UNIVERSITA' DEGLI STUDI DI PADOVA

DIPARTIMENTO DI SCIENZE ECONOMICHE ED AZIENDALI
"M. FANNO"

CORSO DI LAUREA MAGISTRALE IN
ECONOMICS AND FINANCE

TESI DI LAUREA

"EFFICIENZA DEI PORTAFOGLI DELLE FAMIGLIE ITALIANE CONDIZIONATA ALL'ABITAZIONE"

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MATRICOLA N. 1106072

ANNO ACCADEMICO 2016 – 2017

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UNIVERSITY OF PADOVA

DEPARTMENT OF ECONOMICS AND MANAGEMENT
"M. FANNO"

GRADUATE DEGREE IN ECONOMICS AND FINANCE

THESIS

"EFFICIENCY OF ITALIAN HOUSEHOLD PORTFOLIOS CONDITIONAL ON HOUSING"

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A. Y. 2016 – 2017

Abstract

When considering household wealth, the most important asset is housing. However, standard tests of portfolio efficiency neglect the existence of illiquid wealth, and are hence biased when housing returns correlate with financial market returns. This is also true if housing stock adjustments are costly and therefore infrequent. Optimal portfolios in periods of no adjustment are affected by housing price risk through a hedge term and tests for portfolio efficiency of financial assets must be run conditionally upon housing wealth.

We use Italian household portfolio data from SHIW 2014 and time series data on financial assets and housing price returns to assess whether actual portfolios are efficient. We first consider purely financial portfolios and then show that when housing is included in the analysis as an unconstrained asset, most portfolios fail the standard efficiency test. We then consider housing as predetermined and test for conditional efficiency. In our empirical analysis we find that illiquid wealth plays an important role in determining whether portfolios chosen by homeowners are efficient.

Finally, we compare Italian household portfolios prior to and after the financial crisis and show how 2008 portfolios result inefficient when the test is computed with information set up to 2007 and mostly efficient when we do the test with data up to 2014. A possible interpretation is that households correctly anticipated that there would be a sudden drop in housing returns after a prolonged upward trend.

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Introduction

This thesis investigates the efficiency of Italian portfolios through the impact of housing on portfolios' allocation. Household wealth is allocated into financial and real assets, but analysis on portfolio allocations have usually focused mainly on financial assets. Many empirical studies have been developed in recent years but, so far, they have not shown a systematic relationship between housing and portfolio choices.

Owner-occupied housing is the single most important consumption good as well as the dominant asset in most household portfolios (Flavin & Yamashita, 2002) and it must be considered as both consumption and a risky asset. The demand for owner-occupied housing is thus the result of both intra-temporal consumption choice and inter-temporal portfolio choice (Cocco, 2000), (Yao & Zhang, 2005). Housing should be included in the market portfolio, and thus changes the CAPM (Kullmann (2003)). In addition, as owner-occupied housing changes the marginal utility of non-durable consumption, if the utility function is non-separable in non-durable consumption and housing, it also changes the consumption based CAPM (Grossman & Laroque, 1990).

Due to the large transaction costs of buying and selling a house, there is an important dimension of illiquidity or irreversibility in the home investment. Moreover, the price of housing fluctuates considerably over time, and with it the value of the home investment and the wealth of homeowners (Cocco, 2000).

Considering the mean-variance framework developed by Markowitz (1959) and Merton (1973), we argue that household portfolios cannot be considered efficient in the standard sense, as housing asset is not considered. Indeed, they do not allow for one of the assets to enter the utility function as a consumption good and neither they consider that an asset could be subject to liquidity constraint. We include housing as an exogenous and pre-determined asset, assuming that households' prior choice is to select the optimal level of housing that maximize their consumption benefits. Given the housing constraint, households invest their remaining wealth in other financial assets. Moreover, when housing and financial returns are correlated, house owning creates a hedging demand for financial assets.

Following the work by Pelizzon and Weber (2008), we show that optimal portfolios should be conditionally mean-variance efficient, that is mean variance efficient when housing wealth is treated as given but stochastic. A conditional test of mean-variance efficiency, that treats housing wealth as predetermined, was first suggested by Gourieroux and Jouneau (1999).

To implement the test, we use data on Italian household portfolios from the Bank of Italy Survey on Household Income and Wealth (SHIW) for 2014 and time series data on financial assets returns, as well as housing stock returns provided by OECD House Price database, from 1990 to 2015.

This thesis is organized as follows. In Chapter 1 we present a review of the relevant literature, in Chapter 2 we present a theoretical model on optimal portfolio choice developed by Pelizzon and Weber (2008) and discuss related econometric issues. In Chapter 3 we display the characteristics of Italian household portfolios and show how we group assets and obtain asset moments. In Chapter 4 we analyze what are the implications of housing constraint on possible portfolio allocations and in Chapter 5 we present the results of efficiency test on household portfolios and try to understand how expectations on returns and covariances modify the efficiency results. In Chapter 6 we report on a comparison with 2008 household portfolios to understand how households have reacted to the financial crisis and how this affects our efficiency results. In Chapter 7 we conclude regressing the computed test statistic on household characteristics and income as a way to investigate possible causes for inefficient portfolio allocations.

1. Literature review

Grossman and Laroque (1990) first examine the problem of portfolio choice and asset pricing in the presence of housing constraints in a continuous time framework, with the simplifying assumptions that agents care only about housing consumption, but not non-durable good, and that house price is constant. Their conclusion is that two-fund separation theorem still holds and that market portfolio is mean-variance efficient even in the presence of durable consumption goods.

Fougère et al. (1997), develop their analysis in the mean–variance Markowitz (1959) portfolio model framework. To model zero holdings, they assume that households cannot short sell risky assets. Solving the model in the case of three assets, and assuming that the expected excess returns on the risky assets are positive, they show that each household should choose its portfolio on the mean–variance efficient frontier on the basis of the Sharpe performances of the two risky assets and of the correlation coefficient between their excess gains.

Cocco (1999), using Panel Study of Income Dynamics (PSID) data on labor income and house price, estimates a large positive correlation between income shocks and house price shocks, and a large negative correlation between house prices and interest rates. He observes that homeownership serves as a hedge against fluctuations in the cost of consumption, because decreases in the price of housing, and in the wealth of homeowners, tend to be accompanied by a decrease in the implicit rental cost of housing. He considers several frictions that are usually of concern to home-buyers, including large transaction costs, uninsurable labor income risk and borrowing constraints. He finds that both labor income and interest rate risk crowd out housing investment, but due to the highly leveraged nature of investors' portfolios, the welfare and portfolio implications of the interest rate risk are much larger. The characterization of hedging demands for the housing asset emphasizes the role of liquidity constraints.

In another paper Cocco (2000) argues that due to investment in housing, younger and poorer investors have limited financial wealth to invest in stocks, which reduces the benefits of equity market participation. House price risk crowds out stockholdings, but this crowding out effect is larger for low financial net-worth. Transaction costs of changing houses reduce the frequency of house trades and lead investors to reduce their exposure to stocks.

Roon, Eichholtz and Koedijk (2002) analyze the effects of residential property holdings on optimal investment portfolios. Using a mean-variance framework, they show that residential real estate offers significant diversification benefits relative to investments in stocks and bonds for US investors. Risk averse investors that hold residential real estate for investment purposes have future wealth that is less volatile. In addition to this diversification effect, they find that stocks and bonds do not provide a good hedge for positions in real estate, implying that the relative demand for either is not significantly affected by home ownership (Roon, et al., 2002). This last finding, however, is not supported by more recent research.

Englund, Hwang and Quigley (2002) use the rich source of data on housing price in Stockholm to analyze the investment implications of housing choices and find that there are large potential gains from policies or institutions that would permit households to hedge their investment in housing. They argue that the low correlation between housing and other assets suggest that housing should contribute to diversifying the portfolio and lowering risk.

Chetty and Szeidl (2005) focus on infrequency of housing consumption adjustment, and show that the housing commitment mechanism can potentially resolve the equity premium puzzle. Chetty and Szeidl (2007) show that households leave homeownership in place, but cut consumption, after small shocks, while consumption and homeownership are reallocated only after major shocks.

Hu (2005) studies the interaction of the housing investment and financial assets investments in a dynamic lifecycle model. He considers that wealth comes from an uncertain stream of labor income and from savings in both liquid and illiquid assets. The level of housing is treated as endogenous and he includes the possibility of renting. He finds that introducing frictions associated with housing into standard models can partially resolve the portfolio choice puzzle. This because the owner-occupied house is a risky asset and it substitutes for stocks, while bonds provide liquidity to save for a house and make mortgage payments in case of income shortfalls.

Yao and Zhang (2005) expand the model to include housing adjustment costs, refinancing charges, and default penalties. The analysis demonstrates that household liquid wealth is the most important determinant of both home and stock ownership. Their results also suggest that high levels of home equity have an overall negative effect on stock market participation, because of the limited availability of liquid assets to pay the costs of investing in the stock market for those households who have a large (but rather illiquid) proportion of their wealth tied up in home equity (Beaubrun-Diant & Maury, 2016). They also find that when stock and housing returns are correlated, there is a hedging demand for holding stocks and, if this is induced by a positive correlation between stock and housing returns, it reduces homeowners' stockholding and yet raises renters' equity proportion.

An important second strand of literature is related to the contribution of Flavin and Yamashita (2002). In Flavin and Yamashita (2002), Yamashita (2003), Flavin and Nakagawa (2008) and Flavin and Yamashita (2011), the assumptions made in Grossman and Laroque (1990) are relaxed: both non-durable consumption and housing enter the utility function in a non-separable way, and house prices are explicitly modelled as a stochastic geometric Brownian motion. Showing that the market portfolio return has very low correlation with housing return, Flavin and Yamashita assume that the covariance matrix of the asset returns (including housing return) is block diagonal, thus imposing that housing has zero correlations with all stock returns. They conclude that the market portfolio is mean-variance efficient and traditional CAPM holds.

Flavin and Yamashita (2002) consider household demand for real estate as “overdetermined”. They argue that the level of real estate ownership that is optimal from the point of view of the consumption of housing services may differ from the optimal level of housing assets from a portfolio point of view. They assume that, the preferential tax treatment of owner-occupied housing and the frictions due to transactions costs and agency costs involved in the rental market for housing, effectively constrain the household to include in its asset portfolio the level of housing consistent with its consumption of housing services. With the addition of owner-occupied housing to the list of assets, and assuming that the quantity of housing held is predetermined by the household consumption demand for housing services, an additional constraint is imposed on the household portfolio allocation problem. At any given moment, both the value of housing owned, and the total net wealth of the household are fixed, and therefore the ratio of house value to net wealth is a fixed value. The household optimal holdings of financial assets will depend on both the value of the housing constraint (that is, the ratio of house value to net wealth) and on their degree of risk aversion (Flavin, 2011).

Following Pelizzon and Weber (2008) and Chu (2008), we remove the block-diagonal covariance matrix assumption. In fact, even if the market portfolio shows little covariance with housing, it is not the case that every financial asset has very small covariance with housing. Given that owner-occupied housing is the dominant asset in most household portfolios, even small correlation between financial assets return and housing return would significantly change the portfolio choice of assets (Chu, 2008).

Chu (2008) focuses on cross-sectional implication of owner-occupied housing on asset pricing, modelling both housing and non-durable consumption and allowing house price to follow a diffusion process. Using market portfolio return and housing return as pricing factors, and a Cobb-Douglas aggregate utility function, he shows first that both two-fund separation and CAPM fail with owner-occupied housing and second, that both non-durable consumption to wealth and non-durable consumption to housing ratio enter the stochastic discount factor linearly (Chu, 2008).

1. Literature review

Diaz and Luengo-Prado (2008) show that buying and selling costs affect homeownership in different ways. Higher buying costs delay homeownership over the life cycle since it amounts to a higher down payment; selling costs discourage young households from becoming owners since they face higher income uncertainty and move more frequently than older households. Selling costs also lowers the frequency at which homeowners upgrade or downgrade their houses.

In a more recent work, Diaz and Luengo-Prado (2010) review the main economic factors that determine tenure choice: the main benefits of homeownership are preferential tax treatment of owner-occupied housing services, access to collateralized credit and the insurance role of owner-occupied housing against rental price risk. Houses are however subject to substantial transaction costs that render them bad instruments to shield consumption against negative shocks, particularly when house prices are falling and owners mortgage debt is high.

Waggle and Johnson (2010), using a mean-variance utility function, consider the impact of homeownership and mortgage loan financing on the optimal asset allocation decisions of individuals and show that, in general, the higher the home-to-net worth ratio, the higher the optimal portfolio allocation to stock.

Huang (2010) calibrates an optimal dynamic asset allocation model with housing rental market, housing adjustment costs, mortgage collateral borrowing requirement, and studies the relationship between homeownership and household portfolio choices. His result shows that homeownership is hump-shaped in age, and that the liquidity of housing has significant impact on optimal portfolio choice of households. Homeowners tend to be more risk averse in financial investment than renters who are not given a homeownership choice. Down payment ratio crowds out the housing position at the early stage of life-cycle for saving more wealth to buy a house, and descends the stockholdings of young households because of illiquid home equity restrictions. With the increase of transaction cost on housing, a homeowner wants to invest more in stock to acquire more benefit to pay for the increasing potential adjustment cost, and tend to hold their house for a longer period of time (Huang, 2010).

As underlined by Miniaci and Pastorello (2010), the literature surveyed so far assumes that households are informed about financial markets and that they share the same expectations on future financial market performance. However, they notice there is growing evidence that such assumptions are not consistent with actual household behavior. They quote Lusardi and Mitchell (2007), van Rooij et al. (2007), and Guiso and Jappelli (2005, 2006), who raise serious concerns about the ability of households to gather and process the necessary information in order to consciously invest their money. Lack of information and financial illiteracy of individuals are likely to be one of the causes of observed heterogeneity in household expectations (Miniaci and Pastorello (2010)). Vissing and Jorgensen (2003) document that expectations on the stock market returns vary considerably with the demographic characteristics of respondents. We decide to ignore this evidence, though important for further consideration and research.

Fougère and Poulhes (2012) and Chetty and Szeidl (2012), using respectively French and US data on households, propose to estimate the effect of housing on portfolio choice by distinguishing between the effect of mortgage debt and the effect of home equity and by endogenizing these two variables. They find that an increase in mortgage debt (respectively, in home equity) reduces (respectively, raises) stockholding. However, while in the US the wealth effect of holding more home equity is cancelled out by the risk effect of owning a more expensive house, in France the wealth effect dominates the risk effect (Fougère and Poulhes, 2012).

Following the work of Flavin and Yamashita, Mayordomo, Rodriguez-Moreno and Peña (2012) study the investment decisions of Spanish households using the Spanish Survey of Household Finance (EFF). They propose a theoretical model in which households, given a fixed investment in housing, allocate their net wealth across bank time deposits and stocks. They find that households significantly under-invest in stocks and deposits while the optimal and actual mortgage investments agree and show that the households headed by highly financially sophisticated, older, retired, richer, and unconstrained persons are the ones investing more efficiently. They also find that mortgage and housing are closely interconnected such that the lower the proportion of housing, the lower the proportion of mortgage. Our research on Italian households will show very similar results, with the exception that homeowners over-invest in deposits.

Marekwica, Schaefer and Sebastian (2013) study the dynamic consumption-portfolio problem over the life cycle, with respect to tax-deferred investing for investors who acquire housing services by either renting or owning a home. The joint existence of these two investment vehicles creates potential for tax arbitrage, as investors can deduct mortgage interest payments from taxable income, while simultaneously earning interest in tax-deferred accounts tax-free. Their model predicts that investors with higher retirement savings choose higher loan-to-value ratios to exploit this tax arbitrage opportunity. However, many households could benefit from more effectively taking advantage of tax arbitrage. They also show that, as the investors purchase owner-occupied homes, they substitute risky equity with risky homes, as in Yao and Zhang (2005). That is, like the results in Cairns et al. (2006), investors decrease their equity exposure in order not to end up with a portfolio that is over-invested in risky assets. According to previous research, they also find that at a young age, the typically low wealth level, combined with the high probability of a forced move, makes investments in owner-occupied homes unattractive, due to the high transaction costs.

Beaubrun-Diant and Maury (2016) empirically analyze the simultaneous decisions of households to participate in the stock market and/or own their home, only focusing on the causal impact of home tenure on stockholding decisions, and show that households acquiring one asset (either home or stocks) acquire the other at an earlier stage in their life cycles, implying that some households become trapped in a no-stockholding, renting position. They find a significant effect of age of household heads (both linear and quadratic terms) on the marginal probability ratios of becoming an owner rather than a renter.

In this thesis, we follow the model developed by Pelizzon and Weber (2008): relaxing Flavin and Yamashita (2002) assumptions, they find that there are significant partial correlations between housing and financial returns, justifying the need for a hedge term in homeowners' portfolios, and thus showing that optimal portfolios should be conditionally mean-variance efficient, that is mean variance efficient when housing wealth is treated as pre-determined.

We assume that housing choice is predetermined and that all households live in the home that give them the optimal consumption benefits. Thus, portfolio choices are conditional on previous housing decision and we test efficiency of portfolios after they are constrained by the optimal level of housing chosen by every household.

2. A theoretical model

This section closely relies on Pelizzon and Weber (2008) paper, which builds on Flavin and Nakagawa's (2004) analysis of the dynamic optimization problem with housing, and use the same notation for comparison's sake. We keep both model structure and notation.

Flavin and Nakagawa generalize Grossman and Laroque (1990) model, by making utility a function of both a durable good, a house (H), and a non-durable good (C). The non-durable good is infinitely divisible and costlessly adjustable. House (the durable good) is instead subject to an adjustment cost proportional to its value and is therefore adjusted infrequently. This generalization is of great relevance for the analysis of portfolio choice because it allows us to consider explicitly the relation between the real rate of return on housing investment and the real rates of return on financial assets (Pelizzon & Weber, 2008).

The household maximizes expected lifetime utility:

(1) $$\mathrm{U} = E_0 \int_0^{\infty} e^{-\delta t}\, u(H_t\, C_t)\, dt$$

For analytical simplicity, the house is not subject to physical depreciation.

Then define,

(2) $$P_t = \textit{house price (per square meter)}$$

We assume that wealth can be held only in the form of financial assets and housing. Household can invest in a risk-less asset and in any of *n* risky financial assets, whose holdings can be adjusted with zero transaction cost.

Wealth is given by:

(3) $$W_t = P_t H_t + B_t + \underline{X_t}\underline{1}$$

where $\underline{X}_t$ = (1 x *n*) is the vector of amounts held of the risky assets and $\underline{l}$ = (*n* x 1) is a vector of ones. B_t is the amount held in the form of the risk-less asset. The *i*-th element of $\underline{X}_t$ in equation is given by $X_{it} \equiv N_{it}\, b_{it}$, where N_{it} is the number of shares held of asset *i*. Since asset prices, b_{it}, are taken as exogenous, the household determines $\underline{X}_{it}$ by its choice of N_{it}.

2. A theoretical model

All financial assets, including the risk-less asset, may be held in positive or negative amounts (this assumption of no constraints is held only in this theoretic framework).[1]

Dividends or interest payments are reinvested, so that all returns are received in the form of appreciation of the value of the asset. Let b_{it} = the value (per share) of the *i-th* risky asset, and assume that asset prices follow an n-dimensional Brownian motion process:

(4) $$db_{it} = b_{it}((\mu_i + r_f)dt + d\omega_{it})$$

where $\underline{\mu}$ = (n x 1) is the vector of expected excess returns on risky financial assets, $\underline{\mu} = (\mu_1, \mu_2, \ldots, \mu_n)$, r_f is the risk-less interest rate, and the vector $\underline{\omega}_{it} \equiv (\omega_{1t}, \omega_{2t}, \ldots, \omega_{nt})$ follows an n-dimensional Brownian motion with zero drift and with instantaneous covariance matrix Σ.

House prices also follow a Brownian motion:

(5) $$dP_{it} = P_{it}((\mu_H + r_f)dt + d\omega_{Ht})$$

where μ_H is expected excess return on house price and ω_{Ht} is a Brownian motion with zero drift and instantaneous variance σ_P^2.

We then define:

(6) $$d\underline{\omega}_t = \begin{bmatrix} d\omega_{1t} \\ \cdots \\ \cdots \\ d\omega_{nt} \\ d\omega_{Ht} \end{bmatrix}$$

which has instantaneous ((n + 1) x (n + 1)) covariance matrix Ω:

(7) $$\Omega = \begin{bmatrix} \Sigma & \Gamma_{b,P} \\ \Gamma_{b,P}^T & \sigma_P^2 \end{bmatrix}$$

where:

(8) $$\Gamma_{bP} = \begin{bmatrix} \sigma_{b1P} \\ \vdots \\ \sigma_{bnP} \end{bmatrix}$$

[1] We will not deal with labor income wealth or human capital wealth in this model for sake of simplicity. We remind to the work of Hu, 2005.

Here we depart from Flavin, as we remove the block diagonal covariance matrix assumption and allow for covariances between financial assets and housing to be different from zero.
Under these assumptions, the optimal holding of risky financial assets, is given by:

(9) $$X_0^T = \left[\frac{-\frac{\partial V}{\partial W}}{\frac{\partial^2 V}{\partial W^2}}\right] \Sigma^{-1} \underline{\mu} - P_0 H_0 \Sigma^{-1} \Gamma_{bP}$$

and the amount held of the risk-less asset is:

(10) $$B_0 = W_0 - P_0 H_0 - X_0 l$$

The expression in square brackets in equation (9)[2] is the reciprocal of the coefficient of absolute risk aversion:

(11) $$\text{ARA} \equiv -\frac{\partial^2 V(W_t, H_t)}{\partial W_t^2} \Big/ \frac{\partial V(W_t, H_t)}{\partial W_t} > 0$$

As Pelizzon and Weber point out, risk aversion affects the first term on the RHS of equation (9) but not the second term that bears the interpretation of a hedge portfolio. In Flavin's analysis this second term disappears, because she assumes $\Gamma_{bp} = 0$, and therefore she can prove that the traditional CAPM holds.
We now consider a static mean-variance analysis framework and consider the existing housing stock as an additional constraint to the optimization problem. Households can invest in a risk-less asset, n unconstrained and one constrained risky assets. Given $\underline{\mu}$ the expected excess return of unconstrained risky assets and $\underline{m} = \begin{pmatrix} \mu \\ \mu_H \end{pmatrix}$, the first two moments of asset returns are $\underline{m} + r_f$ and Ω. Portfolio allocation in risky assets is given by:

(12) $$Z = \begin{pmatrix} \underline{x}_o \\ h_0 \end{pmatrix}$$

where $\underline{x}_0 \equiv \frac{X_0}{W_0}$ and $h_0 \equiv \frac{H_0 P_0}{W_0}$ and $(1\text{-}Z)^T \underline{1}$ is invested in the risk-less asset ($\underline{1}$ is an n+1 vector of ones).

[2] Which is derived in the Appendix.

2. A theoretical model

We then assume that this investor is constrained in his h_0 (that is h_0 is given and thus predetermined, and equal to h_0), but otherwise behaves according to the mean-variance model. The investor optimization problem becomes:

$$(13) \qquad \begin{cases} \min_{Z} Z^T \Omega Z \\ s.t. \begin{cases} Z^T \underline{m} + r_f = \underline{m}^* \\ h_0 = \overline{h_0} \end{cases} \end{cases}$$

where m* is a given level of expected return. By defining the lagrangian:

$$(14) \qquad \Lambda = (\underline{x}_0 \Sigma \underline{x}_0^T + h_0^2 \sigma_P^2 + 2\underline{x}_0 h_0 \Gamma_{bP}) - 2\gamma(\underline{x}_0 \underline{\mu} + h_0 \mu_H + r_f - m^*)$$

where γ is the Lagrange multiplier of the constraint on the expected return and has the standard relative risk aversion interpretation defined in Samuelson, 1970.

The first order conditions are:

$$(15) \qquad \frac{\partial \Lambda}{\partial \underline{x}_0} = (2\Sigma \underline{x}_0^T + 2h_0 \Gamma_{bP}) - 2\gamma[\underline{\mu}] = 0$$

$$(16) \qquad \frac{\partial \Lambda}{\partial \gamma} = \underline{x}_0 \underline{\mu} + h_0 \mu_H + r_f - m^* = 0$$

With solution:

$$(17) \qquad \underline{x}_0 = \gamma \Sigma^{-1} \underline{\mu} - h_0 \Sigma^{-1} \Gamma_{bP}$$

Investors have thus to be efficient with respect to the risky financial assets and choose the efficient Markowitz portfolio according to their risk aversion (Markowitz (1992)), but they also use the risky financial assets to hedge their expositions on housing (the constrained asset). If $\Gamma_{bP} = 0$, the problem is the same as in Flavin and Yamashita (2002) and portfolio choice can be separated between financial and real assets.

2.1. Econometric Issues

The notion of efficiency of household portfolios depends on the assumption that we make on the nature of housing investment. If housing investment is costless, then the efficient frontier should be computed using all financial assets returns, as well as the return on housing. If transaction costs affect housing investment, then the analysis differs according to the correlation between housing and financial returns (Pelizzon & Weber, 2008). If this correlation is zero, household portfolios will be mean-variance efficient in the usual sense (i.e., with respect to the standard financial assets frontier) as in Flavin and Yamashita (2002). If this correlation is instead non-zero, household portfolios will be mean-variance efficient once we condition on the value of the housing stock as shown in equation (17). In this section, we show how we can test for the efficiency of the observed household portfolios in all cases discussed above, following the work by Gouriéroux & Jouneau, 1999. To do this, we use time-series data on asset returns for 1990-2015 to estimate the mean-variance frontier, assuming rational expectations and normal return distributions. We use exponentially weighted moving average (EWMA) means and covariances to estimate expected excess returns and risk (i.e., the first two unconditional moments). The weights are a declining function of the time distance from the end of the sample period (λ=0.97).

Gibbons, Ross, and Shanken (1989) have also proposed a test of the significance of the difference between the actual portfolio held by an investor and a corresponding efficient portfolio. This test is based on the difference between the slopes of arrays from the origin through the two portfolios in the expected return standard deviation space: if the actual portfolio is an efficient portfolio, the two slopes will be the same; if the actual portfolio is inefficient, the slope of the efficient portfolio will be significantly greater.

Gourièroux and Jouneau (1999) derive efficiency tests for the conditional or constrained case, thus for the case where a subset of asset holdings is potentially constrained (housing in our case). They define the Sharpe ratio of the unconstrained risky financial assets portfolio as:

(18) $$S = \underline{\mu}^T \Sigma^{-1} \underline{\mu}$$

The Sharpe ratio for the observed (constrained) portfolio made of the first *n* (financial) assets is instead defined as:

(19) $$S_1(Z) = \frac{\left[\underline{\mu}^T \nu_1\right]^2}{\left[\nu_1^T \Sigma \nu_1\right]}$$

where $\nu_1^T = \underline{x}_0^T + h_0 \Sigma^{-1} \Gamma_{bP}$, that is the actual risky portfolio after eliminating the hedge term.

Gourièroux and Jouneau (1999) show that, when all asset returns are normally distributed, the Wald statistic:

(20) $$\xi_1 = T \frac{\hat{S}_1 - \hat{S}_1(Z)}{1 + \hat{S}_1(Z) \frac{Z^T \Omega Z}{\nu_1^T \Sigma \nu_1}}$$

where T is the number of observations for excess returns, is distributed as a $\chi^2(n-1)$, under the null hypothesis that the risky financial assets portfolio (after eliminating the hedge term) lies on the financial efficient frontier. This statistic is defined as a function of sample estimates of the first two moments of the rates of return distribution and takes observed portfolio shares as given.

Gourièroux and Jouneau also show that a test for the efficiency of the whole portfolio can be derived as a special case by setting $\nu_1 = Z$. The test statistic becomes:

(21) $$\xi_e = T\frac{\hat{S}-\hat{S}(Z)}{1+\hat{S}(Z)}$$

where:

$$\hat{S} = \underline{m}^T \Omega^{-1} \underline{m} \qquad \text{and} \qquad \hat{S}(Z) = \frac{[\underline{m}^T Z]^2}{Z^T \Omega Z}$$

ξ_e is distributed as a $\chi^2(n)$ under the null hypothesis that the mean and standard deviation of the observed portfolio lie on the efficient frontier. In this special case, the test is asymptotically equivalent to the test derived by Gibbons, Ross, and Shanken (1989).

The standard test for portfolio efficiency is based on (the square of) the Sharpe ratio. The Sharpe ratio is in fact the same along the whole efficient frontier (except for the intercept). This test breaks down when one asset is taken as given because the efficient frontier in the mean variance space corresponding to all assets is no longer a line, but rather a curve. However, equation (17) implies that we can reconduct to the standard case when the analysis is conditional on a particular asset, once the hedge term component is subtracted from the observed portfolio. That is, a Sharpe ratio can be used to test for efficiency in the mean-variance space corresponding to the "unconstrained" assets after allowance has been made for the presence of the same hedge term in all efficient portfolios (Pelizzon & Weber, 2008).

We will compute efficiency test statistics (either ξ_e or ξ_1) for each household in our sample. The standard test (ξ_e) is computed twice: once for the financial portfolio (as in standard practice), and once for the whole portfolio (inclusive of housing). In this latter case, we also compute the constrained test (ξ_1).

3. Data

Data on Italian households are taken from the Survey of Household Income and Wealth (SHIW), conducted by the Bank of Italy. The SHIW has been providing data about the financial conditions of Italian households since 1965. The elementary data, which have been reorganized in the SHIW historical database are only available from 1977 onwards. The data are annual up to 1987 (excluding 1985) and two-yearly afterwards (but with a three-year report covering 1995-98). The set of information collected in the survey has been gradually expanded and refined, and the sample size has progressively increased. In 1987 the SHIW started collecting data on household wealth more systemically, supplementing the information on real estate, which has been collected since 1977, with data on the main financial assets and liabilities held by households. In 1995 the way the data on financial assets were collected was firmly established, so it is possible to compare the data over time. Despite its shortcomings and discontinuities in the time series, the elementary data make it possible to quite accurately describe the evolution of Italian households' financial conditions and behavior.

