UNIVERSITÀ DEGLI STUDI DI PADOVA

Dipartimento di Fisica e Astronomia "Galileo Galilei"

Master Degree in Physics

Final Dissertation

Effective theory for fermionic extensions of the Standard Model

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Academic Year 2019/2020

Abstract

Many BSM theories include extensions of the fermionic content of the Standard Model. Typically the new particles are very heavy, so that a smart way to deal with them is by means of the effective theory approach. In this thesis we are interested in new fermions admitting Yukawa-like couplings ΨHq between the Higgs doublet and any Standard Model fermion. After a general overview of the possible extensions compatible with $SU(3)_c \times SU(2)_W \times U(1)_Y$ gauge invariance, we'll focus on a particular vector-like quark of charges $(3_c, 2_W, 7/6_Y)$. For this model we'll first perform a review of the known results present in literature and then move towards the derivation of the effective theory, which will be computed up to 1-loop by means of the functional integral method. Finally by confronting the obtained Wilson coefficients with their experimental constraints we will obtain new bounds on the coupling constants of this model.

Contents

Introduction

The Standard Model of particle physics is one of the best tested theories not only of physics, but of all science. It has successfully passed all the experimental tests of the past fifty years, finding its coronation with the discovery of the Higgs boson in 2012. However there are still several open problems which suggest that the Standard Model is *not* the ultimate theory of particle physics, but more probably just a piece of a broader story. The certainly most known issues are the impossibility to include gravity in a coherent way, the origin of dark matter and the mistery of neutrino masses, but there are also many other questions which has not found yet a satisfactory answer. This has made the research in the field of physics Beyond the Standard Model one of most the active branches in the last years: both experimentally, with the increasing number of proposal for more and more advanced high energy colliders, but also of a more theoretical nature. Among the many theories that aim to extend and solve some of the open problems a recurring presence is that of new, exotic fermions coupling with the Standard Model particles.

The heaviest particle known today is the top quark, for which its big mass $m_t \simeq 173$ GeV [1] makes it natural to guess that the new fermions should be at least heavier than that, placing themselves in the TeV range. No such particles have been discovered yet in high energy colliders, also because of the high suppression in the production of such massive states, so an alternative way to check for the presence of this kind of new physics is by means of an effective theory approach. Effective field theories have been widely used in the past (Fermi theory, in the context of particle physics, is the most known example) and have always provided important insight about the underlying "complete" model. Indeed thanks to them it is possible to make predictions for low energy observables, which are typically precisely measured and any discrepancy between the theoretical and the experimental value put stringent constraints on the model of interest.

This is exactly the procedure we will adopt throughout this work. In chapter 1, after a qualitative discussion about the models that predict this kind of new particles, we will make a brief review of the Standard Model. Employing a phenomenological approach, then, we will see what kind of new fermions can be introduced that both respect $SU(3)_c \times SU(2)_W \times U(1)_Y$ gauge invariance and couple to the SM matter content. Once this classification will be completed a particularly interesting exotic fermion will be picked and studied in detail: a vector-like quark which charges $(\mathbf{3}_c, \mathbf{2}_W, 7/6_Y)$. In chapter 2 we will recap the studies present in literature about this particle, justifying the need of an effective theory approach. After a general review of Effective Field Theories we will derive the tree-level EFT for that model, thanks to which the known bounds on the new coupling constants will be improved. To proceed further we will need some operators that are generated only at 1-loop. The relevant ones will be selected thanks to an estimate that makes use of the powerful method of spurion analysis combined with $\hbar-$counting. Also, in this chapter we will discuss how this model relates to CP violation.

Chapter 3 will be devoted to the computation of the 1-loop operators. We will adopt the functional integral method, using the new technique of Covariant Derivative Expansion to calculate the functional traces. Having computed the Wilson coefficients at 1-loop, we will make use of the Renormalization Group Equation to bring the coefficients to the scale at which the experimental constraints are given. Through this comparison we will be able to obtain new, improved bounds on the couplings of this model that will be summarized in the Conclusions, 4.

Chapter 1

Fermionic extensions

1.1 Motivations

As of today, the Standard Model has passed all the experimental tests to which it has been subjected. Incredibly high precision measurements have been performed, and the result have almost always been in agreement with the theoretical predictions.
Nevertheless there is still a number of open problems that demand for a solution. Many theoretical attempts have been made to solve these issues, and a typical way to do so is to incorporate the Standard Model in a wider scenario. In fact, a way to explain how the SM makes such accurate predictions is that there exist new physics at a scale $\Lambda_{UV} \gg \Lambda_{EW}$, undergoing then some kind of mechanism at lower energies such that it reduces to the ordinary SM enriched however with a set of new particles interacting with the matter content known today. These states are usually very heavy, but lighter ones are possible provided they don't modify too much the precision observables currently measured. In this context fermions are favoured candidates, since chiral symmetries make it possible for them to have smaller masses with respect to the scale Λ_{UV}.

Actually many extensions predict for the new sector the presence of exotic fermions, which can have very different properties with respect to the usual SM ones.
This is the case, for example, for the *neutrino mass* problem. Experimentally the mass of the neutrinos is found to be not null, contrary to the SM prediction. A well known solution that avoids the necessity of an unnaturally small Yukawa coupling is given by the see-saw mechanism, which introduces a right-handed majorana fermion interacting with the Higgs and lepton doublets through a Yukawa term.
Another set of theories in which new fermions are typically found are the ones that aim to solve the *naturalness problem*. One way to avoid fine tunings of the Higgs mass is to introduce new physics around the TeV scale. These theories usually include a large spectrum of new particles, among which also fermions. In many Composite Higgs Models, for instance, a number of vector-like fermions are predicted.
Even solutions to the *strong CP problem* consider the addition of new fermions: other than the Peccei-Quinn solution with the well known scalar axion, mechanisms such as the Nelson-Barr one need the introduction of vectorlike quarks coupling to the Higgs

doublet and the SM quarks.
Finally new fermions are often found also in flavor models, which try to explain the origin of the fermion masses hierarchy, and in supersymmetric extensions of the SM.

From these qualitative examples it's clear that new fermions are a commonly shared feature of BSM theories. Without specifying any UV completion, in this work we will just assume the existence of new fermions at the TeV scale and study their phenomenology. In this way our study can be used as a reference for any UV completion that predicts this kind of particles.
Before starting the quantitative discussion about the possible fermionic extensions, however, it is useful to perform a general review of the main principles that guide the construction of the Standard Model.

1.2 The Standard Model

The Standard Model is a quantum field theory invariant under the gauge group $SU(3)_c \times SU(2)_W \times U(1)_Y$.
Let's start analysing the properties of the particle content under the $SU(2)_W$ group. The fermionic content is chiral, so the left and rights components transform in different representations. We define the quark and lepton left handed doublets to be

$$Q_{L,i} = \begin{pmatrix} u_i \\ d_i \end{pmatrix}, \quad L_{L,i} = \begin{pmatrix} \nu_i \\ \ell_i \end{pmatrix}. \tag{1.1}$$

These are in the two dimensional representation of $SU(2)_W$. The index $i = 1, 2, 3$ is the flavor index and distinguishes between different generations of fermions. The right handed counterparts

$$u_{R,i}, \quad d_{R,i}, \quad \ell_{R,i} \tag{1.2}$$

are in the singlet representation. Note that neutrinos do not have a right handed component.
We have an additional complex scalar field doublet, the Higgs boson doublet

$$H = \begin{pmatrix} H^+ \\ H^0 \end{pmatrix}. \tag{1.3}$$

Finally the gauge bosons live in the adjoint representation of the corresponding gauge group, and these are B_μ for the $U(1)_Y$ group, $W^i_\mu (i = 1, 2, 3)$ for $SU(2)_W$ and $A^a_\mu (a = 1 \ldots 8)$ for $SU(3)_c$. In this way the covariant derivative of a field f is given by

$$D_\mu f = (\partial_\mu - i g_s A^a_\mu t^a_f - i g W^i_\mu t^i_f - i g' Y_f B_\mu) f \tag{1.4}$$

where $t^a_f = (\lambda^a, 0)$ if f is in the triplet or singlet representation of $SU(3)_c$; t^a are the Gell-Mann matrices. Then $t^i_f = (\sigma^i/2, 0)$ for the doublet or singlet representation of

$SU(2)_W$, σ^i being the Pauli matrices. Y_f is called hypercharge and labels the $U(1)_Y$ representation of f.

The transformation properties of the previously introduced fields are summarized in table 1.1.

Field	$SU(3)$	$SU(2)$	$Y_{U(1)}$
Q_L	**3**	**2**	1/6
u_R	**3**	**1**	2/3
d_R	**3**	**1**	-1/3
L_L	**1**	**2**	-1/2
ℓ_R	**1**	**1**	-1
H	**1**	**2**	1/2

Table 1.1: Transformation properties of the Standard Model matter fields.

Now we can introduce the full lagrangian, split in three different parts:

$$\mathcal{L}_{SM} = \mathcal{L}_{bosonic} + \mathcal{L}_{fermionic} + \mathcal{L}_{yukawa} \tag{1.5}$$

- the bosonic part contains the kinetic terms for the gauge bosons and the Higgs, and also the Higgs potential

$$\begin{aligned}\mathcal{L}_{bosonic} = &-\frac{1}{2}\mathrm{tr}G_{\mu\nu}G^{\mu\nu} - \frac{1}{2}\mathrm{tr}W_{\mu\nu}W^{\mu\nu} - \frac{1}{4}\ B_{\mu\nu}B^{\mu\nu}\\ &+ D_\mu H^\dagger D^\mu H + \mu^2 H^\dagger H - \lambda(H^\dagger H)^2\end{aligned} \tag{1.6}$$

  with the field strength for a gauge field f defined as

$$f_{\mu\nu} = \frac{i}{g_*}[D_\mu, D_\nu] = \partial_\mu f_\nu - \partial_\nu f_\mu - i g_* [f_\mu, f_\nu] \tag{1.7}$$

  where $g_* = g, g', g_s$ for W^i_μ, B_μ, A^a_μ;

- the fermionic part is simply given by the sum of the kinetic terms of all the fermions

$$\mathcal{L}_{fermionic} = \bar{Q}_L i D_\mu \gamma^\mu Q_L + \bar{u}_R i D_\mu \gamma^\mu u_R + \bar{d}_R i D_\mu \gamma^\mu d_R + \bar{L}_L i D_\mu \gamma^\mu L_L + \bar{\ell}_R i D_\mu \gamma^\mu \ell_R. \tag{1.8}$$

  Note that explicitling the covariant derivates we can readily see the interactions between the fermions and the gauge bosons of the Standard Model;

- the Yukawa sector is

$$\mathcal{L}_{yukawa} = -\left(\bar{u}_R y_u \tilde{H}^\dagger Q_L + \bar{d}_R y_d H^\dagger Q_L + \bar{\ell}_R y_\ell H^\dagger L_L + h.c.\right) \tag{1.9}$$

  The sum over the flavor indices is implicitly understood. The field $\tilde{H} = i\sigma^2 (H^\dagger)^T$ has been introduced, and $y_{u,d,\ell}$ are generic 3x3 complex matrices in the flavor space.

The Standard Model undergoes a phase transition, called spontaneous symmetry breaking. The Higgs field chooses a value of the vacuum manifold, breaking the $SU(2)_W \times U(1)_Y$ symmetry into an electromagnetic $U(1)_{EM}$. Explicitly, in the unitary gauge the Higgs field can be written as

$$H = \begin{pmatrix} 0 \\ \frac{v+h(x)}{\sqrt{2}} \end{pmatrix}, \quad \lambda v^2 = \mu^2. \tag{1.10}$$

The kinetic term of the Higgs then provides mass to the gauge bosons, and redefining them as the physical fields we have

$$\begin{aligned} W^{\pm\mu} &= \frac{W^{\mu,1} \mp i W^{\mu,2}}{\sqrt{2}}, & m_W^2 &= \frac{g^2 v^2}{4} \\ \begin{pmatrix} Z^\mu \\ A^\mu \end{pmatrix} &= \begin{pmatrix} c_w & -s_w \\ s_w & c_w \end{pmatrix} \begin{pmatrix} W^{\mu,3} \\ B^\mu \end{pmatrix}, & m_Z^2 &= \left(\frac{g^2 + g'^2}{4} \right) v^2, \quad m_\gamma^2 = 0. \end{aligned} \tag{1.11}$$

where the Weinberg angle has been introduced, $c_w = g/\sqrt{g^2 + g'^2}$.
This mechanism provides mass not only to the gauge bosons , but also to the fermions via the Yukawa sector. The mass of the fermions is given by $m_f = \hat{y}_f v/\sqrt{2}$, where $\hat{y}_f$ are the eigenvalues of the complex matrices appearing in the Yukawa sector. To diagonalise these matrices we must perform a different, unitary rotation for every fermion

$$\begin{aligned} &d_L \to D_L d_L, \quad u_L \to U_L d_L, \quad d_R \to D_R d_R, \quad u_R \to U_R u_R \\ &\nu_L \to L_L \nu_L, \quad \ell_L \to L_L \ell_L, \quad \ell_R \to L_R \ell_R. \end{aligned} \tag{1.12}$$

In this way we have given mass to all the fermions, except for the neutrinos. The fact that neutrinos are massless makes it possible to have flavor diagonal interactions between the bosons $W^\pm, A, Z$ and the leptons, for we have made the same rotation between ℓ_L and ν_L.

In the quark sector this is not possible, and the interactions provide a mixing between different generations of quarks[1]. The mixing is regulated by the matrix $V_{CKM} = U_L^\dagger D_L$. In the three flavor case this matrix has three angles and one phase, that phase being responsible for the violation of the CP symmetry. In Wolfenstein parametrization, to order λ^3 the matrix reads

$$V_{CKM} = \begin{pmatrix} 1 - \frac{\lambda^2}{2} & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - \frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1 \end{pmatrix} \tag{1.13}$$

where experimentally $\lambda \simeq 0.22, A \simeq 0.81, \rho \simeq 0.13, \eta \simeq 0.35$.

[1]Note that the mixing is present only for the $W^\pm$ mediated interactions.

For future reference it is convenient to report the complete form of the Standard Model lagrangian density after the symmetry breaking

$$
\begin{aligned}
\mathcal{L}_{SM} = & \sum_{fermions} \bar{\psi}_f (i\partial\!\!\!/ - m_f)\psi_f + \frac{1}{2}\partial_\mu h \partial^\mu h - \mu^2 h^2 - \\
& \frac{1}{4}\left[\partial_\mu A_\nu - \partial_\nu A_\mu - ie(W_\mu^- W_\nu^+ - W_\mu^+ W_\nu^-)\right]^2 \\
& - \frac{1}{4}\left[\partial_\mu Z_\nu - \partial_\nu Z_\mu + ig' c_w (W_\mu^- W_\nu^+ - W_\mu^+ W_\nu^-)\right]^2 \\
& - \frac{1}{2}\left[\partial_\mu W_\nu^+ - \partial_\nu W_\mu^+ - ie(W_\mu^+ A_\nu - W_\nu^+ A_\mu) + ig' c_w (W_\mu^+ Z_\nu - W_\nu^+ z_\mu)\right]^2 - \frac{1}{2}\mathrm{tr} G_{\mu\nu} G^{\mu\nu} \\
& - \frac{g}{\sqrt{2}}(W_\mu^+ \bar{u}_L \gamma^\mu V_{CKM} d_L + W_\mu^+ \bar{\nu}_L \gamma^\mu \ell_L + h.c.) - \sum_{fermions} Q_f \bar{\psi}_f A\!\!\!/ \psi_f \\
& - \frac{g}{c_w} Z^\mu \sum_{fermions} \left((t_f^3 - s_w^2 Q_f)\bar{\psi}_{fL}\gamma^\mu \psi_{fL} - s_w^2 Q_f \bar{\psi}_{fR}\gamma^\mu \psi_{fR}\right) \\
& - \left(1 + \frac{h}{v}\right)^2 \left(m_W^2 W^{+\mu} W_\mu^- + \frac{1}{2} m_Z^2 Z_\mu Z^\mu\right) - \sum_{fermions} \frac{y_f}{\sqrt{2}} h \bar{\psi}_f \psi_f - \lambda v h^3 - \frac{\lambda}{4} h^4
\end{aligned}
\tag{1.14}
$$

where $e = g s_w$ and t_f^3 is the value of the t^3 component of the fermion relative to its $SU(2)$ representation.
For further reading about the Standard Model an excellent reference is [2].

1.3 Exotic fermions

Having reviewed the main features of the Standard Model we will now have a look at its possible fermionic extensions. As a guiding principle we will rely on $SU(3)_c \times SU(2)_W \times U(1)_Y$ gauge invariance, since there's no evidence that this should not be respected in BSM theories.
Experiments suggest that generic flavor and CP violating new physics can be present only at scales $\Lambda_{UV} \gg \Lambda_{EW}$. However non-generic models may admit particles of mass $m \ll \Lambda_{UV}$, even of TeV order. This is the case we will consider throughout this work. We will look for renormalizable BSM lagrangians, since any term of mass dimension greater than four would be suppressed from the new scale Λ_{UV} and so would induce effects too small to be detected.

The first kind of extensions includes models where the new fermions do not couple at all with the SM. This means that they must be gauge singlets and there must be no term in the lagrangian involving them and any other SM fermion or boson. These fermions are "invisible" and the only constraints on these kind of models can come from cosmological consideration. In this work we will mainly discuss bounds coming from low energy experiments where the new particles must leave a detectable trace, and therefore must have some kind of interaction with the SM content; since no information about invisible particles is available here, we're not discussing further these kind of models.

Focusing our attention, then, on a scenario with non-vanishing couplings with the SM, the number of possibilities is rather constrained. At renormalizable level the interactions with the gauge bosons can only appear through the covariant derivative. Indeed gauge invariance implies that different kind of terms must necessarily include $f_{\mu\nu}$ and putting this together with Lorentz invariance, which imposes the presence of at least two fermions, the "minimal" interaction must be of the form $\sim \psi\sigma_{\mu\nu}\chi f^{\mu\nu}$ that has already mass dimension $2 + \frac{3}{2} \cdot 2 = 5$ and is not renormalizable.

Besides interactions with the gauge bosons the only other renormalizable coupling between the new fermion and the SM must be of Yukawa type. On general ground the terms without the Higgs doublet must be of the form $\sim \Psi q$, where Ψ is the new fermion and q any SM fermion. Here Ψ must be "mirror-like" to q, meaning the charges under the gauge group must be the same but for opposite chiralities. This kind of interaction basically represents a mass mixing term.
Terms with only the Higgs doublet $H/\tilde{H}$ and two new fermions must be of the type $\Psi' H \Psi, \Psi' \tilde{H} \Psi$. For these models there are many possibilities; two Dirac fermions (see 1.3.2) can be in an arbitrary $SU(3)_c$ representation and in any $SU(2)_W \times (1)_Y$ such that $\dim_{SU(2)}(\Psi') = \dim_{SU(2)}(\Psi) + 1, Y_{\Psi'} = Y_\Psi \pm \frac{1}{2}$; for two chiral fermions the mechanism is the same as for the SM; one Dirac and one Majorana fermion is also admitted upon specific requirements for the gauge group representations of these particles (the usual for Majorana and consistently for the Dirac to allow the coupling); and so on. A feature of this kind of interactions is the presence of a Z_2 symmetry $(\Psi, \Psi') \to -(\Psi, \Psi')$ which forbids any decay of the new fermions to SM particles. This strongly constraints the models in which this is the only renormalizable interaction to the SM.
We will therefore focus our attention on fermions whose couplings involve both the Higgs doublet and any already present SM fermion:

$$\Delta\mathcal{L}_{BSM} \sim \Psi H q,\ \Psi \tilde{H} q. \tag{1.15}$$

To proceed quantitatively in the classification of the new particles allowing the interactions (1.15) a distinction must be made between two classes of Ψ: it can either be chiral or vector-like (non-chiral) with respect to the SM gauge group. In the following sections we will explore these two possibilities and their compatibility with the experimental results present at today.

1.3.1 Chiral extensions

A chiral fermion is a fermion for which the left and right component behave differently under the gauge group. This kind of particles can acquire mass only through the Higgs mechanism, which is what the new interaction is providing. This means that they should be present already around the EW scale. New physics in that range is ruled out from low energy precision tests and direct searches at LHC, closing any possibility for the realization of these models.

As a concrete example to show how much chiral extensions are bound to be excluded let's consider the addition of a fourth generation of quarks and leptons. To analyse this model we just need to extend the flavor index $i = 1, \dots 3 \to i = 1, \dots 4$ for any flavor multiplet already present in the Standard Model. Now y_u, y_d, y_ℓ will be 4x4 matrices in the flavor space, referring to equation (1.9).

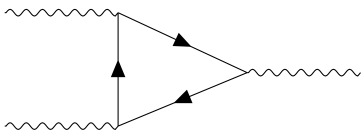


Figure 1.1: Anomaly diagram.

A first important observation that can be made is that we can't introduce a new generation of quarks without a new generation of leptons and vice-versa. This bound comes from anomaly cancellation ([3], [4]). Diagrams like the one in figure 1.1, with two $SU(2)$ gauge bosons and one $U(1)$ gauge boson, would generate local anomalies if the condition

$$\mathrm{Tr}Q = 0 \tag{1.16}$$

is not fullfilled. Q is the electric charge of the particles circulating in the loops, so exploiting the values for the up-type and down-type quarks and for the charged leptons this condition can be written

$$N_c \cdot \left(\frac{2}{3} - \frac{1}{3}\right) \times N^q_{gen} - N^l_{gen} = 0 \to N^q_{gen} = N^l_{gen}. \tag{1.17}$$

The strongest bounds that exclude this extension come from the Higgs boson searches. There are many reviews summarizing the experimental results that lead to this conclusion, as [5]. However it is worth exploring a particular "historical" bound coming from the cross-section for the Higgs production via the gluon-gluon fusion mechanism , computed first in [6]. At the leading order the cross-section for the diagram in figure 1.2 is given by

$$\sigma_{LO}(pp \to h) = \tau_H \frac{d\mathcal{L}^{gg}}{d\tau_H} \cdot G_\mu \frac{\alpha_s^2(\mu_R^2)}{288\sqrt{2}\pi} |\frac{3}{4} \sum_f A^H_{1/2}(\tau_f)|^2 \tag{1.18}$$

where the first part involves the parton distrubution function; then we have the variable $\tau_Q = m_h^2/4m_f^2$, for which $A^H_{1/2} \to 4/3$ when $\tau_f \to 0$ and is null in the opposite limit.
We can safely assume that the particles circulating in the loop coming from the new quark generation (t', b') must be heavier than the top quark, for which already $\tau_f \to 0$. If this wasn't the case, these particles should already have been observed.

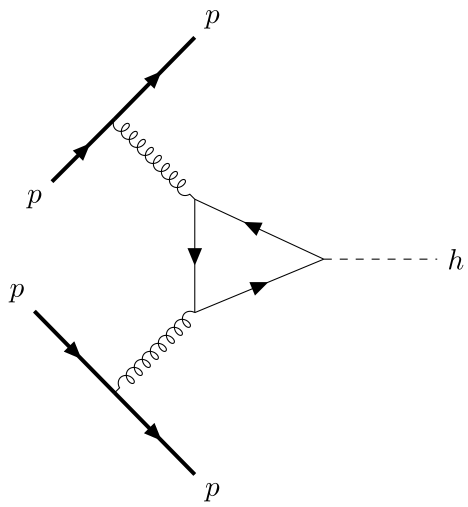


Figure 1.2: Gluon-gluon fusion process for the Higgs production. Only coloured fermions enter in the loop.

In this limit the main contributions to (1.18) come from the t, t', b' quarks and the deviation with respect to the Standard Model prediction reads

$$r = \frac{\sigma_{LO,BSM}(pp \to h) - \sigma_{LO,SM}(pp \to h)}{\sigma_{LO,SM}(pp \to h)} \simeq |\sum_{t,t',b'} 1|^2 - |\sum_{t} 1|^2 = 8. \tag{1.19}$$

This cross-section has been studied in great detail at LHC, and it has been found to be compatible with the Standard Model expectation within 10% [7]. From (1.19) we see that the deviation induced by a fourth generation of flavor would be far too big and incompatible with LHC results.

Other processes that would be significantly affected by this fourth generation are the Higgs decays [5]. The channel $h \to \gamma\gamma$ is very much constrained (4σ) and would be modified by a factor of five with respect to the SM prediction. Also the $h \to \tau\tau$ has a similar constrain and would be hugely modified, as well as the Higgs-strahlung process.

Putting all these informations together, the addition of chiral fermions is severely excluded both from theoretical consistency of the theory and experimental evidences. The most reasonable thing to do is then to consider the addition of non-chiral fermions to the Standard Model particle content.

