UNIVERSITA' DEGLI STUDI DI PADOVA

DIPARTIMENTO DI SCIENZE ECONOMICHE ED AZIENDALI "M.FANNO"

CORSO DI LAUREA MAGISTRALE IN ECONOMICS AND FINANCE

TESI DI LAUREA

"TITOLO"

Agency Problems in PPP Investment Projects: The effects of Moral Hazard and Bargaining Power on Investment Timing

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ANNO ACCADEMICO 2019 – 2020

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INTRODUCTION

During the last two decades of the twentieth century, several industrialized countries went through a process of privatization of many and significant sectors of their economies. Starting in the 1980s various policymakers, reflecting a general change in economic thinking, embraced the idea that governments should focus on a narrow set of "core" activities (e.g. maintenance of public order, defense, market regulation), delegating to the private sector the management of other economic areas such as telecommunication and transportation. This trend was shown not only by OECD countries but also, and more vigorously, by developing countries and Eastern Europe post-communist states transiting to market economies (Recent Trends in Privatization, OECD).

Even though there is no common definition of privatization the OECD defines with that name "any material transaction by which the state's ultimate ownership of corporate entities is reduced." Therefore, the transfer of specific tasks or activities from a state-owned enterprise to a private firm cannot be categorized as privatization. In some contexts, privatization appears as a disproportionate reaction to the inefficiencies of the public sector (if any) even when accompanied by the due reforms. An increasing number of policymakers thus opted for a more pragmatic approach, trying to foster the cooperation between the public and the private sector without necessarily transferring the ownership of entire corporate entities from the state to private investors. This idea materializes in the delegation of specific tasks in the provision of public services to the private sector when it has a competitive advantage in their completion. Examples of these public services range from more traditional ones, such as transportation, energy, and water, to others such as IT services, leisure facilities, military training, and prisons.

One of the contractual forms that have been widely used by governments to entrust the provision of a public service to a private party goes under the name of Public-Private Partnership (PPP). This type of arrangement is characterized by the bundling of the different tasks inherent to the provision of the service. Examples are financing the project, building the infrastructure, operating, and maintaining it. Once bundled the tasks are assigned to a single private entity to be carried out. The typical PPP agreement is a long term contract, transfers a substantial proportion of risk to the private party, entails a large and irreversible initial investment, and once the project is operational it starts generating uncertain streams of cash and public benefit flows[1].

[1] PPP agreements are complex, a clear description of their dynamics and characteristics is presented in the next chapter.

The literature on PPP contracts has focused on two main issues. The first relates to the presence of asymmetries of information that hinders the correct and efficient functioning of these agreements. In the delegation process, the private firm has the chance to acquire private information or to hide its action from the principal. For this reason, problems of adverse selection and moral hazard can arise. Several authors have studied these problems using concepts from mechanism design and contract theory. Another problem faced by PPP contracts is related to the considerable level of uncertainty that surrounds these projects. What emerges from the existing literature is that unrealistic demand expectations, cost inflation, changes in user preferences, regulatory requirements, weak institutions, environmental corruption, and interactions among stakeholders are all factors that have the potential to jeopardize the good outcome of the project. In this context, embedding flexibility provisions in the contracts can help the parties to promptly react when unexpected changes occur. A strand of literature has emphasized the real option nature of these provisions. Various scholars applied mathematical methods used in the valuation of financial options to assess the value created by the flexibility clauses embedded in the contract. Finally, a third strand of literature has tried to incorporate contract design into a real options framework.

An aspect that instead has not received the due attention in the present literature on PPP is the bargaining power of the parties. Most of the scholars discussing PPP contracts allocate all the negotiating power to the government and assume it formulates a take-it-or-leave-it offer for the agent. Nonetheless, we think that, given the significant amount of risk on the agent's shoulders, it is more appropriate to presume that also the private party will claim some bargaining power over at least some of the variables at stake.

Our contribution departs from the paper by Buso et al. (2020). The authors model the optimal design of a PPP agreement considering a moral hazard problem into a real option framework. Also, they assume two possible types of agent, one who has negotiating power and another who does not. We expand their model by allowing for any possible allocation of the bargaining power among the parties. The bargaining power is modeled in two ways (Demougin and Helm, 2006). In the first set of models, we use a Nash bargaining solution in a two-stage maximization problem. We apply to a PPP framework the approach used by Banerjee et. al (2014) in the study of a joint venture. In the second, we model the bargaining power of the agent in terms of his reservation utility, the higher the latter the greater the former.

Our main finding from the Nash bargaining solution is that the result by Buso et. al (2020) for an agent with no negotiating power is confirmed. Allowing for the bargaining of the parties on several combinations of variables, we go back to the optimal contract found by the authors. The main limit of the Nash bargaining model is that it does not allow to draw the principals' payoff as a function of the agent's bargaining power. In this regard, the model based on the agent's reservation utility is effective. It allows integrating the previous results with an insight on the effect of the agent's negotiating power on the principal's payoff. As expected, the higher the reservation utility of the private firm the lower the government's payoff.

The thesis is articulated as follows. The next three chapters provide the theoretical knowledge to understand our models and define the perimeter of our contribution. In chapter 4, the benchmark model is presented defining the setup and the notation that will be used throughout the rest of the work. Chapter 5 presents the Nash bargaining model in its various forms. Chapter 6 discusses the reservation utility model. The last section concludes.

Chapter 1 – PUBLIC-PRIVATE PARTNERSHIPS

1.1 - Public-Private Partnerships: a definition

The provision of a public service involves a complex array of tasks and, when it comes to delegating it to a private entity, the government is faced with a plethora of different contractual forms that could define its interaction with the private sector. In the first place, the decision between hiring multiple private entities and assign each of them to a specific task or entrust a single entity to complete the entire process for the provision of the service emerges. For the sake of clarity, the activities needed for the provision of a public service can be narrowed down to building the infrastructure and operate the asset as efficiently as possible. In terms of contractual arrangement, if the government hires different firms to build and operate the asset we have what is called a traditional procurement contract while, when the tasks are bundled and assigned to a single entity, the arrangement is known as Public-Private Partnership[2]. An example can be useful for a better understanding.

Consider a toll road and the social utility that it exerts in terms, for instance, of travel time savings. The government, to provide the citizens with this service, does not complete the highway on its own but turns to the private sector. Two different contractual alternatives with the private counterparty can be taken into consideration. The traditional procurement, where the government entrusts two different private entities with different core competencies to build the infrastructure and to operate the asset. The PPP, where the public entity contracts with a unique counterparty to build and operate the toll road (Hoppe and Schmitz, 2012).

Besides, Public-Private Partnerships compared to other forms of outsourcing are identified by characteristics other than the bundling of different tasks. Auriol and Picard (2009), in their paper, assumed outsourcing contracts to be simple in terms of ex-ante arrangements. The initial contract specifies just what they called a franchise fee (a transfer from the private entity to the government to have access to the monopolistic market) and a statement with which the government declares of having no rights or decisional power on the future market outcome and structure. However, they recognize another possible interpretation of an outsourcing contract, the PPP. It is characterized by a precise description of how payments change depending on different contingencies related to output quality and technological uncertainties. The greater amount of details, laid down before the investment decision of the firm, considerably increases the complexity of the contract. Moreover, in PPPs, the private entity must acquire or build the

[2]This is not the only characteristic that defines a PPP. Later, a more formal definition will be provided.

asset and, in exchange for control and cash-flow rights, to bear a substantial share of the risk deriving from the investment.

A relevant contribution for a clear outline of the main characteristics of a PPP agreement was provided by Iossa and Matrimort (2015). They schematize the attributes of a PPP under three main components:

I. *Bundling.* As anticipated above, the projects for which PPP are used involves a multitude of tasks: design, build, finance, and operate. All of them are bundled and assigned to the private entity the government is contracting with. Given the differences among the activities and the resulting need for diverse competences, the project is usually assigned not to a single firm but to what is called a consortium. It is made up of a construction company and a facility-management company that are in charge of the totality of the project.

II. *Risk transfer.* The government, in delegating the provision of the public service to the consortium, uses a system of output specifications leaving a great deal of freedom on how to achieve the pre-defined goal. Moreover, the compensation of the private is related to the cash flows of the project and therefore to its quality. The design, construction, and operational risk are thus essentially transferred to the private sector party, being it responsible for delivering the service meeting the pre-specified.

III. *Long-term contracts.* The typical length of a PPP is between 20 and 35 years. During this period, the payments to the private-sector party are typically made by the users of the facility or, seldom, by the government. The long-term of PPPs generates another source of risk for the consortium, as it is impossible, at the time of the investment, to have visibility on the future value of many key variables that affect revenues and costs of the project.

Once defined the main characteristics of the Public-Private Partnerships it is worth saying that, in practice, PPPs can take on different structures depending on the final ownership of the asset and on the number of tasks that are delegated to the private-sector party. Among the most used forms of PPPs, there are the DBFO model (“Design”, “Build”, “Finance”, and “Operate”), the BOT (“Build”, “Operate” and “Transfer”) where the ownership of the asset is transferred from the consortium to the state at the expiration of the concession, the BOO model (“Build”, “Own”, and “Operate”) where the ownership of the infrastructure is retained by the private sector (Iossa, Martimort, 2015).

1.2 - Regulation or outsourcing?

Before discussing what to take into consideration when choosing between a Public-Private Partnership and a traditional procurement agreement, it is worth to take a step back and focus on the question of how the government should interact with a firm that is in charge of providing a public service. The rest of the work, focusing on PPP, will assume the public authority has already opted for delegating the service to a private entity. However, the decision on whether privatizing the service is upstream the one on how to do it. For the sake of completeness, we discuss some of the academic contributions over the choice of privatizing or not the provision of a public service. Being the literature on this theme extremely wide, an exhaustive discussion of it is beyond the scope of this study. The goal of the present paragraph is to give a sense to the reader of the direction that the literature has taken on this complex topic.

On the one hand, the government can decide to nationalize the firm that is devoted to providing the service, in this case, the firm is state-owned. On the other hand, the government can reach a contractual agreement with the firm thus leaving it privately-owned. These two options can be analyzed through a parallel between the theory of the firm and the theory of privatization (Hart, 2003).

Consider two firms which have the chance to merge and become vertically integrated, call them A and B. Assume the two have a reason for entertaining a long-lasting relationship. For instance, firm A is an airplane producer and B is specialized in some aircraft engine components. They are faced with two options: stipulate an arms-length contract or merging into a single entity. Transferring it to the theory of privatization, firm A can be considered as the government while firm B as the firm that is in charge of supplying the society with some good or service. Again, A and B can stipulate a contract, or the government can nationalize (buy) the firm. The two situations are similar but not identical. Consider, for instance, that the decision to privatize or nationalize are generally remarkably political while vertical integration, as private entities are involved, resides in the economic sphere. Additionally, the government can pursue different goals than a private firm. While the former is concerned with social welfare the latter focuses on personal profits (exceptions such as nonprofit and cooperatives make the distinction less sharp). Despite some discrepancies, it is undeniable that the issues of vertical integration and nationalization are highly comparable (Hart, 2003).

The literature on the theory of the firm and the one on the theory of privatization, even if studying two logically similar problems, have focused on two different sources of inefficiency. The first considers the incompleteness of contracts as the main cause of a suboptimal outcome, whereas the latter has assumed a complete contracting environment where inefficiency stems from moral hazard and asymmetric information. An interesting insight deriving from the theory of the firm is that in a complete contracting environment the structure of ownership does not affect the outcome. The owner of the asset does not enjoy any special power since the contract describes perfectly, and from the beginning, all the details of the relationship with the other party. On the contrary, when the contract is incomplete the ownership structure matters as the owner has decisional power on what is not included in the agreement.

Even if the two theories are related it does not mean that the just mentioned result can be applied as it is to the case of privatization. However, looking at the problem from the firm perspective helps to understand the trade-off the government is faced with. Take the example of a prison. If the government decides to own it, the previous manager of the asset, being now just an employee, will not have any incentive to act entrepreneurially, to come up with new ideas, and to invest in the asset. Is this a desirable or undesirable effect? To answer, assume that the owner can undertake two forms of investments: one that raises the quality of the prison services, and another that reduces both operational costs and quality of the prison. If the government does not privatize the firm both types of investment are encouraged, comes on its own that the former is worth being promoted while the latter is not. Under privatization, instead, none of them will receive incentives. Depending on which side of the trade-off is valued the most, the policymaker will opt either for privatizing the firm or not.

Another visual angle on the choice between contracting and privatization can be taken looking at the case where the market structure is non-competitive. Consider, as an example, sectors such as infrastructure, water, and waste management, public transport, mail services, information, and communications technology services that are traditionally characterized by a low level of competition. The government can deal with these sectors under two regimes: regulation and outsourcing (Auriol and Picard, 2009). Under the regulation, the firm operating in the monopolistic market is state-owed and is run by a public manager who has private information on the firm's cost. The government has control and cash flow rights over the firm. It is entitled to set investment and production decisions. Nonetheless, control rights come with accountability, meaning the government taxes the firm profits but, in case of losses, it must subsidize the firm. On the contrary, in the outsourcing regime, an unregulated privately-owned firm provides the commodity or the public service. It has control and cash-flow rights and can charge a laissez faire monopoly price. However, to have access to the market the monopolistic firm can be required to pay a franchise fee to the government and must bear a sunk investment to begin production. If the laissez faire regime does not result to be optimal the only instrument the government can use to improve welfare is an ex-post contract. It is called ex-post because it can be offered only once the sunk investment has been carried out and some uncertainties have been solved. Keep in mind that the monopolist will accept the contract only if it offers a payoff greater or equal than the one under the laissez faire regime.

To sum up, among the advantages of outsourcing we have that it allows the government not to subsidize a money-losing company, it yields franchise fees and it relaxes the reduction in incentive compatibility constraint. On the other hand, the government has no decisional power on the strategy the firm will follow and no rights on the cash flow it will generate if profitable. To make the discussion slightly more technical, the benefits from outsourcing can be split into two effects, the fiscal effect and the economic surplus effect (Auriol and Picard, 2009). The first one is given by the government savings as it does not subsidize a money-losing firm and by the revenues from the collection of the franchise fee. The second occurs when production is greater under outsourcing than under regulation. A key variable that defines the magnitude of these effects is the shadow cost of public funds (Auriol and Picard, 2009). When it is low, outsourcing is always preferred. As the shadow cost of public funds rises, outsourcing is superior to regulation only if the franchise fee is high enough (Auriol and Picard, 2009).

These results provide a useful framework to compare outsourcing and Public-Private Partnerships to the regulation regime. Several practical conclusions can be drawn. Outsourcing is more appropriate when it comes to delivering high technology products in advanced economies or to cover low profitability market segments (e.g. postal and transportation services in areas with few inhabitants). Finally, in developing countries, it takes the extreme form of laissez faire, and it is cost-efficient when applied to low profitability segments of natural monopolies, in contrast to money-making segments that are worth to be maintained under public control (Auriol and Picard, 2009).

The choice between outsourcing and regulation is worth to be studied also in a more specific setting. Instead of considering a general form of outsourcing, it is possible to focus on a specific form of PPP, i.e. the BOT (recall it stands for “Built”, “Operate”, and “Transfer”). Under this type of outsourcing regime, the government entrusts a private company to build and operate an asset of public interest. However, the ownership of the asset is always in the hands of the government and the firm has the right to operate the facility for a pre-defined interval of time. Once the concession is over, the government delegates the operation of the asset to a public manager.

The characteristics of the BOT agreement affect the variables in play when studying which option is best between outsourcing and regulation. Auriol and Picard (2013) have assumed the government and the firm does not share the same information on the operation characteristics of the facility. Under this assumption, they have focused on how effective BOT contracts are under various degrees of information asymmetries and levels of transferability of the project at the end of the concession. While information asymmetries are common to every form of outsourcing, the transferability is specific to BOT agreements. Once the concession ends, the competencies of the private management are not always easily transferable to the public firm's manager. For instance, in sectors such as water and waste management, who operates the asset in the first place can develop considerable expertise and know-how that are difficult to be transferred to the new managers. On the contrary, when considering a canal or tunnel concession, less advanced managerial knowledge is needed and therefore it is also more transferable when the management changes.

As in the case of a non-competitive market structure, the shadow cost of public funds has a vital role in making one alternative superior to the other. Besides, there is a trade-off between the cost the government has to afford to build and operate the facility and the loss of consumer surplus generated by the higher prices that the concession holder will charge (Auriol and Picard, 2013). It has been shown that when the shadow cost of public funds, the business risk, and the information asymmetries between the government and the private party increases there are better incentives to opt for BOT. When the shadow cost of public funds is high the government is more prone to use BOT concession contracts not only in developing countries but also in advanced economies. Also, as anticipated above, when the project characteristics ease the transferability of the facility at the end of the concession the use of BOT contracts is more likely. In conclusion, also the fact that the two parties (public and private) face the same uncertainty about the profitability of the project at the time of the concession signature fosters the use of BOT agreements (Auriol and Picard, 2013).

1.3 - Bundling or unbundling tasks

If the policymaker opts for outsourcing the provision of a public good or service to a private entity, a natural question regarding how to define the relationship will follow. For a better understanding of the issue, it is useful to remember that the provision of most public services requires to perform a complex array of tasks, from building the facility to operate it. In traditional procurement, as anticipated above, different tasks are assigned to different entities while in PPPs the tasks are bundled. Besides, PPPs transfer risk from the public to the private sector and are characterized by a long term.

A strand of literature on the topic has discussed tasks bundling as the discrimen to choose between PPP and traditional procurement. A recurring argument relates to the presence of positive or negative externalities between the building and operation phases. Two cases are feasible and documented by practitioners. In the case of a positive externality, investing in a high-quality design has a positive impact on operating costs as it allows to provide the service more efficiently. Think about a prison, the layout of the rooms and corridors has a vital role in meeting certain safety standards. In the case of a negative externality, an innovative design increases the operational cost. If the facility differs too much form the standard it will probably require the management to re-define certain processes and routines. In the short run, at least, the operational costs will be higher.

Positive and negative externalities become even more relevant when studied under asymmetry of information and incomplete contracting. The government, when the effort of the agent is not observable (moral hazard), faces a trade-off. It is interested in encouraging the builder to improve the quality of the infrastructure and, simultaneously, must provide appropriate insurance against adverse shocks on the realized quality. Reducing the incentives on high quality allows saving money on insurance. However, if a reduced quality has a negative impact on the operating costs the operator of the building will be hurt and will be more demanding for the government to make her willing to participate in the agreement. When the eternality is positive, bundling building and operation provides an efficient solution to the problem. The prospect of reduced operating costs is attractive for the builder who will also operate the facility and thus encourages to exert effort to improve the quality of the asset. On the contrary, if the externality is negative, merging the builder and the operator is not desirable. The builder (who will be the operator) is aware that the higher the quality of the facility the higher the operating costs will be and has no incentive to exert effort. For a negative externality, the two tasks are better to be split (Martimort and Pouyet, 2008; Iossa and Martimort 2015). The decision of bundling when the externality is positive, besides encouraging the investment on asset quality, automatically transfers more risk to the private party. It follows that the induced cost-reducing effort comes at the cost of a higher risk premium. In addition, transferring operational risk promotes the quality-enhancing effort. This points out how closely risk transfer and bundling are related and it is in line with the evidence of more risk transfer and greater risk premiums that characterizes PPPs (Iossa and Martimort, 2015).

To address the choice between bundling and unbundling from a different perspective, one can focus on what aspect of the project can be better specified in the contract between the public authority and the private party (Hart, 2003). Consider three dates 0, 1, and 2. At 0 the government and the builder sign the agreement, between 0 and 1 the facility is built, and between 1 and 2, it is operated either by the same entity who has built it or by a third party. The builder can exert two types of effort. The first, i, is called productive as enhances the quality of the infrastructure and reduces the operating costs. The second, e, is unproductive. It reduces the operating costs together with the quality of the asset. Under unbundling the agreement between the parties specifies just the minimum quality level of the asset. The builder does not internalize the cost-reducing externality related to i and has no incentive to build a facility qualitatively higher than the minimum contractual level (only effort e is exerted). It will build the cheapest facility possible. In the case of bundling, the contract specifies the quality of the service. Here the agent will exert effort i together with effort e since it realizes that an increase in quality induces operational savings in the second period (between 1 and 2). In conclusion traditional procurement (unbundling) is preferable if the quality of the building can be effectively specified while the one of the service cannot be. Here, underinvestment in i under traditional procurement is not a serious issue while overinvestment in e under PPP could be. Differently, PPPs are desirable when the quality of the service can be effectively specified while the one of the building cannot be. Here, underinvestment in i under traditional procurement could be a serious issue, whereas overinvestment in e under PPP is not (Hart, 2003).

1.4 - Asymmetry of information

Many authors have rightfully described the interaction between the parties of a Public-Private Partnership as a Principal-Agent (P-A) problem. A key issue in the relationship between the government and the firm, to which the provision of the public service is delegated, is the asymmetry of information and the inefficiencies that can follow. Delegation in PPPs can be justified either by the benefits in terms of increasing project returns coming from the efficient division of tasks, by the principal's inability or lack of time to carry out the project on her own, and by the principal's limited rationality when facing problems beyond a certain level of complexity. The literature on P-A problems has pointed out that inefficiencies arise because of two types of agency problems to which any issue in delegation can be led back to adverse selection and moral hazard.

Adverse selection arises every time in the delegation process the agent can acquire some private information not available to the principal. For this reason, it is also defined as a problem of hidden information. In PPP, the private company can be better informed than the government on several aspects related to the project. Before the contract signature, the firm can have better access to information on the building costs of the infrastructure. During the building phase, it is likely that the builder will acquire information on the operational costs of the service. Also, the constructor has better knowledge of how expensive would be to modify the infrastructure if new needs emerge in the operation phase. In any of these cases, the additional information in the hands of the agent has some vital implications for the design of the contract to be signed by the parties. For an efficient use of economic resources, the principal needs to extract the agent's private information. To do so, she must afford an additional cost defined as information rent that adds up to the typical technological cost required to carry out the investment. The greater expenditure incurred by the government to smooth the asymmetry of information distorts the contractual equilibrium between the two parties, it deviates from the optimal that would be achieved under symmetric information.

Moral hazard arises when the designated firm for the provision of the service has the chance to deliberately choose a course of action that will materially affect the outcome of the project. Delegating the task, the principal loses any ability to control the agent's action either if it is not observable or if it cannot be proven in front of a Court of Justice which enforces it. This is also called a problem of hidden action. Typically, in PPPs, the agent's action takes the form of effort exertion and its effect is either to increase the probability of having a high-quality project outcome or facing lower costs. In both cases, effort enhances the principal payoff. It is worth to emphasize that moral hazard, as adverse selection, are issues only if the objective function of the firm and the one of the government are not the same. A key feature of effort exertion is that it produces disutility for the agent, it is expensive in terms of time and money. The agent has no incentive to exert effort as his payoff is negatively affected by doing so. Essential for the agency cost to arise is the asymmetry of objective between the principal and the agent. On the one hand, effort exertion has a positive effect on the principal payoff. On the other, the agent has only an extra cost and no benefit from the effort. To solve the agency conflict between the parties, the principal must deign a contract such that the agent will find it more convenient to exert effort than not to. It is important to keep in mind that the quality of the project cannot be used as a perfect proxy for the agent's effort since the relation between the two is not deterministic. This uncertainty is the key to a clear understanding of the moral hazard problem. The random output of the project is the result of the agent effort and the realization of pure luck. The principal thus offers a contact where, bearing a cost, it induces the agent to take the desired action even if it is not directly or indirectly observable. If doing so is too expensive the principal will either opt to stay in the partnership knowing the agent will not exert effort or will decide not to take part in the agreement[3].

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[3] For a clear theoretical discussion of Adverse Selection and Moral Hazard we suggest chapters 2 and 4 of "The theory of incentives I: The principal-agent model" by J.J. Laffont and D. Martimort (2001).

Another aspect of PPP that creates scope for adverse selection and moral hazard is that the relationship between the parties can be considered as one-shot. Even if the delegation of the service to the private sector is characterized by a long term, in many PPPs the parties must define how their interaction will develop at the beginning it. Once the parties have drafted and signed the contract, they typically do not have the chance to change it in a second moment. In addition, it is likely the government will not hire the same company for different PPP projects. This makes the relationship defined in each project one-shot. Thus, the principal and the agent cannot rely on the repetition of their interaction to achieve an efficient allocation of resources and smooth information asymmetry issues.

Given the primary importance of the issues related to asymmetric information, they have been the focus of many scholars in the analysis of PPPs. As anticipated, some of them point out that, greater delegation to the private party could create scope for information gathering on the agent side (Baron and Besanko, 1984; Iossa and Martimort, 2012; Hoppe and Schmitz, 2013).

