thought to have the verical spacing null, see Figure 2.3 below.

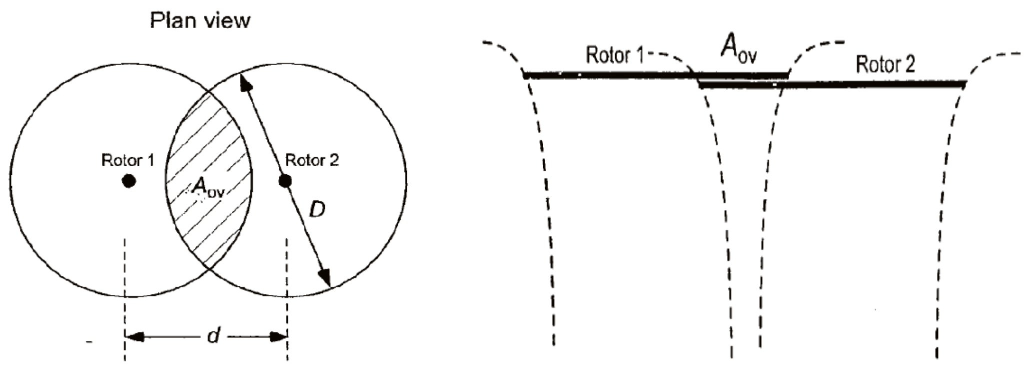


Figure 2.3 Flow model used for tandem rotor analysis, with the consideration that the two rotors are in the same plane ([19] pag. 145 fig. 2.32)

Using the geometry, Figure 2.3 left, results ([1] pag. 72 eq.2.130):

$$m = (\frac{2}{\pi}) \cdot [\theta - \frac{d}{D} \cdot sin\theta] \qquad \text{where} \qquad \theta = \cos^{-1}(\frac{d}{D}) \tag{57}$$

If T_1 and T_2 are the thrusts of the two rotors, that can be also equal, results that the thrust of the overlapped region is $m \cdot (T_1+T_2)$. Assuming that the inflow is uniform, the induced power on each of the three areas (Figure 2.3 left) described by the rotors is:

$$P_1 = \frac{((1-m) \cdot T_1^{(\frac{3}{2})})}{\sqrt{(2 \cdot \rho \cdot A)}} \tag{58}$$

$$P_2 = \frac{((1-m)\cdot T_2^{(\frac{3}{2})})}{\sqrt{(2\cdot\rho\cdot A)}} \tag{59}$$

$$P_{ov} = \frac{(m\cdot(T_1+T_2)^{(\frac{3}{2})})}{\sqrt{(2\cdot\rho\cdot A)}} \tag{60}$$

where the ideal induced power of the tandem design is: $(P_I)_{tot} = P_1 + P_2 + P_{ov}$.

If the two rotors are isolated, *m* goes to zero and the ideal induced power of the system results ([1] pag. 72 eq.2.131):

$$P_I = P_1 + P_2 = \frac{(T_1^{(\frac{3}{2})} + T_2^{(\frac{3}{2})})}{\sqrt{(2\cdot\rho\cdot A)}} \tag{61}$$

The induced power overlap factor, *kov*, for the tandem design, as for the coaxial case with *kint*, is deduced from the ratio:

$$kov = \frac{(P_I)_{tot}}{P_I} = \frac{((1-m)\cdot T_1^{(\frac{3}{2})} + (1-m)\cdot T_2^{(\frac{3}{2})} + m\cdot(T_1+T_2)^{(\frac{2}{3})})}{(T_1^{(\frac{3}{2})} + T_2^{(\frac{3}{2})})} \tag{62}$$

If $T_1 = T_2$ means that the two rotors create equal thrust and results:

$$kov = \frac{(P_I)_{tot}}{P_I} = 1 + (\sqrt{2}-1)\cdot m = 1 + 0.4142\cdot m \tag{63}$$

where *m* is defined in the relation (57).

Another approximation of *kov* was proposed by Harris (1999) ([1] pag. 72 eq. 2.135):

$$kov \approx [\sqrt{2} - (\frac{\sqrt{2}}{2})\cdot(\frac{d}{D}) + (1-(\frac{\sqrt{2}}{2}))\cdot(\frac{d}{D})^2] \tag{64}$$

where *d* is the spacing between the two rotors axes and *D* is the rotor diameter (Fig. 2.3).

For $m \rightarrow 1$ in the equation (63), that is the coaxial case, results $kov \rightarrow \sqrt{2}$. If $m \rightarrow 0$, when the two rotors are isolated and don't present overlap, results $kov \rightarrow 1$.

In the Figure 2.4, is plotted the variation of *kov* in function of the ratio *d/D*. So, bigger is the distance between the two rotor shafts, more isolated result the two rotors and *kov* is going close to one.

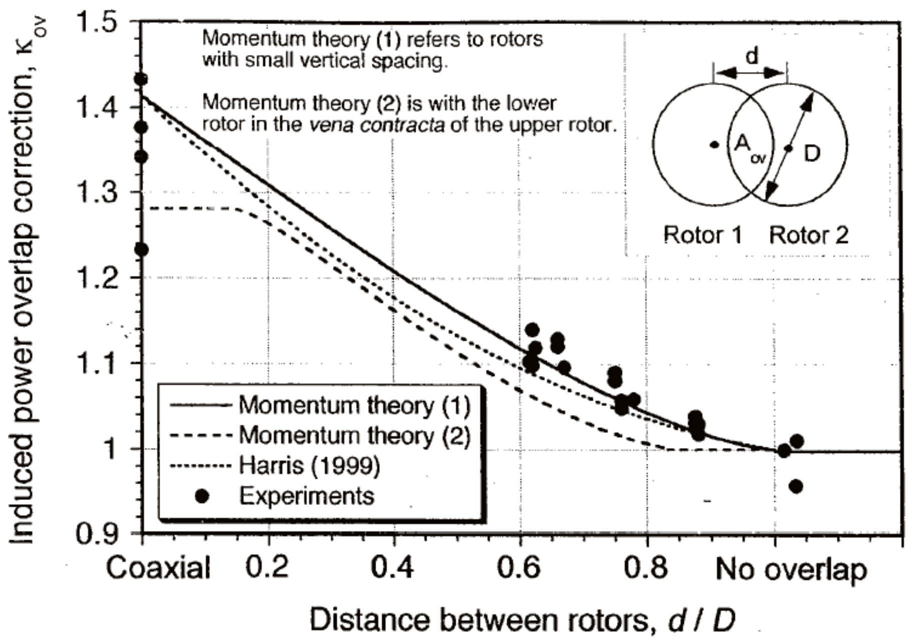


Figure 2.4 Tandem rotor overlap induced power correction, that derives from the momentum theory and compared to measurements ([19] pag. 106 fig. 2.31)

The total induced power needed for the hover (valid also for the forward flight) for a tandem configuration helicopter is described by ([19] pag. 107 eq. 2.168):

$$Pi = 2 \cdot kov \cdot k \cdot \frac{T^{\left(\frac{3}{2}\right)}}{\sqrt{(2 \cdot \rho \cdot A)}} \tag{65}$$

where $T_1 + T_2 = 2 \cdot T$ (in this case is considered $T_1 = T_2 = T = W/2$, where W is the weight of the helicopter) is the total system thrust generated by the two rotors and A is the area of each of the rotors. Was assumed the same k for each rotor in order to semplify the model.

The profile power of the tandem helicopter in hover, like for the coaxial case (46), has the following expression:

$$P0 = \frac{1}{8} \cdot (2 \cdot \sigma) \cdot Cd0 \cdot (\rho \cdot A \cdot Vtip^3) \tag{66}$$

There is $2 \cdot \sigma$ because the profile power considers both rotors of the tandem configuration.

2.1.3.2 Horizontal flight regime with a given altitude

The induced power for hover/level-flight for a tandem configuration can be expressed as in relation (65), but in this work we prefered the following relation ([1] pag. 177 eq. 5.51):

$$Pi = k \cdot Tf \cdot vif + kov \cdot k \cdot Tr \cdot vir \tag{67}$$

Where *Tf* and *vif* are the thrust and the induced speed of the front rotor, and, respectively, *Tr* and *vir* are the thrust and the induced speed of the rear rotor. The induced power overlap factor *kov*, assumed for the rear rotor, was considered in all this thesis equal to 1.14.

In this work we considered that *Tf* = *Tr* = *T/2 (= W/2)*, as for the coaxial case. Each rotor provides equal thrust, the tip speeds of the rotors are considered equal *Vtipf* = *Vtipr* and results *vif* = *vir*.

Summarizing the induced power *Pi* of the tandem case (valid for any value of *vi*) results:

$$Pi = k \cdot \left(\frac{T}{2}\right) \cdot vi + kov \cdot k \cdot \left(\frac{T}{2}\right) \cdot vi = (1 + kov) \cdot k \cdot \left(\frac{T}{2}\right) \cdot vi \tag{68}$$

where *vi* is the induced velocity for small values of α ([1] pag. 65):

$$vi = \lambda \cdot (\Omega \cdot R) \tag{69}$$

From the following expression can be obtain the solution for the inflow ratio λ :

$$\lambda = \frac{(\lambda_h)^2}{(\sqrt{(\mu^2 + \lambda^2)})} \tag{70}$$

where λ_h is the inflow ratio from the hover case:

$$\lambda_h = \sqrt{\left(\frac{Ct}{2}\right)} \tag{71}$$

The coefficient of thrust for the tandem design, as for the coaxial design, is:

$$Ct = \frac{(\frac{T}{2})}{(\rho \cdot A \cdot Vtip^2)} \tag{72}$$

where A and $Vtip$ are the area of the each rotor and the blade tip speed of the tandem helicopter.

As for the conventional helicopter (21), to solve for λ can be used, for example, the Newton-Raphson method.

The expression of the induced power of the tandem configuration (valid for any value of vi), similar to the coaxial case (24) is:

$$Pi = (1 + kov) \cdot k \cdot \frac{(\frac{T}{2})^2}{(2 \cdot \rho \cdot A \cdot Vinf)} = (1 + kov) \cdot k \cdot (\frac{T}{2}) \cdot vi \tag{73}$$

From the relation (73) can be expressed the the induced velocity vi :

$$vi = \frac{(\frac{T}{2})}{(2 \cdot \rho \cdot A \cdot Vinf)} \tag{74}$$

For the calculation of the profile power $P0$ for the tandem design, as for the coaxial case (55), the formula is:

$$P0 = \frac{(2 \cdot \sigma \cdot Cd0)}{8} \cdot (1 + K \cdot \mu^2) \cdot (\rho \cdot A \cdot Vtip^3) \tag{75}$$

The parasitic power Pp for all the cases of helicopters, previously given (27) to explain the expression of the trust of the tail rotor of a conventional helicopter (29), is recapitulated below ([1] p. 166, eq. 5.24):

$$Pp = \frac{1}{2} \cdot (\frac{f}{A}) \cdot \mu^3 \cdot (\rho \cdot A \cdot Vtip^3) \tag{76}$$

So the total power in horizontal flight of the coaxial and the tandem helicopters is given by the expression:

$$P = (Pi + P0 + Pp) \tag{77}$$

2.2 The Blade Element Theory

The Blade Element Theory (or the Strip Theory), was first proposed by Drzewiecki in 1892 to study the marine and later the airplane propellers [16]. The method consists of dividing the blade elements (strips) in the radial direction. The idea is to assume the each element to act as a two-dimensional airfoil that produces moments and aerodynamic forces.

The rotor is influenced by the wake and there results a non-uniform induced velocity distribution. This is represented by an induced component of the angle of attack at each element (Figure 2.5.b).

The integration of the contributions from all the elements, along the blade radius, can be use to obtain predictions of the performance of a certain rotorcraft.

The blade element theory divides the rotor blade into infinitesimally small, uniformly distributed, blade elements as represented in the Figure 2.5.a. The blade element at the rotor disk plane are influenced by the velocity components, U_T and U_R.

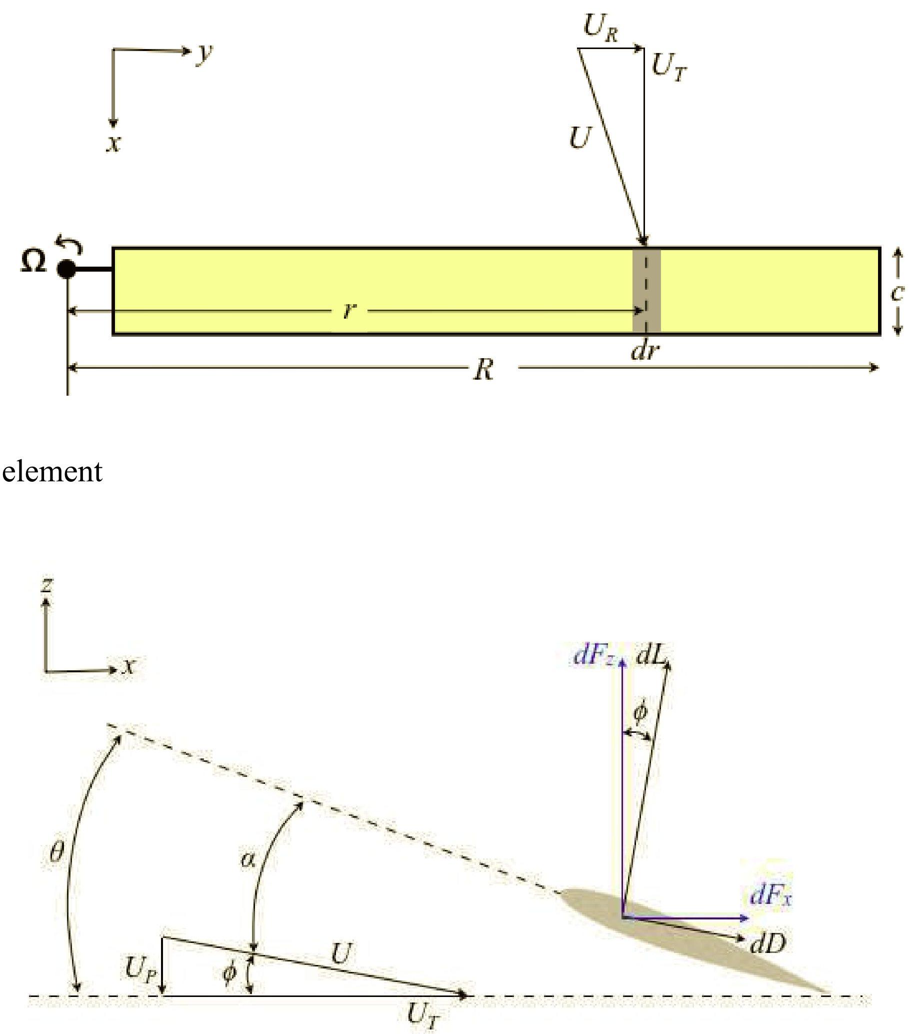
2.2.1 Hover regime

The blade element theory used for the hover analysis considers the blade elements as 2-D airfoil sections that provide aerodynamic forces and moments. U_R is negleted since it is a 2-D analisys and it has no influence on the aerodynamic characteristics of the blade element.

$$U_T = \Omega \cdot y, \quad U_R \approx 0 \tag{78}$$

where $y = R \cdot r$.

(a) Top view



(b) Blade element

Figure 2.5 Velocities and aerodynamic forces at a typical blade element ([16] fig. 2.2)

The Figure 2.5.b illustrates the geometry, the velocities and the aerodynamic forces at the airfoil section plane of the blade element. The blade is considered rigid and undeformed. Considering the climb velocity Vc null, in the Figure 2.5.b is represented the vertical component of the velocity, U_P, and is given by:

$$U_P = Vc + vi = \lambda \cdot \Omega \cdot r \qquad (79)$$

where λ is the induced inflow ratio at the blade element, calculated using by choosing the inflow model.

The resultant of the velocity at the blade element is:

$$U = \sqrt{(U_T^2 + U_R^2 + U_P^2)} \approx \sqrt{(U_T^2 + U_P^2)} \tag{80}$$

The relative inflow angle (or the induced angle of attack) at the blade element, also shown in the Figure 2.5, for small angles results:

$$\phi = \tan^{-1}(\frac{U_P}{U_T}) \approx \frac{U_P}{U_T} \tag{81}$$

so also we have that $sin\phi = \phi$ and $cos\phi = 1$. The relation (81) is not valid for small U_T (near the blade root). But since this is the root cut-out zone this zone is normally neglected.

Now, considering θ as the pitch angle at the blade element, the effective – aerodynamic angle of attack is given by:

$$\alpha = \theta - \phi = \theta - \frac{U_P}{U_T} \tag{82}$$

The elemental aerodynamic forces per unit span on the blade, *dL* and *dD*, are defined as normal and parallel to the velocity vector, respectively, and described below:

$$dL = \frac{1}{2} \cdot \rho \cdot U^2 \cdot c \cdot Cl \cdot dy \qquad dD = \frac{1}{2} \cdot \rho \cdot U^2 \cdot c \cdot Cd \cdot dy \tag{83}$$

where *Cl* and *Cd* are the lift and drag coefficients per blade section, taken from specific airfoil tables. *c* is the local blade chord.

The aerodynamic forces can be solved, represented in Figure 2.5, perpendicular and parallel to the plane of the rotor disk, respectively. The equations, that describe the forces *dFz* and *dFx*, employ the local lift and drag forces (Eq. (83)) using simple geometric transformation:

$$dFz = dL \cdot cos\phi - dD \cdot sin\phi \qquad dFx = dL \cdot sin\phi + dD \cdot cos\phi \tag{84}$$

The thrust, the torque, and the power contribution from a blade element at a radial station r is expressed like below:

$$dT = Nb \cdot dFz = Nb \cdot (dL \cdot cos\phi - dD \cdot sin\phi) \quad (85)$$

$$dQ = Nb \cdot dFx \cdot y = Nb \cdot (dL \cdot sin\phi + dD \cdot cos\phi) \cdot y \quad (86)$$

$$dP = Nb \cdot dFx \cdot \Omega \cdot y = Nb \cdot (dL \cdot sin\phi + dD \cdot cos\phi) \cdot \Omega \cdot y \quad (87)$$

where $y = R \cdot r$ and $dy = R \cdot dr$ (see Figure 2.5.a).

Using the equations (85), (86), 87), and the expressions for the lift and drag for the blade element, given in Eq. (83), the thrust, the torque, and the power can be non-dimensionalized in the next way ([16] eqs. 2.14, 2.15, 2.16):

$$dCt = \frac{dT}{(\rho \cdot A \cdot (\Omega \cdot R)^2)} = \frac{1}{2} \cdot \sigma \cdot (Cl \cdot cos\phi - Cd \cdot sin\phi) \cdot r^2 \cdot dr \quad (88)$$

$$dCq = \frac{dQ}{(\rho \cdot A \cdot (\Omega \cdot R)^2 \cdot R)} = \frac{1}{2} \cdot \sigma \cdot (Cl \cdot sin\phi + Cd \cdot cos\phi) \cdot r^3 \cdot dr \quad (89)$$

$$dCp = \frac{dP}{(\rho \cdot A \cdot (\Omega \cdot R)^3)} = dCq \quad (90)$$

where σ is the local solidity, defined previously (13).

The local blade lift coefficient, based on the linearized aerodynamics, is expressed by:

$$Cl = Cl\alpha \cdot (\alpha - \alpha 0) = Cl\alpha \cdot (\theta - \alpha 0 - \phi) \quad (91)$$

where $Cl\alpha$ is the 2-D lift-curve-slope of the airfoils of the rotor blades and $\alpha 0$ is the zero-lift angle.

To obtain the total coefficient of thrust and of power/torque, Ct and $Cp = Cq$, the contributions of each element are integrated over the entire blade:

$$Ct = \int_0^1 dCt \qquad \text{and} \qquad Cq = Cp = \int_0^1 dCp \quad (92)$$

In this work was not considered a root cut-out of the blades.