1.3.2 Vector-like extensions

Vector-like fermions are fermions for which a mass term in the lagrangian is allowed before the spontaneous symmetry breaking, meaning they must be non-chiral under the

SM gauge group. These particles can be of two types: Majorana fermions and Dirac fermions.

Majorana fermions

From the Dirac point of view, a Majorana fermion is a fermion for which the left and right component are present. Basically, a Majorana fermion has the same degrees of freedom a single massive Weyl spinor. The existence of such a fermion is perfectly possible and its non-interacting lagrangian in the two-components notation reads

$$\mathcal{L}_{maj} = i\bar{\chi}\bar{\sigma}^{\mu}D_{\mu}\chi - \frac{m}{2}\left(\chi\chi + \bar{\chi}\bar{\chi}\right) \tag{1.20}$$

with $\chi\chi = \chi^{\alpha}\chi_{\alpha} = \epsilon^{\alpha\beta}\chi_{\alpha}\chi_{\beta}$, $\bar{\chi}_{\dot{\alpha}} = \chi^{\dagger}_{\alpha}$ and $\bar{\sigma}^{\mu} = \left(1, -\sigma^{i}\right)$. The mass m can always be chosen real[2].

Without having introduced yet the interaction, the invariance of the lagrangian (1.20) under the Standard Model gauge group already strongly constraints the possible charges of χ:

1. $U(1)_Y$: since this symmetry is realized as $\chi \to e^{i\alpha(x)Y_{\chi}}\chi$, the mass term imposes $Y_{\chi} = 0$,

2. $SU(2)_W$: only representations of odd dimension are allowed ($R_W \sim \mathbf{1}, \mathbf{3}, \mathbf{5}, \dots$). Indeed the mass term is symmetric, thus the tensor product of the representations must give symmetric singlets;

3. $SU(3)_c$: the representation must be real, so $R_c \sim \mathbf{1}, \mathbf{8}, \dots$.

Once the interaction with the Higgs doublet and the SM fermion χHq is introduced, only two possibilities are left:

$$\chi \sim (\mathbf{1}_c, \mathbf{1}_W, 0_Y), \qquad \chi \sim (\mathbf{1}_c, \mathbf{3}_W, 0_Y). \tag{1.21}$$

The first possibility represents a sterile particle, while the second is in the triplet representation of $SU(2)_W$. In both cases the only allowed coupling is with the lepton doublet

$$\mathcal{L}_{int} = -\tilde{y}\bar{\chi}\tilde{H}^{\dagger}L_L + hc \tag{1.22}$$

and gives origin to the well known see-saw mechanism (type I and III). This extension can explain the smallness of neutrino masses while keeping their Yukawa couplings of order one, but only provided that the mass of the new Majorana fermion is around 10^{14} GeV. This means that any low-energy effect would be too much suppressed to be detectable, since it would be proportional to the inverse of that mass. For this reason we shall not investigate further this model. An excellent comprehensive review about the full theoretical scenario of this mechanism and the status of the experimental searches is given in [8].

[2] Indeed the more general mass term $(m\chi\chi + m^{*}\bar{\chi}\bar{\chi})$ can always be cast in the form of (1.20) upon redefining the field $\chi = \exp(-i\gamma/2)\chi\prime$ for $m = |m|\exp(i\gamma)$.

Dirac fermions

Dirac fermions are fermions whose left and right components behave in the same way under the Standard Model gauge group. This opens up the possibility for the addition of a Dirac mass term[3] to the Standard Model lagrangian in the pre-SSB phase:

$$\mathcal{L}_{VL} = \bar{\Psi}\left(i\not{D} - m\right)\Psi + \mathcal{L}_{int}. \tag{1.23}$$

The fact the mass is not originated by the Higgs mechanism leaves m unconstrained, and as such it can take arbitrarily big values.

At this point Ψ can have any charge of the gauge group, so in order to classify the possibilities the interaction must be introduced. The interaction of interest is again the Yukawa's $\Psi H q$. Since H is an $SU(2)_W$ doublet and q can be either a singlet or a doublet, the possible $SU(2)_W$ representations for the Dirac fermions are $\mathbf{1}, \mathbf{2}, \mathbf{3}$. Furthermore, the Higgs is colorless while q can be a lepton or a quark. This fixes the $SU(3)_c$ representation to be $\mathbf{1}$ or $\mathbf{3}$. The hypercharge Y has no constraints and depends on the specific fermion q involved in the interaction.

The possible models have been listed in two different tables. In the classification procedure, first the $SU(3)_c$ representation has been fixed, distinguishing between coloured and colorless fermions. Then also the $SU(2)_W$ one has been selected. Once the compatible interactions have been written down, the hypercharge was finally fixed by forcing the $U(1)_Y$ invariance. With this procedure, the lists below have been derived. Table 1.2 contains the so called vector-like leptons, table 1.3 the vector-like quarks. In the tables, the $SU(2)_W$-multiplet components of Ψ have been explicited; they have electric charges $Q(X) = 5/3, Q(T) = 2/3, Q(B) = -1/3, Q(Y) = -4/3$ and $Q(N) = 0, Q(E) = -1, Q(F) = -2$.

Ψ^T	$SU(3)$	$SU(2)$	$Y_{U(1)}$	Interaction
(N,E,F)	$\mathbf{1}$	$\mathbf{3}$	-1	$\bar{L}_L H \Psi$
(N,E)	$\mathbf{1}$	$\mathbf{2}$	$-1/2$	$\bar{\ell}_R H^\dagger \Psi$
(E,F)	$\mathbf{1}$	$\mathbf{2}$	$-3/2$	$\bar{\ell}_R \tilde{H}^\dagger \Psi$
N	$\mathbf{1}$	$\mathbf{1}$	0	$\bar{L}_L \tilde{H} \Psi$
E	$\mathbf{1}$	$\mathbf{1}$	-1	$\bar{L}_L H \Psi$

Table 1.2: Vector-like leptons that provide a consistent extension of the Standard Model.

Some of the multiplets are actually copies of the Standard Model ones, with the only difference that these fermions are non-chiral. This means that for these particles also a coupling with its SM relative and without the Higgs, $\bar{q}\Psi$, is possible. These are $\bar{L}_L(N,E)^T, \bar{\ell}_R E$ for the leptons and $\bar{Q}_L(T,B)^T, \bar{u}_R T, \bar{d}_R B$ for the quarks. The new ones, instead, contain the exotic fermions X, Y, F which have unusual electric charges; these particles, therefore, do not mix with the SM fermions after the spontaneous symmetry breaking.

[3] Again, modulo a field redefinition m can always be chosen to be real.

Ψ^T	$SU(3)$	$SU(2)$	$Y_{U(1)}$	Interaction
(X,T,B)	**3**	**3**	$2/3$	$\bar{Q}_L H \Psi$
(T,B,Y)	**3**	**3**	$-1/3$	$\bar{Q}_L \tilde{H} \Psi$
(X,T)	**3**	**2**	$7/6$	$\bar{u}_R H^\dagger \Psi$
(T,B)	**3**	**2**	$1/6$	$\bar{u}_R \tilde{H}^\dagger \Psi, \bar{d}_R H^\dagger \Psi$
(B,Y)	**3**	**2**	$-5/6$	$\bar{d}_R \tilde{H}^\dagger \Psi$
T	**3**	**1**	$2/3$	$\bar{Q}_L \tilde{H} \Psi$
B	**3**	**1**	$-1/3$	$\bar{Q}_L H \Psi$

Table 1.3: Vector-like quarks that provide a consistent extension of the Standard Model.

Some of these fermions have already been studied in detail, in particular the vector-like leptons ([9]) for which flavor changing processes like $\mu^- \to e^-\gamma$, that would be induced from the interactions in table 1.2, put important constraints.
The quark sector, instead, is the one which is more interesting. Here we have bounds both from flavor violation and LHC. The most relevant bounds for flavor violation come from B_s^0, B_d^0, K^0 meson oscillations for the down-quark, and D^0 meson oscillations for the up quarks. In particular the up-quark sector has the weakest experimental constraints, and as such is the perfect environment in which new approaches should be tried. For this reason in the rest of this work the attention will be entirely focused on a particularly interesting and yet not much explored model: the vector-like quark (X,T).

Chapter 2

Vector-like quark (X, T)

In chapter one the possibilities for non-chiral fermionic extensions of the Standard Model have been listed. The new fermion Ψ can have different combination of charges under the SM gauge group, and each representation will provide a different Yukawa interaction.
Even if general ground considerations on this class of models can be done in a qualitative fashion, a quantitative analysis is needed to obtain meaningful physical considerations. This kind of analysis is possible only upon choosing between one of the vector-like fermions listed in the tables 1.2 and 1.3.
In this chapter the focus will be on a particular vector-like quark.
The first part contains an overview of the results from high-energy physics experiments. This will give the possibility to justify the necessity of an effective field theory approach, that will be derived at tree-level and discussed in the second part of this chapter.

2.1 The model of interest

The model that has been selected to be studied is the one involving a new vector-like quark of charges $\Psi \sim (\mathbf{3}_c, \mathbf{2}_W, 7/6_Y)$, so actually a doublet of particles (X, T) with electric charges $Q = +5/3, +2/3$.
The full ultraviolet lagrangian for this model is

$$\mathcal{L}_{UV} = \mathcal{L}_{SM} + \bar{\Psi}\left(i\not{D} - m\right)\Psi - \left(\bar{u}_{Ri} y_i H^\dagger \Psi + \bar{\Psi} H y_i^\dagger u_{Ri}\right). \tag{2.1}$$

This model appears to be simple but already shows some interesting features:

- the new interaction in (2.1) is the only one compatible with the Standard Model gauge group for Ψ having the charges listed above. With this interaction the lepton number conservation is automatically ensured. This is not the case for the baryon number, for which we can recover the symmetry by implying the transformation $\Psi \to \exp(i\alpha/3)\,\Psi$ under $U(1)_B$;

- the number of new parameters introduced is minimal: m can always be chosen to be a real scalar number, while y is generally a 3-dimensional complex vector in the flavor space. However, through a rotation of Ψ, one of the phases can be eliminated. In this way the total number of newly introduced parameters is 6. Looking at the tables 1.2 and 1.3, it's straightforward to see that any other possible fermion must bring at least this number of new variables, if not even more;

- the fact that y is a complex, three dimensional vector has many fundamental implications. First, the two physical phases give a new source of CP violation that adds up to the already present Standard Model one.
Secondly it provides flavor violation even at tree-level, so we are dealing with a model that does not respect the Minimal Flavor Violation assumption. This could be very dangerous since there are strong bounds on flavor violation in the leptonic and down-quark sector. However Ψ does not couple to any lepton nor directly to any down quark, so this model does not suffer particularly from these constraints. Indeed, that's one of the reasons for which this fermion has been selected to be studied;

- after the Spontaneous Symmetry Breaking, the UV lagrangian reads

$$\begin{aligned}\mathcal{L}_{UV}^{SSB} &= \mathcal{L}_{SM} + \bar{X}(i\partial\!\!\!/ - m)X + \bar{T}i\partial\!\!\!/ T \\ &+ \left(\frac{5}{3}\right) e\bar{X}A\!\!\!/ X + \left(\frac{2}{3}\right) e\bar{T}A\!\!\!/ T + \bar{X}Z\!\!\!/ X\frac{g}{c_w}\left(\frac{1}{2} - \frac{5}{3}s_w^2\right) + \bar{T}Z\!\!\!/ T\frac{g}{c_w}\left(-\frac{1}{2} - \frac{2}{3}s_w^2\right) \\ &+ \frac{g}{\sqrt{2}}\left(\bar{X}W\!\!\!/^{+}T + \bar{T}W\!\!\!/^{-}X\right) + g_s\bar{X}A\!\!\!/^{a}t^aX + g_s\bar{T}A\!\!\!/^{a}t^aT \\ &- m\bar{T}T - \frac{(v+h)}{\sqrt{2}}\left(\bar{u}_R yT + \bar{T}y^\dagger u_R\right). \end{aligned} \tag{2.2}$$

The new Yukawa interaction in the last row provides a mass mixing term between the Standard Model up quarks and the T fermion:

$$\mathcal{L}_{UV}^{mm} = -\begin{pmatrix}\bar{u}_L & \bar{T}_L\end{pmatrix}\begin{pmatrix}\frac{vy_u}{\sqrt{2}} & \emptyset \\ \frac{vy}{\sqrt{2}} & m\end{pmatrix}\begin{pmatrix}u_R \\ T_R\end{pmatrix} + hc. \tag{2.3}$$

In principle this term should be diagonalized in order to move to the mass eigenstates of this matrix, slightly splitting X and T's masses.
However we will see later that $|y|v/m \ll 1$. In this limit the mixing can be ignored, as will be proved now, and we can treat the interaction as an usual perturbative term.
Indeed, considering a simplified case with Ψ interacting only with the top quark, the mixing (2.3) is described by a 2×2 matrix that can be diagonalized with two

unitary matrices, $\hat{M} = U_L M U_R^\dagger$, as shown in [10]. The eigenvectors of this matrix are a mixture of t_R, T_R, t_L, T_L regulated basically by one angle

$$\sin\theta_R = \frac{|y|v}{\sqrt{2}}\left(m^2\left(1-\frac{m_t'^2}{m^2}\right)^2+\frac{y^2v^2}{2}\right)^{-1/2}, \qquad \sin\theta_L = \frac{m_t'}{m}\sin\theta_R$$
$$m_t^2 = m_t'^2\left(1+\frac{y^2v^2}{2m^2}\frac{1}{1-m_t'^2/m_T^2}\right), \qquad m_T^2 = m^2\left(1+\frac{y^2v^2}{2m^2}\frac{1}{1-m_t'^2/m^2}\right) \tag{2.4}$$

where the physical, observable parameters are m_t', m_T and $\theta_{R/L}$.
The diagonalization of the mass matrix in the more general case of eq. (2.1) is just computationally more involved, but the qualitative reasoning is the same: in the limit $|y|v/m \ll 1$ the mixing angle becomes null, justifying the perturbative expansion in the calculations that will follow.

The fact that the mass mixing term can be treated as a small perturbation has an important consequence regarding the possible values that y can assume. As quantified in the works [11], [12] the rotation to the mass basis would induce already at tree-level many processes not present in the Standard Model, among which also flavor changing in the neutral Z and higgs currents that are severely suppressed from experimental evidence. These constraints are usually used to set the coupling between the new fermion and the up and charm quarks to zero, for the interaction with the top quark has the weakest constraints. Actually, as we can see from (2.4), the mixing angle is suppressed as m_q/m in the left-handed sector. This would be sufficient to argue that it is not needed to put $y_{1,2} = 0$ to respect flavor violation bounds. The problems may actually come from the right handed sector that is suppressed "only" as $|y|v/m$. However, the peculiarity of the model we've chosen is that it only affects up-quark physics. This means that the bounds on flavor violation coming from Kaon and B-mesons mixing, which are the strongest ones, are avoided.
In [12] a full tree-level analysis of the effects induced by the mixing has been developed. Since it is a quite comprehensive work, the main results that they obtained will be schematically reported here:

- the most stringent constraint comes from the measurements in the D meson sector. The oscillation $D_0 - \bar{D}_0$ gets modified by the FCNC generated from the presence of the new (X, T) quarks, and it has been found out that the most important bound reads

$$|y_1||y_2|\frac{v^2}{2m^2} < 3.2\times10^{-4} \rightarrow |y_1||y_2| < 1.02\times10^{-2}\cdot\left(\frac{m}{\text{TeV}}\right)^2. \tag{2.5}$$

  Later on we will show that a stronger bound can be obtained from the study of the four fermions operators generated at tree-level and 1-loop;

- a milder bound comes from atomic parity violation. The mass basis rotation modifies also the diagonal coupling of the u quark to the Z, which is experimentally well known by the measurements of the weak charge of the ^{133}Cs, [13].
  The derived result is

$$\frac{|y_1|v}{\sqrt{2}m} < 7.8 \times 10^{-2} \rightarrow |y_1| < 4.4 \times 10^{-1} \cdot \left(\frac{m}{\text{TeV}}\right); \tag{2.6}$$

- the modifications of the up-quarks couplings to the Z are constrained also from the measurements of the Z partial widths at LEP [14]. We will perform the same analysis in section 2.3 using the LHC value for m_h [1], not known at the time, so the results will be reported directly there.

These values are the most stringent constraints on the couplings of this model present at today; still, it's definitely not enough to justify the common choice $y_1 = y_2 = 0$.
Later in this work the effective field theory will be derived in a pre SSB form, in which the basis is not the mass one and we can look for complementary bounds on y directly from there.

Direct searches for this kind of vector-like quarks in high-energy experiments have been performed extensively during the last years, particularly in ATLAS and CMS experiments at LHC.
From the lagrangian (2.1) we see that Ψ couples to all the Standard Model gauge bosons. This means that in a pp collission, such as at LHC, an important production mechanism is pair production through the exchange of gluons. Pair production is a particularly interesting mechanism since it does not depend on the strength of the newly introduced Yukawa coupling, making it possible to obtain complementary informations about the mass m and y. In figure 2.1 it's reported a representative diagram for this process, where also the possible decay modes

$$T \rightarrow W^+b, \quad T \rightarrow Ht, \quad T \rightarrow Zt \tag{2.7}$$

are shown, assuming a mixing with the third generation only. In high energy experiments this is the standard assumption.

This very process has been studied at ATLAS, [15], using the LHC data of 2015 and 2016 runs at $\sqrt{s} = 13$ TeV: the results of the analysis are shown in figure 2.2. The analysis has been done separately for every decay channel, which is directly written in the graph, and then the results have been put together. From this figure it is evident that the production cross-section decreases very rapidly with the mass of the new fermion, becoming undetectable for masses above 1.4 TeV. In particular, assuming the branching ratios $\mathcal{B}(T \rightarrow Ht) + \mathcal{B}(T \rightarrow Zt) + \mathcal{B}(T \rightarrow Wb) = 1$, a lower limit on the mass of T has been found:

$$m_T > 1.3 \text{ TeV}. \tag{2.8}$$

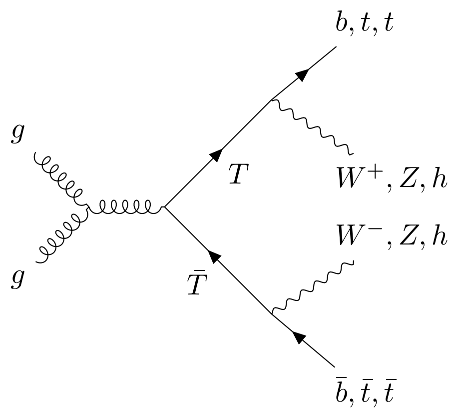


Figure 2.1: Pair production of the T fermion via gluon-gluon fusion, along with the possible decay modes, studied in [15].

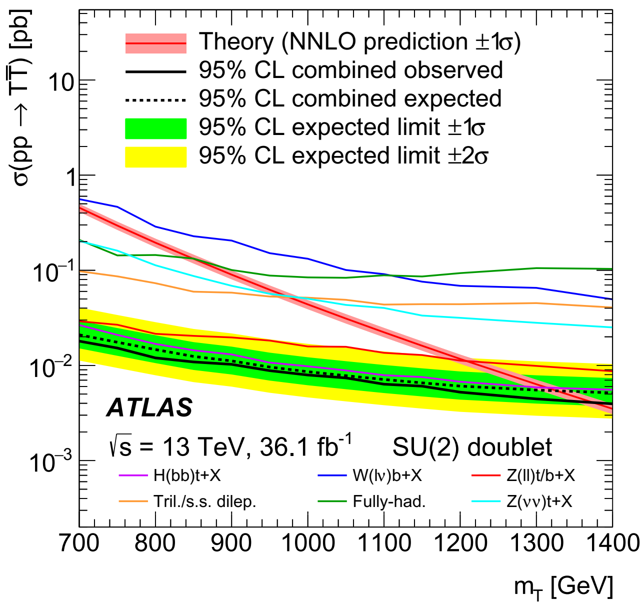


Figure 2.2: Pair production cross-section as a function of T quark's mass found in [15].

Also searches for the associated single production of the T quark have been performed at LHC; however these studies always assume a model for which T is an $SU(2)$ singlet, instead of being pared with X in the (X, T) doublet. This induces a difference in the possible decay channels of T, since in the doublet model also the channel $T \to XW^-$ is present and this would clearly have an impact on the analysis of the experimental data. Even if this may seem an important difference, actually it is not so big: the almost equal masses of X and T give a huge suppression on this decay, such that we can still obtain relevant informations from these experiments.

In [16] the flavor-violating single production process in figure 2.3 has been studied. Again, in this work it is assumed that the new physics couples only to the top quark. The results of this paper are reported in figure 2.4, where the value of the mixing angle is

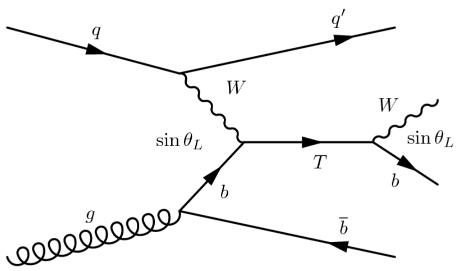


Figure 2.3: Single production of T studied in [16].

drawn as a function of m_T. Also the already known bounds coming from S, T electroweak precision parameters are shown together with the new results, making it clear that these observations are compatible.

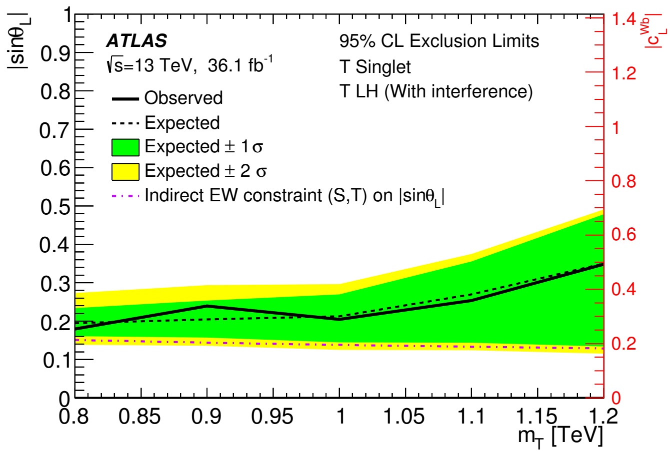


Figure 2.4: Mixing angle vs mass of T found in [16] in the limit of mixing with the third generation only.

We can use this plot to get an approximate bound on y_3, since it's the only parameter which has not yet been constrained. Using the formulae (2.4) with $m_T = 1.3$ TeV and $\sin\theta_R \approx 0.3$[1], we get the reference value $|y_3| \lesssim 0.7$. Although this is not the result of a

[1] We are using an opposite convention with respect to [16], so their $\sin\theta_L$ is actually our $\sin\theta_R$.

precise simulation, it gives an idea about the magnitude of the constraints on the mixing with the third generation.

All in all, for large values of m the production cross-section becomes virtually null and there is no hope for this particle to be directly detected at the energies available today. The bounds coming from single production searches are for very specific models that slightly differ from the one of interest, and anyway give a weak bound which is still not satisfactory. The most interesting constraints come from the study of flavor violation, which however are all mixing-induced tree-level results and basically tell nothing about y_3.

This suggests the use of a completely different kind of approach: look for indirect effects induced at low energy by the presence of this particle. Low energy observables are measured with a very high degree of accuracy and thus can be important sources of information. This strategy is well known and brings us to the realm of Effective Field Theories.

In the next sections the concept of EFT will be introduced and applied to the (X, T) model.

2.2 Effective field theories

The idea behind effective field theories is to be able to perform calculations for a process without using or even knowing the exact "full theory". This allows us to compute experimentally measurable quantities with a finite, improvable degree of accuracy. Indeed an EFT is typically given in terms of a small parameter, δ, for which the expansion is truncated at some order n; in this way the "error" is of order δ^{n+1} and the precision at which observables are computed can be controlled.

In the contex of quantum field field theories, if the EFT is the low-energy limit of a complete ultraviolet theory, as we are interested in, the parameter δ is usually the inverse of a mass scale Λ. At that scale the physics is described by a full microscopic theory that we call fundamental. Moving to an energy scale $E \ll \Lambda$, we should be able to describe the interactions in terms of an effective action. In general this will contain non-renormalizable operators, which will be suppressed by the scale Λ. Therefore in a four-dimensional spacetime we can write

$$S_{EFT}(\Lambda) = \int d^4x \left(\mathcal{L}_{d\leq 4} + \sum_{d>4,\, j} \frac{c_j}{\Lambda^{d-4}} \mathcal{O}_j^{(d)} \right) \tag{2.9}$$

where the index j lists the possible operators of dimension d, each one coming with its dimensionless coefficient c_j, called *Wilson coefficient*.