Precisely, Hoppe and Schmitz (2013) maintain the private entity is likely to acquire better information on how costs will change in the operation phase if unforeseen events happen. It is thus worth considering how incentives to exert effort on improving the quality of the infrastructure and to not acquire private information impact the performances of PPP compared to traditional procurement. As usual, the authors consider two periods: the building and the operation phase. They assume that, when the parties sign the contract, they can only specify the basic features of the service to be provided. Later, once the infrastructure is in place (end of the first phase) the parties realize how the basic design of the infrastructure can be improved to provide a better service. If the agent succeeds in providing an innovative design for the infrastructure, the principal can add the new features at a low cost, if not, the costs are high. The quality of the infrastructure depends on the innovation effort of the agent. However, it is not directly observed by the principal, she only obtains a noisy signal. Additionally, consider that the builder, in the first stage, can spend resources to acquire private information on the future costs of enhancing the service. This can have only a strategic purpose as the same information can be gathered by the operator for free in the second stage. It follows that it is a socially wasteful activity that the government will be interested in discouraging.

Under traditional procurement, the only way the government can promote innovation effort is by rewarding the agent based on the verifiable but noisy signal it gets. On the contrary, under PPPs, the consortium will see the second-period information rent it will realize by providing an innovative infrastructure and will be encouraged to exert innovation effort. Therefore, PPPs have the advantage that the public authority does not need to pay a limited liability rent to induce innovation effort. Nonetheless, the disadvantage, is that the consortium will lever the information rent obtained in the first phase. Thus, making it more costly for the principal to carry out the second-stage service improvements. It can be concluded that the government will prefer PPPs when the signal on the agent's effort is particularly noisy to avoid the high limited liability rent that would have to be paid under traditional procurement. On the opposite, a strong signal on the effort exerted by the agent will make the traditional procurement preferable. In addition, if the wasteful cost of information gathering is relatively low the principal will opt for a Public-Private Partnership because the benefit of indirect effort incentives will exceed the social cost of inefficient information gathering (Hoppe and Schmitz, 2013).

Besides Hoppe and Schmitz (2013), also Iossa and Martimort (2012) focused on how the organizational form is related to the project implementation uncertainty when studying delegated project management. They conclude that PPPs are superior to traditional procurement when operational risk and informational asymmetries are limited. If this is not the case, the private party can gain a disproportionate advantage through private information. Similarly to Hoppe and Schmitz, bundling favors innovative design and is more cost-efficient when demand risk is low, when a precise measure for service standards exists, when the quality of the infrastructure is not easily verifiable, and when unforeseen circumstances can be addressed with an affordable adaptation of the used technology. Furthermore, the provision of a radically new service can hinder the PPP-related efficiency gains due to the lack of knowledge needed to anticipate the impact of unfamiliar design, procedures, or technology on the cost of operations. Differently, consider a service that has been provided for a long time under more traditional forms. If there is scope to modernize the way of provision keeping the level of risk relatively low, PPPs turn out to be suitable. Having the previous provision of the service, no matter in which form, generating the appropriate expertise to foresee the contingencies that may arise during operations (Iossa and Martimort, 2012).

An interesting perspective on the moral hazard issue in PPP projects is taken by Engel et al. (2014). In their analysis they introduce the figure of the sponsor, who is the equity investor in charge of bidding, developing, and managing the agreement. The authors study the life cycle of PPP finance and relate the moral hazard issues to the type of financing that is more suitable for the project. Initially, in the construction phase, the project outcome depends widely on the action of the sponsor and the private firm. The uncertainty surrounding the outcome quality, and the major design changes that can occur create scope for moral hazard. At this stage, the role of banks acting as mitigators on the moral hazard issues is of vital importance. They have the power to enforce strict control over any change in the project contract. Moreover, banks can monitor the behavior of the parties involved by providing funds progressively and only at project steps completion. In the operation phase of the project, after the building of the infrastructure, the project related risk decreases significantly and the scope for moral hazard too. The cash flow from the project is still susceptible to risk but it depends to a lower extent on the action of the parties involved. In this stage, since the control over moral hazard is not as important as before, bond finance becomes the most suitable source of financing. Bondholders are interested only in the events that can considerably affect the project cash flow underpinning the debt but do not directly take part in the PPP management. The operation phase of PPPs is then open to institutional and other passive investors for whom the high-risk of the construction phase was prohibitive.

Examining the literature on PPPs concerning asymmetric information and contract design emerges that most of the contributions focus on moral hazard issues (Martimort and Pouyet, 2008; Iossa and Martimort, 2012 and 2015; Engel et al., 2014; Danau and Vinella, 2015; Marimort and Straub, 2016; Buso et al., 2020) rather than adverse selection (Auriol and Picard, 2009 and 2013). If the selection of the agent happens in a competitive environment using a fair auction the assumption of no adverse selection can be reasonably justified. The empirical evidence shows the procurement process of PPP contracts tend to be longer and more onerous than in the traditional procurement. Precisely, between the 5 and 10% of the project costs derive from the bidding phase (Yescombe, 2007), and the bidding process lasts an average of 34 months (data on PFI projects closed between 2004 and 2006) (NAO, 2007). This evidence allows for maintaining that the adverse selection issue is likely to play a minor role in PPP contracts.

However, the problem of moral hazard, being related to the hidden action of the agent, is more difficult to be ruled out. As the agent’s effort is likely to affect the project outcome and it is difficult to be assessed by the principal, many authors have decided to assume no adverse selection and focus on the more compelling problem of moral hazard.

Chapter 2 – THE REAL OPTIONS APPROACH

2.1 - Irreversibility, magnitude, and uncertainty

In the opening paragraph of this chapter, we focus on three characteristics of PPP projects that considerably affect the way this kind of agreement should be modeled. They are the irreversibility of PPP investments, their magnitude, and the high level of uncertainty coming both form the project itself and from the environment surrounding it. These three aspects make the traditional method of the net present value (NPV)[4] unsuitable for the evaluation of PPP and require the use of a more sophisticated technique such as the real-option approach. The reason for this, the explanation of what a real option is and how it can be applied in the analysis of PPPs will be investigated later in the chapter.

In general, a decision is considered irreversible if it is not possible to modify its outcome once it has been taken. In other words, for a long time, it narrows down the spectrum of available choices (Henry, 1974). For instance, suppose to have the choice to chop down one hectare of the Amazonian forest in favor of croplands. The decision to preserve the forest is not irreversible as the option to free space for croplands is still available. On the contrary, once the trees are chopped down it is not possible to have the forest as it was for a long time; this is an irreversible choice. Moreover, choices differ in their degree of irreversibility, and investments do the same. Consider investing in a power plant, if oil is the only fuel that can be burned the investment is more irreversible than building an equivalent power plant where oil, as well as other types of fuels, can be used.

One of the most common reasons for investment irreversibility is that the capital is firm or industry-specific. This meaning that the capital invested in one firm cannot be used productively in other companies or sectors. In this case, if the investment turns out to be non-profitable the company does not have the chance to divest. The assets do not have value for other firms and thus the price at which they can be sold (if one) does not allow recovering from the expense incurred. The expenditure on firm-specific assets is a sunk cost.

Sector-specific investments are typical of Public-Private Partnerships. According to the data of the World bank group on private participation in infrastructures (PPI)[5] in 2019 this type of investment was concentrated in four main sectors: Transport, Energy, Water, and Municipal Solid Waste (MSW). Of the total US$96.7 billion invested across 409 projects in the world,

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[4] Invest in a project only when the present value of its expected cash flows is greater than, or at least equal to, the one of its costs.

[5] Annual PPI Database Global Report, 2019.

US$ 47.8 billion were invested in the transport sector, US$ 40.1 billion in the energy sector, and the remaining US$ 8.8 were almost equally split between MSW and Water (*Figure 1*).

FIGURE 7
Share of Sectoral Investment Commitments in Infrastructure Projects with Private Participation in EM-DEs, 2010-2019

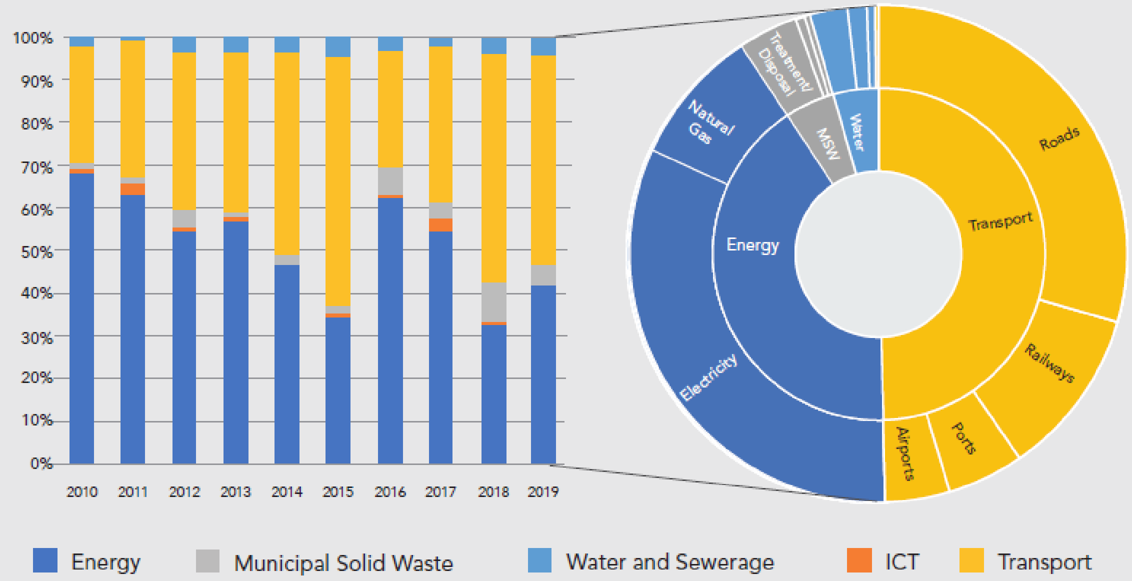

Figure 1, Source: Annual PPI Database Global Report, 2019, World Bank Group

All the just mentioned industries tend to require investment in assets that have no value outside the specific sector for which they were initially meant. Consider, for instance, railways, ports, and airports, if for some reason they are not as profitable as expected and it is not economically convenient to keep them in operation it is almost impossible to recover the initial investment. Even dismantling the infrastructure, selling (if possible) the raw materials, or looking for alternative uses of the assets the chance of recouping the original expense seems at least very optimistic. The same is valid for power plants in the natural gas and electricity sector where the technology used is not employable for any other application.

Besides being irreversible, PPPs tend to be particularly onerous. According to the data of the World bank group on PPI, between 2014 and 2019, the average project size ranged between US$ 229 in 2016 and US$ 324 in 2014 million. Moreover, in 2019 the maximum amount invested in a single project was US$ 8.6 billion while the maximum recorded in the entire period is about US$ 37.7 billion in 2015 (*Table 1*). In general, a project is considered "small" when its size is below US$ 100 million. In Emerging Market and Developing Economies (EMDEs), small projects were about half of the total both in 2018 and 2019, whereas middle-size project (US$ 100 million – US$ 500 million) went from 39 percent in 2018 to 35 percent in 2019.

Finally, large projects (US$ 500 million - US$ 1 billion) rose from 4 to 8 percent in 2019 (*Figure 2).*

TABLE 1: FREQUENCY DISTRIBUTION OF PROJECT SIZES, 2014-2019				
Year	**No. of Projects**	**Mean**	**Median**	**Maximum (2018 US$ millions)**
2014	389	324	80	11,146
2015	385	318	77	37,702
2016	353	229	71	5,430
2017	386	261	76	7,050
2018	428	232	96	3,804
2019	409	238	92	8,637

Table 1, Source: Annual PPI Database Global Report, 2019, World Bank Group.

FIGURE 2

Size Frequency Distribution of Infrastructure Projects with Private Participation in EMDEs, 2018 and 2019

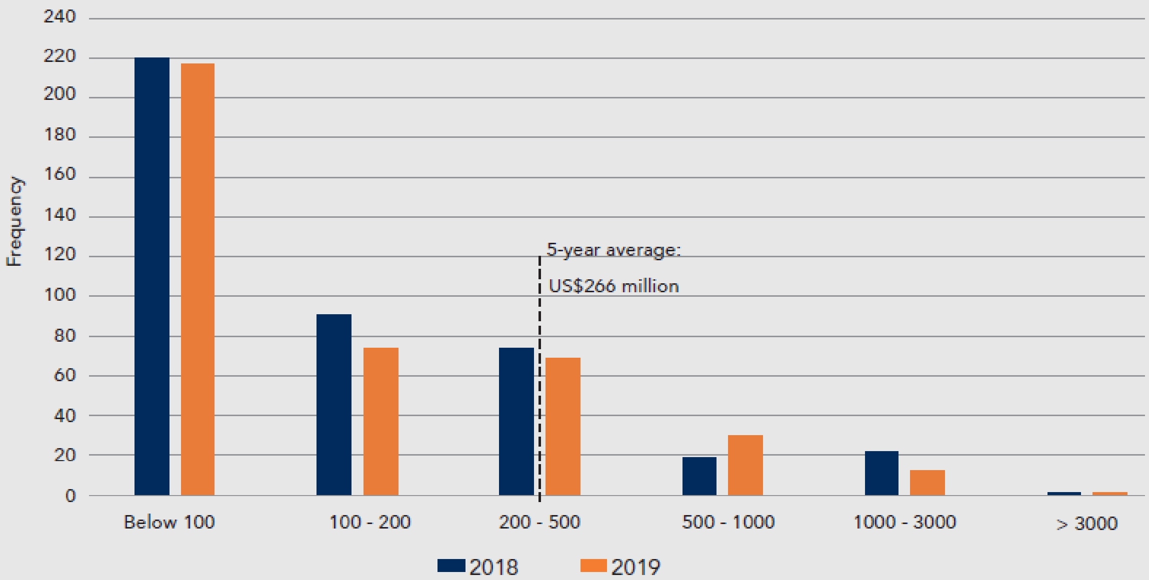

Figure 2, Source: Annual PPI Database Global Report, 2019, World Bank Group.

In conclusion, PPPs are characterized by a considerable level of uncertainty. First, there is the uncertainty related to the project itself. The parties do not know in advance the quality of the outcome, it is usually assumed to depend both on the agent’s effort and pure luck. The cash flows generated by the project is itself uncertain and subject to unforeseeable demand fluctuations. The construction and operation costs can be estimated before the project is initiated but are likely to differ from the initial assessment. Second, also the environment surrounding PPP is a source of uncertainty. Capital cost, regulatory requirements, political risk, and technological changes are just some of the variables that have the power to affect the project outcome. These factors have the potential to alter the proper functioning of PPP contracts, and in some cases, completely jeopardize the good outcome of the project.

2.2 - Flexibility and investment timing

The investment irreversibility, the considerable project size, and the pervasive uncertainty surrounding PPP have the potential to seriously hinder the use of these agreements. Also, considering the long term of the contract, scholars and PPP stakeholders have indicated the injection of more flexibility in PPP agreements as a feasible way to make them more attractive for the parties involved. In this context, the term "flexibility" is used to describe provisions that, once embedded in the contract, allow for an immediate reaction by the parties to unforeseen changes in the assumptions on which the contract was based (Demirel et al., 2017). Flexibility clauses in PPP investments can take the form of guarantees on the project cash flows or key variables for the project profitability. Alternatively, they can allow the private party to early terminate the contract or to defer the payments for the initial investment. These are just examples but the ways to inject flexibility in PPP contracts are virtually infinite.

As most major investments, PPPs have an important characteristic that can have a great impact on the decision to invest. The parties can decide to delay the investment to acquire more information on prices, project costs, and other market conditions before committing resources. On some occasions, the investment must be carried out immediately as strategic or social needs require to do so. However, in most cases, the parties of the PPP agreement have the chance to decide the investment timing. The cost to delay is given by the forgone cash flow and social benefit. The gain is the possibility of acquiring important information on the project variables. Given the high level of uncertainty, and the significant and irreversible commitment entailed by PPP projects, the balance between benefits and costs can be positive, and the decision to postpone the investment optimal.

Irreversibility impairs the theoretical foundation of the standard NPV rule for investment decisions: "An investment is worth to be undertaken when the difference between the present value of its expected cash flows and the one of its costs is at least nonnegative". This principle does not consider that once an investment is carried out, the investor loses the option to wait and acquire more information. Irreversible investments are particularly sensitive to the uncertainty over the future project cash flows, operating costs, and interest rates. This enhances the value of waiting to solve some of the mentioned uncertainties. In other words, the traditional NPV rule does not consider the opportunity cost of exercising the option to invest. Including it in the computation, one can still use the “positive NPV” terminology. However, traditionally the NPV rule has always overlooked the value of delaying the investment.

The characteristics of an irreversible investment can be seen in analogy with a financial call option. The holder of a call option has the right (in an interval of time) to receive a financial asset (e.g. a stock) at a prespecified price called exercise price. A firm having an investment opportunity is faced with an analogous choice. It can afford a known initial investment to have an asset of some future uncertain value. If the asset value rises, the payoff of the investment increases. If it falls, the firm can decide not to invest and loses just what it spent to obtain the investment opportunity (Pindyck, 1990). This analogy is the basic intuition of the real options approach to investment. Being central in our work, it is explained in greater detail in the next paragraph.

2.3 - Real options

To understand what real options are, it is useful to state a basic definition of traditional financial options. First, financial options are derivative instruments, meaning that they do not have value for themselves but derive their value from the underlying instrument they are written on. For example, a stock option derives its value from the one of the stock, which is the so-called underlying. Two basic types of options exist: call and put options. Starting from these two a virtually infinite number of more complicated compound options can be created. A call option gives the holder the right, but not the obligation, to buy a certain amount of an instrument on a certain date in the future, at a price agreed today. A put option gives the holder the right, but not the obligation, to sell a certain amount of an instrument on a certain date in the future, at a price agreed today. The option is said to be exercised when the holder buys (call) or sells (put) the underlying instrument covered by the option contract. The investor will decide to exercise the option and buy (sell) the underlying asset only if the market price of it is higher (lower) than the one agreed in the option contract. At that point, and this is important to our purpose, the investor will have the gain deriving from the difference between the current price and the agreed price of the underlying but will lose the chance to exercise the option in the future. The option can be exercised only at its maturity date it is of the European type while American options can be exercised at any time until the maturity date. Finally, since financial options give just the option but not the obligation to buy or to sell the underlying, they do not come for free but at a premium.

The term “real option” was first coined by Myers (1977) who noticed the analogies between financial options and future investments by corporations. The firm that holds the option to make and investment can arbitrarily choose to undertake it or not. The mechanism is, thus, analogous to a financial option with the only difference that in one case the underlying is a financial asset (financial options), whereas in the other it is a real asset (real options). To fully understand the analogy between real and financial options it is worth going true the main variables on which their value hinges upon (Martins et al., 2015):

1) The *time to maturity* as meant in financial options is translated, in real options, as the span in which the investment opportunity is available to the firm. Real options are often seen as American options since in most cases the company has the option to invest in an interval of time and not on a specific maturity date.
2) The *exercise price* in real options is the initial sunk cost that the company must bear to begin the investment, or, in the divestment case, the amount of money received by the firm.
3) The *volatility* of the stock return in financial options corresponds to the variability of growth in project value when considering real options. In both cases, once the option is exercised, the investor owns (in the case of a call option) either the stock or the asset, the value of which will fluctuate stochastically. This uncertainty captures the unpredictability of the cash flows coming from the project.
4) Today *share price* is associated to the present value of the expected cash flow of the real asset.
5) The *risk-free interest rate over the life of the option* has the same meaning in both real and financial options, being the theoretical interest rate returned in case of a riskless investment.
6) The *dividends* of the traditional stock option parallel the value lost waiting to invest in the asset. While the company is waiting to invest it is bearing the opportunity cost of the missing cash flows. This aspect is particularly important to understand the trade-off entailed by the decision to wait. The more the firms wait before investing the more information it will gather on the future cash flows. However, the cash flows forgone by not initiating the project must be considered in the decision. When the marginal benefit equals the marginal cost of waiting the optimal timing to perform the investment is found.

Also, Dixit and Pindyck (1994) explain the analogy between real and financial options. In brief, they say that the option holder has the right (need not to be exercised) to bear the investment cost (exercise price) and receive a project (the stock) the value of which fluctuates stochastically. Form this analogy, it is possible to apply financial option pricing methods to the study of the optimal investment rule. This intuition has inspired other scholars who have seen the potential of its application. PPP agreements, being subject to irreversibility and uncertainty have also been studied through this lens.

2.4 - Applications to PPP of the real options approach

The flexibility clauses introduced in PPP to allow the parties to adapt to unforeseen changes in the project environment show real option characteristics. Every time a provision is included in the contract it generates some value for the parties, that value can be quantified using the real options approach. The number of arrangements the parties can find is almost unlimited. For this reason, the literature has focused on the most common provisions introduced in PPP contracts and observed in real-world applications.

Some scholars have considered the introduction of flexibility in PPP agreements in the form of guarantees. One example is the granting of a minimum revenue guarantee (Huang and Chou, 2006). Since revenues in infrastructure projects are often subject to a considerable level of risk, the government can agree to recognize a minimum revenue guarantee (MRG) to limit the downside risks. When this clause is active, if the operating revenue of the private company falls below a certain threshold the government is obligated to cover the shortfall. The MRG can enhance the chances the concessionaire will be willing to participate in the project and will also improve the creditworthiness of it. Having a guaranteed cash flow will ensure a minimum level of debt coverage.

Alternatively, in the case of transportation services, one can consider guaranteeing a minimum level of usage of the infrastructure. This is the case of what is called a minimum traffic guarantee (MTG) (Blank et al., 2016). When the operating revenues of the company are tied to the demand for it, the public party can agree to cover the revenue loss if the traffic goes below a pre-defined level. This type of guarantee can also be associated with a maximum traffic ceiling that allows the government to control for a higher-than-expected return placing a cap on revenues deriving from extraordinary traffic volumes.

Other types of flexibility clauses that can be included in PPPs are the abandonment or cancellation options (Huang and Chou, 2006; Alonso-Conde et al., 2007; Blank et al., 2016; Busoet al., 2020) that allow the concessionaire to abandon the project before the natural end of the concession or to the government to terminate the concession period early. Finally, also the possibility of deferring payments has been considered to make PPPs more attractive (Alonso-Conde et al., 2007).

Overall, from the literature emerges that the above-mentioned options do generate value in PPP contracts. However, when more options are considered together in the same project, for a correct assessment of their value, an analysis of the interactions among options must be carried out. It can be the case that two or more options complement each other value or that their values are substitutes and thus they counteract each other, and their values are reduced.

Huang and Chow (2006) consider as a numerical example the Taiwan High-Speed Rail Project to value a minimum revenue guarantee and the option to abandon in the pre-construction stage of the project. The first model they propose considers each option independently, it is a single option pricing model. The second accounts for the interaction among option values, it is a compound option model. The option to abandon allows the private party to abandon the project during the pre-construction phase while the MRG works as explained above. From the single option pricing model, both options can create substantial values. On the contrary, under the compound option model, the two options have a negative effect on each other value. As the MRG is increased the option to abandon decreases in value. Economically speaking, as the government is willing to guarantee a higher revenue the private party has fewer reasons to abandon the project. The minimum revenue can be raised to a point where the firm has no reason to abandon given the strong downside risk protection it is enjoying. In that case, the abandonment option is valueless. However, rising the MRG involves considerable expenses on the side of the government, its benefits and costs need to be carefully evaluated and justified. On the contrary, granting an abandonment option seems a better alternative under binding budgetary constraints, also, its justification is easily understandable.

Interaction among options is considered also in the analysis of the real options embedded in the PPP for the construction and operation of the Melbourne CityLink project (Alonso-Conde et al., 2007). It is the first example of a motorway network characterized by fully automated tolling. The options included in the agreement are two. An option to defer the payments and the chance of anticipated cancellation of the investment. The former is held by the private company and it is shown it can be modeled as a put option. The latter is in the hands of the government which can decide to terminate the concession period early and, in the model, it is treated as a call option. The authors have used a Monte Carlo simulation approach to value the options and their interaction under different scenarios. They conclude there is no monotonic relation between the option value and variations in the project volatility. This is justified by the complex path-dependent nature of the options. However, the value added by including the two options in the contract is measured as being over 10% of the value of the company. The downside of providing financial incentives to foster private investments in infrastructures is again their impact on the government's budget. As policy advice, this aspect needs to be carefully analyzed at the time the revenue guarantee is defined in the contract.

To conclude this section, two works that studied minimum traffic guarantees are analyzed. The first examines the MTG alone and no interaction with other options is taken into consideration. The second studies a case of a Public-Private Partnership where the MTG is offered in combination with a maximum traffic ceiling and an abandonment option.