The rotor incremental power can be written as ([1] pag. 84 eq. 3.27):

$$dCp = dCq = \frac{\sigma}{2} \cdot (\phi \cdot Cl + Cd) \cdot r^3 \cdot dr \qquad (93)$$

Using the relation $\lambda = \phi \cdot r$ and expanding the previous expression (93), results:

$$dCp = \frac{\sigma}{2} \cdot \phi \cdot Cl \cdot r^3 \cdot dr + \frac{\sigma}{2} \cdot Cd \cdot r^3 \cdot dr \qquad (94)$$

$$dCp = \frac{\sigma}{2} \cdot Cl \cdot \lambda \cdot r^2 \cdot dr + \frac{\sigma}{2} \cdot Cd \cdot r^3 \cdot dr = dCpi + dCp0 \qquad (95)$$

where *dCpi* is the induced power and *dCp0* is the profile power.

From the relation (91), where the lift coefficient is expressed, results:

$$Cl = Cl\alpha \cdot (\theta - \alpha 0 - \phi) = Cl\alpha \cdot (\theta 0 - \frac{\lambda}{r}) \qquad (96)$$

where $\theta = \theta 0$ because the blade, in this work, is considered untwisted and $\alpha 0 = 0$ because the airfoils are assumed symmetric. Due to the small angle approximation, *Cd* is assumed to be equal to *Cd0*, the profile (viscous) drag coefficient of the airfoils of the rotor blades.

The pitch angle $\theta 0$, in terms of thrust, is given by the relation ([1] pag. 83 eq. 3.24):

$$\theta 0 = \frac{(6 \cdot Ct)}{(\sigma \cdot Cl\alpha)} + \frac{3}{2} \cdot \sqrt{(\frac{Ct}{2})} \qquad (97)$$

The first term is the pitch of the blade needed to create thrust and the second term is the additional pitch that compensates the inflow created by this thrust.

Recapping and putting together all the relations (96, 97), the power coefficient *dCp* needed for a helicopter can be expressed as follows:

$$Cp = \int_0^1 dCp = \int_0^1 (\frac{\sigma}{2} \cdot Cl \cdot \lambda \cdot r^2 \cdot dr) + (\frac{\sigma}{2} \cdot Cd \cdot r^3 \cdot dr) \qquad (98)$$

$$Cp = \int_0^1 ((\frac{\sigma}{2} \cdot Cl\alpha \cdot (\theta 0 - \frac{\lambda}{r}) \cdot \lambda \cdot r^2) + (\frac{\sigma}{2} \cdot Cd0 \cdot r^3)) \cdot dr \qquad (99)$$

And results the integral made of three therms, the first two terms stand for the induced power and the last one stands for the profile power:

$$Cp = \int_0^1 ((\frac{\sigma}{2} \cdot Cl\alpha \cdot \theta 0 \cdot \lambda \cdot r^2) - (\frac{\sigma}{2} \cdot Cl\alpha \cdot \lambda^2 \cdot r) + (\frac{\sigma}{2} \cdot Cd0 \cdot r^3)) \cdot dr \quad (100)$$

In the transition from hover to forward flight ($0.0 \leq \mu \leq 0.15$) of a rotor, the induced velocity in the plane of the rotor is not uniform. But for a higher forward flight velocity ($\mu > 0.15$) the inflow becomes more linear so an approximation of the induced inflow ratio of the rotor, proposed by Glauert (1926), is given ([1] pag. 116 eq. 3.165):

$$\lambda i = \lambda 0 \cdot (1 + kx \cdot \frac{x}{R}) = \lambda 0 \cdot (1 + kx \cdot r \cdot cos\psi) \quad (101)$$

where $\lambda 0$ is the mean induced inflow ratio at the center of the rotor, as given by the momentum theory ([1] pag. 116 eq. 3.166):

$$\lambda i = \lambda 0 = \frac{Ct}{(2 \cdot \sqrt{(\mu^2 + \lambda i^2)})} \quad (102)$$

An approximation of the weighting factors, proposed by Coleman (1945) and Johnson (1980) is to use the rigid cylindrical vortex wake theories. So kx can be expressed in the following way:

$$kx = \tan(\frac{\chi}{2}) \quad (103)$$

where χ is the angle of the wake skew and expressed as:

$$\chi = \tan^{-1}(\frac{\mu x}{(\mu z + \lambda i)}) \quad (104)$$

where μx and μz are the advance ratios (Figure 2.6), respectively, parallel and perpendicular to the disk of the rotor.

From the Figure 2.6, for μ>0.2, the wake starts to be flat and results that for high forward speed kx reaches 1. In this work, the contribution of μz will be neglected because, in the relation (104) μx is much higher than μz.

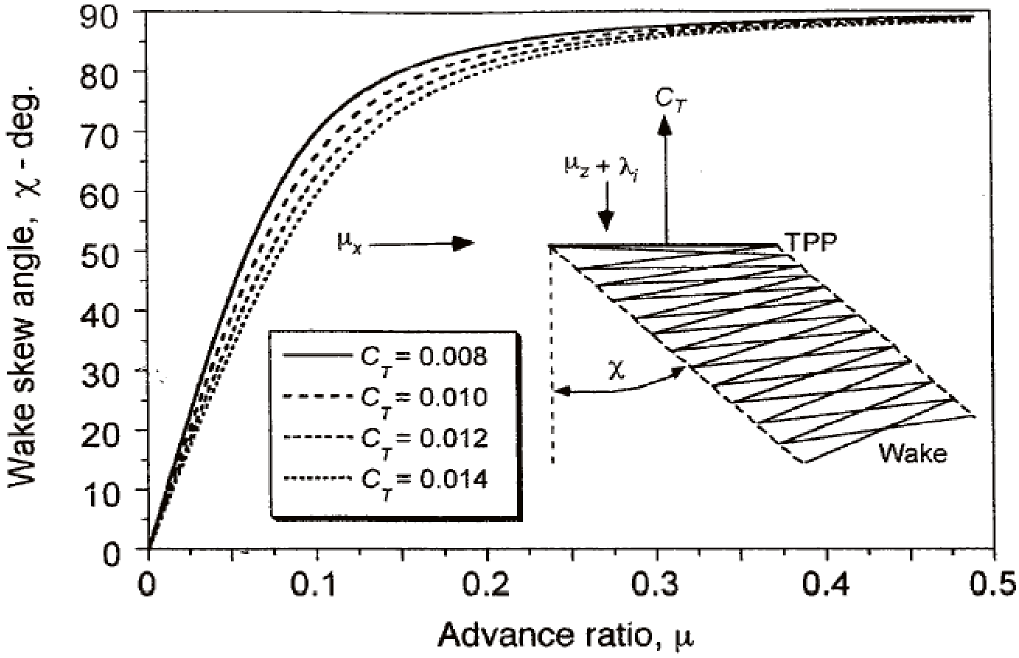


Figure 2.6 Variation in the rotor wake skew angle with the coefficient of thrust *Ct* and the advance ratio μ ([19] pag. 160 fig. 3.28)

Also other authors suggested an approximation of *kx*, listed in the Table 2.1 ([19] pag. 160 Table 3.1):

Author(s)	k_x	k_y
Coleman et al. (1945)	$\tan(\chi/2)$	0
Drees (1949)	$(4/3)(1 - \cos\chi - 1.8\mu^2)/\sin\chi$	-2μ
Payne (1959)	$(4/3)[\mu/\lambda/(1.2 + \mu/\lambda)]$	0
White & Blake (1979)	$\sqrt{2}\sin\chi$	0
Pitt & Peters (1981)	$(15\pi/23)\tan(\chi/2)$	0
Howlett (1981)	$\sin^2\chi$	0

Table 2.1 Estimated values of the first harmonic inflow

where *kx* and *ky* represent the deviation of the inflow from the value predicted with the momentum theory. As told in the relation (101), the Coleman (1945) model was used in this work, but if Drees (1945) model is used, for example, the expression of induced inflow ratio λi (101) uses also *ky* and becomes:

$$\lambda i = \lambda 0 \cdot (1 + kx \cdot \frac{x}{R} + ky \cdot \frac{y}{R}) = \lambda 0 \cdot (1 + kx \cdot r \cdot cos\psi) + ky \cdot r \cdot sin\psi$$

(105)

So recapping, the expression of the λi, (101), can be completed using the last consideration done:

$$\lambda i = \lambda 0 \cdot (1 + kx \cdot r \cdot cos\psi) = \lambda 0 \cdot (1 + \tan(\frac{\chi}{2}) \cdot r \cdot cos\psi) \qquad (106)$$

so λi results:

$$\lambda i = \lambda 0 \cdot (1 + \tan(\frac{(\tan^{-1}(\frac{\mu x}{(\mu z + \lambda i)}))}{2}) \cdot r \cdot cos\psi) \qquad (107)$$

And now, the induced inflow ratio λi, is used to calculate the integral of the coefficient of power of the rotor in the relation (100). The expression (107) has on the both sides λi, so it has to be calculated iteratively, for example, with the Newton – Raphson method. Inside the relation (107) r and ψ are also present, that are the variables along which the integral of dCp is made.

Since an average over the azimuthal angle is needed, the next expression has to be divided by 2π. So, summarizing, for the one rotor helicopter, the coefficient of power for hover is given by (100):

$$Cp_{i0} = Cpi + Cp0 = \frac{1}{2\pi} \cdot (\int_0^{2\pi}\int_0^1 dCpi \cdot dr \cdot d\psi + \int_0^{2\pi}\int_0^1 dCp0 \cdot dr \cdot d\psi)$$

(108)

or better:

$$Cp_{i0} = \frac{1}{2\pi} \cdot (\int_0^{2\pi}\int_0^1 ((\frac{\sigma}{2} \cdot Cl\alpha \cdot \theta 0 \cdot \lambda \cdot r^2) - (\frac{\sigma}{2} \cdot Cl\alpha \cdot \lambda^2 \cdot r)) \cdot dr \cdot d\psi + \int_0^{2\pi}\int_0^1 (\frac{\sigma}{2} \cdot Cd0 \cdot r^3) \cdot dr \cdot d\psi)$$

(109)

And the power of the one rotor helicopter in hover is given by:

$$P_{i0} = Cp_{i0} \cdot \rho \cdot A \cdot Vtip^3 \qquad (110)$$

where $Vtip = \Omega \cdot R$.

For an one rotor helicopter in forward flight, the parasitic power of the fuselage also appears, and it is expressed with the same relation as for the momentum theory (76). So the total power of the one rotor helicopter in forward flight becomes:

$$P = P_{i0} + Pp = Pi + P0 + Pp = Cp_{i0} \cdot \rho \cdot A \cdot Vtip^3 + \frac{1}{2} \cdot \left(\frac{f}{A}\right) \cdot \mu^3 \cdot (\rho \cdot A \cdot Vtip^3)$$

(111)

2.2.1.1 Conventional helicopter

In a conventional helicopter the tail rotor is present, so it's attached to a single rotor helicopter, described by the relation (111) and, like for the momentum theory (29), the thrust provided by the tail rotor is given by:

$$Ttr = \frac{P}{(\Omega \cdot dtr)} = \frac{(P_{i0} + Pp)}{(\Omega \cdot dtr)} = \frac{(Cp_{i0} \cdot \rho \cdot A \cdot Vtip^3 + \frac{1}{2} \cdot (\frac{f}{A}) \cdot \mu^3 \cdot (\rho \cdot A \cdot Vtip^3))}{(\Omega \cdot dtr)}$$

(112)

and the coefficient of thrust of the tail rotor, like for momentum theory (34), results:

$$Cttr = \frac{Ttr}{(\rho \cdot Atr \cdot Vtiptr^2)} \tag{113}$$

The pitch angle $\theta 0$ of the tail rotor, in terms of thrust, is given by the relation, like for the mai rotor (97), is:

$$\theta 0 = \frac{(6 \cdot Cttr)}{(\sigma \cdot Cl\alpha)} + \frac{3}{2} \cdot \sqrt{\left(\frac{Cttr}{2}\right)} \tag{114}$$

The coefficient of power of the tail rotor is calculated with the same formula used for the main rotor (108), where all the variables are of the tail rotor, and results:

$$Cptr = Cpitr + Cp0tr = \frac{1}{2\pi} \cdot \left(\int_0^{2\pi} \int_0^1 dCpitr \cdot dr \cdot d\psi + \int_0^{2\pi} \int_0^1 dCp0tr \cdot dr \cdot d\psi\right)$$

(115)

and, finally, results, like for the main rotor (109):

$$Cptr = \frac{1}{2\pi}\cdot(\int_0^{2\pi}\int_0^1((\frac{\sigma tr}{2}\cdot Cl\alpha tr\cdot\theta 0tr\cdot\lambda tr\cdot r^2)-(\frac{\sigma tr}{2}\cdot Cl\alpha tr\cdot\lambda tr^2\cdot r))\cdot dr\cdot d\psi)$$
$$+\ \frac{1}{2\pi}\cdot(\int_0^{2\pi}\int_0^1(\frac{\sigma tr}{2}\cdot Cd0tr\cdot r^3)\cdot dr\cdot d\psi) \tag{116}$$

The power of the tail rotor is given by:

$$Ptr = Pitr + P0tr = Cptr\cdot\ \rho\cdot\ Atr\cdot Vtiptr^3 \tag{117}$$

In this work, to simplify the calculations, it is considered that the tip speed of the tail rotor is equal to the tip speed of the main rotor $Vtiptr = Vtip$ (see relation (35)) and due to the relation (18) results that $\mu = \mu tr$. In case the two tip speeds are different, the program developed, in this thesis, take it into account for calculations by using the relation (35).

The total power of a conventional helicopter in forward flight, like in the momentum theory (17), is given by the following relation:

$$P = (P_{i0} + Ptr + Pp) = (Pi + P0 + Ptr + Pp) \tag{118}$$

2.2.1.2 Coaxial/Tandem helicopters

The coefficient of thrust of the coaxial/tandem configurations, like for the momentum theory (52), (72), is given by the relation:

$$Ct = \frac{(\frac{T}{2})}{(\rho\cdot\ A\cdot\ Vtip^2)} \tag{119}$$

The pitch angle $\theta 0$, in terms of thrust, is given by the relation (97):

$$\theta 0 = \frac{(6\cdot Ct)}{(\sigma\cdot Cl\alpha)}+\frac{3}{2}\cdot\sqrt{(\frac{Ct}{2})} \tag{120}$$

The coefficient of thrust for hover of each rotor of a coaxial helicopter, as for the one rotor helicopter (109), is given by:

$$(Cp_{i0})_{Coax} = \frac{1}{2\pi} \cdot (\int_0^{2\pi}\int_0^1 kint \cdot ((\frac{\sigma}{2} \cdot Cl\alpha \cdot \theta 0 \cdot \lambda \cdot r^2) - (\frac{\sigma}{2} \cdot Cl\alpha \cdot \lambda^2 \cdot r)) \cdot dr \cdot d\psi)$$
$$+ \frac{1}{2\pi} \cdot (\int_0^{2\pi}\int_0^1 (\frac{\sigma}{2} \cdot Cd0 \cdot r^3) \cdot dr \cdot d\psi)$$

(121)

where *kint* is the induced power interference factor between the rotors, described previously in the momentum theory.

So the total power of the coaxial system, in forward flight, is given by:

$$P = (P_{i0})_{Coax} + Pp = 2 \cdot (Pi_{Coax} + P0_{Coax}) + Pp = 2 \cdot ((Cp_{i0})_{Coax} \cdot \rho \cdot A \cdot Vtip^3) + Pp$$

(122)

where the sum of the induced power and the profile power for one rotor, given from the formula (110), is multiplied per two because there are two rotors and they are on the same axis.

The coefficient of thrust, for hover, of the front rotor of a tandem helicopter, as for the one rotor helicopter (109), is given by:

$$(Cp_{i0})_{TandemFront} = \frac{1}{2\pi} \cdot (\int_0^{2\pi}\int_0^1 ((\frac{\sigma}{2} \cdot Cl\alpha \cdot \theta 0 \cdot \lambda \cdot r^2) - (\frac{\sigma}{2} \cdot Cl\alpha \cdot \lambda^2 \cdot r)) \cdot dr \cdot d\psi)$$
$$+ \frac{1}{2\pi} \cdot (\int_0^{2\pi}\int_0^1 (\frac{\sigma}{2} \cdot Cd0 \cdot r^3) \cdot dr \cdot d\psi)$$

(123)

The coefficient of thrust, for hover, of the rear rotor of a tandem helicopter, as for the front rotor (123), is given by:

$$(Cp_{i0})_{TandemRear} = \frac{1}{2\pi} \cdot (\int_0^{2\pi}\int_0^1 kov \cdot ((\frac{\sigma}{2} \cdot Cl\alpha \cdot \theta 0 \cdot \lambda \cdot r^2) - (\frac{\sigma}{2} \cdot Cl\alpha \cdot \lambda^2 \cdot r)) \cdot dr \cdot d\psi)$$
$$+ \frac{1}{2\pi} \cdot (\int_0^{2\pi}\int_0^1 (\frac{\sigma}{2} \cdot Cd0 \cdot r^3) \cdot dr \cdot d\psi)$$

(124)

where *kov* is the induced power overlap correction between the rotors, described previously in the momentum theory.

So the total power of the tandem system, in forward flight, is given by:

$$P = ((P_{i0})_{TandemFront} + (P_{i0})_{TandemRear}) + Pp = (Pi_{TandemFront} + P0_{TandemFront}) + \\ + (Pi_{TandemRear} + P0_{TandemRear}) + Pp$$

(125)

and, finally, results:

$$P = ((Cp_{i0})_{TandemFront} \cdot \rho \cdot A \cdot Vtip^3) + ((Cp_{i0})_{TandemRear} \cdot \rho \cdot A \cdot Vtip^3) + Pp$$

(126)

The parasitic power, *Pp,* has the same expression, (76), for all the designs and for the both theories used in this thesis, momentum theory and blade element theory.

All the variables present in the previous relations are caracterizing the each helicopter design. Certain variables are considered as an input, like the masses, the altitude, the number of blades ect, that has to be provided. But the other variables are calculated from empirical formulas that will be provided below.