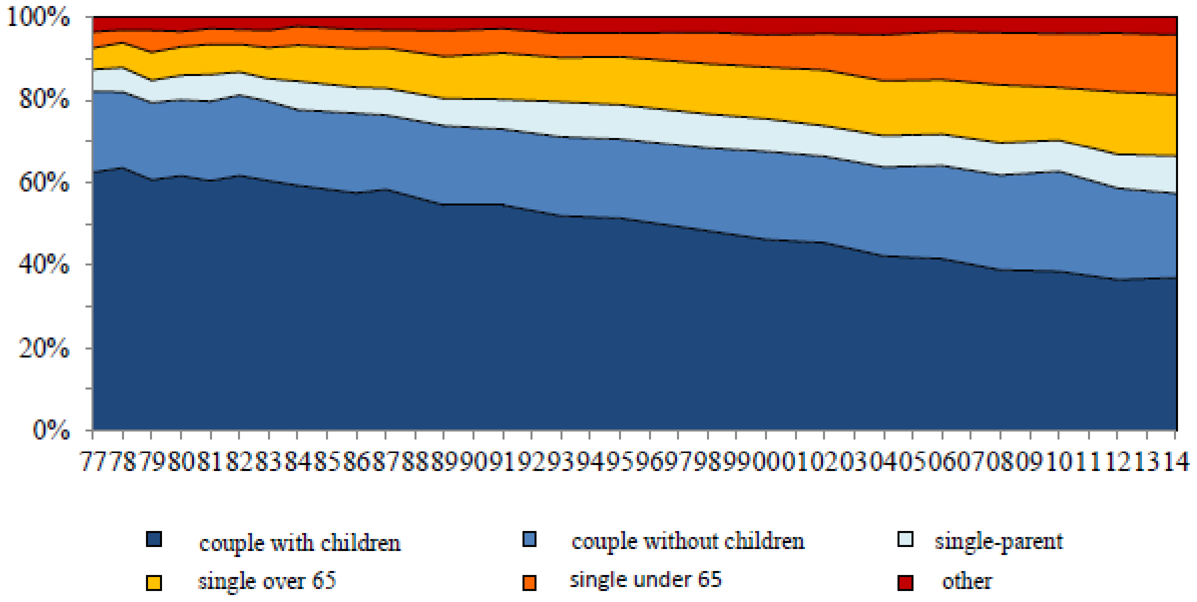
The sample for the survey is drawn in two stages, with municipalities as primary sampling unit and households as secondary. Before municipalities are selected, they are stratified by region and population. Within each stratum, the municipalities in which interviews are selected to include all those with a population of more than 40,000 and those with panel households (self-representing municipalities), while the smaller towns are selected on the basis of probability proportional to size. This method produces a self-weighted two-stage sample when the sample size is constant among strata. In fact, by fixing the number of households to be interviewed in each municipality, the higher probability of a large municipality being included in stage one is exactly offset by the lower probability of units in that municipality being drawn in stage two. The individual households to be interviewed are then selected randomly from the civic register.

An issue that comes from this kind of research, is that non-response is not random and is more frequent among wealthy households. Non-participation is a problem for statistical surveys because it may produce samples in which the less co-operative sections of the population become underrepresented, causing selectivity bias. The quality of estimates is also affected by the reluctance of households to report their sources of income or the real and financial assets they hold. The set of weights provided in the SHIW account for the non-response process. Weights are corrected in order to consider attrition in the panel and the autocorrelation in income and wealth observed for panel households. Finally, weights are adjusted to replicate the same characteristics as the population in terms of sex, age, municipality size and geographical area.

We use the survey conducted in 2014, which collects data on 8,156 households and regards household income and its distribution, wealth, financial assets and means of payment, housing and household indebtedness in Italy. The households were picked from the registry offices of 371 municipalities.

Italian society has changed considerably since the first survey conducted: the population has aged, the average level of education has increased, and women's participation in the job market has risen. The survey shows how these developments affected the structure and financial conditions of households. According to Istat, the share of persons over 64 doubled, going from 11% to 22% of total population, while the share of young people under age 14 dropped from 22% to 14%. This realignment of the population is due to the combined effect of gains in longevity and a drop in the birth rate. The share of persons holding an upper secondary school certificate, or a university degree, increased from under 20% to about 35%. In the age group 20 to 34, the share rose from 35% to two thirds. The female employment rate increased from around one third to about half of the female population aged 15 to 64.

Figure 3.1: Types of households (*per cent*) [3]



[3] Source: based on data from the Historical Database for the Survey of Household Income and Wealth, version 9.0, available athere: http://www.bancaditalia.it/statistiche/tematiche/indagini-famiglieimprese/bilanci-famiglie/index.html (Bank of Italy, 2015).

The drop in the average size of households was accompanied by a significant change in the types of households (Figure 3.1): the share of couples with children halved (from 63% to 34%), while the share of one-person households tripled (from 9% to 30%) and that of single-parent households doubled (from 5% to 9%). In households where the head is in the central age groups, the number of earners increased from one every three family members to one every two. Table 3.1 shows the percentages of households in SHIW 2014 with given demographic and social characteristics.

Table 3.1: Households, earners and individuals by demographic characteristics [3]

Characteristics	**Households** (%)	**Earners** (%)	**Individuals** (%)
***Gender*:**			
Male	64.8	53.9	48.6
Female	35.2	46.1	51.4
***Age*:**			
<34	9.3	14.4	34.6
35-44	17.9	17.7	14.2
45-54	21.4	20.4	16.2
55-64	17.2	16.0	13.0
65<	34.3	31.5	22.0
***Work sector*:**			
Agriculture	2.4	2.6	1.7
Industry	10.9	10.1	6.5
Public	12.6	12.9	8.3
Other sector	30.4	31.0	19.8
Unemployed	43.8	43.4	63.7
***Work category*:**			
Employee:			
Blue-collar worker	23.4	23.6	15.1
Office worker	17.6	18.7	12.0
Manager. executive	4.7	3.6	2.3
Total	45.8	45.9	29.4
Self-employed:			
Business owner. member of profession	4.7	4.6	2.9
Other self-employed	5.7	6.1	3.9
Total	10.4	10.7	6.8

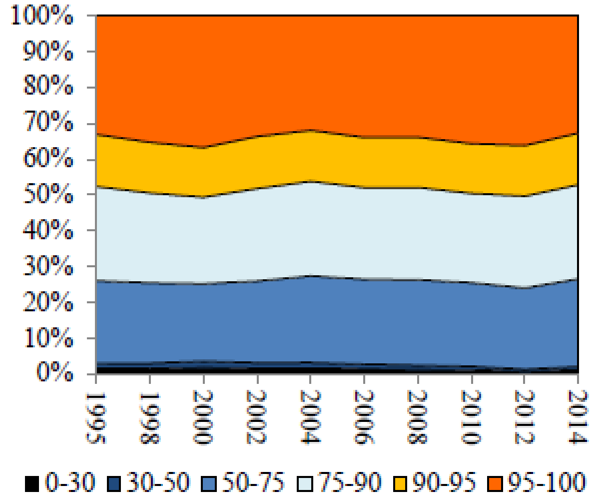
Not employed:			
Retired	38.2	36.3	23.3
Other	5.6	7.0	40.5
Total	43.8	43.4	63.7
Educational qualification:			
None	3.7	3.8	11.7
Primary school certificate	18.9	18.1	17.3
Lower secondary school certificate	37.1	36.3	35.5
Upper secondary school diploma	26.5	27.5	24.6
University degree	13.8	14.2	10.8
Town size:			
Up to 20.000 inhabitants	46.2	48.2	47.0
20.000 – 40.000	14.2	14.2	14.5
40.000 – 500.000	27.0	26.4	27.0
More than 500.000	12.5	11.2	11.4
Geographical area:			
North	47.4	49.2	46.2
Centre	20.2	20.2	19.2
South and Islands	32.4	30.7	34.6

3.1. Household wealth [4]

Between 1995 and 2014, mean net household wealth increased by approximately 8 percentage points in real terms. The median value increased by twice as much. The share of total net wealth owned by the richest 5% of households remained stable around 30%, a share similar to that held by the poorest three quarters of households (Figure 3.2). Households between the 90^{th} and the 95^{th} quantile of richness own about 15% of total net wealth, while households with richness over the mean (from 50^{th} to 75^{th} quantile) own almost 25% of net wealth. Households between the 30^{th} and the 50^{th} quantile own approximately 20% of wealth.

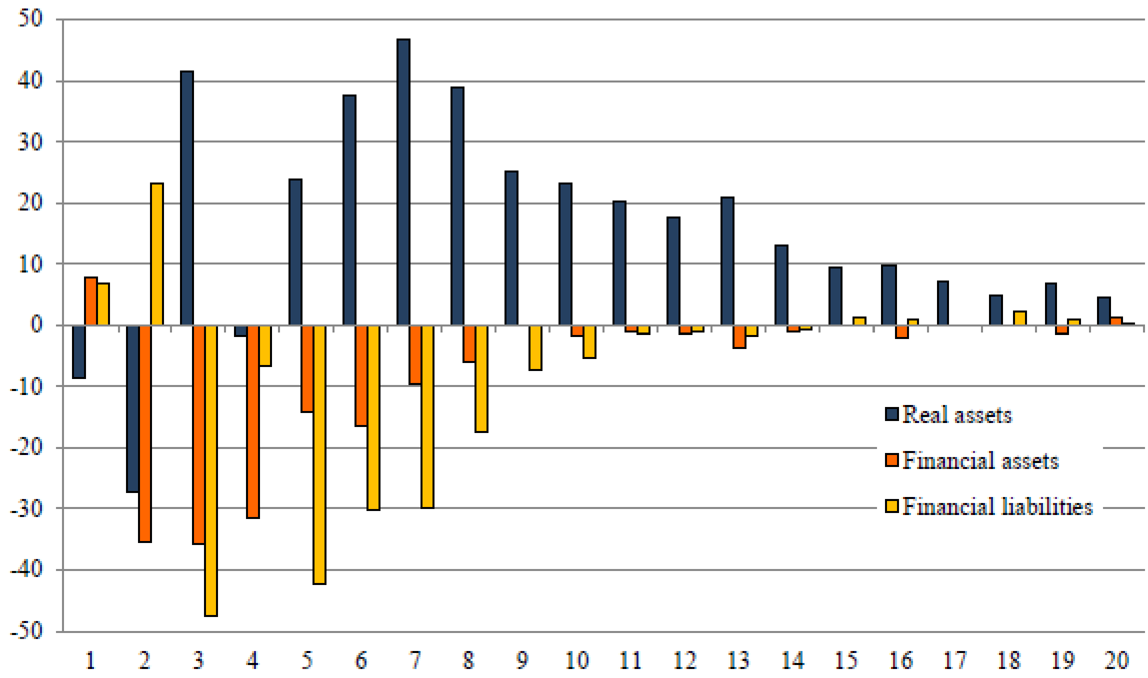
[4] Source: Bank of Italy, 2015. Household income and wealth in 2014. *Supplements to the statistical bulletin*.

Figure 3.2: Share of net weatlh held by household (*per cent*) [3]



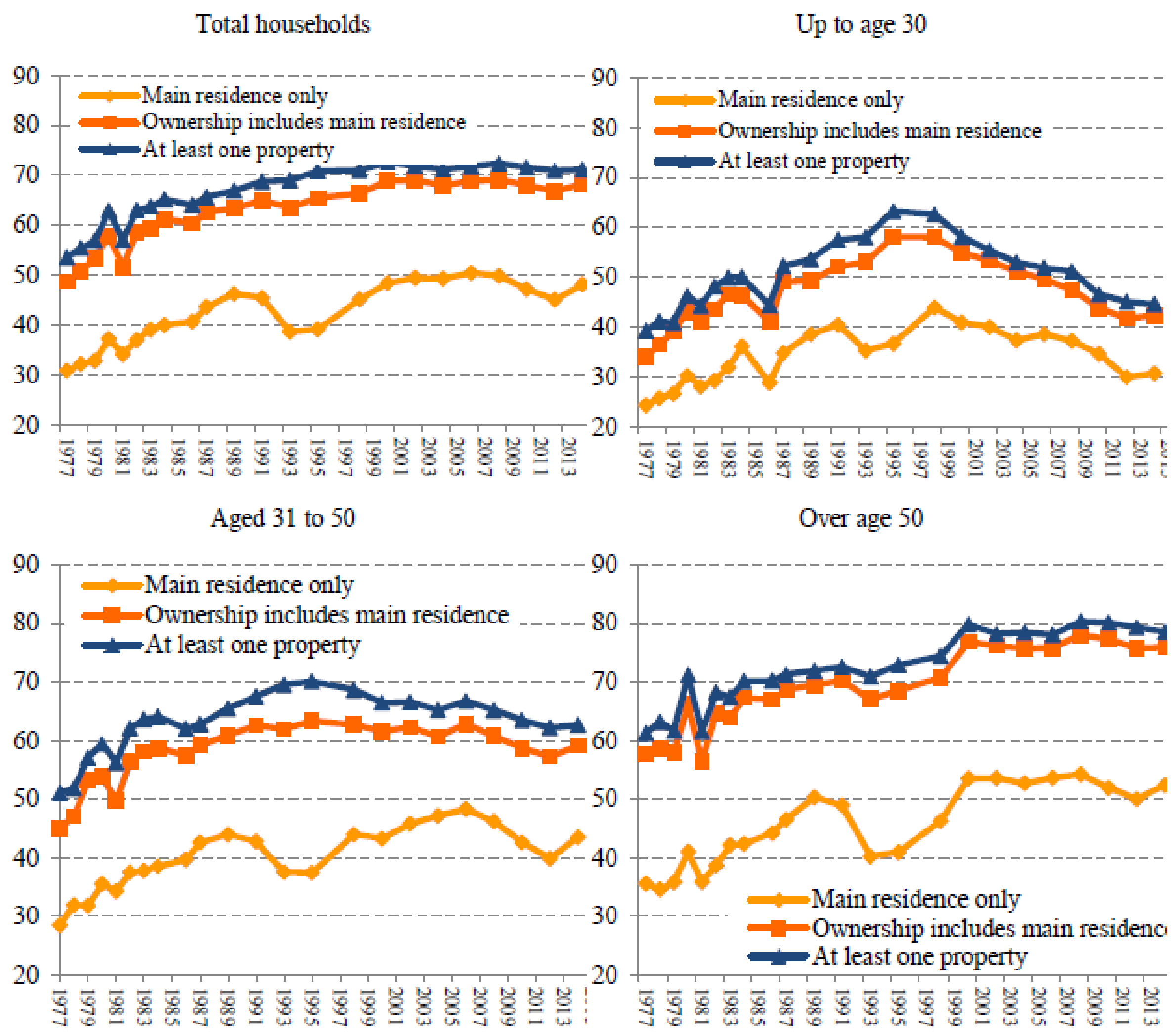
The overall performance of household wealth growth was driven by different components along its distribution (Figure 3.3). The growth in real estate value mainly sustained the wealth of households below the median value, except for the lowest quantile. For these households, indebtedness expanded while the value of financial assets contracted. Growth in real assets value for wealthier households was still positive, but of a lesser amount. This explained why net wealth of richer households remained quite stable during the observation period.

Figure 3.3: Contributions of net wealth to growth by household quantile (1995- 2014) [3]



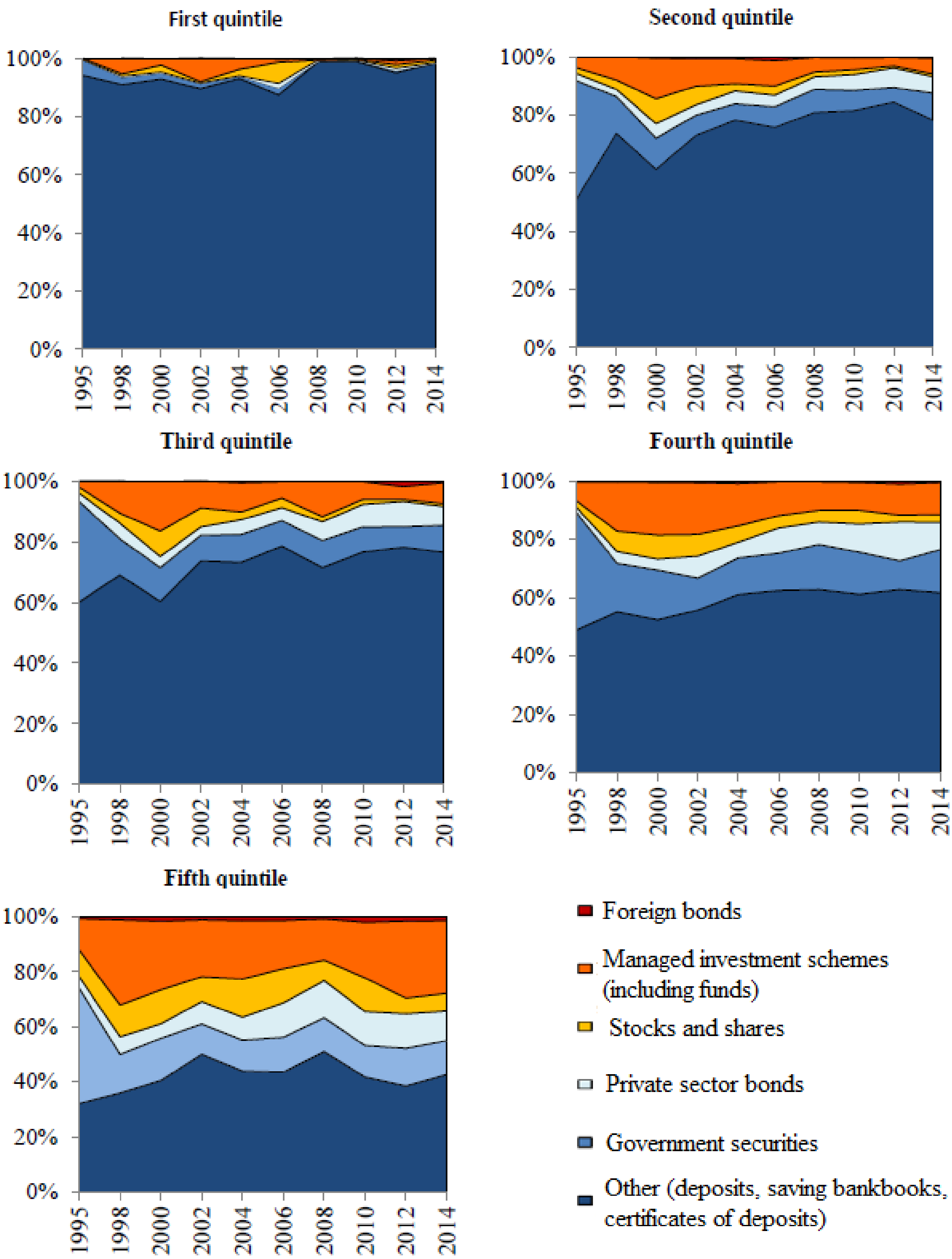
For most Italian households, wealth is mainly composed by real estate. The share of households owning residential and non-residential buildings rose from 55% in 1977 to over 70% in the early 2000s, and has stabilized at those levels. The overall expansion in real estate ownership also reflected the growing weight of the older segments of the population. The share of households owning real estate grew steadily only for those whose head is over age 50; for younger households, where the head is under age 30, the share had grown by 25 percentage points (from 40% to 65%) between the late 1970s and the late 1990s, but returned to the initial levels in the following fifteen years (Figure 3.4).

Figure 3.4: Ownership of real estate by age group of household head (*per cent*) [3]



As shown in Figure 3.5, the importance of financial assets in the overall wealth of better-off households, which was already limited in the mid-1990s, decreased even further in the following twenty years. Conversely, for the poorer households in the first quintile of wealth, financial assets, mainly bank deposits, continue to represent almost all their wealth. Poorer households increased the share of risky assets (stocks, private-sector bonds and funds) from around 6% in 1995 to over 12% in 2006, then went back to exclusively liquid instruments in the following years.

Figure 3.5: Distribution of financial assets by quintile of net wealth [3]

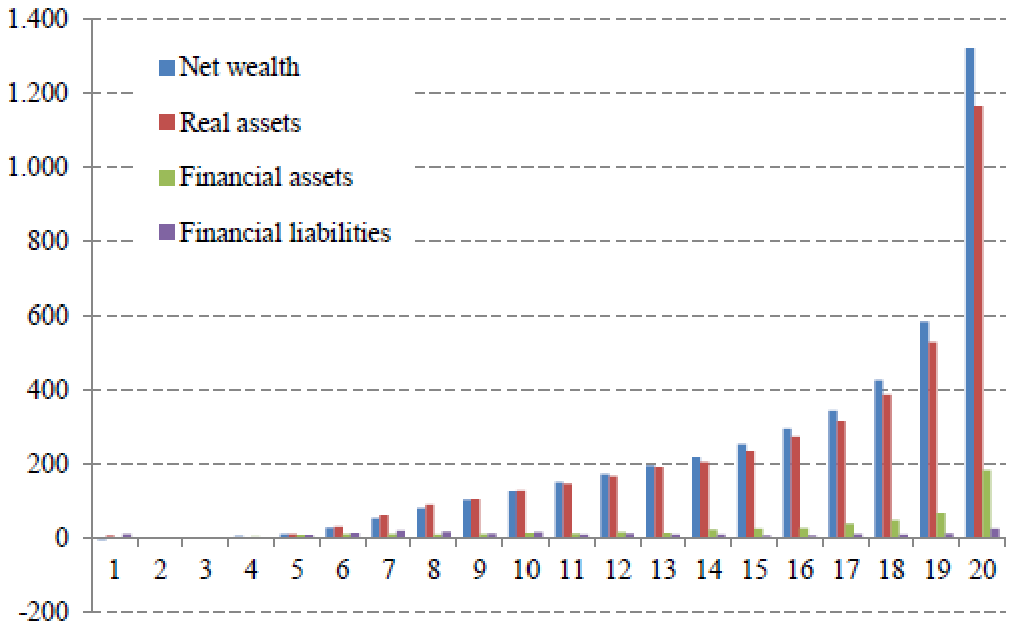


Wealthier households gradually redirected their investment choices from government bonds (which dropped from 40% to 15% between 1995 and 2014) to private-sector bonds, managed investment schemes and, for the richer segment, stocks. The weight of investment funds and asset management schemes peaked in 2000 but gradually declined in the following years, despite the share of wealth allocated to these instruments at the end of 2014 were significantly higher than in 1995. For the wealthiest households, managed assets represent the largest share of financial wealth net of that allocated to risk-free assets (about 22% on average in the last 20 years). For these households, this type of investment increased sharply, especially in the period 1995-1998 (20% points of financial wealth).

In 2014 Italian households had a mean net wealth of €218,000, calculated as the sum of real and financial assets net of financial liabilities. The median value, which separates the bottom 50% of poorer households from the top 50% of wealthier ones, was markedly lower than the average, standing at €138,000. Figure 3.6 shows mean net wealth and its components.

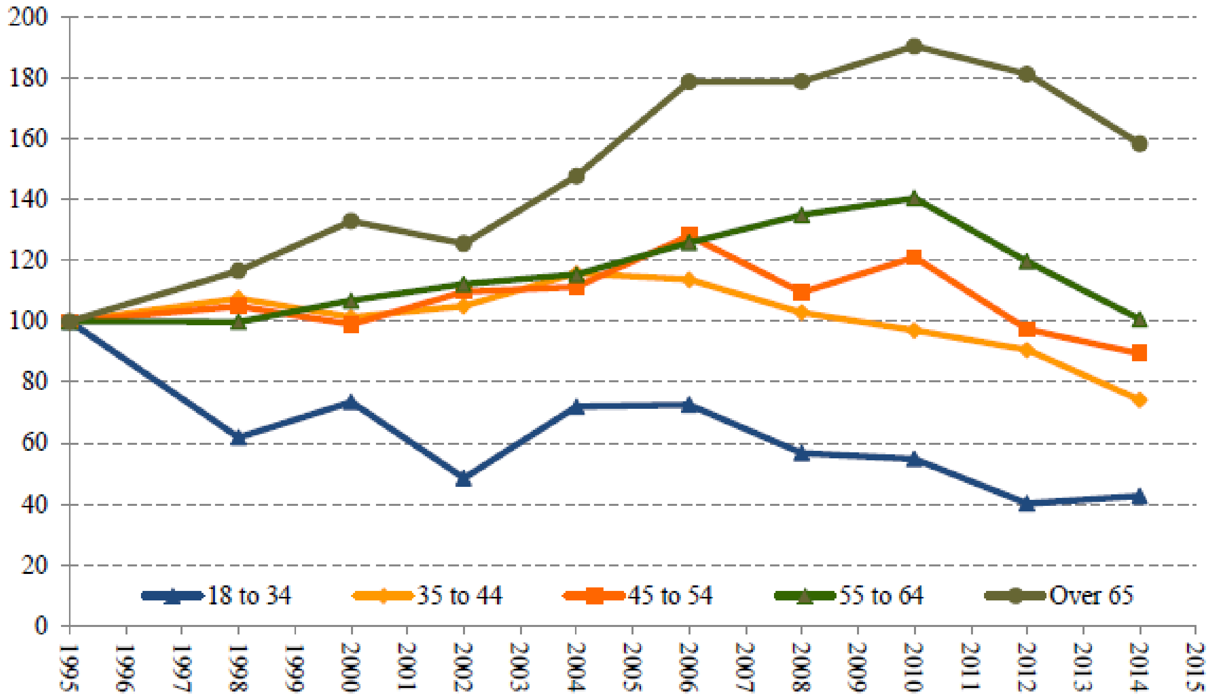
The wealth held by the poorest 30% of Italian households represented less than 1% of total wealth, while the share of total net wealth owned by the richest 5% of households remained stable around 30%, a share similar to that held by the poorest three quarters of households. Between 2012 and 2014 households' average net wealth declined by 11% in real terms, owing to a significant drop affecting the wealthiest households (-15% for the top quintile), which was largely due to a decrease in real estate prices. For households whose wealth is below the median, the average net wealth increased by 4%, and this was almost entirely due to a decline in financial liabilities reflecting both the lower average exposure of borrowers and the lower number of borrowers.

Figure 3.6: Mean net wealth and components (*quantiles of net wealth; thousands of euros*) [3]



In the last twenty years the wealth gap between younger and older households, which partly reflects the natural accumulation of savings during people's lives, has gradually widened. In real terms, the mean wealth of households whose head is aged 18 to 34 is less than half that recorded in 1995, while the mean wealth of households whose head is aged 65 or over increased by about 60%. The ratio of wealth held by households whose head is over 65 to the wealth held by households whose head is aged 18 to 34 rose from less than 1 to more than 3 (Figure 3.7).

Figure 3.7: Mean net wealth by age of head of household (*constant prices, 1995=100*) [3]



3.2. Housing [4]

The composition of net household wealth is largely determined by real assets, thus real estate, firms and valuables. The value of real estate represents more than 80% of household wealth, and accounts for the largest share in all quantiles of wealth, except in the lowest ones.

In 2014, 70% of households owned at least one residential property. The share of households who owned their main home was only slightly lower, at 67.7%. 20.7% of households were tenants, and the remaining 11.6% occupied their home free of charge, in usufruct or under a redemption agreement.

Despite real property being widespread, property value is far more concentrated, with 59% of it owned by the wealthiest 20% of households. As shown by Table 3.2, ownership of the main home is also not equally distributed among population groups: it concerns three quarters of households whose head is either 55 or older, holds a degree or is self-employed, and 70% of those are resident in smaller municipalities or in the Centre. The share drops to 21.9% for foreign-born households. In the first quintile of household income, only a third of households own their home, compared with 90% in the top two quintiles.

Table 3.2: Main residence by tenure (*per cent of households*)

Characteristics	Owned	Rented or sublet	Redemption agreement	Usufruct. free of charge. etc.
Gender:				
Male	70.4	19.7	0.5	9.4
Female	62.9	22.5	0.5	14.1
Age:				
<34	43.9	36.4	0.2	19.6
35-44	56.5	29.0	0.6	13.9
45-54	68.7	19.4	0.6	11.4
55-64	76.3	16.4	0.4	6.9
65<	75.2	15.0	0.6	9.2
Work sector:				
Agriculture	69.1	22.7	0.0	8.2
Industry	66.0	26.7	0.1	7.2
Public	73.4	14.3	0.4	11.9
Other sector	60.5	26.2	0.5	12.8
Unemployed	71.5	17.1	0.7	10.8
Work category:				
Employee:				
Blue-collar worker	50.8	36.3	0.5	12.3
Office worker	74.2	15.4	0.2	10.2
Manager. executive	84.2	6.8	0.3	8.7
Total	63.3	25.2	0.4	11.1
Self-employed:				
Business owner. member of profession	74.6	15.5	0.0	9.9
Other self-employed	69.2	16.0	0.8	14.0
Total	71.6	15.8	0.4	12.2

Not employed:				
Retired	75.9	14.9	0.8	8.4
Other	41.4	31.7	0.2	26.7
Total	71.5	17.1	0.7	10.8
Educational qualification:				
None	57.7	23.2	2.4	16.7
Primary school certificate	69.1	20.2	0.3	10.4
Lower secondary school	61.6	27.0	0.6	10.8
Upper secondary school	72.6	16.6	0.3	10.5
University degree	75.8	11.4	0.4	12.5
Income quintiles:				
1st quintile	35.5	47.3	0.5	16.7
2nd quintile	54.2	30.0	0.8	15.0
3rd quintile	73.7	14.4	0.7	11.2
4th quintile	83.7	8.9	0.5	7.0
5th quintile	91.7	2.7	0.1	5.6
***Town size**:*				
Up to 20.000 inhabitants	71.1	16.0	0.3	12.6
20.000 – 40.000	70.0	21.2	0.4	8.4
40.000 – 500.000	63.3	25.7	0.7	10.4
More than 500.000	62.3	26.5	1.2	9.9
***Geographical area**:*				
North	66.8	23.1	0.3	9.8
Centre	71.3	17.2	0.6	10.9
South and Islands	66.9	19.3	0.8	13.1
Dwelling surface:				
up to 60 sq.m.	40.4	43.9	0.3	15.3
60 - 80 sq.m.	54.4	31.5	0.8	13.3
80 - 100 sq.m.	72.2	15.6	0.6	11.6
100 - 120 sq.m.	84.8	7.6	0.2	7.3
more than 120 sq.m.	91.0	2.3	0.4	6.3

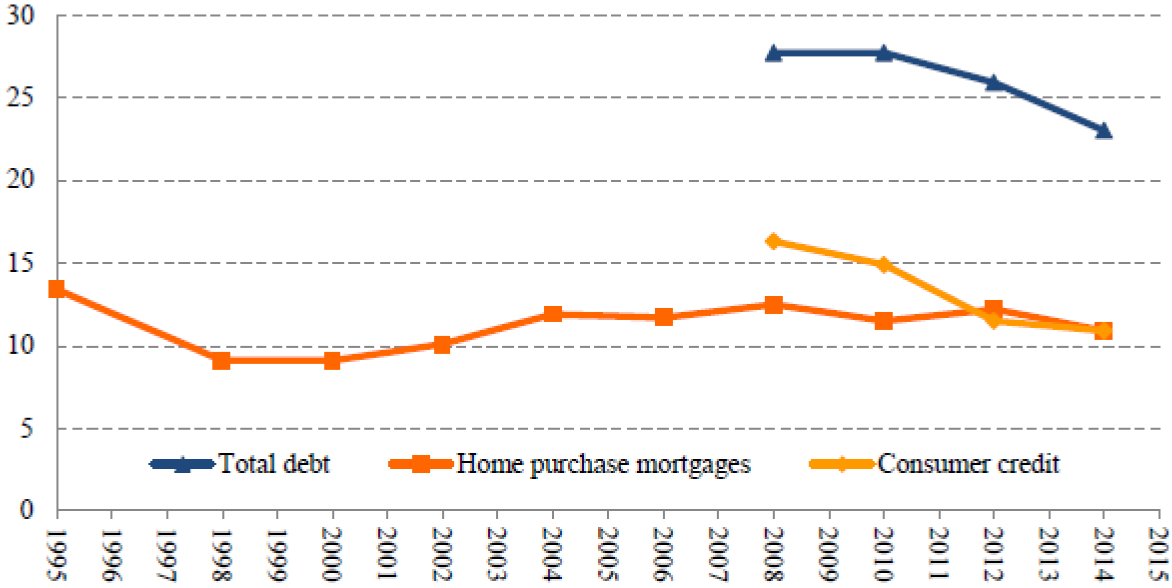
Main homes occupied by their owners have a mean value of €220,000, while homes that are rented have a lower average mean value (€122,000), mainly due to the smaller surface on average. The value of the housing service stemming from ownership of the main home, i.e.

imputed rent, is on average almost 20% of the owner's income, and implies a rate of return of about 3%.

3.3. Household indebtedness [4]

In 2014 the share of indebted households decreased further, continuing a trend that began in 2010 (Figure 3.8). At the end of 2014 about 23% of Italian households were indebted for an average amount slightly over €44,000[5]; in 2012 the proportion of indebted households was 25.9% and the average amount was €51,500, while in 2010 they were 27.7% and with average debt about €44,500.