Studying the MTG in isolation, the authors conclude that having the government sharing part of the risk with the private party is necessary to make some classes of highly risky projects attractive to private investors. Intuitively, the higher the risk of the project the higher the need for a guarantee to make private companies willing to participate in the partnership. The main limit to MTG is the outlay required by the government. To reduce the risk of too high liabilities on the government side, an effective solution is the use of caps on the total expense associated with a specific level of MTG. Limiting the outlay on the single project allows the government to split its resources on a wider range of projects thus leveraging its investment capability (Brandao and Saravia, 2008).

The second work offers a different perspective on how the MTG value changes when combined with other options. The authors study a hypothetical PPP contract for a 25-year toll road concession. In this contract, the profit of the private party depends on the future level of traffic on the toll road (the greater the traffic the higher the profit). However, the amount of traffic is not known in advance. If it falls below a certain threshold, the project is no longer profitable, and the concessionaire has an incentive to break the agreement. To smooth the uncertainty regarding the future profitability of the project and to make it more attractive the government enriches the agreement with three options: a traffic guarantee, a traffic ceiling, and an abandonment option. From the point of view of the concessionaire, the first two work respectively as a long position on a put option and a short position in a call option. The underlying asset is the traffic. The strike "price" for the put is the traffic guarantee (the level below which the concessionaire receives a payment form the government). The strike "price" for the call is the traffic ceiling; if it is exceeded, the company will pay the government the excess profit. Finally, there is the abandonment option, which allows the firm to breach the contract and walk away paying at most a very low abandonment fee. The interactions between these options are studied by the authors using a simulation and an analytical method. What results is that the government can shape the traffic guarantees and the abandonment option in such a way that three main goals are achieved: the private party is willing to undertake the project, the probability of abandonment is lowered to an optimal level, and the risk exposition of the government is minimized. Working on these three aspects will allow maximizing the overall value of the options (Blank et al., 2016).

2.5 - Contract design in a real options framework

At this point, two strands of the literature on PPPs have been presented. The first, analyzed in the first chapter, focuses on asymmetric information, principal-agent problems, and the theory of incomplete contracts. The real options embedded in the agreement and their importance on risk allocation and project valuation are ignored. The second strand, studied in the previous paragraph, on the contrary, has the specific goal of valuing the flexibility clauses included in PPP (e.g. minimum revenue guarantee, deferred payments, abandonment) considering their real option characteristics. Doing so, it determines the optimal investment policy in a real options framework. Nonetheless, it does not consider contract design and performance incentives. Given the significance of both aspects of PPPs, a third strand of literature has tried to bridge the gap between the other two. It aims at finding the optimal specification of PPP contracts giving the proper attention to the real options that are commonly observed to be part of these agreements.

A recurring feature in the third strand of literature is the attention to the timing of the investment that is determined inside the model instead of being taken exogenously. Silaghi and Sarkar (2018) design a clear example of a model that merges the real option aspects of PPPs with the contract design theory. The two authors focus on a hidden action problem. The government is assumed to have an option to invest in a specific project and to delegate the exercise decision (when to begin the project) to the agent. The information asymmetry arises because the government is not able to directly observe the effort of the private party. The only parameter observed is the quality of the outcome, when the firm exerts effort it enhances the chances of having a high-quality project. Being in the best interest of the government to obtain a public service of high quality, it will have to design a contract that makes it more convenient for the firm to exert (costly) effort. The offer of the government takes the form of a menu of contracts, and the variables at stake are the investment triggers (that define the timing at which the project is initiated and begins to produce cash flows) and the compensation of the firm that is expressed as a fraction on the cash flows generated by the project. A combination of investment triggers and firm compensation will be offered for each quality level (high or low) of the project. With this menu of contracts, the government has the chance to induce the firm to initiate the investment at the first best trigger. However, to do so, the principal must pay an informational rent to the firm. The basic model is then extended including an additional abandonment option. The private party, exercising the option, has the right to require a bailout from the government if the situation deteriorates. From this setting, the optimal contract, the social loss, and the effect of the bailout option are illustrated using a numerical example (Silaghi and Sarkar, 2018).

Investment timing is also central in the analytical work by Takashima et al. (2010). The paper investigates the interaction of the government and a private firm over the investment timing in a PPP project. Differently from Silaghi and Sarkar (2018), the authors consider perfect symmetry of information between the parties. In this framework, the effect of a government guarantee and cost-sharing on the investment timing is shown. The authors define first-best and second-best investment timing. The former maximizes the sum of the firm and government payoffs. The latter maximizes the value of the private firm only. The result is that when the value of the government guarantee is high and/or the cost-sharing rate of the private party is low, the first-best exercise timing is later than the second-best.

The assumption of symmetric information is relaxed in the paper by Soumare and Lai (2016). The private company has more information on the project quality than the government does (the so-called plum problem). Compared to Silaghi and Sarkar (2018) this is a problem of hidden information instead of hidden action. The authors study the support of the government in PPPs project in two forms: loan guarantee and direct investment. In the first case, the government commits to pay for the loan that is used to finance the project, if the private party turns out to be insolvent (i.e. default). The government reduces the interest payments that are tax-deductible and, thus, increases the taxable income. The loan guarantee is seen as a put option written by a backer to a bondholder. Under the second form, the government participates directly in the investment and in return receives a share of the project. The number of shares received by the government act as a bargaining tool. The higher the number of shares in the project the more the government can successfully ask for additional information. When the asymmetry of information is particularly pronounced the government must consider requiring a higher share in the project to smooth their negative effect. Also, Soumare and Lai (2016) conclude that a loan guarantee is more effective in diminishing the project’s borrowing rate. In the case of high asymmetric information, the project sponsors should prefer, ceteris paribus, a loan guarantee to a direct investment, if not they will have to give up more control over the project and gain lower earnings.

Another paper considering the problem of hidden information in a PPP agreement is the one by Buso et al. (2020). In their contribution the authors model a case where the government entrusts a private party to build, maintain, and operate an infrastructure project offering as payment the collection of user charges. To encourage investment the government offers a one-time grant and an early-exit option. The latter can be exercised by the private party at any time but entails a termination fee defined at the contract formation. If the option is exercised, the government will resume the project undertaking the responsibility for the provision of the service. Also, the contract defines at which moment the investment must be initiated. Again, the investment timing is treated as an endogenous variable. A continuous-time setting is assumed, and the information asymmetry arises from the fact that the private company holds exclusive information on operating profits. According to the authors' findings, the grant of the exit option encourages a quicker implementation of the project (and thus exertion of the public benefits) with lower costs relative to a lock-in contract. Moreover, the combination of the exit-option with direct subsidies (the one-time grant) alleviates the problems related to the asymmetry of information for particularly risky projects. In so doing, it reduces the government's expenses even considering the extra cost borne in the case it has to take charge of a loss-making project upon the exercise of the abandonment option. The paper proves that, if properly accounted for during the contract formation, the project's abandonment by the private party may not be too harmful.

Chapter 3 – INTRODUCTION TO THE MODELS

3.1 - The bargaining power of the parties

In the present literature on PPPs, an aspect that has received only marginal attention is the bargaining power of the participants. Most of the papers that consider the principal-agent problem between the government and the public firm just assume that one of the parties (usually the principal) makes a take-it-or-leave-it offer to the other. In that case, there is not a bargaining phase as the entire bargaining power is in the hands of who makes the offer. However, we believe that in a typical PPP agreement the bargaining power is likely to be allocated to both parties. For several reasons it is not realistic to assume that the agent has no decisional power on any of the variables involved in the contract. First, consider that in a PPP contract the agent must bear the initial sunk cost. To do so, since the considerable size of the average PPP project is likely to exceed the financial means of a private company, the agent has the burden of finding the proper sources of financing committing to repay the lender once the project starts to generate cash flows. Second, the competencies required to carry out a PPP project are highly specific. In most cases just a few firms will be able to handle the technical complexities related to a PPP project, reducing the competition on the private party side. Third, as pointed out at the beginning of the chapter, PPPs are characterized by a substantial level of risk bore by the agent and by a considerable level of uncertainty deriving both from the project itself and from the environment.

All these elements make the agent an active part of the contractual relationship with the principal. It is likely that bearing significant responsibilities, the private company will not passively let full decisional power to the government over the variable at stake. The agent will claim an active role over the decision of at least some of the variables. This calls for introducing in the models for PPPs a bargaining phase between the government and the private party. Differently, if it were the government financing the project it is reasonable to assume that it will decide autonomously the contractual offer to the private company. It will formulate a take-it-or-leave-it offer over every variable at stake. In this case, the problem does not require to introduce the bargaining between the parties and can be limited to study the classical issues related to the asymmetries of information.

The contractual relationship between the parties involved in a PPP should not be merely seen as the principal hiring the agent but as a more complex interaction in which the two partners reach an agreement for the completion of a complex task. Even if the interests of the public and private entities are usually divergent, it is advantageous for both to find a compromise that allows them to profitably carry out the project. The allocation of bargaining power has been proven to affect the joint surplus deriving from the contract (Pitchford, 1998). Depending on how the bargaining power is split, the joint surplus varies drawing an efficient frontier.

3.2 - Flexibility values

Given the contractual relationship that incurs among the government and the private party, the introduction of flexibility provisions in the contract can serve as a powerful tool to ease the reaching of a mutually profitable agreement. The flexibility clause will more effectively play this role if it is advantageous for both parties. However, most of the literature previously analyzed has focused on provisions that were beneficial for just one of the parties. Consider the above-mentioned minimum revenue guarantee, the minimum traffic guarantee, the maximum traffic ceiling, the abandonment option, the option to delay payments, and the early-cancellation option. In all these cases is one party gaining value while the other is losing it.

Most of the time the provisions are meant as a "bribe" offered by the government to the firm to induce its participation in the PPP; featuring a sort of hierarchical interaction between the entities involved. This view is not in line with the contractual relationship incurring among the two parties. If we intend the private firm and the government negotiating to complete a project that would be individually infeasible, the most effective form of provision should be meant to create contractual value for both parties. In this regard introducing flexibility over the investment timing is pertinent. It is not advantageous just for the firm, which gains access to information that counteracts the project-related uncertainty, but also for the government that improves the chances of obtaining a better project.

Introducing the investment timing, as a variable in the contractual arrangement, justifies the adoption of the real options approach. If the government allows to postpone the investment, and the parties find optimal to carry it out in a date different form today, option value is created. On the contrary, if the government imposes to invest immediately for strategic reasons the possibility of delaying the investment is not on the table, and the traditional NPV rule is suitable to assess the value of the agreement.

3.3 - Our contribution

From the analysis of the literature, two aspects have emerged as remarkable in the study of PPP.

One concerns the issues related to the asymmetry of information among the parties. As the government delegates to the firm the completion of the project, the latter is likely to acquire private information or to be in the condition of hiding its action form the principal. For this reason, PPPs are subject to inefficiencies deriving from adverse selection and moral hazard.

The principal, to solve the asymmetry of information, must pay what is called an information rent that makes the agent revealing its private information or action. In general, PPPs tend to suffer more of moral hazard than adverse selection issues. If the process for selecting the agent takes place in a competitive environment using, for instance, a fair auction, one can reasonably assume that the problem of adverse selection is solved. As done by most of the PPP literature our work focuses on the moral hazard problem and assumes no adverse selection in the delegation process.

The other concerns the flexibility provisions embedded in the PPP contracts and their option-like characteristics. To assess how they alter the value of the contract scholars have used the so-called real options approach. It is based on the analogy between financial options and investment opportunities and allows applying financial option pricing methods to the study of the optimal investment rule. In our contribution, we inject flexibility in the PPP agreement treating the investment timing as endogenous and defined by the parties. In other words, the principal and the agent can decide if they want to begin the project immediately or to postpone the investment to acquire more information on the uncertain variables of the project. If they opt to do so there is some option value of waiting to consider in the project evaluation. To account for this, we use the real options approach.

An aspect that is instead missing in the current literature on PPPs is the study of the bargaining power of the parties. Typically, the scholars studying PPP contracts assume that the principal has all the negotiating power and formulates a take-it-or-leave-it offer for the agent. However, we believe that, given the significant responsibilities of the agent in the project, it is more appropriate to assume that also the private company will have bargaining power over at least some of the variables at stake.

A central aspect of the study of bargaining power is the way it can be modeled. Three ways are described in the literature. First, one can consider a standard Principal-Agent model and increase the reservation utility of the agent to raise his bargaining power. Second, the same moral hazard problem can be studied using a Nash-bargaining solution. Third, the bargaining power can be studied in an alternating offer game with moral hazard. Demougin and Helm (2006) compared the three models in a setting with risk-neutral parties and a financially constrained agent. They show that the same set of contracts arises in each of the three cases. In our contribution, the first two ways of modeling the bargaining power are used to describe the interaction between the government and the private firm. We replicate the study by Demougin and Helm (2006) showing how the principal payoff depends on the bargaining power of the agent in a PPP framework.

The foundation for the present work is to be found in the paper by Buso et al. (2020). In the model presented by the three authors and that we use as a benchmark, a public authority (principal) has the option to undertake an investment for the provision of a public service and decides to delegate the exercise of the option to a private firm (agent) that has relevant expertise. For the delegation, a PPP contract is used. As usual in moral hazard problems (hidden action), the quality of the project depends on the unobservable effort of the agent. What is observable and thus, contractible, is the realization of the stochastic process that defines the cash flow deriving from the project. To induce effort the principal formulates an offer based on the investment timing (that takes the form of a realization of the cash flow) and a fixed transfer. The paper is thus combining real options theory with agency conflicts in the study of PPPs.

Departing from Buso et al. (2020) we first expand the model defining the interaction between the two parties in a two-stage process (Benarjee et al., 2014). First, there is the bargaining between the parties over some of the variables at stake. Subsequently, the government formulates a take-it-or-leave-it offer on the remaining variables. A new variable is introduced as the government is now receiving a share of the cash flow of the project. In this setting, two cases are studied. In the first, the government and the firm bargain on how to share the cash flow, and then the government formulates a take-it-or-leave-it offer on the investment trigger and the fixed transfer. In the second, the government and the firm bargain on how to share the cash flow, and on the investment timing, then the government formulates a take-it-or-leave-it offer on the fixed transfer. Form this second case emerges that the option to invest is considered as jointly held by the two parties. In Buso et al. (2020) the government delegates the option to invest to the private party and then offers a contract to make the investment timing socially optimal. Differently, in our contribution, the two parties bargain on the timing of the investment. As in a jointly held option, the government and the firm must agree on the investment timing to be able to exercise the option.

The second way used to describe the bargaining power of the parties is by introducing the reservation utility in the benchmark model. Here the process is not in two stages anymore as the government makes a take-it-or-leave-it offer on the investment trigger and the fixed transfer. However, the offer must grant a certain level of utility to the agent, the higher that level of utility is, the more demanding is for the government to achieve it. It follows that as the bargaining power of the agent rises, the payoff of the government will shrink in favor of the one of the agent. The model can offer interesting insights into the dependence of the payoff function of the agent's reservation utility. First, the payoff function is divided into two segments. After a certain reservation utility level, the government is not able to induce effort anymore, the agent will have no incentive to make effort and the probability of a high-quality outcome will be lower. Also, the shape of the function is worth to be studied. It results that, as the reservation utility goes up the payoff of the agent goes down. Moreover, the latter decreases at a decreasing rate as the function is convex.

The rest of the work is organized as follows. In the next chapter, the benchmark model is presented defining the setup and the notation that will be used throughout the rest of the thesis. Subsequently, chapters 5 and 6 present, in the order, the Nash-bargaining model and the reservation utility model. The last section concludes.

Chapter 4 - THE BENCHMARK

The present chapter presents the model by Buso et al. (2020) that is used as a benchmark for the analytical part of this work. Their paper is a contribution to that strand of the literature on PPPs that combines the studies on mechanism design and contract theory with the models that focus on the value of the real options embedded into PPP agreements.

The two parties, whose interaction is studied in the model are, as usual in the PPP literature, a public authority (principal), and a private company (agent). The first has the opportunity to invest in an infrastructure for the provision of a public service and is contemplating the idea of engaging in a PPP with the private party. To the latter will be assigned the goal of completing a project that will start generating a cash and a public benefit flow upon delivery. The efficient outcome of the partnership is hindered by a moral hazard problem. Exerting effort the agent could enhance the probability of providing a high-quality service that translates into a greater public benefit and cash flow. However, the effort is expensive for the agent and the principal cannot observe it directly. To enhance the probability of having a high-quality outcome the government has to offer a menu of contracts to the agent that will provide the right incentives to exert effort. To be contractible a variable must be observable by both parties. The optimal menu of contracts is studied under two scenarios. First, the agent has no negotiating power and will accept to participate in the project as long as his NPV does not fall below zero. In this case, the moral hazard issue does not affect the investment timing, so the first-best timing is guaranteed, the surplus of the project is shared among the two parties, and the project efficiency is not harmed. Second, the agent agrees to participate in the partnership only if his net present value is strictly positive. In this occurrence, the project efficiency is less than optimal, the first-best investment timing is not always achieved, and the private party obtains a rate of return higher than the hurdle rate. Considering the investment timing in their work, the authors prove that the agent's negotiating power not only transfer surplus from the principal to the agent but also harms project efficiency.

4.1 - Model set up

The model involves two parties. A public authority (principal) and a private firm (agent). The principal has the option to invest in a facility that will generate a social benefit flow (b), a cash flow (x_t) and will require an initial investment (I). The firm, instead, possesses a relevant expertise in designing and managing the facility. As an assumption, the infrastructure will require no time to be carried out, starts immediately to generate a cash flow, and has an infinite lifetime. To help figuring the situation, a good example of the infrastructure could be a new highway. Besides the private benefit enjoyed by direct users, it generates a social benefit as it helps to unload the urban traffic and to control local pollution. The tolls paid for the use of the highway are the mentioned cash flows.

The timeline along which the interaction between the principal (she) and the agent (he) develops is as follows. At the beginning, there is a pre-contractual phase ($t < 0$) where the principal selects the more suitable firm to carry out the project. The two parties bargain on the compensation required by the agent to participate in the partnership. It is assumed that an agent with negotiating power will successfully claim a compensation that makes his net present value at the time of the investment strictly positive. Differently, if the agent has no negotiating power, he will accept a rate of return equal to the project's hurdle rate. Defining which is the appropriate compensation entails a careful evaluation of the level of riskiness of the project.

After the pre-contractual phase, the parties enter in the contractual phase. The government, based on what defined earlier, submits a contract to the agent. At this stage, the asymmetry of information between the parties plays a critical role. The public benefit b is a function of a random binary variable called θ that indicates the quality of the project. If the outcome of the project is of high quality, θ takes the value θ_1, if the quality is low it takes the value θ_2; where $\theta_1 > \theta_2$, and $\Delta\theta = \theta_1 - \theta_2 > 0$. The probability distribution of the variable θ depends on the agent's effort. If the firm exerts effort, a draw of θ_1 has probability $q_H \in (0,1)$, while with complementary probability $1 - q_H$, θ will take the value θ_2. Differently, if no effort is exerted, θ_1 will be observed with probability $q_L \in (0,1)$, θ_2 with probability $1 - q_L$. Where $q_H > q_L$, and thus $\Delta q = q_H - q_L > 0$. The effort exerted by the agent comes at a cost $\xi > 0$. Once θ assumes one of the two possible values θ_1 or θ_2, it then stays constant throughout the life of the project and, as a consequence, the value of the social benefit flow $b(\theta)$ does not change over time.

The source of the moral hazard problem faced by the principal is the unobservability of the agent's effort. Observing the realization of $b(\theta)$ she cannot infer whether the agent has exerted effort or not. Also, $b(\theta)$ is observable only after the contract is signed so it is not a contractible variable. What can be made part of the contract is the cash flow of the project, described by the variable $x_t \in (0, \infty)$. It is assumed to fluctuate over time according to a risk-neutral geometric Brownian motion:

$$1. \quad \frac{dx_t}{x_t} = \mu dt + \sigma dz_t, x_0 = x$$

Where the initial value is $x_0 = x$, μ gives the risk-neutral rate of drift, σ is the positive constant volatility, while dz_t is the increment of a standard Wiener process under the risk-adjusted measure and satisfies the conditions $E(dz_t) = 0$ and $E(dz_t^2) = dt.$ A positive rate of return shortfall δ is assumed, and so the risk-free interest rate r is greater than μ: $\delta = r - \mu > 0.$ Once the infrastructure is carried out it yields a rate of cash flow of δx_t to the stakeholders either as dividends or liabilities. δ is the rate of return that an investment with the same risk profile would yield to a potential investor. Given δ, when the agent has negotiating power, he will be able to gain from the project a rate of return higher than δ, on the contrary, if the agent has no negotiating power, he will receive exactly δ. If the return of the agent falls below δ he will decide not to take part in the partnership with the government. What makes the variable x_t contractible is the assumption that its realization is observable in each moment by the parties. Besides x_t, the other variable that is used by the principal in the menu of contracts offered to the agent is a fixed transfer called ω. In the contractual phase, the principal will come up with two contracts to be offered to the agent. One specifies the investment trigger and the transfer to be received in the case of a high-quality project (x_1, ω_1), the other the same variables if the quality of the project is low (x_2, ω_2). The investment trigger is a key variable in this setting, as it defines the investment timing[6]. Once the stochastic process 1. hits the value specified in the contract the agent must invest, the trigger will be different if the quality of the project is high or low.

At $t = 0$ the principal has offered the menu of contracts to the agent $\{x_i, \omega_i\}, i \in (1,2)$. Right after the contract is signed, the parties observe the realization of the binary random variable θ and know if the outcome of the project will be qualitatively good or bad. In the first case, the investment will be carried out when the stochastic process hits x_1 otherwise when it hits x_2. Once the investment is initiated the private party will receive respectively ω_1 or ω_2. In the contract, the investment timing is identified in terms of the realization of the stochastic process 1. The stochastic nature of the process does not allow defining a date when to carry out the investment. However, once 1. hits the investment trigger, we can identify an instant in time for the beginning of the investment. In our notation, that moment in the future when the investment is undertaken is indicated as τ, such that $\tau > 0$[7]. From the contract offered and the

[6] In the contract the principal wants to specify the time of the investment. However, being the process 1. stochastic, it is not possible to determine a date in terms of days or years. Thus, the government defines the moment in which to invest in terms of the stopping times x_1 and x_2 which are realizations of 1. When the project outcome is of high (low) quality and the stochastic process hits the value x_1 (x_2) the investment is carried out. See Appendix, part A.

[7] $\tau = \inf(t > 0 \mid x_t = x_\tau)$

characteristics of the stochastic process 1. will follow the agent's NPV and the principal's NPV at time τ. The former is[8]:

$$2. \quad F^B\big(x_\tau, \omega(x_\tau)\big) = E_\tau\big[\int_\tau^\infty x_\tau e^{-r(t-\tau)} dt\big] + \omega(x_\tau) - I = \frac{x_\tau}{\delta} + \omega(x_\tau) - I$$

The latter is:

$$3. \quad Y^B\big(\omega(x_\tau)\big) = E_\tau\big[\int_\tau^\infty b(\theta) e^{-r(t-\tau)} dt\big] - \omega(x_\tau) = \frac{b(\theta)}{r} - \omega(x_\tau)$$

In conclusion, notice that the variables I and x_t are assumed to be independent of θ. On the contrary $b(\theta)$ is not. Thus, exerting effort has no effect on the agent's payoff but only on the principal's one. This assumption defines a perfect asymmetry of objectives between the principal and the agent.

Departing from this setting the authors proceed as follows. First, they describe the first best solution and the principal-agent setting. Then they find the optimal contracts considering agents with and without negotiating power.

4.2 - First best

The first best contract is the one that the principal would offer if she were able to observe the agent's action. In this section $b(\theta)$ is assumed to be constant to ease the notation. As explained above, the optimal investment rule says to invest if the value x_t is at or above a specific threshold that here will be indicated as x_τ. In other words, the optimal investment timing is the first instant in which the process for x_t reaches the critical value x_τ[9]. The contract specifies the investment trigger x_τ and the fixed transfer $\omega(x_\tau)$. The objective function of the principal is her own payoff at time τ discounted back at $t = 0$, it will be indicated as $R^B(x_\tau, \omega)$. She will maximize it subject to the participation of the private party, i.e. the agent's NPV at time τ discounted back at $t = 0$ must be nonnegative.

More formally the maximization problem is:

$$4. \quad \max_{x_\tau, \omega} R^B(x_\tau, \omega) = \max_{x_\tau, \omega} \left(\frac{x}{x_\tau}\right)^\beta Y^B(\omega)$$

Subject to,

[8] Where B stands for benchmark.

[9] The value of x_0 is assumed to be low enough such that is not optimal to invest immediately, $\tau > 0$. If it were not the case, the option value of waiting would go to zero and the use of a real options framework for the analysis of the interaction between the parties would lose meaning.

$$5.\quad V^B(x_\tau,\omega) = \left(\frac{x}{x_\tau}\right)^\beta F^B(x_\tau,\omega) \geq 0$$

Where $Y^B(\omega)$ and $F^B(x_\tau,\omega)$ are defined in 3. and 2. respectively.