2.2.2 Horizontal flight regime with a given altitude

The coefficient of the power of the rotor (see relation (93)), for the horizontal flight regime, can be developed in the next way:

$$dCp = \frac{dP}{(\rho \cdot A \cdot (\Omega \cdot R)^3)} = \frac{(Nb \cdot (dL \cdot sin\phi + dD \cdot cos\phi) \cdot \Omega \cdot y)}{(\rho \cdot (\pi \cdot R^2) \cdot \Omega^3 \cdot R^3)} = \\ = \frac{(Nb \cdot (\frac{1}{2} \cdot \rho \cdot U^2 \cdot c \cdot Cl \cdot \phi \cdot dy + \frac{1}{2} \cdot \rho \cdot U^2 \cdot c \cdot Cd \cdot dy) \cdot \Omega \cdot y)}{(\rho \cdot (\pi \cdot R^2) \cdot \Omega^3 \cdot R^3)}$$

(127)

where:

- dL and dD are given by the relation (83),
- for small angles results $sin\phi = \phi$ and $cos\phi = 1$,
- $A = \pi \cdot R^2$, the area of the rotor,
- $y = R \cdot (r + \mu \cdot sin\psi)$ and results $dy = R \cdot (1 + \mu \cdot cos\psi) \cdot dr \cdot d\psi$,
- from the relations (78) and (80), neglecting Ur and considering Up null for the horizontal fight results: $U = U_T = \Omega \cdot y = \Omega \cdot R \cdot (r + \mu \cdot sin\psi)$,

And results:

$$dCp = \frac{(Nb \cdot (\frac{1}{2} \cdot \rho \cdot (\Omega \cdot R \cdot (r+\mu \cdot sin\psi))^2 \cdot c \cdot Cl \cdot \phi \cdot R \cdot (1+\mu \cdot cos\psi)) \cdot \Omega \cdot R \cdot (r+\mu \cdot sin\psi) \cdot dr \cdot d\psi)}{(\rho \cdot (\pi \cdot R^2) \cdot \Omega^3 \cdot R^3)} +$$

$$+ \frac{(Nb \cdot (\frac{1}{2} \cdot \rho \cdot (\Omega \cdot R \cdot (r+\mu \cdot sin\psi))^2 \cdot c \cdot Cd \cdot R \cdot (1+\mu \cdot cos\psi)) \cdot \Omega \cdot R \cdot (r+\mu \cdot sin\psi) \cdot dr \cdot d\psi)}{(\rho \cdot (\pi \cdot R^2) \cdot \Omega^3 \cdot R^3)} =$$

$$= \frac{(\frac{\sigma}{2} \cdot ((r+\mu \cdot sin\psi)^3 \cdot Cl \cdot \phi) \cdot \Omega^3 \cdot R^4 \cdot (1+\mu \cdot cos\psi) \cdot dr \cdot d\psi)}{(\Omega^3 \cdot R^4)} +$$

$$+ \frac{(\frac{\sigma}{2} \cdot ((r+\mu \cdot sin\psi)^3 \cdot Cd) \cdot \Omega^3 \cdot R^4 \cdot (1+\mu \cdot cos\psi) \cdot dr \cdot d\psi)}{(\Omega^3 \cdot R^3)} =$$

$$= \frac{\sigma}{2} \cdot ((r+\mu \cdot sin\psi)^3 \cdot Cl \cdot \phi) \cdot (1+\mu \cdot cos\psi) \cdot dr \cdot d\psi +$$

$$+ \frac{\sigma}{2} \cdot ((r+\mu \cdot sin\psi)^3 \cdot Cd) \cdot (1+\mu \cdot cos\psi) \cdot dr \cdot d\psi = dCpi + dCp0$$

(128)

where:

- σ is given by the relation (13),

- from the relation (96) the lift coefficient is given by: $Cl = Cl\alpha \cdot (\theta - \alpha 0 - \phi)$, where $\theta = \theta 0$ because the blade is considered untwisted and $\alpha 0 = 0$ because the airfoils are assumed symmetric,

- $Cd = Cd0$, as for hover condition,

- the relative inflow angle (or the induced angle of attack) at the blade element, see (Figure 2.5), for small angles given in the relation (81) in this case:

$$\phi = \tan^{-1}(\frac{U_P}{U_T}) \approx \frac{U_P}{U_T} = \frac{(Vc+vi)}{(\Omega \cdot y)} = \frac{(vi)}{(\Omega \cdot R \cdot (r+\mu \cdot sin\psi))} = \frac{(\Omega \cdot R \cdot \lambda)}{(\Omega \cdot R \cdot (r+\mu \cdot sin\psi))}$$

(129)

and results:

$$\phi = \frac{\lambda}{(r + \mu \cdot sin\psi)} \tag{130}$$

- the inflow rate λ is given by the relation (107),

And the lift coefficient for forward flight condition results:

$$Cl = Cl\alpha \cdot (\theta 0 - \frac{\lambda}{(r + \mu \cdot sin\psi)}) \tag{131}$$

Summarizing, the relation (128), can be expressed in the next way:

$$dCp = \frac{\sigma}{2} \cdot (r+\mu \cdot sin\psi)^3 \cdot Cl\alpha \cdot (\theta 0 - \frac{\lambda}{(r+\mu \cdot sin\psi)}) \cdot \frac{\lambda}{(r+\mu \cdot sin\psi)} \cdot (1+\mu \cdot cos\psi) \cdot dr \cdot d\psi +$$
$$+ \frac{\sigma}{2} \cdot ((r+\mu \cdot sin\psi)^3 \cdot Cd0) \cdot (1+\mu \cdot cos\psi) \cdot dr \cdot d\psi =$$
$$= \frac{\sigma}{2} \cdot (r+\mu \cdot sin\psi)^2 \cdot Cl\alpha \cdot \theta 0 \cdot \lambda \cdot (1+\mu \cdot cos\psi) \cdot dr \cdot d\psi -$$
$$- \frac{\sigma}{2} \cdot (r+\mu \cdot sin\psi) \cdot Cl\alpha \cdot \lambda^2 \cdot (1+\mu \cdot cos\psi) \cdot dr \cdot d\psi + \frac{\sigma}{2} \cdot (r+\mu \cdot sin\psi)^3 \cdot Cd0 \cdot (1+\mu \cdot cos\psi) \cdot dr \cdot d\psi$$

(132)

So the relation (109), for the forward flight condition, results:

$$Cp_{i0} = Cpi + Cp0 = \frac{1}{2\pi} \cdot (\int_0^{2\pi} \int_0^1 (\frac{\sigma}{2} \cdot (r+\mu \cdot sin\psi)^2 \cdot Cl\alpha \cdot \theta 0 \cdot \lambda) \cdot (1+\mu \cdot cos\psi) \cdot dr \cdot d\psi) -$$
$$- \frac{1}{2\pi} \cdot (\int_0^{2\pi} \int_0^1 (\frac{\sigma}{2} \cdot (r+\mu \cdot sin\psi) \cdot Cl\alpha \cdot \lambda^2) \cdot (1+\mu \cdot cos\psi) \cdot dr \cdot d\psi) +$$
$$+ \frac{1}{2\pi} \cdot (\int_0^{2\pi} \int_0^1 (\frac{\sigma}{2} \cdot (r+\mu \cdot sin\psi)^3 \cdot Cd0) \cdot (1+\mu \cdot cos\psi) \cdot dr \cdot d\psi)$$

(133)

As for hover condition, the first two terms of the expressions (132) and (133) are characterising the induced power and the last term stays for the profile power of the rotor.

2.3 Important advance velocities

Many important characteristics of the helicopter in forward flight, like the climb or the autorotation, can be estimated from the curves of the power in function of the level flight speed, see the next figure. From these curves, the airspeed for maximum rate of climb and endurance can be obtained, as well as the vertical velocity of the helicopter for the maximum rate of climb and endurance, the airspeed for the maximum range, the maximum level flight speed, etc.

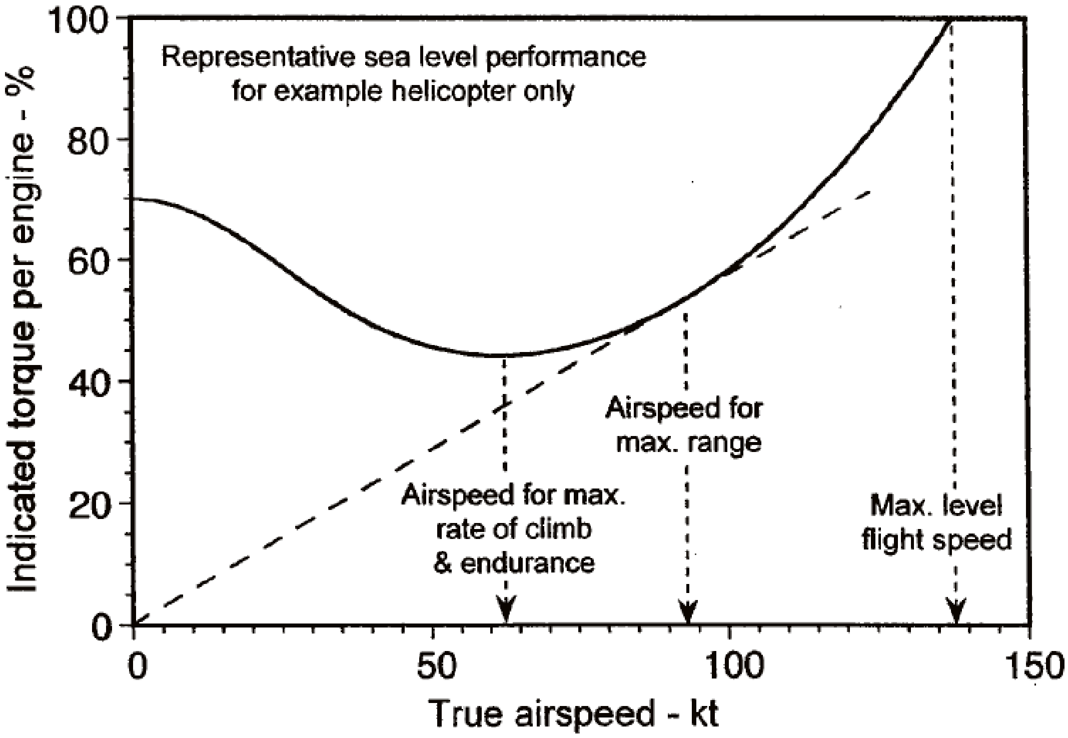


Figure 2.7 The representation, with the torque (power) curve, of the speed to fly for maximum rate of climb, maximum range and maximum level flight speed ([19] pag. 235 fig. 5.17)

2.3.1 Velocity for minimum power, Vmp

Vmp is the forward velocity for the minimum power of a helicopter, also called the forward velocity for which the climb rate is maximum at that height. This situation is verified at a relatively low speed, in the range of 60-80 kts (110-150 km/h), see Figure 2.7.

The maximum possible rate of climb is also the optimum speed to fly for minimum autorotative rate of descent. In this case, if a mechanical failure occurs, the pilot is able to translate the potential energy stored into the translational kinetic energy and so to save the machine and the crew. This is possible because the autorotative rate of descent results lower. In this thesis was not used the condition of the autorotation, this can be an objective to treat in a possible future improvement.

The velocity for the minimum power *Vmp* determines also the airspeed ratio to obtain the maximum flying time of the helicopter, the endurance.

To estimate *Vmp*, the expression of the coefficient of power of the helicopter ([1] pag. 172 eq. 5.37), is used where $\lambda c = 0$, so to look for the point of minimum power and the point of minimum *P/Vinf*. In the relation (134) is added also the contribution due to the tail rotor for the convetional design:

$$Cp = Cpmr + Cptr = (\frac{(k \cdot Cw^2)}{(2 \cdot \mu)} + \frac{(\sigma \cdot Cd0)}{8} \cdot (1 + K \cdot \mu^2) + \frac{1}{2} \cdot (\frac{f}{A}) \cdot \mu^3) + Cptr$$

(134)

where *Cpmr* is the coefficient of power of the main rotor plus the parasitic power and *Cptr* is the coefficient of power of the tail rotor.
And results:

$$Cp = \frac{(k \cdot Cw^2)}{(2 \cdot \mu)} + \frac{(\sigma \cdot Cd0)}{8} \cdot (1 + K \cdot \mu^2) + \frac{1}{2} \cdot (\frac{f}{A}) \cdot \mu^3 + (\frac{(k \cdot Cttr^2)}{(2 \cdot \mu)} + \frac{(\sigma tr \cdot Cd0tr)}{8} \cdot (1 + K \cdot \mu^2))$$

(135)

where $Ct = Cw$ is the coefficient of weight and equals the thrust needed by the main rotor:

$$Cw = \frac{(W)}{(\rho \cdot A \cdot Vtip^2)} \qquad (136)$$

where *W* is the weight of the helicopter at takeoff. *Cttr* is the coefficient of thrust due to the tail rotor and is given by the relation (29), so using the first three terms of the relation (135), results:

$$\begin{aligned} Cttr &= \frac{P}{(\Omega \cdot dtr)} \cdot \frac{1}{(\rho \cdot Atr \cdot Vtip^2)} = (\frac{(Cpmr \cdot \rho \cdot A \cdot Vtip^3)}{(\Omega \cdot dtr)}) \cdot \frac{1}{(\rho \cdot Atr \cdot Vtip^2)} = \\ &= \frac{((\frac{(k \cdot Cw^2)}{(2 \cdot \mu)} + \frac{(\sigma \cdot Cd0)}{8} \cdot (1 + K \cdot \mu^2) + \frac{1}{2} \cdot (\frac{f}{A}) \cdot \mu^3) \cdot (\rho \cdot A \cdot Vtip^3))}{((\Omega \cdot dtr) \cdot (\rho \cdot Atr \cdot Vtip^2))} = \\ &= \frac{((\frac{(k \cdot Cw^2)}{(2 \cdot \mu)} + \frac{(\sigma \cdot Cd0)}{8} \cdot (1 + K \cdot \mu^2) + \frac{1}{2} \cdot (\frac{f}{A}) \cdot \mu^3) \cdot (A \cdot Vtip))}{(\Omega \cdot dtr \cdot Atr)} \end{aligned}$$

(137)

where P is the power of the main rotor plus the parasitic power of the helicopter.

$$Cp = \frac{(k \cdot Cw^2)}{(2 \cdot \mu)} + \frac{(\sigma \cdot Cd0)}{8} \cdot (1+K \cdot \mu^2) + \frac{1}{2} \cdot (\frac{f}{A}) \cdot \mu^3 + (\frac{(k \cdot Cttr^2)}{(2 \cdot \mu)} + \frac{(\sigma tr \cdot Cd0tr)}{8} \cdot (1+K \cdot \mu^2))$$

(138)

Differentiating the expression (135) respect to μ results:

$$\frac{dCp}{d\mu} = \frac{(-k \cdot Cw^2)}{(2 \cdot \mu^2)} + \frac{(\sigma \cdot Cd0)}{4} \cdot K \cdot \mu + \frac{3}{2} \cdot (\frac{f}{A}) \cdot \mu^2 + \frac{d}{d\mu}(\frac{(k \cdot Cttr^2)}{(2 \cdot \mu)}) + \frac{(\sigma tr \cdot Cd0tr)}{4} \cdot K \cdot \mu = 0$$

(139)

that equals to zero to obtain a minimum.

The fourth member of the relation (139) can be developed in the next way:

$$\frac{d}{d\mu}(\frac{(k \cdot Cttr^2)}{(2 \cdot \mu)}) = \frac{d}{d\mu}\{\frac{k}{(2 \cdot \mu)} \cdot [\frac{(k \cdot Cw^2)}{(2 \cdot \mu)} + \frac{(\sigma \cdot Cd0)}{8} \cdot (1+K \cdot \mu^2) + \frac{1}{2} \cdot (\frac{f}{A}) \cdot \mu^3]^2 \cdot (\frac{(A \cdot Vtip)}{(\Omega \cdot dtr \cdot Atr)})^2\} =$$

$$= \frac{d}{d\mu}\{\frac{k}{(2 \cdot \mu)} \cdot Z\} \cdot (\frac{(A \cdot Vtip)}{(\Omega \cdot dtr \cdot Atr)})^2 = Ymp \cdot (\frac{(A \cdot Vtip)}{(\Omega \cdot dtr \cdot Atr)})^2$$

(140)

Where Z results to be:

$$Z = Cpmr^2 = [\frac{(k \cdot Cw^2)}{(2 \cdot \mu)} + \frac{(\sigma \cdot Cd0)}{8} \cdot (1+K \cdot \mu^2) + \frac{1}{2} \cdot (\frac{f}{A}) \cdot \mu^3]^2 \qquad (141)$$

Ymp results from doing the square of the quadrinomial, multiplying per $k/(2\mu)$ and after differentiating for μ the relation:

$$Ymp = \frac{d}{d\mu}\{\frac{k}{(2 \cdot \mu tr)} \cdot Z\} = \frac{d}{d\mu}\{\frac{k}{(2 \cdot \mu \cdot \frac{Vtip}{Vtiptr})} \cdot Z\} = \frac{(-3 \cdot k^3 \cdot Cw^4)}{(8 \cdot \mu^4)} - \frac{(k \cdot \sigma^2 \cdot Cd0^2)}{(128 \cdot \mu^2)} +$$

$$+ \frac{(3 \cdot \sigma^2 \cdot k \cdot Cd0^2 \cdot K^2 \cdot \mu^2)}{128} + \frac{(5 \cdot k \cdot f^2 \cdot \mu^4)}{(8 \cdot A^2)} - \frac{(k^2 \cdot Cw^2 \cdot \sigma \cdot Cd0)}{(8 \cdot \mu^3)} + \frac{(k^2 \cdot Cw^2 \cdot f)}{(4 \cdot A)} +$$

$$+ \frac{(k \cdot K \cdot \sigma^2 \cdot Cd0^2)}{64} + \frac{(k \cdot \sigma \cdot Cd0 \cdot f \cdot \mu)}{(8 \cdot A)} + \frac{(k \cdot \sigma \cdot Cd0 \cdot K \cdot f \cdot \mu^3)}{(4 \cdot A)}$$

(142)

For the coaxial/tandem configurations, in the relation (134), $Cptr = 0$ because there is no a tail rotor. Cw results in this case:

$$Cw = \frac{(\frac{W}{2})}{(\rho \cdot A \cdot Vtip^2)} \tag{143}$$

because it's considered that each rotor has to counteract a half of the weight.

For a coaxial helicopter the relation (139) results:

$$\frac{dCp}{d\mu} = \frac{(-2 \cdot k \cdot kint \cdot Cw^2)}{(2 \cdot \mu^2)} + \frac{(2 \cdot \sigma \cdot Cd0)}{4} \cdot K \cdot \mu + \frac{3}{2} \cdot (\frac{f}{A}) \cdot \mu^2 = 0 \tag{144}$$

For a tandem helicopter the relation (139) results:

$$\frac{dCp}{d\mu} = \frac{(-k \cdot (1+kov) \cdot Cw^2)}{(2 \cdot \mu^2)} + \frac{(2 \cdot \sigma \cdot Cd0)}{4} \cdot K \cdot \mu + \frac{3}{2} \cdot (\frac{f}{A}) \cdot \mu^2 = 0 \tag{145}$$

where *kint* and *kov* are, respectively, the induced power and the overlap interference factors.

The equations (139), (144) and (145) can be solved numerically for μ, for example, with Newton Raphson method.

And the maximum rate of climb of a helicopter, *Vmp*, results:

$$Vmp = \mu \cdot Vtip \tag{146}$$

where μ is the advance ratio corresponding to the minimum power, calculated previously in the relation, respectively, (139), (144) and (145). *Vtip* is the speed of the blade tip of the helicopter design chosen.

2.3.2 Vertical velocity for minumum power, Vvmp

If $\lambda c \neq 0$, the vertical velocity for minimum power is not null any more. So the coefficient of power, from the relation (134), can be expressed this way:

$$Cp_{max} = Cp_{fly} + \lambda c \cdot Cw \tag{147}$$

and the induced inflow ratio for the minimum power λc is given by:

$$\lambda c = \frac{(Cp_{max} - Cp_{fly})}{Cw} \tag{148}$$

and, finally, the vertical velocity for the maximum rate of climb and endurance, *Vvmp*, results:

$$Vvmp = \lambda c \cdot Vtip \tag{149}$$

The coefficient of weight, *Cw*, for the conventional and for the coaxial/tandem helicopters are defined, respectively, in the previous relations (136) and (143).

Cp_{max} is the coefficient of the power installed on the helicopter and is given by the formula ([19] pag. 67 eq. 2.33):

$$Cp_{max} = \frac{P_{installed}}{(\rho \cdot A \cdot Vtip^3)} \tag{150}$$

where $P_{installed}$ is the power of the engines installed on the aircraft, specified by the constructor.

Cp_{fly} changes for every configuration of helicopters, so summarizing results:

- for the conventional helicopter:

$$\begin{aligned}(Cp_{fly})_{Conv} = &\frac{(k \cdot Cw^2)}{(2 \cdot \mu)} + \frac{(\sigma \cdot Cd0)}{8} \cdot (1 + K \cdot \mu^2) + \frac{1}{2} \cdot (\frac{f}{A}) \cdot \mu^3 \\ &+ (\frac{(k \cdot Cttr^2)}{(2 \cdot \mu)} + \frac{(\sigma tr \cdot Cd0tr)}{8} \cdot (1 + K \cdot \mu^2))\end{aligned} \tag{151}$$

where *Cttr* is given by the relation (137).