By dimensional analysis we can see that the amplitude for a dimension d operator is scaling as

$$\mathcal{A} \sim \left(\frac{E}{\Lambda} \right)^{d-4} \tag{2.10}$$

from which it is evident that at small energy scales the contribution from higher-dimension operators is more and more suppressed. For this reason the operators with $d < 4$ are called *relevant*, with $d = 4$ *marginal* and with $d > 4$ *irrelevant*. In this way the expansion in terms of Λ is well defined and we can truncate it at some order $n > 4$ for which the effects of the underlying UV theory become negligible.

The case of our interest is when the microscopic effects are due to a heavy field with mass $m \gg m_Z$, where m_Z is the EW scale at which the light fields are defined. The EFT is built out of the light fields only, so we can determine *a priori* what kind of operators can appear at a definite order of the expansion. This is done imposing the symmetries that we want our effective theory to respect, and looking for the operators of dimension d compatible with these symmetries. In this way the operators expansion is reduced to the determination of the Wilson coefficients. The computation of these is generally very involved, and can be performed in two ways:

1. *amplitudes matching.* This is the most intuitive approach and in this way it's possible to clearly see what is happening.
   In this procedure a particular process that is present both in the UV theory and in the EFT is chosen. Since we want to integrate out the heavy field, the process must involve it in some way that is not as a final state or initial state, i.e. it must appear only via a propagator. The procedure is composed of three steps:

   - first, the amplitude of the process is computed in the effective theory. This will give something depending on the scale Λ and on the Wilson coefficients c_j
   $$\mathcal{A}_{EFT} = f(c_j, \frac{E}{\Lambda}). \tag{2.11}$$
   At tree level f is given by $f_{TL}(c_j, \frac{E}{\Lambda}) = \sum_{d,j} \frac{c_j}{\Lambda^{d-4}} \langle f| \mathcal{O}_j^{(d)} |i\rangle$, where $O_j^{(d)}$ are defined in (2.9). However when considering loop diagrams the amplitude can involve more complicated functions of $c_j, E/\Lambda$;
   - the amplitude is computed in the full UV theory and is expanded in powers of $1/\Lambda$;
   - the results of the two calculations are matched order by order
   $$\hat{\mathcal{A}}_{UV} = \mathcal{A}_{EFT} \tag{2.12}$$
   the hat meaning the expansion in terms of the new physics scale.

   By considering a sufficient number of processes a system of equations will be obtained, from which it possible to read the values for the Wilson coefficients c_j.
   This method is the most straightforward and direct one, although it can be lengthy in some cases and requires special care for the selection of the most convenient basis of operators and observables.

2. *functional integral.* This is the most formal approach and requires some technicalities. However it can be much faster than matching amplitudes, in particular for cases where the contributions for a process are given by many diagrams and it can be difficult to exactly identify and compute each one of these contribution in the right way. In fact the effective operators are directly generated from the evaluation of functional determinants, so it is not required to identify them before starting the calculation.
   The formalism for the computation of the operators generated at 1-loop will be fully developed in chapter 3; however the tree-level result in this approach is simple and well-known. Calling the set of heavy and light fields Φ, ϕ, the effective lagrangian at this order is simply obtained by substituting Φ with its classical solution, i.e. the value for which it satisfies the classical equation of motion:

$$\mathcal{L}^{(0)}_{EFT}(\phi) = \hat{\mathcal{L}}_{UV}(\phi, \Phi_c(\phi)). \tag{2.13}$$

   The hat, again, signifies the expansion in terms of the new physics scale Λ. Indeed the classical solution $\Phi_c(\phi)$ is usually given in terms of non-local operators, that must be expanded in order to get the right Wilson coefficients.

Now that the general formalism of effective field theories is known, the application of all this to the case of our interest is straightforward. The key point is that the number of the possible operators of a given dimension is fixed, and these are determined a priori and independently of the underlying UV theory generating them. Once this set of operators is found, it is possible to make predictions in terms of their Wilson coefficients. Finally, the comparison between experimental results and these predictions constrains the values of the Wilson coefficients.
Since we are proceeding in a top-down approach, the c_j's are given in terms of the new physics parameters. In this way the constraints are translated in bounds on the UV model, that is what we are ultimately interested in.

2.3 Tree-level EFT

Using the concepts of section 2.2 we are now ready to derive the EFT for the (X, T) model. The computation will generate both relevant, marginal and irrelevant operators. New physics effects are mostly included in the last class, which however get less and less important as energy decreases, according to the scaling in equation (2.10). For this reason the expansion will be stopped at dimension six operators.
Furthermore we need to set the limits in which the expansion is defined:

1. $E \ll m$. We are interested in physics at energies well below the mass of the new fermions ($m > 1.31$ TeV). This is the limit on which a general effective field theory relies on;

2. $|y|^2 \ll 16\pi^2$. This limit states that we are in a perturbative regime, so loop expansion is well defined. This is a reasonable assumption that will be also confirmed later;

3. $|y|v/m \ll 1$. This is the null-mixing limit already described in section 2.1, and will be verified *a fortiori*. Note that this limit is perfectly coherent with 1. and 2.. In particular it would make no sense to diagonalize the interaction while stopping the expansion at 1-loop: the order of magnitude of the two processes is the same, so it would be equivalent to evaluate "all the loops" in one side, while stopping at the first order on the other side.

Having set the limits of validity of our effective theory, we can finally begin its computation. We start by substituting Ψ with its classical value in the UV lagrangian. The equations of motion for Ψ can be read from the lagrangian (2.1)[2]:

$$\begin{cases} (i\not{D} - m)\Psi = Hy^\dagger u_R \\ \bar{\Psi}(i\overleftarrow{\not{D}} + m) = -\bar{u}_R y H^\dagger. \end{cases} \tag{2.14}$$

Defining $O = (i\not{D} - m)$, the solution is formally given by $\Psi = O^{-1}Hy^\dagger u_R$. Substituting this in the original lagrangian we get the non-local form:

$$\mathcal{L}_{TL} = \mathcal{L}_{SM} - \bar{u}_R y H^\dagger O^{-1}\left(Hy^\dagger u_R\right). \tag{2.15}$$

To get the effective lagrangian we need to expand (2.15) in powers of $1/m$, coherently with the limits in which we are deriving the effective theory. As such we can write

$$O^{-1} = -\frac{1}{m}\left(1 + \frac{i\not{D}}{m} + \dots\right). \tag{2.16}$$

The 0th order is null upon substitution because of the projectors product $P_L P_R = 0$. Thus the effective lagrangian at tree level is given by:

$$\mathcal{L}^{(0)}_{EFT} = \mathcal{L}_{SM} + \frac{1}{m^2}\bar{u}_R y y^\dagger (i\not{D} u_R) H^\dagger H + \frac{1}{m^2}\bar{u}_R y \gamma^\mu y^\dagger u_R H^\dagger D_\mu H. \tag{2.17}$$

At this point, in order to confront the coefficients of these dimension six operators with their experimental values, a specific basis needs to be chosen. The most common choices are the Warsaw [17] and the SILH [18] bases. In this work we will stick to the last one, since the constraints that are interesting for our model are usually given in that basis. Indeed, that basis has been built specifically for the study of the Higgs physics. The SILH lagrangian is actually just a piece of the most general dimension six lagrangian compatible with the Standard Model gauge symmetries. In fact the full lagrangian[3] can be split into

[2] From now on the compact notation $\bar{u}_{Ri} y_i \equiv \bar{u}_R y$ will be used.

[3] Actually $\mathcal{L}_{SILH}, \mathcal{L}_V$ contain only CP-even operators. The most general lagrangian should include also odd operators, however later on we will show these are not generated at 1-loop, so we are not reporting them.

$$\mathcal{L}_{d=6} = \mathcal{L}_{SILH} + \mathcal{L}_{cc} + \mathcal{L}_{dipole} + \mathcal{L}_V + \mathcal{L}_{4f} \tag{2.18}$$

with

$$\begin{aligned}
\mathcal{L}_{SILH} =& \frac{\bar{c}_H}{2v^2}(\partial_\mu |H|^2)^2 + \frac{\bar{c}_T}{2v^2}\left(H^\dagger \overleftrightarrow{D_\mu} H\right)^2 - \frac{\bar{c}_6 \lambda}{v^2}(H^\dagger H)^3 \\
&+ \left(\frac{\bar{c}_u}{v^2} y_u |H|^2 \bar{u}_R \tilde{H}^\dagger Q_L + \frac{\bar{c}_d}{v^2} y_d |H|^2 \bar{d}_R H^\dagger Q_L + \frac{\bar{c}_\ell}{v^2} y_\ell |H|^2 \bar{\ell}_R H^\dagger L_L + hc\right) \\
&+ \frac{i\bar{c}_W g}{2m_W^2}\left(H^\dagger \sigma^i \overleftrightarrow{D_\mu} H\right) D_\nu W^{i\mu\nu} + \frac{i\bar{c}_B g'}{2m_W^2}\left(H^\dagger \overleftrightarrow{D_\mu} H\right)\partial_\nu B^{\mu\nu} \\
&+ \frac{i\bar{c}_{HW} g}{m_W^2}\left(D_\mu H^\dagger \sigma^i D_\nu H\right) W^{i\mu\nu} + \frac{i\bar{c}_{HB} g'}{m_W^2}\left(D_\mu H^\dagger D_\nu H\right) B^{\mu\nu} \\
&+ \frac{\bar{c}_\gamma g'^2}{m_W^2}|H|^2 B_{\mu\nu} B^{\mu\nu} + \frac{\bar{c}_g g_s^2}{m_W^2}|H|^2 G^{a\mu\nu} G^a_{\mu\nu}
\end{aligned} \tag{2.19}$$

$$\begin{aligned}
\mathcal{L}_{cc} =& \frac{i\bar{c}_{HQ}}{v^2}\bar{Q}_L \gamma^\mu Q_L \left(H^\dagger \overleftrightarrow{D_\mu} H\right) + \frac{i\bar{c}'_{HQ}}{v^2}\bar{Q}_L \sigma^i \gamma^\mu Q_L \left(H^\dagger \sigma^i \overleftrightarrow{D_\mu} H\right) + \frac{i\bar{c}_{Hu}}{v^2}\bar{u}_R \gamma^\mu u_R \left(H^\dagger \overleftrightarrow{D_\mu} H\right) \\
&+ \frac{i\bar{c}_{Hd}}{v^2}\bar{d}_R \gamma^\mu d_R \left(H^\dagger \overleftrightarrow{D_\mu} H\right) + \left(\frac{i\bar{c}_{Hud}}{v^2}\bar{u}_R \gamma^\mu d_R \left(\tilde{H}^\dagger \overleftrightarrow{D_\mu} H\right) + hc\right) \\
&+ \frac{i\bar{c}_{HL}}{v^2}\bar{L}_L \gamma^\mu L_L \left(H^\dagger \overleftrightarrow{D_\mu} H\right) + \frac{i\bar{c}'_{HL}}{v^2}\bar{L}_L \sigma^i \gamma^\mu L_L \left(H^\dagger \sigma^i \overleftrightarrow{D_\mu} H\right) + \frac{i\bar{c}_\ell}{v^2}\bar{\ell}_R \gamma^\mu \ell_R \left(H^\dagger \overleftrightarrow{D_\mu} H\right)
\end{aligned} \tag{2.20}$$

$$\begin{aligned}
L_{dipole} =& \frac{\bar{c}_{uB} g'}{m_W^2} y_u \bar{u}_R \sigma^{\mu\nu} \tilde{H}^\dagger Q_L B_{\mu\nu} + \frac{\bar{c}_{uW} g}{m_W^2} y_u \bar{u}_R \sigma^{\mu\nu} \tilde{H}^\dagger \sigma^i Q_L W^i_{\mu\nu} + \frac{\bar{c}_{uG} g_s}{m_W^2} y_u \bar{u}_R \sigma^{\mu\nu} \tilde{H}^\dagger \lambda^a Q_L G^a_{\mu\nu} \\
&+ \frac{\bar{c}_{dB} g'}{m_W^2} y_d \bar{d}_R \sigma^{\mu\nu} H^\dagger Q_L B_{\mu\nu} + \frac{\bar{c}_{dW} g}{m_W^2} y_d \bar{d}_R \sigma^{\mu\nu} H^\dagger \sigma^i Q_L W^i_{\mu\nu} + \frac{\bar{c}_{dG} g_s}{m_W^2} y_d \bar{d}_R \sigma^{\mu\nu} H^\dagger \lambda^a Q_L G^a_{\mu\nu} \\
&+ \frac{\bar{c}_{\ell B} g'}{m_W^2} y_\ell \bar{\ell}_R \sigma^{\mu\nu} H^\dagger L_L B_{\mu\nu} + \frac{\bar{c}_{\ell W} g}{m_W^2} y_\ell \bar{\ell}_R \sigma^{\mu\nu} H^\dagger \sigma^i L_L W^i_{\mu\nu}
\end{aligned} \tag{2.21}$$

$$\begin{aligned}
\mathcal{L}_V =& \frac{\bar{c}_{2W} g^2}{m_W^2}\left(D^\mu W_{\mu\nu}\right)^i \left(D_\rho W^{\rho\nu}\right)^i + \frac{\bar{c}_{2B} g'^2}{m_W^2}\left(\partial^\mu B_{\mu\nu}\right)\left(\partial_\rho B^{\rho\nu}\right) + \frac{\bar{c}_{2G} g_s^2}{m_W^2}\left(D^\mu G_{\mu\nu}\right)^a \left(D_\rho G^{\rho\nu}\right)^a \\
&+ \frac{\bar{c}_{3W} g^3}{m_W^2}\varepsilon^{ijk} W^i_{\mu\nu} W^{j\nu\rho} W^{k\mu}_\rho + \frac{\bar{c}_{3G} g_s^3}{m_W^2} f^{abc} G^a_{\mu\nu} G^{b\nu\rho} G^{c\mu}_\rho
\end{aligned} \tag{2.22}$$

and $\mathcal{L}_{4f}$ includes all the operators involving 4 fermions, which we are not going to list explicitly here.

The coefficients $\bar{c}_X$ and the Yukawa's $y_{u/d/\ell}$ are matrices in flavor space[4], and the lagrangian has been split in a way that the operators belonging to different classes have different primary effects. Indeed, $\mathcal{L}_{SILH}$ affects mainly Higgs physics, $\mathcal{L}_{cc}$ modifies the couplings of the fermions to the vector bosons, $\mathcal{L}_{dipole}$ includes all the dipole operators and $\mathcal{L}_V$ is made of pure gauge operators that give oblique corrections.

Since we have chosen to stay in this basis, we must convert the tree-level lagrangian (2.17). To do so it is just needed to perform the local field redefinition

$$u_{Ri} \to u_{Ri} - \frac{(yy^\dagger)_{ij}}{m^2} u_{Rj} H^\dagger H. \tag{2.23}$$

From S-matrix equivalence theorem the amplitudes won't be affected by this change, and with this substitution we can rewrite the tree-level result in the SILH format:

$$\begin{aligned} \mathcal{L}^{(0)}_{EFT} =& \mathcal{L}_{SM} + \frac{i}{2m^2}\bar{u}_R y \gamma^\mu y^\dagger u_R (H^\dagger \overleftrightarrow{D}_\mu H) \\ & - \frac{1}{2m^2}|H|^2 \left(\bar{u}_R (yy^\dagger y_u) \tilde{H}^\dagger Q_L + \bar{Q}_L \tilde{H} (y_u^\dagger y y^\dagger) u_R \right). \end{aligned} \tag{2.24}$$

In this form the corresponding coefficients can be read immediately:

$$\bar{c}_{Hu} = \frac{yy^\dagger}{2}\left(\frac{v^2}{m^2}\right), \qquad \bar{c}_u = -\frac{yy^\dagger}{2}\left(\frac{v^2}{m^2}\right). \tag{2.25}$$

Now we can start to analyse the experimental constraints on the tree-level EFT. The operator $\hat{O}_{Hu}$ of (2.24) modifies the coupling of the up-quarks to the Z boson:

$$\hat{O}_{Hu} = \frac{i}{2m^2}\bar{u}_R y \gamma^\mu y^\dagger u_R (H^\dagger \overleftrightarrow{D}_\mu H) \xrightarrow{SSB} \frac{v^2}{2m^2}\frac{g}{c_w}\bar{u}_R y \not{Z} y^\dagger u_R + \ldots \tag{2.26}$$

This has been studied accurately at LEP, where they found [1]

$$R_h^{exp} = \frac{\Gamma(Z \to \text{hadrons})}{\Gamma(Z \to \mu^+\mu^-)} = 20.785 \pm 0.033. \tag{2.27}$$

We can obtain an approximate bound on $|y_{1,2}|$ by computing this observable in our model. At tree-level this is given by

$$\begin{aligned} R_h &= \sum_{i,j=d,s,b,u,c} \frac{\Gamma(Z \to q_i \bar{q}_j)}{\Gamma(Z \to \mu^+\mu^-)} \\ &= R_h^{SM} + N_c \left(\frac{g_{Ru}\frac{|y_1|^2 v^2}{m^2} + g_{Ru}\frac{|y_2|^2 v^2}{m^2} + \frac{|y_1|^4 v^4}{4m^4} + \frac{|y_2|^4 v^4}{4m^4} + \frac{|y_1|^2|y_1|^2 v^4}{2m^4}}{g_{Le}^2 + g_{Re}^2} \right) \end{aligned} \tag{2.28}$$

[4]The notation here is a bit sloppy, but for the sake of clarity all the coefficients have been moved in front of the operators omitting the flavor indices.

where g_{Ru}, g_{Re}, g_{Le} are the SM coupling of the Z to the up-quarks and the leptons and can be read off from (1.14). The comparison with the experimental result at 2σ constraints the parameters to lie in circles whose radius grows with m, as depicted in figure 2.5. On the axes the bounds read

$$|y_1| < 5.4 \times 10^{-1} \cdot \left(\frac{m}{\mathrm{TeV}}\right), \qquad |y_2| < 5.4 \times 10^{-1} \cdot \left(\frac{m}{\mathrm{TeV}}\right). \tag{2.29}$$

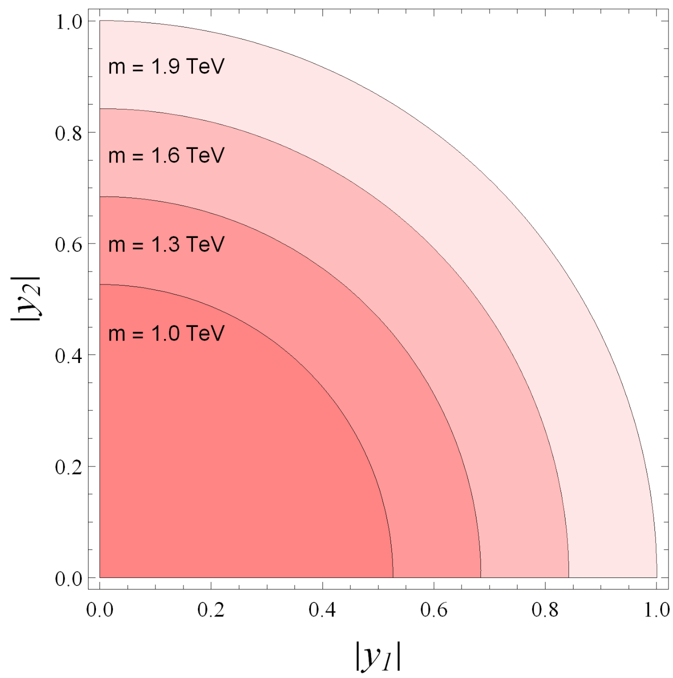


Figure 2.5: Regions allowed by R_h^{exp} for the new couplings.

A more sophisticated analysis has been performed in the work [19], in which they found $-0.008 < \bar{c}_{Hu(11)} < 0.02, -0.01 < \bar{c}_{Hu(22)} < 0.02$. This translates into the bounds

$$|y_1| < 4.4 \times 10^{-1} \cdot \left(\frac{m}{\mathrm{TeV}}\right), \qquad |y_2| < 4.4 \times 10^{-1} \cdot \left(\frac{m}{\mathrm{TeV}}\right) \tag{2.30}$$

which are slightly better than the results (2.29) of our raw analysis, as expected.
The interaction (2.26) generates also four fermions operators inducing $\Delta F = 2$ transitions. Indeed integrating out at the lowest order the Z boson from the diagram in figure 2.6 we obtain an effective vertex in the lagrangian:

$$\mathcal{L}_{EFT}^{(0)} \supset \frac{1}{2}\frac{v^4}{4m^4}\frac{g^2}{c_w^2}\frac{1}{m_Z^2}\left(\bar{u}_R y \gamma^\mu y^\dagger u_R\right)\left(\bar{u}_R y \gamma_\mu y^\dagger u_R\right) = \frac{1}{2}\frac{v^2}{m^4}\left(\bar{u}_R y \gamma^\mu y^\dagger u_R\right)\left(\bar{u}_R y \gamma_\mu y^\dagger u_R\right). \tag{2.31}$$

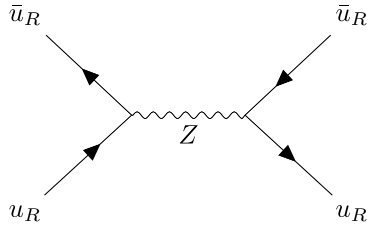


Figure 2.6: Diagram generating the effective four fermions interaction (2.31).

The constraints on this operator can be found in the work [20], which uses the data coming from $\Delta F = 2$ processes involving B_s^0, B_d^0, K^0 and D^0 mesons oscillations. The strongest constrain on (2.31) is on the mixing between the u and c quarks and reads $c^u_{(1212)} < (28_{\text{Re}}, 0.83_{\text{Im}}) \times 10^{-8}$ TeV^{-2}. By confronting with our coefficient we obtain the bounds

$$\begin{aligned}
&\text{Re}\left\{(y_1 y_2^*)^2\right\} \frac{v^2}{2m^4} < 28 \times 10^{-8} \text{ TeV}^{-2} \to |y_1||y_2|\sqrt{|\cos 2(\phi_1 - \phi_2)|} < 3.0 \times 10^{-3} \left(\frac{m}{\text{TeV}}\right)^2 \\
&\text{Im}\left\{(y_1 y_2^*)^2\right\} \frac{v^2}{2m^4} < 8.3 \times 10^{-9} \text{ TeV}^{-2} \to |y_1||y_2|\sqrt{|\sin 2(\phi_1 - \phi_2)|} < 5.2 \times 10^{-4} \left(\frac{m}{\text{TeV}}\right)^2
\end{aligned} \tag{2.32}$$

These bounds are stronger than the ones (2.5) from the work [12]. This is due to the fact that in that paper they did not use the data about CP violation in $D^0 - \bar{D}^0$ mixing. Indeed in the last years the measurements in the D mixing sector have become more accurate, and this is the reason for which our result (2.32) is more tight.

The operator $\hat{O}_u$ of the tree-level EFT (2.24) can generate $\Delta F = 1$ and $\Delta F = 2$ transitions in the Higgs mediated processes. However these will not improve the bounds obtained from $\hat{O}_{Hu}$, as we will show now.

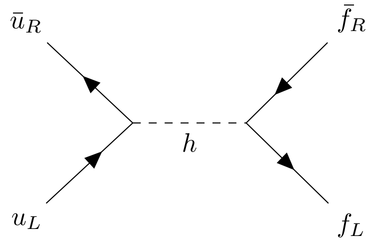


Figure 2.7: Diagram generating a $\Delta F = 1$ process. The u-quark vertex is given by $\hat{O}_u$, the other by a usual SM Yukawa coupling.

The $\Delta F = 1$ processes are given by diagrams like the one in figure 2.7, where the u-quark vertex is given by $\hat{O}_u$ and the other is a usual Yukawa coupling of a SM fermion. If we integrate out the virtual Higgs boson we are left with an effective four fermion $\Delta F = 1$

operator of the form $\bar{u}_R u_L \bar{f}_R f_L$ with coefficient $c_{ijkl}^{\Delta F=1} = \frac{(yy^\dagger y_u)_{ij} v^2}{4\sqrt{2}m^2} \cdot \frac{1}{m_h^2} \frac{(y_f)_{kl}}{\sqrt{2}}$ (obtained for example from amplitudes matching), and is clearly diagonal in the indices (kl). The bounds present in literature for the $\Delta F = 1$ operators usually constrain m to be in the TeV range; this translated on $c^{\Delta F=1}$ implies $\frac{\sqrt{(yy^\dagger y_u) y_f}}{m} \cdot 7 \times 10^{-1} \lesssim 1$ TeV^{-1}, which is always satisfied using the known bounds on y from the $\hat{O}_{Hu}$ analysis (2.30), (2.32) and from direct searches ($y_3 \lesssim 0.7$), and thus brings no new information.
The $\Delta F = 2$ processes are generated when in the diagram of figure 2.7 we use two $\hat{O}_u$ vertexes mediated by the Higgs boson. In this case the coefficient of the effective four fermion operator reads $c_{ijkl}^{\Delta F=2} = \frac{(yy^\dagger y_u)_{ij}(yy^\dagger y_u)_{kl}}{32} \frac{v^4}{m^4} \frac{1}{m_h^2}$. For the $c - u$ mixing we have $c_{1212}^{\Delta F=2} = (y_1 y_2^*)^2 y_c^2 \frac{v^2}{32\lambda m^4}$, which is basically the coefficient of (2.31) multiplying the factor $\frac{y_c^2}{16\lambda}$. This factor gives a $\sim 10^{-4}$ suppression to the coefficient, meaning that the best bound coming by $\Delta F = 2$ transitions is still the one from $\hat{O}_{Hu}$ (2.32).