In the maximization problem 4.-5. the term $\left(\frac{x}{x_\tau}\right)^\beta$ is used to discount both $Y^B(\omega)$ and $F^B(x_\tau,\omega)$. The optimal investment threshold lies in the future (recall $\tau > 0$, i.e. $x_\tau > x_0$) but the terms of the contract have to be defined at $t = 0$, so a proper discount rate is needed. The term $\beta > 1$ is the positive root of the characteristic equation $\Psi(\zeta) \equiv \frac{1}{2}\sigma^2\zeta(\zeta-1) + \mu\zeta - r = 0$. See part A of the Appendix for the derivation of $\left(\frac{x}{x_\tau}\right)^\beta$, β, and $\Psi(\zeta)$.

The objective function 4. can be also expressed as the difference between the project's total welfare $W(x_\tau)$ and the agent's NPV $V(x_\tau,\omega)$ at $t = 0$. $W(x_\tau)$ is the sum of the payoffs of the principal and the agent at time 0. As ω is a transfer from the former to the latter it cancels out and does not appear in $W(x_\tau)$ that depends only on the present value of future cash and social benefit flows, and on the investment expenditure.

The maximization problem becomes:

$$6.\quad \max_{x_\tau,\omega} R^B(x_\tau,\omega) = \max_{x_\tau,\omega}[W^B(x_t) - V^B(x_t,\omega)]$$

Where,

$$7.\quad W^B(x_\tau) = \left(\frac{x}{x_\tau}\right)^\beta \left(\frac{x_\tau}{\delta} - \left(I - \frac{b(\theta)}{r}\right)\right)$$

Here the agent is assumed to have no negotiating power. Thus, the principal will appropriate the entire surplus from the project while the agent merely expects to break even. The government will find optimal to offer an ω that brings the firm's NPV to zero[10]:

$$8.\quad F^B(x^{BFB},\omega^{BFB}) = 0 \rightarrow\ \omega^{BFB} = I - \frac{x^{BFB}}{\delta}$$

The x_τ that maximizes eq. 7. is

$$9.\quad x^{BFB} = \delta\frac{\beta}{\beta-1}\left(I - \frac{b(\theta)}{r}\right) = \delta'\left(I - \frac{b(\theta)}{r}\right)$$

Where $\delta' = \frac{\beta}{\beta-1}\delta$ and $\frac{\beta}{\beta-1} > 1$ is the option multiplier that captures the effect of uncertainty. See part A of the Appendix for its derivation.

[10] BFB stands for benchmark first best.

Now, substituting 9. into 8. it is obtained:

$$10.\ \omega^{BFB} = I - \frac{x^{BFB}}{\delta} = \frac{\beta\frac{b(\theta)}{r} - I}{\beta - 1}$$

Summarizing, the optimal contract offered by the principal is[11]:

$$11.\ \{x^{BFB}, \omega^{BFB}\} = \left\{\delta'\left(I - \frac{b(\theta)}{r}\right), \frac{\beta\frac{b(\theta)}{r} - I}{\beta - 1}\right\}$$

From 11. results:

$$12.\ W^B(x^{FB}) = \left(\frac{x}{x^{FB}}\right)^{\beta} \frac{1}{\beta - 1}\left(I - \frac{b(\theta)}{r}\right)$$

4.3 - The investment problem under moral hazard

In this section as in the rest of the work, the action of the agent is hidden, and the principal faces a moral hazard problem. For studying how the parties interact under asymmetric information, $b(\theta)$ is assumed to be a random variable as described in the model set up. Based on the contract she is offering, the principal designs a mechanism to induce the agent to exert effort. The two contractible variables are the cash flow x_t and the fixed transfer ω. The menu of contracts offered to the agent takes the form $\{x_i, \omega_i\}, i \in (1,2)$. The contract maximizes the principal's ex-ante investment option value guaranteeing the participation of an effort exerting agent in the partnership. The maximization problem is as follows:

$$13.\ \max_{\{\omega_i, x_i\}, i \in \{1,2\}} R(x_1, x_2, \omega_1, \omega_2) = q_H \left(\frac{x}{x_1}\right)^{\beta} Y_1^B + (1 - q_H)\left(\frac{x}{x_2}\right)^{\beta} Y_2^B$$

Where $Y_i^B = \frac{b(\theta_i)}{r} - \omega_i, i \in \{1,2\}$.

The objective function must be maximized under two orders of constraints, ex-ante and ex-post. The first are:

$$14.\ q_H \left(\frac{x}{x_1}\right)^{\beta} F_1^B + (1 - q_H)\left(\frac{x}{x_2}\right)^{\beta} F_2^B - \xi \geq q_L \left(\frac{x}{x_1}\right)^{\beta} F_1^B + (1 - q_L)\left(\frac{x}{x_2}\right)^{\beta} F_2^B$$

$$15.\ q_H \left(\frac{x}{x_1}\right)^{\beta} F_1^B + (1 - q_H)\left(\frac{x}{x_2}\right)^{\beta} F_2^B - \xi \geq 0$$

Where $F_i^B = \frac{x_i}{\delta} + \omega_i - I,\ i \in \{1,2\}$.

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[11] For the contract $\{x^{BFB}, \omega^{BFB}\}$ to make sense it is required $\beta\frac{b(\theta)}{r} > I > \frac{b(\theta)}{r}$. Also, for $\tau > 0$ it is needed that $x < \delta'\left(I - \frac{b(\theta)}{r}\right)$.

Constraint 14. is the ex-ante incentive compatibility constraint. It ensures that the agent will weakly prefer to induce effort. Thus, it ensures there is no unobservable action on the agent's side at the time of the investment.

Constraint 15. is the ex-ante participation constraint. It guarantees that the private company will be willing to take part in the partnership, by requiring that its payoff is nonnegative. If this constraint is not satisfied there is no reason for the agent to abide by the principal's choice of transfers and investment triggers.

The ex-post constraints are:

$$16.\ F_1^B \geq 0$$

$$17.\ F_2^B \geq 0$$

$$18.\ \omega_1 \geq 0$$

$$19.\ \omega_2 \geq 0$$

The inequalities 16. and 17. guarantee to the agent that, irrespective of the realization of θ, his payoff will be nonnegative. The inequalities 18. and 19. follows from the assumption that the transfer to the private company that is undertaking the investment cannot be negative. In other words, the government can, under certain circumstances, grant no transfer to the firm but it is not reasonable to think that it will tax the company carrying the investment out.

4.4 - Optimal contracts when the agent does not have negotiating power

An agent with no negotiating power fails to claim $F_i^B > 0, i \in \{1,2\}$. The problem 13.-19. can be slightly reduced. The authors prove that in this circumstance equations 15. and 16. are always slack. The objective function 13. is maximized subject to 14. that can be simplified to:

$$20.\ \left(\frac{x}{x_1}\right)^\beta F_1^B - \left(\frac{x}{x_2}\right)^\beta F_2^B \geq \frac{\xi}{\Delta q}$$

and inequalities 17.-19.

The result is[12]:

Benchmark proposition 1 *When the agent does not have negotiating power, the principal offers the following menu of contracts:*

$$21.\ \{x_1^{\blacktriangle}, \omega_1^{\blacktriangle}(x_1^{\blacktriangle})\} = \{x_1^{BFB}, \omega_1^{BFB} + \Phi_1\}$$

[12] The proof of this proposition is in the appendix of the original paper by Buso, Moretto, and Zormpas (2020).

$$22.\ \{x_2^{\blacktriangle}, \omega_2^{\blacktriangle}(x_2^{\blacktriangle})\} = \{x_2^{BFB}, \omega_2^{BFB}\}$$

where $x_i^{BFB} = \delta'\left(I - \frac{b(\theta_i)}{r}\right), \omega_i^{BFB} = \frac{\beta\frac{b(\theta_i)}{r} - I}{\beta - 1}, i \in \{1,2\}$ *and* $\Phi_1 \equiv \frac{\xi}{\Delta q}\left(\frac{x}{x_1^{BFB}}\right)^{-\beta} > 0.$

To be noticed that the transfers $\omega_1^{\blacktriangle}$ and $\omega_2^{\blacktriangle}$ are expressed as functions of the investment triggers $x_1^{\blacktriangle}$ and $x_2^{\blacktriangle}$. This is used to highlight that $x_t^{\blacktriangle}$ is the only contractible variable in the PPP agreement and that in the contract the triggers and the transfers are defined in couples. Also, the variables x_i^{BFB} and ω_i^{BFB} are strictly related to x^{BFB} and ω^{BFB}, they are their respective realizations for $\theta = \theta_i, i \in \{1,2\}$.

Form the menu of contract 21.-22. it follows that the principal is always able to obtain the optimal investment timing, i.e. $x = x_i^{BFB}, i \in \{1,2\}$. When the outcome of the project is of low-quality, she does so by setting $F_2^B(x_2^{\blacktriangle}, \omega_2^{\blacktriangle}) = 0$. The agent, having no negotiating power, accepts a contract offering a net present value of zero at the time of the investment. If the outcome is of high quality, the public authority must pay an information premium $\Phi_1 > 0$ to induce the agent's effort and thus $F_1^B(x_1^{\blacktriangle}, \omega_1^{\blacktriangle}) > 0$. Guaranteeing a higher payoff in the case of a high-quality outcome is essential to have the agent exerting effort. If the private company receives an equal payoff for any manifestation of θ, it does not have any incentive to afford the cost of exerting effort and will decide not to. The condition under which it is more convenient for the principal to exert effort is:

$$23.\ \left(\frac{I - \frac{b(\theta_1)}{r}}{\beta - 1}\right)\left(\frac{x}{x_1^{BFB}}\right)^{\beta} - \left(\frac{I - \frac{b(\theta_2)}{r}}{\beta - 1}\right)\left(\frac{x}{x_2^{BFB}}\right)^{\beta} \geq \frac{q_H \xi}{\Delta q \Delta q}$$

Alternatively,

$$24.\ \Delta q A \geq \xi + \frac{q_L \xi}{\Delta q}$$

Where $A = \left(\frac{x}{x_1^{BFB}}\right)^{\beta}\left(\frac{b(\theta_1)}{r} - \omega_1^{BFB}\right) - \left(\frac{x}{x_2^{BFB}}\right)^{\beta}\left(\frac{b(\theta_2)}{r} - \omega_2^{BFB}\right)$ is the difference between the project total welfare in the first best when θ takes respectively the value θ_1 or θ_2. The left-hand side is the gain from effort exertion. It follows from the fact that the overall welfare is higher when θ_1 is observed and the effort enhances the chance of having good quality. The right-hand side is the total social cost of effort, ξ paid by the agent and $\frac{q_L \xi}{\Delta q}$ paid by the principal.

4.5 - Optimal contracts when the agent has negotiating power

The negotiating power of the agent is modeled assuming slack the constraint on his payoff, i.e. $F_i^B > 0, i \in \{1,2\}$. He can successfully claim a strictly positive NPV at the time of the investment. The maximization problem 13.-19. is solved assuming that the constraints 16. and 17. are slack. The solution is[13]:

Benchmark proposition 2 *When the agent has negotiating power, the principal offers the following menu of contracts:*

$$25.\ \{x_1^{\blacktriangle\blacktriangle}, \omega_1^{\blacktriangle\blacktriangle}(x_1^{\blacktriangle\blacktriangle})\} = \{x_1^{BFB}, \omega_1^{BFB} + \Phi_1 + \Phi_2\}$$
$$26.\ \{x_2^{\blacktriangle\blacktriangle}, \omega_2^{\blacktriangle\blacktriangle}(x_2^{\blacktriangle\blacktriangle})\} = \left\{x_2^{BFB} + \frac{\delta'}{q_H}\frac{b(\theta_2)}{r}, 0\right\}$$

where $x_i^{BFB} = \delta'\left(I - \frac{b(\theta_i)}{r}\right), \omega_i^{BFB} = \frac{\beta\frac{b(\theta_i)}{r} - I}{\beta - 1}, i \in \{1,2\}$ *and* $\Phi_2 \equiv \left(\frac{1-q_H}{q_H}\frac{\beta}{\beta-1}\frac{b(\theta_2)}{r} + \frac{I}{\beta-1}\right)\left(\frac{x_1^{BFB}}{x_2^{\blacktriangle\blacktriangle}}\right)^{\beta} > 0.$

Differently from the previous case, the principal faces an insurance-efficiency trade-off. When the quality of the outcome is low, she will postpone the investment by setting a trigger higher than the optimal one. Simultaneously, the fixed transfer $\omega_2^{\blacktriangle\blacktriangle}$ is brought down to the minimum, i.e. 0. The principal, not being able to guarantee the optimal timing, decides to save at least on the fixed transfer. In this case, the positivity of the agent's payoff depends only on the cash flow of the project. The principal allows the agent to postpone the investment, thus obtaining a higher cash flow, but grants no transfer. On the contrary, under θ_1 the optimal investment timing is achieved. However, to do so the principal faces a higher cost due to the negotiating power of the agent. As before, $\Phi_1 > 0$ is the rent that must be paid to solve the agency conflict. On the other hand, $\Phi_2 > 0$ accounts for the cost deriving from the agent's ability to claim a rate of return higher than δ. Ultimately, we have that $\omega_1^{\blacktriangle\blacktriangle} > \omega_1^{\blacktriangle}$, $\omega_2^{\blacktriangle\blacktriangle} < \omega_2^{\blacktriangle}$, $x_1^{\blacktriangle\blacktriangle} = x_1^{BFB}$, $x_2^{\blacktriangle\blacktriangle} > x_2^{BFB}$. The transfer is higher than the no-negotiating-power case if the outcome is qualitatively good, it is lower if the outcome is qualitatively poor. The optimal investment timing is achieved only if $\theta = \theta_1$.

The principal will find it convenient to induce effort if the following condition holds:

$$27.\ \Delta q A \geq \xi + \frac{q_L \xi}{\Delta q} + \Delta R^B$$

The left-hand side is, as before, the gain from effort exertion. To change is the right-hand side as the cost to induce effort is increased. The term $\Delta R^B > 0$ is the difference between the principal's payoff under no agent's negotiating power $R^B(x_1^{\blacktriangle}, x_2^{\blacktriangle}, \omega_1^{\blacktriangle}, \omega_2^{\blacktriangle})$ and the same

[13] The proof of this proposition is in the appendix of the original paper by Buso, Moretto, and Zormpas (2020).

measure when the agent has negotiating power, i.e. $R^B(x_1^{**}, x_2^{**}, \omega_1^{**}, \omega_2^{**})$. This difference gives the magnitude of the agency cost associated with the agent's ability to claim a return higher than δ. It follows that inducing effort exertion in this case is more demanding than under condition 24. and the principal will decide to do so less often.

4.6 - Remarks

Buso et al. (2020) have successfully endogenized the possibility of choosing the investment timing describing it as a real option embedded in the PPP agreement. Moreover, they have found a contract design method to solve the agency conflict between the public authority and the private party involved in the PPP agreement. Yet, this is no guarantee on the achievement of the first best solution. The negotiating power of the agent creates inefficiencies that hinder the use of these contracts, it may lead to underinvestment in PPPs and inhibit the use of incentive contracts by public administrators.

From their results emerges that the bargaining process between the parties involved in PPPs could have primary importance in determining the success of these agreements. However, the modeling of a bargaining game between the two parties goes beyond the scope of the paper. This aspect is the one our contribution wants to investigate. To do so, a model of Nash bargaining and a P-A framework that allows for increasing agent's reservation utility levels are developed. Both are designed as extensions to the model described in this chapter. It will be used as a reference in terms of methods and notation throughout the rest of this work and its results will be the benchmark to which compare ours.

Chapter 5 - NASH BARGINING

Differently from the previous chapter, here we introduce the bargaining between the government and the firm by modelling it as a Nash bargaining[14]. The setting from which we depart is the one by Buso et al. (2020) described in the previous chapter, also the notation follows from there. For a better understanding of the methodology used here, it is worth to briefly introduce the contribution by Banerjee et al. (2014).

The three authors examine the exercise of a jointly held real option to make an investment. The parties involved are assumed to be two for simplicity. They can be two firms that have the option to start a joint venture or an entrepreneurial firm, owning the project, and an investor providing financing. Their identities notwithstanding, for the exercise of the option they must agree on how to share the proceeds from the project and on the investment timing. Their interaction is modeled in two stages. One is the bargaining phase over how to share the revenues from the investment. The other is the choice of the investment timing, this decision is assumed to be in the hands of only one of the parties. In the reference paper both the possible orders of the two stages are considered. The more relevant for the purpose of the present work is the one where, at first, the two parties bargain on how to split the revenues, and successively, the one entitled decides the timing to exercise the option. In this case, at the end of the bargaining phase on how to split the proceeds, the option of one of the firms (call it T) to decide the investment timing is still alive. Once the investment terms have been set, firm T will decide the timing that maximizes its own payoff. However, during the bargaining phase, the parties are aware of how T will act for each possible allocation of the proceeds in the first phase and will anticipate that when bargaining. To model this interaction a two-stage maximization problem is used, as the investment timing to be chosen is anticipated during the bargaining, the problem is solved backward. First, the payoff function of the firm deciding the timing is maximized with respect to when initiating the investment. Second, the timing decision is substituted into the bargaining function that is maximized to obtain how the parties will agree to share the revenues from the investment project.

The situation described in the model by Banerjee et al. (2014) differs from the interaction between a public authority and private firm in a PPP under many aspects. First, two firms taking part in a joint venture act with the only intent of maximizing their own payoff while, when the government is involved, it will bring into play the public benefit besides the private advantages deriving from the project. Second, the relation between two private parties is generally different

[14] The original work is “The bargaining problem”, Nash, 1950.

from one where a public authority interacts with the private sector as the former has regulatory power over the latter. Third, the authors do not consider any issue related to the asymmetry of information between the parties, while it is a vital aspect to be included when studying PPPs as pointed out by many scholars (Martimort and Pouyet, 2008; Iossa and Martimort, 2012 and 2015; Hoppe and Schmitz, 2013). However, the framework of Banerjee et al. (2014) can be adjusted to be successfully applied to model the interaction between the public and private parties involved in a PPP agreement.

Precisely, in the present chapter we describe the interaction between the government and the private entity as a two-stage process. First, there is a bargaining phase over some of the variables at stake. Subsequently, the government formulates a take-it-or-leave-it offer on the remaining variables. The variables included in each of the two stages change with our assumptions on which aspects are the object of the bargaining. In general, the contractual arrangement is based on the investment triggers $x_i, i \in (1,2)$, the fixed transfer granted to the firm $\omega_i, i \in (1,2)$, and a new variable indicating the share of the cash flow from the project that goes in the pockets on the firm. The assumption that the government and the firm will share the cash flow of the project works as a bridge between the interests of the two parties as now both have a gain coming from the cash flow. This makes the option to invest a jointly held real option, compared to the pure delegation due to the total asymmetry of objectives between the parties in Buso et al. (2020). The share received by the principal is indicated as $1-\alpha$, the one of the agent as α, where $0 \leq \alpha \leq 1$.

The models are presented in the following order. First, the first-best solution when the parties bargain only over α and the principal formulates her offer on $x_1, x_2, \omega_1, \omega_2$ is found, assuming the principal can observe the effort exerted by the agent. Second, the model under moral hazard is solved with the two parties bargaining only over α. Third, the parties are allowed to bargain over the optimal triggers x_1, x_2, and the share of cash flows α is assumed to be 1. Fourth, the assumption of $\alpha = 1$ is relaxed and the parties bargain over x_1, x_2, and α.

In each model the two-stage maximization problem is solved backward. The parties are assumed to foresee during the bargaining phase what the offer of the government will be. The first step is thus to maximize the government payoff with respect to the variables not included in the bargaining. This gives the value of the variables of the take-it-or-leave-it offer as a function of the others. Plugging them in the bargaining function and maximizing with respect to the bargaining variables allows finding the optimal contract. The goal of each considered version of the model is to find the optimal contract, i.e. the values of: $x_1, x_2, \omega_1, \omega_2$, and α.

Depending on how the interaction among the parties is designed they will bargain only over α, and then the government will have the power of deciding $x_1, x_2, \omega_1, \omega_2$, or they will jointly decide also the timing of the investment defining together the optimal triggers x_1, x_2, and the principal will be entitled to make a take-it-or-leave-it offer only on the fixed transfers ω_1, ω_2. Every model will consider the moral hazard problem as described in Buso et al. (2020).

5.1 - First best

In this part it is delineated the contract that the principal submits to the agent when it can observe his effort exertion. For simplicity, the level of θ is assumed constant and, with it, the level of $b(\theta)$. The principal has to define the transfer she wants to grant to the firm and the optimal time for the investment. As all the information on how the process 1. evolves is contained in x_t, the moment in which to invest takes the form of a threshold on the variable x. When the stochastic variable x_t hits the threshold x_τ, it is the optimal moment to carry out the investment. As an assumption, the initial value of x is lower than x_τ (i.e. $\tau > 0$), it is not optimal to invest immediately and this guarantees the option to invest has a positive value. The problem solved by the principal is the maximization of her expected payoff (that is the value of her option to invest) under the condition that the agent with no negotiating power will participate.

More formally the maximization problem is:

$$28.\ \max_{x_\tau,\omega} R(x_\tau,\omega) = \max_{x_\tau,\omega} \left(\frac{x}{x_\tau}\right)^\beta Y(\omega)$$

Subject to,

$$29.\ V(x_\tau,\omega) = \left(\frac{x}{x_\tau}\right)^\beta F(x_\tau,\omega) \geq 0$$

Where

$$30.\ Y(\omega) = E_\tau\left[\int_\tau^\infty (b(\theta) + (1-\alpha)x_t)e^{-r(t-\tau)}dt\right] - \omega = \frac{b(\theta)}{r} - \omega + \frac{(1-\alpha)x_t}{\delta}$$

is the principal's payoff at time τ and

$$31.\ F(x_\tau,\omega) = E_\tau\left[\int_\tau^\infty \alpha x_\tau e^{-r(t-\tau)}dt\right] + \omega - I = \frac{\alpha x_\tau}{\delta} + \omega - I$$

is the agent's net present value (NPV) at time τ.

In the maximization problem 28.-29. the term $\left(\frac{x}{x_\tau}\right)^\beta$ is used to discount both $Y(\omega)$ and $F(x_\tau,\omega)$.

The objective function 28. can be also expressed as the difference between the project's total welfare $W(x_\tau)$ and the agent's NPV $V(x_\tau,\omega)$. The maximization problem becomes:

$$32.\ \max_{x_\tau,\omega} R(x_\tau,\omega) = \max_{x_\tau,\omega} W(x_t) - V(x_t,\omega)$$

Where,

$$33.\ W(x_t) = \left(\frac{x}{x_\tau}\right)^\beta \left(\frac{\alpha x_\tau}{\delta} - \left(I - \frac{b(\theta)}{r}\right) + \frac{(1-\alpha)x_\tau}{\delta}\right) = \left(\frac{x}{x_\tau}\right)^\beta \left(\frac{x_\tau}{\delta} - \left(I - \frac{b(\theta)}{r}\right)\right)$$

The government will find optimal to offer an ω that brings the firm's NPV to zero[15]:

$$34.\ F(x^{FB},\omega^{FB}) = 0 \rightarrow\ \omega^{FB} = I - \frac{\alpha x^{FB}}{\delta}$$

The x_τ that maximizes eq. 33. is

$$35.\ x^{FB} = \delta \frac{\beta}{\beta-1}\left(I - \frac{b(\theta)}{r}\right) = \delta' \left(I - \frac{b(\theta)}{r}\right)$$

Where $\delta' = \frac{\beta}{\beta-1}\delta$ and $\frac{\beta}{\beta-1} > 1$ is the standard option multiplier that captures the effect of uncertainty. See appendix, part A.

Now, substituting 35. into 34. it is obtained:

$$36.\ \omega^{FB}(x^{FB}) = \frac{I(\beta-1-\alpha\beta)+\alpha\beta\frac{b(\theta)}{r}}{\beta-1}$$

Summarizing, the optimal contract offered by the principal is[16]

$$37.\ \{x^{FB},\omega^{FB}\} = \left\{\delta'\left(I - \frac{b(\theta)}{r}\right), \frac{I(\beta-1-\alpha\beta)+\alpha\beta\frac{b(\theta)}{r}}{\beta-1}\right\}$$

From 37. results:

$$38.\ W(x^{FB}) = \left(\frac{x}{x^{FB}}\right)^\beta \frac{1}{\beta-1}\left(I - \frac{b(\theta)}{r}\right)$$

The contract obtained in 37. is closely related but not equal to the one found in the benchmark model by Buso et al. (2020) and reported in 11. The optimal investment timing is the same in both models, $x^{BFB} = x^{FB}$. When the agent is not receiving the entire cash flow, the principal

[15] FB stands for first best.