- for the coaxial helicopter:

$$(Cp_{fly})_{Coax} = \frac{(k \cdot kint \cdot Cw^2)}{(2 \cdot \mu)} + \frac{(2 \cdot \sigma \cdot Cd0)}{8} \cdot (1 + K \cdot \mu^2) + \frac{1}{2} \cdot (\frac{f}{A}) \cdot \mu^3 \tag{152}$$

- for the tandem helicopter:

$$(Cp_{fly})_{Tandem} = \frac{(k \cdot (1+kov) \cdot Cw^2)}{(2 \cdot \mu)} + \frac{(2 \cdot \sigma \cdot Cd0)}{8} \cdot (1+K \cdot \mu^2) + \frac{1}{2} \cdot (\frac{f}{A}) \cdot \mu^3 \tag{153}$$

where the advance ratio for the maximum rate of climb and endurance, μ, for the conventional, coaxial and tandem designs is found, respectively, from the relations (139), (144), (145).

2.3.3 Velocity for maximum range, Vmr

The range of an aircraft is the maximum distance the aircraft can fly at a given takeoff weight and a given quantity of fuel. The best range can be obtained at the speed when the ratio *P/V* results the minimum, this means that the aircraft has the best lift-to-drag ratio. The speed for maximum range *Vmr* can be obtained from the *P/V* curve (Figure 2.7) using the line drawn through the origin and tangent to the *P/V* curve. The speed for maximum range is usually higher than the speed for minimum power.

The ratio *P/V*, for the conventional design, can be approximated by dividing the relation (138) per μ:

$$\begin{aligned}\frac{Cp}{\mu} &= \frac{(k \cdot Cw^2)}{(2 \cdot \mu^2)} + \frac{(\sigma \cdot Cd0)}{8} \cdot (\frac{1}{\mu} + K \cdot \mu) + \frac{1}{2} \cdot (\frac{f}{A}) \cdot \mu^2 \\ &+ (\frac{(k \cdot Cttr^2)}{(2 \cdot \mu^2)} + \frac{(\sigma tr \cdot Cd0tr)}{8} \cdot (\frac{1}{\mu} + K \cdot \mu))\end{aligned} \tag{154}$$

where *Cw* is given by the relation (136) and *Cttr* by the relation (137).

Differentiating this expression respect to μ results:

$$\begin{aligned}\frac{(d(Cp/\mu))}{d\mu} &= \frac{(-k \cdot Cw^2)}{(\mu^3)} + \frac{(\sigma \cdot Cd0)}{8} \cdot (-(\frac{1}{\mu^2}) + K) + (\frac{f}{A}) \cdot \mu \\ &+ \frac{d}{d\mu}(\frac{(k \cdot Cttr^2)}{(2 \cdot \mu^2)}) + \frac{(\sigma tr \cdot Cd0tr)}{8} \cdot (-(\frac{1}{\mu^2}) + K) = 0\end{aligned} \tag{155}$$

that equals to zero to obtain a minimum.

The fourth member of the previous relation (155) can be developed in the next way:

$$\frac{d}{d\mu}\left(\frac{(k \cdot Cttr^2)}{(2 \cdot \mu^2)}\right) = \frac{d}{d\mu}\left\{\frac{k}{(2 \cdot \mu^2)} \cdot \left[\frac{(k \cdot Cw^2)}{(2 \cdot \mu)} + \frac{(\sigma \cdot Cd0)}{8} \cdot (1+K \cdot \mu^2) + \frac{1}{2} \cdot \left(\frac{f}{A}\right) \cdot \mu^3\right]^2 \cdot \left(\frac{(A \cdot Vtip)}{(\Omega \cdot dtr \cdot Atr)}\right)^2\right\} =$$
$$= \frac{d}{d\mu}\left\{\frac{k}{(2 \cdot \mu^2)} \cdot Z\right\} \cdot \left(\frac{(A \cdot Vtip)}{(\Omega \cdot dtr \cdot Atr)}\right)^2 = Ymr \cdot \left(\frac{(A \cdot Vtip)}{(\Omega \cdot dtr \cdot Atr)}\right)^2$$

(156)

So developing Z, given by the relation (141), multiplying per $k/(2\mu^2)$ and differentiating, the next expressione called Ymr, results:

$$Ymr = \frac{d}{d\mu}\left\{\frac{k}{(2 \cdot \mu^2)} \cdot Z\right\} = \frac{(-k^3 \cdot Cw^4)}{(2 \cdot \mu^5)} - \frac{(k \cdot \sigma^2 \cdot Cd0^2)}{(64 \cdot \mu^3)} + \frac{(\sigma^2 \cdot k \cdot Cd0^2 \cdot K^2 \cdot \mu)}{64} +$$
$$+ \frac{(k \cdot f^2 \cdot \mu^3)}{(2 \cdot A^2)} - \frac{(3 \cdot k^2 \cdot Cw^2 \cdot \sigma \cdot Cd0)}{(16 \cdot \mu^4)} - \frac{(k^2 \cdot Cw^2 \cdot \sigma \cdot Cd0 \cdot K)}{(16 \cdot \mu^2)} +$$
$$+ \frac{(k \cdot \sigma \cdot Cd0 \cdot f)}{(16 \cdot A)} + \frac{(3 \cdot k \cdot \sigma \cdot Cd0 \cdot K \cdot f \cdot \mu^2)}{(16 \cdot A)}$$

(157)

For a coaxial helicopter, the relation (155), results:

$$\frac{(d(Cp/\mu))}{d\mu} = \frac{(-2 \cdot k \cdot kint \cdot Cw^2)}{(\mu^3)} + \frac{(2 \cdot \sigma \cdot Cd0)}{8} \cdot \left(-\left(\frac{1}{\mu^2}\right) + K\right) + \left(\frac{f}{A}\right) \cdot \mu = 0$$

(158)

For a tandem helicopter, the relation (156), results:

$$\frac{(d(Cp/\mu))}{d\mu} = \frac{(-k \cdot (1+kov) \cdot Cw^2)}{(\mu^3)} + \frac{(2 \cdot \sigma \cdot Cd0)}{8} \cdot \left(-\left(\frac{1}{\mu^2}\right) + K\right) + \left(\frac{f}{A}\right) \cdot \mu = 0$$

(159)

where Cw, for the coaxial/tandem configurations, is given by the relation (143).

The equations (155), (158) and (159) can be solved numerically for μ, like the relations (139), (144) and (145), with Newton Raphson method.

And the airspeed for maximum range of a helicopter, Vmr, results:

$$Vmr = \mu \cdot Vtip \qquad (160)$$

where μ is the advance ratio corresponding to the power obtained by using the line through the origin and tangent to the P/V curve. μ is calculated from the previous relations (155), (158) or (159).

2.3.4 Range and Endurance

The maximum range and the maximum endurance of a helicopter depends of the characteristics of the engine. The power required varies with the gross mass of the helicopter and with the density altitude. McCormick (1995) proposed an analysis of the range for an aircraft, that can be also applied for a helicopter. So the next differential formula expresses the fuel flow rate *dMf* respect the range *R* ([1] pag. 174 eq 5.48):

$$\frac{dMf}{dR} = \frac{(P \cdot SFC)}{V} \tag{161}$$

where *SFC* is the specific fuel consumption of the engine(s).

The mass *Mf* decreases as the fuel is burned during the flight, so the equation (161) has to be integrated to obtain the range. The mass of the fuel is, usually, a small part of the entire gross mass and the specific range results linear respect the mass, so the equation (161) can be applied, as approximation, at the moment of the cruise of the helicopter when the gross mass is:

$$M = M_{gtow} - \frac{Mf}{2} \tag{162}$$

so the range for the mass $M_{gtow} - Mf/2$ can be found from the next formula ([1] pag. 174 eq 5.49):

$$R_{(M_{gtow} - \frac{Mf}{2})} = Mf \cdot [\frac{Vmr}{(P_{(Vmr)} \cdot SFC)}] \tag{163}$$

and the endurance for the mass $Mgtow - Mf/2$ results ([1] pag. 174 eq 5.50):

$$E_{(M_{gtow} - \frac{Mf}{2})} = Mf \cdot [\frac{1}{(P_{(Vmp)} \cdot SFC)}] \tag{164}$$

$P_{(Vmr)}$ and $P_{(Vmp)}$ are the total power of the helicopter calculated, respectively, for the velocity for maximum range *Vmr* (160) and for the velocity for minimum power *Vmp* (146). These power are calculate for the mass *M* given by the relation (162), respectively, with the momentum theory or with the blade element theory. *Mf* is the mass of the fuel present in the tanks of the helicopter.

CHAPTER 3

IMPLEMENTATION OF THE PROGRAM

The program was written in Matlab and uses the theory exposed in the previous chapter (the momentum theory and the blade element theory).

3.1 Empirical formulas

In the program were used empirical formulas for certain parameters that describe the helicopter. This formulas were studied and developed, for the conventional configuration, in the work of Rand Omri and Khromov Vladimir, at the Technion – Israel Institute of Technology [20]. The studies are based on a database for more than 180 conventional rotor helicopter configurations and for the analysis was used The Multiple Regression Analysis.

Each regression expression is multiplied by the adjustment coefficient K_h, which can be changed by the user and have values around 1. The units of all quantities present in the regressions are in SI units.

The main rotor diameter, in [m], is described by:

$$D = K_h \cdot 9.133 \cdot M^{0.380} \cdot (Vmax)^{-0.515} \tag{165}$$

where *Vmax* is the maximum speed of the helicopter in [km/h].

The average chord of the main rotor, in [m], is described by:

$$\bar{c} = K_h \cdot 0.0108 \cdot M^{0.539} \cdot Nb^{-0.714} \tag{166}$$

where *Nb* is the number of blades of the mai rotor and *M* is the mass of the helicopter in [kg].

The angular speed of the main rotor (see Figure 3.1) :

$$\Omega = K_h \cdot 2672.881 \cdot D^{-0.829} \tag{167}$$

where Ω is expressed in [rpm] and the diameter of the main rotor *D* in [m].

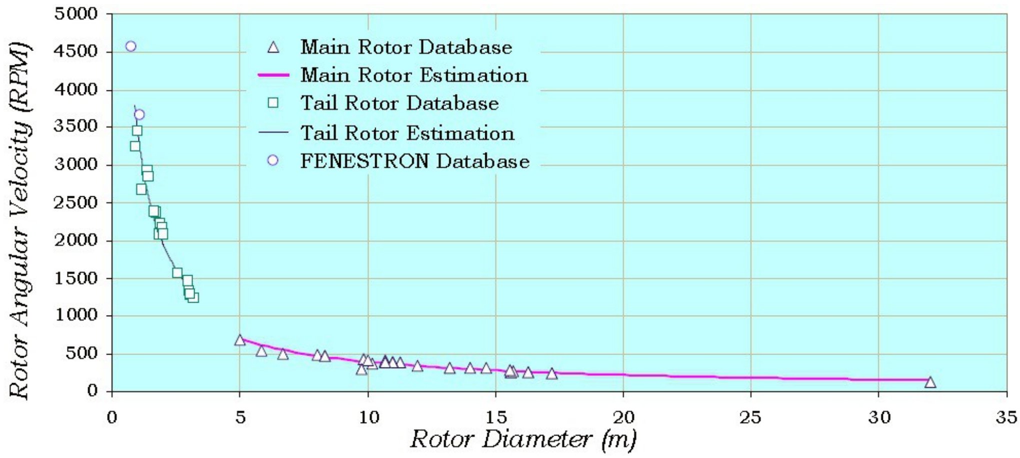


Figure 3.1 Main & tail rotor angular velocities in function of the rotor diameter ([20] pag. 13)

The tip speed of the blades of the main rotor, in [m/s], is described by (Figure 3.2):

$$Vtip = K_h \cdot 140 \cdot D^{0.171} \tag{168}$$

The diameter of the tail rotor, in [m], is described by:

$$Dtr = K_h \cdot 0.0895 \cdot M^{0.391} \tag{169}$$

where M is the mass of the helicopter in [kg].

The average chord of the tail rotor, in [m], is described by:

$$\bar{ctr} = K_h \cdot 0.0058 \cdot M^{0.506} \cdot Nbtr^{-0.72} \tag{170}$$

The angular speed of the tail rotor, in [rpm], is given by (Figure 3.1):

$$\Omega tr = K_h \cdot 3475 \cdot Dtr^{-0.828} \tag{171}$$

The tip speed of the blades of the tail rotor is, in [m/s], described by (Figure 3.2):

$$Vtiptr = K_h \cdot 182 \cdot Dtr^{0.172} \tag{172}$$

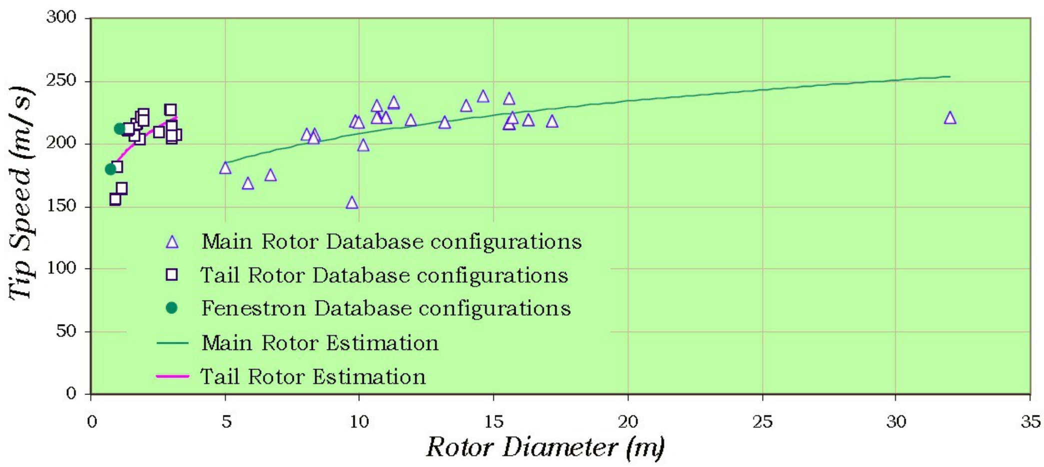


Figure 3.2 Main & tail rotor tip speeds of the blades in function of the rotor diameter ([20] pag. 15)

The empirical formulas for the main rotor previously exposed (165 - 168), can be used for the coaxial and tandem helicopters. In this thesis, for the coaxial design, the formulas (165 – 168) are used to define the rotor diameter, the chord of the blades, the angular speed of the rotors and the tip speed of the blades.

For the tandem design, in this work, was decided to create a formula that approximates linearly the diameter of the rotors in function of the mass. The next graph was created from data of already existing tandem helicopters (Figure 3.3, blue graphic)

([21] - [27]). In the figure are plotted the mass of the helicopters in function of the diameter of the rotors.

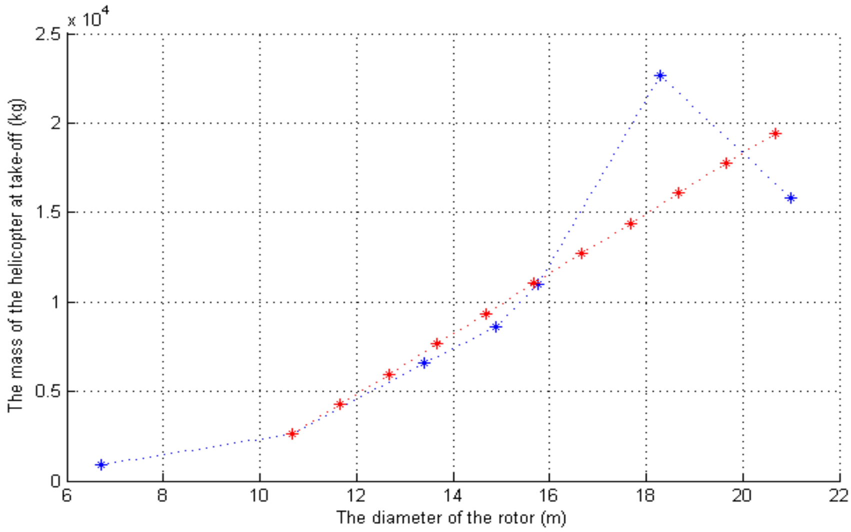


Figure 3.3 Curves of the mass at take-off of the tandem helicopter in function of the diameter of the rotors (blue: data of existing helicopters, red: linear approximation of the blue curve)

The highest point in the blue graph represents the particular tandem helicopter CH-47 Chinook (Boeing) [26], that has a particularly big take-off mass for a relatively small diameter of the rotors. And the first blue point represents one of the smallest tandem helicopter realized, in terms of take-off mass and dimension of the rotor, the MC-4 (McCulloch) [21].

The idea is to approximate the blue graph with a straight line:

$$y - y_0 = m \cdot (x - x_0) \tag{173}$$

where x is the diameter of the rotors to find and, respectively, x_0 the optimal initial diameter of the rotors. x_0, in this case, was chosen of the HUP Retriever (Piasecki) [22]. y is the mass at take-off of the tandem helicopter that is designing and y_0 is, respectively, the optimal initial mass at take-off of the helicopter, that is the HUP Retriever (Piasecki) [22]. m is the the angular coefficient of the straight line needed for the approximation.

Rewritting the relation (173) results:

$$M - 2608 = m \cdot (D_{tandem} + 10.67) \quad (174)$$

where *m* is given by the relation:

$$m = \frac{(x - x_0)}{(y - y_0)} = \frac{(20000 - 2608)}{(21 - 10.67)} = 1.6836 \cdot 10^3 \quad (175)$$

where “21” is the diameter, in meters, of the helicopter associated to the last point of the blue curve [27]. The mass of this helicopter (Yak-24, Yakovlev [27]) is 15830kg , but to obtain a better inclination of the red straight line (represented in the Figure 3.3) this number was increased to 20000 in the relation (175).

So summarizing the relations (174) and (175), the diameter of the rotors of the tandem helicopter is given by:

$$D_{tandem} = \frac{(M - 2608)}{(1.6836 \cdot 10^3)} + 10.67 \quad (176)$$

To obtain the length of the chord, the angular speed and the tip speed of the rotors of the tandem configuration are used the same formulas of the main rotor in the conventional case, (166), (167) and (168).

3.2 Numerical solutions

3.2.1 Numerical solution to inflow equation for the momentum theory

A common numerical approach to solve for λ the inflow equation, the relation (21), is a Newton – Raphson procedure ([19] pag. 97 eq. 2.134). The advantage, for example respect to a simple fixed – point approach, is that for the price of calculating the first derivative the convergence is much faster. The iteration procedure consists of:

$$\lambda_{(n+1)} = \lambda_{(n)} - [\frac{(f(\lambda))}{(f'(\lambda))}]_n \quad (177)$$

where "n" is the iteration number. The equation (21) can be rearranged in the form *f(λ)=0* and results:

$$f(\lambda) = \lambda - \frac{(\lambda_h)^2}{(\sqrt{(\mu^2+\lambda^2)})} = \lambda - \frac{Ct}{(2 \cdot \sqrt{(\mu^2+\lambda^2)})} = 0 \tag{178}$$

Differentiating this relation results:

$$f'(\lambda) = 1 + \frac{Ct}{2} \cdot (\mu^2+\lambda^2)^{-(\frac{3}{2})} \cdot \lambda = 0 \tag{179}$$

The Newton – Raphson approach can be sensitive to the initial conditions, usually the hover value $\lambda_0 = \lambda_h$ (see the relation (22)), gives good results, with only 3-4 iterations. In fact, with the fixed – point approach up to 10 more iterations are required to reach the tolerance.