In order to proceed further in the determination of new constraints on this model we should carry on and look for the next terms in the effective field theory. This means we should start the derivation of the operators generated at 1-loop.
Before even introducing the formalism needed for the loop computations, it's important to first understand what terms will really be relevant and could bring improved bounds with respect to the ones already derived. Indeed an enormous quantity of operators will be generated, and the mechanical calculation of all of them would be meaningless. To understand what will be the relevant terms, an estimate of the order of magnitude of the coefficients must be given. This can be performed in two ways:

1. diagrammatically. With this method the operation is easily done simply by using the rules of the UV lagrangian (2.1) and drawing the diagram that generates the concerned operator. Apart from numerical coefficients of order one, the couplings appearing in the vertexes are the ones that will be found in the coefficient. Putting this together with the usual loop suppression factor $1/16\pi^2$ and placing enough powers of m (that is, the only relevant UV scale in this model) to correct the dimensionality of the lagrangian, the rough estimate of the coefficient is given;

2. by combining the $\hbar$-counting with the spurion analysis.
   The $\hbar$-counting basically consists in checking that the dimensionality of the term of interest is correct. In fact, the condition $[S] = \hbar$ together with $c = 1$ imposes the dimension of the scalar fields to be $[\phi] = \hbar^{1/2}\ell^{-1}$ and of the fermionic ones to be $[\psi] = \hbar^{1/2}\ell^{-3/2}$. From this it's possible to read that the dimensions of the gauge couplings and of the Yukawa couplings must be $[g_*] = [y_*] = \hbar^{-1/2}$, while the Higgs quartic interaction must have $[\lambda] = \hbar^{-1}$. The last piece of information comes from the fact that the loop expansion is actually an expansion in $\hbar$, and as such each loop factor brings $\hbar/16\pi^2$. In this way, depending on the kind of the fields appearing in the operator, it's possible to determine the number of couplings that must enter in the coefficient.

As an example, let's consider a four fermions operator. The four fermionic fields bring $\hbar^2$. If the operator is generated at 1-loop then an additional $\hbar$ is present, making the term of dimension $\hbar^3$. This implies that the coefficient must go as $\hbar^{-2}$, and this is possible for $y_*^n g_*^m \lambda^k$ with $n + m + k/2 = 4$.

This method alone, however, does not fix the structure of the coefficient. Indeed, this must be integrated with the spurion analysis.
The concept of spurion analysis is simple: it consists in promoting the coupling constants to fields that transform in a very specific way, in order to have an additional symmetry in the lagrangian. The coefficient of the operator of interest, then, must respect this new symmetry and as such is very much constrained.
For example, by promoting the Standard Model leptons' Yukawa to a field transforming as $U_L y_\ell U_\ell^\dagger$ we get an additional $U(3)_L \times U(3)_R$ global symmetry: $\bar{L}_L H y_\ell \ell_R \to \bar{L}_L H U_L^\dagger U_L y_\ell U_\ell^\dagger U_\ell \ell_R = \bar{L}_L H y_\ell \ell_R$. In this way every operator containing L_L, ℓ_R must have a coefficient respecting this symmetry, and this together with the informations from the $\hbar$-counting basically fixes the term.

The spurion analysis relevant for our model concerns the Yukawa couplings; in this case the couplings transformations are

$$y_\ell \to U_L y_\ell U_\ell^\dagger, \quad y_u \to U_u y_u U_Q^\dagger, \quad y_d \to U_d y_d U_Q^\dagger, \quad y \to U y \tag{2.33}$$

with the fields transforming as

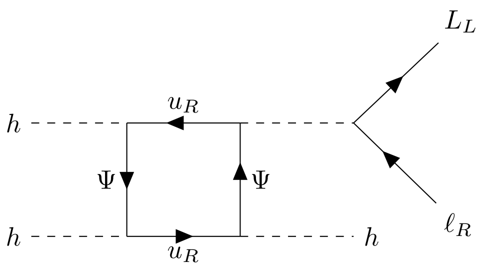
$$Q_L \to U_Q Q_L, \quad u_R \to U_u u_R, \quad d_R \to U_d d_R, \quad L_L \to U_L L_L, \quad \ell_R \to U_\ell \ell_R. \tag{2.34}$$

Once the coefficient has been fixed, apart from order one factors, with either one of the two methods above, the numerical estimate is given by substituting y with the saturated bounds (2.30). If the constraints present in literature on this coefficient are weaker than its estimated value, no new information is added and its exact computation would be pointless.

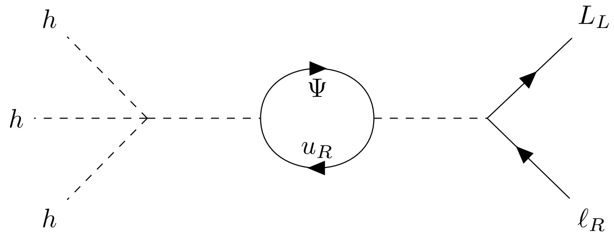
Proceeding in this direction, we should give an estimate of all the coefficients of the general dimension six lagrangian (2.18). These must be generated from processes in which Ψ is present, and this already rules out many of these operators. In fact the new interaction involves only u_R, Ψ and H and the new fermion can only be present as virtual. This is sufficient to exclude most of the operators involving leptons, since there is no way they can be generated at 1-loop.
Anyway a more specific analysis is necessary, so in the rest of this section every term of (2.18) will be separately analysed and depending of the outcome of the order-of-magnitude estimate the operators worth to be derived will be chosen. Particular attention will be paid on the possible improvements of the bounds on y_3, since for the moment is the less constrained quantity.

- $\mathcal{L}_{SILH}$: the terms in this lagrangian have relevant constraints since they modify Higgs physics. For the flavor invariant operators, spurion analysis allows the coefficients to be of the form $|y|^p\lambda^q g_*^r(\mathrm{tr} y_f^\dagger y_f)^s, y^\dagger y_u y_u^\dagger y, \ldots$. A direct estimate of $|y| = \sqrt{|y_1|^2+|y_2|^2+|y_3|^2}$ has not yet been given, so it's definitely worth deriving every single one of the terms with this structure.
  Now let's consider the flavor violating $\hat{O}_u, \hat{O}_d, \hat{O}_\ell$. $\hat{O}_u$ is already present at tree-level, so the bounds from the 1-loop analysis would be of the same kind but weaker. For $\hat{O}_d, \hat{O}_\ell$ by $\hbar$-counting we get that the combination of couplings in $\bar{c}_d, \bar{c}_\ell$ must have dimension $\hbar^{-2}$, since the $\bar{c}_d, \bar{c}_\ell$ are defined to be adimensional. In fact if the operators are generated at one loop then it must be $\bar{c}_{d,\ell} \sim \frac{v^2}{m^2} \times \frac{\hbar}{16\pi^2} \times [\hbar]^{-2}$, where $[\hbar]^{-2}$ denotes the function of the couplings with dimension $\hbar^{-2}$. Here we used $[v^2] = \hbar^1$, obtained from $\hbar^0 = [m_h^2] = [\lambda v^2]$.
  From now on we will go back to natural units where $\hbar = 1$, but the reasoning just exposed still holds. Exploiting the spurion analysis, the two potentially relevant possibilities for the coefficient of the operator involving leptons are $\bar{c}_\ell \sim \frac{v^2}{m^2}\frac{1}{16\pi^2}|y|^4, \frac{v^2}{m^2}\frac{1}{16\pi^2}|y|^2\lambda$. These come from the non 1PI diagrams of figure 2.8: it's just either one of the 4-Higgs operators described above for which an external leg is attached to the SM Yukawa vertex y_ℓ or a process already present in the SM in which the self-energies corrections are applied to the Higgs propagator. Since these operators are not 1PI they are not directly generated from the EFT computation, but can emerge in the process of change of basis (in particular eliminating some two Higgs operators as $D^2H^\dagger D^2 H$, see section 3.2.3). Anyway we can use the bound on $\bar{c}_\ell$ in [21], obtained by looking at the modification induced by this operator to the Higgs couplings to the fermions, vector bosons and itself. This reads $\bar{c}_\ell \lesssim 10^{-1}$. The strongest inequality we can get is $10^{-3}\frac{|y|^4}{m^2(\mathrm{TeV})} < 10^{-1}$ which doesn't give an interesting bound on $|y|$.
  The situation for $\hat{O}_d$ is analogous, but there is an additional possibility: also the non-trivial flavor structure $\bar{c}_d \sim \frac{v^2}{m^2}\frac{1}{16\pi^2}y_u^\dagger y y^\dagger y_u$ is admitted. This case is similar to the one at the beginning of this section for $\hat{O}_u$ that was however generated at tree level, meaning the constraints now will be even weaker. Indeed with this structure the final term in front of the operator $\hat{O}_d$ is $y_d\bar{c}_d/v^2 \approx (y_b y_t^2 V_{tb} y^2/m^2)\cdot 10^{-2} \sim 10^{-4}(\ \mathrm{TeV}^{-2})$, while the typical values in literature are $\sim 10^{-2}(\ \mathrm{TeV}^{-2})$, so no new informations can be inferred from this coefficient.
  Summing up, all the operators in $\mathcal{L}_{SILH}$ except $\hat{O}_u, \hat{O}_d, \hat{O}_\ell$ are worth to be studied and will be derived in the next chapter.

- $\mathcal{L}_{cc}$: in this sector the situation for the leptonic operators is similar as for $\mathcal{L}_{SILH}$: $\bar{c}_{HL,HL',H\ell} \sim \frac{v^2}{m^2}\frac{1}{16\pi^2}\left(|y|^4, |y|^2\lambda, |y|^2 y_\ell^2, |y|^2 g_*^2, \ldots\right)$. These operators are not 1PI, and their flavor structure is diagonal. The constraints in literature [19] are at best of order 10^{-3} and thus in the less suppressed case give, exploiting the analysis of the $\mathcal{L}_{SILH}$ discussion, $\frac{|y|^4}{m^2(\mathrm{TeV})} \lesssim 1$. Better bounds will be provided by $\mathcal{L}_{SILH}$. This analysis holds also for $\bar{c}_{Hd}$, for which the situation is the same (even the



(a) 4-Higgs irreducible diagram with attached a Yukawa vertex y_ℓ to one of the external Higgs.



(b) Diagram already present in the SM with the 1-loop correction to the Higgs propagator coming from Ψ.

Figure 2.8: Diagrams generating the operator O_ℓ.

constraints are of the same order).
The operator $\hat{O}_{Hu}$ is already present at tree-level, and shall not be derived at 1-loop.
The other operators are responsible for $\Delta F = 1$ transitions at tree-level, and thus can be potentially relevant. The only possible flavor violating coefficients are $\bar{c}_{HQ}, \bar{c}'_{HQ} \sim \frac{v^2}{m^2}\frac{1}{16\pi^2} y_u^\dagger y y^\dagger y_u$ and $\bar{c}_{Hud} \sim \frac{v^2}{m^2}\frac{1}{16\pi^2} y y^\dagger y_u y_d^\dagger$. However we already stated that constraints from $\Delta F = 1$ transitions are weak, so the 1-loop study of these effects wouldn't give any new information. Indeed, in [22] the best experimental bound on FV is $|\bar{c}_{HQ,32}|/v^2 < 1.4\times 10^{-2}$ TeV^{-2}. In our model $|\bar{c}_{HQ,32}|/v^2 \sim |y_t y_c y_3 y_2^*|/16\pi^2 m^2$, so we get the ridiculous bound $|y_3 y_2| \lesssim 10^4 \left(\frac{m}{\text{TeV}}\right)^2$. Deriving this operator would be useless, since we already got stronger bounds from tree-level considerations.
In conclusion, no operator in $\mathcal{L}_{cc}$ is worth to be derived at 1-loop.

- $\mathcal{L}_{dipole}$: this lagrangian includes (chromo)electric and (chromo)magnetic dipole operators.
  In this term the operators involving leptons can, again, only have the flavor invariant invariant structure as in $\mathcal{L}_{SILH}, \mathcal{L}_{cc}$ and thus are not interesting. Indeed, as shown later in section 2.4, the potentially relevant leptonic electric dipoles are related to CP violation that can occur only at three or four-loops.

A simple spurion analysis shows that also the ones involving down quarks must have that same structure. This means that the only possibly relevant dipoles are the ones with up quarks, for which flavor violation is allowed: $\bar{c}_{uV} \sim \frac{m_W^2}{m^2}\frac{1}{16\pi^2}yy^\dagger$. From the analysis in [23], the coefficient of chromo-magnetic dipole operator of the top quark has been found to be $|\tilde{\mu}_t|m_t < 0.05$. After SSB this gets contributions from $\frac{m_t v}{m_W^2}\bar{c}_{uG,33} \sim |y_3|^2 m_t^2/16\sqrt{2}\pi^2 m^2$, from which we can read $|y_3| \lesssim 2.1\left(\frac{m}{\text{TeV}}\right)$. This is not an improvement with respect to the result from direct searches $|y_3| \lesssim 0.7$.
Also bounds on the flavor violating component are present in literature, particularly for the top mixing. However, the work [22] provides $|\bar{c}_{uW,32}| < 0.15$, which gives $|y_3 y_2| \lesssim 24\left(\frac{m}{\text{TeV}}\right)^2$ and again is not an improvement.
Because of these estimates, then, no dipole operator will be computed at 1-loop.

- $\mathcal{L}_V$: the "pure gauge" term. These operators involve only the gauge fields and give oblique corrections to their propagators. These corrections are tightly constrained, since they can be linked to the Y, W, Z parameters ([24]) that are severely bounded from the electroweak precision tests.
  Diagrammatically these terms are given by self-energy diagrams that involve only the couplings of the heavy fermion with the gauge bosons, so they are always present. As such, the coefficient of these operators will not involve y. This is extremely useful because in this way it's possible to obtain bounds on m that are independent on the new Yukawa coupling.
  Needless to say, these operators are fundamental and will be derived in the next chapter. In particular the (X,T) model is charged both under $SU(3)_c, SU(2)_W$ and $U(1)_Y$, so every term of (2.22) will be generated.

- $\mathcal{L}_{4f}$: this term hasn't been written explicitly because of its length, but it's actually very important.
  It contains the operators involving four fermions, so of the form $\bar{f}_{i,L/R} f_{j,R/L} \bar{f}_{k,L/R} f_{l,R/L}$ and $\bar{f}_{i,L/R}\gamma^\mu f_{j,L/R}\bar{f}_{k,R/L}\gamma_\mu f_{l,R/L}$ with all the possible colour index contractions and the possible insertions of the generators $\lambda^a/2, \sigma^i/2$ between the fermions.
  The coefficients c^X_{ijkl} are typically much constrained (note that these are not defined in an adimensional way, $\mathcal{L}_{4f} \supset c^X_{ijkl}\hat{O}^X_{ijkl}$). They can have arbitrary flavor indices, therefore generating $\Delta F = 2$ processes which have been experimentally much investigated, as we've seen in the tree-level analysis.
  In our model the generated 1PI operators are $\hat{O}^{uf}_{ijkl} = \bar{u}_{i,R}\gamma^\mu u_{j,R} f_{k,L/R}\gamma_\mu f_{l,L/R}$, and from spurion analysis the possible flavor violating coefficients must be $c^{uf}_{ijkl} \sim (yy^\dagger)_{ij}(y_f y_f^\dagger)_{kl}/m^2$. From works like [20] we see that the constraints are strong on the real and on the imaginary part of these coefficients. This makes it worth the derivation of these terms, so also this lagrangian will be computed.

All in all, thanks to this analysis we have identified the operators which could give new, interesting bounds. These are the ones in $\mathcal{L}_{SILH}$ (except $\hat{O}_u, \hat{O}_d, \hat{O}_\ell$), $\mathcal{L}_V$, $\mathcal{L}_{4f}$. At this point everything is ready to perform the exact calculation at 1-loop, that will be the subject of the next chapter.

2.4 CP violation

Before jumping in the 1-loop computation of the EFT, let's justify an assumption we made in section 2.3.
In writing the most general dimension 6 lagrangian (2.18) the CP-odd operators of $\mathcal{L}_{SILH}, \mathcal{L}_V$ have been omitted upon the claim that they are not generated at 1-loop. Indeed, thanks to the $\hbar-$counting and the spurion analysis tools introduced in section 2.3, we can prove this statement in a quantitative way.

First let's review how CP acts. An explicit form of the charge-conjugation and the parity operators' action on fermions is given by

$$\mathcal{C} : \psi \to i\gamma^2\gamma^0\psi^*, \quad \mathcal{P} : \psi \to \gamma^0\psi. \tag{2.35}$$

so a fermionic bilinear transforms as[5]

$$\bar{\psi}_i\chi_j \xrightarrow{\mathcal{CP}} \bar{\chi}_j\psi_i. \tag{2.36}$$

If a model contains this kind of interactions the check for the presence of CP violation is easily done: calling A_{ij} the coupling and X a generic CP-invariant scalar field, the lagrangian transforms as

$$\begin{aligned}\left(\bar{\psi}A\chi + \bar{\chi}A^\dagger\psi\right) X &\xrightarrow{\mathcal{CP}} \left(\bar{\chi}A^T\psi + \bar{\psi}_(A^\dagger)^T\chi\right) X \\ &= \left(\bar{\psi}(A^*)\chi + \bar{\chi}(A^*)^\dagger\psi\right) X.\end{aligned} \tag{2.37}$$

From this it is clear that the general requisite for the CP symmetry to be violated is $A \neq A^*$.
This is indeed the case for the Standard Model, where the Yukawa interactions are responsible for the violation since y_u, y_d, y_ℓ are general complex matrices.

In the (X, T) model the coefficient y is a complex vector, meaning that CP violation is allowed in principle. The lowest order CP violating and flavor invariant observable θ can be found from the condition $\theta \neq \theta^\dagger$[6], where θ must be a combination of y, y_u, y_d, y_ℓ that must be invariant when the coupling constants are promoted to spurion fields.
In pure Standard Model that observable is proportional to the Jarlskog invariant [25] and is given by $C = \det[y_u y_u^\dagger, y_d y_d^\dagger]$, however since now also y is present new combinations

[5]Proof: $\bar{\psi}_i\chi_j \to (i\gamma^2\psi_i)^T i\gamma^2\chi_j^* = -\psi_i^T\gamma^{2T}\gamma^0\gamma^2\chi_j^* = -\psi_i^T\gamma^0\chi_j^* = \bar{\chi}_j\psi_i$, where it has been used $\{\psi^\alpha, \chi^{*\beta}\} = 0$ and $(\gamma^2)^2 = -\mathbb{1}$.

[6]Since θ must be a scalar the CP violating condition $\theta \neq \theta^*$ is the same as $\theta \neq \theta^\dagger$.

are possible. Indeed it's easy to see that the minimal CP-odd quantity involving y is given by

$$\theta = y^\dagger \left[y_u y_u^\dagger, y_u y_d^\dagger y_d y_u^\dagger \right] y. \tag{2.38}$$

In the interaction basis this quantity can be written as

$$\begin{aligned} \theta &= y^\dagger \hat{y}_u \left[\hat{y}_u^2, V_{CKM} \hat{y}_d^2 V_{CKM}^\dagger \right] \hat{y}_u y \\ &= \sum_{i<j} \hat{y}_{u,i} \hat{y}_{u,j} (\hat{y}_{u,i}^2 - \hat{y}_{u,j}^2)(y_i^* y_j B_{ij} - y_i y_j^* B_{ij}^*) \end{aligned} \tag{2.39}$$

where $B = V_{CKM} \hat{y}_d^2 V_{CKM}^\dagger$. Using Wolfenstein parametrization for the CKM matrix, an estimate of this quantity (assuming $y_i y_j^* - y_i^* y_j$ is approximately the same for all the three generations) is given by

$$\theta \approx y_t^3 y_c y_b^2 A \lambda^2 (y_i y_j^* - y_i^* y_j) \sim 10^{-8} (y_i y_j^* - y_i^* y_j). \tag{2.40}$$

As we can see θ is very small (Jarlskog invariant is $\sim 3 \times 10^{-5}$), meaning CP violation in this model is highly suppressed.

This result can be used to determine at how many loop the electron dipole moment operator is generated. This is a quantity that is experimentally very much constrained, so it's important to check if the theory prediction is consistent with the data.
With a slightly improper notation, this operator is given by

$$\hat{O}_{EDM} = i \frac{c}{m^2} \bar{\ell}_R \gamma^5 \sigma^{\mu\nu} H^\dagger L_L F_{\mu\nu}. \tag{2.41}$$

From spurion analysis and NDA the CP-odd coefficient c can only be [7] $c \sim e y_\ell \theta$, up to $\mathcal{O}(1)$ constants.
Then $\hbar$-counting implies $[c] = \hbar$. Since $[e y_\ell] = \hbar^{-1}$ and $[\theta] = \hbar^{-4}$ we need a factor $(\hbar/16\pi^2)^4$, meaning that the EDM is generated at four loops.
Note that this reasoning holds only in the limits $m_q/m \ll 1$, with m_q the mass of SM quarks, and $\frac{yv}{m} \ll 1$. We have already seen that these conditions are respected from this model. If the second assumption would not be satisfied then y could enter through the adimensional combination yv/m, so the operator would be generated already at three loops. The first condition, instead, ensures that the SM Yukawa's in θ don't come from the adimensional ratio $m_q/m = y_f v/m$.

[7] This is true because we are working in an EFT, where the infrared divergences of the loop integral generating the operator get cancelled in the matching procedure (they are the same in both the theories). In this way the only term of the integral that survives is the one analytic in the light masses (and so in the Yukawa's y_f). This can be expanded, meaning that the CP violating coefficient can always be written as a polynomial in the couplings.

Thanks to this result it's possible to give an estimate of the numerical prediction of the EDM in this model, which is proportional to the coefficient of the EDM operator:

$$|d_e| \sim e\frac{m_e}{m^2}\frac{|\theta|}{(16\pi^2)^4} \approx 10^{-40}\left(\frac{\text{TeV}}{m}\right)^2 |y_i y_j| \sin(\varphi_i - \varphi_j)\, e \cdot \text{cm} \tag{2.42}$$

The most recent measurement [26] gives the bound $|d_e^{exp}| < 1.1 \times 10^{-29} e \cdot \text{cm}$, which is by far respected plugging the saturated values of m, y in (2.42).

Coming back to the general question about the possible CP odd operators generated at 1-loop, the answer is readily given following the reasoning just exposed.
The operators not involving fermions (as $HHG\tilde{G}, GG\tilde{G}, \dots$) must contain θ and are therefore excluded. For the other ones, either they have a trivial flavor structure (as the ones involving leptons) and so they are generated at three or four loops, or the non-trivial possible structures have already been listed with the help of spurion analysis and it has been shown that the bounds they provide are not particularly interesting.

Chapter 3

EFT at 1-loop

In chapter 2 the operators generated at 1-loop that can give improvements to the already known bounds on this model have been selected.
In order to get quantitative results, the exact computation of these operators is needed. For this reason this chapter is dedicated to that calculation. In section 3.1 the formalism for the EFT derivation by means of the functional integral method is introduced, and it will be concretely applied to the case of interest in the remaining sections.

3.1 Functional integral method

In order to introduce the functional integral method for the EFT computation, let's have a short review of the necessary formalism of the path integral in the context of quantum field theories.