[16] For the contract $\{x^{FB},\omega^{FB}\}$ to make sense it is required $\frac{-\alpha\beta\frac{b(\theta)}{r}}{\beta-1-\alpha\beta} > I > \frac{b(\theta)}{r}$ and $\alpha > \frac{\beta-1}{\beta}$. Also, for $\tau > 0$ it is needed that $x < \delta^{'}\left(I - \frac{b(\theta)}{r}\right)$.

compensates him by increasing the fixed transfer but holding the investment timing optimal. Precisely, ω^{FB} oscillates between I, when $\alpha = 0$, and ω^{BFB}, when $\alpha = 1$. As expected, the first derivative of ω^{FB} with respect to α is negative[17], the higher the share of the cash flow in the hands of the firm the lower the fixed transfer from the principal to the agent. The reason why the principal can always increase the fixed transfer and stick to the optimal investment timing is that, even at its maximum, ω^{FB} is affordable. Indeed, for $\alpha = 0$ it coincides with I, that is the maximum transfer the government can grant to the firm. If the transfer outlay exceeds I, the government does not find convenient to hire a private firm to undertake the investment as it can afford the sunk investment I on its own and save money. If it were the case, that for some value of α, the fixed transfer to the agent exceeded the threshold of I, the principal could have decided either not to take part in the partnership or to postpone the investment offering to the agent a transfer lower than I. Postponing the investment means, under our framework, to increase the investment trigger. If it is done, the expected cash flow when the investment is initiated, i.e. x_τ[18], rises. Thus, the cash flow to be split among the principal and the agent is greater and the agent will be willing to accept a lower fixed transfer since the value of his share in the cash flow has risen. In other words, the principal can enhance the agent's payoff in two ways, either by postponing the investment or by increasing the transfer. Depending on the situation, the principal will use one of this two instruments to maximize her payoff still inducing the participation of the agent.

Now consider the bargaining maximization problem over α:

$$39. \max_{\alpha} \frac{\left[\left(\frac{x}{x^{FB}}\right)^{\beta} Y(\omega^{FB})\right]^{\eta}}{\left[\left(\frac{x}{x^{FB}}\right)^{\beta} F(x^{FB},\omega^{FB})\right]^{\eta-1}}$$

Where the parameter $\eta \geq 0$ indicates the bargaining power of the principal and $1-\eta$ the one of the agent. Notice that from 34. we have that $F(x^{FB}, \omega^{FB}) = 0$. It follows that the bargaining function 39. is null and α cannot be determined.

5.2 - Moral hazard and bargaining over α

In this section, it is considered the case where the two parties bargain on how to share the cash flow generated by the project, i.e. they bargain over α. The problem is solved backward as in Banerjee et al. (2014). First, α is assumed to be exogenous and the objective function of the

[17] The first derivative of ω^{FB} with respect to α is $\dot{\omega}^{FB} = -\frac{\beta}{\beta-1}\left(I - \frac{b(\theta)}{r}\right) < 0$

[18] The geometric Brownian motion followed by x is a Markov process, therefore the best forecast for the future is the current value of x.

government is maximized with respect to x_1, x_2, ω_1, and ω_2. Second, the optimal values for x_1, x_2, ω_1, and ω_2 are plugged in the bargaining function that is then maximized with respect to α.

The first maximization problem is:

$$40. \max_{\{\omega_i, x_i\}, i\in\{1,2\}} q_H \left(\frac{x}{x_1}\right)^{\beta} Y_1 + (1-q_H)\left(\frac{x}{x_2}\right)^{\beta} Y_2$$

Where $Y_i = \frac{b(\theta_i)}{r} - \omega_i + \frac{(1-\alpha)x_i}{\delta}, i \in \{1,2\}$.

Subject to:

$$41. q_H \left(\frac{x}{x_1}\right)^{\beta} F_1 + (1-q_H)\left(\frac{x}{x_2}\right)^{\beta} F_2 - \xi \geq q_L \left(\frac{x}{x_1}\right)^{\beta} F_1 + (1-q_L)\left(\frac{x}{x_2}\right)^{\beta} F_2$$

$$42. q_H \left(\frac{x}{x_1}\right)^{\beta} F_1 + (1-q_H)\left(\frac{x}{x_2}\right)^{\beta} F_2 - \xi \geq 0$$

$$43.\ F_1 \geq 0$$

$$44.\ F_2 \geq 0$$

$$45.\ \omega_1 \geq 0$$

$$46.\ \omega_2 \geq 0$$

Where $F_i = \frac{\alpha x_i}{\delta} + \omega_i - I, i \in \{1,2\}$ and 41. can be rewritten as:

$$47. \left(\frac{x}{x_1}\right)^{\beta} F_1 - \left(\frac{x}{x_2}\right)^{\beta} F_2 \geq \frac{\xi}{\Delta q}$$

When the agent has no negotiating power, the constraints 42. and 43. can be proven to be always slack (see Appendix). Reducing the problem and solving it the result is:

Proposition 1 *After the bargaining phase over α, the principal offers the following menu of contracts.*

$$48.\ \{x_1^*, \omega_1^*\} = \left\{x_1^{FB}, \omega_1^{FB} + \frac{\varepsilon}{\Delta q}\left(\frac{x}{x_1^{FB}}\right)^{-\beta}\right\}$$

$$49.\ \{x_2^*, \omega_2^*\} = \left\{x_2^{FB}\frac{1-q_H}{\alpha-q_H}, \frac{\alpha}{\alpha-q_H}\frac{\beta\frac{b(\theta_2)}{r}-I}{\beta-1} - \frac{q_H}{\alpha-q_H}\omega_2^{FB}\right\}$$

Where $x_i^{FB} = \delta\frac{\beta}{\beta-1}\left(I - \frac{b(\theta_i)}{r}\right), \omega_i^{FB} = \frac{I(\beta-1-\alpha\beta)+\alpha\beta\frac{b(\theta_i)}{r}}{\beta-1}, i \in \{1,2\}$.

For the proof see the Appendix, part B.

Given that x_1^{FB} and ω_1^{FB} are positive, it follows that also x_1^* and ω_1^* are positive. The condition for the positivity of x_2^* is $\alpha > q_H$ while the one for $\omega_2^* \geq 0$ is $\alpha \geq q_H * \frac{I(\beta-1)}{\beta\frac{b(\theta_2)}{r}-I+q_H\beta\left(I-\frac{b(\theta_2)}{r}\right)}$. Finally, it can be easily verified that the always slack inequations 42. and 43. are satisfied by the solution $\{x_i^*, \omega_i^*\}, i \in (1,2)$.

Comparative static on α

Before solving the second part of the model related to the bargaining over α, this section presents some comparative static on α. As expected, the results for the optimal menu of contracts $\{\omega_i^*, x_i^*\}$ depend on the parameter α. The sensitivity analysis can help to understand the dynamics of this dependence.

First, recall that x_i^* is assumed to be bigger than the initial value of x, and positive. The former guarantees the option value to be greater than zero, the latter assumes the government will never undertake a project that has a negative expected cash flow[19]. For this assumption to be valid, the value of α must be greater than q_H, if not x_2^* goes below zero. Besides, also the value of ω_i^* has to be non-negative: it is possible to have no transfer from the government to the firm undertaking the project, but it is meaningless to consider a situation where the firm is required to pay a tax to the government (i.e. $\omega_i^* < 0$). For the non-negativity of ω_2^* it is required that $\alpha \geq q_H * \frac{I(\beta-1)}{\beta\frac{b(\theta_2)}{r}-I+q_H\beta\left(I-\frac{b(\theta_2)}{r}\right)}$. This condition is more stringent than the previous one[20] and defines a lower bound for the parameter α. Finally, to have a non-negative ω_i^{FB}, α needs to be greater or equal than $\frac{\beta-1}{\beta}$. Which of the two constraints on α will be binding depends on the value of the parameters, both need to be verified in the analytical part of the bargaining over α.

Second, while x_1^* and ω_1^* coincide with the results from the initial model (Buso et al. 2020), x_2^* and ω_2^* do not. When the principal is maximizing not only the social benefit but also her share of the cash flow of the project and she observes a poor outcome quality she opts for deviating from the first best (i.e. x_2^{FB}, ω_2^{FB}). Specifically, she postpones the investment setting a higher x_2^*[21] , and simultaneously it grants a lower fixed transfer to the firm[22]. Initiating the project when the forecasted cash flow is higher makes the firm willing to accept a lower transfer ω_2^*.

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[19] The geometric Brownian motion followed by x is a Markov process, therefore the best forecast for the future is the current value of x.

[20] Note that $q_H < q_H * \frac{I(\beta-1)}{\beta\frac{b(\theta_2)}{r}-I+q_H\beta\left(I-\frac{b(\theta_2)}{r}\right)}$.

[21] Note that $\frac{1-q_H}{\alpha-q_H} > 1$.

[22] $\omega_2^* < \omega_2^{FB}$

The solution obtained when the outcome is of low quality (x_2^*, ω_2^*) can be placed in between the cases with and without negotiating power of our benchmark model. The former brings the solution to the extreme where the time of the investment is delayed in such a way that the fixed transfer is brought to zero $(\omega_2^{\blacktriangle\blacktriangle} = 0)$, the latter keeps it at the first best. If the quality of the facility is low, both when the principal receives $(1-\alpha)x$ and when the agent has negotiating power, the government delays the investment and lowers the fixed transfer.

Bargaining over α

This is the second step of the model. As introduced above, after finding the optimal contract offered by the principal to the agent, the optimal values for $\{x_i, \omega_i\}, i \in \{1,2\}$ are plugged in the bargaining function that is then maximized with respect to α. The maximization problem is the following:

$$50.\ \max_{\alpha} \frac{\left[q_H\left(\frac{x}{x_1^{FB}}\right)^{\beta}\left(\frac{b(\theta_1)}{r}-\omega_1^*+\frac{(1-\alpha)x_1^{FB}}{\delta}\right)+(1-q_H)\left(\frac{x}{x_2^*}\right)^{\beta}\left(\frac{b(\theta_2)}{r}-\omega_2^*+\frac{(1-\alpha)x_2^*}{\delta}\right)\right]^{\eta}}{\left[q_H\left(\frac{x}{x_1^{FB}}\right)^{\beta}\left(\frac{\alpha x_1^{FB}}{\delta}+\omega_1^*-I\right)+(1-q_H)\left(\frac{x}{x_2^*}\right)^{\beta}\left(\frac{\alpha x_2^*}{\delta}+\omega_2^*-I\right)-\xi\right]^{\eta-1}}$$

Where η indicates the bargaining power of the principal and $1-\eta$ the one of the agent. With $\eta = 0$, the Nash bargaining product equals the agent's payoff, while if $\eta = 1$ the principal's payoff.

Maximizing the logarithm of the function in 50., it results that the optimal α is equal to 1.

Proposition 2 *When the parties share the cash flow of the project and bargain on how to split it, they opt to give it entirely to the private party. The optimal contract is:*

$$51.\ \{x_1^*, \omega_1^*\} = \{x_1^{BFB}, \omega_1^{BFB} + \Phi_1\}$$

$$52.\ \{x_2^*, \omega_2^*\} = \{x_2^{BFB}, \omega_2^{BFB}\}$$

Recall, $x_i^{BFB} = \delta'\left(I - \frac{b(\theta_i)}{r}\right), \omega_i^{BFB} = \frac{\beta\frac{b(\theta_i)}{r}-I}{\beta-1}, i \in \{1,2\}$ *and* $\Phi_1 \equiv \frac{\xi}{\Delta q}\left(\frac{x}{x_1^{BFB}}\right)^{-\beta} > 0$

The proof of *Proposition 2* is given by substituting the value $\alpha = 1$ obtained form 50. in the contract of *Proposition 1*. For its simplicity, it is not included in the appendix.

This result is the same as the *Benchmark proposition 1*. Allowing for different allocations of the cash flows among the parties will not affect the validity of the benchmark model. In the bargaining phase the parties' optimal choice is $\alpha = 1$, i.e. the entire cash flow goes to the private party. This result mirrors the assumption of the benchmark model. Allowing for no bargaining on how to split the cash flow Buso et al. (2020) are imposing that it will go entirely to the private party.

This result ($\alpha = 1$) shows that the government during the bargaining can achieve the optimal investment timing offering an adequate fixed transfer to the firm. Besides, notice that the agent is indifferent between any of the feasible values of α. He is aware that his payoff will be 0 when the outcome is of low quality, and that the payoff for a high-quality project does not depend on the allocation of the cash flows. Thus, the bargaining is solved with the best option for the principal who prioritizes the optimal investment timing.

5.3 - Moral hazard and bargaining over x_1 and x_2 when $\alpha = 1$

In this section the case where the two parties bargain over the optimal triggers x_1, and x_2 is considered. The problem is solved backward as in Banerjee et al. (2014). First, x_1 and x_2 are assumed to be exogenous and the objective function of the government is maximized with respect to ω_1, and ω_2. Second, the optimal values for ω_1 and ω_2 are plugged in the bargaining function that is then maximized with respect to x_1 and x_2. For simplicity, α is assumed equal to 1 in this initial model.

The first maximization problem is:

$$53.\ \max_{\omega_1,\omega_2} q_H \left(\frac{x}{x_1}\right)^\beta \left(\frac{b(\theta_1)}{r} - \omega_1\right) + (1 - q_H) \left(\frac{x}{x_2}\right)^\beta \left(\frac{b(\theta_2)}{r} - \omega_2\right)$$

Subject to:

$$54.\ \left(\frac{x}{x_1}\right)^\beta F_1^B - \left(\frac{x}{x_2}\right)^\beta F_2^B \geq \frac{\xi}{\Delta q}$$

$$55.\ q_H \left(\frac{x}{x_1}\right)^\beta F_1^B + (1 - q_H) \left(\frac{x}{x_2}\right)^\beta F_2^B - \xi \geq 0$$

$$56.\ F_1^B \geq 0$$

$$57.\ F_2^B \geq 0$$

$$58.\ \omega_1 \geq 0$$

$$59.\ \omega_2 \geq 0$$

Where $F_i^B = \frac{x_i}{\delta} + \omega_i - I, Y_i^B = \frac{b(\theta_i)}{r} - \omega_i, i \in \{1,2\}$.

The solution to this maximization problem is:

$$60.\ \omega_1^{\square} = \frac{\xi}{\Delta q}\left(\frac{x}{x_1}\right)^{-\beta} - \frac{x_1}{\delta} + I$$

$$61.\ \omega_2^{\square} = I - \frac{x_2}{\delta}$$

Bargaining over x_1 *and* x_2

This is the second stage of the model. As introduced above, in the first stage the optimal values for ω_1 and ω_2 that maximize the principal's payoff are found. Then, they are plugged in the bargaining function that is maximized with respect to α. The maximization problem is the following:

$$62.\ \max_{x_1,x_2} \frac{\left[q_H\left(\frac{x}{x_1}\right)^{\beta}\left(\frac{b(\theta_1)}{r}-\omega_1^{\square}\right)+(1-q_H)\left(\frac{x}{x_2}\right)^{\beta}\left(\frac{b(\theta_2)}{r}-\omega_2^{\square}\right)\right]^{\eta}}{\left[q_H\left(\frac{x}{x_1}\right)^{\beta}\left(\frac{x_1}{\delta}+\omega_1^{\square}-I\right)+(1-q_H)\left(\frac{x}{x_2}\right)^{\beta}\left(\frac{x_2}{\delta}+\omega_2^{\square}-I\right)-\xi\right]^{\eta-1}}$$

Maximizing the logarithm of the function we obtain:

Proposition 3 *When the parties bargain on the investment triggers the optimal contract is:*

$$63.\ \{x_1^{\square}, \omega_1^{\square}\} = \{x_1^{BFB}, \omega_1^{BFB} + \Phi_1\}$$

$$64.\ \{x_2^{\square}, \omega_2^{\square}\} = \{x_2^{BFB}, \omega_2^{BFB}\}$$

Recall, $x_i^{BFB} = \delta'\left(I - \frac{b(\theta_i)}{r}\right), \omega_i^{BFB} = \frac{\beta\frac{b(\theta_i)}{r}-I}{\beta-1}, i \in \{1,2\}$ *and* $\Phi_1 \equiv \frac{\xi}{\Delta q}\left(\frac{x}{x_1^{BFB}}\right)^{-\beta} > 0$

For the proof see the Appendix, part B.

As in *Proposition 2*, the optimal contract coincides with the one of the *Benchmark Proposition 1*. Not including α in the bargaining and assuming it equal to 1 the optimal contract perfectly coincides with the benchmark. The parties during the bargaining phase opt for implementing the project with the optimal timing, irrespective of the realization of θ. However, to induce effort, the principal differentiates the fixed transfer depending on the quality of the project outcome. This justifies the presence of a positive information premium Φ_1. Again, the results of Buso et al. (2020) are valid even considering the parties' bargaining.

5.4 - Moral hazard and bargaining over x_1, x_2, and α

Now consider the more general case where the government and the firm share part of the cash flow of the project. The parameter α indicates the share of the firm, while $1-\alpha$ the one of the government. The problem is solved backward as before. First, x_1, x_2, and α are assumed to be exogenous and the objective function of the government is maximized with respect to ω_1, and

ω_2. Second, the optimal values for ω_1 and ω_2 are plugged in the Nash bargaining function that is then maximized with respect to x_1, x_2, and α.

The first maximization problem is:

$$65.\ \max_{\omega_1,\omega_2} q_H \left(\frac{x}{x_1}\right)^{\beta} Y_1 + (1-q_H)\left(\frac{x}{x_2}\right)^{\beta} Y_2$$

Subject to:

$$66.\ \left(\frac{x}{x_1}\right)^{\beta} F_1 - \left(\frac{x}{x_2}\right)^{\beta} F_2 \geq \frac{\xi}{\Delta q}$$

$$67.\ q_H \left(\frac{x}{x_1}\right)^{\beta} F_1 + (1-q_H)\left(\frac{x}{x_2}\right)^{\beta} F_2 - \xi \geq 0$$

$$68.\ F_1 \geq 0$$

$$69.\ F_2 \geq 0$$

$$70.\ \omega_1 \geq 0$$

$$71.\ \omega_2 \geq 0$$

Where $F_i = \frac{\alpha x_i}{\delta} + \omega_1 - I, Y_i = \frac{b(\theta_i)}{r} - \omega_i + \frac{(1-\alpha)x_i}{\delta}, i \in \{1,2\}$.

The solution to this maximization problem is:

$$72.\ \omega_1^{\star} = \frac{\xi}{\Delta q}\left(\frac{x}{x_1}\right)^{-\beta} - \frac{\alpha x_1}{\delta} + I$$

$$73.\ \omega_2^{\star} = I - \frac{\alpha x_2}{\delta}$$

Bargaining over x_1, x_2, and α

This is the second stage of the model. As introduced above, in the first stage the optimal values for ω_1 and ω_2 that maximize the principal's payoff are found. Then, they are plugged in the Nash bargaining function that is maximized with respect to x_1, x_2, and α. The maximization problem is the following:

$$74.\ \max_{x_1,x_2,\alpha} \frac{\left[q_H\left(\frac{x}{x_1}\right)^{\beta}\left(\frac{b(\theta_1)}{r} - \omega_1^{\star} + \frac{(1-\alpha)x_1}{\delta}\right) + (1-q_H)\left(\frac{x}{x_2}\right)^{\beta}\left(\frac{b(\theta_2)}{r} - \omega_2^{\star} + \frac{(1-\alpha)x_2}{\delta}\right)\right]^{\eta}}{\left[q_H\left(\frac{x}{x_1}\right)^{\beta}\left(\frac{\alpha x_1}{\delta} + \omega_1^{\star} - I\right) + (1-q_H)\left(\frac{x}{x_2}\right)^{\beta}\left(\frac{\alpha x_2}{\delta} + \omega_2^{\star} - I\right) - \xi\right]^{\eta-1}}$$

Substituting 72. and 73. in 74. α simplifies. The bargaining function does not depend on α and does not allow to find an optimal value for it. Thus, the optimal contract in *Proposition 4* will be only on the investment triggers and the fixed transfers.

Maximizing the logarithm of the function we obtain:

Proposition 4 *When the parties bargain on how to share the cash flow of the project, and on the investment triggers the optimal contract is:*

$$75.\ \{x_1^\star, \omega_1^\star\} = \{x_1^{FB}, \omega_1^{FB} + \Phi_1\}$$
$$76.\ \{x_2^\star, \omega_2^\star\} = \{x_2^{FB}, \omega_2^{FB}\}$$

Where $x_i^{FB} = \delta \frac{\beta}{\beta-1}\left(I - \frac{b(\theta_i)}{r}\right), \omega_i^{FB} = \frac{I(\beta-1-\alpha\beta)+\alpha\beta\frac{b(\theta_i)}{r}}{\beta-1}, i \in \{1,2\}$ *and* $\Phi_1 \equiv \frac{\xi}{\Delta q}\left(\frac{x}{x_1^{FB}}\right)^{-\beta} > 0$

For the proof see the Appendix, part B.

The contract that emerges from the *Proposition 4* mirrors the one of the *Benchmark Proposition 1*. However, it is not identical. While we have that $x_1^{FB} = x_1^{BFB}$ and x_2^{FB}=x_1^{BFB}, the optimal transfers ω_1^{FB} and $\omega_2^{FB}$are respectively equal to ω_1^{BFB} and ω_2^{BFB} only if $\alpha = 1$. Differently, for $\alpha < 1$, we observe $\omega_1^{FB} > \omega_1^{BFB}$ and $\omega_2^{FB} > \omega_2^{BFB}$. As the agent is not receiving the entire cash flow, the principal must compensate him in some other way to induce his participation. The optimal compensation turns out to be in terms of a greater fixed transfer. It is worth to notice that the information premium Φ_1 does not depend on α, indeed it is exactly equal to the one of the *Benchmark Proposition 1*. Sharing the project cash flow among the parties does not help in smoothing the moral hazard problem, and the effort-inducing information rent is not affected. As anticipated, a limit of the model presented in this section is that it does not allow to find the optimal α that would emerge from the bargaining process.

5.5 - Bargaining over x_1, x_2, and α imposing $0 \leq \omega_i \leq I$

In this section we complicate the previous model. The bargaining function is maximized subject to new constraints over α. First, we impose a limited liability constraint on the government side. Indeed, the fixed transfer to the firm cannot exceed the investment expenditure I otherwise the government will find more convenient to afford the investment cost than to pay the private company. Second, as already mentioned, the fixed transfer cannot be negative since it is not realistic to imagine that the government will tax the firm that already bears the burden of the investment cost. Introducing these constraints over α we are also trying to obtain the optimal value of the parameter that was not found in the previous version of the model.

The new constraints are:

$$77.\ \omega_1^\star \geq 0 \rightarrow \frac{\xi}{\Delta q}\left(\frac{x}{x_1}\right)^{-\beta} - \frac{\alpha x_1}{\delta} + I \geq 0$$

$$78.\ \omega_1^\star \leq I \rightarrow \frac{\xi}{\Delta q}\left(\frac{x}{x_1}\right)^{-\beta} - \frac{\alpha x_1}{\delta} + I \leq I$$

$$79.\ \omega_2^\star \geq 0 \rightarrow\ I - \frac{\alpha x_2}{\delta} \geq 0$$

$$80.\ \omega_2^\star \leq I \rightarrow I - \frac{\alpha x_2}{\delta} \leq I$$

Inequality 80. is always satisfied as $\frac{\alpha x_2}{\delta}$ is a non-negative quantity. The other three constraints are added to the maximization of the Nash bargaining function that becomes:

$$81.\ \max_{x_1,x_2,\alpha} \frac{\left[q_H\left(\frac{x}{x_1}\right)^\beta\left(\frac{b(\theta_1)}{r}+\frac{x_1}{\delta}-I\right)-q_H\left(\frac{\xi}{\Delta q}\right)+(1-q_H)\left(\frac{x}{x_2}\right)^\beta\left(\frac{b(\theta_2)}{r}+\frac{x_2}{\delta}-I\right)\right]^\eta}{\left[q_L\frac{\xi}{\Delta q}\right]^{\eta-1}}$$

Subject to:

$$82.\ \frac{\xi}{\Delta q}\left(\frac{x}{x_1}\right)^{-\beta} - \frac{\alpha x_1}{\delta} + I \geq 0$$

$$83.\ \frac{\xi}{\Delta q}\left(\frac{x}{x_1}\right)^{-\beta} - \frac{\alpha x_1}{\delta} + I \leq I$$

$$84.\ I - \frac{\alpha x_2}{\delta} \geq 0$$

The objective function 81. is the same as the one obtained substituting 72. and 73. into 74. Indeed, it does not depend on α. Differently, the constrains include α and consequently, the Lagrangian associated with the maximization problem does it too.