3.2.2 Numerical solution to inflow equation for the blade element theory

To find the induced inflow ratio λi used for the blade element theory (see relation (107)) the Newton – Raphson procedure, exposed in the relation (177), can be used. The equation (107) can be rearranged in the form *f(λ) = 0* and results:

$$f(\lambda i) = \lambda i - \lambda 0 \cdot (1 + \tan(\frac{1}{2} \cdot (\tan^{-1}(\frac{\mu}{\lambda i})))) \cdot r \cdot cos\psi = 0 \tag{180}$$

Differentiating this relation results:

$$f'(\lambda i) = 1 - \lambda 0 \cdot (1 + (\tan(\frac{1}{2} \cdot \tan^{-1}(\frac{\mu}{\lambda i})))^2 \cdot (\frac{1}{(1+(\frac{\mu}{\lambda i})^2)}) \cdot (\frac{-\mu}{\lambda i})) \cdot r \cdot cos\psi = 0 \tag{181}$$

where μz is neglected and $\mu = \mu x$.

3.2.3 Numerical solution to the dimensionless airspeed for minimum power

To find the dimensionless airspeed for minimum power the Newton – Raphson procedure, exposed in the relation (177), can be used and results:

$$\mu_{(n+1)} = \mu_{(n)} - [\frac{(f(\mu))}{(f'(\mu))}]_n \tag{182}$$

The equations (139), (144), (145) can be rewritten in the form $f(\mu) = 0$:
- for the conventional configuration:

$$\begin{aligned} f(\mu) = \frac{dCp}{d\mu} = \frac{(-k \cdot Cw^2)}{(2 \cdot \mu^2)} + \frac{(\sigma \cdot Cd0)}{4} \cdot K \cdot \mu + \frac{3}{2} \cdot (\frac{f}{A}) \cdot \mu^2 \\ + \frac{d}{d\mu}(\frac{(k \cdot Cttr^2)}{(2 \cdot \mu)}) + \frac{(\sigma tr \cdot Cd0tr)}{4} \cdot K \cdot \mu = 0 \end{aligned} \tag{183}$$

where the fourth element of the expression (183) was developed in the relation (140) and results:

$$\begin{aligned} \frac{d}{d\mu}(\frac{(k \cdot Cttr^2)}{(2 \cdot \mu)}) = \frac{d}{d\mu}\{\frac{k}{(2 \cdot \mu)} \cdot [\frac{(k \cdot Cw^2)}{(2 \cdot \mu)} + \frac{(\sigma \cdot Cd0)}{8} \cdot (1 + K \cdot \mu^2) + \frac{1}{2} \cdot (\frac{f}{A}) \cdot \mu^3]^2 \cdot (\frac{(A \cdot Vtip)}{(\Omega \cdot dtr \cdot Atr)})^2\} = \\ = \frac{d}{d\mu}\{\frac{k}{(2 \cdot \mu)} \cdot Z\} \cdot (\frac{(A \cdot Vtip)}{(\Omega \cdot dtr \cdot Atr)})^2 = Ymp \cdot (\frac{(A \cdot Vtip)}{(\Omega \cdot dtr \cdot Atr)})^2 \end{aligned} \tag{184}$$

where Z is given by (141) and by Ymp (142).

Differentiating the relation (183) results:

$$\begin{aligned} f'(\mu) = \frac{(d^2 Cp)}{(d\mu^2)} = \frac{(k \cdot Cw^2)}{(\mu^3)} + \frac{(\sigma \cdot Cd0)}{4} \cdot K + 3 \cdot (\frac{f}{A}) \cdot \mu \\ + \frac{d^2}{d\mu^2}(\frac{(k \cdot Cttr^2)}{(2 \cdot \mu)}) + \frac{(\sigma tr \cdot Cd0tr)}{4} \cdot K = 0 \end{aligned} \tag{185}$$

where the fourth element of this expression results:

$$\begin{aligned} \frac{d^2}{d\mu^2}(\frac{(k \cdot Cttr^2)}{(2 \cdot \mu)}) = \frac{d^2}{d\mu^2}\{\frac{k}{(2 \cdot \mu)} \cdot [\frac{(k \cdot Cw^2)}{(2 \cdot \mu)} + \frac{(\sigma \cdot Cd0)}{8} \cdot (1 + K \cdot \mu^2) + \frac{1}{2} \cdot (\frac{f}{A}) \cdot \mu^3]^2 \cdot (\frac{(A \cdot Vtip)}{(\Omega \cdot dtr \cdot Atr)})^2\} = \\ = \frac{d^2}{d\mu^2}\{\frac{k}{(2 \cdot \mu)} \cdot Z\} \cdot (\frac{(A \cdot Vtip)}{(\Omega \cdot dtr \cdot Atr)})^2 = \frac{d}{d\mu}\{Ymp\} \cdot (\frac{(A \cdot Vtip)}{(\Omega \cdot dtr \cdot Atr)})^2 = Ymp' \cdot (\frac{(A \cdot Vtip)}{(\Omega \cdot dtr \cdot Atr)})^2 \end{aligned} \tag{186}$$

so *Ymp'* results:

$$Ymp' = \frac{(3 \cdot k^3 \cdot Cw^4)}{(2 \cdot \mu^5)} + \frac{(k \cdot \sigma^2 \cdot Cd0^2)}{(64 \cdot \mu^3)} + \frac{(3 \cdot \sigma^2 \cdot k \cdot Cd0^2 \cdot K^2 \cdot \mu)}{64} + \\ + \frac{(5 \cdot k \cdot f^2 \cdot \mu^3)}{(2 \cdot A^2)} + \frac{(3 \cdot k^2 \cdot Cw^2 \cdot \sigma \cdot Cd0)}{(8 \cdot \mu^4)} + \\ + \frac{(k \cdot \sigma \cdot Cd0 \cdot f)}{(8 \cdot A)} + \frac{(3 \cdot k \cdot \sigma \cdot Cd0 \cdot K \cdot f \cdot \mu^2)}{(4 \cdot A)}$$

(187)

- for the coaxial configuration:

$$f(\mu) = \frac{dCp}{d\mu} = \frac{(-2 \cdot k \cdot kint \cdot Cw^2)}{(2 \cdot \mu^2)} + \frac{(2 \cdot \sigma \cdot Cd0)}{4} \cdot K \cdot \mu + \frac{3}{2} \cdot (\frac{f}{A}) \cdot \mu^2 = 0$$

(188)

Differentiating this relation results:

$$f'(\mu) = \frac{(d^2 Cp)}{(d\mu^2)} = \frac{(2 \cdot kint \cdot k \cdot Cw^2)}{(\mu^3)} + \frac{(2 \cdot \sigma \cdot Cd0)}{4} \cdot K + 3 \cdot (\frac{f}{A}) \cdot \mu = 0$$

(189)

- for the tandem configuration:

$$f(\mu) = \frac{dCp}{d\mu} = \frac{(-k \cdot (1+kov) \cdot Cw^2)}{(2 \cdot \mu^2)} + \frac{(2 \cdot \sigma \cdot Cd0)}{4} \cdot K \cdot \mu + \frac{3}{2} \cdot (\frac{f}{A}) \cdot \mu^2 = 0$$

(190)

Differentiating this relation results:

$$f'(\mu) = \frac{(d^2 Cp)}{(d\mu^2)} = \frac{(k \cdot (1+kov) \cdot Cw^2)}{(\mu^3)} + \frac{(2 \cdot \sigma \cdot Cd0)}{4} \cdot K + 3 \cdot (\frac{f}{A}) \cdot \mu = 0$$

(191)

3.2.4 Numerical solution to the dimensionless airspeed for the maximum range

To find the dimensionless airspeed for the maximum range was used the Newton – Raphson procedure exposed in the previous relation (177).

The equation (155), (158) and (159) can be rewritten in the form $f(\mu) = 0$:

- for the convetional configuration:

$$f(\mu) = \frac{(d(Cp/\mu))}{d\mu} = \frac{(-k \cdot Cw^2)}{(\mu^3)} + \frac{(\sigma \cdot Cd0)}{8} \cdot (-(\frac{1}{\mu^2}) + K) + (\frac{f}{A}) \cdot \mu$$
$$+ \frac{d}{d\mu}(\frac{(k \cdot Cttr^2)}{(2 \cdot \mu^2)}) + \frac{(\sigma tr \cdot Cd0tr)}{8} \cdot (-(\frac{1}{\mu^2}) + K) = 0 \quad (192)$$

where the fourth term of the relation (192), expressed already in the relation (156) results:

$$\frac{d}{d\mu}(\frac{(k \cdot Cttr^2)}{(2 \cdot \mu^2)}) = \frac{d}{d\mu}\{\frac{k}{(2 \cdot \mu^2)} \cdot [\frac{(k \cdot Cw^2)}{(2 \cdot \mu)} + \frac{(\sigma \cdot Cd0)}{8} \cdot (1 + K \cdot \mu^2) + \frac{1}{2} \cdot (\frac{f}{A}) \cdot \mu^3]^2 \cdot (\frac{(A \cdot Vtip)}{(\Omega \cdot dtr \cdot Atr)})^2\} =$$
$$= \frac{d}{d\mu}\{\frac{k}{(2 \cdot \mu^2)} \cdot Z\} \cdot (\frac{(A \cdot Vtip)}{(\Omega \cdot dtr \cdot Atr)})^2 = Ymr \cdot (\frac{(A \cdot Vtip)}{(\Omega \cdot dtr \cdot Atr)})^2 \quad (193)$$

where Z is given by (141) and Ymr is given by (157).

Differentiating this relation results:

$$f'(\mu) = \frac{(d^2(Cp/\mu))}{(d\mu^2)} = \frac{(3 \cdot k \cdot Cw^2)}{(\mu^4)} + \frac{(\sigma \cdot Cd0)}{(4 \cdot \mu^3)} + \frac{f}{A}$$
$$+ \frac{d^2}{d\mu^2}(\frac{(k \cdot Cttr^2)}{(2 \cdot \mu^2)}) + \frac{(\sigma tr \cdot Cd0tr)}{(4 \cdot \mu^3)} = 0 \quad (194)$$

the fourth term of the relation (192) results:

$$\frac{d^2}{d\mu^2}(\frac{(k \cdot Cttr^2)}{(2 \cdot \mu^2)}) = \frac{d^2}{d\mu^2}\{\frac{k}{(2 \cdot \mu^2)} \cdot [\frac{(k \cdot Cw^2)}{(2 \cdot \mu)} + \frac{(\sigma \cdot Cd0)}{8} \cdot (1 + K \cdot \mu^2) + \frac{1}{2} \cdot (\frac{f}{A}) \cdot \mu^3]^2 \cdot (\frac{(A \cdot Vtip)}{(\Omega \cdot dtr \cdot Atr)})^2\} =$$
$$= \frac{d^2}{d\mu^2}\{\frac{k}{(2 \cdot \mu^2)} \cdot Z\} \cdot (\frac{(A \cdot Vtip)}{(\Omega \cdot dtr \cdot Atr)})^2 = Ymr' \cdot (\frac{(A \cdot Vtip)}{(\Omega \cdot dtr \cdot Atr)})^2 \quad (195)$$

so *Ymr'* is given by:

$$Ymr' = \frac{(5 \cdot k^3 \cdot Cw^4)}{(2 \cdot \mu^6)} + \frac{(3 \cdot k \cdot \sigma^2 \cdot Cd0^2)}{(64 \cdot \mu^4)} + \frac{(\sigma^2 \cdot k \cdot Cd0^2 \cdot K^2)}{64} + \frac{(3 \cdot k \cdot f^2 \cdot \mu^2)}{(2 \cdot A^2)} + \\ + \frac{(3 \cdot k^2 \cdot Cw^2 \cdot \sigma \cdot Cd0)}{(4 \cdot \mu^5)} + \frac{(k^2 \cdot Cw^2 \cdot \sigma \cdot Cd0 \cdot K)}{(8 \cdot \mu^3)} + \frac{(3 \cdot k \cdot \sigma \cdot Cd0 \cdot K \cdot f \cdot \mu)}{(8 \cdot A)} \tag{196}$$

- for the coaxial configuration:

$$f(\mu) = \frac{(d(Cp/\mu))}{d\mu} = \frac{(-2 \cdot kint \cdot k \cdot Cw^2)}{(\mu^3)} + \frac{(2 \cdot \sigma \cdot Cd0)}{8} \cdot (-(\frac{1}{\mu^2}) + K) + (\frac{f}{A}) \cdot \mu = 0 \tag{197}$$

Differentiating this relation results:

$$f'(\mu) = \frac{(d^2(Cp/\mu))}{(d\mu^2)} = \frac{(3 \cdot (2 \cdot kint \cdot k) \cdot Cw^2)}{(\mu^4)} + \frac{(2 \cdot \sigma \cdot Cd0)}{(4 \cdot \mu^3)} + \frac{f}{A} = 0 \tag{198}$$

- for the tandem configuration:

$$f(\mu) = \frac{(d(Cp/\mu))}{d\mu} = \frac{(-k \cdot (1 + kov) \cdot Cw^2)}{(\mu^3)} + \frac{(2 \cdot \sigma \cdot Cd0)}{8} \cdot (-(\frac{1}{\mu^2}) + K) + (\frac{f}{A}) \cdot \mu = 0 \tag{199}$$

Differentiating this relation results:

$$f'(\mu) = \frac{(d^2(Cp/\mu))}{(d\mu^2)} = \frac{(3 \cdot k \cdot (1 + kov) \cdot Cw^2)}{(\mu^4)} + \frac{(2 \cdot \sigma \cdot Cd0)}{(4 \cdot \mu^3)} + \frac{f}{A} = 0 \tag{200}$$

Cw for the conventional case is given by (136) and *Cw* for the coaxial/tandem cases is given by (143).

3.3 Geometrical technique to find the airspeed ratios for minimum power and for maximum range

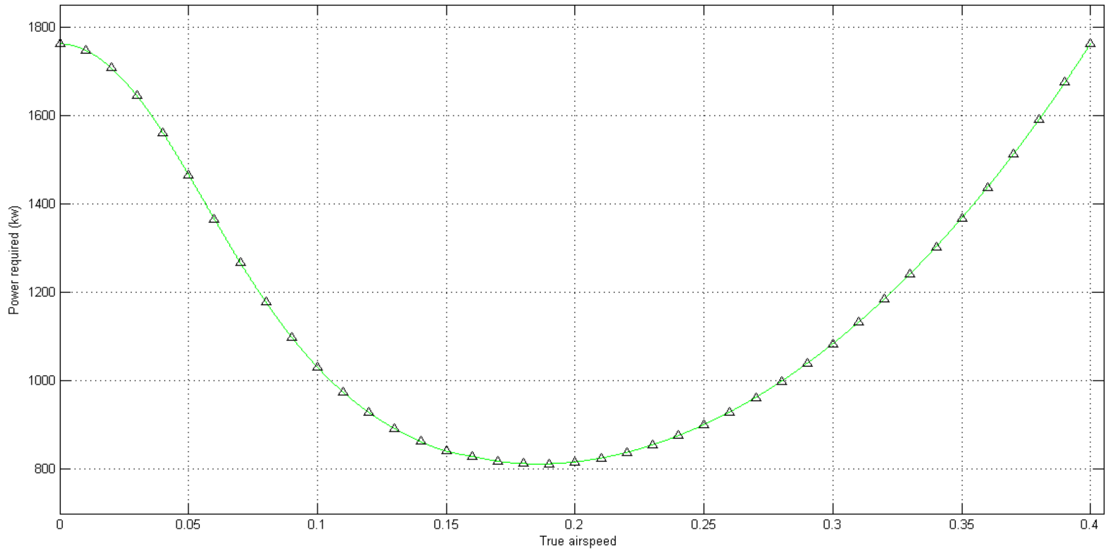


Figure 3.4 Power curve of a helicopter in level flight

If the power curve for a level flight of a helicopter is given, for example, by the previous graph, where the triangles are the points that can be calculated with the momentum or the blade element theories, to find the the airspeed ratios for the minimum power and for the maximum range a geometrical approach can be used.

To find the airspeed ratio of the minimum power, it's enough to choose the point in the graph that has the minimum y value (in this case the power in kW), so automatically, the respective x value, or the airspeed needed is found. In the program code, developed in this work, is easy to do this operation because all the coordinates of the power points are saved in two different vectors, that stands for the x and, respectively, for the y axes. So once found the minimum value in the y vector, using the function "find", can be found the position of this value in the y vector. As soon as, the two vectors, x and y, have the same length, results that for the position found in the y vector corrisponds a value in the x vector, that is exactly the airspeed ratio needed.

To find the airspeed for the maximum range, as defined in the chapter 2.3, Figure 2.7, can be obtained from the *P/V* curve (in this case the *P/μ* curve) using the line drawn through the origin and tangent to the *P/V* curve (*P/μ* curve). In our possesion are a group of points that aproximate the power curve of the helicopter in level flight. So in this case the goal is to find the right point from this group that will be the closest to the point in which the line drawn through the origin is tangent to the P/μ curve. In the Figure 3.5 are drawn the lines from the origin to the point of minimum power and the parallel to this line that passes through the next point.

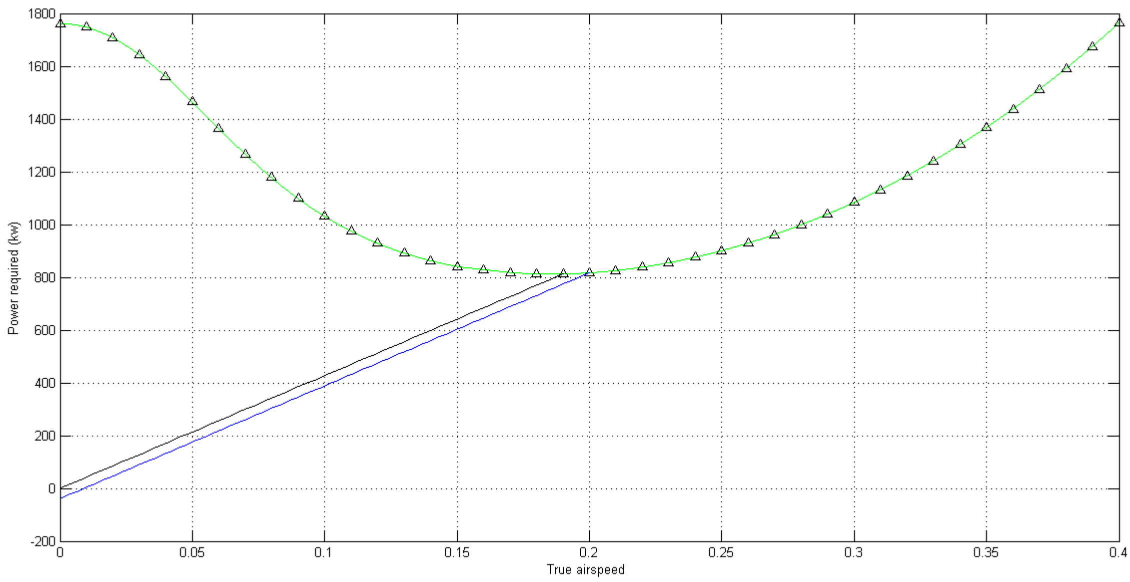


Figure 3.5 Power curve of a helicopter in level flight and two parallel curves needed to find the airspeed ratio for maximum range

The Figure 3.6, which is the zoom of Figure 3.5, is a graphical rappresentation behind the reason for this point.