Given a set of fields $[\phi]$, an ensemble of currents $[J]$ is introduced in the generating functional

$$Z[J] = \int [D\phi]\, e^{i\int d^4x(\mathcal{L}(\phi)+J\phi)}. \tag{3.1}$$

In this way the correlation functions are computed by differentiation

$$\langle \phi(x_1)\dots\phi(x_n)\rangle = \left(-i\frac{\delta}{\delta J(x_1)}\right)\dots\left(-i\frac{\delta}{\delta J(x_n)}\right)\log Z[J]|_{J=0}. \tag{3.2}$$

All the building blocks needed to compute any amplitude are actually encoded in the 1-particle irreducible diagrams, that are not generated by (3.1), but from its Legendre transform. This functional is defined in terms of the classical field ϕ_b:

$$\phi_b = -i\frac{\delta \log Z[J]}{\delta J(x)}. \tag{3.3}$$

Then the quantum 1PI action is given by

$$\Gamma[\phi_b] = -i \log Z[J] - \int d^4x J(x)\phi_b(x) \tag{3.4}$$

where $J(x)$ must be given as a function of $\phi_b(x)$: $J(x) = J(\phi_b(x))$. The 1PI diagrams can be computed by differentiation

$$\langle \phi(x_1)\ldots\phi(x_n)\rangle_{1PI} = i\frac{\delta^n\Gamma[\phi]}{\delta\phi(x_1)\ldots\delta\phi(x_n)}. \tag{3.5}$$

At this point we have all the necessary informations needed to build an EFT given a UV theory using the functional integral. Let's consider a theory with two sets of light and heavy particles (ϕ, Φ) and with a lagrangian $\mathcal{L}_{UV}(\phi, \Phi)$. The task is to compute an effective theory valid for energies much smaller than the heavy particles' masses. This will give a lagrangian $\mathcal{L}_{EFT}(\phi)$ valid for low energies and given in terms of the light particles only. We say that Φ has been integrated out.
The matching between the UV theory and the EFT can be given entirely in terms of the 1PI quantum action. Given the partition function for both the theories

$$\begin{aligned} Z_{UV}[J_\phi, J_\Phi] &= \int [D\phi][D\Phi]\, e^{i\int d^4x\left(\mathcal{L}_{UV}(\phi,\Phi)+J_\phi\phi+J_\Phi\Phi\right)} \\ Z_{EFT}[J_\phi] &= \int [D\phi]\, e^{i\int d^4x\left(\mathcal{L}_{EFT}(\phi)+J_\phi\phi\right)} \end{aligned} \tag{3.6}$$

the correlation functions for the light fields in the UV theory are entirely given in terms of $Z_{UV}[J_\phi, 0]$, while in the EFT by means of $Z_{EFT}[J_\phi]$. For practical reasons it is more convenient to consider the 1PI quantum action. Therefore, following the reasoning just exposed, the matching condition simply reads

$$\Gamma_{UV}[\phi, 0] = \Gamma_{EFT}[\phi]. \tag{3.7}$$

The condition (3.7) is actually stronger than we need, since we only need S-matrix element equivalence and not both on and off-shell correlation functions; thus field redefinitions will be allowed once $\mathcal{L}_{EFT}$ will be evaluated.
The key point is that the condition (3.7) must be satisfied only up to some order $1/\Lambda$. This allows to perform a perturbative expansion and match the condition order by order. In this work we are interested in computing the EFT up to 1-loop, so let's see how to match the condition in this case.

Let's consider first the EFT. First, we perform the change of variables $\phi = \phi_b + \phi'$, where ϕ_b is defined as in (3.3). Note that since ϕ_b solves the equation of motion with an arbitrary source J_ϕ, the terms linear in ϕ' are absent (at tree level). Then the 1-loop partition function reads

$$Z_{EFT}[J_\phi] = e^{i\int d^4x\left(\mathcal{L}_{EFT}(\phi_b)+J_\phi\phi_b\right)} \int [D\phi']\, e^{-\frac{i}{2}\int d^4x \bar{\phi}' Q_{EFT}\phi'} \tag{3.8}$$

where a generic notation $\bar{\phi}'$ has been used, since no assumption were made about the nature of the fields. What has been done is basically an approximation of the action at the second order, where the functional appearing at the exponent is defined as

$$Q_{EFT} = -\frac{\delta^2 \mathcal{L}_{EFT}}{\delta\phi\delta\bar{\phi}}|_{\phi=\phi_b}. \tag{3.9}$$

The formula for the gaussian integral at the exponent is well known, and depends on the kind of the field that needs to be integrated. Labeling the appropriate coefficient with c_s the result is

$$\begin{aligned} Z_{EFT}[J_\phi] &= e^{i\int d^4x\left(\mathcal{L}_{EFT}(\phi_b)+J_\phi\phi_b\right)} \cdot [\det Q_{EFT}]^{-\frac{c_s}{2}} + \dots \\ \to \Gamma_{EFT} &= \int d^4x \mathcal{L}_{EFT} + \frac{i}{2} c_s \log\det Q_{EFT} + \dots . \end{aligned} \tag{3.10}$$

with $c_s = 1$ for scalar fields $c_s = -2$ for fermionic fields.

On the UV side the calculation is similar. However since the source relative to the heavy fields has been set to zero, $J_\Phi = 0$, the equation of motion for Φ are exactly the classical ones and thus it becomes a function of the light fields only: $\Phi_b = \Phi_c(\phi)$. Therefore the quantum 1PI functional is given by

$$\Gamma_{UV}[\phi] = \int d^4x \mathcal{L}_{UV}|_{\Phi=\Phi_c(\phi)} + \frac{i}{2} c_s \log\det Q_{UV} + \dots . \tag{3.11}$$

Ordering the EFT side in terms of loop expansion, $\mathcal{L}_{EFT} = \mathcal{L}^{(0)}_{EFT} + \mathcal{L}^{(1)}_{EFT} + \dots$, the matching condition (3.7) immediately tells us the tree-level EFT

$$\mathcal{L}^{(0)}_{EFT}(\phi) = \hat{\mathcal{L}}_{UV}(\phi, \Phi_c(\phi)) \tag{3.12}$$

where the hat means the expansion of $\mathcal{L}_{UV}$ in $1/\Lambda$. This is the well known result for the tree-level EFT, which has been used also in chapter 2, 2.2.
For the 1-loop matching some care is required in evaluating the functional determinant on the EFT side. If we want to get a 1-loop result we must take into account only $\mathcal{L}^{(0)}_{EFT}$ in the functional determinant, otherwise we would get results at 2 or more loops. Explicitly, this means taking

$$Q^{(1)}_{EFT} = -\frac{\delta^2 \mathcal{L}^{(0)}_{EFT}}{\delta\phi\delta\bar{\phi}}|_{\phi=\phi_b}. \tag{3.13}$$

In this way it's straightforward to obtain the formula for the effective lagrangian up to 1-loop:

$$\int d^4x \mathcal{L}^{1-loop}_{EFT} = \int d^4x \hat{\mathcal{L}}_{UV}(\phi, \Phi_c(\phi)) + \frac{i}{2} c_s \log\det Q_{UV} - \frac{i}{2} c_s \log\det Q^{(1)}_{EFT}. \tag{3.14}$$

The result (3.14) is reliable and very general. The tree-level piece, again, is well known and widely used; on the contrary the 1-loop result is seldom found. It involves the calculation of functional determinants that can turn out to be a quite complex task.

The formula of equation (3.14) is not so straightforward to apply in case fields of different nature are present. Nevertheless, it is possible to obtain a usable expression. In a completely general way, let's write the second order lagrangian as

$$\delta^2\mathcal{L} = \frac{1}{2}\left(\xi^T A\xi + \bar{\eta}B\eta - \eta^T B^T\bar{\eta}^T + \xi^T\bar{\Gamma}\eta - \eta^T\bar{\Gamma}^T\xi + \bar{\eta}\Gamma\xi - \xi^T\Gamma^T\bar{\eta}^T\right) \tag{3.15}$$

where ξ, η are collections of bosonic and fermionic fields. The quantities A, B are bosonic operators, while $\Gamma, \bar{\Gamma}$ are fermionic operators in the relative spaces and depend on the classical solution of the fields.
We would like to evaluate the functional integral

$$\int [D\xi][D\eta][D\bar{\eta}]e^{\frac{i}{2}\int d^4x\delta^2\mathcal{L}} \tag{3.16}$$

to get the explicit expression to plug in (3.14); however the Hessian 3.15 is not diagonal in the two kind of fields, so the integral is not well defined. In order to disentangle these two, we square the second order lagrangian

$$\begin{aligned}\delta^2\mathcal{L} =& \frac{1}{2}\left(\bar{\eta} + \xi^T\bar{\Gamma}B^{-1}\right)B\left(\eta + B^{-1}\Gamma\xi\right) + \frac{1}{2}\left[\left(\bar{\eta} + \xi^T\bar{\Gamma}B^{-1}\right)B\left(\eta + B^{-1}\Gamma\xi\right)\right]^T\\ &+ \frac{1}{2}\xi^T\left(A - \bar{\Gamma}B^{-1}\Gamma + \Gamma^T B^{-1T}\bar{\Gamma}^T\right)\xi.\end{aligned} \tag{3.17}$$

At this point we first integrate over the fermionic fields. By shifting $\eta \to \eta - B^{-1}\Gamma\xi$ we are left with a simple gaussian integral, which yields the expression[1]

$$\begin{aligned}\int [D\xi][D\eta][D\bar{\eta}]e^{-\frac{1}{2}\int d^4x\delta^2\mathcal{L}} &= c\cdot\det B\int [D\xi]e^{-\frac{1}{2}\int d^4x\xi^T\left(A-\bar{\Gamma}B^{-1}\Gamma+\Gamma^T B^{-1T}\bar{\Gamma}^T\right)\xi}\\ &= c\cdot\det B\cdot\left[\det\left(A - \bar{\Gamma}B^{-1}\Gamma + \Gamma^T B^{-1T}\bar{\Gamma}^T\right)\right]^{-1/2}\end{aligned} \tag{3.18}$$

where we finally integrated also over the bosonic fields ξ, and c is an irrelevant constant. This is the familiar result found for example in [27], [28].
Applying this to (3.14), the final expression for the 1-loop EFT lagrangian is given by

$$\begin{aligned}\int d^4x\mathcal{L}_{EFT}^{1-loop} = \int d^4x\hat{\mathcal{L}}_{UV}(\phi, \Phi_c(\phi)) &- i[\log\det B_{UV} - \log\det B_{EFT}]\\ &+ \frac{i}{2}\left[\log\det\left(A - \bar{\Gamma}B^{-1}\Gamma + \Gamma^T B^{-1T}\bar{\Gamma}^T\right)_{UV}\right.\\ &\left.\quad - \log\det\left(A - \bar{\Gamma}B^{-1}\Gamma + \Gamma^T B^{-1T}\bar{\Gamma}^T\right)_{EFT}\right].\end{aligned} \tag{3.19}$$

[1] In evaluating the integral we have moved to the Euclidean space. In this way can use the expression for the Gaussian integrals and the result is well defined. Then we can move back to the usual Minkowski space through a Wick rotation.

Equation (3.19) is the starting point for the calculation of our 1-loop EFT and will be used all throughout this chapter. It includes fluctuations of scalar, vector and fermionic fields; since we are studying a new fermion in addition to the SM particles, we will need to integrate all over these fields. In this regard, gauge bosons will be considered as collections of scalar fields.

3.2 Computation of the operators

Before jumping in the concrete derivation there are some very useful considerations that can be done in the case of this particular model, thanks to which the calculations will become more doable.

First, it is convenient to perform a field redefinition for Ψ:

$$\Psi = \Psi_{cl} + \delta\Psi = O^{-1} H y^\dagger u_R + \delta\Psi \tag{3.20}$$

where O^{-1} is given by (2.16). In this way the lagrangian (2.1) becomes

$$\begin{aligned}\mathcal{L}_{UV} =& \mathcal{L}_{SM} + \delta\bar{\Psi} O \Psi_{cl} + \bar{\Psi}_{cl} O \delta\Psi - \bar{u}_R y H^\dagger \delta\Psi - \delta\bar{\Psi} H y^\dagger u_R + \delta\bar{\Psi} O \delta\Psi \\ &+ \bar{\Psi}_{cl} O \Psi_{cl} - \bar{u}_R y^\dagger \Psi_{cl} - \bar{\Psi}_{cl} H y^\dagger u_R \\ =& \mathcal{L}_{SM} - \bar{u}_R y H^\dagger O^{-1} (H y^\dagger u_R) + \delta\bar{\Psi} O \delta\Psi. \end{aligned} \tag{3.21}$$

This manipulation is extremely useful because in this way the Ψ contribution to the Hessian is separated from the rest of the Standard Model particles. Indeed let's immediately see this by starting the computation of the UV action at the second order.
We begin by evaluating the fermionic piece. With the redefined UV lagrangian (3.21) the part involving the new fermion is factorized, meaning $B_{UV} = (i\not{D} - m) \otimes \tilde{B}_{UV}$, where $\tilde{B}_{UV}$ includes the variations with respect to the SM fermions. Indeed B_{UV} reads

$$B_{UV} = \begin{array}{c c} & \begin{array}{cccccc} \delta\Psi & \delta u_R & \delta d_R & \delta Q_L & \delta\ell_R & \delta L_L \end{array} \\ \begin{array}{c} \delta\bar{\Psi} \\ \delta\bar{u}_R \\ \delta\bar{d}_R \\ \delta\bar{Q}_L \\ \delta\bar{\ell}_R \\ \delta\bar{L}_L \end{array} & \begin{pmatrix} i\not{D} - m & 0 & 0 & 0 & 0 & 0 \\ 0 & UU & 0 & -y_u \tilde{H}^\dagger & 0 & 0 \\ 0 & 0 & i\not{D} & -y_d H^\dagger & 0 & 0 \\ 0 & -y_u^\dagger \tilde{H} & -y_d^\dagger H & i\not{D} & 0 & 0 \\ 0 & 0 & 0 & 0 & i\not{D} & -y_\ell H^\dagger \\ 0 & 0 & 0 & 0 & -y_\ell^\dagger H & i\not{D} \end{pmatrix} \end{array} \tag{3.22}$$

$$UU = i\not{D} - y H^\dagger O^{-1} y^\dagger H.$$

Next we have to write down $A_{UV}, \Gamma_{UV}, \bar{\Gamma}_{UV}$. We will consider only the Higgs contribution because none of the operators that we consider are generated by diagrams with virtual gauge bosons. Under this approximation A_{UV} becomes

$$A_{UV} = \begin{array}{c} \\ \delta H^\dagger \\ \delta H^T \end{array} \begin{array}{c} \begin{array}{cc} \delta H & \delta H^{\dagger T} \end{array} \\ \begin{pmatrix} a & -\frac{\lambda}{2} H H^T \\ -\frac{\lambda}{2} H^{\dagger T} H^\dagger & a^T \end{pmatrix} \end{array} \tag{3.23}$$

$$a = -D^2 + \mu^2 - \frac{\lambda}{2}(H^\dagger H + H H^\dagger) - \bar{u}_R y \frac{1}{i\not{D} - m} y^\dagger u_R.$$

Similar considerations apply to $\Gamma, \bar{\Gamma}$. With the redefined lagrangian (3.21) Ψ is factorized with respect to the SM particles, so the matrices read

$$\bar{\Gamma}_{UV} = \begin{array}{c} \\ \delta H^\dagger \\ \delta H^T \end{array} \begin{array}{c} \begin{array}{ccccc} \delta u_R & \delta d_R & \delta Q_L & \delta \ell_R & \delta L_L \end{array} \\ \begin{pmatrix} UH' & 0 & -\bar{d}_R y_d & 0 & -\bar{\ell}_R y_\ell \\ 0 & -\bar{Q}_{L,i} y_d^\dagger & \bar{u}_R y_u \epsilon & -\bar{L}_{L,i} y_\ell^\dagger & 0 \end{pmatrix} \end{array} \tag{3.24}$$

$$\Gamma_{UV} = \begin{array}{c} \\ \delta \bar{u}_R \\ \delta \bar{d}_R \\ \delta \bar{Q}_L \\ \delta \bar{\ell}_R \\ \delta \bar{L}_L \end{array} \begin{array}{c} \begin{array}{cc} \delta H & \delta H^{\dagger T} \end{array} \\ \begin{pmatrix} UH & 0 \\ 0 & -y_d Q_{L,i} \\ -y_d^\dagger d_R & -\varepsilon y_u^\dagger u_R \\ 0 & -y_\ell L_{L,i} \\ -y_\ell^\dagger \ell_R & 0 \end{pmatrix} \end{array} \tag{3.25}$$

$$\begin{aligned} UH &= y_u Q_{L,i} \varepsilon_{ji} - H^\dagger y \frac{1}{i\not{D} - m} y^\dagger u_R \\ UH' &= -\varepsilon_{ij} \bar{Q}_{L,i} y_u^\dagger - \bar{u}_R y \frac{1}{i\not{D} - m} H y^\dagger. \end{aligned} \tag{3.26}$$

where the index i refers to the $SU(2)$ components.

Now that the UV action at the second order has been computed we should derive also the EFT action at second order. However there is smart observation that can be made, stated also in [27], thanks to which we won't need that Hessian. Indeed, after the field redefinition for Ψ the UV lagrangian reads (3.21). We can note that the EFT at tree level is simply obtained by expanding the non-local operator O^{-1} and setting $\delta\Psi = 0$:

$$\begin{aligned} \mathcal{L}_{UV} &= \mathcal{L}_{SM} + \delta\bar{\Psi}(i\not{D} - m)\delta\Psi - \bar{u}_R H^\dagger y \left(\frac{1}{i\not{D} - m} \right) H y^\dagger u_R \\ \mathcal{L}_{EFT}^{(0)} &= \mathcal{L}_{SM} - \bar{u}_R H^\dagger y \left(-\frac{1}{m} + \frac{i\not{D}}{m^2} \right) H y^\dagger u_R \end{aligned} \tag{3.27}$$

Now, the UV action at second order is factorized between $\delta\Psi$ and the SM particles. This means that the difference between the UV and the EFT functional determinants is entirely given by the difference between O^{-1} and its expansion, so its non-local part O_{NL}:

$$\frac{1}{i\not{D}-m} = -\frac{1}{m} - \frac{i\not{D}}{m^2} + O_{NL} \to O_{NL} = \frac{1}{m^2}\frac{(i\not{D})^2}{i\not{D}-m} \tag{3.28}$$

The practical relevance of this consideration is that we don't need to compute separately the two functional determinants of the UV and the EFT actions and then subtract them. We can just compute the determinant of the UV action and drop the local counterparts of O^{-1}. This is equivalent to computing the UV and EFT determinants and then subtract them. Concretely this means substituting *any* O^{-1} with O_{NL} in the functional traces we must compute. In our case we will encounter only two kind of traces, so lets see explicitly how that applies. We will indicate the operation of dropping the local counterparts with the subscript d. So for generic function of the fields $P(x), Q(x), R(x)$ the d operation reads

$$\begin{aligned}
&\cdot)\ \mathrm{tr}\left\{P(x)\frac{1}{i\not{D}-m}Q(x)\right\}_d = \frac{1}{m^2}\mathrm{tr}\left\{P(x)\frac{(i\not{D})^2}{i\not{D}-m}Q(x)\right\}\\
&\cdot)\ \mathrm{tr}\left\{P(x)\frac{1}{i\not{D}-m}Q(x)\frac{1}{i\not{D}-m}R(x)\right\}_d = \frac{1}{m^2}\mathrm{tr}\left\{P(x)\frac{(i\not{D})^2}{i\not{D}-m}Q(x)\frac{1}{i\not{D}-m}R(x)\right\}\\
&\quad+\frac{1}{m^2}\mathrm{tr}\left\{P(x)\frac{1}{i\not{D}-m}Q(x)\frac{(i\not{D})^2}{i\not{D}-m}R(x)\right\} - \frac{1}{m^4}\mathrm{tr}\left\{P(x)\frac{(i\not{D})^2}{i\not{D}-m}Q(x)\frac{(i\not{D})^2}{i\not{D}-m}R(x)\right\}.
\end{aligned} \tag{3.29}$$

The second relation can be derived by explicitly computing the difference between the trace with O^{-1} and the one with its expansion.

At this point we have all the tools that are needed to start the computation. We will look at the traces coming from the UV functional determinant and apply the d prescription, as explained above, to obtain the EFT at 1-loop. The next sections we will be devoted to the complete derivation of the operators marked in chapter 2 as relevant.

3.2.1 Pure gauge

The first class of operators to be derived is the one of the pure gauge dimension six operators, present in $\mathcal{L}_V$. As already stated these affect the gauge bosons self-energies, which are constrained by the electroweak precision tests.
Diagrammatically, these operators are generated by loops where only the heavy fermion is present, as in figure 3.1, and many external gauge bosons can be attached to the propagators entering the loop.

This class is entirely encoded in the variations of the action with respect to the new fermion Ψ only. From the central formula (3.19) only $-i(\log\det B_{UV} - \log\det B_{EFT})$

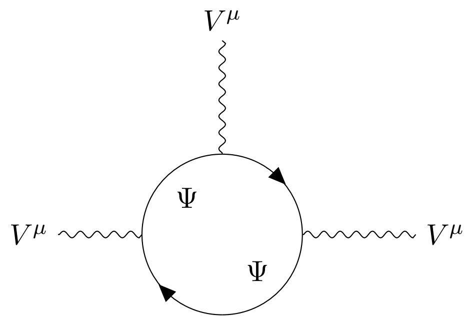


Figure 3.1: Example of a diagram generating the pure gauge operators. V^μ is a generic vector boson. Many external gauge bosons can be attached to the Ψ propagators.

generate these operators, since double variations with respect to fermions are present only in B. Besides B_{EFT} does not contain variations with respect to Ψ, so we just have to look at B_{UV}. From the explicit form of B_{UV} (3.22) we see that the pure gauge operators are simply included in the factorized $(i\not{D} - m)$:

$$\int d^4x\mathcal{L}_V \subset -i\log\det(i\not{D} - m). \tag{3.30}$$

To proceed with the evaluation of this functional determinant we have to handle a little bit this expression. The determinant is invariant under flipping the sign of the γ^μ matrices, so we can write

$$\begin{aligned}\int d^4x\mathcal{L}_V \subset &= -i\log\det(i\not{D} + m) \\ &= -\frac{i}{2}\log\det\left[(-i\not{D} + m)(i\not{D} + m)\right]. \\ &= -\frac{i}{2}\log\det\left(D^2 + m^2 + U\right). \end{aligned} \tag{3.31}$$

Here we have used

$$(-i\not{D} + m)(i\not{D} + m) = \not{D}^2 + m^2 = D^2 + m^2 - \frac{i}{2}\sigma^{\mu\nu}G'_{\mu\nu} \tag{3.32}$$

where $\sigma^{\mu\nu} = -\frac{i}{2}[\gamma^\mu, \gamma^\nu]$ and the properties of the γ^μ matrices have been exploited: $\gamma^\mu\gamma^\nu = \eta^{\mu\nu} - i\sigma^{\mu\nu}$. Finally $G'_{\mu\nu} = [D_\mu, D_\nu] = -ig_sG_{\mu\nu} - igW_{\mu\nu} - iYg'B_{\mu\nu}, Y = 7/6$.

Functional determinants like the one in (3.31) are not new and the result is well known in literature, [29]. The formula is obtained through the Covariant Derivative Expansion technique, explained in appendix A. The full expression truncated at dimension six reads

$$\begin{aligned}\log\det(D^2+m^2+U) \supset \int d^4x \left(-\frac{i}{16\pi^2}\right) \mathrm{tr}\Bigg\{ & m^2\left[(1+\frac{1}{\epsilon}+\log\frac{\mu^2}{m^2}+\log 4\pi-\gamma)U\right] \\ & + m^0\left[\frac{1}{2}(\frac{1}{\epsilon}+\log\frac{\mu^2}{m^2}+\log 4\pi-\gamma)U^2\right. \\ & \left.+\frac{1}{12}(1+\frac{1}{\epsilon}+\log\frac{\mu^2}{m^2}+\log 4\pi-\gamma)G'^2_{\mu\nu}\right] \\ & + \frac{1}{m^2}\left[-\frac{U^3}{6}+\frac{1}{12}(D_\alpha U)^2-\frac{1}{12}UG'^2_{\mu\nu}+\frac{1}{60}(D^\mu G'_{\mu\nu})^2\right. \\ & \left.\left. -\frac{1}{90}G'_{\mu\nu}G'^{\nu\rho}G'^{\mu}_{\rho}\right]\right\}\end{aligned} \tag{3.33}$$

and has been computed in dimensional regularization with $\epsilon = (4-d)/2$. The trace must be taken in the space where the heavy field used to live: we have a trace over the spinorial space, $SU(2)$ and $SU(3)$.
We immediately see that $\mathrm{tr}U \propto \mathrm{tr}\sigma^{\mu\nu} = 0$. Then we must trace also U^2, U^3 over the spinorial indices:

$$\mathrm{tr}U^2 = -\frac{1}{4}\mathrm{tr}G'_{\mu\nu}G'_{\rho\sigma}\mathrm{tr}\sigma^{\mu\nu}\sigma^{\rho\sigma} = -\frac{1}{4}(8\mathrm{tr}G'_{\mu\nu}G'^{\mu\nu}) = -2\mathrm{tr}G'_{\mu\nu}G'^{\mu\nu} \tag{3.34}$$

$$\mathrm{tr}U^3 = \frac{i}{8}\mathrm{tr}G'_{\mu\nu}G'_{\rho\sigma}G'_{\gamma\delta}\mathrm{tr}\sigma^{\mu\nu}\sigma^{\rho\sigma}\sigma^{\gamma\delta} = \frac{i}{8}(-32i\mathrm{tr}G'_{\mu\nu}G'^{\nu\rho}G'^{\mu}_{\rho}) = 4\mathrm{tr}G'_{\mu\nu}G'^{\nu\rho}G'^{\mu}_{\rho}. \tag{3.35}$$

Finally the operators not involving any $\sigma^{\mu\nu}$ are proportional to the identity in the spinorial space; this gives a factor $\mathrm{tr}\mathbb{1}_{4\times4} = 4$.