Figure 3.8: Household indebtedness (*per cent*) [3]



The drop in the share of indebted households reflects the lower incidence of home purchase or renovation loans (from 12.2% in 2012 to 10.9% in 2014) as well as that of consumer credit[6]. The share of households indebted because of the latter was also 10.9% in 2014, and had already fallen considerably between 2008 (the first year it was included in the survey) and 2012, from 16.3% to 11.5%. Revolving credit cards and current account overdrafts, which represent a flexible form of consumer credit, were used in 2014 by 1.2% and 4.2% of

[5] Households are defined as indebted when they have at least one of the following financial liabilities: a home purchase or renovation loan, a loan from a financial intermediary for the purchase of durable or non-durable goods, a loan from friends or relatives, trade debts or bank loans in connection with a sole proprietorship or family business, a current account overdraft, or a negative credit card balance (Bank of Italy, 2015).

[6] The definition of consumer credit used in the SHIW survey includes loans for the purchase of means of transport, other durable goods (e.g. furniture and household appliances) and non-durable goods. It also includes current account overdrafts and debt on revolving credit cards as at the end of the year.

households respectively. The use of these two debt instruments remained basically stable in six years.

Unlike financial and real assets, total liabilities are distributed less unevenly between wealth groups: the richest 20% of households own 28% of total debt, while the poorest 20% only own 7%, corresponding to an average amount of less than €4,000 (the overall average for indebted households is €18,000).

The ratio of total debt to annual monetary income[7], which is an indicator of sustainability showing how many years' income is needed to pay off the debt, decreased from 80% in the 2012 survey to 73% in 2014 one for the median indebted household, corresponding to slightly less than nine months' income. In 2014, about 26% of households with a monetary income above the median were indebted, and their yearly instalment payments were on average equal to €6,000, or 16% of income. Conversely, less than 10% of households with a monetary income below the median were indebted, but their yearly instalment payments, on average equal to €3,800, represented a 30% share of their income.

Among households in the bottom quartile of income, only 5.9% were indebted but the mean annual instalment payment was equal to 40% of their monetary income (Table 3.3). Financially vulnerable households, defined as those with a monetary income below the median and debt service payments equal to more than 30% of their income, accounted for 11.4% of indebted households and 2% of total households. Vulnerability was concentrated among lower-income households. In 2014 about 56.8% of indebted households in the first quartile of income were financially vulnerable, but the same was true for only one third of those in the second quartile.

Table 3.3: Household financial vulnerability per income quartile (*per cent; euros*) [4]

Monetary income	Indebted households	Mean annual inst.	Median value of inst.	Median inst. to income	Mean annual inst.	Mean inst./mean income	Vulnerable households	Vulnerable households on tot. pop.
		Indebted households only						
1st q.	5.9	205	2.900	34.2	3,492	40.7	56.8	3.3
2nd q	13.5	567	3.800	23.3	4,186	24.6	34.3	4.6
3rd q.	22.4	1,132	4.800	18.4	5,056	19.6	0.0	0.0
4th q.	28.3	1,995	6.000	13.3	7,055	14.3	0.0	0.0
Total	**17.5**	**974**	**4.800**	**17.1**	**5,564**	**17.3**	**11.4**	**2.0**

[7] Monetary income is defined as the household income net of imputed rents but including financial costs.

3.4. Household portfolio characteristics

Table 3.4 shows the proportion of households reporting positive holdings of each asset recorded in SHIW 2014, as well as the way each asset is classified for the purpose of our efficiency analysis.

Table 3.4: Participation Decision. Individual Financial and Real Assets

Asset	Participation	Classification
Deposits	82.07	*Risk-free*
Certificates of deposits	2.06	*Short-term*
Repos	1.25	*Short-term*
Postal saving certificates	6.06	*Long-term*
BOT	5.54	*Short-term*
CCT	1.63	*Short-term*
BTP	2.15	*Long-term*
BTPI	0.44	*Long-term*
CTZ	0.18	*Short-term*
Other government bonds	0.52	*Long-term*
Corporate bonds	2.07	*Corporate bonds*
Financial corporate bonds	6.47	*Corporate bonds*
Investment funds (liquidity)	2.17	*Short-term*
Investment funds (bonds)	1.86	*Long-term*
Investment funds (balanced)	2.37	*Long-term (1/2) Stocks (1/2)*
Investment funds (stocks)	1.02	*Stocks*
Investment funds (foreign currency)	0.36	*Long-term*
Listed shares	4.07	*Stocks*
Unlisted shares	0.49	*Stocks*
Limited Liability Company	0.21	*Stocks*
Shares of Partnerships	0.10	*Stocks*
Managed accounts	1.01	*Long-term (1/3) Corporate bonds (1/3) Stocks (1/3)*
Foreign certificates	0.33	*Long-term (1/2) Corporate bonds (1/2)*
Foreign bonds	0.25	*Corporate bonds*
Foreign shares	0.26	*Stocks*

Other foreign financial assets	0.34	*Long-term (1/2) Corporate bonds (1/2)*
Loans to Coop.	1.53	*Stocks*
Other financial assets	0.21	*Long-term (1/3) Corporate bonds (1/3) Stocks (1/3)*
House	71.43	*House*
Mortgage	9.05	*Long-term (Negative position)*
Debt	3.5	*Long-term (Negative position)*

Table 3.5 shows the proportion of households holding various combinations of assets. Mortgages and debt are treated as negative positions in risky assets.

Table 3.5: Asset combinations

Classification	**%**
Risk-free	23.55
Risk-free + House	40.65
Risk-free + Risky	3.40
Risk-free + Risky + House	32.40

Italian households traditionally hold poorly diversified financial portfolios. Since 1990s, stock exchange has grown considerably, and mutual funds have become a commonly held financial instrument. At the end of 2014, a quarter of Italian households held financial assets other than a bank or post office deposit, marking a slight increase compared with the end of 2012. In about three quarters of cases, these consisted exclusively of direct investments, mainly bonds, whereas one tenth of households only had managed portfolios (investment funds and asset management portfolios).[8]

We analyze household portfolios structure taking into consideration different characteristics of Italian population, such as age profile, working sector and category, education and income quantiles.

[8] The definition of a financial asset used does not include pension funds and supplementary pension schemes. The SHIW survey found that in 2014 around 13.2% of households were paying in to a pension fund or to supplementary pension schemes in addition to the state pension scheme; about a third did not know how much these forms of investment had capitalized. Supplementary pension funds are more popular in the North (17%) and in the Centre (12%) than in the South and Islands (9%), among those with a higher educational qualification (20.2% in the case of households where the head holds a degree) or where the head of household's work status is that of manager or executive (40.5%). As the national accounts show, the value of the insurance technical provisions, which include both the pension funds and the supplementary pension schemes, is equal to about 20% of households’ total financial wealth (Bank of Italy, 2015).

3. Data

During the last decades, fertility rate in Italy decreased dramatically, while population median age increased, due to higher longevity owed to better life condition. Proportion of households with reference person aged 70 or more steadily grows and we observe an almost proportional decrease of young household heads. At the same time, average household size decreased during the observed period, with a rise in the number of single households and couples with respect to families with more than 2 children (Figure 3.1).

In the SHIW are collected data on 8,156 households. When we consider homeowners, we remain with 5,826 households (about 68.7%). In Table 3.6 we consider the mean portfolio weights of homeowners grouped by age class profiles:

- Until 34: 3.89% (227 households);
- 35-44: 9.65% (562 households);
- 45-54: 19% (1,107 households);
- 55-64: 20.91% (1,218 households);
- Over 65: 46.55% (2,712 households);

Table 3.6: Mean portfolio weights by age class

Assets (%)	House	Deposits	Short-term	Long-term	Corporate	Stocks
<34	1.7907	0.0716	0.0086	-0.8818	0.0038	0.0070
35-44	1.3540	0.0642	0.0088	-0.4470	0.0095	0.0069
45-54	1.1961	0.0575	0.0110	-0.2880	0.0109	0.0107
55-64	0.9176	0.0621	0.0179	-0.0249	0.0156	0.0101
65<	0.8998	0.0586	0.0171	0.0046	0.0128	0.0072

As expected, young homeowners have the majority of their wealth invested in housing (about 180% of total wealth) and at the same time they mostly resort to debt, with an average mortgage that counts for the 88% of total wealth. With ageing, households accumulate wealth, thus reducing housing weight in their portfolio and repaying mortgage, thus reducing debt. Indeed, Couley, Pavlov and Schwarz (2007), show that the effect of a homeownership constraint is largest at the beginning, i.e., for young households who have smaller net worth relative to current income. As individuals accumulate wealth, the homeownership constraint becomes less binding. Because of this constraint, young households, who should invest more in stocks, as they have a longer investment horizon to offset risk, are crowded out. The average shares of wealth kept in deposits go from 7.2% for younger households, up to 5.9% for retired. Investments in risky financial assets grow steadily with ageing but remain low if compared with house weight in total wealth. Average short-term shares go from 0.86% to

1.79%, corporate-bonds present lower shares, from 0.38% to 1.56%, and stocks represent the smallest part of household wealth, from 0.7% to 1.07%. Pre-retired households seem the ones who held the biggest shares in risky financial assets. Table 3.7 shows mean asset value held by homeowners.

Table 3.7: Mean asset value by age class (*standard deviation in parenthesis*)

Assets (€)	House	Mortgage	Deposits	Short-term	Long-term	Corporate	Stocks	Tot
<34	201,035 (136,856)	22,493 (44,077)	13,189 (28,350)	2,118 (8,683)	-21,274 (45,143)	969 (5,450)	150 (1,080)	196,186 (156,179)
35-44	209,624 (133,862)	26,707 (50,618)	11,400 (23,222)	1,863 (10,119)	-25,774 (50,659)	3,281 (36,713)	799 (7,525)	201,194 (151,673)
45-54	231,182 (187,061)	17,962 (45,614)	16,195 (75,033)	3,329 (32,240)	-16,179 (47,770)	3,906 (26,293)	1,817 (21,574)	240,301 (252,995)
55-64	232,006 (172,493)	5,342 (19,449)	16,505 (32,111)	4,828 (23,595)	40 (50,282)	5,589 (35,220)	2,063 (30,459)	261,033 (214,528)
65<	213,064 (170,967)	956 (8,038)	15,082 (46,752)	4,090 (17,835)	2,484 (21,942)	4,116 (22,902)	2,878 (70,052)	241,718 (218,080)

Home value shows a curve shape with respect to ageing, reaching a maximum when household head is aged between 55-64 years, probably due to family needs and wealth accumulation. Even the value of most risky assets reaches its maximum at pre-retirement age, with exception for stocks, whose value grows with ageing, reaching its maximum when households are probably retired from work. This contrary to common theory, which suggest that young people should invest more in stocks as they have a longer time horizon to offset stock market volatility. As previously suggested, this low participation of young homeowners to the risky market could be explained by crowding out effect of owning a house or by the uncertainty that relates with their future income streams.

Table 3.8: Mean portfolio risk and return by age class

	Standard deviation	**Annual return**
<34	3.8228 %	4.5662 %
35-44	2.9136 %	3.8749 %
45-54	2.5863 %	3.6365 %
55-64	2.0163 %	3.1672 %
65<	1.9850 %	3.1541 %

Table 3.8 shows risk and return of different age class portfolios. Mean risk and return of portfolios decrease with ageing, as a smaller share of wealth is invested in housing. Even if ageing homeowners increase their quote of risky assets, like corporate bonds and stocks, which have the greater expected returns and volatility, they maintain a low risk profile as most of wealth is still invested in house and low-risk assets.

Table 3.9 and 3.10 display household portfolio characteristics when we group homeowners by work sector. Homeowners employed in primary sector are on average the ones who less invest in financial assets. It could be the case that they are less financially educated. They are also the households who have the lower wealth share in housing (about 94%) and thus the lower debt. Homeowners who keep the largest share of wealth in housing seem to be the ones employed in the industry sector, with house representing about 127% of wealth, and they hold also the portfolios with highest mean risk and return. Share of wealth kept in deposits amount on average to 6%, while the share invested in short-term assets remains low and similar to the one invested in corporate bonds (about 1.2% of wealth). Stocks are still the asset in which less shares of wealth are employed, going from 0.71% for homeowners whose heads are employed in primary sector, to 1.3% for individuals who work in sector different from the ones considered.

Table 3.9: Mean portfolio weights by work sector

Assets (%)	House	Deposits	Short-term	Long-term	Corporate	Stocks
Agriculture	0.9426	0.0517	0.0126	-0.0218	0.0078	0.0071
Industry	1.2688	0.0595	0.0134	-0.3669	0.0122	0.0091
Public	1.2238	0.0753	0.0122	-0.3343	0.0146	0.0082
Other	1.1798	0.0617	0.0136	-0.2815	0.0116	0.0133

Table 3.10: Mean portfolio risk and return by work sector

	Standard deviation	**Annual return**
Agriculture	2.0755 %	3.2460 %
Industry	2.7359 %	3.7446 %
Public	2.6381 %	3.6353 %
Other	2.5490 %	3.5936 %

When we consider work categories, as shown in Table 3.11, workers are on average the ones with highest share of wealth invested in house and thus have the highest debt burden. Workers are also the ones who less invest in risky financial assets.

On the contrary, executives and entrepreneurs are the ones who invest the biggest share of wealth in the risky financial market and particularly in stocks, reaching 3.14% of wealth invested, thus could be the case that they are the most financially educated. They also invest large shares in deposits, about 8% of wealth, about 2% in short-term assets and 2.4% and 2.2% respectively in corporate bonds and stocks. Self-employed and retired have in mean the lowest share of wealth invested in housing, respectively 101% and 90%, while other homeowners invest in house more than 108% of wealth. Retired are the only ones that on average do not own debt.

Table 3.11: Mean portfolio weights by work category

Assets (%)	House	Deposits	Short-term	Long-term	Corporate	Stocks
Worker	1.3140	0.0502	0.0095	-0.3851	0.0040	0.0025
Employee	1.1931	0.068	0.0111	-0.2953	0.0145	0.0087
Executive	1.1935	0.0806	0.0202	-0.3410	0.0244	0.0223
Entrepreneur	1.0838	0.0775	0.0208	-0.2301	0.0165	0.0314
Self-employed	1.0132	0.0645	0.0158	-0.1165	0.0129	0.0101
Retired	0.9029	0.0582	0.0174	0.0015	0.0128	0.0065

In Table 3.12 we observe again that homeowners who have the largest share of wealth in the main home, hold the portfolios with highest expected return and the highest risk. If we compare employees and executives, who have similar shares in housing, we notice that employees' portfolios have on average higher return. This because, even if executives' portfolios are in mean more differentiated, employees incur in lower debt.

Table 3.12: Mean portfolio risk and return by work category

	Standard deviation	**Annual return**
Worker	2.8418 %	3.85 %
Employee	2.5774 %	3.6076 %
Executive	2.5650 %	3.5538 %
Entrepreneur	2.3455 %	3.3717 %
Self-employed	2.2092 %	3.3192 %
Retired	1.9920 %	3.1596 %

Looking at the effect of education on homeowner portfolios, Table 3.13 and 3.14 show that, the more educated are the household heads, the higher the share of wealth they invest in risky assets on average. Homeowners whose heads have a 3 or 5 years University Degree are the ones with the highest share of wealth invested in housing (125% and 114%) and have also the highest debt burden (36% and 29% of wealth).

When we consider portfolio characteristics for different level of education of household head, we should remember some peculiarities of Italian households: more educated homeowners are commonly younger, thus their portfolio characteristics could mainly reflect their age profile; more educated investors are also the most financially educated; in Italy, homeowners with a tertiary education represent a low share of households (about 14% in our sample), though this share is steadily grown in recent years. Less educated homeowners, which we suppose are also the older ones, keep the smallest share of wealth in financial assets, and have also a home value which amounts for 93% of wealth, for the ones whose heads have elementary education, and 105% for the ones with middle school education.

Table 3.13: Mean portfolio weights by education profile

Assets (%)	House	Deposits	Short-term	Long-term	Corporate	Stocks
Elementary	0.9327	0.0508	0.0137	-0.0064	0.0072	0.0019
Middle	1.0545	0.0512	0.0131	-0.1340	0.0089	0.0036
Vocational	1.1359	0.0565	0.0129	-0.2267	0.0139	0.0076
Secondary	1.0436	0.0662	0.0164	-0.1560	0.0152	0.0133
Degree (3 years)	1.2534	0.0697	0.0168	-0.3609	0.0069	0.0141
Degree (5 years)	1.1404	0.0828	0.0228	-0.2917	0.0245	0.0211
Post degree	0.9250	0.0804	0.0205	-0.0586	0.0203	0.0125

Table 3.14: Mean portfolio risk and return by education profile

	Standard deviation	**Annual return**
Elementary	2.0617 %	3.2362 %
Middle	2.3055 %	3.4276 %
Vocational	2.4657 %	3.5481 %
Secondary	2.2679 %	3.3586 %
3 years degree	2.6970 %	3.6863 %
5 years degree	2.4552 %	3.4599 %
Post degree	2.0213 %	3.1258 %

In Table 3.15 and 3.16 we consider homeowner portfolio characteristics by income profile. Portfolio diversification grows with income, as well as the share of wealth invested in risky assets. The value of home relative to wealth decreases as income grows, while the contrary happens for debt, with the exception of the 4th quintile, which shows homeowners with home value amounting to 106% of wealth and a debt which reaches 17% of total wealth.

Nonetheless, homeowners in the 4th income quintile present in mean well diversified portfolios if compared with households with lower income.

In the 5th quintile we find on average that homeowners keep 94% of wealth in housing, 8% in deposits, 2.3% in short-term assets, 2.9% in corporate bonds and 2.5% in stocks, with a thin debt burden of about 10%.

Table 3.15: Mean portfolio weights by income profile

Assets (%)	House	Deposits	Short-term	Long-term	Corporate	Stocks
1st quintile	1.0950	0.0424	0.0028	-0.1478	0.0014	0.0010
2nd quintile	1.0722	0.0515	0.0141	-0.1456	0.0054	0.0025
3rd quintile	1.0183	0.0605	0.0169	-0.1115	0.0109	0.0049
4th quintile	1.0661	0.0639	0.0180	-0.1721	0.0151	0.0091
5th quintile	0.9401	0.0826	0.0231	-0.0994	0.0289	0.0246

Table 3.16: Mean portfolio risk and return by education profile

	Standard deviation	**Annual return**
1st quantile	2.3973 %	3.5234 %
2nd quantile	2.3437 %	3.4590 %
3rd quantile	2.2262 %	3.3425 %
4th quantile	2.3175 %	3.4069 %
5th quantile	2.0503 %	3.1336 %

3.4.1 Financial portfolios

To analyze the composition of household financial portfolios, we exclude real estates and mortgages. This in order to focalize on investment choices for what is defined as liquid wealth. We will maintain the previous classifications for homeowners to see how these characterize investment choices. Table 3.17 shows the composition of homeowner portfolios using the data retrieved from the SHIW in 2014. We separate our previously aggregations of assets in all their components to better observe what are the weights of every single asset in the portfolios.

Table 3.17: Composition of household financial wealth. Aggregate financial accounts

	Age class				
Assets (%)	*<34*	*35-44*	*45-54*	*55-64*	*65<*
Deposits	65.60	52.20	47.23	38.20	39.82
Certificates of deposits	2.56	3.60	2.36	1.52	2.25
Repos	1.29	0.30	2.27	1.36	1.11
Postal saving certificates	5.06	2.15	1.72	3.67	3.91
BOT	6.32	3.90	4.15	6.35	6.09
CCT	0.37	0.54	0.86	1.84	1.29
BTP	0.45	1.80	2.48	5.32	4.43
BTPI	0	0.24	0.44	1.00	0.34
CTZ	0	0.20	0.07	0.11	0.06
Other government bonds	0.55	0.08	0.56	2.23	0.40
Corporate bonds	1.05	8.31	2.47	2.36	1.63
Financial corporate bonds	3.76	6.68	7.95	10.08	9.05
Investment funds (liquidity)	0.91	1.37	3.95	4.82	3.72
Investment funds (bonds)	4.76	1.67	2.49	3.92	2.36
Investment funds (balanced)	1.39	4.07	3.87	3.10	4.02
Investment funds (stocks)	0.07	0.41	1.83	1.75	1.71
Investment funds (foreign)	1.1	0.04	0.95	0.28	0.64
Listed shares	0.64	3.33	4.57	2.61	3.63
Unlisted shares	0.11	0.13	0.36	2.11	3.63
Limited Liability Company	0	0.57	0.21	1.77	0.04
Shares of Partnerships	3.29	0.29	1.62	0	0.07
Managed accounts	0.66	5.90	3.59	3.30	7.85
Foreign certificates	0	1.18	0.06	0.16	0.35
Foreign bonds	0	0.04	0.97	0.49	0.19
Foreign shares	0	0.20	0.52	0.05	0.34
Other foreign financial assets	0	0.57	1.11	0.13	0.45
Loans to Coop.	0.11	0.08	0.80	0.69	0.30
Other financial assets	0	0.20	0.55	0.76	0.30

Older household participation and share of risky assets are clearly superior to the ones of younger ones. Moreover, the share of wealth invested in deposits decreases as households become older, while the share invested in risky assets sharply increases.

Assets which seem to have greater importance in household portfolios are financial corporate bonds (with share from 3.76% to 10.08%) and managed accounts, while direct investment in stocks shows an increase with ageing but remains low. Investment funds' shares seem quite constant, while the shares in different fund categories, i.e. liquidity, bonds, balanced and stocks, change over time, with younger households investing mostly in funds which invest mostly in government bonds and differentiating when ageing. An important share of liquid wealth in all age classes, in particular for younger and older ones, is that of safe government securities, like BOT and postal saving certificates, while share in BTP shows the already noticed curve shape.

Table 3.18: Participation to stock market and risky assets owning by age class

Age group	**Stock market (%)**		**Risky assets[9] (%)**	
	Participation	*Share*	*Participation*	*Share*
<34	2.64	0.75	11.89	15.44
35-44	5.69	3.66	18.51	35.77
45-54	6.68	5.45	19.51	37.72
55-64	6.65	4.78	24.88	42.73
65<	5.16	7.60	18.66	44.41

Table 3.18 shows that participation in both stock market and risky assets exhibits a curve shape, with a maximum in pre-retirement period, while share of liquid wealth invested in stocks and risky assets sharply increases with ageing. While younger homeowners who participate in risky market invest there about 15% of their liquid wealth, pre-retired and retired homeowners have almost 40% of their financial portfolios composed by risky assets.

Considering work sector in Table 3.19, people employed in agriculture mostly hold risk-less assets as deposits (54% of portfolio value on average) or postal saving certificates (7.3%). Interestingly they hold also relevant shares in financial corporate bonds (7%) and shares of partnership (6%). Financial corporate bonds are also held by employed in public sector and in sector different from agriculture, industry and public with a share of about 9% of liquid wealth. Employed in industry seem to hold high shares in investment funds (almost 25%).

[9] Risky assets: stocks, long term government bonds, other bonds, mutual funds and managed investment accounts

Table 3.19: Financial portfolios' shares by work sector of the household head

	Work sector			
Assets (%)	*Agriculture*	*Industry*	*Public*	*Other*
Deposits	54.86	37.83	48.61	39.87
Certificates of deposits	0	3.89	3.15	1.93
Repos	1.57	2.20	2.91	1.02
Postal saving certificates	7.32	0.03	3.46	1.56
BOT	0.89	0.46	3.79	4.08
CCT	4.28	1.60	0.86	2.13
BTP	0.27	7.65	2.00	5.32
BTPI	0	2.91	0.46	0.31
CTZ	0	1.60	0.13	0.12
Other government bonds	0.93	2.88	0.83	0.17
Corporate bonds	4.68	0.39	2.20	3.61
Financial corporate bonds	7.11	0.32	9.10	9.14
Investment funds (liquidity)	2.68	3.58	1.62	4.43
Investment funds (bonds)	2.05	18.06	3.17	4.30
Investment funds (balanced)	0.66	0.08	3.77	5.13
Investment funds (stocks)	3.57	0	0.66	1.64
Investment funds (foreign)	0.96	5.28	0.39	0.76
Listed shares	0	0.12	3.49	4.75
Unlisted shares	0	0.05	0.16	0.51
Limited Liability Company	0	0.15	0.04	2.27
Shares of Partnerships	6.24	0.51	0	1.87
Managed accounts	0	0.73	5.72	2.28
Foreign certificates	0	0.59	0.15	0.32
Foreign bonds	0	3.89	0.90	0.36
Foreign shares	0	2.20	0.50	0.29
Other foreign financial assets	1.30	0.03	0.53	0.67
Loans to Coop.	0	0.46	0.54	0.98
Other financial assets	0	1.60	0.85	0.01

According to Table 3.20, participation in stock market concerns 6-7% of homeowners, while share invested in stocks clearly vary with work sector, going to less than 1% for homeowners whose head is employed in agriculture, up to 21.7% for the ones employed in the industry sector. Participation to entire risky market goes from 17,4% for agriculture up to 22.8% for public sector, while share invested in risky assets goes from 33% for primary sector to 49% for employed in industry.

Table 3.20: Participation to stock market and risky assets owning by work sector

Sector	**Stock market (%)**		**Risky assets [9] (%)**	
	Participation	*Share*	*Participation*	*Share*
Agriculture	6.1	0.96	17.39	33.17
Industry	6.1	21.79	20.59	49.47
Public	6.0	4.15	22.78	35.19
Other	7.3	5.55	22.02	43.69

In 2014 households headed by a payroll employee (46% of Italian households) owned 35.6% of total household financial assets; among these households, those headed by a blue-collar worker (almost a quarter of the total) owned only 8% of financial wealth, mainly as bank or post office deposits. Households whose head was self-employed (one tenth of Italian households) held 19.1% of total household financial assets and nearly half of the stocks. Households whose head was a pensioner represented 38.2% of the total and owned more than half of the value of both Italian government securities and indirect investments (Bank of Italy, 2015).

As regard peculiarities of different work categories, shown in Table 3.21, financial portfolios of executives and entrepreneurs are the most differentiated and the ones with lower shares in deposits, near to 30%. While executives mostly invest in government bonds (for about 24% of liquid wealth), entrepreneurs hold bigger shares in investment funds (21%) and in listed (2.8%) and unlisted stock shares (7.8%).

Workers keep the bigger quote of financial wealth in deposits (60%) and short-term government bonds (about 11%), probably because of safeness and liquidity of these assets. They also own significant shares in financial corporate bonds (5.7%) and in low risk investment funds (8.7%). Self-employed invest about 10% of their liquid wealth in financial corporate bonds and 7% in corporate bonds, while retired mostly invest in government bonds (16%), and corporate bonds (4%). Retired seem to have also relevant shares in limited liability company and in loans to cooperative.

Table 3.21: Financial portfolios' shares by work category of the household head

	Work category					
Assets (%)	*Worker*	*Employee*	*Executive*	*Entrepreneur*	*Self-employed*	*Retired*
Deposits	60.01	50.15	32.53	35.96	47.72	41.07
Certificates of deposits	1.97	0.59	1.45	2.88	0.24	4.46
Repos	0.89	3.53	2.36	2.75	1.42	7.10
Postal saving certificates	4.32	1.37	0.91	2.64	5.58	1.22
BOT	10.24	0.04	8.65	0.09	2.12	4.16
CCT	0.60	0.20	1.09	0.12	0.94	0.31
BTP	1.07	4.64	0.15	0.07	0.31	0.06
BTPI	0.10	9.31	0.74	1.22	0.03	1.33
CTZ	0.12	1.98	2.70	4.80	0.74	1.98
Other government bonds	0.64	2.46	13.31	2.87	2.43	9.19
Corporate bonds	1.36	3.31	3.10	2.83	7.81	4.25
Financial corporate bonds	5.72	0.76	5.29	4.28	10.55	2.18
Investment funds (liquidity)	3.03	0.22	4.60	1.12	2.92	3.27
Investment funds (bonds)	2.97	3.39	1.78	1.30	6.28	1.88
Investment funds (balanced)	2.77	0.38	0.10	3.37	1.01	0.46
Investment funds (stocks)	0.14	0.04	6.50	13.34	0.21	3.05
Investment funds (foreign)	0.47	0	0.47	4.05	3.63	1.13
Listed shares	1.16	3.21	0.03	2.84	0.39	0.02
Unlisted shares	0	0.68	0	7.80	0	0
Limited Liability Company	0	0.24	3.20	0	1.21	7.32

Shares of Partnerships	0	0.24	0.06	0.19	1.70	0.35
Managed accounts	1.23	0.21	1.25	0.03	0	0.37
Foreign certificates	0.10	1.25	0.93	0.68	0	0.27
Foreign bonds	0.04	0.67	1.29	0.16	0	0.43
Foreign shares	0	0.59	0.25	1.03	0	0.32
Other foreign financial assets	0	3.53	0.69	2.88	0.64	0.29
Loans to Coop.	1.03	1.37	1.45	2.75	0	4.46
Other financial assets	0	0.04	2.36	2.64	0.24	7.10

Table 3.22: Participation to stock market and risky assets owning by work category

Category	**Stock market (%)**		**Risky assets [9] (%)**	
	Participation	*Share*	*Participation*	*Share*
Worker	1.11	1.16	9.49	20.70
Employee	6.88	4.01	22.33	34.75
Executive	18.73	7.90	45.94	54.88
Entrepreneur	12.41	16.74	32.27	46.53
Self-employed	4.39	4.02	20.27	38.60
Retired	5.13	4.44	18.88	41.62

Table 3.22 shows that participation in stock market reaches 18.7% for homeowners whose heads are executives, while the highest share held in stocks belongs to entrepreneurs, with 16.7% of liquid wealth. Both participation and share in risky assets reach their maximum for executives, who participate for the 45.9% and invest almost 54%, while workers are the ones with lowest participation and shares owned.

Besides the head of household work status, the structure of financial portfolios also reflects educational qualifications (Table 3.23): homeowners whose heads have a degree hold more sophisticated portfolios, featuring significant shares of a variety of instruments, while households whose heads have no educational qualification tend to focus almost exclusively on bank or post office deposits. In particular, the table shows that homeowners whose heads have a 5 years University Degree or a post-Degree education, hold the lowest shares of financial assets in deposits (about 30%), while investing relevant shares in corporate bonds (8.5% and

4.75% respectively), investment funds for 5 years Degree homeowners and listed shares for ones with post-Degree education. Homeowners with an education level below secondary school still hold more shares in short-term government bonds and financial corporate bonds.