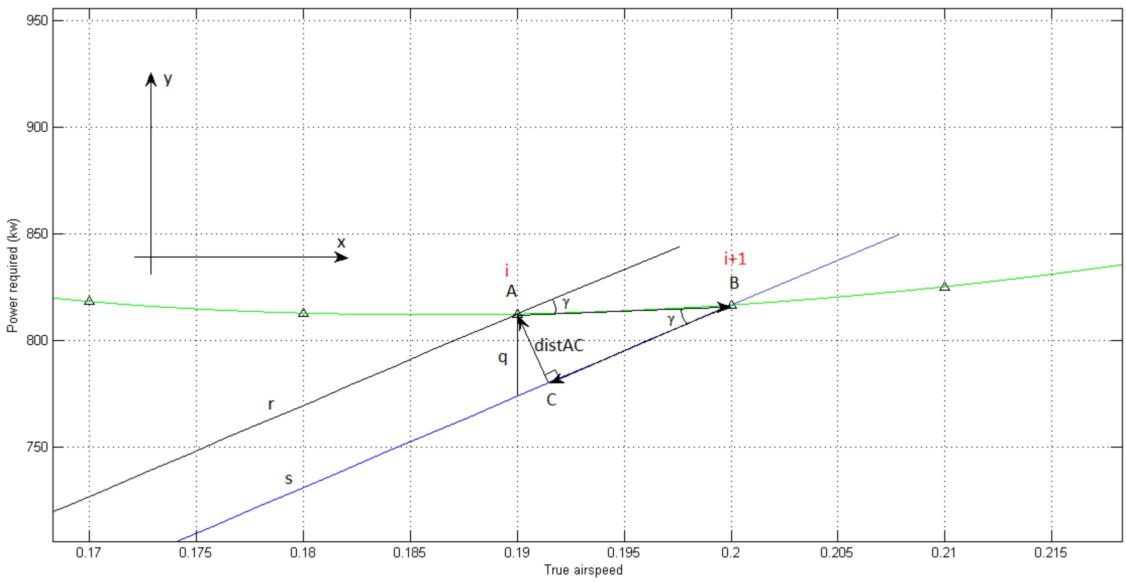


Figure 3.6 The zoom of the Figure 3.5 where is rappresented the angle that is the graphical condition to find the airspeed for the maximum range of the helicopter

The line that passes for the origin and the point A was called *r*. A and B, in this work, are the start points of the search of the airspeed ratio for maximum range. So the first A, for simplicity, was chosen to be the point for minimum power, exposed at the begining of this chapter.

$$r: \; y_{(i)} = m \cdot x_{(i)} \tag{201}$$

The line parallel to *r* and which passes through the next point, B, was called *s*.

$$s: \; y_{(i+1)} = m \cdot x_{(i+1)} + q \tag{202}$$

The angular coefficient *m* of the two lines is the same, so expressing for *m* the relation (201) and substituting it in the expression of the line *s*, results:

$$y_{(i+1)} = m \cdot x_{(i+1)} + q = \frac{y_{(i)}}{x_{(i)}} \cdot x_{(i+1)} + q \tag{203}$$

So expressing for *q* the relation (203), results:

$$q = y_{(i+1)} - \frac{y_{(i)}}{x_{(i)}} \cdot x_{(i+1)} \tag{204}$$

Using the formula of the distance between a point and a line, to find the distance between the point A and the point C on the line *s*, and substituting the relation (204) inside results:

$$\begin{aligned} dist_{AC} &= \frac{\left| -m \cdot x_{(i)} + y_{(i)} - q \right|}{\sqrt{(m^2+1)}} = \frac{\left| -\left(\frac{y_{(i)}}{x_{(i)}}\right) \cdot x_{(i)} + y_{(i)} - q \right|}{\sqrt{\left(\frac{y_{(i)}}{x_{(i)}}\right)^2 + 1}} = \\ &= \frac{\left| -y_{(i)} + y_{(i)} - \left(y_{(i+1)} - \frac{y_{(i)}}{x_{(i)}} \cdot x_{(i+1)}\right) \right|}{\sqrt{\left(\frac{y_{(i)}}{x_{(i)}}\right)^2 + 1}} = \frac{\left| -y_{(i+1)} + \frac{y_{(i)}}{x_{(i)}} \cdot x_{(i+1)} \right|}{\sqrt{\left(\frac{y_{(i)}}{x_{(i)}}\right)^2 + 1}} \end{aligned} \tag{205}$$

Using the relation of the distance between two points results:

$$dist_{AB} = \sqrt{\left(x_{(i)} - x_{(i+1)}\right)^2 + \left(y_{(i)} - y_{(i+1)}\right)^2} \tag{206}$$

And now to find the angle between the line r and the vector AB or the line s and the vector AB it's enough to do the sine function between the two distances from the relations (205) and (206):

$$\sin(\gamma) = \frac{dist_{AC}}{dist_{AB}} \tag{207}$$

And expressing for γ results:

$$\gamma = \arcsin\left(\frac{dist_{AC}}{dist_{AB}}\right) \tag{208}$$

So to find the airspeed ratio for maximum range it's enough to find the minimum angle γ between the AB vector and the line r. As soon as the γ vector, in the program, has the same length as the μ (the airspeed ratio) vector, determining the position, with the function “find”, of the minimum γ value in the γ vector, for this position corresponds a value of μ in the μ vector, that is the airspeed ratio needed.

3.4 Blade element theory implementation

For a numerical implementation of the blade element theory (see Chapter 2.2), this work divides the rotor blade into a finite number of uniformly distributed blade elements, while integration along the blade span is approximated numerically using summation. The key component in blade element analysis lies in modeling the induced velocity on the rotor disk.

3.5 Validation

To test the program code of this thesis and to validate it experimental data of real helicopter rotors are used. For the single rotor, coaxial and tandem configurations are used the data from the technical note of Richard C. Dingeldein [28], in which is described the work done at the Langley fullscale tunnel. The research program described in the paper treats the different rotor arrangements on the basis of relative aerodynamic efficiency and involve measuring for various flight conditions the power required, the blade motions, the flow angles in the rotor wake, and the rotor static stability.

3.5.1 Coaxial rotor

The coaxial rotor system, represented in the Figure 3.7, was part of an actual helicopter and has a diameter of 25 feet (7.62 m) and a rotor spacing equal to 19 percent of its radius. Each rotor has two blades, and the total solidity of the coaxial configuration, based on the projected area, is 0.054. For this small coaxial helicopter is used an equivalent flat-plate parasite-drag area of 10 square feet (0.92903 square metres). Were used a profile drag coefficient Cd0 = 0.0087 and a curve slope of the profile of the blades $C\alpha$ = 5.73. The coefficient of thrust and the tip speed for the coaxial experiment are Ct = 0,0048 and $\Omega \cdot R$ = 469 FPS (142.95 m/s).

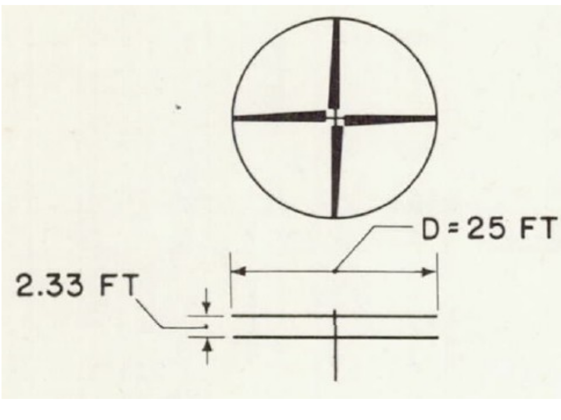


Figure 3.7 Coaxial rotor configuration tested ([28], pag. 8)

From early flow-visualization studies and also from the general rotor theory is known that the air flow through and around rotors operating near one another can be a lot different from what happens for a single rotor. In the Figure 3.8 is represented the flow through a coaxial rotor model in the hovering condition. To define the flow lines associated with the blade-tip-vortex filaments were introduced in the air, above the rotors, the dust of balsa-wood. Can be noticed that the lower rotor, from about 0.8 of the radius, is affected by a strong downflow.

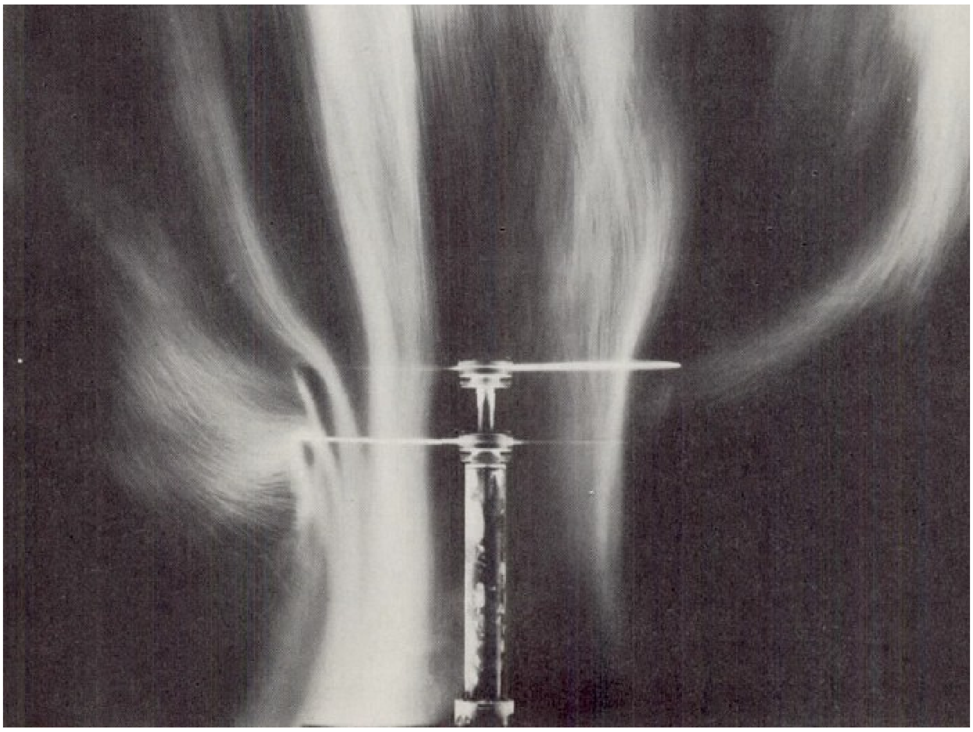


Figure 3.8 Air flow through a coaxial rotor model in static thrust ([28], pag. 8, fig. 2)

So the tips of the lower rotor may stall at higher thrust coefficients. Due to the unsymmetrical downflow over the lower rotor, the single-rotor theory results more complicated and need corrections to study a performance analysis of this configuration.

The power required for a coaxial helicopter in function of the tip-speed ratio μ, using the rotor system tested in the wind tunnel, is represented in the Figure 3.9. The system is operating in level flight mode, at a constant rotor thrust coefficient and tip speed (values indicated higher).

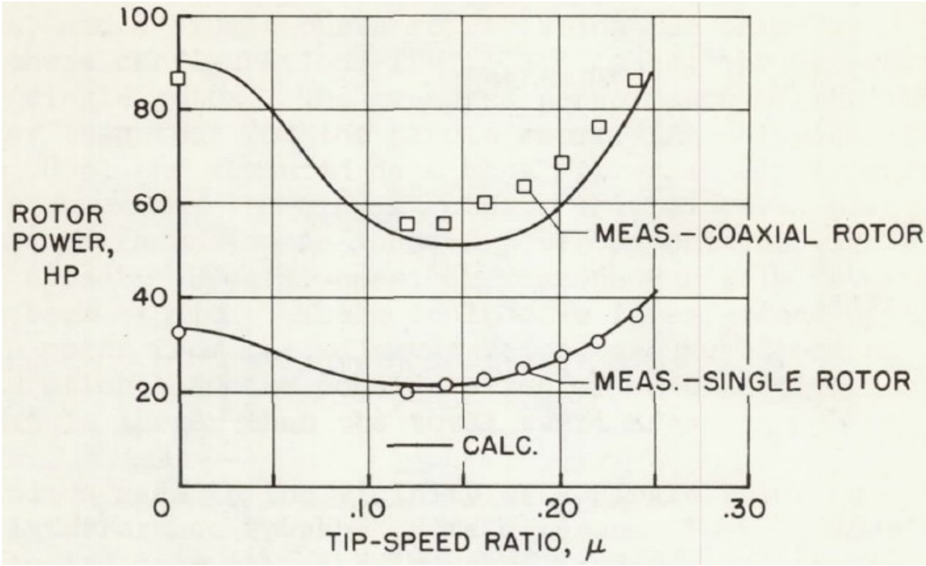


Figure 3.9 Level - flight performance with coaxial and single rotors ([28], pag.10, fig.5)

The next graph gives the power curves of the coaxial configuration with the momentum (green curve) and blade element (blue curve) theories, used in the program. The red points in the Figure 3.10 are the measured power points in function of the tip speed ratio, made in the wind-tunnel of the coaxial rotor model, the squares in the Figure 3.9.

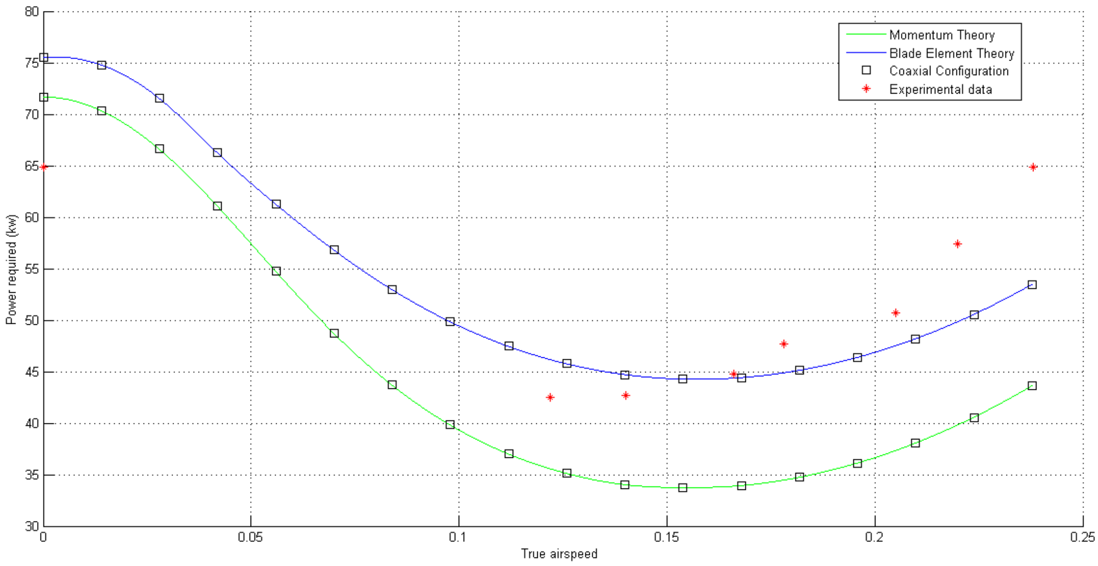


Figure 3.10 Momentum and blade element theories resulting curves of coaxial helicoter design

From the Figure 3.10 results that, for the true airspeed $\mu \leq 0.16$, the blade element theory (blue curve) overestimates the experimental values of the power arriving to a maximum error of 14.5% around $\mu = 0$. For $\mu > 0.16$ the blade element theory underestimates the power necessary for the level flight arriving to a maximum error of 17.5% around $\mu = 0.24$.

The momentum theory (green curve) overestimates the power for small values of the airspeed ratio μ, and around $\mu = 0$ shows a maximum error of 9.7%. Between $0.04<\mu<0.08$ occure the change from the overestimation to the underestimation of the power using the momentum theory. For $\mu>0.08$ the inclination of the green curve changes less fast respect to the inclination of the red experimental points, so around $\mu=0.24$ can be noticed the maximum error of 32%. It's interesting to notice that at $\mu=0.12$, where the experimental red point is placed between the two curves, the error due to the overestimation of the blade element theory value results of 7.6% and the error due to the underestimation of the momentum theory value results of 15.3%.

3.5.2 Single - rotor

In the Figure 3.9 is represented also the power curve of the single-rotor design in function of the tip speed ratio. The input variables are the same of the coaxial rotor configuration tested in the wind tunnel. Changes only the solidity σ = 0.027 and the thrust coefficient that is half of the coaxial case Ct = 0.0024.

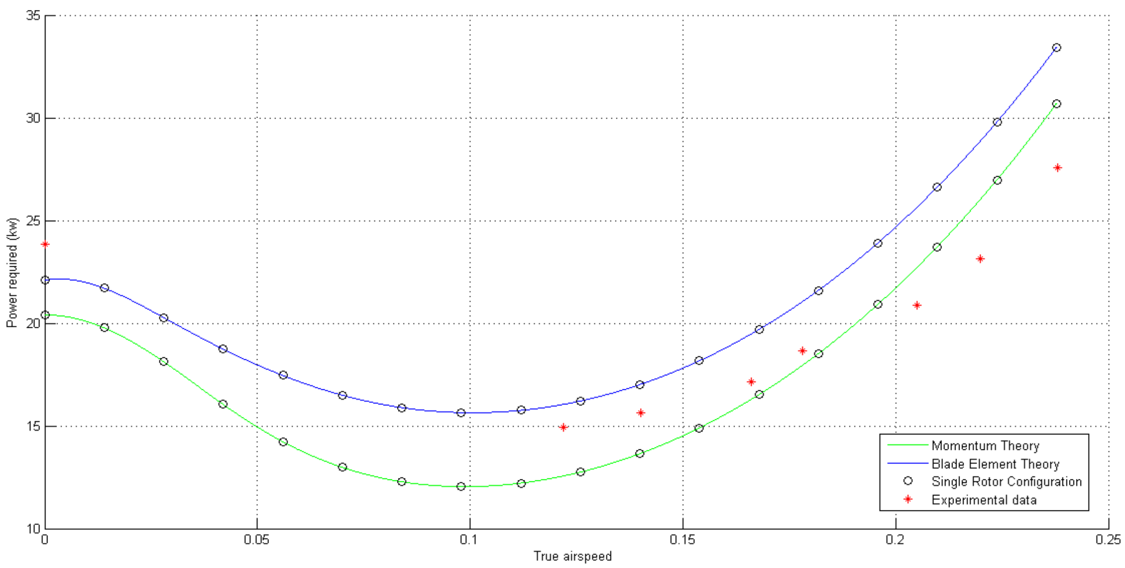


Figure 3.11 Momentum and blade element theories curves for single rotor

The red points in the Figure 3.11, are the measured power points in function of the tip speed ratio, made in the wind-tunnel of the single rotor model, the circles of the lower curve in the Figure 3.9. From the Figure 3.11 can be observed that the two theories are in a good agreement with the experimental values (red points). From the graph can be intuited that, for $\mu < 0.07$, the blade element theory (blue curve) underestimates the power needed for the level flight with a maximum error of 6.5% around $\mu = 0$. For $\mu \geq 0.07$ the blade element theory overestimates the power needed with a maxium error of 17.6% verified around $\mu = 0.24$.

The momentum theory (green curve), for $\mu < 0.19$, underestimates the power needed with a maximum error of 14.6% around $\mu = 0$. For $\mu \geq 0.19$ the momentum theory overestimates the power with a maximum error of 11% around $\mu = 0.24$.

It's interesting to notice that, for $0.14 < \mu < 0.16$, the two theories result to have the same error respect the experimental points and arrive at a balanced value of 8.5% of error around $\mu = 0.15$.

3.5.3 Tandem rotor

The tandem configuration model used to validate the software developed for this thesis has the rotor-shafts spaced 3 percent greater than the rotor diameter and the rotors lie in the same plane. The tandem model (see Figure 3.12) has two rotors with two blades each and the diameter of the rotors is 15 feet (4.57 meters). The rotor shafts are parallel and the solidity of each rotor is 0.054. The blades used are untwisted and untapered and the airfoil section are NACA 0012.