The next trace to be taken is the one on the internal gauge group[2]. Since $\mathrm{tr}\sigma^i = 0, \mathrm{tr}\lambda^a = 0$ the surviving terms after this operation are, ignoring for a moment how they are Lorentz contracted,

$$\mathrm{tr}G'G' = -g_s^2G^aG^a - \frac{3}{2}g^2W^iW^i - 6Y^2g'^2BB \tag{3.36}$$

$$\mathrm{tr}G'G'G' = -\frac{1}{2}g_s^3f^{abc}G^aG^bG^c - \frac{3}{4}g^3\varepsilon^{ijk}W^iW^jW^k \tag{3.37}$$

where the $SU(N)$ identities $\mathrm{tr}G^aG^bt^at^b = G^aG^b/2$ and $\mathrm{tr}G^aG^bG^ct^at^bt^c = \frac{i}{4}f^{abc}G^aG^bG^c$ have been used (for W we just exchange $f^{abc}\to\varepsilon^{abc}$).

We can then compute the traces of all the terms contained in (3.33):

- $\frac{1}{2}\mathrm{tr}U^2 = g_s^2G^{a\mu\nu}G^a_{\mu\nu} + \frac{3}{2}g^2W^{i\mu\nu}W^i_{\mu\nu} + 6Y^2g'^2B^{\mu\nu}B_{\mu\nu}$;
- $\frac{1}{12}\mathrm{tr}G'^2_{\mu\nu} = -\frac{g_s^2}{12}G^{a\mu\nu}G^a_{\mu\nu} - \frac{3}{24}g^2W^{i\mu\nu}W^i_{\mu\nu} - \frac{1}{2}Y^2g'^2B^{\mu\nu}B_{\mu\nu}$;
- $\frac{1}{12}\mathrm{tr}D_\alpha UD^\alpha U = \frac{g_s^2}{6}D_\alpha G^{a\mu\nu}D^\alpha G^a_{\mu\nu} + \frac{g^2}{4}D_\alpha W^{i\mu\nu}D^\alpha W^i_{\mu\nu} + Y^2g'^2\partial_\alpha B^{\mu\nu}\partial^\alpha B_{\mu\nu}$;

[2] As for the trace over the spinorial space, operators not involing directly generators of the gauge group are meant to be proportional to the identity in that space. This gives a factor $\mathrm{tr}\mathbb{1}_{SU(2)} = 2, \mathrm{tr}\mathbb{1}_{SU(3)} = 3$.

- $\frac{1}{60}\mathrm{tr}(D_\mu G'^{\mu\nu})^2 = -4g_s^2 D_\mu G^{a\mu\nu} D_\rho G^{a\rho}_\mu - 6g^2 D_\mu W^{i\mu\nu} D_\rho W^{i\rho}_\mu - 24Y^2 g'^2 \partial_\mu B^{\mu\nu}\partial_\rho B^\rho_\mu$;
- $-\mathrm{tr}\frac{U^3}{6} = \frac{g_s^3}{3} f^{abc} G^a_{\mu\nu} G^{b\nu\rho} G^{c\mu}_\rho + \frac{g^3}{2}\varepsilon^{ijk} G^i_{\mu\nu} G^{j\nu\rho} G^{k\mu}_\rho$;
- $-\frac{1}{90}\mathrm{tr}G'_{\mu\nu} G'^{\nu\rho} G'^{\mu}_\rho = \frac{g_s^3}{45} f^{abc} G^a_{\mu\nu} G^{b\nu\rho} G^{c\mu}_\rho + \frac{g^3}{30}\varepsilon^{ijk} G^i_{\mu\nu} G^{j\nu\rho} G^{k\mu}_\rho$;

Looking at the list above it is immediate to see that we have three kind of operators of dimension six:

$$D_\mu G^a_{\mu\nu} D_\rho G^{\rho a}_\nu, \quad D_\alpha G^a_{\mu\nu} D^\alpha G^{a\mu\nu}, \quad f^{abc} G^a_{\mu\nu} G^{b\nu\rho} G^{c\mu}_\rho \tag{3.38}$$

and the same holds for W, while for B the triple field strength interaction is not present. However only two out of the three operators in (3.38) are really independent. The decision is to eliminate $D_\alpha G^a_{\mu\nu} D^\alpha G^{a\mu\nu}$: this is simply dictated by the fact that in the $\mathcal{L}_V$ lagrangian (2.22) this operator is not present.
We can eliminate that operator exploiting Bianchi identity: it can be proved that the following relation holds for any $SU(N)$ gauge field:

$$\frac{1}{2} D_\alpha F^a_{\mu\nu} D^\alpha F^{a\mu\nu} = D_\mu F^a_{\mu\nu} D_\rho F^{\rho a}_\nu - g_* f^{abc} F^a_{\mu\nu} F^{b\nu\rho} F^{c\mu}_\rho. \tag{3.39}$$

This expression holds also for $B_{\mu\nu}$ by setting $f^{abc} = 0$ ($U(1)$ is Abelian, the structure constants are null).

By making use of all the considerations reported above the final result is obtained just by a straightforward sum of the traces of all the terms contained in the expression (3.33). The pure gauge effective lagrangian at 1-loop then reads

$$\begin{aligned}
\mathcal{L}^{(1)}_{EFT,V} =& -\frac{1}{4}\left[1 - \frac{g_s^2}{12\pi^2}\left(\log\frac{m^2}{\mu^2} - \log 4\pi - \frac{1}{\epsilon} + \gamma + \frac{1}{2}\right)\right] G^a_{\mu\nu} G^{a\mu\nu} \\
& -\frac{1}{4}\left[1 - \frac{g^2}{8\pi^2}\left(\log\frac{m^2}{\mu^2} - \log 4\pi - \frac{1}{\epsilon} + \gamma + \frac{1}{2}\right)\right] W^a_{\mu\nu} W^{a\mu\nu} \\
& -\frac{1}{4}\left[1 - \frac{Y^2 g'^2}{2\pi^2}\left(\log\frac{m^2}{\mu^2} - \log 4\pi - \frac{1}{\epsilon} + \gamma + \frac{1}{2}\right)\right] B_{\mu\nu} B^{\mu\nu} \\
& - \frac{1}{16\pi^2}\frac{1}{m^2}\left[\frac{2}{15} g_s^2 D^\mu G^a_{\mu\nu} D^\rho G^{a\nu}_\rho + \frac{g_s^3}{90} f^{abc} G^a_{\mu\nu} G^{b\nu\rho} G^{c\mu}_\rho + \frac{1}{5} g^2 D^\mu W^a_{\mu\nu} D^\rho W^{a\nu}_\rho \right.\\
& \left. + \frac{g^3}{60}\varepsilon^{abc} W^a_{\mu\nu} W^{b\nu\rho} W^{c\mu}_\rho + \frac{4}{5} g'^2 Y^2 \partial_\mu B^{\mu\nu}\partial_\rho B^\rho_\nu\right].
\end{aligned} \tag{3.40}$$

The divergences in the corrections to the kinetic terms are reported to make explicit the contribution of the new fermion to the beta function of the coupling constants, and may be eliminated by adding the usual counterterms. Also, the matching between the UV theory and the EFT is performed at $\mu = m$; in this way the logarithms in the expression above are null.

3.2.2 Four fermions

The next set of operators that is going to be derived is the one comprehending four fermions operators.
These operators are generated from the diagrams in figure 3.2, that are of two kinds: they may have four y vertices or two y and two Standard Model y_f vertices. In both cases the flavor structure is non-trivial.

(a) 4-fermions operator involing four y vertices.

(b) 4-fermions operator involving two y and two SM's y_f vertices.

Figure 3.2: Diagrams generating the 4-fermions operators.

With reference to the formula (3.19) and looking at the matrices $B_{UV}, A_{UV}, \Gamma_{UV}, \bar{\Gamma}_{UV}$ we note that these operators are encapsulated in the bosonic term:

$$\begin{aligned} \int d^4x \mathcal{L}_{4f} &\subset \frac{i}{2} \log\det\left(A_{UV} - \bar{\Gamma} B_{UV}^{-1} \Gamma_{UV} + \Gamma_{UV}^T B_{UV}^{-1T} \bar{\Gamma}_{UV}^T\right)_d \\ &= \frac{i}{2}\log\det(A_{UV})_d + \frac{i}{2}\log\det\left(1 - A_{UV}^{-1}\bar{\Gamma}_{UV} B_{UV}^{-1}\Gamma_{UV} + A_{UV}^{-1}\Gamma_{UV}^T B_{UV}^{-1T}\bar{\Gamma}_{UV}^T\right)_d \end{aligned} \tag{3.41}$$

where we used the d for the UV-EFT difference, as explained in section 3.2.

The first term in (3.41), $\log\det(A_{UV})_d$, generates the y^4 operator $\hat{O}_{4f,y^4}$. The matrix A_{UV} can be rewritten as

$$\begin{aligned} A_{UV} &= (-D^2 - m_h^2)\left(\mathbb{1} + \frac{1}{-D^2 - m_h^2}\begin{pmatrix} -\bar{u}_R y \frac{1}{i\not{D} - m} y^\dagger u_R & \\ 0 & -\left(\bar{u}_R y \frac{1}{i\not{D}-m} y^\dagger u_R\right)^T \end{pmatrix} + \dots\right) \\ &\equiv (-D^2 - m_h^2)\left(\mathbb{1} + \frac{1}{-D^2 - m_h^2}\Omega_A\right) \end{aligned} \tag{3.42}$$

where the ... contain terms involving the Higgs boson that do not contribute to the four fermions operators, and we defined Ω_A. Exploiting the logarithm properties we have

$$\log\det A_{UV} = \log\det(-D^2 - m_h^2) + \log\det(1 + (-D^2 - m_h^2)^{-1}\Omega_A).$$

Since we are working in a perturbative regime, the coupling constants are supposed to be small. In this way, using the well known equality $\log\det M = \operatorname{tr}\log M$, we have

something of the form $\log(1+x)$ which can be expanded.
Indeed, the determinant of A_{UV} becomes

$$\log\det A_{UV} = \log\det(-D^2 - m_h^2) + \mathrm{tr}\frac{1}{-D^2-m_h^2}\Omega_A - \frac{1}{2}\mathrm{tr}\frac{1}{-D^2-m_h^2}\Omega_A\frac{1}{-D^2-m_h^2}\Omega_A + \dots \tag{3.43}$$

The y^4 four fermion operator is part of the $\sim \Omega_A^2$ trace:

$$\begin{aligned}\hat{O}_{4f,y^4} &= -\frac{i}{2}\cdot\frac{1}{2}\mathrm{tr}\begin{pmatrix}\left(\frac{1}{-D^2-m_h^2}\left(\bar{u}_R y\frac{1}{i\not{D}-m}y^\dagger u_R\right)\right)^2 & 0 \\ 0 & \left(\frac{1}{-D^2-m_h^2}\left(\bar{u}_R y\frac{1}{i\not{D}-m}y^\dagger u_R\right)^T\right)^2\end{pmatrix}_d \\ &= -\frac{i}{2}\mathrm{tr}\left\{\frac{1}{-D^2-m_h^2}\left(\bar{u}_R y\frac{1}{i\not{D}-m}y^\dagger u_R\right)\frac{1}{-D^2-m_h^2}\left(\bar{u}_R y\frac{1}{i\not{D}-m}y^\dagger u_R\right)\right\}_d\end{aligned} \tag{3.44}$$

where the property $\mathrm{tr}M = \mathrm{tr}M^T$ has been exploited together with the antisymmetry of the fermionic operators.
Now we just need to apply the d prescription explained in section 3.2. In particular we need the formula (3.29) for $P(x) = 1$, $Q(x) = R(x)$, which is even simpler[3]: we can just evaluate one piece containing O_{NL} and multiply it by two, and then subtract the two O_{NL} contribution.
The evaluation of this trace is performed by means of the CDE technique, as explained in appendix A. Since the four fermionic fields already sum up to mass dimension six, we must stop the expansion at the lowest order. In this way the one-O_{NL} term gives

$$\begin{aligned}\hat{O}_{4f,y^4,a} &= -\frac{i}{m^2}\mathrm{tr}\left\{\frac{1}{-D^2-m_h^2}\left(\bar{u}_R y\frac{(i\not{D})^2}{i\not{D}-m}y^\dagger u_R\right)\frac{1}{-D^2-m_h^2}\left(\bar{u}_R y\frac{1}{i\not{D}-m}y^\dagger u_R\right)\right\} \\ &= -\frac{i}{m^2}\mathrm{tr}\left\{\frac{1}{-D^2-m_h^2}\left(\bar{u}_R y\frac{(i\not{D}+m)(i\not{D})^2}{-\not{D}^2-m}y^\dagger u_R\right)\frac{1}{-D^2-m_h^2}\left(\bar{u}_R y\frac{(i\not{D}+m)}{-\not{D}^2-m}y^\dagger u_R\right)\right\} \\ &= -\frac{4i}{m^2}\mathrm{tr}\int d^4x d^4p\frac{1}{p^2-m_h^2}\bar{u}_R y\frac{\not{p}p^2}{p^2-m^2}y^\dagger u_R\frac{1}{p^2-m_h^2}\bar{u}_R y\frac{\not{p}}{p^2-m^2}y^\dagger u_R \\ &= -\frac{4i}{m^2}\mathrm{tr}\int d^4x\left(\bar{u}_R y\gamma^\mu y^\dagger u_R\bar{u}_R y\gamma^\nu y^\dagger u_R\right)\int d^4p\frac{p_\mu p_\nu p^2}{(p^2-m_h^2)^2(p^2-m^2)^2} \\ &= \frac{1}{16\pi^2 m^2}\cdot\frac{1}{2}\left(\frac{1-6m_h^2/m^2+m_h^4/m^4}{(1-m_h^2/m^4)^2} + 2\frac{m_h^4}{m^4}\frac{(3-m_h^2/m^2)}{(1-m_h^2/m^2)^3}\log\left(\frac{m^2}{m_h^2}\right)\right. \\ &\quad \left. + \frac{2}{\epsilon} + 2\log\left(\frac{\mu^2}{m^2}\right) - 2\gamma + 2\log 4\pi\right)\times\mathrm{tr}\left(\bar{u}_R y\gamma^\mu y^\dagger u_R\bar{u}_R y\gamma_\mu y^\dagger u_R\right).\end{aligned} \tag{3.45}$$

The factor of 4 in the third lines comes from the fact that we must apply $(-D^2-m_h^2)^{-1}$ to all the fields, giving that combinatorial factor. Finally, the momentum integral has

[3] Thanks to the cyclicity of the trace $\mathrm{tr}O_{NL}Q(x)O^{-1}Q(x) = \mathrm{tr}O^{-1}Q(x)O_{NL}Q(x)$.

been performed in dimensional regularization with $\epsilon = (4-d)/2$.
The two-O_{NL} term coming by (3.44) has a similar expression:

$$
\begin{aligned}
\hat{O}_{4f,y^4,b} &= -\frac{i}{2m^4}\mathrm{tr}\left\{\frac{1}{-D^2-m_h^2}\left(\bar{u}_R y\frac{(i\not{D})^2}{i\not{D}-m}y^\dagger u_R\right)\frac{1}{-D^2-m_h^2}\left(\bar{u}_R y\frac{(i\not{D})^2}{i\not{D}-m}y^\dagger u_R\right)\right\} \\
&= -\frac{i}{2m^4}\mathrm{tr}\left\{\frac{1}{-D^2-m_h^2}\left(\bar{u}_R y\frac{(i\not{D}+m)(i\not{D})^2}{-\not{D}^2-m}y^\dagger u_R\right)\frac{1}{-D^2-m_h^2}\left(\bar{u}_R y\frac{(i\not{D}+m)}{-\not{D}^2-m}y^\dagger u_R\right)\right\} \\
&= -\frac{4i}{2m^4}\mathrm{tr}\int d^4x d^4p\frac{1}{p^2-m_h^2}\bar{u}_R y\frac{\not{p}p^2}{p^2-m^2}y^\dagger u_R\frac{1}{p^2-m_h^2}\bar{u}_R y\frac{\not{p}p^2}{p^2-m^2}y^\dagger u_R \\
&= -\frac{4i}{m^4}\mathrm{tr}\int d^4x\left(\bar{u}_R y\gamma^\mu y^\dagger u_R\bar{u}_R y\gamma^\nu y^\dagger u_R\right)\int d^4p\frac{p_\mu p_\nu p^4}{(p^2-m_h^2)^2(p^2-m^2)^2} \\
&= \frac{1}{16\pi^2 m^2}\cdot\frac{1}{2}\left(\frac{2-3m_h^2/m^2-3m_h^4/m^4+2m_h^6/m^6}{(1-m_h^2/m^4)^2}+2\frac{m_h^6}{m^6}\frac{(2-m_h^2/m^2)}{(1-m_h^2/m^2)^3}\log\left(\frac{m^2}{m_h^2}\right)\right. \\
&\left.+\left(1+m_h^2/m^2\right)\left(\frac{2}{\epsilon}+2\log\left(\frac{\mu^2}{m^2}\right)-2\gamma+2\log 4\pi\right)\right)\times\mathrm{tr}\left(\bar{u}_R y\gamma^\mu y^\dagger u_R\bar{u}_R y\gamma_\mu y^\dagger u_R\right).
\end{aligned}
\tag{3.46}
$$

Finally we must evaluate the last trace over the remaining degrees of freedom $SU(2)$, that gives just a factor of two (the only degree of freedom of the Higgs field, since we are taking the variation with respect to it).
By subtracting the second piece from the first and stopping at the lowest order in $(m_h/m)^2$ we obtain the expression for the first 4-fermion operator:

$$
\hat{O}_{4f,y^4} = -\frac{1}{16\pi^2 m^2}\left(\bar{u}_R y\gamma^\mu y^\dagger u_R\bar{u}_R y\gamma_\mu y^\dagger u_R\right). \tag{3.47}
$$

Note that the matching has been performed at the scale $\mu = m$ and we employed the $\overline{\mathrm{MS}}$ scheme, subtracting then $(\frac{1}{\epsilon}-\gamma+\log 4\pi)$.

The $y^2 y_f^2$ operators are enclosed in the right term of (3.41). To compute this determinant we need A_{UV}^{-1} and $\tilde{B}_{UV}^{-1}$ [4]. Exploiting the perturbativity, A_{UV}^{-1} can be written as

$$
A_{UV}^{-1} = \left((-D^2-m_h^2)\left(1+\frac{1}{-D^2-m_h^2}\Omega_A\right)\right)^{-1} = \left(1-\frac{1}{-D^2-m_h^2}\Omega_A+\dots\right)\cdot\frac{1}{-D^2-m_h^2} \tag{3.48}
$$

and $\tilde{B}_{UV}^{-1}$ as

$$
\tilde{B}_{UV}^{-1} = \left((i\not{D})\left(1+\frac{1}{i\not{D}}\Omega_B\right)\right)^{-1} = \left(1-\frac{1}{i\not{D}}\Omega_B+\dots\right)\cdot\frac{1}{i\not{D}}. \tag{3.49}
$$

[4] Since B_{UV} is factorized between the variations with respect to the new fermion and the SM fermions we we only need $\tilde{B}_{UV}$.

Expanding the right determinant of (3.41), the first order already contains the operators we are interested in:

$$\begin{aligned}
\hat{O}_{4f,y^2y_f^2} &\subset \frac{i}{2}\log\det\left(1 - A_{UV}^{-1}\bar{\Gamma}_{UV}B_{UV}^{-1}\Gamma_{UV} + A_{UV}^{-1}\Gamma_{UV}^{T}B_{UV}^{-1T}\bar{\Gamma}_{UV}^{T}\right)_d \\
&= -\frac{i}{2}\mathrm{tr}\left(A_{UV}^{-1}\bar{\Gamma}_{UV}B_{UV}^{-1}\Gamma_{UV} + A_{UV}^{-1}\Gamma_{UV}^{T}B_{UV}^{-1T}\bar{\Gamma}_{UV}^{T}\right)_d + \dots \\
&= -2\frac{i}{2}\mathrm{tr}\left(A_{UV}^{-1}\bar{\Gamma}_{UV}B_{UV}^{-1}\Gamma_{UV}\right)_d + \dots
\end{aligned} \tag{3.50}$$

where the property $A = A^T$ has been used. By making use of the explicit expressions for the UV matrices (3.22),(3.23),(3.25) in the formulae above with A_{UV}^{-1} at the 1st order and $\tilde{B}_{UV}^{-1}$ at the 0-th order (i.e. $\tilde{B}_{UV}^{-1} = (i\not{D})^{-1}$) we get

$$\begin{aligned}
\hat{O}_{4f,y^2y_f^2} \subset \sum_{f_L,f_R} & i\,\mathrm{tr}\left\{\bar{u}_R y\frac{1}{i\not{D}-m}y^\dagger u_R\frac{1}{-D^2-m_h^2}\left(\bar{f}_R y_f\frac{1}{i\not{D}}y_f^\dagger f_R\right)\right\}_d \\
&+ i\,\mathrm{tr}\left\{\bar{u}_R y\frac{1}{i\not{D}-m}y^\dagger u_R\frac{1}{-D^2-m_h^2}\left(\bar{f}_L y_f^\dagger\frac{1}{i\not{D}}y_f f_L\right)\right\}_d
\end{aligned} \tag{3.51}$$

with $f_{L,R}$ being the Standard Model fermions (we are working in the pre SSB phase) and y_f the relative Yukawa coupling.
The evaluation of these terms is straightforward. The d prescription (3.29) here just consists in substituting $O^{-1} \to O_{NL}$.Furthermore we just need the CDE at the lowest order, so that for the right handed fermions we have

$$\begin{aligned}
\hat{O}_{4f,y^2y_f^2,f_R} \subset & \frac{i}{m^2}\mathrm{tr}\left\{\bar{u}_R y\frac{(i\not{D}+m)(i\not{D})^2}{-\not{D}^2-m}y^\dagger u_R\frac{1}{-D^2-m_h^2}\left(\bar{f}_R y_f\frac{i\not{D}}{-\not{D}^2}y_f^\dagger f_R\right)\right\} \\
&= \frac{2i}{m^2}\mathrm{tr}\int d^4x d^4p\frac{1}{p^2-m_h^2}\bar{u}_R y\frac{\not{p}p^2}{p^2-m^2}y^\dagger u_R\frac{1}{p^2-m_h^2}\bar{f}_R y_f\frac{\not{p}}{p^2}y_f^\dagger f_R \\
&= \frac{2i}{m^2}\mathrm{tr}\int d^4x\left(\bar{u}_R y\gamma^\mu y^\dagger u_R\bar{f}_R y_f\gamma^\nu y_f^\dagger f_R\right)\int d^4p\frac{p_\mu p_\nu}{(p^2-m_h^2)^2(p^2-m^2)} \\
&= \frac{1}{16\pi^2 m^2}\left(\frac{3}{4}\frac{1-m_h^2/3m^2}{1-m_h^2/m^2} - \frac{m_h^2}{m^2}\frac{1-m_h^2/2m^2}{(1-m_h^2/m^2)^2}\log\left(\frac{m^2}{m_h^2}\right)\right. \\
&\left. + \frac{1}{2\epsilon} + \frac{1}{2}\log\left(\frac{\mu^2}{m^2}\right) - \frac{\gamma}{2} + \frac{1}{2}\log 4\pi\right)\times\left(\bar{u}_R y\gamma^\mu y^\dagger u_R\bar{f}_R y_f\gamma_\mu y_f^\dagger f_R\right).
\end{aligned} \tag{3.52}$$

Now again we keep the lowest order in $(m_h/m)^2$, we take the final trace and we match at $\mu = m$, subtracting the divergences with the $\overline{\mathrm{MS}}$ scheme. The operator then becomes

$$\hat{O}_{4f,y^2y_f^2,f_R} = \frac{1}{16\pi^2 m^2}\cdot\frac{3}{2}\left(\bar{u}_R y\gamma^\mu y^\dagger u_R\bar{f}_R y_f\gamma_\mu y_f^\dagger f_R\right). \tag{3.53}$$

The expression for the left-handed SM fermions is identical upon the substitution $f_R \to f_R$ and inverting the order of $y_f, y_f^\dagger$ in the expression above.