Table 3.23: Financial portfolios' shares by education profile of the household head

	Level of education						
Assets (%)	*Elementary school*	*Middle school*	*Vocational school*	*Secondary school*	*3 Years Degree*	*5 Years Degree*	*Post-Degree*
Deposits	49.21	49.25	50.73	45.62	49.78	31.87	34.64
Certificates of deposits	2.47	2.38	3.58	2.04	1.15	1.54	1.22
Repos	2.42	1.22	0.54	1.04	0.91	1.51	1.47
Postal saving certificates	7.52	5.66	4.27	2.87	0.87	3.01	0.22
BOT	11.84	9.10	5.94	5.21	5.90	0.62	0
CCT	0.98	1.73	0.42	2.08	0.81	6.23	2.30
BTP	1.02	4.37	2.32	3.16	0	1.14	1.11
BTPI	0.13	0.21	0.28	0.17	0	0.07	9.99
CTZ	0	0.06	0	0.14	0	0.68	2.16
Other government bonds	0.78	0.22	0.55	1.41	0	3.10	6.47
Corporate bonds	1.81	2.51	1.85	1.89	0.11	8.57	4.75
Financial corporate bonds	8.73	8.72	13.96	8.73	0.76	6.13	3.76
Investment funds (liquidity)	1.53	2.28	2.53	3.15	5.98	3.20	0.13
Investment funds (bonds)	2.94	2.96	2.05	2.15	1.30	4.21	1.64

Investment funds (balanced)	2.45	2.10	3.59	4.15	11.14	1.05	0.30
Investment funds (stocks)	0.44	0.50	0.10	3.38	0.27	0.46	0
Investment funds (foreign)	1.58	0.24	0	0.70	0	5.36	0
Listed shares	0.93	2.44	2.68	2.98	4.69	5.04	21.29
Unlisted shares	0	0.13	0.76	1.83	0	0.12	0.21
Limited Liability Company	0	0.08	0.10	1.51	0	0.85	0
Shares of Partnerships	0	0.57	0.48	0.05	0	9.68	0
Managed accounts	2.23	2.47	1.87	3.38	10.78	0.61	0
Foreign certificates	0.05	0	0.15	0.20	0	0.86	0.26
Foreign bonds	0.11	0.04	0	0.25	0	0.57	0
Foreign shares	0.11	0	0	0.24	0	0.77	1.22
Other foreign financial assets	0.60	0.08	0	0.31	5.43	0.24	1.47
Loans to Coop.	0.135	0.67	1.25	0.65	0.11	0.62	0.21
Other fin. assets	0	0	0	0.75	0	1.54	0

From Table 3.24 we see that participation to stock market and risky assets grows with education level of household heads, going from respectively 1% and 8% participation rate for homeowners with elementary school education, up to 15.5% and about 40% for the ones with 3-years University Degree. Also, the share invested in risky instruments steadily increases with education, reaching 11% of liquid wealth invested in stocks and 56.5% in risky assets for homeowners with a 3-years Degree.

Table 3.24: Participation to stock market and risky assets owning by education profile

Education	Stock market (%)		Risky assets[9] (%)	
	Participation	*Share*	*Participation*	*Share*
Elementary school	1.12	1.039	8.8	25.303
Middle school	2.988	2.574	14.343	29.068
Vocational school	4.245	3.437	19.575	32.415
Secondary school	8.218	5.042	25.659	37.879
Degree (3 years)	7.576	4.686	24.242	40.461
Degree (5 years)	15.493	10.964	40.845	56.519
Post-degree	13.235	1.945	41.176	55.577

In Table 3.25 we group homeowner heads on the basis of their income. As regard deposits, homeowners invest less in them as income grows, and the same happens for postal saving certificates. Poorest homeowners focus almost exclusively on bank or post office deposits, with 60-70% invested in deposits and about 10% in postal savings certificates. As income of household heads grows, the portfolio becomes more differentiated and the investment in funds and stocks, both listed and unlisted, increases steadily. While investment in BOT decreases as income grows, going from 11% to 1%, investments in BTP, financial corporate bonds, investment funds, stocks and managed accounts increase with income.

Table 3.25: Financial portfolios' shares by income profile of the household head

	Income quantile					
Assets	*<10*	*10-25*	*25-50*	*50-75*	*75-90*	*90<*
Deposits	76.48	63.56	55.94	46.31	46.73	29.68
Certificates of deposits	0	1.61	3.04	3.16	1.66	1.75
Repos	0.39	0.54	1.13	1.87	0.54	1.63
Postal saving certificates	10.97	9.20	6.07	4.71	3.37	0.96

BOT	7.60	7.89	11.30	7.79	4.73	2.90
CCT	0	0.37	1.64	1.59	2.05	0.80
BTP	0	0.44	0.96	4.27	1.96	6.50
BTPI	0.74	0	0.07	0.23	0.36	0.92
CTZ	0	0	0.09	0.12	0.03	0.09
Other government bonds	0	0	0.07	0.80	1.16	1.16
Corporate bonds	0	0.64	1.54	2.03	2.26	3.05
Financial corporate bonds	2.46	4.34	6.83	10.08	8.51	9.75
Investment funds (liquidity)	0	0.48	0.89	4.14	3.56	5.27
Investment funds (bonds)	0	2.02	2.30	2.44	3.37	3.01
Investment funds (balanced)	0.41	0.93	2.96	2.83	6.81	3.34
Investment funds (stocks)	0	3.15	0.50	1.38	1.66	2.00
Investment funds (foreign)	0	0.08	1.23	0.09	0.04	0.95
Listed shares	0.62	0.82	1.13	2.71	4.73	4.39
Unlisted shares	0	0	0.34	0.19	0.28	5.49
Limited Liability Company	0	0	0.02	0.02	0.31	1.14
Shares of Partnerships	0	0	0	0	0.25	0.91
Managed accounts	0	3.46	1.11	2.01	2.95	10.73
Foreign certificates	0.33	0.11	0	0.11	0.31	0.49
Foreign bonds	0	0	0.14	0.05	0.60	0.61
Foreign shares	0	0	0	0.11	0.15	0.57
Other foreign financial assets	0	0	0	0.34	0.69	0.69
Loans to Coop.	0	0.36	0.64	0.60	0.63	0.30
Other financial assets	0	0	0.07	0	0.30	0.94

Both risky assets participation and share increase sharply with income (Table 3.26). More differentiated portfolios and more participation and shares in risky assets can be due both to the higher values of liquid wealth of high income households and both to the higher education, which normally characterized richer.

The value of the assets held by less wealthy 40% of Italian households (those with the lowest net wealth just over €5,000 on average) represents 8% of total financial wealth, while the assets held by the wealthiest 20% of Italian households (averaging around €84,000) represent two thirds of the total, half of which is owned by the top 5%.

Table 3.26: Participation to stock market and risky assets owning by income profile

Income quantile	**Stock market (%)**		**Risky assets [9] (%)**	
	Participation	*Share*	*Participation*	*Share*
<10	0.172	0.615	0.858	3.814
10-25	0.915	0.825	4.348	16.47
25-50	2.129	1.474	10.58	20.01
50-75	4.941	3.015	22.65	33.60
75-90	10.88	5.152	35.97	39.04
90<	21.61	10.45	54.03	57.99

In 2014 poorer households (those in the first quintile of net wealth) held almost exclusively deposits, certificates of deposit and repos. In addition to these, households in the central groups of net wealth invested a large part of their assets in government securities, private-sector bonds and asset management portfolios. The richest quintile displayed more diversified financial portfolios, with more than a quarter managed by financial brokers. These households owned two thirds of the value of the stock of government securities held by households, 70% of private-sector bonds and over 80% of stocks and managed investment schemes (Bank of Italy, 2015).

3.5. Asset moments

To show the implications of our theoretical analysis, we use data on asset returns and Italian household portfolios. We group Italian portfolio assets in five categories: deposits (risk-free), short-term assets, long-term assets, corporate bonds, stocks and house. Data on returns are so obtained:

- Deposits: mean interest rates on retail bank deposit (Bank of Italy). We will consider deposits as the risk-free asset;
- Short-term bonds: fixed income value-weighted average returns of BOT, CCT and CTZ (Bank of Italy);
- Long-term bonds: fixed income value-weighted average returns of BTP with time to maturity of 3, 5, 10 and 30 years (Bank of Italy);
- Corporate bonds: Merrill Lynch EMU Corporates Total Return Index 3-5 years (Eikon dataset);
- Italian stocks: FTSE MIB Total Return Index for the Italian stock market (Eikon dataset);
- House: Italy house price index (OECD);

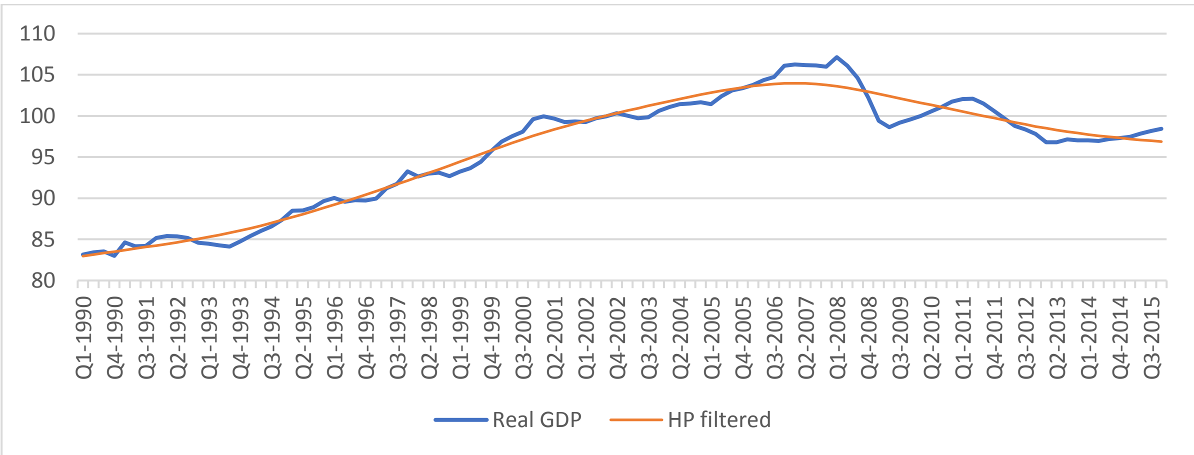
Returns are taxed at relative nominal tax rate and deflated using the private consumption deflator from the national account statistics, so we obtain real returns for all assets. As regard returns on house prices, according to the formula:

$$r_{H,t} = \frac{P_t - P_{t-1}}{P_{t-1}} + \frac{D_t - COM_t}{P_{t-1}} = \frac{P_t - P_{t-1}}{P_{t-1}} + \kappa \tag{22}$$

where D denotes rent and COM maintenance costs, we add a 5% annual net of taxes return as consumption benefits (κ), following the guidelines of Pelizzon and Weber (2008) and Flavin and Yamashita (2002). We underline that the choice of κ is immaterial in the analysis of the constrained case efficiency, as κ is a fixed number (see equation (20)). It would be important in the case where housing is treated as unconstrained, given that it affects its expected return directly.

We consider quarterly returns for all assets, starting from 1990 to 2015, thus we have 102 observations. These years saw a low economic growth in Italy, as current real GDP is very close to the value it had at the end of the last century (Figure 3.9). In 2008 GDP reached its peak, before financial crisis presented its effects on Italian economy. From Figure 3.10 we can observe real GDP quarterly growth rate (detrended from the HP filtered real GDP series[10]) and the downfall between 2008 and 2009 is clear.

Figure 3.9: Real GDP Index 1990-2015 [11]

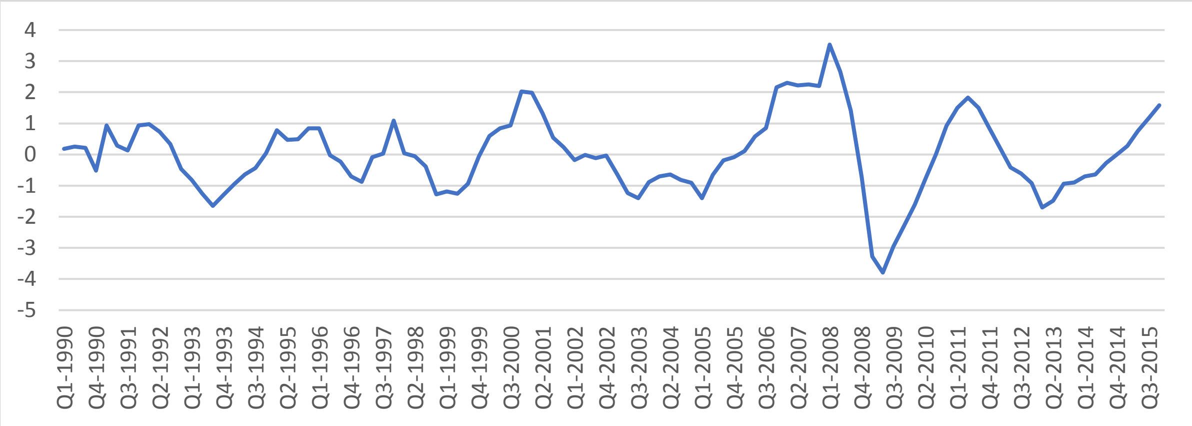


[10] The Hodrick-Prescott (HP) filter is used in macroeconomics, especially in real business cycle theory, to remove the cyclical component of a time series from raw data. It obtains a smoothed-curve representation of a time series, more sensitive to long-term than to short-term fluctuations. HP filter identifies as trend the series x_t^{trend} that minimizes $\sum_{t=1}^{T}(x_t - x_t^{trend})^2 + \lambda \sum_{t=2}^{T-1}(\Delta x_{t+1}^{trend} - \Delta x_t^{trend})^2$. The first term is lower the closer to one another are the actual and trend series; the second term is lower the 'smoother' is the trend series. Therefore, reducing one term implies increasing the other (unless the actual series is on a straight line). The balance of the two contrasting objectives depends on 'lambda': for any give data frequency, the higher lambda, the smoother the trend series. For quarterly series, a lambda = 1600 can be agreeable.

[11] Index based on OECD data. Q1 2010=100.

Figure 3.10: Real GDP per cent quarterly growth 1990-2015

(*detrended from HP filtered series*)



From 1990 we have observed a growth in the equity market, especially in indirect participation through investment funds and managed accounts, with peaks in 2000 and 2006, paralleled to a sharp decrease in the importance of bank accounts and short-term government debt in household portfolios, but this trend reversed in recent years. While stocks market participation is steadily grown during the years observed, its capitalization presents a cyclical component, with a very low observable positive trend (Figure 3.11). Guiso and Jappelli (2002) show that between 1989 and 1998 the fraction of people investing directly in stocks almost doubled, while the number of households investing in mutual funds or holding corporate bonds increased from 2.84% to more than 10%, and from 1 to 6%, respectively. In the Figure 3.11 we can see the evolution of the FTSE MIB Index. After the shock of the "dot-com" bubble in 2001-2002, the FTSE MIB index steadily starts growing, reaching its historical peak in 2007, before collapsing as a consequence of the financial crisis. Even today, its level is far below the level it was in 1999.

Figure 3.11: FTSE MIB Total Return Index 1990-2015 [12]

[12] Source: Eikon Dataset

Figure 3.12 shows the evolution of the real house price index derived from OECD dataset. House prices reached a peak in 1992, then decreased till 1997, starting in 1998 a positive trend which ended in 2007 with the burst of house bubble. Before the financial crisis, house prices steadily increased, reaching very high values. This was essentially due to the evolution of securitization process, especially in more financially developed countries (i.e. US and England), that caused a strong increase in housing demand because of the easier access to the credit system. Securitization in US carried on with mortgages credit fragmentations through Mortgage Backed Securities (MBS) and Collateralized Debt Obligation (CDO), which offered better returns than government securities, along with attractive ratings from rating agencies. House prices steadily increased as a consequence of speculation on these financial instruments, and the bubble burst with the subprime mortgage crisis between 2007 and 2008, due to the high percentage of low quality subprime mortgages.

Figure 3.12: Real house prices index 1990-2015 [13]

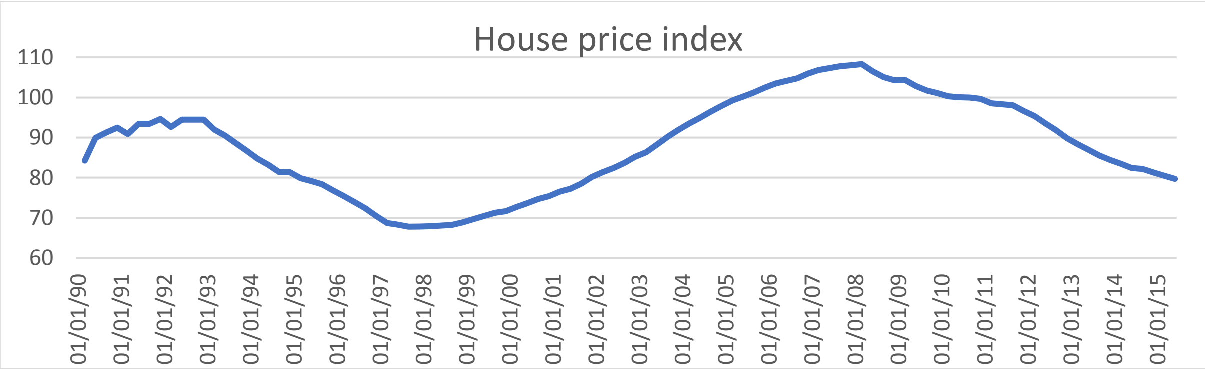
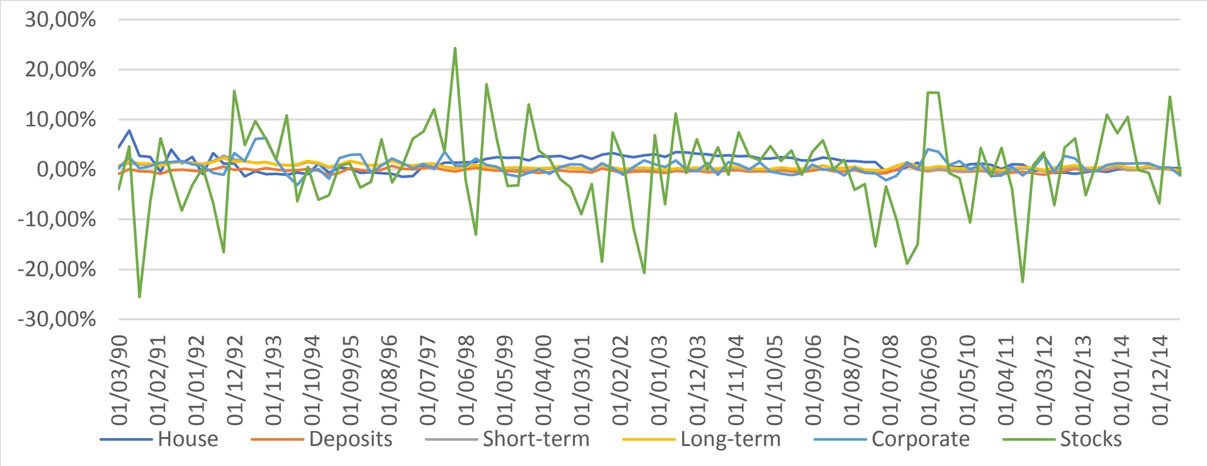


Figure 3.13 shows quarterly real returns for financial assets and house prices index. We express all returns net of withholding tax on the assumption that for most investors other tax distortions are relatively minor. Returns for all assets were more volatile before the introduction of the euro currency, in particular around 1992 crisis, due to devaluation of Lira (secured to the European Exchange Rate Mechanism) and due to political issues ("*Tangentopoli*"). Increase in volatility occurred also during the years of the financial crisis, from mid-2007-2008, and between 2011-2012, due to the sovereign debt crisis. This last crisis was particularly hard for Italy, because of the great stock of public debt and for the feeble situation of many Italian banks. We saw a mild recovery of the economy only in 2014 and 2015.

As expected, we can notice that stocks' returns are the most volatile, followed by housing returns and corporate bonds.

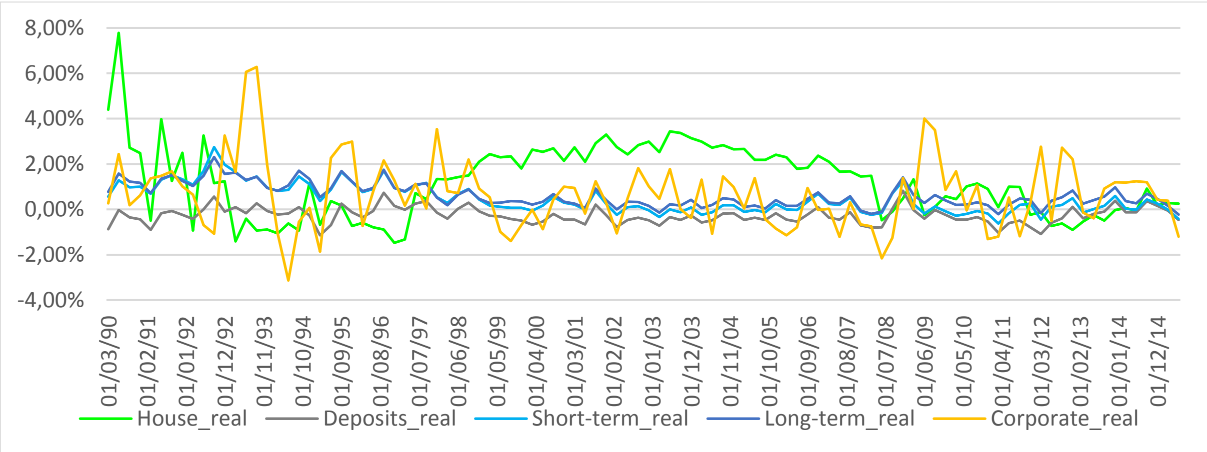
[13] Source: OECD Dataset. Seasonally adjusted, index based in 2010

Figure 3.13: Quarterly real returns 1990-2015



Excluding stocks returns from the graph (Figure 3.14), we see that house price returns and interest rates on corporate bonds are the ones who show highest returns, but also highest volatility, as confirmed by variances in Table 3.27 (the diagonal bold values). We observe that interest rates on deposits, short-term and long-term government bonds seem highly correlated. These latter also show a steadily decreasing pattern as a consequence of the introduction of Euro currency, but with some sharp peaks during financial and sovereign debt crises

Figure 3.14: Quarterly real returns 1990-2015 (excluding stocks returns)



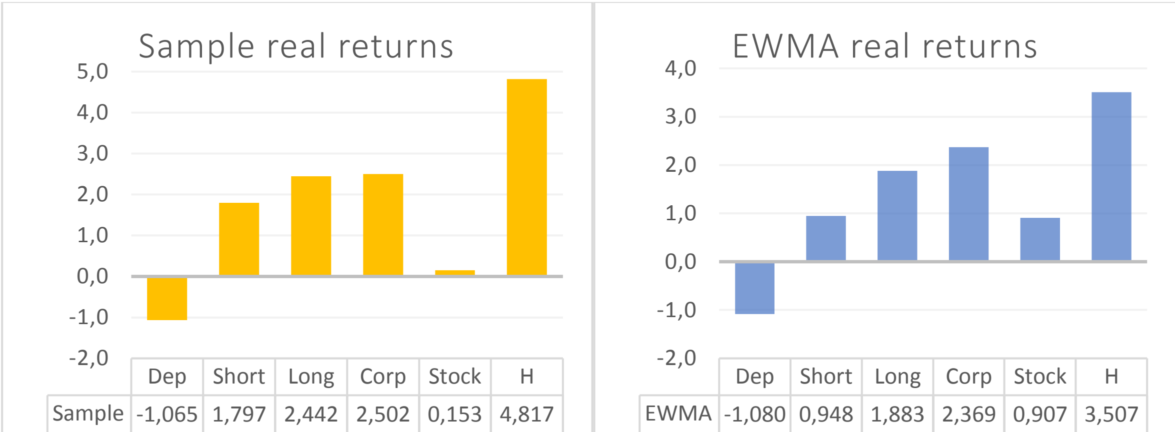
To estimate mean returns and covariance matrix we use an exponentially weighted moving average (EWMA[14]) of sample data, thus a moving average of the sample estimator with weights decreasing over time, giving higher weights to most recent observations. Table 3.27 displays covariances between assets and variances (in bold). Figure 3.15 shows expected real returns calculated with sample returns and with EWMA.

[14] We compute $r_t = (1 - \Lambda)*r_{t-j} + \Lambda r_{t-1}$ and $\Sigma_t = (1 - \Lambda)*r_{t-j}*r'_{t-j} + \Lambda\Sigma_{t-1}$. Decay parameter Λ=0.97.

Table 3.27: Covariance matrix 1990-2015 (real returns with EWMA method)

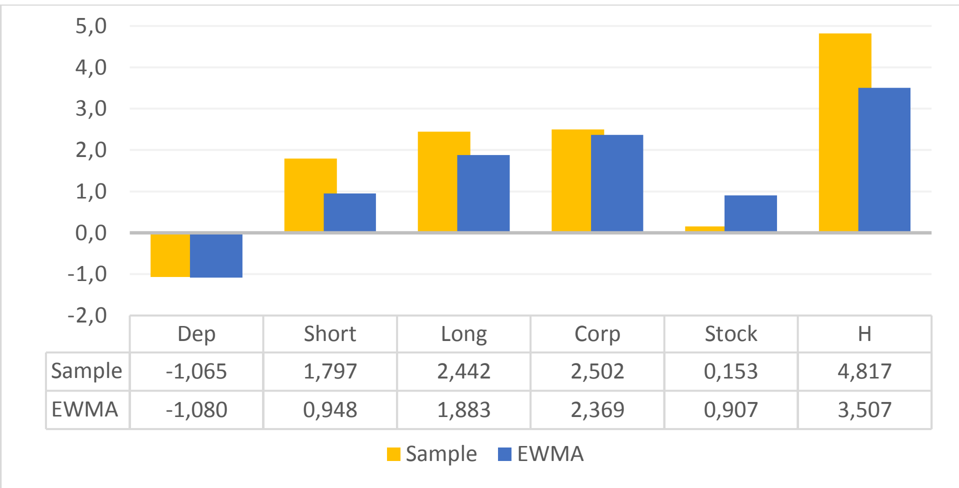
Covariance matrix	Deposit	Short-term	Long-term	Corporate bonds	Stocks	House
Deposit	**0.415855**	0.140548	0.0017170	-0.0004613	0.0669153	-0.557994
Short-term	0.140548	**0.325192**	0.3355416	0.3174003	-0.7250433	0.1431694
Long-term	0.001717	0.335542	**0.5052107**	0.6471648	-0.2448355	0.4610649
Corp. bonds	-0.000461	0.3174003	0.6471648	**4.3463602**	9.2839306	0.1759585
Stocks	0.066915	-0.725043	-0.244835	9.2839306	**151.501666**	-2.844831
House	-0.557995	0.1431694	0.461065	0.1759585	-2.844831	**4.955113**

Figure 3.15: Expected real returns 1990-2015 (*annual %*)



EWMA method adapts returns to more recent observations. Being real returns, we notice that returns on deposits are negative (-1.08%). Housing mean returns are the highest (3.5% annual real mean return), followed by corporate bonds (2.37%) and long-term bonds (1.88%). Returns from corporate bonds are slightly above long-term bonds ones.

Figure 3.16: Mean real returns comparison 1990-2015 (*annual %*)



In Table 3.16 we compare mean returns with sample and EWMA methods for calculation. The differences with respect to sample moments returns can be explained by the changed economic environment: before financial crisis, house prices were experiencing a strong growing period, while returns on Italian stocks' market have suffered for the *Dot-com Bubble* in 2000s and for the financial crisis of 2007. Moreover, Italian stocks' returns have never been characterized by long periods of growth, maybe due to the climate of political uncertainty that is specific of our country. As regard government bonds, we have already said that after Euro introduction their interest rate returns have follow a decreasing pattern. Real interest rates on deposits seem to be unchanged, regardless the evaluation method. These can be explained by the changes in inflation rates behavior: till 2002, Italy experienced a period of high inflation, joined with the continuous Lira devaluation, while after the creation of the monetary union, inflation decreased till dropping to zero and becoming also negative after the financial crisis. Hence it seems that decrease in nominal interest rates on deposits were compensated by an equal decrease in inflation.

As regard correlations (Table 3.28), we see that deposit interest rates are strongly positively correlated with both short-term bonds ($\rho = 0.64$) and long-term bonds ($\rho = 0.67$), while are negatively correlate with stocks ($\rho = -0.0069$) and housing returns ($\rho = -0.1693$). Short-term bonds and long-term bonds returns seem to be almost perfectly correlated (ρ=0.97). House returns seem to be slightly negatively correlated with all assets. Also, stock returns show negative correlations with all asset except corporate bonds (ρ=0.27).

Table 3.28: Correlation matrix for real returns 1990-2015

Correlation	Deposits	Short-term	Long-term	Corporate	Stocks	House
Deposits	1	0.6401	0.67477	0.31734	-0.00689	-0.16925
Short-term		1	0.97070	0.29944	-0.10163	-0.14783
Long-term			1	0.33216	-0.11352	-0.16132
Corporate				1	0.27043	-0.11721
Stocks					1	-0.12315
House						1

To evaluate the efficiency of the household portfolios, we need to determine the expected return and the expected variance-covariance matrix of the assets. We use nominal returns to estimate expected excess returns for all assets. Interest rates on deposits are considered as risk-free and are subtracted from returns of other assets to obtain excess returns. Figure 3.17 shows quarterly excess returns for the assets considered.

Figure 3.17: Quarterly excess returns 1990-2015

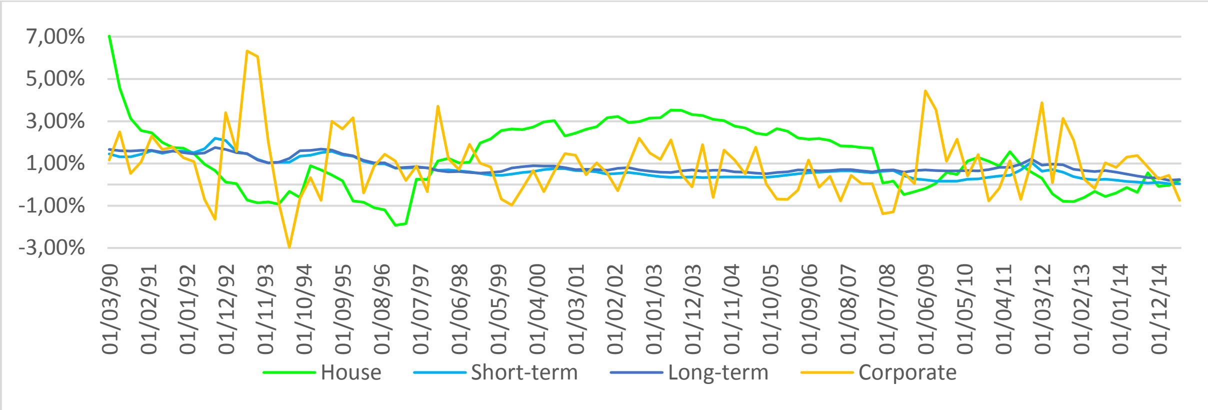


Table 3.29 displays the variance-covariance matrix of excess returns and presents similar variances to the ones for real returns, while covariances seem quite different.

Table 3.29: Covariance matrix for excess returns 1990-2015 (EWMA method)

Covariance	Short-term	Long-term	Corp. bonds	Stocks	House
Short-term	**0.4621699**	0.61187625	0.5971093	-0.5202624	0.7948733
Long-term	0.6118762	**0.92135658**	1.0676039	0.0980216	1.1285212
Corp. bonds	0.5971093	1.0676039	**4.8212958**	9.7442235	0.712555
Stocks	-0.5202623	0.0980216	9.7442235	**153.13037**	-1.503146
House	0.79487334	1.12852116	0.71255504	-1.5031458	**5.2929631**

In Table 3.30, which shows correlation between assets, we observe the same patterns of the real returns correlation matrix: short-term bonds and long-term bonds are highly positively correlated (ρ = 0.93); corporate bonds are positively correlated with all assets except housing; stocks are negatively correlated with all assets except corporate bonds (ρ = 0.28); housing is slightly negatively correlated with all assets, but exhibits smaller values in modulus than in real returns correlation matrix.