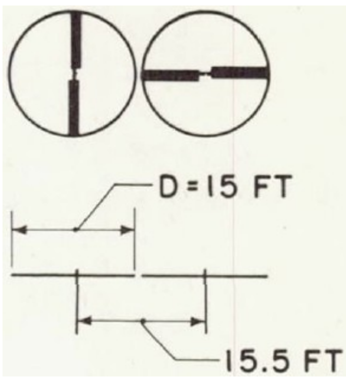


Figure 3.12 Tandem rotor configuration tested ([28], pag. 8)

The tandem system has no rotor overlap or vertical offset, but the rotors can be moved toward each other to mesh the blades up to 75 percent of the radius and can be offset vertically. The thrust coefficient, for this tandem design case, is around 0.0034 and the equivalent flat-plate parasite-drag used in this test is 2 square feet (0.186 square meter).

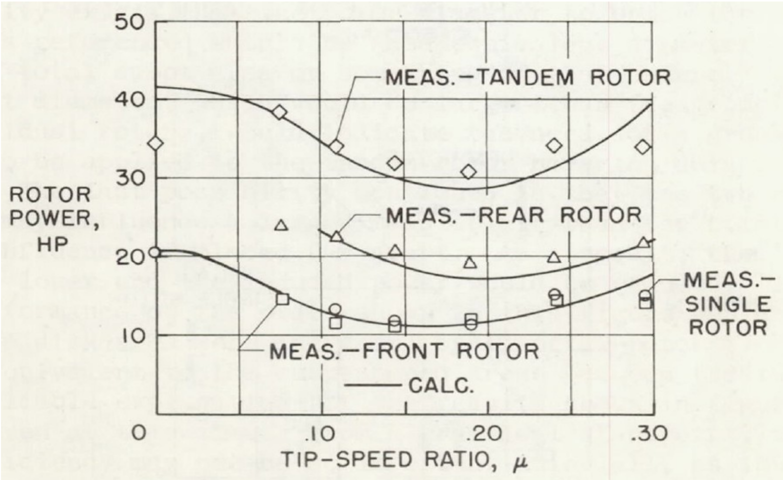


Figure 3.13 Level flight performance with tandem and single rotors ([28], pag.10, fig.6)

The curve of the forward flight performance obtained by using the tested tandem rotor configuration, as well as a loss of the power absorbed by the front and rear rotors, are represented in the Figure 3.13.

The agreement is excellent for the single rotor. The measured performance of the tandem rotor is much better than that for the single rotor. The circles designate the points obtained with one of the rotors removed.

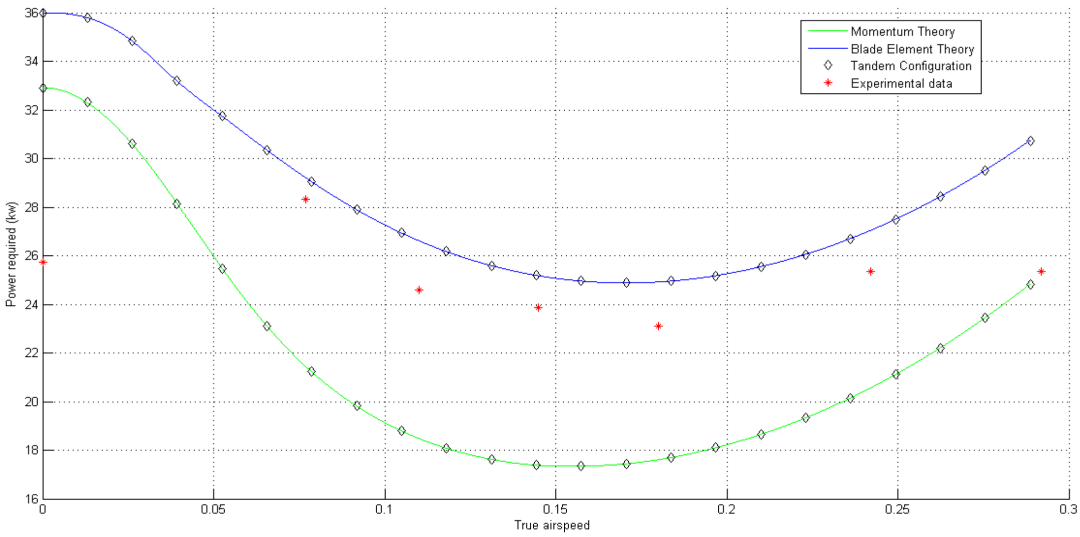


Figure 3.14 Momentum and blade element theories resulting curves for tandem rotor design

The red points in the Figure 3.14, are the measured power points in function of the tip speed ratio of the tandem rotor model, the diamonds in the Figure 3.13. The two curves, green and blue, in the Figure 3.14 are obtained with the momentum and, respectively, blade element theories. The idea is to use the experimental values of tandem model presented to verify and to validate the program code developed. The momentum theory curve (green), overestimates the power needed for small airspeed ratios with a maximum error of 21.8% at $\mu = 0$. For $\mu > 0.06$ the momentum theory curve underestimates the power and the maximum error of 27% is verified around $\mu=0.14$.

The blade element theory overestimates the power, needed by the tandem configuration and presents a maximum difference from the experimental data of 28.3%, verified at $\mu = 0$. For $0 < \mu \leq 0.8$ the difference decreases to a minimum of 3.1% around $\mu = 0.08$. For $\mu > 0.8$ the difference from the experimental points oscillates and rises to a value of 17.8% at $\mu = 0.28$.

3.5.4 Conventional rotor helicopter:

For the validation of the conventional design case are used the data of the helicopter model in level flight mode described in the book of J. Gordon Leishman, "Principles of helicopter aerodynamics"([19], pag.227). Are assumed the mass at takeoff of 7256kg and a the maximum operating altitude of 1585m of the helicopter. The equivalent flat-plate area, f, is 2.137 square meters. Are also assumed, for the both main and tail rotors, that k = 1.15 and Cd0 = 0.008 and the distance between the rotors is 9.9m. In the Figure 3.15 the points represented are the experimental values, measured in the wind tunnel, of the total power of the conventional helicopter model.

The theoretical curves present in the Figure 3.15 are the induced, profile, parasitic and tail-rotor powers that compound the total power needed by the helicopter for the level flight. The Figure 3.16 presents the resulting momentum theory (green) and the blade element theory (blue) curves, in comparison with the flight test data of the same helicopter model. The red points in the Figure 3.16 are the black dots from the Figure 3.15.

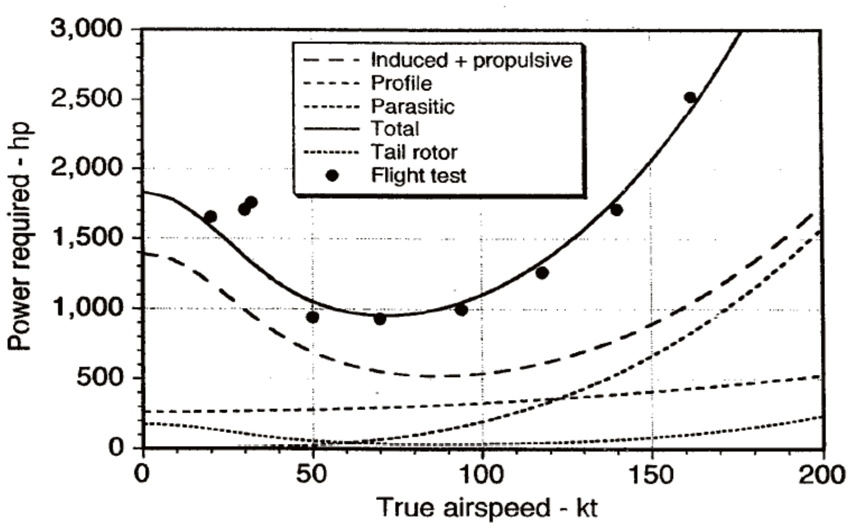


Figure 3.15 Level flight performance of a conventional rotor design ([19], pag.227, fig.5.10)

As soon in the reference of the experimental data are not specified some variables needed, using the input data given of the aircraft, and introducing them into the empirical equations (165), (166), (168), (169), (170) and (172), can be obtained reasonable diameters, chords and tip speeds needed for the calculations. The number of the blades is chosen to be three for the main rotor and two for the tail rotor. So the diameters of the main and tail rotors result to be 14.23m and, respectively, 2.9m. The chords of the main and tail rotors result to be 0.59m and, respectively, 0.32m. The values of the blade tip speed of the main rotor *Vtip* and of the tail rotor *Vtiptr* result 220.5 [m/s] and, respectively, 218.5 [m/s].

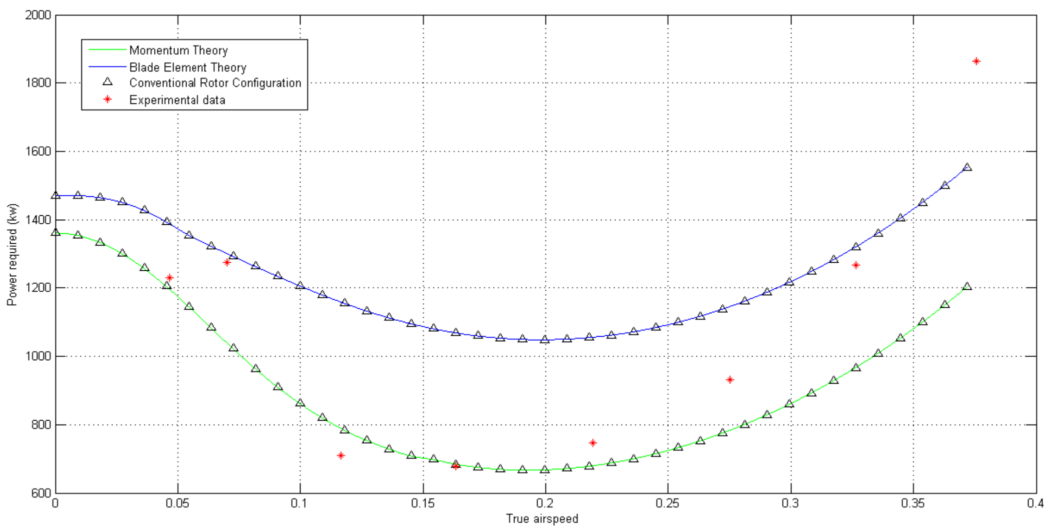


Figure 3.16 Momentum and blade element theories resulting curves for conventional rotor design

The momentum theory (green curve), predicts the power very well for low airspeed ratios, except around $\mu = 0.07$, where the error is about 18%. For $0.1<\mu<0.25$, the green curve passes very close to the experimental values with a maximum error of 9.5% around $\mu = 0.12$ and $\mu = 0.218$. For $\mu > 0.2$ the momentum theory curve has a lower variation of the inclination respect to the sequence of the experimental points, so the difference continues to rise till a value of 35% for $\mu = 0.37$.

The blade element theory (blue curve) overestimates the power needed by the conventional configuration for $\mu \leq 0.34$. The maximum error of 37% is verified around $\mu = 0.12$. For $\mu > 0.34$, the blade element theory underestimates the power necessary for the level flight of the conventional design and the maximum error, of 16.3%, is verified for $\mu = 0.37$. It's interesting to notice how, around $\mu = 0.3$, the difference of the two theories respect the experimental values results to be around 14.5%.

CHAPTER 4

THE DESIGN OF A HELICOPTER WITH THE DEVELOPED PROGRAM

4.1 Input values in common

After validating the program code, the next idea of this thesis work is to create and test three helicopter configurations. It's also important to chose right the input data because the main object is to compare the three designs between them, from the point of view of the power required (the two theories), the specific velocities, the endurance and the range. The common input data for all the configurations and the two theories are:

- The maximum mass at take-off: M = 11000 [kg],
- The maximum speed: Vmax = 80 [m/s],
- The step of variation of Vmax: dv = 2 [m/s],
- The coefficient of drag of the rotor: Cd0 = 0.008,
- The number of blades of each rotor: Nb = 3,
- The flat plate parasitic drag: f = 3.5,
- The height from the sea level: h = 0 [m],
- The parameter (varies from 4 in hover, to 5 at mu = 0.5): K = 4.7,
- The induced power factor: k = 1.15,
- The adjustment coefficient: Kh = 1,
- The chord of the blades of the rotor: c = 0.74 [m],
- The tip speed of each rotor: Vtip = 200 [m/s],
- The curve slope of the profile of the blades of the rotor: Clalpha = 5.73,

• The step of integration of the adim. blade length in the blade element theory:
dr = 0.05,

• The step of integration in the blade element theory due to the azimuthal angle:
dxi = 0.628,

• The power installed: Pintstalled = 2500 [kW],

• The specific fuel consumption: SFC = 1.3609e-04 [kg/(W·s)],

• The mass of the fuel present onboard: Mf = 2000 [kg],

4.2 Conventional configuration:

The additional input data for the conventional helicopter are:

• The diameter of the main rotor: D = 22 [m],

• The diameter of the tail rotor: Dtr = 3 [m],

• The chord of the blades of the tail rotor: ctr = 0.29 [m],

• The distance between the two rotor shafts: dtr = 13 [m],

In the Figure 4.1 is represented the power in function of the airspeed ratio required by the conventional helicopter. The green curve is obtained with the momentum theory and the blue one with the blade element theory.

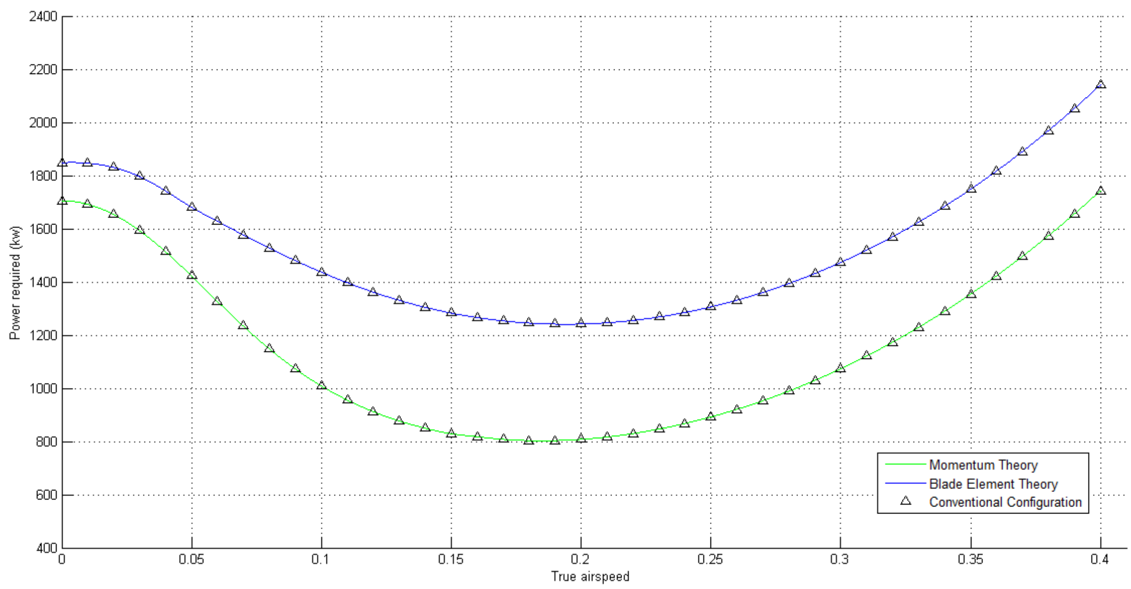


Figure 4.1 Output power curves of conventional helicopter with momentum and blade element theories

From the Figure 4.1 it results that for the hover condition (μ = 0), the difference between the blade element theory curve and the momentum theory curve is around 7.7%, for μ = 0.18 arrives to a maximum difference of 35.5% and decrease to 18.6% for μ = 0.4.

4.3 Coaxial configuration:

The additional input data for the coaxial helicopter are:

• The diameter of the rotors: D = 16 [m],

• The induced power interference factor between the rotors: kint = 1.16,

• The stall of the blade tips starts: rtiploss = 1,

In the next figure is represented the power in function of the airspeed ratio required by the coaxial helicopter:

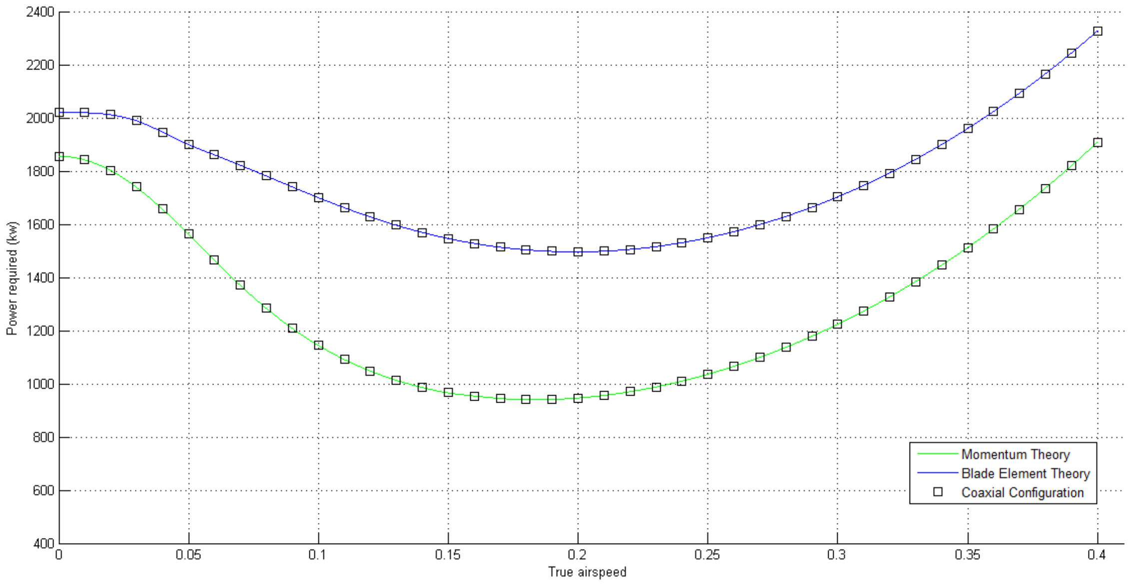


Figure 4.2 Output power curves of coaxial helicopter with momentum and blade element theories

From the Figure 4.2 results that for the hover condition (μ = 0), the difference between the blade element theory curve (blue) and the momentum theory curve (green) is around 8.1%, for μ = 0.18 arrives to a maximum difference of 37.4% and decrease to 18% for μ = 0.4.

4.4 Tandem configuration:

The additional input data for the coaxial helicopter are:

• The diameter of the rotors: D = 16 [m],

• The induced power overlap factor between the rotors: kov = 1.14,

In the next figure is represented the power in function of the airspeed ratio required by the tandem helicopter.

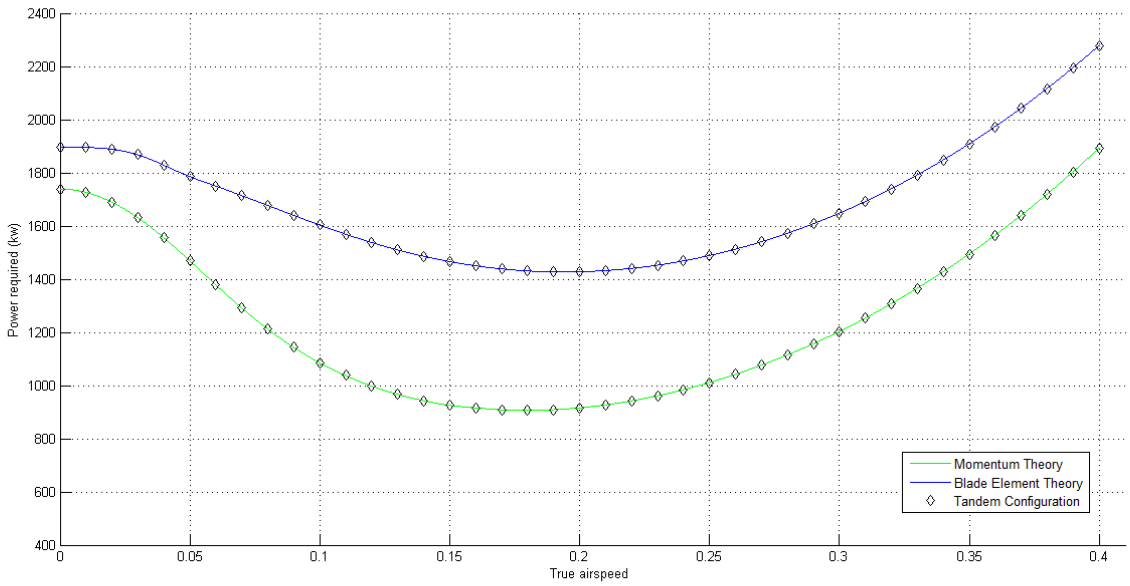


Figure 4.3 Output power curves of tandem helicopter with momentum and blade element theories

From the Figure 4.3 results that for the hover condition ($\mu = 0$), the difference between the blade element theory curve (blue) and the momentum theory curve (green) is around 8.3%, for $\mu = 0.18$ arrives to a maximum difference of 36.6% and decrease to 16.9% for $\mu = 0.4$.