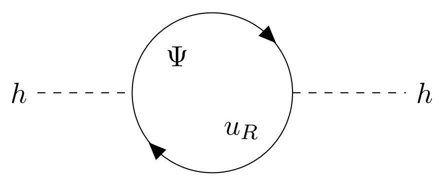
In the end, the final 4-fermions lagrangian at 1-loop reads

$$\begin{aligned}\mathcal{L}^{(1)}_{EFT,4f} = \frac{1}{16\pi^2 m^2}\Bigg[&\left(-yy^\dagger\cdot yy^\dagger + \frac{3}{2}yy^\dagger\cdot y_u y_u^\dagger\right)\bar{u}_R\gamma^\mu u_R \bar{u}_R\gamma_\mu u_R \\ &+\left(\frac{3}{2}yy^\dagger\cdot y_d y_d^\dagger\right)\bar{u}_R\gamma^\mu u_R \bar{d}_R\gamma_\mu d_R \\ &+\left(\frac{3}{2}yy^\dagger\cdot y_u^\dagger y_u + \frac{3}{2}yy^\dagger\cdot y_d^\dagger y_d\right)\bar{u}_R\gamma^\mu u_R \bar{Q}_L\gamma_\mu Q_L \\ &+\left(\frac{3}{2}yy^\dagger\cdot y_\ell y_\ell^\dagger\right)\bar{u}_R\gamma^\mu u_R \bar{\ell}_R\gamma_\mu \ell_R + \left(\frac{3}{2}yy^\dagger\cdot y_\ell^\dagger y_\ell\right)\bar{u}_R\gamma^\mu u_R \bar{L}_L\gamma_\mu L_L\Bigg].\end{aligned} \tag{3.54}$$

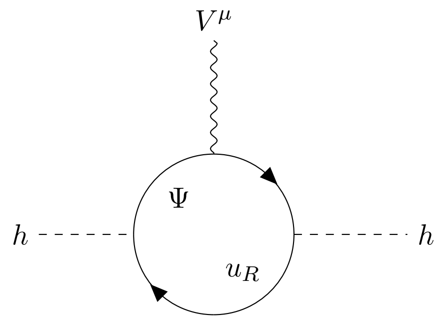
The flavor structure is the same of (3.47),(3.53) but for brevity the coefficient has been moved in front of the operator.

3.2.3 Two Higgs

This and the next section will be dedicated to the derivation of the relevant operators in $\mathcal{L}_{SILH}$. In this section the focus will be on the operators involving two Higgs, while in the next one we will compute the four Higgs fields terms.



(a) Diagram involving only two external Higgs.



(b) Diagram involving two external Higgs and one vector boson.

Figure 3.3: Example of diagrams contributing to the 2 Higgs operators.

The two Higgs operators are generated from diagrams like the ones in figure 3.3. The ones involving two external Higgs and one or more gauge bosons are obtained by inserting the external leg of the vector field between the u_R or Ψ propagators.

With reference to the formula (3.19), these operators are encoded in the first order of the $-i\left(\log\det\tilde{B}_{UV} - \log\det\tilde{B}_{UV}\right)$ term of (3.19) since we need two external Higgs boson:

$$\begin{aligned}\hat{O}_{2h} \subset -i\left(\log\det\tilde{B}_{UV} - \log\det\tilde{B}_{UV}\right) &= -i\mathrm{tr}\log\left\{i\not{D}(1+(i\not{D})^{-1}\Omega_B)\right\}_d \\ &\simeq -i\mathrm{tr}\left((i\not{D})^{-1}\Omega_B\right)_d + \dots \\ \to \hat{O}_{2h} \subset \frac{i}{m^2}\mathrm{tr}\left\{\frac{1}{i\not{\partial}}\left(H^\dagger y\frac{(i\not{D})^2}{i\not{D}-m}y^\dagger H\right)\right\}\end{aligned} \tag{3.55}$$

where the d prescription (3.29) has already been applied. Since we have a gauge singlet in the brackets the external covariant derivative has been substituted with a partial derivative $D_\mu \to \partial_\mu$.

In order to obtain all the operators up to dimension six we must apply the CDE to the denominators of (3.55). To do this, first we bring the γ^μ matrices to the numerator

$$\hat{O}_{2h} \subset \frac{i}{m^2}\mathrm{tr}\left\{\frac{i\not{\partial}}{(i\partial)^2}\left(H^\dagger y\frac{(i\not{D}+m)}{-D^2-m^2-U}(i\not{D})^2 y^\dagger H\right)\right\}. \tag{3.56}$$

The operator U comes from $-\not{D}^2 = -D^2 - U$, $U = -\frac{i}{2}\sigma^{\mu\nu}G'_{\mu\nu}$.
Then the formalism of the CDE must be applied. The consequences of the shift $i\not{D} \to i\not{D} - \not{p}$ are

$$\begin{cases}(i\not{D})^2 = p^2 - 2ip^\mu D_\mu - D^2 - U \\ \frac{1}{(i\partial)^2} = \sum_n \frac{1}{(p^2)^{n+1}}\left(2ip^\mu\partial_\mu + \partial^2\right)^n \\ \frac{1}{-D^2-m^2-U} = \sum_n \frac{1}{(p^2-m^2)^{n+1}}\left(2ip^\mu D_\mu + D^2 + U\right)^n\end{cases} \tag{3.57}$$

where n labels the order of the denominator's expansion. Also, the shift in the numerators reads $(i\not{\partial}) \to (i\not{\partial} - \not{p}), (i\not{D}+m) \to (i\not{D} - \not{p} + m)$.

The full calculation of all the traces resulting from this expansion is lengthy, so the derivation will be reported only schematically. Indeed, let's have a closer look at the equation 3.56. First we focus on the term with two $\not{p}$'s at the numerator [5], where the slashed momentum comes from the shift described above. We call this term $\hat{O}_{2h,1}$. A compact notation will be employed, including $\int d^4x d^4p$ in the definition of trace. In this way we must compute the operators up to dimension six generated by [6]

$$\hat{O}_{2h,1} = \frac{2i}{m^2}\mathrm{tr}\left\{\frac{\not{p}}{(iD-p)^2}\left(H^\dagger y\frac{\not{p}(p^2-2ip^\mu D_\mu - D^2 - U)}{(iD-p)^2-m^2-U}y^\dagger H\right)\right\}. \tag{3.58}$$

The various orders of the denominators' expansion can be labelled by $n = n_L + n_R$, where n_L and n_R are the orders at which the left and the right denominators are expanded. Then we can look at the generated operators at fixed n:

[5] The term with the m from $(i\not{D}-m)$ at the numerator will never contribute, since with this term one would always have $\mathrm{tr}\#\mathrm{odd}\gamma_\mu = 0$.

[6] The factor of 2 comes from the Leibniz rule. In fact actually the CDE must be performed after applying the operator $(i\not{D})^{-1}$ to $\sim\left(H^\dagger O_{NL}H\right)$, bringing two identical terms with the $\not{p}$ in front of everything. So we explicitly opened the derivative, performed the CDE, and resummed.

- $n = 0$: here we obtain $H^\dagger H$ and $H^\dagger D^2 H$, without gauge fields since $\mathrm{tr}\not{p}\not{p}U = \mathrm{tr}p^2 U = 0$;

- $n = 1$: we can have either $n_L = 1$ or $n_R = 1$:

$$\begin{aligned}&\frac{2i}{m^2}\mathrm{tr}\left\{\frac{\not{p}}{p^4}\left(2ip^\mu\partial_\mu + \partial^2\right)\left(H^\dagger y\frac{\not{p}(p^2 - 2ip^\mu D_\mu - D^2 - U)}{p^2 - m^2}y^\dagger H\right)\right\}\\ &+\frac{2i}{m^2}\mathrm{tr}\left\{\frac{\not{p}}{p^2}\left(H^\dagger y\frac{\not{p}\left(2ip^\mu D_\mu + D^2 + U\right)(p^2 - 2ip^\mu D_\mu - D^2 - U)}{(p^2 - m^2)^2}y^\dagger H\right)\right\}.\end{aligned} \tag{3.59}$$

It is immediate to see that the left expansion is useless: the numerator $\left(2ip^\mu\partial_\mu + \partial^2\right)$ always generates terms that are total derivatives, that can be neglected in the $\int d^4x$ of the action. For this reason we can always keep $n_L = 0$, and focus only on the expansion of the right denominator.
Having in mind that, this term then generates $H^\dagger D^2 H, H^\dagger D^4 H$ and also $\mathrm{tr}H^\dagger UUH = -2\mathrm{tr}H^\dagger G'_{\mu\nu}G'^{\mu\nu}H$;

- $n = 2$: in this case we have, keeping $n_L = 0$ as explained before,

$$\frac{2i}{m^2}\mathrm{tr}\left\{\frac{\not{p}}{p^2}\left(H^\dagger y\frac{\not{p}\left(2ip^\mu D_\mu + D^2 + U\right)\left(2ip^\mu D_\mu + D^2 + U\right)(p^2 - 2ip^\mu D_\mu - D^2 - U)}{(p^2 - m^2)^3}y^\dagger H\right)\right\}. \tag{3.60}$$

generating $H^\dagger D^2 H, H^\dagger D^4 H, H^\dagger D_\mu D^2 D^\mu H, H^\dagger G'_{\mu\nu}G'^{\mu\nu}H$. The operator with $D_\mu U$ is not present since $\mathrm{tr}\#\mathrm{odd}\ p^\mu = 0$;

- $n = 3$: at these orders we can generate only operators of dimension six or greater:

$$\begin{aligned}&\frac{2i}{m^2}\mathrm{tr}\left\{\frac{\not{p}}{p^2}\left(H^\dagger y\frac{\not{p}\left(2ip^\mu D_\mu + D^2 + U\right)\left(2ip^\mu D_\mu + D^2 + U\right)}{(p^2 - m^2)^3}\right.\right.\\ &\qquad\left.\left.\cdot\left(2ip^\mu D_\mu + D^2 + U\right)(p^2 - 2ip^\mu D_\mu - D^2 - U)y^\dagger H\right)\right\}.\end{aligned} \tag{3.61}$$

The possible combinations of the expansions are many. They generate $H^\dagger D^4 H$, but also the new operator $H^\dagger D_\mu D_\nu D_\mu D_\nu H$. This comes from

$$\int d^4p\frac{p^\mu p^\nu p^\rho p^\sigma}{p^2(p^2 - m^2)^3}H^\dagger D_\mu D_\nu D_\rho D_\sigma H$$

which provides two metric tensors gg with all the possible symmetric permutation of the indices;

- $n = 4$: this is the last order of the expansion that is needed:

$$\frac{2i}{m^2}\mathrm{tr}\left\{\frac{\not p}{p^2}\left(H^\dagger y\frac{\not p\left(2ip^\mu D_\mu + D^2 + U\right)\left(2ip^\mu D_\mu + D^2 + U\right)}{(p^2-m^2)^3}\right.\right.$$
$$\left.\left.\cdot\left(2ip^\mu D_\mu + D^2 + U\right)\left(2ip^\mu D_\mu + D^2 + U\right)(p^2 - 2ip^\mu D_\mu - D^2 - U)y^\dagger H\right)\right\}. \tag{3.62}$$

The only contribution comes from the $p^\mu p^\nu p^\rho p^\sigma$, since all the other combinations generate operators of dimension eight or greater.

The next and last term to evaluate is the one involving $i\not D$ at the right numerator of (3.56). Indeed, keeping $i\not\partial$ at the left numerator would mean having again total derivative terms, which are to be ignored upon the spacetime integration. For the same reason we must still keep $n_L = 0$. So we just need to evaluate

$$\hat{O}_{2h,2} = \frac{2i}{m^2}\mathrm{tr}\left\{\frac{\not p}{p^2}\left(H^\dagger y\frac{i\not D(p^2 - 2ip^\mu D_\mu - D^2 - U)}{(iD-p)^2 - m^2 - U}y^\dagger H\right)\right\}. \tag{3.63}$$

till dimension six. In this case the expansion stops at $n = 3$, since we already have $\sim H^\dagger DH$ at zero-th order; indeed, the lowest order generated operator here is $H^\dagger D^2 H$. The calculations are similar to the previous case, so providing the terms $H^\dagger D^4 H, H^\dagger D^2 H$, $H^\dagger D_\mu D^2 D^\mu H, H^\dagger D_\mu D_\nu D^\nu D^\mu H$. However in this case there are also some new operators involving one gauge boson field strength. Indeed, these kind of contributions must pass by the trace over the spinorial space $\mathrm{tr}\left(\gamma^\mu\gamma^\nu\sigma^{\alpha\beta}\right)\mathrm{tr}\left(X^a_{\mu\nu}G'^a_{\alpha\beta}\right)$, where X includes all the Higgs fields and the covariant derivatives. In the case with two $\not p$ at the numerator the quantity X had to be symmetric in the Lorentz indices because the gamma matrices come from $\gamma^\mu p_\mu \gamma^\nu p_\nu$. Then using $\frac{-i}{2}\mathrm{tr}\left(\gamma^\mu\gamma^\nu\sigma^{\alpha\beta}\right)G'^a_{\alpha\beta} = -4G'^a_{\mu\nu}$ we are left with the contraction $\mathrm{tr}G'^a_{\mu\nu}X^{a\mu\nu}$, which is null because the field strength is antisymmetric by definition. This does not hold for the case with $i\not D$ at the right numerator, for which $X_{\mu\nu}$ is not symmetric: consequently, in this computation also the terms $H^\dagger D_\mu G'^{\mu\nu} D_\nu H$, $H^\dagger D_\mu D_\nu G'^{\mu\nu} H$, $H^\dagger G'^{\mu\nu} D_\mu D_\nu H$ are present.

Once the computation of the two cases are completed the results must be added up. The "raw" terms that we get are:

$$\begin{aligned}\hat{O}_{2h} = &\left(\mu^2 + \frac{3}{2\pi^2}|y|^2 m^2\right)H^\dagger H + \left(1 + \frac{15|y|^2}{8\pi^2}\right)D_\mu H^\dagger D^\mu H \\ &- \frac{2|y|^2}{16\pi^2 m^2}\mathrm{tr}\left\{H^\dagger D^4 H\left(-\frac{5}{9}\right) + H^\dagger D_\mu D_\nu D^\mu D^\nu H\left(-\frac{2}{9}\right) + H^\dagger D^\mu D^2 D_\mu H\left(\frac{1}{9}\right)\right. \\ &+ H^\dagger G'_{\mu\nu}G'^{\mu\nu}H(1) + H^\dagger D_\mu D_\nu G'^{\mu\nu}H(-1) + H^\dagger G'^{\mu\nu}D_\mu D_\nu H(-2) \\ &\left.+ H^\dagger D_\nu G'^{\mu\nu} D_\mu H(1)\right\}.\end{aligned} \tag{3.64}$$

Here the matching has been performed at $\mu = m$, so all the $\log(\mu^2/m^2)$ factors coming from the divergent integrals are null. Also, the divergences have already been eliminated using the $\overline{\text{MS}}$ scheme.

Now this result must be converted in the SILH basis. First we notice that in (3.64) a lot of operators are redundant, and can be eliminated by means of the following identities[7]

$$\begin{cases} H^\dagger D_\mu D_\nu D_\mu D_\nu H = D^2 H^\dagger D^2 H + D_\mu H^\dagger G'^{\mu\nu} D_\nu H \\ H^\dagger D^\mu D^2 D_\mu H = 2 D_\mu H^\dagger G'^{\mu\nu} D_\nu H + D_\mu H^\dagger D_\nu G'^{\mu\nu} H + D^2 H^\dagger D^2 H \end{cases} \tag{3.65}$$

together with integration by parts.
Then the operators $(H^\dagger \overleftrightarrow{D}_\mu H) D_\nu G'^{\mu\nu}$ can be obtained using the following decomposition

$$ax + by = \frac{1}{2}(a+b)(x+y) + \frac{1}{2}(a-b)(x-y)$$

$$\begin{aligned} &x = D_\mu H^\dagger D_\nu G'^{\mu\nu} H, && y = H^\dagger D_\nu G'^{\mu\nu} D_\mu H, \\ &(x+y) = -\frac{1}{2} H^\dagger [D_\mu, D_\nu] G'^{\mu\nu} H = H^\dagger G'_{\mu\nu} G'^{\mu\nu} H, && (x-y) = (H^\dagger \overleftrightarrow{D}^a_\mu H) D_\nu G'^{a\mu\nu}. \end{aligned} \tag{3.66}$$

where the result for $(x+y)$ comes from integration by parts:
$D_\mu H^\dagger D_\nu G'^{\mu\nu} H = -H^\dagger D_\mu D_\nu G'^{\mu\nu} H - H^\dagger D_\nu G'^{\mu\nu} D_\mu H$ and $D_\mu D_\nu G'^{\mu\nu} = \frac{1}{2} G'_{\mu\nu} G'^{\mu\nu}$.
Putting this together with the final trace we obtain the following dimension six operators:

$$\begin{aligned} \hat{O}_{2h|dim.6} = -\frac{2|y|^2}{16\pi^2 m^2} \Big\{ & D^2 H^\dagger D^2 H\,(-2) + ig \left(D_\mu H^\dagger \sigma^i D_\nu H \right) W^{\mu\nu i}\,(-3) \\ & + ig' \left(D_\mu H^\dagger D_\nu H \right) B^{\mu\nu}\,(-6Y) + ig \left(H^\dagger \sigma^i \overleftrightarrow{D}_\mu H \right) D_\nu W^{i\mu\nu} \left(-\frac{5}{6} \right) \\ & + ig' \left(H^\dagger \overleftrightarrow{D}_\mu H \right) \partial_\nu B^{\mu\nu} \left(-\frac{5}{3} Y \right) + g^2 |H|^2 W^{i\mu\nu} W^i_{\mu\nu} \left(-\frac{1}{24} \right) \\ & + g'^2 |H|^2 B_{\mu\nu} B^{\mu\nu} \left(-\frac{1}{6} Y^2 \right) + g g' (H^\dagger \sigma^i H) W^i_{\mu\nu} B^{\mu\nu} \left(-\frac{1}{12} Y \right) \\ & + g_s^2 |H|^2 G^{\mu\nu a} G^a_{\mu\nu} \left(-\frac{1}{36} \right) \Big\}. \end{aligned} \tag{3.67}$$

The parameter Y is the new fermion hypercharge, $Y = 7/6$.
Three operators not belonging to SILH basis are left: $|H|^2 W^{\mu\nu i} W^i_{\mu\nu}$, $H^\dagger \sigma^i H W^{i\mu\nu} B_{\mu\nu}$ and $D^2 H^\dagger D^2 H$. The first two can be converted using the identities

$$\begin{cases} \frac{ig}{2} \left(H^\dagger \sigma^i \overleftrightarrow{D}_\mu H \right) D_\nu W^{i\mu\nu} = & ig \left(D_\mu H^\dagger \sigma^i D_\nu H \right) W^{\mu\nu i} \\ & + \frac{1}{4} \left(g^2 |H|^2 W^{i\mu\nu} W^i_{\mu\nu} + g g' \left(H^\dagger \sigma^i H \right) W^i_{\mu\nu} B^{\mu\nu} \right) \\ \frac{ig'}{2} \left(H^\dagger \overleftrightarrow{D}_\mu H \right) \partial_\nu B^{\mu\nu} = & ig' \left(D_\mu H^\dagger D_\nu H \right) B^{\mu\nu} \\ & + \frac{1}{4} \left(g'^2 |H|^2 B^{\mu\nu} B_{\mu\nu} + g g' \left(H^\dagger \sigma^i H \right) W^i_{\mu\nu} B^{\mu\nu} \right) \end{cases}. \tag{3.68}$$

[7] These can be derived using integration by parts and $[D_\mu, D_\nu] = G'_{\mu\nu}$.

The last one, instead, can be eliminated using the equations of motion of the Higgs field

$$D^2 H = \mu^2 H - 2\lambda H|H|^2 + j_H, \quad j_H = -\bar{d}_R y_d Q_L - \bar{l}_R y_l L_l + \bar{Q}^m_{Lj}\epsilon_{ij} y^*_{u,nm} u_{Rn}. \tag{3.69}$$

However the substitution in $D^2H^\dagger D^2 H$ generates operators classified as non relevant in section 2.3. This means that this term can be dropped. Also, the corrections to the Higgs mass and to the quartic interaction λ can be reabsorbed in the definition of the EFT parameters.

All in all, the final expression for the two Higgs operators in the SILH basis reads

$$\begin{aligned}
\hat{O}_{2h} = &\left(\mu^2 + \frac{3}{2\pi^2}|y|^2 m^2\right) H^\dagger H + \left(1 + \frac{15|y|^2}{8\pi^2}\right) D_\mu H^\dagger D^\mu H \\
&+ \frac{|y|^2}{16\pi^2 m^2}\left\{ ig\left(H^\dagger \sigma^i \overleftrightarrow{D_\mu} H\right) D_\nu W^{i\mu\nu}\left(\frac{11}{6}\right) + ig'\left(H^\dagger \overleftrightarrow{D_\mu} H\right)\partial_\nu B^{\mu\nu}\left(\frac{11}{3}Y - \frac{1}{6}\right)\right. \\
&+ ig\left(D_\mu H^\dagger \sigma^i D_\nu H\right) W^{\mu\nu i}\left(\frac{35}{6}\right) + ig'\left(D_\mu H^\dagger D_\nu H\right) B^{\mu\nu}\left(\frac{35}{3}Y + \frac{1}{6}\right) \\
&\left.+ g'^2 |H|^2 B_{\mu\nu} B^{\mu\nu}\left(\frac{1}{3}\right)\left(Y^2 - \frac{Y}{2} + \frac{1}{4}\right) + g_s^2 |H|^2 G^{\mu\nu a} G^a_{\mu\nu}\left(\frac{1}{18}\right)\right\}.
\end{aligned} \tag{3.70}$$

3.2.4 Four Higgs

The last operators left to be derived are the ones involving four Higgs fields. These come from diagrams like the ones in the figures 3.4, from which it is clear that there are two kind of contributions. These contributions have coefficients proportional either to y^4 or to $y^2 y_u^2$ and must be summed: $\hat{O}_{4h} = \hat{O}_{4h,y^4} + \hat{O}_{4h,y^2y_u^2}$.

(a) Diagram involving only two external Higgs.

(b) Diagram involving two external Higgs and one vector boson.

Figure 3.4: Example of diagrams contributing to the 4 Higgs operators.

The $|y|^4$ operators are encoded in $-i\left(\log\det\tilde{B}_{UV} - \log\det\tilde{B}_{EFT}\right)$ of (3.19). Indeed the determinant expansion is identical to the $\hat{O}_{2h}$ one (3.55), but this time the second order must be considered:

$$\begin{aligned}
\hat{O}_{4h,y^4} &\subset \frac{i}{2m^2}\mathrm{tr}\left\{(i\not{D})^{-1}\Omega_B (i\not{D})^{-1}\Omega_B\right\}_d \\
&\subset \frac{i}{2m^2}\mathrm{tr}\left\{(i\not{D})^{-1}\left(H^\dagger y \frac{1}{i\not{D} - m} y^\dagger H\right)(i\not{D})^{-1}\left(H^\dagger y\frac{1}{i\not{D} - m} y^\dagger H\right)\right\}_d.
\end{aligned} \tag{3.71}$$

Using the "d" prescription (3.29) to remove the local counterparts and exploiting the cyclicity of the trace the expression that must be evaluated becomes

$$\begin{aligned}\hat{O}_{4h,y^4} \subset & 2 \cdot \frac{i}{2m^2}\mathrm{tr}\left\{(i\not{D})^{-1}\left(H^\dagger y\frac{(i\not{D})^2}{i\not{D}-m}y^\dagger H\right)(i\not{D})^{-1}\left(H^\dagger y\frac{1}{i\not{D}-m}y^\dagger H\right)\right\} \\ & - \frac{i}{2m^4}\mathrm{tr}\left\{(i\not{D})^{-1}\left(H^\dagger y\frac{(i\not{D})^2}{i\not{D}-m}y^\dagger H\right)(i\not{D})^{-1}\left(H^\dagger y\frac{(i\not{D})^2}{i\not{D}-m}y^\dagger H\right)\right\}. \\ & \equiv \hat{O}_{4h,y^4,a} + \hat{O}_{4h,y^4,b} \end{aligned} \tag{3.72}$$

Also in this case the calculations are fairly involved. Focusing on the first term, once we apply the CDE we must compute

$$\begin{aligned}\hat{O}_{4h,y^4,a} =& \frac{i}{m^2}\mathrm{tr}\left\{\frac{(i\not{\partial}-\not{p})}{(i\partial-p)^2}\left(H^\dagger y\frac{(i\not{D}-\not{p}+m)(p^2-2ip^\mu D_\mu - D^2 - U)}{(iD-p)^2-m^2}y^\dagger H\right)\right. \\ & \left.\cdot\frac{(i\not{\partial}-\not{p})}{(i\partial-p)^2}\left(H^\dagger y\,\frac{(i\not{D}-\not{p}+m)}{(iD-p)^2-m^2}y^\dagger H\right)\right\}\end{aligned} \tag{3.73}$$

where the property $i\not{D}(H^\dagger H) = i\not{\partial}(H^\dagger H)$ has been exploited.
We can already tell that no operators involving $G'_{\mu\nu}$ will appear: we start already with a dimension four H^4, so the field strength might appear only as $G^\alpha_\alpha = 0$. This shortens the computations since it means that the term U can be neglected in both the numerators and the denominators.