Table 3.30: Correaltion matrix for excess returns 1990-2015

Correlation	Assets	Short-term	Long-term	Corporate	Stocks	House
	Short-term	1	0.9266905	0.134483	-0.113471	-0.128527
	Long-term		1	0.158395	-0.123969	-0.079544
	Corporate			1	0.2790491	-0.125547
	Stocks				1	-0.086441
	House					1

Expected excess returns in Figure 3.18 reflect the situation we have observed for real returns: financial excess returns with sample and EWMA method show quite similar dynamics, with highest excess return for corporate, followed by long-term bonds and stocks. We choose to use EWMA method for returns calculation. Looking at EWMA excess returns we see that housing has the highest mean excess return (3.6% per year), followed by corporate bonds (3.46%) and long-term bonds (2.97%). Short-term bonds have a mean excess return of 2% per year, while stocks present the lowest mean excess return, about 2% too.

Figure 3.18: Expected excess returns 1990-2015 (*annual %*)

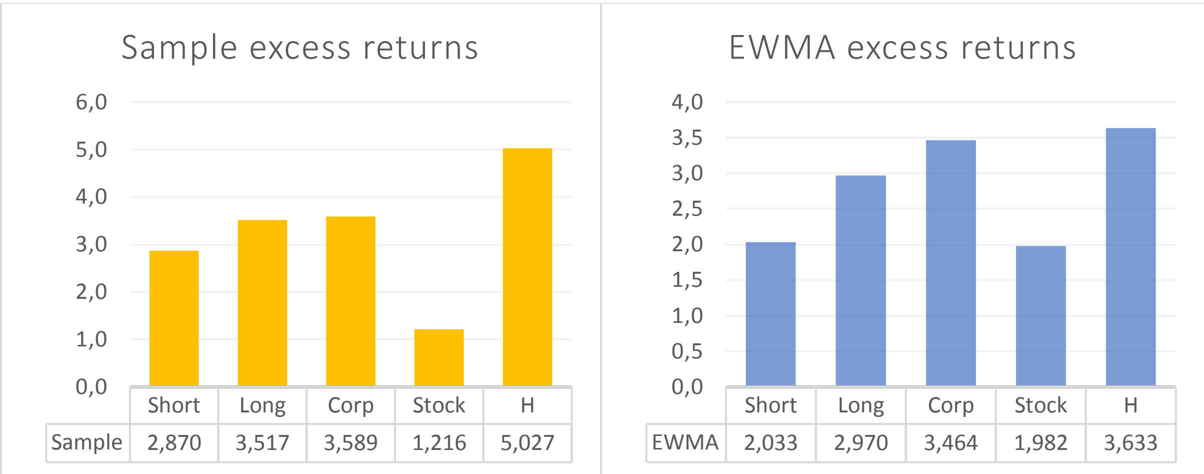


Figure 3.19: Expected excess returns comparison 1990-2015 (*annaul %*)

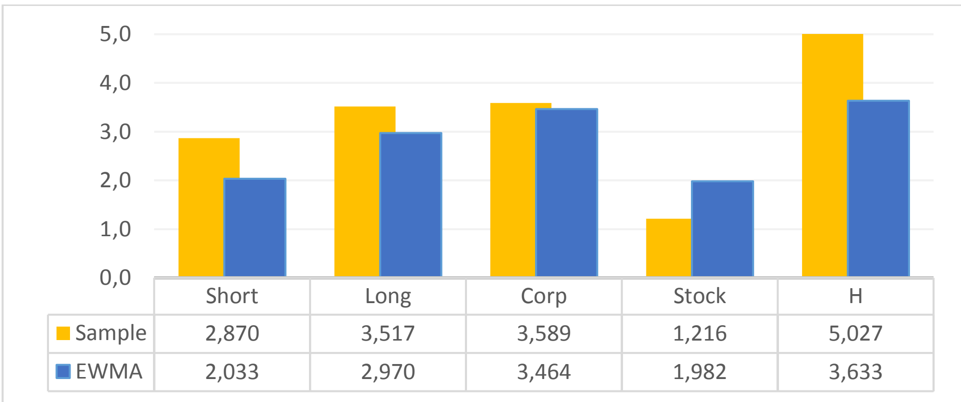


Figure 3.19 shows how expected excess returns calculated with EWMA stand in relation to sample ones. The higher excess mean returns for short-term and long-term assets in sample moments, respectively 2.9% and 3.5%, are explained by the period prior to Euro introduction, characterized by many devaluations of the Lira and high interest rates on Italian public debt. These large increases in public debt costs have been taking place also during the 2007-2008 financial crisis and 2011-2012 sovereign debt crisis. EWMA method gives minor weights to

these events, thus presenting lower value for returns on government bonds. The higher value in expected stocks return shown by EWMA is due to the high growth in stock market capitalization after dot-bubble and after the financial crisis; we jet remember that its actual value is well below 2007 peak. Lower value in house excess return are due to the fact that before financial crisis we have experienced a boom of house prices, while the crisis made the bubble to burst. EWMA method seems thus the one which better explains excess returns for all assets.

The summary statistics shown in previous sections (3.4 and 3.5) clearly state that household financial portfolios have changed a great deal over the years, and that real estate plays a key role in total household wealth. It makes sense to consider the interaction of housing and financial wealth holdings when assessing the efficiency of household portfolios. A financial portfolio may deviate from the mean-variance frontier for financial assets simply as a result of its covariance properties with the return on housing equity. A relevant issue is whether housing wealth is treated as a liquid or as an illiquid asset (Pelizzon & Weber, 2008).

Our interest is on the negative correlation between housing excess returns and other financial assets. The issue arises of whether these correlations are negligible.

3.6. Hedge term coefficients significance

Owner-occupied housing is the dominant asset in most household portfolios, thus even small correlation between financial assets return and housing return would significantly change the portfolio choice.

The issue arises of whether these correlations are negligible, and especially important is to consider partial correlation, as in a multiple asset setting. In order to assess the relevance of partial correlations, we have to estimate the coefficients of the hedge term in equation (17). This can be done by running the regression of housing excess returns on financial assets excess returns, as suggested by de Roon, Eichholtz and Koedijk (2002).

Running the regression using all observations, between 1990 and 2015, none of the parameters seem to be significant (Linear regression 3.1). Only short-term returns seem to be statistically significant, with a p-value for the t-test of 11,7%. Moreover, also the F-test for joint significance presents a p-value of 13.79%, thus it seems that excess return on housing is not even partially correlated with financial assets. However, it may be that these partial correlations, and thus the coefficients of the hedge term, change over time, due to changes in the economic environment.

3. Data

Linear regression 3.1: House excess returns on fin. assets excess returns (1990-2015)

```
. regress House Shortterm Longterm Corporate MSCI, vce(robust)

Linear regression                               Number of obs     =        102
                                                F(4, 97)          =       1.79
                                                Prob > F          =     0.1379
                                                R-squared         =     0.0437
                                                Root MSE          =     .01599
```

House	Coef.	Robust Std. Err.	t	P>|t|	[95% Conf.	Interval]
Shortterm	-1.207767	.7642766	-1.58	0.117	-2.724645	.3091102
Longterm	1.090464	1.178349	0.93	0.357	-1.248233	3.429162
Corporate	-.0784492	.0783099	-1.00	0.319	-.2338727	.0769743
MSCI	-.0090113	.0126068	-0.71	0.476	-.0340324	.0160098
_cons	.0083731	.0056603	1.48	0.142	-.0028609	.0196072

If we indeed consider returns from 1990 to 2008 (Linear regression 3.2), the regression shows that both short-term and long-term assets coefficients are statistically significant almost at the 0.2% level. For that period, also the F-test shows a high joint significance of all parameters. The R-squared is higher than the one considering the whole sample observations. Furthermore, the coefficients for both the significant parameters present high values, negative for short-term asset returns, and positive for long-term ones. Coefficients for corporate bonds and stocks returns are not statistically significant if taken individually, but as already said, the F-test shows joint significance of all parameters.

Linear regression 3.2: House excess returns on fin. assets excess returns (1990-2008)

```
. regress House Shortterm Longterm Corporate MSCI, vce(robust)

Linear regression                               Number of obs     =         76
                                                F(4, 71)          =       7.12
                                                Prob > F          =     0.0001
                                                R-squared         =     0.3210
                                                Root MSE          =     .01438
```

House	Coef.	Robust Std. Err.	t	P>|t|	[95% Conf.	Interval]
Shortterm	-5.435031	1.252681	-4.34	0.000	-7.932805	-2.937257
Longterm	4.995637	1.58645	3.15	0.002	1.832345	8.158928
Corporate	.0107145	.100334	0.11	0.915	-.1893458	.2107749
MSCI	-.0064874	.0131239	-0.49	0.623	-.0326557	.0196809
_cons	.014621	.0059783	2.45	0.017	.0027006	.0265413

Considering the pre-Euro observations (1990-2002), we obtain the Linear regression 3.3, which reflects the results of the previous one, but shows a convergence in modulus of the value of short-term and long-term coefficients. Anyway, the values of the F-test and of the t-statistics are lower but still show high significance for both short-term and long-term returns and jointly significance of all parameters.

Linear regression 3.3: House excess returns on fin. assets excess returns (1990-2002)

```
. regress House Shortterm Longterm Corporate MSCI in 1/48, vce(robust)

Linear regression                               Number of obs     =         48
                                                F(4, 43)          =       3.66
                                                Prob > F          =     0.0119
                                                R-squared         =     0.2027
                                                Root MSE          =     .01689
```

House	Coef.	Robust Std. Err.	t	P>|t|	[95% Conf.	Interval]
Shortterm	-4.299793	1.448232	-2.97	0.005	-7.220432	-1.379154
Longterm	4.152585	1.857125	2.24	0.031	.4073352	7.897834
Corporate	-.0048183	.1108612	-0.04	0.966	-.2283913	.2187547
MSCI	-.0194374	.015319	-1.27	0.211	-.050331	.0114562
_cons	.0107909	.0077523	1.39	0.171	-.0048431	.0264249

If we consider the period 2002-2015, thus we consider all observations after the introduction of the Euro currency, both short-term and long-term assets coefficients are significant at very low level of the p-value (0.3% and 3.7% respectively) and the F-test shows joint significance of all assets at least at the 3% level. Jet the sign of short-term and long-term coefficients are now inverted, respectively they have a plus and a negative sign, but both still present high coefficient values.

Linear regression 3.4: House excess returns on fin. assets excess returns (2002-2015)

```
. regress House Shortterm Longterm Corporate MSCI in 49/102, vce(robust)

Linear regression                               Number of obs     =         54
                                                F(4, 49)          =       2.94
                                                Prob > F          =     0.0294
                                                R-squared         =     0.1090
                                                Root MSE          =     .01365
```

House	Coef.	Robust Std. Err.	t	P>|t|	[95% Conf.	Interval]
Shortterm	3.35257	1.081428	3.10	0.003	1.179359	5.52578
Longterm	-3.023848	1.409024	-2.15	0.037	-5.855389	-.1923072
Corporate	.0162251	.1387091	0.12	0.907	-.2625212	.2949714
MSCI	.0132913	.0184756	0.72	0.475	-.0238369	.0504195
_cons	.022316	.0076657	2.91	0.005	.0069112	.0377208

Coefficients lose significance when we instead consider the period from 2008-2015 (Linear regression 3.5) and this probably can be explained by the high volatility of this last period, with two crises characterizing financial and political environment.

Linear regression 3.5: House excess returns on fin. assets excess returns (2008-2015)

```
. regress House Shortterm Longterm Corporate MSCI in 77/102, vce(robust)
```

Linear regression

Number of obs	=	26
F(4, 21)	=	1.00
Prob > F	=	0.4273
R-squared	=	0.1060
Root MSE	=	.00699

House	Coef.	Robust Std. Err.	t	P>|t|	[95% Conf.	Interval]
Shortterm	-.7474437	1.002469	-0.75	0.464	-2.832192	1.337305
Longterm	1.091385	1.010396	1.08	0.292	-1.009847	3.192618
Corporate	-.0676689	.086886	-0.78	0.445	-.2483582	.1130205
MSCI	-.0087368	.0131884	-0.66	0.515	-.0361637	.0186901
_cons	-.000502	.0034409	-0.15	0.885	-.0076578	.0066537

If we only look at the recent recovery period, since 2012 (Linear regression 3.6), coefficient of short-term assets is significant at least at the 8.2% level, while the one for long-term returns is significant at the 1.2% level. The F-test shows joint significance at the 5.3% level and the R-squared value is high, at 45%. It is worth stressing that we have only 14 observations.

Linear regression 3.6: House excess returns on fin. assets excess returns (2012-2015)

```
. regress House Shortterm Longterm Corporate MSCI in 89/102, vce(robust)
```

Linear regression

Number of obs	=	14
F(4, 9)	=	3.54
Prob > F	=	0.0534
R-squared	=	0.4510
Root MSE	=	.00365

House	Coef.	Robust Std. Err.	t	P>|t|	[95% Conf.	Interval]
Shortterm	1.715263	.8747175	1.96	0.082	-.2634854	3.694011
Longterm	-2.382464	.7612812	-3.13	0.012	-4.104602	-.6603262
Corporate	.0733779	.0971116	0.76	0.469	-.1463039	.2930597
MSCI	-.0076791	.011165	-0.69	0.509	-.0329361	.0175778
_cons	.0093175	.0029294	3.18	0.011	.0026908	.0159442

Thus, even if considering results from regression with the whole sample data, the parameters seem not to be significant, when we part observation in sub-samples, excluding relevant crises, both short-term and long-term coefficients are strongly significant and show relevant coefficients values. F-tests in the sub-samples regressions claim moreover for joint significance of all the parameters. When we drop data from crises periods, regression shows the following.

Linear regression 3.7: House excess returns on fin. assets excess returns (1990-2015)

```
. regress House Shortterm Longterm Corporate MSCI, vce(robust)

Linear regression                               Number of obs     =         82
                                                F(4, 77)          =       2.38
                                                Prob > F          =     0.0593
                                                R-squared         =     0.0814
                                                Root MSE          =     .01666
```

House	Coef.	Robust Std. Err.	t	P>|t|	[95% Conf.	Interval]
Shortterm	-2.445678	1.063878	-2.30	0.024	-4.564129	-.3272262
Longterm	2.501599	1.48437	1.69	0.096	-.454159	5.457356
Corporate	-.0501659	.0866944	-0.58	0.565	-.2227965	.1224647
MSCI	-.0082588	.0144072	-0.57	0.568	-.0369471	.0204295
_cons	.0090509	.0059903	1.51	0.135	-.0028773	.0209791

Considering Linear regression 3.7 and the previous results, we conclude that housing returns present significant correlations with financial assets returns in Italy, and that this provides the basis for introducing a hedge term in household portfolios of homeowners.

When we regress housing excess returns on financial assets excess returns using EWMA returns (Linear regression 3.8), we find that all parameters except corporate bonds are statistically different from zero. Moreover, the F-test shows a very high value, thus all parameters are jointly significant.

Linear regression 3.8: House excess returns on fin. assets excess returns (EWMA)

```
. regress House Shortterm Longterm Corporate Stock, vce(robust)

Linear regression                               Number of obs     =         82
                                                F(4, 77)          =      24.55
                                                Prob > F          =     0.0000
                                                R-squared         =     0.5391
                                                Root MSE          =     .28968
```

House	Coef.	Robust Std. Err.	t	P>|t|	[95% Conf.	Interval]
Shortterm	6.561941	1.281601	5.12	0.000	4.009948	9.113934
Longterm	-8.878186	1.932596	-4.59	0.000	-12.72648	-5.029896
Corporate	-.5524017	.5122116	-1.08	0.284	-1.572345	.4675419
Stock	-.1043682	.0539737	-1.93	0.057	-.2118436	.0031072
_cons	5.092959	.4993893	10.20	0.000	4.098548	6.08737

3. Data

On the basis of these evidences, it is moreover far from obvious that portfolio allocations that could be efficient in a determined period, still remained efficient after a crisis or a change in the economic environment. While households can still change their allocation in financial assets quite easily in order to compensate a shock, their investment in own housing has jet to be considered illiquid.

Financial crisis can thus be exploited to observe if households hedge their position in housing, as house prices coped with dramatically increase in volatility and experienced a strongly decreasing pattern in returns. Our further scope in Chapter 6 will be to show how the financial crisis has influenced household portfolios' efficiency.

4. Portfolio allocations

All the feasible allocations of the portfolio assets can be shown in a mean-standard deviation set of points. The left boundary of a feasible set is called the minimum variance set, since for any value of the mean rate of return, the feasible point with the smallest variance (or standard deviation) is the corresponding left boundary point. The minimum-variance set has a characteristic bullet shape and the point on this set having minimum variance is termed minimum-variance point. The portfolio associated with this minimum-variance point is called Global Minimum-Variance portfolio.

The efficient frontier is the set of optimal portfolios that offers the highest expected return for a defined level of risk or the lowest risk for a given level of expected return. Portfolios that lie below the efficient frontier are sub-optimal, because they do not provide enough return for the level of risk. Portfolios that cluster to the right of the efficient frontier are also sub-optimal, because they have a higher level of risk for the defined rate of return.

If a risk-free asset is available, the opportunity set is larger, and its upper boundary, the efficient frontier, is a straight-line segment emanating from the vertical axis at the value of the risk-free asset return and tangent to the risky-assets-only opportunity set. All portfolios between the risk-free asset and the tangency portfolio are portfolios composed of risk-free assets and the tangency portfolio, while all portfolios on the linear frontier above and to the right of the tangency portfolio are generated by borrowing at the risk-free rate and investing the proceeds into the tangency portfolio. The tangency portfolio that lies on the efficient frontier (originally consisting of only risky assets), with allowance of short sales, can be used with the risk-free security to build any other portfolio that is on the new efficient frontier. The tangency portfolio is also the one which gives the highest expected return per unit of risk and is thus the most “risk-efficient” portfolio.

Considering the Sharpe-ratio, that is the return-risk ratio, defined as:

$$Sharpe\ ratio = \frac{mean}{standard\ deviation}$$

the tangency portfolio is the one with the maximum Sharpe ratio, and thus is called Max Sharpe portfolio. It also represents the so-called market portfolio.

With EWMA quarterly excess returns we calculate asset moments, we create different portfolios with different constraints (lower and upper bounds), hence to observe what are the effects of housing constraint on allocation possibilities of households. We will compute

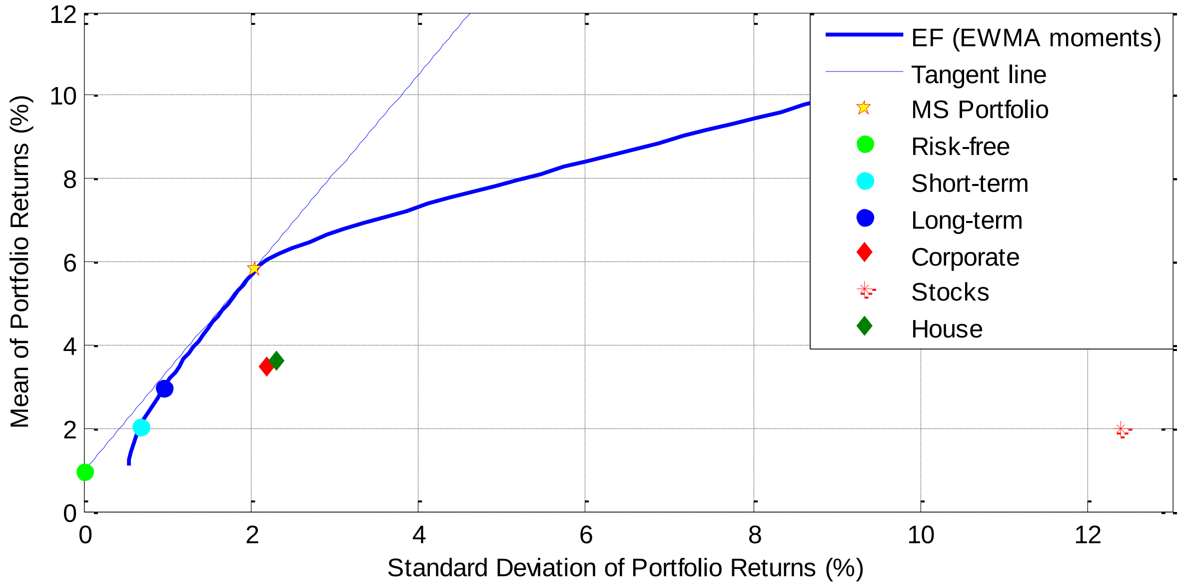
4. Portfolio allocations

Global Minimum Variance and Max Sharpe portfolio weights for all the portfolios and show how efficient frontiers change when further constraints are added.

We define wealth as the sum of all assets in a portfolio (W=1), and H=House/Wealth as the share of house value in the portfolio. Assuming that the quantity of housing held is predetermined by the household consumption demand for housing services, an additional constraint is imposed on the household portfolio allocation problem. At any given moment, both the value of housing owned, and the total net wealth of the household are fixed, and therefore the ratio of house value to net wealth, the H ratio, is a fixed value (Flavin & Yamashita, 2011). Long-term negative position (mortgage) can be taken only on long-term asset and can't be bigger than house value in modulus (Long term >= -House). Risk-free rate is set equal to mean interest rate on deposits.

To show how housing constraint influences homeowner portfolio choices, we construct some exemplifying portfolios. We start from the unconstrained case, where short selling on all assets is allowed until 300% of total wealth. Then we impose a standard set of constraints, whit short selling allowed only on short-term asset (until 1% of total wealth, i.e. c/c overdraft or credit card debt) and on long term-asset (until 300% of total wealth, i.e. mortgage). Gradually we then impose some fixed values of the H ratio, as a way to illustrate housing constraint. We will observe efficient frontiers where home value is set at 65% of total wealth (H=0.65), then progressively at 100% (H=1), 150% (H=1.5) and 200% of wealth (H=2).

Figure 4.1: Unconstrained efficient frontier



Start with an unconstrained household, thus he can take negative or positive position on all assets until 300% of his actual wealth[15]. Figure 4.1 shows the mean-standard deviation space. The risk-free rate is set at 0.97% annual return, while other assets have mean excess returns and standard deviation already calculated in Chapter 3.5. We notice that the points representing most assets are on or close to the mean-variance efficient frontier. Stocks instead appear at the right of the mean variance space, as they have the highest risk but average returns comparable to the short-term assets. House and corporate bonds seem to have a very similar trade-off between risk and return.

In order to make the most of the trade-off between risk and return, an unconstrained household should take negative position on the short-term asset and invest all his wealth plus debt on long-term bonds (Figure 4.2). However, this asset allocation is clearly not possible for real life investors.

Figure 4.2: Portfolio weight for Global Minimum Variance and Max Sharpe portfolios

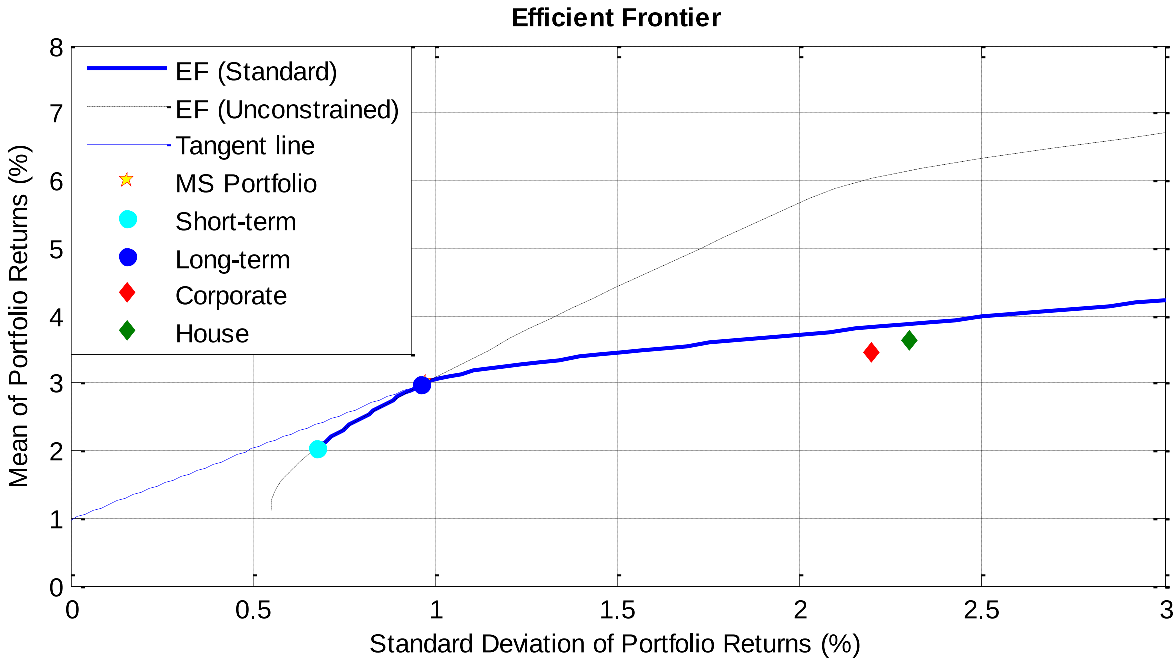
We thus impose some common-sense constraints to asset allocations as normally faced by real households. We assume a negative position is allowed only on short-term asset (up to 1% of total wealth, i.e. c/c overdraft or credit card debt) and on long-term asset (up to 300% of total wealth, i.e. mortgage). Moreover, the negative position on long-term asset cannot be higher in modulus than the house value, which is set as collateral for the debt. This also means that long-term debt can be used only to buy the main residence (as we rule out other real estate). Even these "*standard*" constraints are only a simplification of reality, as usually mortgages are granted up to 80% of the house value, not all households can take negative position in short-term, and so on, but they will useful as an example.

[15] This can be considered only as a hypothetical situation, as real households are constrained in the amount of debt they can engage and must securitize it with mortgage on real property in order to subscribe large loans, hence can never borrow more than the value of the warranty they take in place. Moreover, households can incur in debt only paying higher interest rates that the one they receive normally on deposits or short-term assets.

4. Portfolio allocations

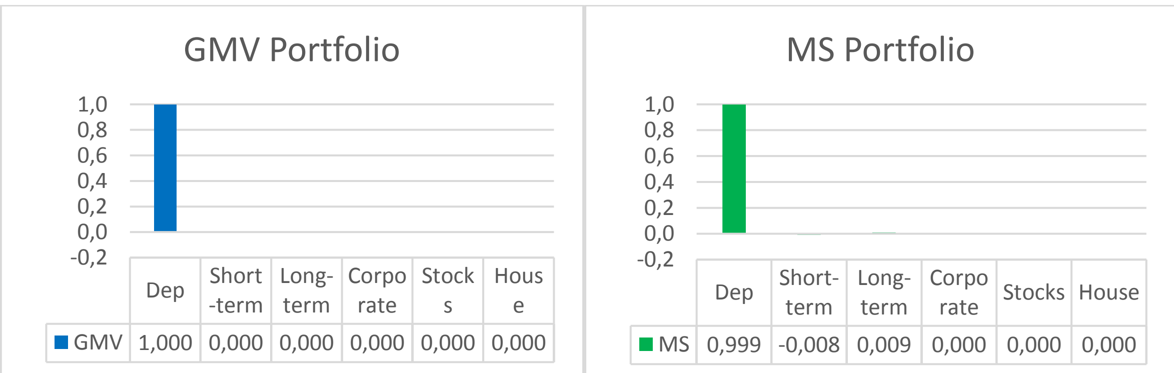
Figure 4.3 shows how the constrained efficient frontier (or "standard frontier" in the graph) compares to the unconstrained one, and thus how possible portfolio allocations change when the standard constraints described above are added.

Figure 4.3: Efficient frontiers with unconstrained and standard constraints



The tangency point between the EF with the risk-free asset and the constrained one, thus the Max Sharpe or market portfolio, lies in proximity of the long-term asset point. As Figure 4.4 shows, households who are not homeowners, should invest all their wealth in long-term assets in order to maximaze the trade-off between risk and return, as in the unconstrained case shown in figure 4.2 [16]. The GMV point of the risky efficient frontier corresponds to the short-term asset. Housing risk-return lies in the right part of the figure, having higher return but also higher variance w.r.t. other assets. Corportate bonds' risk-return is very near to housing one as already observed and also quite close to the EF.

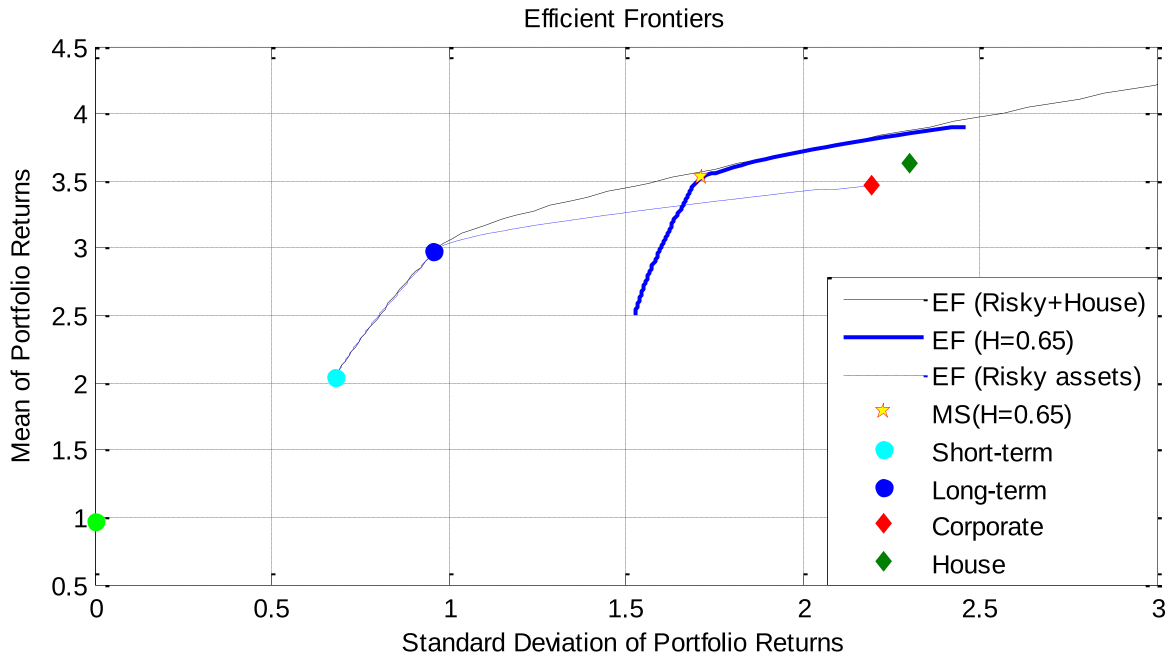
Figure 4.4: GMV and MS portfolios weights for the EF with standard constraints



[16] In order to obtain the portfolio with lowest risk, households should invest all their wealth only in the risk-less asset as in the unconstrained case.

Now we set an addional constraint on household portfolios: we assume that households own their home. As we have seen in Chapter 3, this applies to about 70% of Italian households. Assume that home represents 65% of household wealth. This is still a conservative value, as data from SHIW show that only households whose heads are more than 65 years or very wealthy present such a low value of the housing to wealth ratio (H ratio). Figure 4.5 shows how the constrained EF, that is based on H=0.65, shifts to the right relative to the unconstrained one and hence is dominated. The right shift occurs because homeowners have to maintain a big fraction of their wealth on house investment, hence they increase their portfolio risk. In the figure we can also observe how the EF with only risky financial assets (the blue dotted line) relates to the ones including house as an asset. If compared with the EF with unconstrained housing, we see that the EF with only financial assets is always dominated (at least, to the right of long-term bonds). For level of standard deviation higher than 1.7%, the frontier made by risky financial assets plus house (the black line) dominates the one composed only by risky financial assets. This means that adding housing to household portfolios can increase efficiency even in the constrained case, but forces households to sustain a higher level of risk.