The solutions to the maximization problem 81.-84. are:

$$85.\ x_1^{FB} - x_1 = \frac{1-q_H}{q_H}(\beta-1)\left(\frac{x_1}{x_2}\right)^\beta (x_2^{FB} - x_2)$$

Alternatively,

$$86.\ x_2^{FB} - x_2 = \frac{q_H}{1-q_H}\frac{1}{\beta-1}\left(\frac{x_2}{x_1}\right)^\beta (x_1^{FB} - x_1)$$

Or,

$$87.\ \frac{q_H\left(\frac{x}{x_1}\right)^\beta (x_1^{FB}-x_1)}{(1-q_H)\left(\frac{x}{x_2}\right)^\beta (x_2^{FB}-x_2)} = \beta - 1$$

And,

$$88.\ \alpha = \frac{\delta I}{x_2^{FB} - \frac{q_H}{1-q_H}\frac{1}{\beta-1}\left(\frac{x_2}{x_1}\right)^\beta (x_1^{FB}-x_1)}$$

Complicating the maximization problem, the new constraints do not allow us to express explicitly the optimal values for x_1, x_2, and α. The only possible solution to the problem is numerical. However, α can be expressed implicitly as follows.

Proposition 5 *When the parties bargain on how to share the cash flow of the project, and on the investment triggers the optimal value of α is:*

$$89.\ \alpha^{\star} = \frac{\delta I}{x_2^{FB} - \frac{q_H}{1-q_H}\frac{1}{\beta-1}\left(\frac{x_2}{x_1}\right)^{\beta}\left(x_1^{FB} - x_1\right)}$$

Where $x_i^{FB} = \delta \frac{\beta}{\beta-1}\left(I - \frac{b(\theta_i)}{r}\right), i \in \{1,2\}$

The objective function shows that α does not enter in the payoff function of the agent or in the one of the principal. None of the parties is interested in bargaining on how to split the cash flows. From how we have designed the model, the government and the firm do not have a contrasting interest on α that could motivate bargaining over it. Thus, the only solution we find to the problem is numerical, not analytical. Observe that α enters linearly in our model and this is probably the reason why it simplifies in the payoff functions of the parties. A possible solution is to reformulate the model introducing a nonlinear dependence on α and see if it allows obtaining an explicit optimal value for the parameter. However, this goes beyond the scope of the present work.

Chapter 6 - RESERVATION UTILITY

In the present chapter we present an alternative way of modeling the bargaining power of the parties. We introduce in the P-A benchmark model the agent reservation utility u, i.e. the minimum utility level required by the private party to take part in the partnership. The higher the agent's bargaining power the higher his reservation utility.

Recall that in the benchmark model the authors studied the case of an agent with negotiating power. They modelled it allowing for a private party that can successfully claim a rate of return higher than the hurdle rate of the project. Thus, they adopted a binary framework where either the agent has some negotiating power, or he does not. However, the concept of bargaining power is not necessarily discreet and can be modelled as continuous. Introducing the reservation utility allows us to do so. An interesting insight is provided by studying how the principal's payoff function varies when the agent's negotiating power increases. As expected, the principal's payoff is a decreasing function of u. Moreover, having modelled the bargaining power in two different ways (Nash bargaining and reservation utility) it is possible to compare the optimal contracts obtained under the two circumstances[23].

6.1 - The model

The maximization problem that will be used as a reference for the rest of the chapter is very similar to the one of the benchmark model, 13.-19. However, here the agent's payoff is required to be grater or equal than the reservation utility u. This imposition makes the participation constraint and the restrictions on the non-negativity of ω_1 and ω_2 redundant. Indeed, once the agent's payoff is equal or above the reservation utility, the participation of the agent is guaranteed, and the fixed transfers must be acceptable, i.e. greater than or, at least, equal to zero. The principal's payoff is then maximized subject to the incentive compatibility constraint (needed to induce effort) and the two constraints on the agent's payoff.

The principal's maximization problem is:

$$90.\ \max_{\{\omega_i, x_i\}, i\in\{1,2\}} q_H \left(\frac{x}{x_1}\right)^{\beta} Y_1^B + (1-q_H)\left(\frac{x}{x_2}\right)^{\beta} Y_2^B$$

Subject to:

$$91.\ \left(\frac{x}{x_1}\right)^{\beta} F_1^B - \left(\frac{x}{x_2}\right)^{\beta} F_2^B \geq \frac{\xi}{\Delta q}$$

[23] The approach is similar to Demougin and Helm (2006).

$$92.\ F_1^B \geq u$$

$$93.\ F_2^B \geq u$$

Where $Y_i^B = \frac{b(\theta_i)}{r} - \omega_i, F_i^B = \frac{x_i}{\delta} + \omega_i - I, i \in \{1,2\}$

As stated above, to have the agent taking part in the agreement, the principal is forced to provide him with a net present value at least equal to u for any quality level of the final outcome: either high or low. It follows that there is no economic reason for the government to offer a contract that grants the agent a NPV greater than u with a low-quality outcome. The constraint 93. is then assumed to be binding for any level of u.

The principal, when possible, will find optimal to induce high effort enhancing the chances of having a high-quality outcome[24]. To do so, at the minimum cost, the constraint 91. must be binding. Also, when the outcome is of high quality, the principal will have to ensure a NPV greater than u to make the agent willing to exert effort.

From the binding constraints 91. and 93. it follows:

$$94.\ F_1 = \frac{\xi}{\Delta q}\left(\frac{x}{x_1}\right)^{-\beta} + \left(\frac{x_1}{x_2}\right)^{\beta} F_2 = \frac{\xi}{\Delta q}\left(\frac{x}{x_1}\right)^{-\beta} + \left(\frac{x_1}{x_2}\right)^{\beta} u$$

Equation 94. makes clear that, depending on the level of u, the constraint 92. will be binding or not. There are two possible alternatives:

a. $u < \frac{\frac{\xi}{\Delta q}}{\left(\frac{x}{x_1}\right)^{\beta} - \left(\frac{x}{x_2}\right)^{\beta}} \to F_1 > u$

b. $u \geq \frac{\frac{\xi}{\Delta q}}{\left(\frac{x}{x_1}\right)^{\beta} - \left(\frac{x}{x_2}\right)^{\beta}} \to F_1 = u$

Under alternative a., the constraint 92. is always slack. The principal offers to the agent a NPV at the time of the investment higher than u when the output of the project is qualitatively good. She can differentiate what the agent gains based on the outcome quality and, in so doing, she induces high effort. On the other hand, under alternative b., we have $F_1 = F_2 = u$. When the agent's payoff is the same irrespective of the project quality there is no reason for the agent to exert high effort, and the maximization problem must be modified as below.

[24] The condition under which the principal prefers to induce effort is found and discussed later.

We will solve the two cases separately. First, the case a. where the principal effectively induces effort is considered. The maximization problem is the one from 90. to 93. where 91. and 93. are binding while 92. is slack. Solving we obtain:

Proposition 6 *When the principal induces effort the optimal contract is:*

$$95.\ \{x_1^{\bullet}, \omega_1^{\bullet}\} = \left\{x_1^{BFB}, \omega_1^{BFB} + u\left(\frac{x_1}{x_2}\right)^{\beta} + \Phi_1\right\}$$

$$96.\ \{x_2^{\bullet}, \omega_2^{\bullet}\} = \left\{x_2^{BFB} + u\frac{\beta}{\beta-1}\delta\frac{1}{1-q_H}, \omega_2^{BFB} - u\frac{q_H(\beta-1)+1}{(\beta-1)(1-q_H)}\right\}$$

Where $x_i^{BFB} = \delta'\left(I - \frac{b(\theta_i)}{r}\right), \omega_i^{BFB} = \frac{\beta\frac{b(\theta_i)}{r} - I}{\beta-1}, i \in \{1,2\}$ *and* $\Phi_1 \equiv \frac{\xi}{\Delta q}\left(\frac{x}{x_1^{BFB}}\right)^{-\beta} > 0.$

From *Proposition 6*, we observe that when the project outcome is of high quality the principal chooses to implement the optimal investment timing, $x_1^{\bullet} = x_1^{BFB}$. This comes at a cost as the transfer $\omega_1^{\bullet}$ is greater than ω_1^{BFB}. Precisely, in addition to the information premium Φ_1 of the benchmark model the principal must pay and additional information rent that depends on the agent's reservation utility, i.e. $u\left(\frac{x_1}{x_2}\right)^{\beta}$. In other words, the principal is determined to obtain the optimal timing for the investment. As u grows, she will compensate the agent with a greater fixed transfer but leaving the investment trigger x_1^{BFB} unchanged and optimal. On the contrary, for a low-quality project outcome, the government will work on two sides to meet the constraints on the agent's payoff and to induce effort. On the one hand, as u increases, the investment will be postponed allowing the agent to obtain a higher expected cash flow at the time of the investment. On the other hand, the fixed transfer $\omega_2^{\bullet}$, depends negatively on u and it will decrease as u rises.

The critical value for u can now be expressed as a function of $x_1^{\bullet}$, and $x_2^{\bullet}$, i.e. $\frac{\frac{\xi}{\Delta q}}{\left(\frac{x}{x_1^{\bullet}}\right)^{\beta} - \left(\frac{x}{x_2^{\bullet}}\right)^{\beta}}$. Thus, it can be compared to the value of the exogenous variable u. If u is smaller than the critical value, the principal can induce effort and the maximization problem is 90.-93. On the contrary, when $u \geq \frac{\frac{\xi}{\Delta q}}{\left(\frac{x}{x_1^{\bullet}}\right)^{\beta} - \left(\frac{x}{x_2^{\bullet}}\right)^{\beta}}$ the maximization problem changes. The principal cannot induce effort anymore and the probability of a high-quality project will be lower (q_L). The maximization problem is reduced to:

$$97.\ \max_{\{\omega_i, x_i\}, i\in\{1,2\}} q_L\left(\frac{x}{x_1}\right)^{\beta} Y_1^B + (1-q_L)\left(\frac{x}{x_2}\right)^{\beta} Y_2^B$$

Subject to:

$$98.\ F_1^B = u$$

$$99.\ F_2^B = u$$

Where $Y_i^B = \frac{b(\theta_i)}{r} - \omega_i, F_i^B = \frac{x_i}{\delta} + \omega_i - I, i \in \{1,2\}$.

Solving the problem, we have:

Proposition 7 *When the principal cannot induce effort the optimal contract is:*

$$100.\qquad \{x_1^{\bullet\bullet}, \omega_1^{\bullet\bullet}\} = \left\{x_1^{BFB} + u\frac{\beta}{\beta-1}\delta, \omega_1^{BFB} - \frac{u}{\beta-1}\right\}$$

$$101.\qquad \{x_2^{\bullet\bullet}, \omega_2^{\bullet\bullet}\} = \left\{x_2^{BFB} + u\frac{\beta}{\beta-1}\delta, \omega_2^{BFB} - \frac{u}{\beta-1}\right\}$$

Where $x_i^{BFB} = \delta'\left(I - \frac{b(\theta_i)}{r}\right), \omega_i^{BFB} = \frac{\beta\frac{b(\theta_i)}{r} - I}{\beta-1}, i \in \{1,2\}$.

Proposition 7 tells that the optimal investment timing is never achieved. Both when the outcome quality is high and when it is low, the investment trigger will deviate from the optimum $x_i^{BFB}, i \in \{1,2\}$ by the positive quantity $u\frac{\beta}{\beta-1}\delta$. In addition, the greater the reservation utility of the agent the more severe the deviation from the optimum is. On the contrary, the fixed transfer granted to the agent shrinks as u rises. As u exceeds $\beta\frac{b(\theta_i)}{r} - I, i \in \{1,2\}$ the transfer $\omega_i^{\bullet\bullet}, i \in \{1,2\}$ goes to zero. At that point, the agent will continue to postpone the investment keeping $\omega_i^{\bullet\bullet}, i \in \{1,2\}$ at zero.

6.2 - The principal's payoff as a function of u

Plugging the optimal contracts of *Proposition 6* and *Proposition 7*, respectively in 90. and 97. it is possible to study how the payoff function of the principal behaves as u changes. The function will be divided into two different parts: the first is defined for $u \in (0, \hat{u})$, the second for $u \in [\hat{u}, u^{max}]$. $\hat{u}$ is the value from which the agent is not able to induce effort, i.e. $\frac{\frac{\xi}{\Delta q}}{\left(\frac{x}{x_1^{\bullet}}\right)^{\beta} - \left(\frac{x}{x_2^{\bullet}}\right)^{\beta}}$[25]. While u^{max} is the reservation utility that brings the transfer $\omega_2^{\bullet\bullet}$ down to zero. u^{max} is equal to $\beta\frac{b(\theta_2)}{r} - I$. From now on the payoff of the agent as a function of u will be referred to as $\pi(u)$.

[25] $\hat{u}$ is expressed in terms of $x_1^{\bullet}$ and $x_2^{\bullet}$ because it derives from the constraint 91. that is part of the maximization problem that gives *Proposition 6*.

Substituting the results of *Proposition 6* into the objective function 90. We obtain $\pi(u)$ when $u \in (0, \hat{u})$:

$$102. \qquad \pi(u) = q_H \left(\frac{x}{x_1^{\bullet}}\right)^{\beta} \left(\frac{b(\theta_1)}{r} - \frac{\beta\frac{b(\theta_1)}{r} - I}{\beta-1} - u\left(\frac{x_1}{x_2^{\bullet}}\right)^{\beta} - \left(\frac{x}{x_1^{\bullet}}\right)^{-\beta}\frac{\xi}{\Delta q}\right) + (1-q_H)\left(\frac{x}{x_2^{\bullet}}\right)^{\beta}\left(\frac{b(\theta_2)}{r} - \frac{\beta\frac{b(\theta_2)}{r} - I}{\beta-1} + u\,\frac{q_H(\beta-1)+1}{(\beta-1)(1-q_H)}\right)$$

Simplifying,

$$103. \qquad \pi(u) = q_H\left(\frac{x}{x_1^{\bullet}}\right)^{\beta}\frac{I - \frac{b(\theta_1)}{r}}{\beta-1} + (1-q_H)\left(\frac{x}{x_2^{\bullet}}\right)^{\beta}\frac{I-\frac{b(\theta_2)}{r}}{\beta-1} + \frac{u}{\beta-1}\left(\frac{x}{x_2^{\bullet}}\right)^{\beta} - \frac{q_H\xi}{\Delta q}$$

The principal's payoff function for $u \in [\hat{u}, u^{max}]$ is obtained, similarly, by substituting the results of *Proposition 7* into the objective function 97.:

$$104. \qquad \pi(u) = q_L\left(\frac{x}{x_1^{\bullet\bullet}}\right)^{\beta}\left(\frac{b(\theta_1)}{r} - \frac{\beta\frac{b(\theta_1)}{r}-I}{\beta-1} + \frac{u}{\beta-1}\right) + (1-q_L)\left(\frac{x}{x_2^{\bullet\bullet}}\right)^{\beta}\left(\frac{b(\theta_2)}{r} - \frac{\beta\frac{b(\theta_2)}{r}-I}{\beta-1} + \frac{u}{\beta-1}\right)$$

Simplifying,

$$105. \qquad \pi(u) = q_L\left(\frac{x}{x_1^{\bullet\bullet}}\right)^{\beta}\frac{I-\frac{b(\theta_1)}{r}}{\beta-1} + (1-q_L)\left(\frac{x}{x_2^{\bullet\bullet}}\right)^{\beta}\frac{I-\frac{b(\theta_2)}{r}}{\beta-1} + \frac{u}{\beta-1}\left[q_L\left(\frac{x}{x_1^{\bullet\bullet}}\right)^{\beta} + (1-q_L)\left(\frac{x}{x_2^{\bullet\bullet}}\right)^{\beta}\right]$$

Equations 103. and 105. can give some useful insights on how the principal's expected payoff behaves as u grows. Differentiating eq. 103. with respect to u we have:

$$106. \qquad \dot{\pi}(u) = -\left(\frac{x}{x_2^{\bullet}}\right)^{\beta} < 0$$

As expected, the first derivative is negative. Meaning that as the agent's reservation utility increases the principal has a lower expected payoff. To study the rate at which $\pi(u)$ is decreasing the second derivative is computed:

$$107. \qquad \ddot{\pi}(u) = \beta\left(\frac{x}{x_2^{\bullet}}\right)^{\beta}\left(\frac{\dot{x}_2^{\bullet}}{x_2^{\bullet}}\right) > 0$$

Where $\dot{x}_2^{\bullet} = \frac{\beta}{(\beta-1)}\frac{1}{(1-q_H)}\delta$

The second derivative is positive. The function is convex, meaning that as u increases $\pi(u)$ shrinks at a decreasing rate. Taking the first and the second derivative of eq. 105. it follows:

$$108. \qquad \dot{\pi}(u) = -q_L\left(\frac{x}{x_1^{\bullet\bullet}}\right)^{\beta} - (1-q_L)\left(\frac{x}{x_2^{\bullet\bullet}}\right)^{\beta} < 0$$

$$109. \qquad \ddot{\pi}(u) = q_L\beta\left(\frac{x}{x_1^{\bullet\bullet}}\right)^{\beta}\left(\frac{\dot{x}_1^{\bullet\bullet}}{x_1^{\bullet\bullet}}\right) + (1-q_L)\beta\left(\frac{x}{x_2^{\bullet\bullet}}\right)^{\beta}\left(\frac{\dot{x}_2^{\bullet\bullet}}{x_2^{\bullet\bullet}}\right) > 0$$

Where $\dot{x}_1^{\bullet\bullet} = \dot{x}_2^{\bullet\bullet} = \frac{\beta}{\beta-1}\delta$

The result is analogous to the previous one. Again, the first derivative is negative while the second one is positive. The function is convex, meaning that as u grows the payoff of the principal decreases at a decreasing rate. This is consistent with the fact that, as u exceeds u^{max}, the principal has always the chance of offering a null fixed transfer $\omega_i^{\bullet\bullet}, i \in \{1,2\}$ to the agent, simultaneously rising the investment trigger $x_i^{\bullet\bullet}, i \in \{1,2\}$ to make the expected cash flow of the project high enough to ensure the agent's participation in the partnership. Theoretically, the investment can always be postponed. However, this will make the chosen trigger diverging dramatically from the optimal one and the principal's payoff will go asymptotically to zero.

It is worth to be noticed that equation 106. is less negative than equation 108. This follows from $x_2^{\bullet}$ being greater than both $x_2^{\bullet\bullet}$, and $x_1^{\bullet\bullet}$. Therefore, the first segment of the principal's payoff function will decrease slower than the second.

This information allows studying the condition under which the principal will prefer to induce effort. If for $u = 0$ (the minimum value of u) equation 103. has a greater value than equation 105., the former will be strictly preferred to the latter for the entire interval $(0, \hat{u})$.

Here we compare equation 103. to 105. for $u = 0$. First, notice that from $u = 0$ follows $x_1^{\bullet} = x_1^{\bullet\bullet} = x_1^{BFB}$ and $x_2^{\bullet} = x_2^{\bullet\bullet} = x_2^{BFB}$. This helps the comparison of the two functions and gives the following condition to have 103. greater or equal than 105.:

$$110. \qquad \left(\frac{I-\frac{b(\theta_1)}{r}}{\beta-1}\right)\left(\frac{x}{x_1^{BFB}}\right)^{\beta} - \left(\frac{I-\frac{b(\theta_2)}{r}}{\beta-1}\right)\left(\frac{x}{x_2^{BFB}}\right)^{\beta} \geq \frac{q_H\xi}{\Delta q \Delta q}$$

The present condition coincides exactly with the one reported in 24. That is the condition, in the benchmark model, under which the principal prefers to induce effort in the case of an agent with no negotiating power (i.e. $u = 0$). Assuming 110. to hold, it can be concluded that 103. is superior to 105. for $u \in (0, \hat{u})$. At the critical value $\hat{u}$ equation 103. cease to exist and from that value up to u^{max} equation 105. describes the principal's payoff.

CONCLUSIONS

The present work analyzed the optimal design of a PPP contract modeling different aspects of these complex agreements. First, we assumed that the government faces a problem of moral hazard when delegating the firm to build and operate an infrastructure. Second, we considered the investment timing as endogenous to our model. In other words, the parties can decide when to initiate the investment. This possibility takes the form of a flexibility provision embedded in the contract and is modeled following a real options approach (Dixit and Pindyck, 1994). Third, we studied the bargaining power of the parties allowing it to be split between the government and the firm. This is innovative in the literature on PPP as scholars have typically assumed that the principal has all the negotiating power and formulates a take-it-or-leave-it offer for the agent. Moreover, we model the bargaining power of the parties in two different ways. In one case, we use a Nash bargaining solution. In the other, we introduce the agent's reservation utility to account for his bargaining power, the greater the former the higher the latter.

The importance of allocating at least some bargaining power to the agent derives from the specificities of the contractual relationship that incurs among the parties in a PPP agreement. The private company must finance the project, needs highly specific competencies (this reduces the number of competitors on the agent side), bears a substantial level of risk and uncertainty. Therefore, the agent is likely not to passively accept the decision of the government on the variables at stake and will claim an active role in it.

We choose as a flexibility provision the investment timing as it is valuable both for the government and the private firm. Differently, other clauses such as government guarantees create value for just one of the parties and are used to convince the other to participate in the agreement. It is to be noticed that the public authority and the private entity are negotiating to complete a project that would be individually infeasible. Thus, we believe that a form of provision meant to create value for both parties is more effective in making the contract attractive for the two participants.

The benchmark model for our work is the one designed by Buso et al. (2020). The authors study the optimal design of a PPP considering the moral hazard issue between the public and the private party. Also, they consider two agent types, one with negotiating power and one without. We expand their framework by explicitly introducing the bargaining power of the parties. The first set of models does so using a Nash bargaining solution, the second introducing the agent's reservation utility.

Nash Bargaining results

In the first version of the model, we assume the parties bargain over α and the government makes a take-it-or-leave-it offer to the agent on the investment triggers and the fixed transfer. The results (*Proposition 1* and *Proposition 2*) show that the parties agree to give the entire cash flow to the firm. The optimal contract is exactly equal to the one of the benchmark model. This happens because the agent, anticipating the principal's take-it-or-leave-it offer, realizes that his payoff will not depend on the level of α (it will always be 0 if the project quality is poor and it is independent of α if the project quality is high). Thus, the principal will decide to implement the first best investment timing and pay an informational rent of Φ_1 if the project is of high quality.

In the second version of the model, we assume $\alpha = 1$ and the parties bargaining over the investment triggers x_1, and x_2. The optimal contract (*Proposition 3*) is exactly equal to the one of the benchmark model. The first best timing is achieved, and the principal will pay an information rent of Φ_1 if the project is of high quality.

The third version of the model relaxes the assumption of $\alpha = 1$, the parties are free to split the project cash flow as they prefer. The principal and the agent bargain over α and the investment triggers x_1, and x_2. A limit of the model is that it does not allow to determine univocally the optimal α as the bargaining function does not depend on it. The optimal contract (*Proposition 4*) mirrors the one of the benchmark model but it is not equal to it. While the achieved first best timing is the same in this model and in the benchmark, the optimal transfers coincide only if $\alpha = 1$. For $\alpha < 1$ the principal will instead have to compensate the agent with a fixed transfer higher than the one found in the benchmark model.

Finally, the fourth version of the model differs from the previous one as we introduce in the bargaining problem some constraints over α. Precisely, we impose a limited liability on the government side. The fixed transfer to the firm cannot exceed the investment expenditure I otherwise, the government will find it more convenient to afford the investment cost than to pay the private company. Besides, the fixed transfer cannot be negative since it is not realistic to imagine that the government will tax the firm that already bears the burden of the investment cost. These new constraints were meant to allow finding the optimal value for α. However, the new more complicated bargaining problem does not have an analytical solution but only a numerical one. The only solution we have for the optimal α is implicit and reported in *Proposition 5*.

The introduction of bargaining in the framework of the benchmark model has allowed showing how the results by Buso et al. (2020) are solid to the considered bargaining dynamics. However, the use of a Nash bargaining solution has shown some limitations. In the first place, it did not allow solving for an explicit optimal value of α. Further research is needed to study if introducing in the model a nonlinear dependence on α would allow obtaining explicitly the optimal allocation of the project cash flow. In the second place, we observe that the optimal contracts do not depend on the bargaining power of the parties. In other words, the parameter η is never part of the optimal contract. Thus, not allowing the analysis of how the principal's payoff function evolves according to the agent's bargaining power. To have this efficient frontier we expanded the model in another direction modeling the bargaining power in terms of the agent's reservation utility.

Reservation utility results

The principal's payoff as a function of u is divided into two parts. For $u < \frac{\frac{\xi}{\Delta q}}{\left(\frac{x}{x_1^{\bullet}}\right)^{\beta} - \left(\frac{x}{x_2^{\bullet}}\right)^{\beta}}$ the principal can induce the agent to exert effort. From that value on she cannot, and she offers to the agent the same payoff irrespective of the project quality.