4.5 Momentum theory:

In the Figure 4.4 are grouped the power curves of the conventional (triangles), coaxial (sqares) and tandem (diamonds) helicopters, that are found with the momentum theory (green colour). The Figure 4.4 aims to put in comparison the three configurations from the point of view of the momentum theory, to see the shape of the curves and also to notice the differences in the power required between each design.

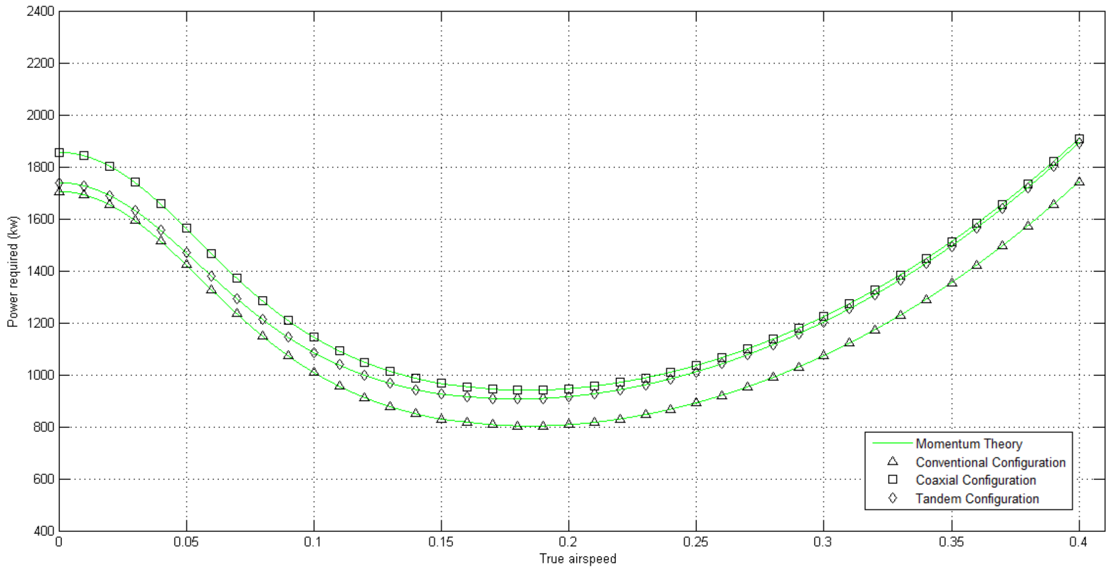


Figure 4.4 Power curves of conventional/coaxial/tandem designs with momentum theory

From the Figure 4.4 can be noticed that coaxial configuration requires more power respect the tandem and the conventional configurations. Results also that the configuration that needs less power for the flight is the conventional one. It's interesting to notice that the tandem design curve assumes, for small μ, a shape close to that of the conventional design with a minimum difference of 1.95%. From $\mu > 0.07$ the tandem design curve is getting close to the coaxial design curve, reaching a minimum difference of 0.8% for μ=0.4.

From the Figure 4.4 results also that for the hover condition ($\mu = 0$), the difference between the coaxial configuration curve and the conventional configuration curve is around 8.1%, for $\mu = 0.18$ arrives to a maximum difference of 14.6% and decreases to 8.7% for $\mu = 0.4$. Can also be noticed that for $\mu = 0.18$, the difference in the power between the coaxial and the tandem cases is about 3.6% and between the tandem and the conventional cases is about 11.4%.

The other important results, needed to study and to compare the various designs, are:

- The velocity for the minimum power: Vmp,
- The vertical velocity for the minimum power: Vvmp,
- The velocity for the maximum range: Vmr,

• The range of the helicopter,

• The endurance of the helicopter,

The velocities Vmp and Vmr are calculated with the momentum theory (see Chapters 2.2 and 3.2). The Vvmp, the range R and the endurance E are found with the relations (149), (163) and, respectively (164). In the Table 4.1 are represented in comparison the output data of each helicopter configuration:

	Conventional:	Coaxial:	Tandem:
Vmp [m/s]	40,3	36,7	35,9
Vvmp [m/s]	8,3	14,4	14,8
Vmr [m/s]	57,4	56,6	55,8
R [km]	784,1	687,7	716,9
E [hr]	3,3	2,96	2,95

Table 4.1 Results table of conventional/coaxial/tandem designs with momentum theory

The results, obtained with the momentum theory and represented in the Table 4.1, show how the tandem and the coaxial configurations have similar behaviors with a difference in range of 4%.

Can also be noticed that the conventional design has the best results in terms of range and of endurance. The conventional configuration represents an increase in range of 12.3% respect the coaxial case and 8.6% respect the tandem case. Results also an increase in endurance of 10.6% respect the coaxial case and 10.3% respect the tandem case.

The results of the velocity for maximum range Vmr (Table 4.1) of the three configurations result to be very similar, but anyaway the lowest value is verified for the tandem helicopter and the highest for the conventional helicopter and the difference is of 2.8%. The Vmr values of the coaxial and of the tandem designs differ from the value of the conventional case about 1.4%.

The highest value of the velocity for minimum power Vmp is verified for the conventional helicopter, in fact, is higher about 8.9% respect the coaxial configuration and about 10.9% respect the tandem configuration. The lowest value of the vertical velocity for the minimum power Vvmp is verified for the conventional configuration. The values of Vvmp of the coaxial and tandem designs are higher respect the conventional design about 42.4% and, respectively, 43.9%.

4.6 Blade element theory:

From the Figure 4.5, as in the Figure 4.4, can be noticed that coaxial configuration (square points) requires more power respect the tandem and conventional configurations. Also with the blade element theory (blue curve) results that the design that requires less power is the conventional one (triangle points).

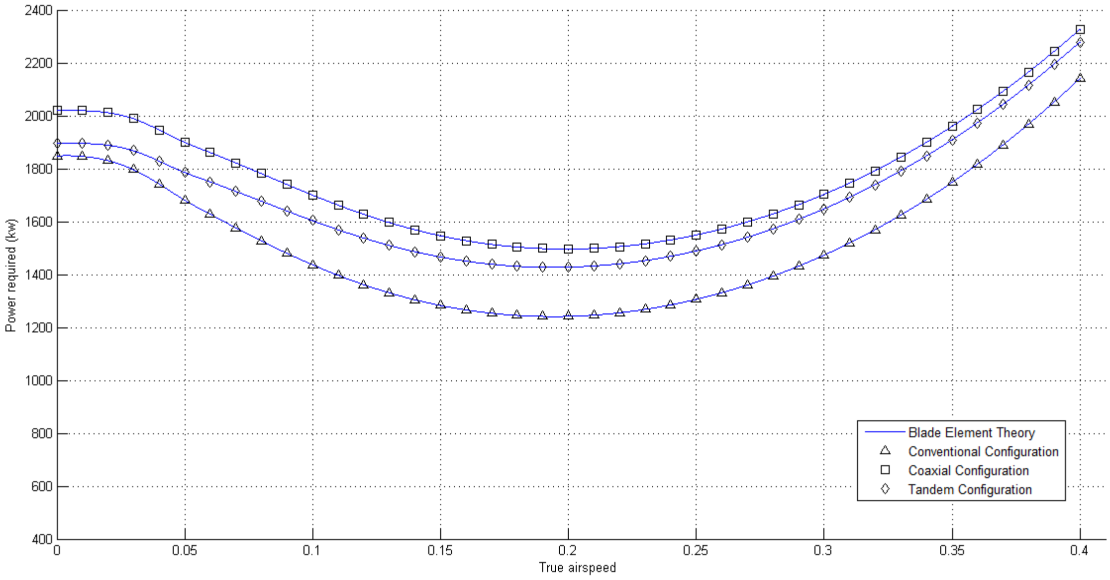


Figure 4.5 Power curves of conventional/coaxial/tandem designs with blade element theory

Can be noticed, as for the momentum theory case (Chapter 4.5), that the tandem design curve, for small μ, passes close to the curve of the conventional design with a minimum difference of 2.52% at μ = 0. For μ > 0.04 the tandem design curve is starting to get close to the coaxial design curve (square points) and reaching a minimum difference of 2.04% for μ = 0.4.

From the Figure 4.5 results also that for the hover condition (μ = 0), the difference between the coaxial configuration curve and the conventional configuration curve is around 8.5%, for μ = 0.18 arrives to a maximum difference of 17.2% and decreases to 8% at μ = 0.4.

It's interesting to noticed from the Figure 4.5 that, for μ = 0.18, the difference in the power between the coaxial and the tandem cases results to be about 4.8% and between the tandem and the conventional cases results to be about 13%. In the next Table 4.2, as in the Table 4.1, are represented the velocities Vmp, Vvmp, Vmr, the range and the endurance of each helicopter configuration calculated for the blade element theory curves (Figure 4.5). The velocities for minimum power and for maximum range are found with the "geometric technique" exposed in the Chapter 3.3.

The Vvmp, and the range/endurance are, as for the momentum theory, calculated with the relations (149), (163) and (164).

	Conventional:	Coaxial:	Tandem:
Vmp [m/s]	38	40	38
Vvmp [m/s]	8,47	14,39	14,74
Vmr [m/s]	62	66	66
R [km]	671,64	619,96	656,28
E [hr]	2,74	2,39	2,54

Table 4.2 Results table of conventional/coaxial/tandem designs with blade element theory

The results obtained with the blade element theory, represented in the Table 4.2, are in agreement with the results obtained with the momentum theory (Table 4.1). The conventional design still has best results, from the point of view of range and endurance, respect the coaxial and tandem designs. The conventional configuration represents an increase in range of 7.7% respect the coaxial case and 2.3% respect the tandem case. Results also an increase in endurance of 12.8% respect the coaxial case and 7.3% respect the tandem case.

The results of the velocity for maximum range Vmr (Table 4.2) of the coaxial and tandem configurations result to be equal. The value of Vmr of the conventional design is lower then the coaxial/tandem designs of 6.1%.

The results of the velocity for minimum power Vmp of the conventional and tandem configurations result to be equal. The value of Vmp of the coaxial design is higher then the conventional/tandem designs of 5%.

The lowest value of the vertical velocity for the minimum power Vvmp, as for the momentum theory (Table 4.1), is verified for the conventional configuration. The values of Vvmp of the coaxial and tandem designs are higher respect the conventional design and results to be close to the values found with the momentum theory (see Table 4.1). The differences from the conventional case are 41% for the coaxial case and, respectively, 42.2% for the tandem case. The value of Vvmp of the tandem case is higher then the coaxial case of 2.04%.

CHAPTER 5

CONCLUSIONS AND POSSIBLE IMPROVEMENTS

5.1 Conclusions

The main objective of creating a program code as a tool for a preliminary design of a helicopter is reached. The tool can be used to design three kind of configurations of helicopter: the conventional, the coaxial and the tandem. For this aim are used the momentum and the blade element theories. The program calculates the power required by the helicopter chosen for the flight with the theory chosen, and plots it in function of the airspeed ratio. The program calculates and displays also the range, the endurance, the velocity for maximum range, the velocity for minimum power and the vertical velocity for minimum power. Depending on the input parameters of the helicopter to be designed and with the two theories used for this purpose the tool gives as output the power required curve and the five output parameters.

Was made the validation of the program by using data collected in wind-tunnel experiments on the three configurations treated in this work. In fact the results are close to the experimental values with acceptable errors.

To study and to see the differences between the three configurations a test was made by giving the compatible inputs to each helicopter design and were put in comparison from the point of view of the momentum theory and, respectively, the blade element theory. It was noticed that the conventional helicopter gives the best results in terms of power requirement for the flight, of the range and of the endurance. On the other side the coaxial configuration shows the highest power consumption and the lowest values of the range and endurance. The tandem configuration results, in terms of the power, range and endurance, lie between the conventional and coaxial configuration values.

Was also noticed that the blade element theory has a much higher time per simulation than the momentum theory.

5.2 Possible improvements

For future applications and for a more complete tool for a preliminary designing of a helicopter can be improved or, at least included, some aspects:

- the power for the vertical flight,
- the fuselage drag due to the rotor wake (or the vertical drag),
- the retreating blade with the reverse flow,
- the twisting along the blade,
- the tapering along the blade,
- the variation of the profile of the blade along the blade,
- the root cut-out of the blades,
- for the conventional configuration, the interaction of the tail rotor and the vertical fin,
- the load distributions and non-linearities in the flow environment (for example: tip losses),
- the deviation of the inflow from the value predicted with the momentum theory *ky* and the other values of the first harmonic inflow (see Table 2.1),
- the autorotation,
- the compressibility of the air,
- the control of the helicopter for a more complicated mision profile then the level flight (from the hover to the maximum speed),
- a more friendly program interface for the insertion and variation of the input variables by the user,
- the application of the theories to multirotor helicopters,

APPENDIX

DESCRIPTION OF THE SOFTWARE

When the program is launched, in the "Command Window" appears a string that asks which configuration of helicopter is wanted:

```
Command Window
fx THE CONFIGURATION OF THE HELICOPTER: CONVENTIONAL [1], COAXIAL [2], TANDEM [3] :
```

Figure A.1 The choice of the configurations of the helicopter

After choosing one of the three configurations, for example typing "1" for the conventional one, appears a new question that asks to choose the theory that is wanted to be applied:

```
Command Window
 THE CONFIGURATION OF THE HELICOPTER: CONVENTIONAL [1], COAXIAL [2], TANDEM [3] : 1

fx The type of Theory you want to apply: Momentum [1], Blade Element [2]:
```

Figure A.2 The choice of the theory

Typing "1" is chosen the momentum theory and results:

```
Command Window
 THE CONFIGURATION OF THE HELICOPTER: CONVENTIONAL [1], COAXIAL [2], TANDEM [3] : 1

 The type of Theory you want to apply: Momentum [1], Blade Element [2]: 1


  THE CONVENTIONAL HELICOPTER CONFIGURATION:


  MOMENTUM THEORY:

fx The maximum mass at takeoff [kg]: M =
```

Figure A.3 The input of data in the momentum theory

At this point, the user is asked to insert the next input values, needed to realize the calculations:

• the maximum mass at take-off M, in [kg];

• the maximum speed Vmax, in [m/s];

• the step of variation of the Vmax, dv, in [m/s];

• the coefficient of drag of the main rotor, Cd0;

• the coefficient of drag of the tail rotor, Cd0tr;

• the number of blades of the main rotor, Nb;

• the number of blades of the tail rotor: Nbtr;

• the distance between the two rotor shafts, dtr, in [m];

• the flat-plate parasite-drag, f;

• the height from the sea level, h, in [m];

• the parameter (varies from 4 in hover, to 5 at mu = 0.5) K;

• the induced power factor k;

• the adjustment coefficient, Kh

• the diameter of the main rotor, D, in [m];

• the diameter of the tail rotor, Dtr, in [m];

• the chord of the blades of the main rotor, c, in [m];

• the chord of the blades of the tail rotor, ctr, in [m];

• The blades tip speed of the main rotor, Vtip: [m/s];

• The blades tip speed of the tail rotor, Vtiptr: [m/s];

If is typed "2", so is chosen the blade element theory:

```
Command Window
THE CONFIGURATION OF THE HELICOPTER: CONVENTIONAL [1], COAXIAL [2], TANDEM [3] : 1

The type of Theory you want to apply: Momentum [1], Blade Element [2]: 2


 THE CONVENTIONAL HELICOPTER CONFIGURATION:


BLADE ELEMENT THEORY:

fx The maximum mass at takeoff [kg]: M =
```

Figure A.4 The input of data in the blade element theory

For the blade element theory are also required the next variables:

• the curve slope of the profile of the blades of the main rotor, Clalpha;

• the curve slope of the profile of the blades of the tail rotor: Clalphatr;

• the step of integration of the adim. blade length, dr;

• the step of integration due to the azimuthal angle, dxi, in [rad];

For all the variables that are asked, except for the mass at take-off M, exist a default value that was set previously in the program and can be used just pushing ENTER when the respective variable is asked. In particular for the diameters, the chords and the tip speed of the rotors, as default, are used the empirical formulas provided by the relations, respectively, (165), (166), (168), (169), (170) and (172).

For the coaxial and tandem configurations for both theories the program requires a single value for Cd0, Nb, D, c, Vtip, Clalpha because in these designs is not present a tail rotor and the two rotors of the coaxial/tandem designs are considered identical.

For the calculations of the power in the coaxial design are required also the next input data:

• the induced power interference factor between the rotors: kint,

• the dimensionless radius at which the blade tips start to stall due to the tip losses: rstall,

For the calculations of the power in the tandem design are required also the next input data:

• the induced power overlap factor between the rotors: kov,

So when the program asks the inputs, the user must insert the value of the mass at take-off of the rotorcraft but for the other input variables can be chosen an own value or used the default one.

At this point the program calculates and plots the power (in [kw]) required by the helicopter in function of the dimensionless forward speed μ. Next the program aks:

```
fx Do you want to calculate the ENDURANCE and the RANGE? Yes = [1], No = [2]
```

Figure A.5 The question about the calculation of the endurance and the range

If is chosen “1” the program calculates the endurance and the range of the helicopter configuration chosen (see Figure A.2) with the theory, respectively, chosen (see Figure

A.3). Also are displayed the velocity for maximum range, the velocity for minimum power and the vertical velocity for minimum power.

The program requires the next inputs:

• the power installed on the helicopter, Pinstalled, in [kwatts];

• the specific fuel consumption, SFC, in [kg/(W·s)];

• the mass of the fuel, Mf, in [kg];

As for the previous input values, also for Pinstalled, for SFC and for Mf are set default values, so the user can choose to insert the proper data or to use the already present ones.

After the display of the results or when the user just doesn't want to calculate the range, the endurance and the velocities by pressing “2” in Figure A.5, the program asks:

```
fx Do you want to start a new calculation in the last configuration/theory? Yes = [1], No = [2]
```

Figure A.6 The question about the repeat of the calculation in the same config/theory

If the user wants to repeat the calculation of the same configuration of the rotorcraft and with the same theory, he has to select “1”. So when the user changes the value of some input data in the new iteration, he can easily compare the results with the previous calculation. If the user doesn't want to repeat the calculation and to quit from the present configuration/theory he has to select “2”.

In the “Command Window” appears the question:

```
fx Do you want to start a new session? Yes = [1], No = [2]
```

Figure A.7 The question to start a new session of calculations

If the user wants to start a new calculation session he has to select “1”. Next appears the same message in Figure A.1, where the user can switch to a different design of helicopter or re-enter to the same configuration to proceede with the different theory. It is comfortable in this way to compare the graphs and the results of different configurations of the helicopter and with different theories. If the user want to quit the program he has just to select “2” at this point.

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