Figure 4.5: Standard EF and EF with housing at 65% of wealth [17]



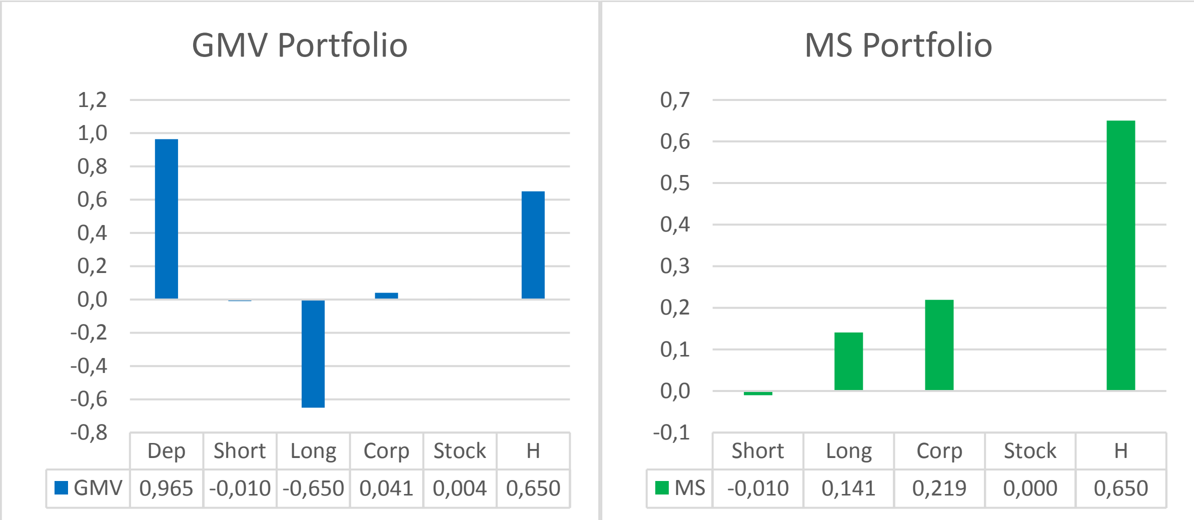
From Figure 4.6 we see that the MS allocation consists now in investing about 21.9% of non-housing wealth in corporate bonds and 14.1% in long-term bonds, while to obtain the lowest

[17] The Efficient Frontier of risky assets + house has to be considered as the one with standard constrained, where house is still considered as a liquid asset and is thus unconstrained.

4. Portfolio allocations

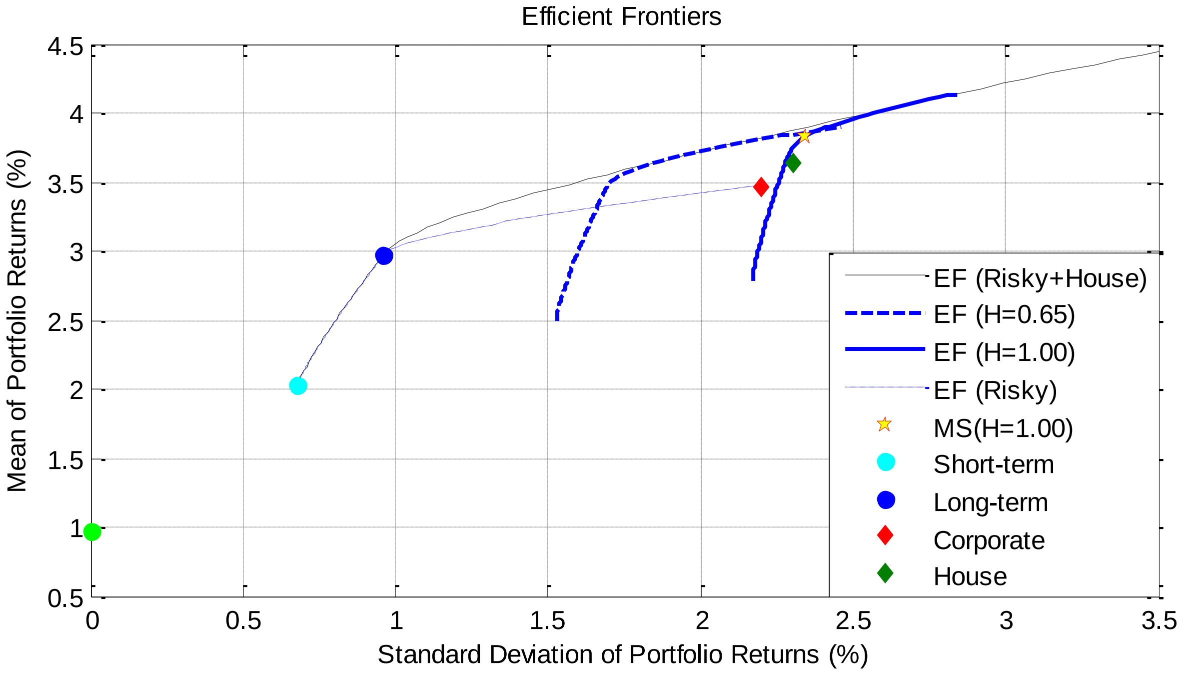
risk portfolio, households should take negative position on the long-term asset, thus take up a mortgage to buy their home for about all of its value, invest 4% of wealth in corporate bonds and keep an amount equal to about 96.5% of their wealth in deposits (the risk-less asset).

Figure 4.6: GMV and MS portfolios weights for the EF with H=0.65



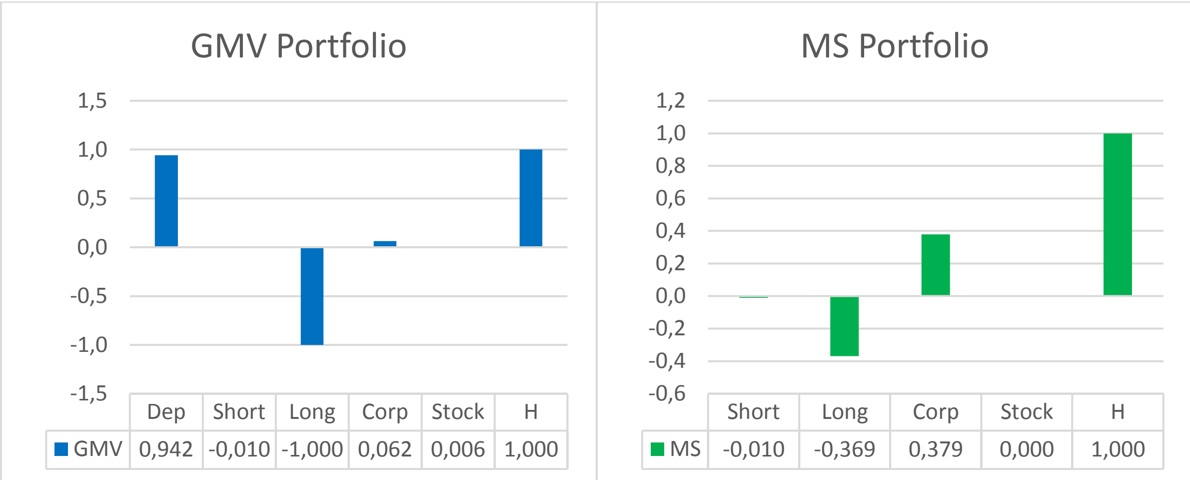
When home value is set equal to the value of a household total wealth as in Figure 4.7, EF (H=1.0, the bold blue line) moves further to the right and is now slightly above the risk-return coordinates of housing. We see that the points of the EFs with house constraint further to the right are almost coincident with the unconstrained standard EF. Thus, for homeowners, riskier allocations are the most efficient ones, if compared to the unconstrained households.

Figure 4.7: Standard EF and EF with housing at 65% and 100% of wealth [17]



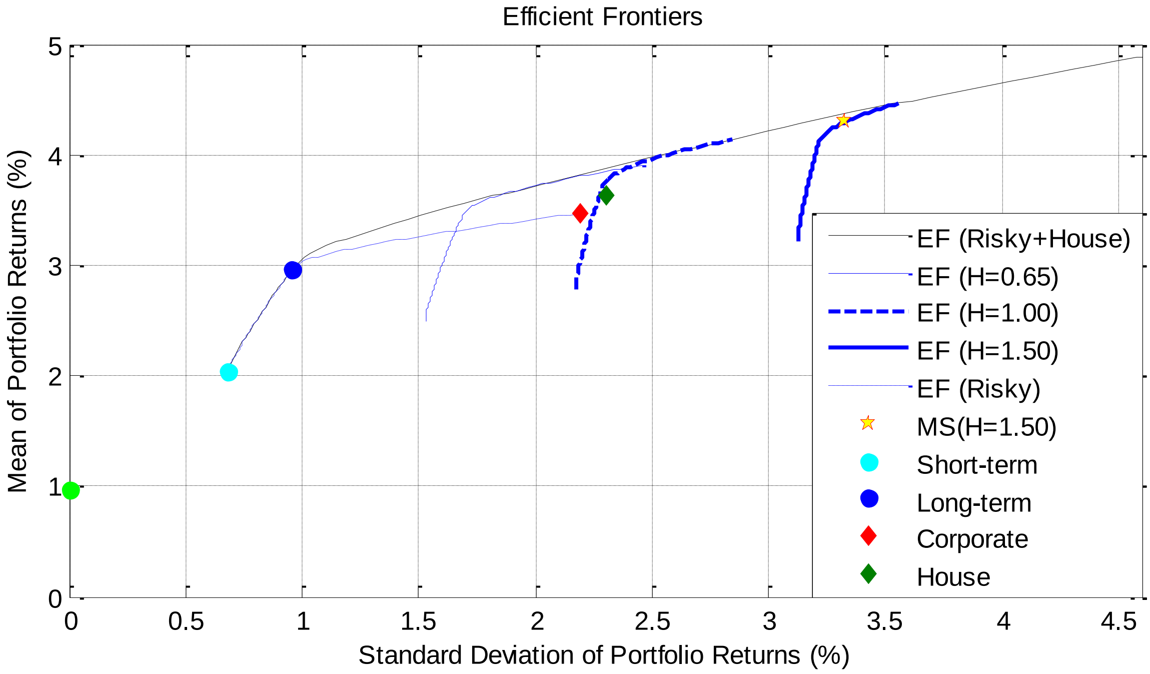
GMV portfolio still implies financing the whole home value with long-term debt and keeping 94% of wealth in deposits and 6.2% in corporate bonds. The market portfolio is instead obtained with a debt for 36.9% of wealth and investing 37.9% of wealth in corporate bonds, thus keeping also 1% in short-term debt.

Figure 4.8: GMV and MS portfolio weights for the EF with H=1.00



If households have low wealth relative to their desired home value, they will have to borrow. Hence their efficient frontier will move far to the right, increasing even more the portfolio risk, but compensating with a relatively small increase in return (Figure 4.9). The right tail of EF with housing set at 150% of wealth is shorter than the right tails of other EFs. This occurs as the debt burden further limits homeowners' investment possibilities.

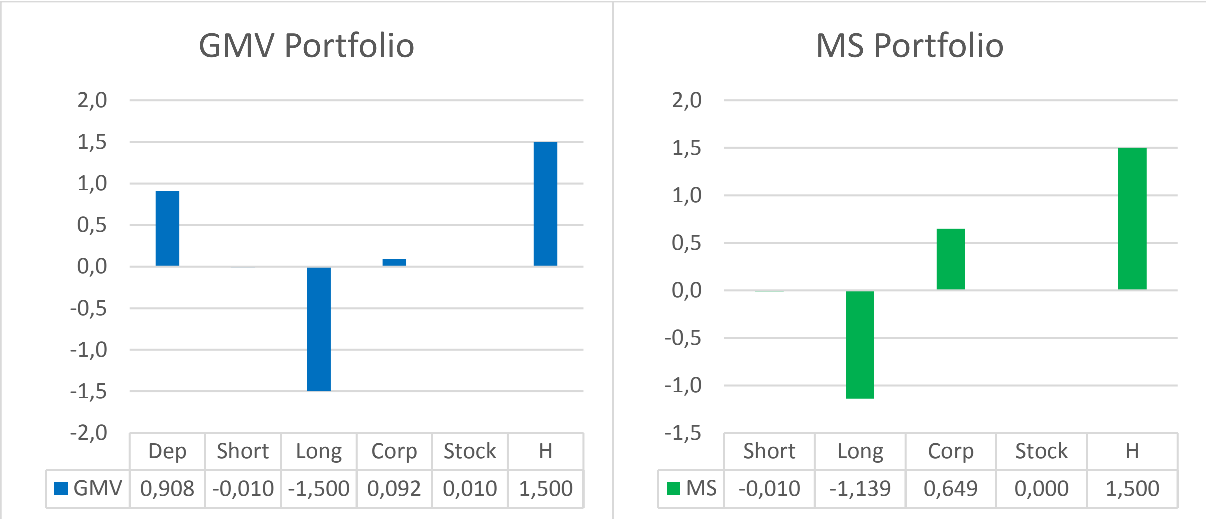
Figure 4.9: Standard EF and EF with different levels of housing [17]



4. Portfolio allocations

Homeowners should finance all the home value with long-term debt and keep 90.8% of their wealth in deposits in order to minimize their portfolio risk (Figure 4.10). Those who instead want to maximize the risk-return trade-off should keep debt at 113.9% of their wealth and invest all they have in corporate bonds, for a fraction of 64.9% of their wealth. This is however an unpleasant solution as we can see from the EFs graph that risk level is very high if compared with single assets risk.

Figure 4.10: GMV and MS portfolio weights for the EF with H=1.50



In the extreme case when a household home weighs 200% of their wealth, as could happen for very poor or young homeowners, the market portfolio lies close below the EF without home constraint (Figure 4.11). The right tail of the EF with H=2.00 is even shorter than the one with H=0.5, as homeowners with such a level of housing relative to total wealth should bump into an excessive debt burden, for at least 100% of total wealth.

Figure 4.12 shows that GMV portfolio is composed by 200% of debt, 87.5% of deposits, 12.2% of corporate bonds and 1.3% of stocks, while the market portfolios is made by a negative position in long-term bonds for 194.5% of wealth and 95.5% invested in corporate bonds, with a negative position of 1% also in short-term assets.

We can conclude that, as homeowners increase the value of their home relative to their total wealth, and thus increase the H ratio, their Max Sharpe portfolio allocation comes closer to the unconstrained EF, while the risk of their portfolios increases dramatically. Their portfolio could thus be considered efficient also in the standard sense, without conditioning on housing. Nevertheless, when adding Italian household portfolios in this mean-variance framework, we can make some further considerations that relate to the efficiency test described in Chapter 5.

Figure 4.11: Standard EF and EF with different levels of housing [17]

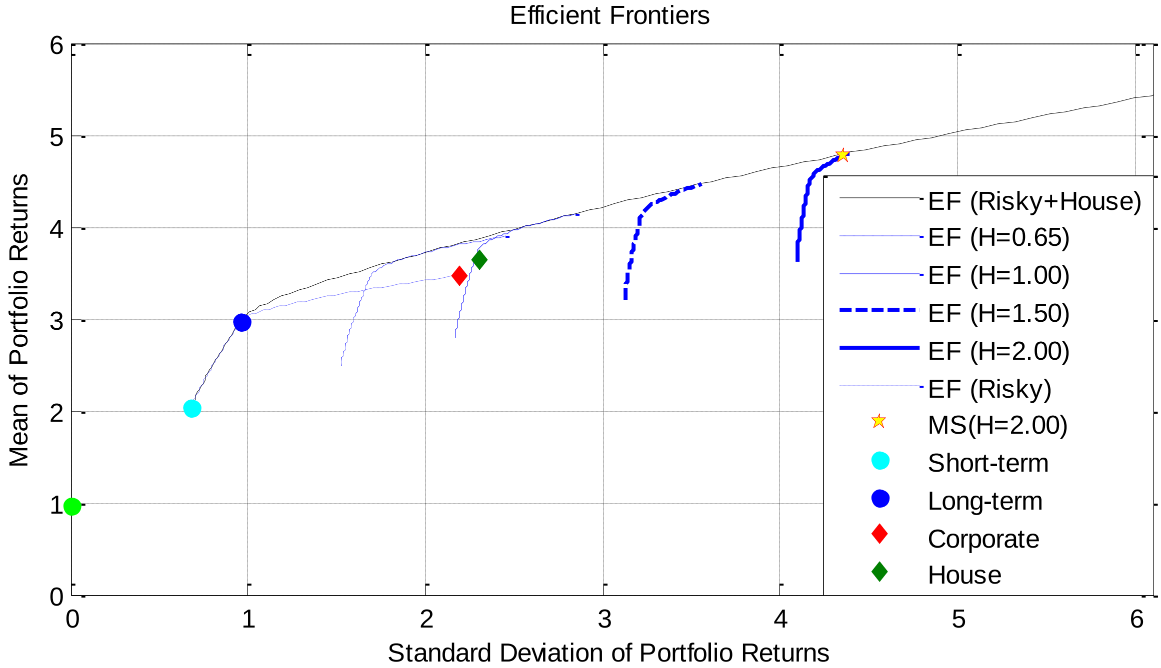
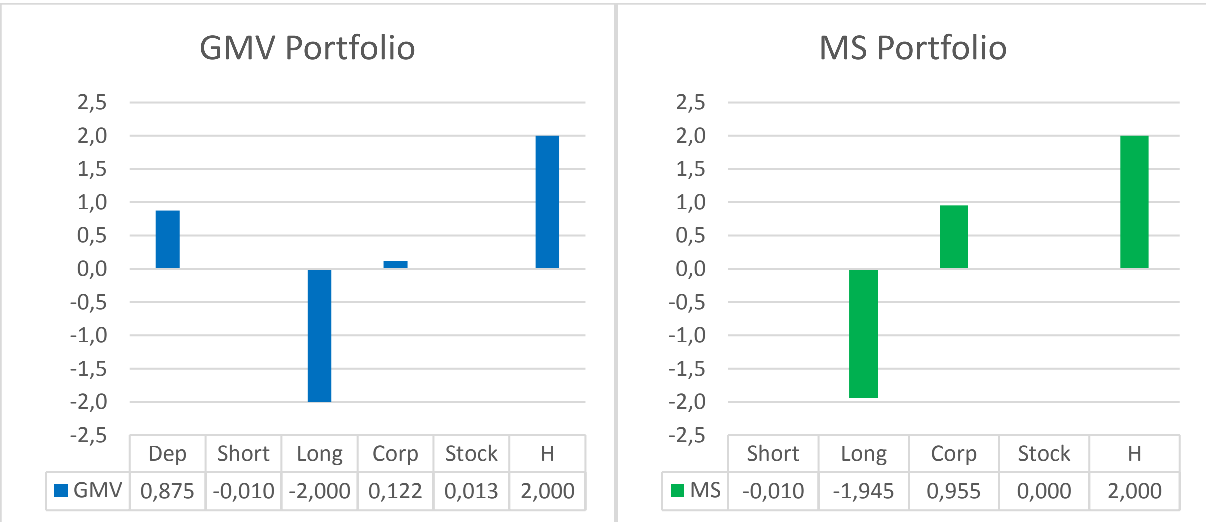


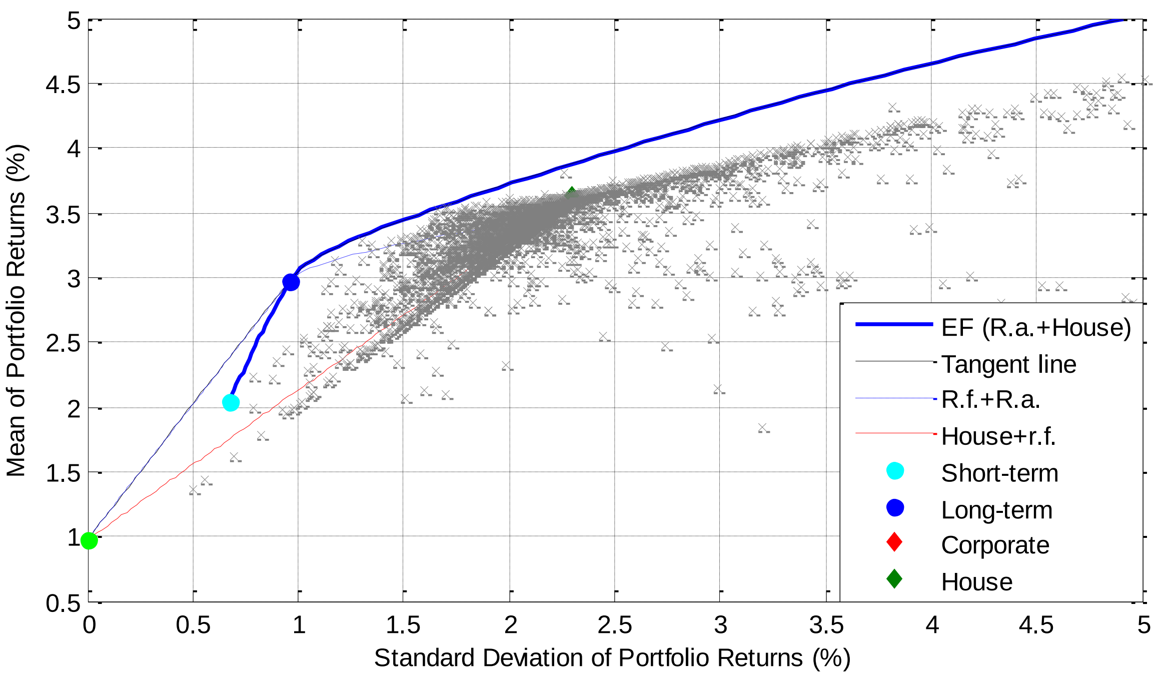
Figure 4.12: GMV and MS portfolio weights for the EF with H=2.00



5. Testing for efficiency

Figure 5.1 shows how portfolios of homeowners (the x dots) stand in relation with the EFs we have seen in the previous chapter. We observe that all homeowner portfolios lie under what we called the "standard" efficient frontier with risky asset and housing (the bold blue line) and many portfolios seem to lie on a straight line that goes from the risk-less asset to housing coordinates (the red line in the graph). This happens because, as we have seen in Chapter 3.4, a large proportion of homeowners keep all their liquid wealth in deposits (about 40% of Italian households hold only deposits and own house).

Figure 5.1: Efficient frontiers with Italian household portfolios [17]



We previously supposed that households could keep efficient portfolios even if it would imply a large value for H ratio and a higher risk portfolio, but from the graph it seems that many portfolios lie far below the standard EF. It is thus clear that we need to compute a statistic to test the efficiency of household portfolios conditional on their housing constraint.

We perform firstly the formal efficiency test ξ_e (only for financial portfolios), described in equation 21 in Chapter 2, and compute the statistics for all households and for sub-samples of those households who own risky assets. We compute the test both with sample and EWMA returns and different values of the test size (90% and 95%).

5. Testing for efficiency

Table 5.1: Efficiency test for financial portfolios

Standard test ξ_e (% of efficient portfolios)				
Test size	**90%**	**95%**	**90%**	**95%**
Number of portfolios	*Sample moments*		*EWMA moments*	
Risk-free (5,654)	100%	100%	100%	100%
Risky (2,471)	2.46%	2.50%	34.02%	53.22%
Total (8,125)	70.29%	70.30%	79.90%	85.75%

Table 5.1 shows results from the standard ξ_e test. Considering only financial portfolios, we disregard mortgages. Looking at statistics with EWMA moments calculation, when considering the full sample size, about 80% of portfolios are considered efficient when the test is computed at the 10% level and almost 85% when ξ_e is computed at the 5% level. This result is however biased by the fact that portfolios composed by risk-less asset only are trivially efficient, and this is the case for a large part of Italian households. When we consider only risky financial portfolios, the results are still good, with about 34% of portfolios considered as efficient at the 10% level and 53% at the 5% level. If we compute the test with sample moments we find that only about 2.5% of risky financial portfolios are considered as efficient.

Markedly different conclusions on the efficiency of household allocations are reached if the investment set is extended to housing and mortgages, and housing is treated as unconstrained (Table 5.2). At any size of the test computed with EWMA returns, there are very few efficient portfolios, while when the test is done using sample returns, no efficient portfolios can be found.

The test when housing is treated as unconstrained (with EWMA returns) shows only 3 efficient portfolios jet and this can be explained looking at the distance of household portfolios from the unconstrained risky frontier as shown in Figure 5.1.

Table 5.2: Efficiency test including house as unconstrained

Standard test ξ_e (% of efficient portfolios)				
Test size	**90%**	**95%**	**90%**	**95%**
Number of portfolios	*Sample moments*		*EWMA moments*	
Home + R.f. (3,722)	0%	0%	0%	0%
Home + R.a. (2,074)	0%	0%	0.14%	0.14%
Tot. homeowners (5,796)	0%	0%	0.000517%	0.000517%
Total (8,125)	0%	0%	0.000369%	0.000369%

We have already argued in Chapter 2 that we must consider the illiquid nature of housing. If households keep a large fraction of their wealth in housing for reasons other than investment (because rental markets are imperfect, due to information asymmetries, pride of ownership, and so on), and do not trade frequently because of high pecuniary and non-pecuniary transaction costs (Flavin and Nakagawa (2004)), then we should estimate their portfolio efficiency conditional on housing. It is, in fact, plausible that their financial decisions are partly dictated by the need to hedge some of the risks connected with their illiquid housing investment (Pelizzon & Weber, 2008). For each household that has non-zero housing wealth, we can compute a specific conditional efficiency test as done by Gouriéroux & Jouneau (1999) that treats housing as constrained. It is obvious that in the constrained case the risk-free portfolio cannot be attained, except trivially (zero housing).

We compute the test statistic for the conditional portfolios, ξ_1 (defined in equation (20), Chapter 2), and calculate for how many portfolios the test fails to reject the null hypothesis of mean-variance efficiency at 90% and 95% significance levels. The test is not defined in the case of portfolios made entirely of risk-free assets (it is a ratio of zero to zero), and is identical to the standard test (ξ_e) for portfolios consisting of just financial assets.

Table 5.3: Efficiency test conditional on housing

Constrained test ξ_1 (% of conditionally efficient portfolios)				
Test size	**90%**	**95%**	**90%**	**95%**
Number of portfolios	*Sample moments*		*EWMA moments*	
Home + R.f. (3,722)	0%	0%	100%	100%
Home + R.a. (2,074)	8.49%	12.49%	82.84%	83.7%
Only homeowners (5,796)	3.04%	4.43%	93.86%	94.17%
Tot. households (8,125)	2.40%	3.57%	94.99%	95.26%

Table 5.3 shows that the test with EWMA moments presents a very high percentage value of efficient household portfolios. Portfolio allocations seem to be efficient for over 94% of households. However, we notice that the percentage of efficient portfolios if the test is computed using sample moments is very low, with about 2.4% of the portfolios considered efficient when the test is computed at the 10% level, and about 3.6% at the 5% level.

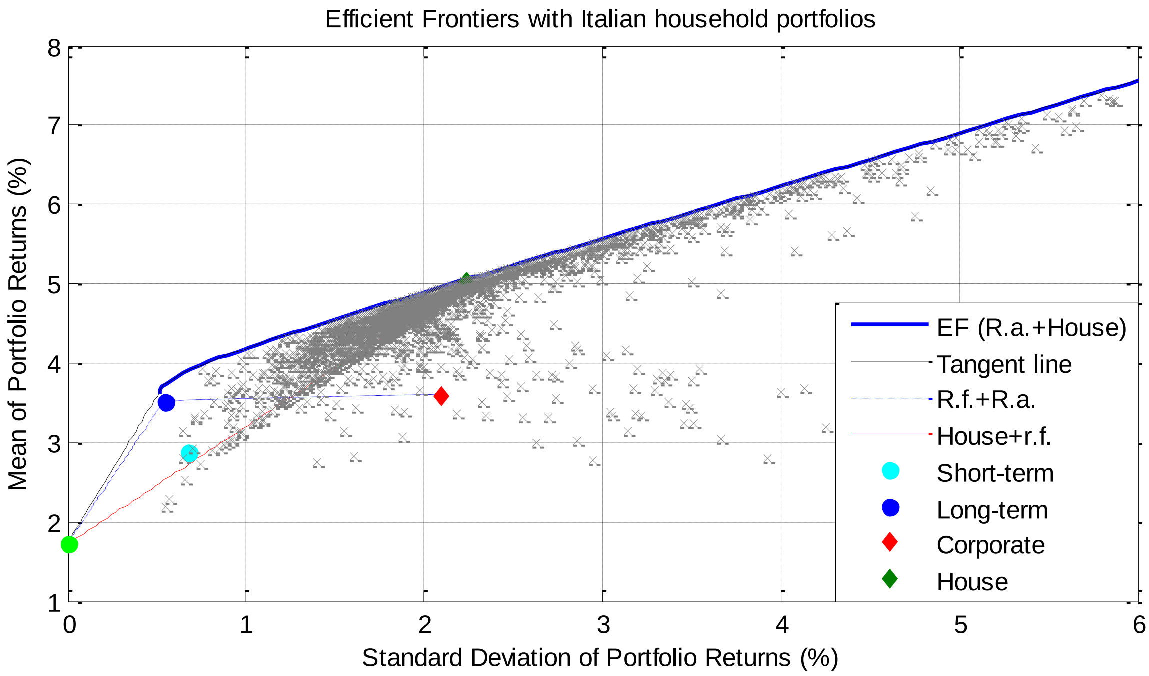
Thus, we should consider an important doubt regarding our results: why do the tests computed with sample moments and with EWMA moments give such a different picture of Italian household portfolios efficiency? Why with sample moments still very few portfolios seem to be efficient while with EWMA almost the 94% are efficient?

5. Testing for efficiency

What matters for the answer to this question is the covariance matrix, and thus the role of the coefficients of the hedge term we have derived in Chapter 2 in determining the efficiency test results.

Up until now we have work using EWMA moments, but let us plot the graph with homeowner portfolios and EFs using sample returns and covariance matrix (Figure 5.2).

Figure 5.2: EF and portfolios obtained using sample returns [17]



As shown in Chapter 3.5, sample moments provide higher average housing returns compared to EWMA. Hence the EF with housing (the bold blue line in Figure 5.2) strongly dominates the one with only financial assets (the blue dot line which starts from long-term asset and ends at corporate bonds one). Many portfolios lie on the red line, that is the EF with only house and risk-free. So why do so many portfolios fail the efficiency test we have jet computed when using sample returns? Comparing the composition of GMV and MS portfolios with sample and EWMA moments, as shown in following figures, helps to explain this result.

When we compare GMV and MS portfolios when the house is unconstrained (Figure 5.3), no differences can be practically found, as households should keep all their wealth in deposits. But when we add housing as a constrained asset, the situation in GMV portfolios changes in a substantial way, as shown in Figure 5.4.