In *Proposition 6* we find the optimal menu of contracts offered to the agent when the principal can induce effort. The optimal investment timing is achieved only when the project quality is high. However, in that case, the fixed transfer is higher than the first best. In addition to the information premium Φ_1, the principal must pay $u\left(\frac{x_1}{x_2}\right)^{\beta}$ that increases with the agent's reservation utility. On the contrary, when the project quality is low the investment is postponed, compared to the first-best, of a factor $u\frac{\beta}{\beta-1}\delta\frac{1}{1-q_H}$ that depends positively on u. The fixed transfer is instead reduced by $u\frac{q_H(\beta-1)+1}{(\beta-1)(1-q_H)}$, the higher the reservation utility the lower the transfer. This is valid for $u \in (0, \hat{u})$ where $\hat{u} = \frac{\frac{\xi}{\Delta q}}{\left(\frac{x}{x_1^{\bullet}}\right)^{\beta} - \left(\frac{x}{x_2^{\bullet}}\right)^{\beta}}$.

When the principal cannot induce effort, she offers to the agent the optimal menu of contracts found in *Proposition 7*. Here, the first best investment timing is never achieved as, regardless of the project quality, the investment is postponed by $u\frac{\beta}{\beta-1}\delta$. Besides, the fixed transfer granted to the agent is smaller than the first best of a quantity $\frac{u}{\beta-1}$. This result is valid for $u \in [\hat{u}, u^{max}]$ where $u^{max} = \beta\frac{b(\theta_2)}{r} - I$ is the value that brings the transfer $\omega_2^{\bullet\bullet}$ down to zero.

In conclusion, we studied the shape of $\pi(u)$. Both segments are decreasing in u and convex. Moreover, the function for the principal's payoff when she does not induce effort is steeper than when the effort is exerted. This meaning that, if at $u = 0$ the latter is greater than the former, the principal will find it convenient to encourage effort as long as she can do so. This is verified when condition 110. is satisfied.

Once u hits $\hat{u}$ there is a jump, the principal cannot induce effort anymore and the principal's payoff is lower. $\pi(u)$ goes from equation 103. to equation 105. Notice that theoretically the investment can always be postponed. When the transfer to the agents reaches 0 and u keeps growing the principal can decide to hold ω_2^{**} at zero and indeterminately postpone the investment. However, this will make the chosen trigger diverging dramatically from the first best and the principal's payoff will go asymptotically to zero.

The model with reservation utility is effective in showing the effect of the agent's bargaining power on the principals' payoff, and results to be an appropriate complement to the model with Nash bargaining. The agent's bargaining power is showed to have a considerable impact on how profitable the PPP contract can be for the principal.

APPENDIX

PART A - The optimal investment choice

Here we will derive the optimal investment rule using the contingent claim analysis. We will show how the results presented by Dixit and Pindyck (1994) have general validity and can be applied in our contribution.

The goal is to find the value of the option to invest $F(x_t)$ that depends on the cash flow x_t. Recall that x_t follows a geometric Brownian motion

$$\text{A.1} \qquad dx_t = \alpha x_t dt + \sigma x_t dB_t, x_0 = x$$

Where α is the expected percentage rate of change of the project cash flow x_t, σ is the positive constant volatility, and dB_t is the increment of a standard Wiener process.

The contingent claims analysis assumes that the market is sufficiently complete to find an asset or a dynamic portfolio of assets (tracking portfolio) that is perfectly correlated with the value of the cash flow x_t. The tracking portfolio is assumed to pay no dividends, the return derives only from the capital gains.

Besides α, the project also generates a positive rate-of-return-shortfall $\delta > 0$. δ plays an important role in the model as it can be interpreted as the opportunity cost to postpone the investment. Suppose x_t is the price of a share of common stocks, δ is to be interpreted as the dividend paid by the stock. If it is null, a call option on the stock will be held until maturity since waiting does not entail any opportunity cost. On the contrary, if the stock pays a dividend, the more the investor waits to exercise the call option the more the foregone dividends are. Therefore, a positive value of δ guarantees that in our setting there is an option value to wait. When $\delta = 0$ the investor will find optimal to never invest, no matter how high the NPV of the project is. Differently, when δ goes to infinite the value of the option goes to zero as the opportunity cost is so high that the investment will be carried out immediately, in this case, the traditional NPV rule applies.

As anticipated, $F(x_t)$ it the value of the option to invest. Consider now the following portfolio. A long position on the option to invest $F(x_t)$ and a short position on *n* units of the cash flow. The value of the portfolio is:

$$\text{A.2} \qquad \Psi = F(x_t) - nx_t$$

The portfolio is dynamic as its value changes with x_t. The short position implies that a payment of $\delta n x_t$ must be done in favor of the counterparty who holds the long position. In an arbitrarily small interval of time dt the value of the portfolio is:

$$\text{A.3} \qquad d\Psi = dF(x_t) - n dx_t - \delta n x_t dt$$

Using the Ito's lemma, we can expand $dF(x_t)$ into:

$$\text{A.4} \qquad dF(x_t) = \tfrac{1}{2}(dx_t)^2 F_{xx}(x_t) + dx_t F_x(x_t)$$

Where $F_x(x_t)$ and $F_{xx}(x_t)$ are respectively the first and the second derivative of $F(x_t)$ with respect to x_t.

The portfolio value then is:

$$\text{A.5} \qquad d\Psi = \tfrac{1}{2}(dx_t)^2 F_{xx}(x_t) + dx_t F_x(x_t) - n dx_t - \delta n x_t dt =$$

$$= \frac{1}{2}\sigma^2 x_t^2 F_{xx}(x_t) dt - \delta n x_t dt + (F_x(x_t) - n) dx_t$$

By assuming $n = F_x(x_t)$ we can get rid of the uncertainty. It follows:

$$\text{A.6} \qquad d\Psi = \left[\tfrac{1}{2}\sigma^2 x_t^2 F_{xx}(x_t) - \delta x_t F_x(x_t)\right] dt$$

As the portfolio is built to edge the risk, its return should be risk-free. We have:

$$\text{A.7} \qquad \tfrac{1}{2}\sigma^2 x_t^2 F_{xx}(x_t) dt - \delta x_t F_x(x_t) dt = r\Psi dt$$

$$\frac{1}{2}\sigma^2 x_t^2 F_{xx}(x_t) dt - \delta x_t F_x(x_t) dt = r[F(x_t) - x_t F_x(x_t)] dt$$

$$\frac{1}{2}\sigma^2 x_t^2 F_{xx}(x_t) + (r - \delta) x_t F_x(x_t) - r F(x_t) = 0$$

The value of the option to invest $F(x_t)$ must satisfy the second-order differential equation found in A.7 Moreover, $F(x_t)$ must also meet the following boundary conditions:

$$\text{A.8} \qquad F(0) = 0$$

$$\text{A.9} \qquad F(x_t^{\clubsuit}) = \tfrac{x_t^{\clubsuit}}{\delta} + \omega - I$$

$$\text{A.10} \qquad F_x(x_t^{\clubsuit}) = \tfrac{1}{\delta}$$

Condition A.8 follows from the stochastic process A.1 which implies that as x_t goes to zero, it will stay at zero. The number A.9 and number A.10 derive from considerations on the optimal investment timing. $x_t^{\clubsuit}$ is the optimal investment trigger, meaning that when the stochastic process for x_t hits $x_t^{♣}$, it is the optimal moment to carry out the investment. The A.9 is the so-called value-matching condition. It states that upon investment, the firm receives exactly $\frac{x_t^{♣}}{\delta} + \omega - I$, i.e. the present value of future expected cash flows and the transfer from the government minus the investment cost. Notice that equation A.7 is a second-order differential equation but to solve it we have three conditions. This is because also the optimal investment trigger $x_t^{♣}$ must be determined in the solution of the problem and so we need the third boundary condition. The A.10 is the "smooth-pasting" condition. It states that $F(x_t)$ must be continuous and smooth at the critical exercise point. Otherwise, it is possible to obtain a higher project value exercising at a different point.

Equation A.9 has some meaningful interpretations. Consider it in the form $\frac{x_t^{♣}}{\delta} + \omega - F(x_t^{♣}) = I$. It shows that the investment expenditure I must be equal to the value received from the investment $\frac{x_t^{♣}}{\delta} + \omega$ minus the value of the option to invest that is lost upon investment. Alternatively, considering it as $F(x_t^{♣}) + I = \frac{x_t^{♣}}{\delta} + \omega$ it shows that the optimal investment timing is when the value of the project $\frac{x_t^{♣}}{\delta} + \omega$ is equal to the investment expenditure plus the value of the option to invest.

The characteristic equation of the differential equation A.7 is:

$$\text{A.11} \qquad \Psi(\zeta) \equiv \frac{1}{2}\sigma^2\zeta(\zeta-1) + \mu\zeta - r = 0$$

It has two solutions, one positive and one negative. Once considered the boundary conditions the only relevant solution is the positive one. It will be called β.

$$\text{A.12} \qquad \beta = \left(\frac{1}{2} - \frac{r-\delta}{\sigma^2}\right) + \sqrt{\left(\frac{1}{2} - \frac{r-\delta}{\sigma^2}\right)^2 + \frac{2r}{\sigma^2}} > 1$$

To solve A.7 guess the form for the solution:

$$\text{A.13} \qquad F(x_t) = Ax_t^{\beta}$$

The solution is:

$$\text{A.14} \qquad \frac{x_t^{♣}}{\delta} + \omega = \frac{\beta}{\beta-1} I$$

$$\text{A.15} \qquad A = \frac{\frac{x_t^{♣}}{\delta} + \omega - I}{x_t^{♣\beta}}$$

Plugging A.15 into A.13 we obtain:

$$\text{A.16} \qquad F(x_t) = \begin{cases} \left(\frac{x_t}{x_t^{\clubsuit}}\right)^{\beta} \left(\frac{x_t^{\clubsuit}}{\delta} + \omega - I\right) & for\ x_t \leq x_t^{\clubsuit} \\ \frac{x_t}{\delta} + \omega - I & for\ x_t > x_t^{\clubsuit} \end{cases}$$

Equations A.13-A.15 give the value of the option to invest and the optimal investment rule. The most important point is that from $\beta > 1$, follows $\frac{\beta}{\beta-1} > 1$ and $\frac{x_t^{\clubsuit}}{\delta} + \omega > I$. The optimal investment timing requires the value received from the project to be greater than the investment cost. This proves that, when the option to delay the investment exists, the NPV rule is wrong and there is a wedge between the critical project value $\frac{x_t^{\clubsuit}}{\delta} + \omega$ and the investment expenditure I.

Now observe the value of the option to invest when $x_t \leq x_t^{\clubsuit}$. If who holds the investment opportunity decide to exercise earlier than $x_t^{\clubsuit}$ the payoff will be what she would have obtained exercising at the optimal stopping time, i.e. $\left(\frac{x_t^{\clubsuit}}{\delta} + \omega - I\right)$, discounted back at the moment she is investing. Following this line of reasoning, the term $\left(\frac{x_t}{x_t^{\clubsuit}}\right)^{\beta}$ must be interpreted as a discount factor. Indeed, observe that if the investment is carried out at the optimal investment timing, i.e. $x_t = x_t^{\clubsuit}$, the term $\left(\frac{x_t}{x_t^{\clubsuit}}\right)^{\beta}$ goes to 1. Also, the smaller x_t the higher the discount[26].

The same value as A.16 can be obtained defining:

$$\text{A.17} \qquad F(x_t) = E_0(e^{-r\tau})[V(x_t) - I + \omega]$$

With

$$\text{A.18} \qquad dx_t = (r - \delta)x_t dt + \sigma x_t dz_t, x_0 = x$$

Where r is the risk-free rate, σ is the positive constant volatility, $\delta = r - \alpha > 0$ is the positive rate-of-return-shortfall, dz_t is the increment of a standard Wiener process under a risk-neutral measure, τ is the investment timing defined as $\tau = \inf(t > 0 \mid x_t = x_\tau)$, where x_τ is the level of cash flows that triggers the investment and, finally, $V(x_\tau)$ is the expected discounted value of future cash flows defined as:

$$\text{A.19} \qquad V(x_\tau) = E_\tau\left[\int_\tau^\infty e^{-rt} x_t dt\right] = \int_\tau^\infty e^{-rt} E_\tau[x_t] dt = \int_\tau^\infty e^{-rt} x_\tau e^{(r-\delta)t} dt = \frac{x_\tau}{\delta}$$

[26] See Dixit and Pindyck, 1994, pp. 315-316.

By substituting $V(x_\tau)$ we get:

$$\text{A.20} \qquad F(x_\tau) = E_0(e^{-r\tau})\left[\frac{x_\tau}{\delta} - I + \omega\right]$$

$$= \left(\frac{x_t}{x_\tau}\right)^\beta \left[\frac{x_\tau}{\delta} - I + \omega\right]$$

Where $\left(\frac{x_t}{x_\tau}\right)^\beta$ is given by $\left(\frac{x_t}{x_t^{\clubsuit}}\right)^\beta$.

PART B – Proposition proofs

Proof of proposition 1

The principal's problem is:

$$\text{B.1} \qquad \max_{\{\omega_i, x_i\}, i\in\{1,2\}} q_H \left(\frac{x}{x_1}\right)^{\beta} Y_1 + (1-q_H)\left(\frac{x}{x_2}\right)^{\beta} Y_2$$

Where $Y_i = \frac{b(\theta_i)}{r} - \omega_i + \frac{(1-\alpha)x_i}{\delta}, i \in \{1,2\}$.

Subject to:

$$\text{B.2} \qquad q_H \left(\frac{x}{x_1}\right)^{\beta} F_1 + (1-q_H)\left(\frac{x}{x_2}\right)^{\beta} F_2 - \xi \geq q_L \left(\frac{x}{x_1}\right)^{\beta} F_1 + (1-q_L)\left(\frac{x}{x_2}\right)^{\beta} F_2$$

$$\text{B.3} \qquad q_H \left(\frac{x}{x_1}\right)^{\beta} F_1 + (1-q_H)\left(\frac{x}{x_2}\right)^{\beta} F_2 - \xi \geq 0$$

$$\text{B.4} \qquad F_1 \geq 0$$

$$\text{B.5} \qquad F_2 \geq 0$$

$$\text{B.6} \qquad \omega_1 \geq 0$$

$$\text{B.7} \qquad \omega_2 \geq 0$$

Where $F_i = \frac{\alpha x_i}{\delta} + \omega_i - I, i \in \{1,2\}$ and B.2 can be written as:

$$\text{B.8} \qquad \left(\frac{x}{x_1}\right)^{\beta} F_1 - \left(\frac{x}{x_2}\right)^{\beta} F_2 \geq \frac{\xi}{\Delta q}$$

From B.8 it follows that B.4 is always slack:

$$\text{B.9} \qquad \left(\frac{x}{x_1}\right)^{\beta} F_1 \geq \frac{\xi}{\Delta q} + \left(\frac{x}{x_2}\right)^{\beta} F_2 \rightarrow \left(\frac{x}{x_1}\right)^{\beta} F_1 \geq \frac{\xi}{\Delta q} > 0$$

B.3 can be written as:

$$\text{B.10} \qquad \left(\frac{x}{x_1}\right)^{\beta} F_1 + \frac{(1-q_H)}{q_H}\left(\frac{x}{x_2}\right)^{\beta} F_2 \geq \frac{\xi}{q_H}$$

From B.9 follows $F_1 \geq \frac{\xi}{\Delta q} \rightarrow \frac{\xi}{\Delta q} > \frac{\xi}{q_H} \rightarrow \left(\frac{x}{x_1}\right)^{\beta} F_1 > \frac{\xi}{q_H}$. We have that B.10 and thus B.3 are always slack.

In summary, the principal's problem is reduced to:

$$\text{B.11} \qquad \max_{\{\omega_i, x_i\}, i\in\{1,2\}} q_H \left(\frac{x}{x_1}\right)^{\beta} Y_1 + (1-q_H)\left(\frac{x}{x_2}\right)^{\beta} Y_2$$

Subject to:

$$\text{B.12} \qquad \left(\frac{x}{x_1}\right)^{\beta} F_1 - \left(\frac{x}{x_2}\right)^{\beta} F_2 \geq \frac{\xi}{\Delta q}$$

$$\text{B.13} \qquad F_2 \geq 0$$

$$\text{B.14} \qquad \omega_1 \geq 0$$

$$\text{B.15} \qquad \omega_2 \geq 0$$

The Lagrangian is:

$$L = q_H \left(\frac{x}{x_1}\right)^{\beta} \left(\frac{b(\theta_1)}{r} - \omega_1 + \frac{(1-\alpha)x_1}{\delta}\right) + (1-q_H)\left(\frac{x}{x_2}\right)^{\beta}\left(\frac{b(\theta_2)}{r} - \omega_2 + \frac{(1-\alpha)x_2}{\delta}\right)$$
$$+ \lambda_1 \left[\left(\frac{x}{x_1}\right)^{\beta}\left(\frac{\alpha x_1}{\delta} + \omega_1 - I\right) - \left(\frac{x}{x_2}\right)^{\beta}\left(\frac{\alpha x_2}{\delta} + \omega_2 - I\right) - \frac{\xi}{\Delta q}\right]$$
$$+ \lambda_2 \left(\frac{\alpha x_2}{\delta} + \omega_2 - I\right) + \lambda_3 \omega_1 + \lambda_4 \omega_2$$

Where the Lagrangian multipliers are $\lambda_1, \lambda_2, \lambda_3$ and λ_4. One for each constraint B.12-B.15.

The FOC with respect to ω_1 gives:

$$\text{B.16} \qquad \lambda_3 = (q_H - \lambda_1)\left(\frac{x}{x_1}\right)^{\beta}$$

Equation B.16 is satisfied in three possible cases:

i. $q_H = \lambda_1 > 0, \lambda_3 = 0$
ii. $q_H > \lambda_1 > 0, \lambda_3 > 0$
iii. $q_H > \lambda_1 = 0, \lambda_3 > 0$

The FOC with respect to ω_2 gives:

$$\text{B.17} \qquad \lambda_2 + \lambda_4 = (1 - q_H + \lambda_1)\left(\frac{x}{x_2}\right)^{\beta}$$

The right-hand side of the equality is strictly positive, so either λ_2, λ_4, or both are positive. There are three possible cases:

a. $\lambda_2 = 0, \lambda_4 > 0$
b. $\lambda_2 > 0, \lambda_4 = 0$
c. $\lambda_2 > 0, \lambda_4 > 0$

The FOC with respect to x_1 gives:

$$\text{B.18} \qquad \frac{\beta}{\beta-1}\delta\left[q_H\left(\omega_1-\frac{b(\theta_1)}{r}\right)-\lambda_1(\omega_1-I)\right]=x_1[(1-\alpha)q_H+\alpha\lambda_1]$$

Finally, the FOC with respect to x_2 gives:

$$\text{B.19} \qquad \frac{\lambda_2\alpha}{\delta}=\left(\frac{x}{x_2}\right)^{\beta}\left\{\frac{\beta}{x_2}\left[(1-q_H)\left(\frac{b(\theta_2)}{r}-\omega_2\right)-\lambda_1(\omega_2-I)\right]+(\beta-1)\frac{(1-q_H)(1-\alpha)-\lambda_1\alpha}{\delta}\right\}$$

Given B.16-B.19 the possible solutions are discussed.

Case ia

The case is characterized by the following relations: $q_H=\lambda_1>0, \lambda_2=0, \lambda_3=0, \lambda_4>0$. In other words, B.13 and B.14 are slack, while B.12 and B.15 are binding. Solving the maximization problem, it follows[27]:

$$\text{B.20} \qquad x_1^{**}=x_1^{FB}$$

$$\text{B.21} \qquad x_2^{**}=x_2^{FB}*\frac{q_H}{q_H-(1-\alpha)}+\frac{\beta}{\beta-1}\delta\frac{1}{q_H-(1-\alpha)}*\frac{b(\theta_2)}{r}$$

$$\text{B.22} \qquad \omega_1^{**}=\omega_1^{FB}+\left(\frac{x_1}{x_2}\right)^{\beta}\left(\omega_2^{FB}\frac{1-q_H}{q_H-(1-\alpha)}+\frac{I\alpha}{(\beta-1)\left(q_H-(1-\alpha)\right)}\right)+\left(\frac{x}{x_1}\right)^{-\beta}\frac{\xi}{\Delta q}$$

$$\text{B.23} \qquad \omega_2^{**}=0$$

Where $x_i^{FB}=\delta\frac{\beta}{\beta-1}\left(I-\frac{b(\theta_i)}{r}\right), \omega_i^{FB}=\frac{I(\beta-1-\alpha\beta)+\alpha\beta\frac{b(\theta_i)}{r}}{\beta-1}, i\in\{1,2\}$.

Some conditions for having $x_1^{**}, x_2^{**}, \omega_1^{**}>0$ needs to be imposed. Recall that for having x_i^{FB}, and ω_i^{FB} positive, it is needed that $\frac{\alpha\beta\frac{b(\theta_i)}{r}}{1+\alpha\beta-\beta}>\beta\frac{b(\theta_i)}{r}>I>\frac{b(\theta_i)}{r}$. Besides, for guaranteeing the positivity of ω_i^{FB} also $\alpha>\frac{\beta-1}{\beta}$ is required. This automatically proves the positivity of x_1^{**}. Besides, for x_2^{**} and ω_2^{**} to be positive, also $\alpha>1-q_H$ is needed., as well as the slack inequations of this case (i.e. B.13, B.14) are satisfied by the solution $\{x_i^{**}, \omega_i^{**}\}, i\in(1,2)$.

Case ib

The case is characterized by the following relations: $q_H=\lambda_1>0, \lambda_2>0, \lambda_3=0, \lambda_4=0$. In other words, B.14 and B.15 are slack, while B.12 and B.13 are binding. Solving the maximization problem, it follows that:

[27] Recall: $x_i^{FB}=\frac{\beta}{\beta-1}\delta\left(I-\frac{b(\theta_i)}{r}\right)$ and $\omega_i^{FB}=\frac{I(\beta-1-\alpha\beta)+\alpha\beta\frac{b(\theta_i)}{r}}{\beta-1}$ where $i\in\{1,2\}$

$$\text{B.24} \qquad x_1^* = x_1^{FB}$$

$$\text{B.25} \qquad x_2^* = x_2^{FB} * \frac{1-q_H}{\alpha - q_H}$$

$$\text{B.26} \qquad \omega_1^* = \omega_1^{FB} + \frac{\varepsilon}{\Delta q}\left(\frac{x}{x_1^{FB}}\right)^{-\beta}$$

$$\text{B.27} \qquad \omega_2^* = \frac{\alpha}{\alpha - q_H} * \frac{\beta\frac{b(\theta_2)}{r} - I}{\beta - 1} - \frac{q_H}{\alpha - q_H} * \omega_2^{FB}$$

From the above-mentioned conditions for the positivity of x_i^{FB}, and ω_i^{FB} follows the one of x_1^* and ω_1^*. For having x_2^* positive it is required to have α greater than q_H. Besides, the condition for the positivity of ω_2^* is: $\alpha \geq q_H * \frac{I(\beta-1)}{\beta\frac{b(\theta_2)}{r} - I + q_H\beta\left(I - \frac{b(\theta_2)}{r}\right)}$. Finally, it can be easily verified that the always slack inequations B.3 and B.4 are satisfied by the solution $\{x_i^*, \omega_i^*\}, i \in (1,2)$.

Case ic

The case is characterized by the following relations: $q_H = \lambda_1 > 0, \lambda_2 > 0, \lambda_3 = 0, \lambda_4 > 0$. In other words, B.14 is slack, while B.12, B.13, and B.15 are binding. Solving the maximization problem, it follows that $x_1^{ic} = x_1^{FB}$ and $\omega_1^{ic} = \omega_1^{FB} + \frac{\varepsilon}{\Delta q}\left(\frac{x}{x_1^{FB}}\right)^{-\beta}$. However, results from the binding inequation B.15 that $\omega_2^{ic} = 0$. Since $\omega_2^{ic} = 0$, it is expected $x_2^{ic} = x$ which is a contradiction since, by assumption, $x_2^{ic} > x$

Cases iia, iib, iic

All these cases have in common that $q_H > \lambda_1 > 0$ and $\lambda_3 > 0$. Form this follows that B.12 and B.14 are always binding. From the first-order condition B.18 we obtain:

$$\text{B.28} \qquad x_1 = \frac{\frac{\beta}{\beta-1}\delta\left[q_H\left(\omega_1 - \frac{b(\theta_1)}{r}\right) - \lambda_1(\omega_1 - I)\right]}{(1-\alpha)q_H + \alpha\lambda_1}$$

Substituting B.28 into the always slack inequation B.4 we have:

$$\text{B.29} \qquad I > \frac{q_H\alpha\beta\frac{b(\theta_1)}{r}}{q_H(\alpha\beta + 1 - \beta) + \alpha(\lambda_1 - q_H)}$$

The inequality B.29 is proven to be stronger than $I > \frac{-\alpha\beta\frac{b(\theta)}{r}}{\beta - 1 - \alpha\beta}$. This result contradicts the condition $\frac{\alpha\beta\frac{b(\theta_i)}{r}}{1+\alpha\beta-\beta} > \beta\frac{b(\theta_i)}{r} > I > \frac{b(\theta_i)}{r}$.