Figure 5.3: GMV and MS portfolios for unconstrained households

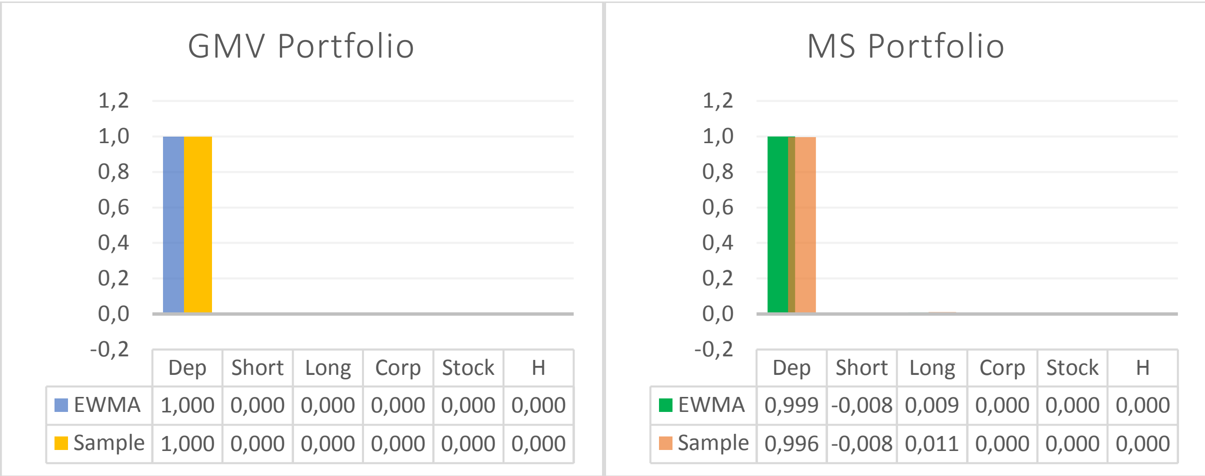
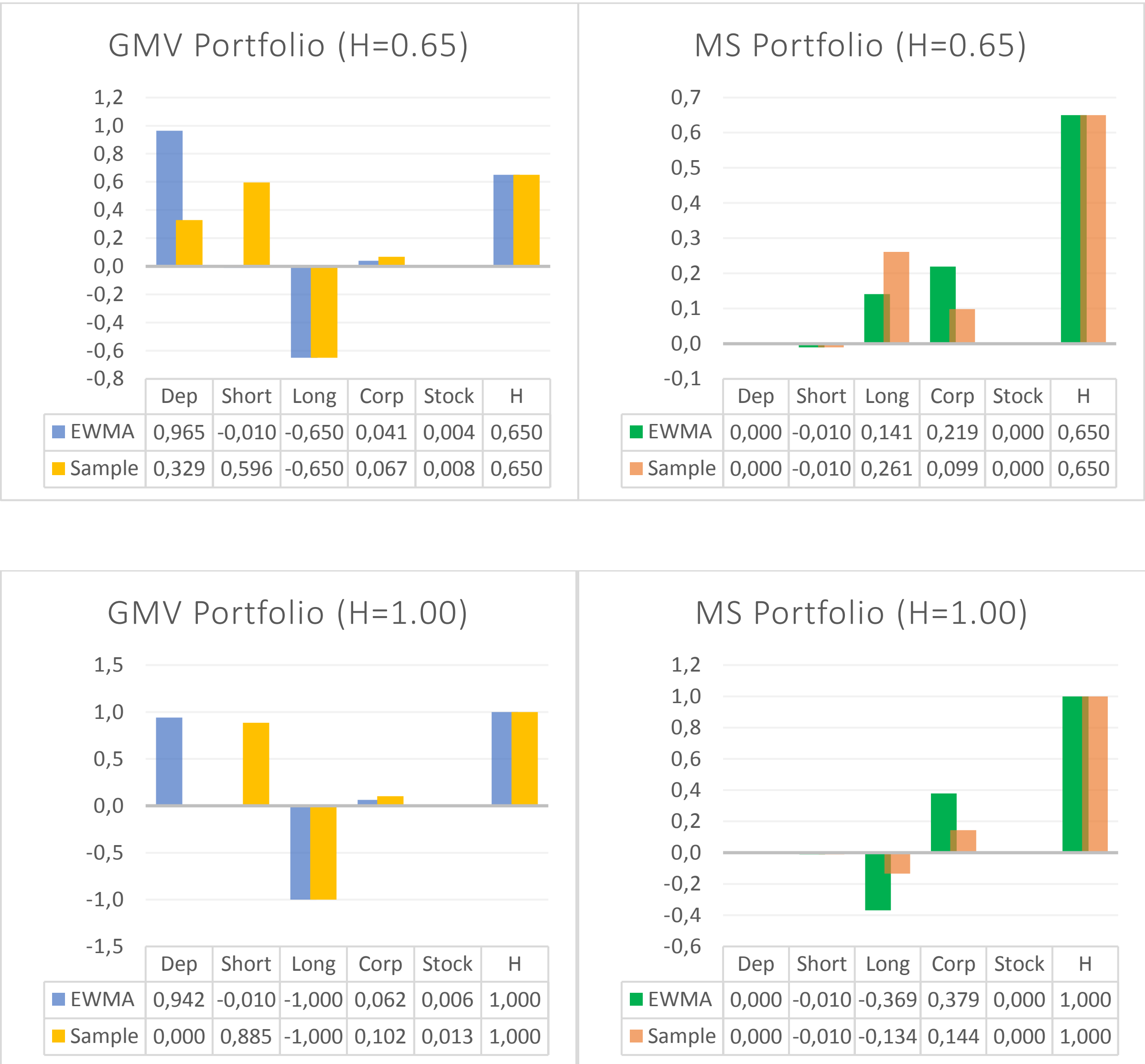
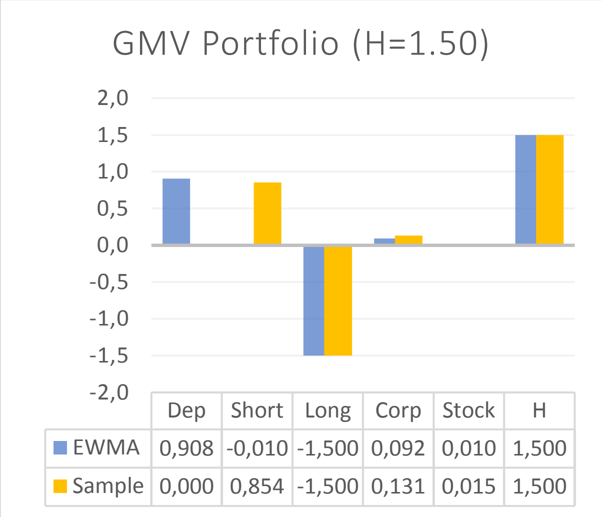
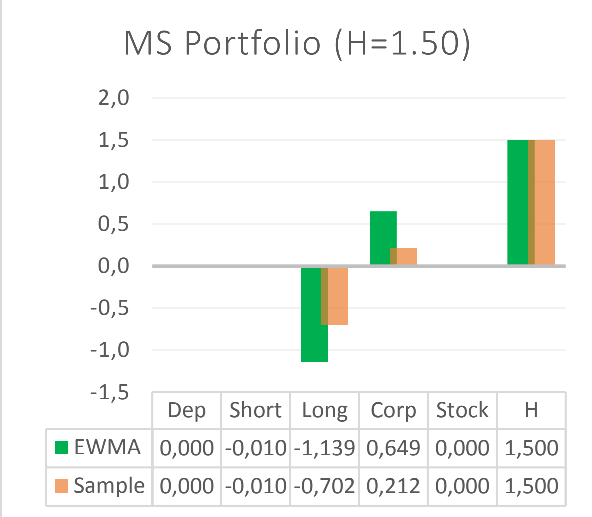


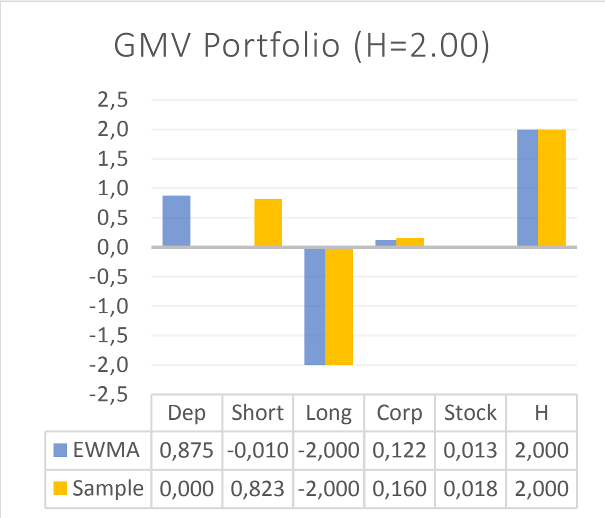
Figure 5.4: GMV and MS portfolios for constrained homeowners (H ratio in parentesis)

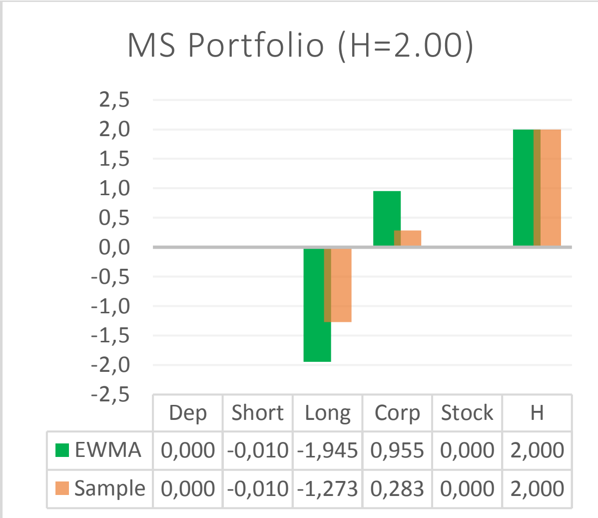


5. Testing for efficiency









Using sample returns, the optimal portfolio for homeowners who want to minimize risk is still the one which finances all the home value with mortgage, but households should not keep most of the wealth in deposits (as is the case for most of Italian households). Instead they should invest a large fraction of wealth in short-term government bonds. This condition is not true for the majority of households (see Chapter 3) and thus the test with sample moments shows so few efficient portfolios when is done conditional to housing.

We now consider the 2,074 fully diversified portfolios, thus for homeowners who keep risk-free asset, at least one risky financial asset and housing. In Tables 5.4 and 5.5, we cross-tabulate diversified financial portfolios and total conditional portfolios according to the efficiency criterion. We use the test results computed at the 10% level for both test statistics.

Even if the tests show very different percentages of efficient portfolios with EWMA and sample returns, we find that in both the tests, the number of conditionally efficient portfolios is bigger than the number of financially efficient portfolios.

Table 5.4: Number of efficient portfolios with EWMA returns

EWMA	Efficient (financial)	Inefficient (financial)	Total
Efficient (conditional)	654	1,064	1,718
Inefficient (conditional)	1	355	356
Total	655	1,419	2,074

Table 5.5: Number of efficient portfolios with sample returns

Sample	Efficient (financial)	Inefficient (financial)	Total
Efficient (conditional)	0	176	176
Inefficient (conditional)	58	1,840	1,898
Total	58	2,016	2,074

With EWMA, only 1 portfolio results to be classified as efficient when housing is neglected, but inefficient when it is considered, while using sample returns 58 portfolios are classified as financially efficient but inefficient conditionally. On the other side, 654 portfolios are considered as conditionally efficient but inefficient financially when the test is computed with EWMA returns, while 176 when using sample returns. This suggests that hedging opportunities are well exploited and could be evidence that these households use financial assets to hedge housing risk, but could also reveal that housing has diversification properties (for homeowners, financial risks are relatively small compared to total wealth). Given the high correlations and the very large weight attached to housing wealth, the failure to exploit hedging opportunities could outweighs the benefits from diversification.

It is of interest to notice that all portfolios that are found to be conditionally efficient when the test is computed with sample returns at the 5% level are found to be conditionally efficient also when computing ξ_1 with EWMA returns both at the 5% and at the 10% level.

The crucial point for the reliability of our efficiency test is thus to use the correct returns for moments estimation. For financial returns, EWMA is commonly adopted both in literature and in financial application, especially for risk valuation. For housing returns, unweighted sample averages are normally taken because of the lack of frequent data streams. We thus follow standard practice in both areas by mixing the two methods. We decide to use EWMA to obtain assets returns, as they better reflect current economic situation and take into account for the effects of financial crisis, while we use sample estimators for covariances. We then compute the test using EWMA returns and sample covariance matrix.

5. Testing for efficiency

Table 5.6 shows the standard test (ξ_e) for only financial portfolios, Table 5.7 the standard test when housing is considered as unconstrained and Table 5.8 the constrained test (ξ_1) conditional on housing. The test results with mixed moments are the red ones.

Notice how the introduction of mixed moments change the test results. While in the case of efficiency for only financial portfolios, percentage values are still near the one calculated with sample returns (as the Σ matrix is the same), when we run the constrained test they show a slight increase in portfolios considered as conditionally efficient, with 11.62% considered as conditionally efficient at the 10% level and 15.24% at the 5% level. Percentages of efficient portfolios are however always very low if compared with the one obtained using EWMA method for obtaining both excess returns and covariances.

Table 5.6: Efficiency test for financial portfolios

Standard test ξ_e (% of efficient financial portfolios)						
Test size	**90%**	**95%**	**90%**	**95%**	**90%**	**95%**
Number of portfolios	*Sample moments*		*EWMA moments*		*Mixed*	
Risk-free (5,672)	100%	100%	100%	100%	100%	100%
Risky (2,484)	2.46%	2.50%	34.02%	53.22%	2.46%	2.46%
Total (8,156)	70.29%	70.30%	79.90%	85.75%	70.29%	70.29%

Table 5.7: Efficiency test including house as unconstrained

Standard test ξ_e (% of efficient portfolios including house as unconstrained)						
Test size	**90%**	**95%**	**90%**	**95%**	**90%**	**95%**
Number of portfolios	*Sample moments*		*EWMA moments*		*Mixed moments*	
Home + R.f. (3,722)	0%	0%	0%	0%	0%	0%
Home + R.a. (2,074)	0%	0%	0.14%	0.14%	0%	0%
Tot. homeowners (5,796)	0%	0%	0.000517%	0.000517%	0%	0%

Table 5.8: Efficiency test conditional on housing

Constrained test ξ_1 (% of conditionally efficient portfolios)						
Test size	**90%**	**95%**	**90%**	**95%**	**90%**	**95%**
Number of portfolios	*Sample moments*		*EWMA moments*		*Mixed moments*	
Home + R.f. (3,722)	0%	0%	100%	100%	0%	0%
Home + R.a. (2,074)	8.49%	12.49%	82.84%	83.7%	11.62%	15.24%
Tot. homeowners (5,796)	3.04%	4.43%	93.86%	94.17%	4.16%	5.45%

In order to understand how expectations on returns and covariance matrix, and thus the coefficient of the hedge term, change over time periods (as we saw in Chapter 3.6 regressing housing excess return on financial assets ones), in the next part we will compare household portfolios efficiency in 2008 (thus before Italian households felt the effects of financial crisis) and 2014. This comparison will be useful to understand how Italian households have reacted to the financial crisis and how they have changed their portfolio allocations. Our interest is to show if households held efficient portfolios before the crisis and if they managed to correctly evaluate its impact on their asset returns, reaching efficiency after the recovery.

6. Effect of the financial crisis on efficiency

We start computing the efficiency test for household portfolios in 2008. For moments calculation we use the same methods as for 2014. We use data until December 2007, thus we have 72 quarterly observations for returns, while portfolios data come from SHIW 2008, which covers 7,977 households. If we exclude outliers we remain with 7,928 household portfolio observations, of which 2,156 invest at least in one risky financial asset.

As far as the standard test for only financial portfolios concerns (excluding mortgages and home) in Table 6.1, results are quite similar to the ones for 2014, but the percentage of efficient portfolios is slightly higher than the one for 2014 portfolios. When ξ_e is computed with sample returns 4.92% risky portfolios are considered efficient both at the 10% and at the 5% level (they were 2.5% in 2014), while with EWMA returns about 54% of risky portfolios seem to be efficient at both 10% and 5% level (34% at the 10% level and 53% at the 5% level in 2014). Hence, when we consider only financial portfolios and compute efficiency with the standard test, it seems that Italian household portfolios were more efficient in 2008 than in 2014.

Table 6.1: Efficiency test for financial portfolios 2008

Standard test ξ_e (% of efficient financial portfolios)				
Test size	**90%**	**95%**	**90%**	**95%**
Number of portfolios	*Sample moments*		*EWMA moments*	
Risk-free (5,772)	100%	100%	100%	100%
Risky (2,156)	4.92%	4.92%	54.45%	54.55%
Total (7,928)	74.14%	74.14%	87.61%	87.64%

Table 6.2 displays the efficiency standard test (ξ_e) for 2008 when housing is included as unconstrained in household portfolios. The test is computed for homeowners, as for non-homeowners the test results are the same as seen in previous table. We end up with 5,605 portfolio observations, of which 1,833 include at least a risky asset (that could be just the mortgage).

Now the results are completely different from the previous case, and only a couple of portfolios are found to be efficient when the test is computed using EWMA returns. This mirrors the situation we have observed for 2014 when computing ξ_e when house is considered as unconstrained.

6. Effect of the financial crisis on efficiency

Table 6.2: Efficiency test including house as unconstrained 2008

Standard test ξ_e (% of efficient portfolios including house as unconstrained)				
Test size	**90%**	**95%**	**90%**	**95%**
Number of portfolios	*Sample moments*		*EWMA moments*	
Home + R.f. (3,722)	0%	0%	0%	0%
Home + R.a. (1,833)	0%	0%	0.16%	0.16%
Tot. homeowners (5,605)	0%	0%	0.0053524‰	0.000535%

Unexpected results come when we compute the constrained test ξ_1 for conditional efficiency on 2008 portfolios (Table 6.3). While in 2014 we found that almost 93% of portfolios were considered efficient computing the test with EWMA returns and about 3% when using sample moments, in 2008 only very few portfolios seem to be conditionally efficient both when the test is computed at the 10% or at the 5% level. These results are somewhat strange, as we would have expected to find a larger proportion of efficient household portfolios before the financial crisis.

Table 6.3: Efficiency test conditional on housing 2008

Constrained test ξ_1 (% of conditionally efficient portfolios)				
Test size	**90%**	**95%**	**90%**	**95%**
N	*Sample moments*		*EWMA moments*	
Home+R.f. (3,722)	0%	0%	0%	0%
Home+R.a. (1,833)	0.00546‰	0.11%	0.22%	0.22%
Tot. homeowners (5,605)	0.00178‰	0.00357‰	0.00714‰	0.00714‰

When we consider the 1,833 fully diversified portfolios in 2008 (risk-free, risky financial assets, and housing) and we cross-tabulate diversified financial portfolios and total conditional portfolios according to the efficiency criterion (at the 10% level for both test statistics), we obtain results shown in Tables 6.4 and 6.5.

As already observed, the number of only conditionally efficient portfolios is almost equal to zero, with only 4 portfolios considered both as financially and conditionally efficient using EWMA returns, while we find that a good number of portfolios, about 988, is considered as financially efficient but inefficient conditional on housing. Finally, 841 portfolios are inefficient both conditionally and financially.

Table 6.4: Number of efficient portfolios with EWMA returns 2008

EWMA	Efficient (financial)	Inefficient (financial)	Total
Efficient (conditional)	4	0	4
Inefficient (conditional)	988	841	1,829
Total	992	841	1,833

When we use sample returns for computing ξ_e and ξ_1, one out of 1,833 portfolios is considered conditionally efficient but inefficient financially, 101 portfolios are only financially efficient, and 1,731 portfolios are found to be both financially and conditionally inefficient.

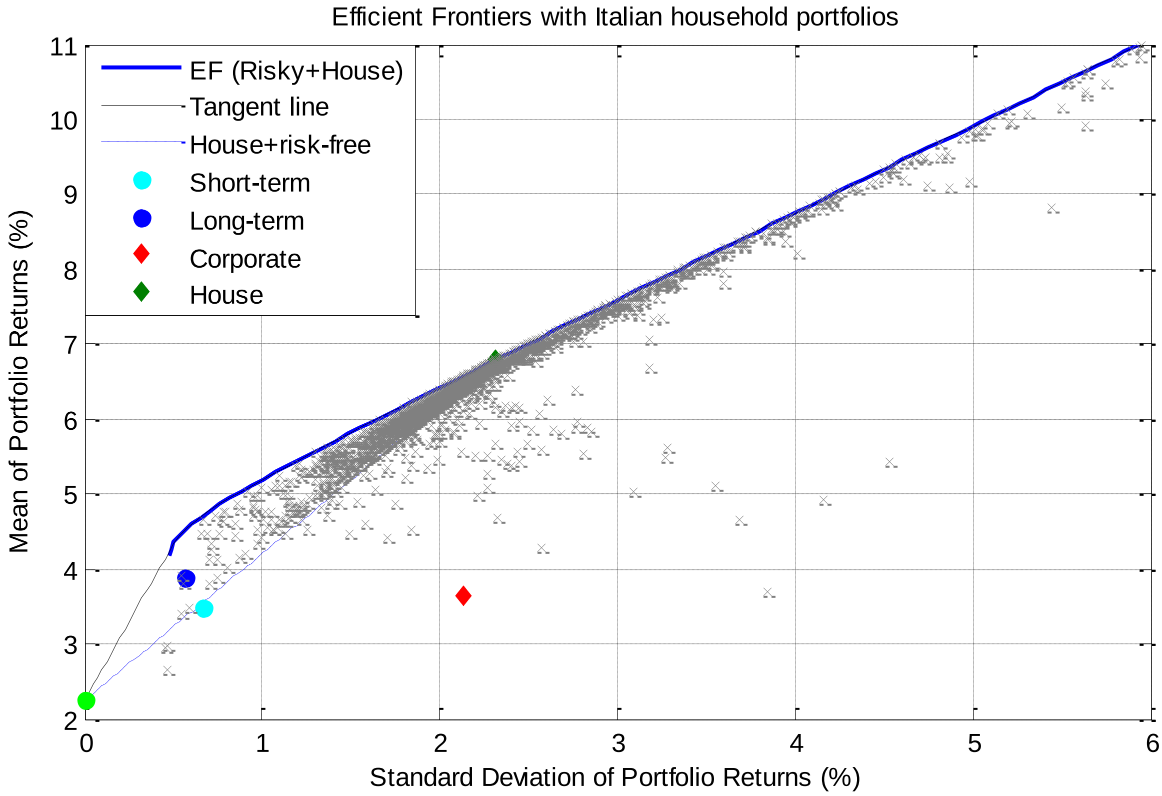
Table 6.5: Number of efficient portfolios with sample returns 2008

Sample	Efficient (financial)	Inefficient (financial)	Total
Efficient (conditional)	0	1	1
Inefficient (conditional)	101	1,731	1,832
Total	101	1,732	1,833

As we have found a greater number of conditional efficient portfolios in 2014 with respect to 2008, it seems that households have better exploited hedging opportunities in the wake of the financial crisis. This could be evidence that households use financial assets to hedge housing price risk, which was dramatically increased after 2007. This situation also suggests that homeowners were not considering 2008 EFs when allocating their assets, in particular as regards housing. It could be the case that homeowners were lacking information about recent years prior to the crisis, or more interestingly, it could be that homeowners were expecting the EFs to change in the near future and thus had already reallocated their assets according to new expectation on returns. In fact, the years before 2008 where characterized by a steadily positive trend in house prices. Households may have predicted that this trend would come to a halt, and therefore could have foreseen different expected returns w.r.t. the ones we observed looking at data until December 2007.

The picture of Italian portfolios relative to EFs in 2008 calculated with sample returns, in Figure 6.1, is quite similar to the one in 2014. However, due to the higher mean expected return of housing prior to the crisis, the EF with all assets is more positively sloped. In 2008 corporate bonds had an expected excess return between the ones of short-term and long-term assets, but had a higher risk, similar to the one in 2014. Short-term assets had lower expected excess return than long-term bonds, but surprisingly had a higher risk (notice the position of the light blue point, thus short-term mean risk-return, in relation to the blue one, thus long-term mean risk-return). We also see that short-term assets are dominated by the EF with only house and risk-free asset, the blue dotted line that goes from the light green point (risk-free) to the green diamond (house), and that many homeowner portfolios, which include only house and deposits, lie on that line. House risk-return point lies on the standard EF, as prior to the burst of housing bubble, housing returns were high relative to house price risk and thus house resulted to be a good investment.

Figure 6.1: Efficient frontiers with Italian household portfolios (sample returns - 2008)



If we compare the EFs in 2008 and 2014 constructed using EWMA returns (Figure 6.2), we immediately notice the difference in slopes, and the fact that 2014 EFs are dominated by 2008 ones. As regards expected returns and risk of assets, all, except corporate bonds, in 2014 have a smaller mean return and standard deviation, in particular housing. A hypothetical investor who owned only his home, with value set at 100% of wealth, would have had an 8% expected annual excess return in 2008, while only of about 3.6% in 2014. The risk-free rate was also different, as in 2008 keeping wealth in deposits would have brought a 1.9% mean annual return, while in 2014 the risk-free interest rate was set at about 1%.

Figure 6.2: Efficient frontiers in 2008 and 2014 comparison (EWMA returns)

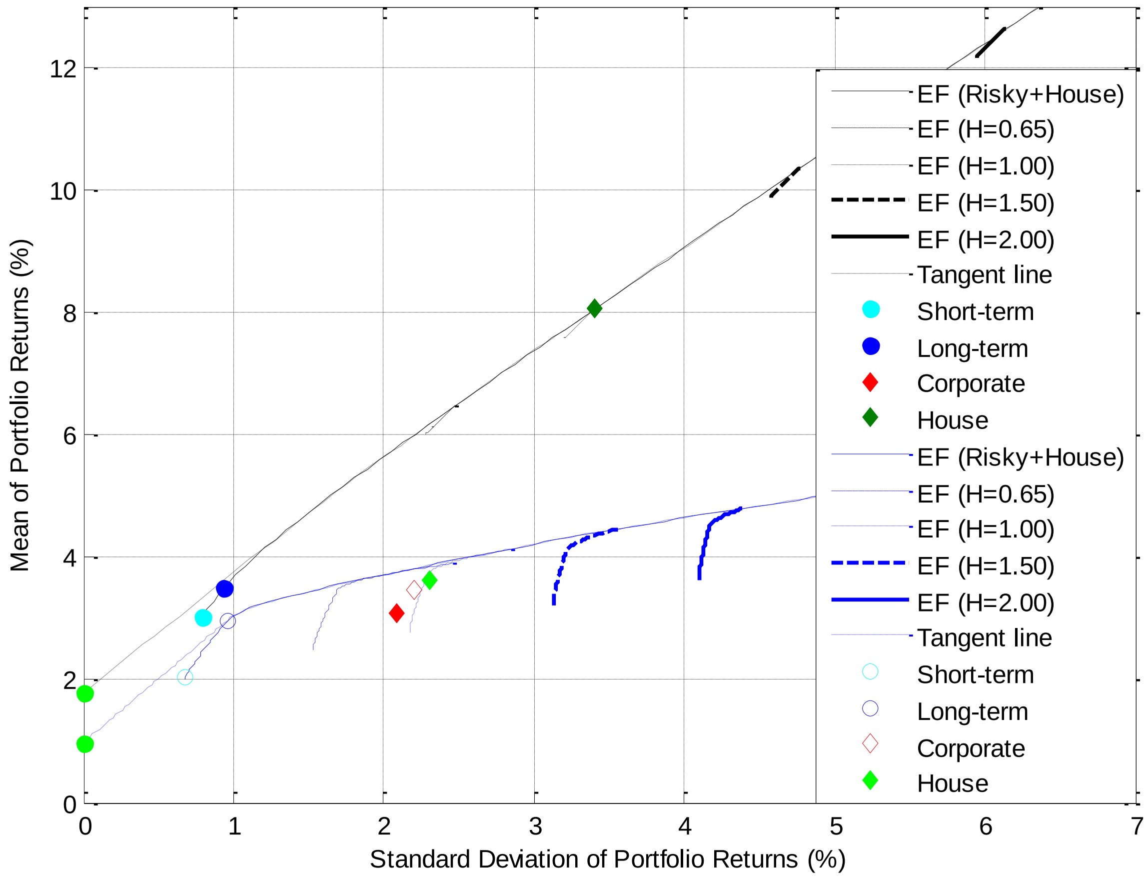
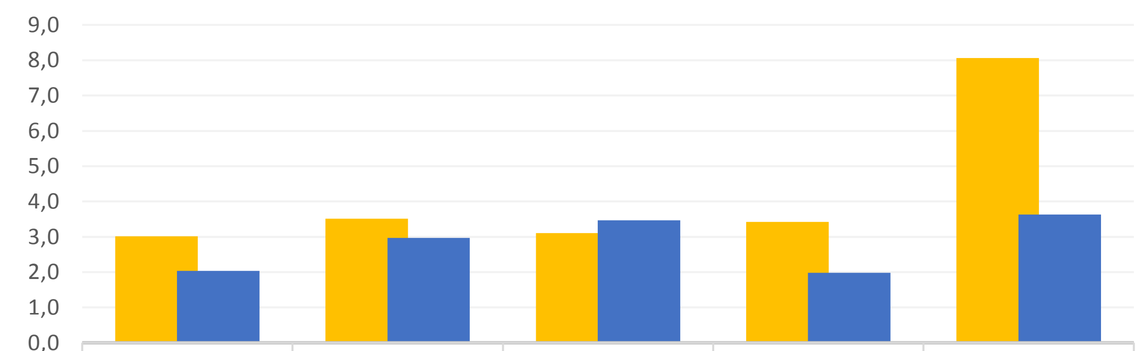


Figure 6.3 shows expected excess return for each asset in 2008 and in 2014. Short-term bonds granted a mean one point higher annual excess return in 2008 (3% vs 2% in 2014), mean annual excess interest rate on long-term assets was half point higher (3.5% vs 1.97% in 2014) while the expected return on corporate bonds was smaller but quite similar to the one in 2014 (3.1% vs 3.46% in 2014). Stock mean excess return was also higher in 2008 (3.4% vs 1.98% in 2014), but as we know few Italian households invest directly in stock market.

Figure 6.3: Comparison of expected excess returns in 2008 and 2014 (*annual %*)



	Short	Long	Corp	Stock	H
2008	3,019	3,511	3,101	3,422	8,066
2014	2,033	2,970	3,464	1,982	3,633

6. Effect of the financial crisis on efficiency

In order to understand what determined so few conditionally efficient portfolios in 2008 we compute the efficiency tests changing the expected excess returns and covariances between 2008 and 2014, thus we compare 2008 Italian household portfolios with the EFs in 2014 and 2014 household portfolios with the EFs in 2008. We then document how Italian household portfolios changed between 2008 and 2014 to further investigate the cause of the widespread inefficiency in 2008.

In Table 6.6 we compute both unconstrained and constrained tests (ξ_e and ξ_1) for 2008 household portfolios, but using returns and covariances with data till 2014.

Table 6.6: Financial and conditional efficiency of 2008 portfolios related to 2014 EFs

Efficiency of 2008 portfolios compared to 2014 efficient frontiers				
Standard test ξ_e (% of efficient financial portfolio)				
Test size	**90%**	**95%**	**90%**	**95%**
	Sample moments		*EWMA moments*	
Risk-free (5,772)	100%	100%	100%	100%
Risky (2,156)	4.92%	4.92%	28.57%	54.41%
Total (7,928)	74.14%	74.14%	80.58%	87.60%
Standard test ξ_e (% of efficient portfolios including house as unconstrained)				
Test size	**90%**	**95%**	**90%**	**95%**
N	*Sample moments*		*EWMA moments*	
Home+R.f. (3,722)	0%	0%	0%	0%
Home+R.a. (1,833)	0%	0%	0.11%	0.11%
Tot. homeowners (5,605)	0%	0%	0.00357‰	0.00357‰
Constrained test ξ_1 (% of conditionally efficient portfolios)				
Test size	**90%**	**95%**	**90%**	**95%**
N	*Sample moments*		*EWMA moments*	
Home+R.f. (3,722)	0%	0%	99.95%	99.97%
Home+R.a. (1,833)	7.69%	11.02%	88%	88.87%
Tot. homeowners (5,605)	2.52%	3.60%	96.04%	96.04%

The tests show that 2008 portfolios are at least as efficient as 2014 portfolios when the efficiency frontiers are computed using returns up to 2014. They in fact present higher percentages of efficient portfolios when the tests are computed both at the 10% and at the 5% level. Standard test ξ_e for financial portfolios shows 74.14% of efficient portfolios with sample returns and almost 80% with EWMA returns, thus a bit higher results than the ones for 2014 portfolios (70% of portfolios considered as financially efficient with sample returns and about 80% with EWMA returns in 2014). Including home as an unconstrained asset leads the ξ_e test to find almost no efficient portfolios. Considering house as constrained and computing the ξ_1 test for conditional efficiency made us find that 2.52% of household portfolios are considered efficient when the test is computed with sample returns at the 10% level and 3.6% at the 5% level. When computing the test with EWMA returns we find 96% of portfolios to be conditional efficient at both 10% and 5% level. Thus, with sample returns we find a few less conditional efficient portfolios than in 2014 (2.5% vs 3% in 2014), while with EWMA returns we find a bit higher percentage of efficiency (96% vs 94% in 2014).

When we instead compute unconstrained and constrained tests (ξ_e and ξ_1) for 2014 household portfolios with returns and covariances using data until 2008, as shown in Table 6.7, we observe very few conditionally efficient portfolios.

Table 6.7: Financial and conditional efficiency of 2014 portfolios related to 2008 EFs

Efficiency of 2014 portfolios compared to 2008 efficient frontiers				
Standard test ξ_e (% of efficient financial portfolio)				
Test size	**90%**	**95%**	**90%**	**95%**
	Sample moments		*EWMA moments*	
Risk-less (5,654)	100%	100%	100%	100%
Risky (2,471)	2.39%	2.39%	53.01%	53.18%
Total (8,125)	70.31%	70.31%	85.71%	85.76%
Standard test ξ_e (% of efficient portfolios including house as unconstrained)				
Test size	**90%**	**95%**	**90%**	**95%**
N	*Sample moments*		*EWMA moments*	
Home+R.f. (3,722)	0%	0%	0%	0%
Home+R.a. (2,074)	0%	0%	0.14%	0.14%
Tot. homeowners (5,796)	0%	0%	0.00518‰	0.00518‰
Constrained test ξ_1 (% of conditionally efficient portfolios)				
Test size	**90%**	**95%**	**90%**	**95%**
N	*Sample moments*		*EWMA moments*	
Home+R.f. (3,722)	0%	0%	0%	0%
Home+R.a. (2,074)	0%	0.00482‰	0.19%	0.19%
Tot. homeowners (5,796)	0%	0.00173‰	0.00690‰	0.00690‰

6. Effect of the financial crisis on efficiency

Standard test ξ_e without home and considering home as an unconstrained asset shows similar results to the ones already observed. Almost 70% of portfolios are found to be financially efficient using sample returns (85% when using EWMA returns) when house is not included in the assets, while almost zero when house is considered as unconstrained.

The situation changes when doing the constrained ξ_1 test on homeowner portfolios, as very few are found to be conditionally efficient both at the 10% and at the 5% level.

We can conclude that the difference in the few number of efficient portfolios in 2008 relative to the high percentages of efficiency in 2014, is due to the differences between the efficient frontiers in the two years and in particular in the constrained EFs. Constrained EFs are mostly determined by house constraint. Our previous hypothesis that in 2008 households could have foreseen that house prices would have changed following a decreasing trend in near future could hence be a possible explanation.

Nevertheless, now we will consider how Italian household portfolios changed between 2008 and 2014, as a way to investigate other entailments for efficiency. Table 6.8 displays average wealth asset shares for homeowner portfolios divided by age class.

Looking at the composition of Italian homeowners' wealth, it seems that in 2008 the age classes between 35 and 54 years, thus the one who are supposed to be the active working part of population, detained a smaller part of their wealth in housing and were also less indebted than in 2014. Shares of wealth held in deposits increased for all age classes except youngest one. Debt burden is higher for all age classes, and also the share of wealth invested in stocks and corporate bonds seem a bit higher in 2014 w.r.t. 2008.

Table 6.8: Mean wealth share on assets for age class

(%)	House		Deposits		Short-term		Long-term		Corporate		Stocks	
	2008	2014	2008	2014	2008	2014	2008	2014	2008	2014	2008	2014
<34	1.8	1.79	0.08	0.07	0.011	0.01	-0.9	-0.9	0.005	0.004	0.003	0.004
35-44	1.12	1.37	0.05	0.07	0.009	0.01	-0.19	-0.46	0.006	0.008	0.005	0.005
45-54	0.97	1.22	0.05	0.06	0.01	0.01	-0.04	-0.31	0.007	0.01	0.007	0.007
55-64	0.93	0.92	0.06	0.06	0.02	0.02	-0.03	-0.03	0.011	0.014	0.006	0.007
65<	0.92	0.90	0.05	0.06	0.016	0.02	0.0006	0.0003	0.008	0.012	0.004	0.006

From Table 6.9, we see that household stock of wealth in 2014 is declined during the financial crisis of about 10% w.r.t. to 2008. This was due mainly because of devaluation of home value, which has reached a peak in 2007, just before the burst of real estate bubble.

Table 6.9: Mean assets value for age class

Assets (€)	Year	House	Mortgage	Dep	Short	Long	Corp	Stocks	Tot
<34	*2008*	223,862	33,542	9,476	2,565	-32,727	9,476	677	205,160
	2014	201,035	22,493	13,189	2,118	-21,274	969	150	196,186
Variation		*-10.20%*	*-32.94%*	*39.18%*	*-17.43%*	*-35.00%*	*-89.77%*	*-77.84%*	*-4.37%*
35-44	*2008*	235,796	23,609	9,994	2,281	-21,644	1,851	1,422	229,698
	2014	209,624	26,707	11,400	1,863	-25,774	3,281	799	201,194
Variation		*-11.10%*	*13.12%*	*14.07%*	*-18.33%*	*19.08%*	*77.26%*	*-43.81%*	*-12.41%*
45-54	*2008*	252,417	13,022	15,425	3,601	-10,330	3,105	2,925	267,143
	2014	231,182	17,962	16,195	3,329	-16,179	3,906	1,817	240,301
Variation		*-8.41%*	*37.94%*	*4.99%*	*-7.55%*	*56.62%*	*25.80%*	*-37.88%*	*-10.05%*
55-64	*2008*	264,591	5,943	17,595	8,406	-2,511	6,332	2,798	297,212
	2014	232,006	5,342	16,505	4,828	40	5,589	2,063	261,033
Variation		*-12.32%*	*-10.11%*	*-6.19%*	*-42.56%*	*-101.59%*	*-11.73%*	*-26.27%*	*-12.17%*
65<	*2008*	227,805	1,001	13,927	6,063	2,317	3,877	1,912	255,901
	2014	213,064	956	15,082	4,090	2,484	4,116	2,878	241,718
Variation		*-6.47%*	*-4.50%*	*8.29%*	*-32.54%*	*7.21%*	*6.16%*	*50.52%*	*-5.54%*

While younger (<34) and older households (<55) decrease the value of debt together to home value, homeowners whose heads age between 35 and 54 years still decrease home value, but at the same time increase their debt position, mainly as a consequence of increased mortgage burden. Mean value of deposits increased for all age classes except from pre-retired homeowners, while mean value of short-term assets in homeowner portfolios decrease for all age classes. Changes in value of corporate bonds owning and stocks seem to be more volatile.

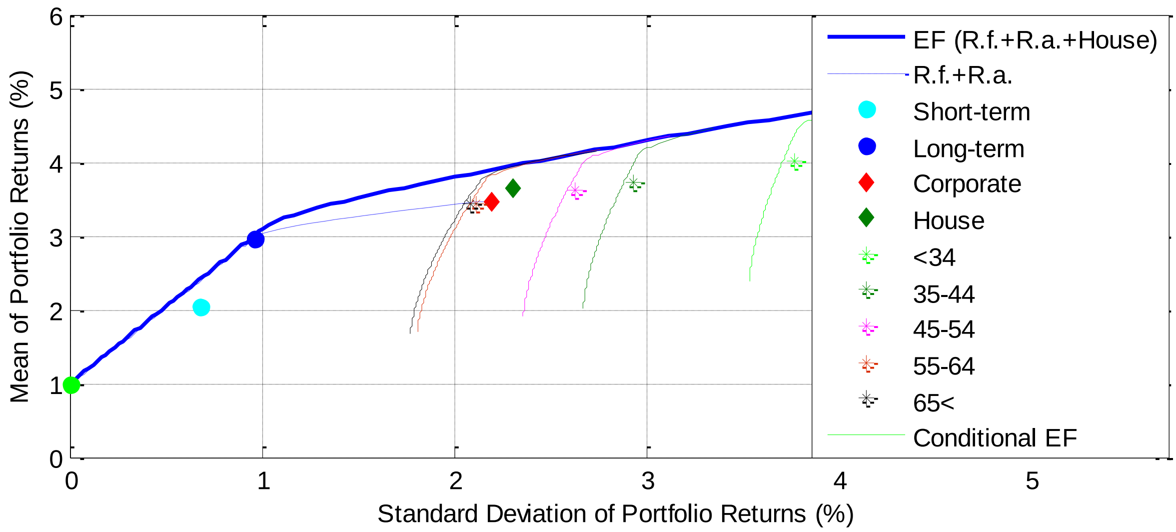
Consider how financial wealth of homeowners is changed as shown in Table 6.10. Starting from deposits, we see that almost all homeowners, except the ones with age between 55 and 64, increase their value in the risk-less asset. As previously said, all age classes decrease their position in short-term bonds, while as regard long-term assets we find the opposite situation for mortgages, thus younger and older homeowners substantially increase their value, while middle aged homeowners strongly decrease long-term bonds owning due to higher mortgages. Also, investments in stocks steadily decrease for all households, except oldest ones.

Table 6.10: Mean financial assets value per age class

Assets (€)	Year	Dep	Short	Long	Corp	Stocks	Tot
<34	*2008*	9,476	2,565	815	9,476	677	23,009
	2014	13,189	2,118	1,219	969	150	17,645
Variation		*39.18%*	*-17.43%*	*49.57%*	*-89.77%*	*-77.84%*	*-23.31%*
35-44	*2008*	9,994	2,281	1,965	1,851	1,422	17,513
	2014	11,400	1,863	933	3,281	799	18,276
Variation		*14.07%*	*-18.33%*	*-52.52%*	*77.26%*	*-43.81%*	*4.36%*
45-54	*2008*	15,425	3,601	2,692	3,105	2,925	27,748
	2014	16,195	3,329	1,783	3,906	1,817	27,030
Variation		*4.99%*	*-7.55%*	*-33.77%*	*25.80%*	*-37.88%*	*-2.59%*
55-64	*2008*	17,595	8,406	3,432	6,332	2,798	38,563
	2014	16,505	4,828	5382	5,589	2,063	34,367
Variation		*-6.19%*	*-42.56%*	*56.82%*	*-11.73%*	*-26.27%*	*-10.88%*
65<	*2008*	13,927	6,063	3,318	3,877	1,912	29,097
	2014	15,082	4,090	3,440	4,116	2,878	29,606
Variation		*8.29%*	*-32.54%*	*3.68%*	*6.16%*	*50.52%*	*1.75%*

In Figure 6.4 we scatter in the mean-variance space mean portfolio allocations for different age classes. The younger the homeowner, the more its conditional EF and its risk-return allocation are shifted to the right in the mean-variance space, with consequent increase in risk and relative small increase in return.

Figure 6.4: Portfolios by age class with conditional EFs (EWMA returns – 2014)



Comparing these changes in assets with GMV and MS portfolios seen in Chapter 4, we observe that: homeowners with portfolios near the GMV point and H ratio bigger than one, increase values of deposits and debt; homeowners with portfolios near the GMV point and H smaller than one (e.g. poor households), increase wealth kept in deposits; homeowners with portfolios near the MS point and with H bigger than one (e.g. middle-aged households), increase shares in debt and corporate bonds; homeowners with portfolios near the MS point and H smaller than one (e.g. older or richer households), increase values of long-term and corporate assets.

When we classify homeowners by income quintiles some differences between 2008 and 2014 portfolio weights can be found, as shown in Table 6.11. In 2008 housing shares were smaller for all income classes. Weights of deposits, corporate bonds and stocks have grown from 2008 and 2014 while share invested in short-term assets is decreased and homeowners have increased debt on long-term bonds.

Table 6.11: Mean wealth share on assets per income quantile

Asset	House		Deposits		Short-term		Long-term		Corporate		Stocks	
(%)	2008	2014	2008	2014	2008	2014	2008	2014	2008	2014	2008	2014
1st q.	1,05	1,10	0,038	0,042	0,003	0,003	-0,09	-0,15	0,001	0,001	0,0007	0,001
2nd q.	1,07	1,07	0,049	0,05	0,01	0,014	-0,14	-0,15	0,003	0,005	0,0014	0,003
3rd q.	1,01	1,02	0,053	0,06	0,014	0,017	-0,087	-0,11	0,005	0,011	0,004	0,005
4th q.	0,93	1,07	0,055	0,06	0,018	0,018	-0,012	-0,17	0,009	0,015	0,005	0,009
5th q.	0,91	0,94	0,07	0,08	0,028	0,023	-0,041	-0,10	0,022	0,03	0,014	0,025

When we look at mean asset values in Table 6.12 we see that average wealth is decreased from 2008 to 2014. Home value is decreased for all classes of income, while mortgages average value has decreased for most income classes, with exception of the 4th quintile.

Focalizing on financial assets, Table 6.13 shows mean asset values for income classes. Mean value of deposits has grown for all classes except for the 4th quintile and short-term asset average stock increased for 1st, 2nd and 3rd quintiles while it decreased for the 4th and 5th ones. All income classes with exception of the 4th quintile decreased the average values of long-term assets. Average values invested in corporate bonds and stocks seem to have increased for all income classes on average.

From evidences shown in the last part of this Chapter, we conclude that homeowners have considerably changed their portfolio compositions between 2008 and 2014. These changes could have happened because of the effects of the financial crisis or due to changes in homeowner profiles. Of our interest is to know if household characteristics, such as age and income, significantly affect portfolios efficiency, and this will be the scope of Chapter 7.

6. Effect of the financial crisis on efficiency

Table 6.12: Mean asset value per income quantile

Assets (€)	Year	House	Mort.	Dep	Short	Long	Corp	Stocks	Tot
1st quintile	*2008*	138,434	5,605	4,164	514	-5,120	242	113	138,347
	2014	132,877	3,477	4,720	569	-2,769	297	347	136,041
Variation		-4.01%	-37.97%	13.35%	10.70%	-45.92%	22.73%	207.08%	-1.67%
2nd quintile	*2008*	188,932	7,883	7,408	1,793	-6,870	610	280	192,154
	2014	171,218	6,060	9,200	3210	-4,099	1,311	565	181,405
Variation		-9.38%	-23.13%	24.19%	79.03%	-40.33%	114.9%	101.79%	-5.59%
3rd quintile	*2008*	221,236	8,458	11,768	3,477	-5,703	1,556	1,078	233,412
	2014	211,204	7,876	12,244	4,491	-4,870	2,842	1,399	227,309
Variation		-4.53%	-6.88%	4.04%	29.16%	-14.61%	82.65%	29.78%	-2.61%
4th quintile	*2008*	267,209	9,141	14,272	5,703	-6,101	3,275	1,664	286,022
	2014	245,093	15,525	13,834	5,458	-7,028	4,319	2,873	266,240
Variation		-8.28%	69.84%	-3.07%	-4.30%	15.19%	31.88%	72.66%	-6.92%
5th quintile	*2008*	392,669	34,080	22,088	15,905	-4,942	13,876	7,821	459,410
	2014	337,949	12,386	34,124	12,236	5,742	16,520	18,395	424,966
Variation		-13.94%	-63.66%	54.49%	-23.07%	-216.2%	19.05%	135.20%	-7.50%

Table 6.13: Mean financial assets value per income quantile

Assets (€)	Year	Dep	Short	Long	Corp	Stocks	Tot
1st quantile	*2008*	4,164	514	7,613	242	113	138,347
	2014	4,720	569	5,491	297	347	136,041
Variation		13.35%	10.70%	-27.87%	22.73%	207.08%	-1.67%
2nd quantile	*2008*	7,408	1,793	9,891	610	280	192,154
	2014	9,200	3,210	8,074	1,311	565	181,405
Variation		24.19%	79.03%	-18.37%	114.92%	101.79%	-5.59%
3rd quantile	*2008*	11,768	3,477	10,466	1,556	1,078	233,412
	2014	12,244	4,491	9,890	2,842	1,399	227,309
Variation		4.04%	29.16%	-5.50%	82.65%	29.78%	-2.61%
4th quantile	*2008*	14,272	5,703	11,149	3,275	1,664	286,022
	2014	13,834	5,458	17,539	4,319	2,873	266,240
Variation		-3.07%	-4.30%	57.31%	31.88%	72.66%	-6.92%
5th quantile	*2008*	22,088	15,905	36,088	13,876	7,821	459,410
	2014	34,124	12,236	14,400	16,520	18,395	424,966
Variation		54.49%	-23.07%	-60.10%	19.05%	135.20%	-7.50%

7. The determinants of efficient portfolios

In this Chapter we try to determine what characteristics make a household more likely to hold an efficient portfolio. To address this question, we run a simple probit regression explaining the efficiency test result (ξ_1 calculated with sample returns at the 95% level) in terms of observable household characteristics such as age, education, employment position of the head, region, and household income.

Efficient portfolios are either made of the risk-free asset alone, or include housing wealth as well as financial assets. Test results are highly affected by the wide-spread presence of efficient portfolios characterized only by risk-free assets and by homeownership (indeed in the sample many households own only the risk-free asset and house). For this reason, we focus our attention on the group of homeowners who own at least a risky financial asset - thus we will consider 2,074 portfolios.

In Table 7.1 we report the results of the PROBIT regression (the dependent variable takes value 1 if the observed portfolio is classified as efficient). As regard regressors, logY stands for the log of the income of the household head, logY2 stands for the square of logY, Age is the age of household head and Age2 is its square. NCOMP stands for the number of household components, sex takes value one when household head is male and zero otherwise. The SETT dummy variables indicate in which working sector household head is employed and A (area) variables locate household region of residence, with living in the Centre as base case.

Not all the variables considered are statistically significant: the variables that are statistically significant at least at the 10% level are income, most of dummy variables relative to work category, NCOMP, sex, civil status and education. F-tests for joint significance of the different variable categories show that dummies for civil status are not jointly significant, dummies for education are jointly significant at the 15% level, and other variable groups (income, age, working sector and categories, and region of residence) are jointly significant at least at the 10% level.

The probability of being classified as efficient at first increases with income, but then decreases. Log(income) is also the parameter that presents the highest coefficient and is significant at the 5.2% level, while log(income)^2 has a p-value of 7.8%. When including total wealth in the regression we find that it influences efficiency the same way as income does.

Being an entrepreneur seems to have a particularly negative influence on efficiency and the same happens for increasing the number of household components. Being male, divorced or widowed negatively influence efficiency of household portfolios, too. Unexpectedly also having better education seems to negatively influence portfolio efficiency.

Table 7.1: PROBIT regression conditional on housing (1 = efficient at 95%; 0 = inefficient)

Probit regression	Number of obs	=	2,074
	Wald chi2(29)	=	70.42
	Prob > chi2	=	0.0000
Log pseudolikelihood = -744.15024	Pseudo R2	=	0.0423

prob5S	Coef.	Robust Std. Err.	z	P>|z|	[95% Conf.	Interval]
logY	8.221045	4.235514	1.94	0.052	-.0804107	16.5225
logY2	-.8014956	.4554661	-1.76	0.078	-1.694193	.0912016
Age	.0106547	.0216541	0.49	0.623	-.0317866	.0530959
Age2	-.0000345	.0001712	-0.20	0.841	-.0003701	.0003011
Worker	-.5989838	.3163139	-1.89	0.058	-1.218948	.0209801
Employee	-.4392037	.2974351	-1.48	0.140	-1.022166	.1437583
Executive	-.4254632	.3123568	-1.36	0.173	-1.037671	.1867449
Entrepreneur	-.7577124	.3209134	-2.36	0.018	-1.386691	-.1287337
Selfemployed	-.4958581	.3238907	-1.53	0.126	-1.130672	.138956
Retired	-.4607071	.2802248	-1.64	0.100	-1.009938	.0885234
SETT_agr	-.0258122	.3233359	-0.08	0.936	-.659539	.6079145
SETT_ind	.082065	.1570716	0.52	0.601	-.2257897	.3899198
SETT_pub	-.0512129	.1495665	-0.34	0.732	-.3443577	.241932
A_NORDW	.1012194	.0999551	1.01	0.311	-.094689	.2971278
A_NORDE	.022853	.1033683	0.22	0.825	-.1797452	.2254512
A_South	-.1328349	.1521292	-0.87	0.383	-.4310026	.1653328
A_IS	.1406563	.1576754	0.89	0.372	-.1683817	.4496943
NCOMP	-.1320647	.0472867	-2.79	0.005	-.224745	-.0393845
sex	-.15197	.0914036	-1.66	0.096	-.3311177	.0271777
Married	-.1815088	.1242677	-1.46	0.144	-.4250691	.0620515
Divorced	-.3243089	.170344	-1.90	0.057	-.6581771	.0095593
Widowed	-.364718	.1508755	-2.42	0.016	-.6604285	-.0690075
Elementary	-.7139796	.3592833	-1.99	0.047	-1.418162	-.0097972
Middle_school	-.7118729	.3641001	-1.96	0.051	-1.425496	.0017501
Vocational	-.8930582	.3813689	-2.34	0.019	-1.640527	-.1455889
High_school	-.8843352	.3652567	-2.42	0.015	-1.600225	-.1684452
Ys3_Degree	-1.126164	.5312911	-2.12	0.034	-2.167475	-.0848524
Ys5_Degree	-1.151092	.376928	-3.05	0.002	-1.889857	-.4123266
Post_Degree	-.8343049	.4628653	-1.80	0.071	-1.741504	.0728945
_cons	-20.61215	9.762852	-2.11	0.035	-39.74699	-1.477313

When we look at asset weights of efficient portfolios when the test ξ_1 is computed with sample returns at the 95% level, we find that only portfolios with H ratio smaller than one are considered as efficient. This mean that portfolios considered as conditionally efficient are only the ones in which home value is smaller than household total wealth. According to the probit regression, income is the main determinant of portfolio efficiency, because higher income makes housing weight relatively smaller if compared to total wealth. Moreover, older households should hold more efficient portfolios, as income and wealth grow with age. The curve shape of home value relative to total wealth ratio also explains why Age has a positive coefficient and Age^2 a negative one. It may be that the higher the home constraint is, the less the probabilities that a household could efficiently allocate his liquid wealth.

As we have shown in Chapter 3, homeownership constraint is largest at the beginning, i.e., for young households who have smaller net worth relative to current income. As individuals accumulate wealth, the homeownership constraint becomes less binding.

NCOMP negatively influence efficiency probably because when the size of the family increase, also the costs to sustain consumption increase, thus reducing liquid wealth, and the risk for further expenses during life cycle increases too, thus obliging homeowners to keep more liquid wealth in safe assets for precautionary motive.

As regards the negative coefficients of education level dummies, we should remember that more educated homeowners are commonly younger, thus their portfolio characteristics could mainly reflect their age profile. Less educated homeowners, who are supposed to be also the oldest ones, keep the smallest share of wealth in financial assets, and have also the lowest home value relative to total wealth ratio.

Conclusion

In this thesis we investigate Italian household portfolios' efficiency. Housing is the most important asset in mostly household portfolios, thus standard tests for efficiency are biased as they neglect existence of illiquid assets. Moreover, Italian portfolios are known to be poorly financially diversified and hence housing should assume even more weight in portfolio analysis.

Using data on Italian households from SHIW 2014 we compute both standard efficiency test, considering only financial assets, and then include housing as unconstrained. We find that almost all portfolios result to be inefficient when house is included as a liquid asset. In fact, optimal portfolios in periods of no housing adjustment are affected by housing price risk through a hedge term and tests for portfolio efficiency of financial assets must be run conditionally upon housing wealth.

When we test for portfolio efficiency conditional on housing we find that many portfolios are considered as efficient. Moreover, we find more conditional efficient portfolios than financially efficient ones. This could be evidence that these households use financial assets to hedge housing risk, but could also reveal that housing has diversification properties (for homeowners, financial risks are relatively small compared to total wealth). Given the high correlations and the very large weight attached to housing wealth, the failure to exploit hedging opportunities could outweighs the benefits from diversification.

We show how the coefficients of the hedge term change through different economic cycles and thus how portfolio efficiency is influenced by economic shocks. When comparing Italian portfolios prior to the financial crisis with portfolios in 2014, we find that 2008 portfolios result inefficient when the test is computed with information set up to 2007 and mostly efficient when we do the test with data up to 2014. This could mean that households correctly expected changes in returns trend.

At the end we perform a probit regression relating the efficiency test results to observable household characteristics as a way to investigate possible causes for inefficient portfolio allocations. We find that income seem to be the main driver for an efficient portfolio. Increasing the number of household components and, unexpectedly, also having better education seem to negatively influence portfolio efficiency.

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Appendix - Derivation of equation (9)

We show here how Pelizzon and Weber (2008) derived the asset allocation rule of equation (9) by suitably extending Flavin's (2002) analysis. They take as given the result that there are finite periods of time when the household decides not to adjust the housing stock, because the benefit from adjusting is smaller than the transaction costs incurred. Their derivations hold for such periods of inaction. Suppose that at time t=0, the household decides that it is not optimal to change the housing stock immediately. During a time interval (0, s) when the possibility of such change is negligible, wealth evolves according to:

(A1) $$dW_t = [P_t H_0(\mu_H + r_f) + \underline{X}_t(\underline{\mu} + r_f) + r_f B_t - C_t]dt + \underline{X}_t d\underline{\omega}_{Ft} + P_t H_0 d\omega_{Ht}$$

or, rewriting to eliminate the term representing risk-free bonds:

(A2) $$dW_t = [r_f W_t + P_t H_0 \mu_H + \underline{X}_t \underline{\mu} - C_t]dt + \underline{X}_t d\underline{\omega}_{Ft} + P_t H_0 d\omega_{Ht}$$

Let V (H, W, P) denote the supremum of household expected utility be twice continuously differentiable, conditional on the current values of the state variables (H, W, P). Bellman's principle of optimality can be stated as:

(A3) $$V(H_0, W_0, P_0) = \sup_{\{\underline{X}_t\},\{C_t\}} E\left[\int_0^s e^{-\delta t} u(H_0, C_t)dt + e^{-\delta s} V(H_0, W_s, P_s)\right]$$

subject to the budget constraint (10) and the process for house prices (5). The term inside the brackets intuitively represents the sum of the rewards on the interval (0, s) and the maximized expected value by proceeding optimally on the interval (s, ∞) with the system started at time s in state (H_0, W_s, P_s)[18].

Subtracting V(H_0, W_s, P_s), dividing by s and taking the limit as s → 0 gives:

(A4) $$0 = \lim_{s \to 0} \sup_{\{\underline{X}_t\}\{C_t\}} E\left[\frac{1}{s}\int_0^s e^{-\delta t} u(H_0, C_t)dt + \frac{1}{s}(e^{-\delta s} V(H_0, W_s) - V(H_0, W_0)\right]$$

[18] We assume that the transversality condition holds such that V (H_0, W_s, P_s) is bounded.

Appendix - Derivation of equation (9)

Evaluating the integral and using Ito's lemma, equation (A4) can be rewritten as:

(A5) $$0 = \sup_{\underline{X}_0, C_0} \Big\{ u(H_0, C_0) - \delta V(H_0, W_0, P_0, P_0{}') + \frac{\partial V}{\partial W}\Big(r_f W_0 + P_0 H_0 \mu_H + \underline{X}_0 \underline{\mu} - C_0\Big) + \frac{\partial V}{\partial P} P_0 \mu_H + \frac{1}{2}\frac{\partial^2 V}{\partial W^2}\left(\underline{X}_0 \Sigma \underline{X}_0^T + P_0^2 H_0^2 \sigma_P^2 + 2 P_0 H_0 \underline{X}_0 \Gamma_{bP}\right) + \frac{1}{2}\frac{\partial^2 V}{\partial P^2} P_0^2 \sigma_P^2 + \frac{\partial^2 V}{\partial W \partial P}\left(P_0^2 H_0 \sigma_P^2 + P_0 \underline{X}_0 \Gamma_{bP}\right)\Big\}$$

that is:

(A6) $$0 = \sup_{C_0} \Big\{ u(H_0, C_0) - C_0 \frac{\partial V}{\partial W}\Big\} - \delta V(H_0, W_0, P_0, P_0{}') + \frac{\partial V}{\partial W}\left(r_f W_0 + P_0 H_0 \mu_H\right) + \frac{\partial V}{\partial P} P_0 \mu_H + \frac{1}{2}\frac{\partial^2 V}{\partial P^2} P_0^2 \sigma_P^2 + \frac{\partial^2 V}{\partial W \partial P}\left(P_0^2 H_0 \sigma_P^2 + P_0 \underline{X}_0 \Gamma_{bP}\right) + \frac{1}{2}\frac{\partial^2 V}{\partial W^2} P_0^2 H_0^2 \sigma_P^2 + \sup_{\underline{X}_0}\Big\{\frac{\partial V}{\partial W}\underline{X}_0 \underline{\mu} + \frac{1}{2}\frac{\partial^2 V}{\partial W^2}\left(\underline{X}_0 \Sigma \underline{X}_0^T + 2 P_0 H_0 \underline{X}_0 \Gamma_{bP}\right)\Big\}$$

Non-durable consumption satisfies the usual first order condition:

(A7) $$\frac{\partial u}{\partial C} = \frac{\partial V}{\partial W}$$

and the necessary first order conditions for the investment in risky financial assets, $\underline{X}_0$, is:

(A8) $$0 = constant + \frac{\partial V}{\partial W}\left(r_f W_0 + P_0 H_0 \mu_H - C_0\right) + \frac{1}{2}\frac{\partial^2 V}{\partial W^2} P_0^2 H_0^2 \sigma_P^2 + \sup_{\underline{X}_0}\Big\{\frac{\partial V}{\partial W}\underline{X}_0 \underline{\mu} + \frac{1}{2}\frac{\partial^2 V}{\partial W^2}\left(\underline{X}_0 \Sigma \underline{X}_0^T + 2 P_0 H_0 \underline{X}_0 \Gamma_{bP}\right) + \frac{\partial^2 V}{\partial W \partial P}\left(P_0 \underline{X}_0 \Gamma_{bP}\right)\Big\}$$

Assuming that $(\partial^2 V)/(\partial W \partial P) = 0$ we can derive the optimal holding of risky financial assets as:

(A9) $$X_0^T = \left[\frac{-\frac{\partial V}{\partial W}}{\frac{\partial^2 V}{\partial W^2}}\right] \Sigma^{-1} \underline{\mu} - P_0 H_0 \Sigma^{-1} \Gamma_{bP}.$$