Cases iiia, iiib, iiic

All these cases have in common that $q_H > \lambda_1 = 0$ and $\lambda_3 > 0$ from which follows $\omega_1^{iii} = 0$. Plugging $\lambda_1 = 0$ and $\omega_1^{iii} = 0$ into the first-order condition B.18 it follows:

$$\text{B.30} \qquad x_1 = \frac{\frac{\beta}{\beta-1}\delta\left(-\frac{b(\theta_1)}{r}\right)}{(1-\alpha)} < 0$$

This generates a contradiction because by assumption, x_1 cannot be negative. It does not make sense that the government decides to start the project when the expected cash flow, in the best scenario, is negative.

The optimal solution

The only two solutions available to the principal are $\{x_i^*, \omega_i^*\}$ and $\{x_i^{**}, \omega_i^{**}\}, i \in (1,2)$. Choosing the former it has $q_H \left(\frac{x}{x_1^{FB}}\right)^\beta \left(\frac{b(\theta_1)}{r} - \omega_1^* + \frac{(1-\alpha)x_1^{FB}}{\delta}\right) + (1-q_H)\left(\frac{x}{x_2^*}\right)^\beta \left(\frac{b(\theta_2)}{r} - \omega_2^* + \frac{(1-\alpha)x_2^*}{\delta}\right)$ as a payoff. A similar expression results if she chooses the latter. Now, refer to the difference between the two as $\Delta R = R^* - R^{**}$.

$$\text{B.31} \qquad \Delta R = \left(\frac{x}{x_2^{**}}\right)^\beta \left(\frac{q_H\left(I - \frac{b(\theta_2)}{r}\right) + \frac{b(\theta_2)}{r}}{\beta - 1}\right) + (1-q_H)\left(\frac{x}{x_2^*}\right)^\beta \left(\frac{b(\theta_2)}{r} - \omega_2^* + \frac{(1-\alpha)x_2^*}{\delta}\right) > 0$$

Given a positive ΔR, it can be concluded that the principal will offer $\{x_i^*, \omega_i^*\}, i \in (1,2)$ to the agent.

The same operation can be repeated for the agent. His payoff, when the offered menu of contracts is $\{x_i^*, \omega_i^*\}, i \in (1,2)$, is $q_H \left(\frac{x}{x_1^{FB}}\right)^\beta \left(\frac{\alpha x_1^{FB}}{\delta} + \omega_1^* - I\right) + (1-q_H)\left(\frac{x}{x_2^*}\right)^\beta \left(\frac{\alpha x_2^*}{\delta} + \omega_2^* - I\right) - \xi$. A similar expression results under $\{x_i^{**}, \omega_i^{**}\}$. Now, refer to the difference between the two as $\Delta V = V^* - V^{**}$.

$$\text{B.32} \qquad \Delta V = q_H \left(\frac{x}{x_1^{FB}}\right)^\beta (\omega_1^* - \omega_1^{**}) - (1-q_H)\left(\frac{x}{x_2^{**}}\right)^\beta \left(\frac{\alpha x_2^{**}}{\delta} - I\right) < 0$$

Given a negative ΔV, it can be concluded that the agent prefers the menu of contract $\{x_i^{**}, \omega_i^{**}\}, i \in (1,2)$.

Proof of proposition 3

The principal's maximization problem is:

$$\text{B.33} \qquad \max_{\omega_1, \omega_2} q_H \left(\frac{x}{x_1}\right)^\beta Y_1^B + (1-q_H)\left(\frac{x}{x_2}\right)^\beta Y_2^B$$

Subject to:

$$\text{B.34} \qquad \left(\frac{x}{x_1}\right)^{\beta} F_1^B - \left(\frac{x}{x_2}\right)^{\beta} F_2^B \geq \frac{\xi}{\Delta q}$$

$$\text{B.35} \qquad q_H \left(\frac{x}{x_1}\right)^{\beta} F_1^B + (1 - q_H)\left(\frac{x}{x_2}\right)^{\beta} F_2^B - \xi \geq 0$$

$$\text{B.36} \qquad F_1^B \geq 0$$

$$\text{B.37} \qquad F_2^B \geq 0$$

$$\text{B.38} \qquad \omega_1 \geq 0$$

$$\text{B.39} \qquad \omega_2 \geq 0$$

Where $F_i^B = \frac{x_i}{\delta} + \omega_i - I, Y_i^B = \frac{b(\theta_i)}{r} - \omega_i, i \in \{1,2\}$.

Assuming B.34 to be binding, it follows:

$$\text{B.40} \qquad \omega_1 = \frac{\xi}{\Delta q}\left(\frac{x}{x_1}\right)^{-\beta} + \left(\frac{x_1}{x_2}\right)^{\beta}\left(\frac{x_2}{\delta} + \omega_2 - I\right) - \frac{x_1}{\delta} + I$$

This value for ω_1 is to be substituted in the objective function. Moreover, B.35 and B.36 are proven to be slack as in the proof of *Proposition 1*.

In summary, the principal's problem is reduced to:

$$\text{B.41} \qquad \max_{\omega_1,\omega_2} q_H \left(\frac{x}{x_1}\right)^{\beta}\left(\frac{b(\theta_1)}{r} - \frac{\xi}{\Delta q}\left(\frac{x}{x_1}\right)^{-\beta} - \left(\frac{x_1}{x_2}\right)^{\beta}\left(\frac{x_2}{\delta} + \omega_2 - I\right) + \frac{x_1}{\delta} - I\right) + (1 - q_H)\left(\frac{x}{x_2}\right)^{\beta}\left(\frac{b(\theta_2)}{r} - \omega_2\right)$$

Subject to:

$$\text{B.42} \qquad F_2^B \geq 0$$

$$\text{B.43} \qquad \omega_1 \geq 0$$

$$\text{B.44} \qquad \omega_2 \geq 0$$

the Lagrangian is:

$$\text{B.45} \qquad L = q_H \left(\frac{x}{x_1}\right)^{\beta}\left(\frac{b(\theta_1)}{r} - \frac{\xi}{\Delta q}\left(\frac{x}{x_1}\right)^{-\beta} - \left(\frac{x_1}{x_2}\right)^{\beta}\left(\frac{x_2}{\delta} + \omega_2 - I\right) + \frac{x_1}{\delta} - I\right) + (1 - q_H)\left(\frac{x}{x_2}\right)^{\beta}\left(\frac{b(\theta_2)}{r} - \omega_2\right) + \lambda_1\left(\frac{x_2}{\delta} + \omega_2 - I\right) + \lambda_2\omega_2$$

The FOC with respect to ω_2 gives:

$$\text{B.46} \qquad \lambda_1 + \lambda_2 = \left(\frac{x}{x_2}\right)^{\beta}$$

The right-hand side of Equation B.46 is always positive, so either λ_1, λ_2, or both are positive. There is a total of three cases.

d. $\lambda_1 > 0, \lambda_2 = 0$
e. $\lambda_1 = 0, \lambda_2 > 0$
f. $\lambda_1 > 0, \lambda_2 > 0$

Case e. and f. can be ruled out. Form the binding inequality B.44 it follows that $\omega_2 = 0$. Since $\omega_2 = 0$, it is expected that $x_2 = x$ which is a contradiction, by assumption, $x_2 > 0$.

Solving case d. it gives:

$$\text{B.47} \qquad \omega_1^{\square} = \frac{\xi}{\Delta q}\left(\frac{x}{x_1}\right)^{-\beta} - \frac{x_1}{\delta} + I$$

$$\text{B.48} \qquad \omega_2^{\square} = I - \frac{x_2}{\delta}$$

The always slack inequalities B.35 and B.36 are easily verified.

Bargaining over x_1 and x_2

The second stage of the problem is the Nash bargaining between the two parties:

$$\text{B.49} \qquad \max_{x_1,x_2} \frac{\left[q_H\left(\frac{x}{x_1}\right)^{\beta}\left(\frac{b(\theta_1)}{r}-\omega_1\right)+(1-q_H)\left(\frac{x}{x_2}\right)^{\beta}\left(\frac{b(\theta_2)}{r}-\omega_2\right)\right]^{\eta}}{\left[q_H\left(\frac{x}{x_1}\right)^{\beta}\left(\frac{x_1}{\delta}+\omega_1-I\right)+(1-q_H)\left(\frac{x}{x_2}\right)^{\beta}\left(\frac{x_2}{\delta}+\omega_2-I\right)-\xi\right]^{\eta-1}}$$

Substituting B.47 and B.48 into B.49 and maximizing the log of the function, the new problem is:

$$\text{B.50} \qquad \max_{x_1,x_2} ln\left\{\frac{\left[q_H\left(\frac{x}{x_1}\right)^{\beta}\left(\frac{b(\theta_1)}{r}+\frac{x_1}{\delta}-I\right)-q_H\left(\frac{\xi}{\Delta q}\right)+(1-q_H)\left(\frac{x}{x_2}\right)^{\beta}\left(\frac{b(\theta_2)}{r}+\frac{x_2}{\delta}-I\right)\right]^{\eta}}{\left[q_L\frac{\xi}{\Delta q}\right]^{\eta-1}}\right\}$$

The FOCs with respect to x_1 and x_2 gives respectively:

$$\text{B.51} \qquad x_1^{\square} = x_1^{BFB}$$

$$\text{B.52} \qquad x_2^{\square} = x_2^{BFB}$$

Where $x_i^{BFB} = \frac{\beta}{\beta-1}\delta\left(I - \frac{b(\theta_i)}{r}\right), i \in \{1,2\}$

Once obtained the optimal values for $x_1^{\square}$ and $x_2^{\square}$ they can be substituted back into B.47 and B.48. The result is:

$$\text{B.53} \qquad \omega_1^{\square} = \omega_1^{BFB} + \frac{\xi}{\Delta q}\left(\frac{x}{x_1}\right)^{-\beta}$$

$$\text{B.54} \qquad \omega_2^{\square} = \omega_2^{BFB}$$

Where $\omega_i^{BFB} = \frac{\beta\frac{b(\theta_i)}{r} - I}{\beta - 1}, i \in \{1,2\}$

These results coincide with the ones obtained by Buso et al. (2020) in the model with no negotiating power.

Proof of proposition 4

The principal's maximization problem is:

$$\text{B.55} \qquad \max_{\omega_1,\omega_2} q_H\left(\frac{x}{x_1}\right)^{\beta} Y_1 + (1 - q_H)\left(\frac{x}{x_2}\right)^{\beta} Y_2$$

Subject to:

$$\text{B.56} \qquad \left(\frac{x}{x_1}\right)^{\beta} F_1 - \left(\frac{x}{x_2}\right)^{\beta} F_2 \geq \frac{\xi}{\Delta q}$$

$$\text{B.57} \qquad q_H\left(\frac{x}{x_1}\right)^{\beta} F_1 + (1 - q_H)\left(\frac{x}{x_2}\right)^{\beta} F_2 - \xi \geq 0$$

$$\text{B.58} \qquad F_1 \geq 0$$

$$\text{B.59} \qquad F_2 \geq 0$$

$$\text{B.60} \qquad \omega_1 \geq 0$$

$$\text{B.61} \qquad \omega_2 \geq 0$$

Where $F_i = \frac{\alpha x_i}{\delta} + \omega_1 - I, Y_i = \frac{b(\theta_i)}{r} - \omega_i + \frac{(1-\alpha)x_i}{\delta}, i \in \{1,2\}$.

Assuming B.56 to be binding it follows:

$$\text{B.62} \qquad \omega_1 = \frac{\xi}{\Delta q}\left(\frac{x}{x_1}\right)^{-\beta} + \left(\frac{x_1}{x_2}\right)^{\beta}\left(\frac{\alpha x_2}{\delta} + \omega_2 - I\right) - \frac{\alpha x_1}{\delta} + I$$

As above, proving B.57 and B.58 to be always slack and substituting B.62 in B.55 the reduced maximization problem is:

$$\text{B.63} \qquad \max_{\omega_1,\omega_2} q_H\left(\frac{x}{x_1}\right)^{\beta}\left(\frac{b(\theta_1)}{r} - \frac{\xi}{\Delta q}\left(\frac{x}{x_1}\right)^{-\beta} - \left(\frac{x_1}{x_2}\right)^{\beta}\left(\frac{\alpha x_2}{\delta} + \omega_2 - I\right) + \frac{x_1}{\delta} - I\right) +$$
$$(1 - q_H)\left(\frac{x}{x_2}\right)^{\beta}\left(\frac{b(\theta_2)}{r} - \omega_2 + \frac{(1-\alpha)x_2}{\delta}\right)$$

Subject to:

$$\text{B.64} \qquad F_2 \geq 0$$

$$\text{B.65} \qquad \omega_1 \geq 0$$

$$\text{B.66} \qquad \omega_2 \geq 0$$

The Lagrangian is:

$$\text{B.67} \qquad L = q_H\left(\frac{x}{x_1}\right)^{\beta}\left(\frac{b(\theta_1)}{r} - \frac{\xi}{\Delta q}\left(\frac{x}{x_1}\right)^{-\beta} - \left(\frac{x_1}{x_2}\right)^{\beta}\left(\frac{\alpha x_2}{\delta} + \omega_2 - I\right) + \frac{x_1}{\delta} - I\right) +$$

$$(1 - q_H)\left(\frac{x}{x_2}\right)^{\beta}\left(\frac{b(\theta_2)}{r} - \omega_2 + \frac{(1-\alpha)x_2}{\delta}\right) + \lambda_1\left(\frac{\alpha x_2}{\delta} + \omega_2 - I\right) + \lambda_2\omega_2$$

The FOC with respect to ω_2 gives:

$$\text{B.68} \qquad \lambda_1 + \lambda_2 = \left(\frac{x}{x_2}\right)^{\beta}$$

The right-hand side of Equation B.68 is always positive, so either λ_1, λ_2, or both are positive. There is a total of three cases.

g. $\lambda_1 > 0, \lambda_2 = 0$
h. $\lambda_1 = 0, \lambda_2 > 0$
i. $\lambda_1 > 0, \lambda_2 > 0$

As above, case h. and i. can be ruled out. Form the binding inequality B.66 it follows that $\omega_2 = 0$. Since $\omega_2 = 0$, it is expected that $x_2 = x$ which is a contradiction, by assumption, $x_2 > 0$.

Solving case g. it gives:

$$\text{B.69} \qquad \omega_1^{\star} = \frac{\xi}{\Delta q}\left(\frac{x}{x_1}\right)^{-\beta} - \frac{\alpha x_1}{\delta} + I$$

$$\text{B.70} \qquad \omega_2^{\star} = I - \frac{\alpha x_2}{\delta}$$

The always slack inequalities B.57 and B.58 are easily verified and both ω_1 and ω_2 are non-negative.

Bargaining over x_1, x_2, and α

The second stage of the problem is the Nash bargaining between the two parties:

$$\text{B.71} \qquad \max_{x_1, x_2, \alpha} \frac{\left[q_H\left(\frac{x}{x_1}\right)^{\beta}\left(\frac{b(\theta_1)}{r} - \omega_1 + \frac{(1-\alpha)x_1}{\delta}\right) + (1-q_H)\left(\frac{x}{x_2}\right)^{\beta}\left(\frac{b(\theta_2)}{r} - \omega_2 + \frac{(1-\alpha)x_2}{\delta}\right)\right]^{\eta}}{\left[q_H\left(\frac{x}{x_1}\right)^{\beta}\left(\frac{\alpha x_1}{\delta} + \omega_1 - I\right) + (1-q_H)\left(\frac{x}{x_2}\right)^{\beta}\left(\frac{\alpha x_2}{\delta} + \omega_2 - I\right) - \xi\right]^{\eta-1}}$$

Substituting B.47 and B.48 in B.49 and maximizing the log of the function, the new problem is:

$$\text{B.72} \qquad \max_{x_1, x_2, \alpha} ln\left\{\frac{\left[q_H\left(\frac{x}{x_1}\right)^{\beta}\left(\frac{b(\theta_1)}{r} + \frac{x_1}{\delta} - I\right) - q_H\left(\frac{\xi}{\Delta q}\right) + (1-q_H)\left(\frac{x}{x_2}\right)^{\beta}\left(\frac{b(\theta_2)}{r} + \frac{x_2}{\delta} - I\right)\right]^{\eta}}{\left[q_L\frac{\xi}{\Delta q}\right]^{\eta-1}}\right\}$$

The FOCs with respect to x_1 and x_2 gives respectively:

$$\text{B.73} \qquad x_1^{\star} = x_1^{FB}$$

$$\text{B.74} \qquad x_2^{\star} = x_2^{FB}$$

Where $x_i^{FB} = \frac{\beta}{\beta-1}\delta\left(I - \frac{b(\theta_i)}{r}\right), i \in \{1,2\}$

Once obtained the optimal values for $x_1^{\star}$ and $x_2^{\star}$ they can be substituted back into B.69 and B.70. The result is:

$$\text{B.75} \qquad \omega_1^{\star} = \omega_1^{FB} + \frac{\xi}{\Delta q}\left(\frac{x}{x_1}\right)^{-\beta}$$

$$\text{B.76} \qquad \omega_2^{\star} = \omega_2^{FB}$$

Where $\omega_i^{FB} = \frac{\beta\frac{b(\theta_i)}{r} - I}{\beta-1}, i \in \{1,2\}$

Proof of Proposition 5

The Nash bargaining maximization problem is:

$$\text{B.77} \qquad \max_{x_1,x_2,\alpha} ln\left\{\frac{\left[q_H\left(\frac{x}{x_1}\right)^{\beta}\left(\frac{b(\theta_1)}{r}+\frac{x_1}{\delta}-I\right)-q_H\left(\frac{\xi}{\Delta q}\right)+(1-q_H)\left(\frac{x}{x_2}\right)^{\beta}\left(\frac{b(\theta_2)}{r}+\frac{x_2}{\delta}-I\right)\right]^{\eta}}{\left[q_L\frac{\xi}{\Delta q}\right]^{\eta-1}}\right\}$$

Subject to:

$$\text{B.78} \qquad \frac{\xi}{\Delta q}\left(\frac{x}{x_1}\right)^{-\beta} - \frac{\alpha x_1}{\delta} + I \geq 0$$

$$\text{B.79} \qquad \frac{\xi}{\Delta q}\left(\frac{x}{x_1}\right)^{-\beta} - \frac{\alpha x_1}{\delta} + I \leq I$$

$$\text{B.80} \qquad I - \frac{\alpha x_2}{\delta} \geq 0$$

The Lagrangian is:

$$\text{B.81} \qquad L = \eta ln\left[q_H\left(\frac{x}{x_1}\right)^{\beta}\left(\frac{b(\theta_1)}{r}+\frac{x_1}{\delta}-I\right) - q_H\left(\frac{\xi}{\Delta q}\right) + (1-q_H)\left(\frac{x}{x_2}\right)^{\beta}\left(\frac{b(\theta_2)}{r}+\frac{x_2}{\delta}-I\right)\right] + (1-\eta)ln\left(q_L\frac{\xi}{\Delta q}\right) + \lambda_1\left(\frac{\xi}{\Delta q}\left(\frac{x}{x_1}\right)^{-\beta} - \frac{\alpha x_1}{\delta} + I\right) + \lambda_2\left(\frac{\alpha x_1}{\delta} - \frac{\xi}{\Delta q}\left(\frac{x}{x_1}\right)^{-\beta}\right) + \lambda_3\left(I - \frac{\alpha x_2}{\delta}\right)$$

The first-order conditions are:

$$\text{B.82} \qquad \frac{\eta q_H}{A}\left(\frac{x}{x_1}\right)^{\beta}\left[-\frac{\beta}{x_1}\left(\frac{b(\theta_1)}{r}+\frac{x_1}{\delta}-I\right)+\frac{1}{\delta}\right] + \left(\frac{\xi}{\Delta q}\beta\left(\frac{x}{x_1}\right)^{-\beta-1}\left(\frac{x}{x_1^2}\right) - \frac{\alpha}{\delta}\right)(\lambda_1-\lambda_2) = 0$$

$$\text{B.83} \qquad \frac{\eta(1-q_H)}{A}\left[\beta\left(\frac{x}{x_2}\right)^{\beta-1}\left(-\frac{x}{x_2^2}\right)\left(\frac{b(\theta_2)}{r}+\frac{x_2}{\delta}-I\right) + \left(\frac{x}{x_2}\right)^{\beta}\frac{1}{\delta}\right] - \frac{\alpha\lambda_3}{\delta} = 0$$

B.84 $x_1(\lambda_2 - \lambda_1) = \lambda_3 x_2$

B.85 $\frac{\xi}{\Delta q}\left(\frac{x}{x_1}\right)^{-\beta} - \frac{\alpha x_1}{\delta} + I = 0$

B.86 $\frac{\alpha x_1}{\delta} - \frac{\xi}{\Delta q}\left(\frac{x}{x_1}\right)^{-\beta} = 0$

B.87 $I - \frac{\alpha x_2}{\delta} = 0$

Where A is equal to $q_H\left(\frac{x}{x_1}\right)^{\beta}\left(\frac{b(\theta_1)}{r} + \frac{x_1}{\delta} - I\right) - q_H\left(\frac{\xi}{\Delta q}\right) + (1 - q_H)\left(\frac{x}{x_2}\right)^{\beta}\left(\frac{b(\theta_2)}{r} + \frac{x_2}{\delta} - I\right)$, the argument of the first logarithm in B.81.

From the equality B.84 three cases follow:

j. $\lambda_3 = 0, \lambda_2 = \lambda_1$
k. $\lambda_3 > 0, \lambda_2 > \lambda_1, \lambda_2 > 0, \lambda_1 > 0$
l. $\lambda_1 = 0, \lambda_2 > 0, \lambda_3 > 0$

Under case j. the problem goes back to the unconstrained one. From case k. follows a contradiction as I goes to zero. Case l. needs to be studied. Considering only B.79 and B.80 as binding we have the following solution:

B.88 $x_1^{FB} - x_1 = \frac{1-q_H}{q_H}(\beta - 1)\left(\frac{x_1}{x_2}\right)^{\beta}(x_2^{FB} - x_2)$

Alternatively,

B.89 $x_2^{FB} - x_2 = \frac{q_H}{1-q_H}\frac{1}{\beta-1}\left(\frac{x_2}{x_1}\right)^{\beta}(x_1^{FB} - x_1)$

Or,

B.90 $\frac{q_H\left(\frac{x}{x_1}\right)^{\beta}(x_1^{FB} - x_1)}{(1-q_H)\left(\frac{x}{x_2}\right)^{\beta}(x_2^{FB} - x_2)} = \beta - 1$

And,

B.91 $\alpha = \frac{\delta I}{x_2^{FB} - \frac{q_H}{1-q_H}\frac{1}{\beta-1}\left(\frac{x_2}{x_1}\right)^{\beta}(x_1^{FB} - x_1)}$

Note that B.89 and B.90 are alternative forms of the same initial equation B.88.

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RINGRAZIAMENTI

In primo luogo, sento di ringraziare il Professor Michele Moretto per avermi guidato, con grande attenzione, dedizione e pazienza, nella stesura di questa tesi.

Uno speciale grazie va poi al Professor Cesare Dosi, che ha altruisticamente dedicato ore del suo tempo al tentativo di chiarire i miei infiniti dubbi sul futuro.

Al di là dell'ambito universitario, rivolgo un pensiero alle persone sulle quali ho sempre potuto contare.

A Papà, che più di chiunque altro è per me un esempio.

A Mamma, che come nessuno sa trasmettermi sicurezza.

Ad Anna e Mariasole, le mie sorelle, che mi offrono prospettive diverse dalle quali guardare il mondo.

E alla nonna Paola, che insieme al nonno Didi ha sempre richiesto che dessi il meglio senza mai farmi mancare tutto il suo affetto.

Non posso poi lasciare da parte Barbara e l'intera famiglia Di Natale, la cui spontanea generosità mi hanno fatto sentire accolto.

Infine, solo per ordine, ci sono Andrea, Gianmarco, Gregorio e Maicol. Gli amici di una vita con i quali non posso immaginare un rapporto più arricchente rispetto a quello che già c'è. Fatto di supporto reciproco, sana competizione, e profondo rispetto.

Grazie, perché senza di voi, nessun traguardo accademico o professionale avrebbe alcun valore.