From now on the calculations are identical to the two Higgs case, except this time more combinations are possible since we have four denominators to be expanded and many $i\not{D}, \not{p}$ alternatives for the numerators can be picked. We can also keep both m in the round brackets of (3.73), generating a $\propto m^2$ trace.
Anyway, the expansion of the denominators must be carried out only up to second order to get dimension six operators. The generated terms will be of the kind H^4 and DDH^4, i.e. all the possible permutations of two covariant derivatives applied to the four Higgs field. The same reasoning applies for $\hat{O}_{4h,y^4,b}$, whose calculation is a little bit longer since we have another set of possible combinations from the CDE of $(i\not{D})^2$ at the numerator. All in all, the raw result obtained from evaluating (3.72) is

$$\begin{aligned}\hat{O}_{4h,y^4} =& \frac{|y|^4}{16\pi^2}\,(4)\,\mathrm{tr}(H^\dagger H)^2 - \frac{|y|^4}{16\pi^2 m^2}\mathrm{tr}\left\{H^\dagger D^2 H H^\dagger H\left(-\frac{1}{3}\right)\right. \\ & \left. + H^\dagger D_\mu H H^\dagger D^\mu H\left(\frac{25}{6}\right)\right\}\end{aligned} \tag{3.74}$$

The operators appearing in (3.74) are to be manipulated a little bit. To move to SILH basis, we first write the D^2 operator using integration by parts

$$H^\dagger D^2 H H^\dagger H = -D_\mu H^\dagger D^\mu H H^\dagger H - H^\dagger D_\mu H D^\mu H^\dagger H - H^\dagger D_\mu H H^\dagger D^\mu H \tag{3.75}$$

The non-Hermitian piece $HDHHDH$ is present since we used the Dirac Lagrangian for the fermions; indeed we started from a non-Hermitian quantity (3.71). However it is

known that the Dirac lagrangian can be rewritten as a symmetric lagrangian, up to total derivatives. As a consequence, also the apparently problematic term must be written in terms of total derivatives and therefore it can be neglected.
The operator $H^\dagger D_\mu H D^\mu H^\dagger H$ can be eliminated by making use of the following identity, which be easily derived using integration by parts

$$4|H^\dagger D_\mu H|^2 = \left(\partial_\mu |H|^2\right)^2 - \left(H^\dagger \overleftrightarrow{D_\mu} H\right)^2 . \tag{3.76}$$

Plugging this relation in the lagrangian (3.74) and performing the last trace what is left is

$$\begin{aligned}\hat{O}_{4h,y^4} =& \frac{12|y|^4}{16\pi^2}(H^\dagger H)^2 - \frac{|y|^4}{16\pi^2 m^2}\Big\{D_\mu H^\dagger D^\mu H H^\dagger\,(1)\\ &\left(\partial_\mu|H|^2\right)^2\left(\frac{1}{2}\right) + \left(H^\dagger \overleftrightarrow{D_\mu} H\right)^2\left(-\frac{1}{4}\right)\Big\}.\end{aligned} \tag{3.77}$$

In this expression it is possible to recognize the $\bar{c}_T, \bar{c}_H$'s operators of the SILH lagrangian. The additional $D_\mu H^\dagger D^\mu H H^\dagger H$ is not independent and can be eliminated exploiting the equations of motion for H, that provide

$$D_\mu H^\dagger D^\mu H H^\dagger H = \left(\frac{1}{2}\partial^2|H|^2 - \mu^2|H|^2 + 2\lambda|H|^4 - \frac{1}{2}H^\dagger j_H - \frac{1}{2}j_H^\dagger H\right)|H|^2. \tag{3.78}$$

The first term contributes to $\bar{c}_H$ and is therefore relevant. The other ones can be neglected. Those proportional to j_H have been argued to provide negligible constraints, the μ^2 enters in the redefinition of the quartic interaction in the EFT and finally H^6 contributes to the coefficient $\bar{c}_6$. The experimental constraints on $\bar{c}_6$ are very weak since it involves Higgs self-interactions, and for this reason we will ignore it. Finally, then, the y^4 lagrangian reads

$$\hat{O}_{4h,y^4} = -\frac{3|y|^4}{4\pi^2}(H^\dagger H)^2 - \frac{|y|^4}{16\pi^2 m^2}\left\{\left(\partial_\mu|H^\dagger H|^2\right)^2\left(\frac{1}{4}\right) + \left(H^\dagger \overleftrightarrow{D_\mu} H\right)^2\left(-\frac{1}{4}\right)\right\}. \tag{3.79}$$

Now the only contribution left to be computed is the one from the diagram 3.4(b). This term involves one Ψ and three u_R propagators, therefore it is again encapsulated in $-i\left(\log\det\tilde{B}_{UV} - \log\det\tilde{B}_{EFT}\right)$ of (3.19). Indeed, the third order of the determinant expansion contains exactly this term

$$\begin{aligned}\hat{O}_{4h,y^2y_u^2} &\subset -i\left(\log\det\tilde{B}_{UV} - \log\det\tilde{B}_{EFT}\right)\\ &= -i\log\det\left\{i\not{D}\left(1+(i\not{D})^{-1}\Omega_B\right)\right\}_d\\ &= i3m^2\mathrm{tr}\left\{3\cdot\frac{1}{i\not{D}}\left(H^\dagger y\frac{(i\not{D})^2}{i\not{D}-m}y^\dagger H\right)\frac{1}{i\not{D}}y_u\tilde{H}^\dagger\frac{1}{i\not{D}}y_u^\dagger\tilde{H}\right\}+\dots\end{aligned} \tag{3.80}$$

Applying the CDE this expression reads

$$\hat{O}_{4h,y^2y_u^2} \subset \frac{i}{m^2} y^\dagger y_u y_u^\dagger y \times \mathrm{tr}\left\{ \frac{i\not{\partial} - \not{p}}{(i\partial - p)^2} \left(H^\dagger \frac{(i\not{D} - \not{p} + m)(p^2 - 2ip \cdot D - D^2)}{(iD - p)^2 - m^2} H \right) \frac{i\not{D} - \not{p}}{(iD - p)^2} H^\dagger \frac{i\not{D} - \not{p}}{(iD - p)^2} H \right\} \tag{3.81}$$

where it has been used $\tilde{H}^\dagger \tilde{H} = -H^T \varepsilon\varepsilon (H^\dagger)^T = H^\dagger H$. The U terms have already been dropped for the same reason as for the y^4 case.

The calculations are very similar to the first case, except here no term involving m at the numerator is present. The denominator must be expanded up to $n = 2$, and the the same kind of operators are generated. In the end, the raw result in $\overline{\text{MS}}$ scheme is

$$\begin{aligned} \hat{O}_{4h,y^2y_u^2} =& -\frac{y^\dagger y_u y_u^\dagger y}{16\pi^2}(4)\mathrm{tr}(H^\dagger H)^2 - \frac{y^\dagger y_u y_u^\dagger y}{16\pi^2 m^2}\mathrm{tr}\left\{ D_\mu H^\dagger D^\mu H H^\dagger H\,(-15) \right. \\ &+ H^\dagger D_\mu H H^\dagger D^\mu H(3) + H^\dagger D_\mu H D^\mu H^\dagger H(-4) \\ &\left. + \left(\partial_\mu |H|^2\right)(-8) \right\}. \end{aligned} \tag{3.82}$$

Now applying the identities (3.75), (3.76),(3.78) and tracing over the gauge group this term reads

$$\begin{aligned} \hat{O}_{4h,y^2y_u^2} =& -\frac{3y^\dagger y_u y_u^\dagger y}{4\pi^2}(H^\dagger H)^2 - \frac{y^\dagger y_u y_u^\dagger y}{16\pi^2 m^2}\left\{ \left(\partial_\mu |H^\dagger H|^2\right)^2 \left(+\frac{9}{2}\right) \right. \\ &\left. + \left(H^\dagger \overleftrightarrow{D_\mu} H\right)^2 (-9) \right\}. \end{aligned} \tag{3.83}$$

At this point the two results (3.79), (3.83) can be put together, leading to the final expression for the four Higgs operators lagrangian:

$$\begin{aligned} \hat{O}_{4h} =& -\frac{3}{4\pi^2}\left[|y|^4 \ + y^\dagger y_u y_u^\dagger y \right](H^\dagger H)^2 \\ &+ \frac{1}{16\pi^2 m^2}\left\{ \frac{1}{2}\left(\partial_\mu |H^\dagger H|^2\right)^2 \left(-\frac{1}{2}|y|^4 - 9y^\dagger y_u y_u^\dagger y\right) \right. \\ &\left. + \frac{1}{2}\left(H^\dagger \overleftrightarrow{D_\mu} H\right)^2 \left(\frac{1}{2}|y|^4 + 18y^\dagger y_u y_u^\dagger y\right) \right\}. \end{aligned} \tag{3.84}$$

We conveniently summarize the coefficients obtained from the tree-level and 1-loop computations in the tables 3.1, 3.2. The notation for the coefficients of the four fermions operators is obvious and refers to (3.54). Note that these are not defined to be adimensional, and the subscripts represent the flavor indices.

Coefficient	Value	Coefficient	Value
$\frac{m^2}{v^2}\bar{c}_{Hu}$	$\frac{yy^\dagger}{2}$	$\frac{m^2}{v^2}\bar{c}_u$	$-\frac{yy^\dagger}{2}$

Table 3.1: Coefficients obtained at tree-level. The basis is the SILH one, (2.18).

Coefficient	Value $\times 16\pi^2$	Coefficient	Value $\times 16\pi^2$
$\frac{m^2}{v^2}\bar{c}_H$	$-\frac{1}{2}\lvert y\rvert^4 - 9y^\dagger y_u y_u^\dagger y$	$\frac{m^2}{v^2}\bar{c}_T$	$\frac{1}{2}\lvert y\rvert^4 + 18y^\dagger y_u y_u^\dagger y$
$\frac{m^2}{m_W^2}\bar{c}_W$	$\frac{11}{3}Y\lvert y\rvert^2$	$\frac{m^2}{m_W^2}\bar{c}_B$	$\left(\frac{22}{3}Y - \frac{1}{3}\right)\lvert y\rvert^2$
$\frac{m^2}{m_W^2}\bar{c}_{HW}$	$\frac{35}{3}\lvert y\rvert^2$	$\frac{m^2}{m_W^2}\bar{c}_{HB}$	$\left(\frac{70}{3}Y + \frac{1}{3}\right)\lvert y\rvert^2$
$\frac{m^2}{m_W^2}\bar{c}_\gamma$	$\left(\frac{Y^2}{3} - \frac{Y}{6} + \frac{1}{12}\right)\lvert y\rvert^2$	$\frac{m^2}{m_W^2}\bar{c}_g$	$\frac{1}{18}\lvert y\rvert^2$
$\frac{m^2}{m_W^2}\bar{c}_{3W}$	$-\frac{1}{60}$	$\frac{m^2}{m_W^2}\bar{c}_{3G}$	$-\frac{1}{90}$
$\frac{m^2}{m_W^2}\bar{c}_{2W}$	$-\frac{1}{5}$	$\frac{m^2}{m_W^2}\bar{c}_{2B}$	$-\frac{4}{5}Y^2$
$\frac{m^2}{m_W^2}\bar{c}_{2G}$	$-\frac{2}{15}$		
$m^2 c^u_{ijkl}$	$-(yy^\dagger)_{ij}(yy^\dagger)_{kl} + \frac{3}{2}(yy^\dagger)_{ij}(y_u y_u^\dagger)_{kl}$	$m^2 c^{ud(1)}_{ijkl}$	$\frac{3}{2}(yy^\dagger)_{ij}(y_d y_d^\dagger)_{kl}$
$m^2 c^{Qu(1)}_{ijkl}$	$\frac{3}{2}(yy^\dagger)_{ij}(y_u^\dagger y_u)_{kl} + \frac{3}{2}(yy^\dagger)_{ij}(y_d^\dagger y_d)_{kl}$		
$m^2 c^{Lu}_{ijkl}$	$\frac{3}{2}(yy^\dagger)_{ij}(y_\ell^\dagger y_\ell)_{kl}$	$m^2 c^{u\ell}_{ijkl}$	$\frac{3}{2}(yy^\dagger)_{ij}(y_\ell y_\ell^\dagger)_{kl}$

Table 3.2: Coefficients obtained at 1-loop. The basis is the SILH one, (2.18). The first two blocks of the table comprehend operators that are part of $\mathcal{L}_{SILH}$, the third of $\mathcal{L}_V$ and the fourth of $\mathcal{L}_{4f}$.

3.3 Experimental bounds

In the previous section all the 1-loop operators classified in 2.3 as potentially relevant have been derived, meaning we should be ready to compare the obtained results with the constraints present in literature in order to get some new insights about the coupling and the mass of the new fermion. However the coefficients have been found by matching the UV theory and the EFT at the new scale $\mu = m$, so one last step needs to be done before performing that comparison. Indeed the constraints are obtained from low energy observables ($E \sim m_Z$), so we should first translate the EFT to that energy scale. This can be done thanks to the Renormalization Group Equation, which tells us how the renormalized quantities vary with the renormalization scale μ. In particular we are interested in seeing how the coefficients of dimension six operators change while moving through the Renormalization Group flow, and so moving from a higher to a lower energy scale. The effect of this scaling is not only a "diagonal" change in the values of the Wilson coefficients but also a mixing between different operators, generating non-trivial combinations of the C_i's and the appearance of new dimension six terms.

In dimensional regularization the running is described by the equation

$$\mu\frac{\partial}{\partial\mu}C_i = \gamma_{ij}C_j, \tag{3.85}$$

where γ_{ij} is the *anomalous dimension matrix.* The computation of γ_{ij} (a 59×59 matrix, not counting flavor indices) is quite a hard task and has been entirely performed at 1-loop in the huge works [30], [31], [32] where the results are given in the Warsaw basis, while only partially in the SILH basis in [33]
In the case of our interest, however, we don't need to run all the coefficients. In fact the operators obtained in this chapter come from a 1-loop computation, so evaluating the running would mean going for a 2-loop result. What we should run, instead, are the two operators obtained at tree-level 2.24 to get a consistent 1-loop analysis. Since this would be a quite involved computation we are going to first see when actually it is the case to perform the running, and then eventually evaluate it thanks to the equation (3.85).

The first bound we're going to look for is on $|y|/m$, to see if there's any improvement with respect to the results that we obtained at tree-level (2.30). These kind of bounds can be obtained from the two-Higgs terms (3.70) and from the four fermions ones.
Let's focus on the first ones. The constraints on the coefficients of all these operators can be found in the recent work [21], which uses electroweak precision data from LEP together with data from LHC Run 1 & 2. The strongest bound is on the Higgs-gluon operator $|H|^2G^{\mu\nu a}G_{\mu\nu a}$, for which $|\bar{c}_g| < 10^{-5}$.
This operator is present in both SILH and Warsaw basis, and the conversion from one basis to the other leaves it untouched. Since also our tree level operators are present in both the bases[8] we can directly look at the RGE from the works [30]- [32], in which the running for C_{HG} (the equivalent of $\bar{c}_g$ in Warsaw) does not involve $\hat{O}_{Hu}, \hat{O}_u$[9].
This means that the running of that coefficient doesn't need to be done at this order, thus we can directly use the result from (3.70): $|\bar{c}_g| = \frac{1}{18}\frac{|y|^2}{16\pi^2}\frac{m_W^2}{m^2}$.
Comparing that with the constraint reported above we obtain

$$|y| < 2.1\cdot\left(\frac{m}{\mathrm{TeV}}\right). \tag{3.86}$$

Although not very stringent, this is the best result we can get from the two Higgs operators computed at 1-loop. Since $|y| = \sqrt{|y_1|^2+|y_2|^2+|y_3|^2}$ a conservative bound for $|y_3|$ can be obtained by using the limit $y_1, y_2 \to 0$: $|y_3| < 2.1\cdot\left(\frac{m}{\mathrm{TeV}}\right)$. This is comparable with the bound coming from direct searches on $|y_3|$ ($\lesssim 0.7$). It is interesting that the two result are very similar despite the completely different ways they have been obtained

[8] Clearly even if the operator is the same the coefficient would be different since in eliminating some SILH operators (as $\partial_\mu B^{\mu\nu}$) this term is generated. However this would induce a 2-loop RGE, and as such can be neglected.

[9] We stick to the SILH notation for clarity, however the operator $|H|^2\bar{u}_R H^\dagger Q_L$ is called O_{uH} in that paper.

(high energy direct searches and low energy indirect measurements).

Now let's move to the analysis of the four fermions operators (3.54).
The constraints on these terms will be read from the work [20], as for the tree-level analysis. The most relevant bounds for the operators we have generated are

$$\begin{aligned} &c^{u}_{(1212)} < (28, 0.83) \times 10^{-8}\ \mathrm{TeV}^{-2} && c^{Qu(1)}_{(1212)} < (13, 0.4) \times 10^{-8}\ \mathrm{TeV}^{-2} \\ &\hat{O}^{u}_{ijkl} = (\bar{u}_R \gamma^{\mu} u_R)_{ij}\, (\bar{u}_R \gamma_{\mu} u_R)_{kl} && \hat{O}^{Qu(1)}_{ijkl} = \left(\bar{Q}_L \gamma^{\mu} Q_L\right)_{ij} (\bar{u}_R \gamma_{\mu} u_R)_{kl}\,. \end{aligned} \tag{3.87}$$

where the brackets distinguish between the real and the imaginary part of the coefficients, and these values are valid only in the y_u-diagonal basis.
These operators have a running that involves the tree-level $\hat{O}_{Hu}, \hat{O}_u$, so in principle we should compute the RGE for their coefficients. However the evolution can only add contributions with the structures $(yy^{\dagger})_{ij}(y_u y_u^{\dagger})_{kl}$ to $\hat{O}^u$ and $(y_d^{\dagger} y_d)_{ij}(yy^{\dagger})_{kl}$ to $\hat{O}^{Qu(1)}$, since a $\sim y^4$ would be an higher-order result (diagrammatically we would need two tree level operators attached in a loop, which means the coefficient would scale as m_h^2/m^4).
Once we move to the y_u-diagonal mass basis, as for the bounds given above, we have that $y_u \to \hat{y}_u$ and $y_d \to \hat{y}_d V_{CKM}$ and so the two structures change:

i) $(yy^{\dagger})_{ij}(y_u y_u^{\dagger})_{kl} \to (yy^{\dagger})_{ij}(\hat{y}_u^2)_{kl}$.
This is diagonal in the indices (kl), so it doesn't contribute to $c^{u}_{(1212)}$.
Therefore we don't need to compute the RGE for $\hat{O}^u$, since (3.87) constraints only the $(yy^{\dagger})_{ij}(yy^{\dagger})_{kl}$ piece of the $\hat{O}^u$ coefficient obtained in the 1-loop computation (3.54).
Following this reasoning, then, we can directly exploit the values (3.87) from which we get:

$$\begin{aligned} &\mathrm{Re}\left\{(y_1 y_2^*)^2\right\} < 4.4 \times 10^{-5} \left(\frac{m}{\mathrm{TeV}}\right)^2 \to |y_1||y_2|\sqrt{|\cos 2(\phi_1 - \phi_2)|} < 6.6 \times 10^{-3} \left(\frac{m}{\mathrm{TeV}}\right) \\ &\mathrm{Im}\left\{(y_1 y_2^*)^2\right\} < 1.3 \times 10^{-6} \left(\frac{m}{\mathrm{TeV}}\right)^2 \to |y_1||y_2|\sqrt{|\sin 2(\phi_1 - \phi_2)|} < 1.1 \times 10^{-3} \left(\frac{m}{\mathrm{TeV}}\right). \end{aligned} \tag{3.88}$$

ii) $(y_d^{\dagger} y_d)_{ij}(yy^{\dagger})_{kl} \to (V_{CKM}^{\dagger} \hat{y}_d^2 V_{CKM})_{ij}(yy^{\dagger})_{kl}$.
This structure contributes to $c^{Qu(1)}_{(1212)}$, so in this case the running should be computed. However let's see if that calculation is worth to be performed. If we estimate the bound using only the 1-loop EFT result, we get

$$\begin{aligned} &\frac{y_1 y_2^*}{16\pi^2 m^2}(V_{CKM}^{\dagger} \hat{y}_d^2 V_{CKM})_{12} \lesssim 10^{-8}\ \mathrm{TeV}^{-2} \\ &\frac{y_1 y_2^*}{16\pi^2\, (m/\mathrm{TeV})^2} \left[(1-\lambda^2/2)\lambda y_d^2 - \lambda(1-\lambda^2/2) y_s^2 - A^2\lambda^5(1-\rho-i\eta) y_b^2\right] \lesssim 10^{-8} \\ &\to \frac{y_1 y_2^*}{(m/\mathrm{TeV})^2} \cdot 10^{-9} \lesssim 10^{-8}. \end{aligned} \tag{3.89}$$

By exploiting the tree level bounds (2.32) this inequality reads $10^{-13} \lesssim 10^{-8}$, by far satisfied and thus bringing no new informations. The contribution coming from the RGE has the same structure multiplying a numerical factor of order one (see [31]), therefore the computation of the running would not change what seen qualitatively above and the calculation is not worth to be pursued.

All in all the constraints on the four fermions operators have brought the relations (3.88). These are very interesting bounds: the quantity y^2/m has never been directly constrained, so this is a real new result. In addition that bound is quite strong and justifies in a more quantitative way the common assumption $y_1 = y_2 = 0$ if added to the one obtained at tree-level (2.32).

The four Higgs operators generated at 1-loop (3.83) have as coefficient a quantity proportional to $|y|^4/m^2$ from which we can indirectly get the last, missing bound cited above. We focus our attention on the operator $\left(H^\dagger \overleftrightarrow{D_\mu} H\right)^2$ for which the bound is the strongest: $\bar{c}_T < 1.3 \times 10^{-3}$ [18]. Indeed the bound on $\bar{c}_H$ can be read off for example from [21], that gives just $\bar{c}_H \lesssim 10^{-1}$.
The RGE of $\hat{O}_T$ involves the tree-level operators (2.24), so this running needs to be evaluated. The exact computation is an hard task, but we can guess what will be the most relevant contribution. Indeed, similarly to the four fermions case, the possible terms coming from the RGE must be of the form $y^2 y_{SM}^2/m^2$, for which the strongest one involves the top Yukawa coupling y_t. The RGE for $\bar{c}_T$ under these assumptions has been calculated in [33], that keeping only the contribution from the terms present in our tree-level lagrangian ($\bar{c}_{Hu}$) reads

$$16\pi^2 \mu \frac{\partial}{\partial \mu} \bar{c}_T = 4 N_c y_t^2 \bar{c}_{Hu(33)}. \tag{3.90}$$

We can solve this equation at first order using the tree-level values, ignoring the evolution of $\bar{c}_{Hu}$ that would lead to a 2-loops result. So, the coefficient at the scale m_Z reads

$$\begin{aligned} \bar{c}_T(m_Z) &= \bar{c}_T(m) - \frac{3 y_t^2 |y_3|^2 v^2}{16\pi^2 m^2} \log\left(\frac{m^2}{m_Z^2}\right) \\ &= \frac{v^2}{16\pi^2 m^2}\left(\frac{|y|^4}{2} + \left(18 - 3\log\left(\frac{m^2}{m_Z^2}\right)\right) y_t^2 |y|^2\right) \end{aligned} \tag{3.91}$$

Now we can finally compare this with the experimental bound. The slowly varying logarithm can be approximated as $\log(m^2/m_Z^2) \approx \log 10^2 \approx 5$ since m is in the TeV range. Thus, the approximated relation we get in the end is

$$\frac{|y|^4}{m^2} + 6\frac{y_t^2 |y_3|^2}{m^2} < 4.1 \times 10^{-1}\ \text{TeV}^{-2}. \tag{3.92}$$

Now, we know that the bounds on $|y_{1,2}|^{(2)}/m$ are much more tight than on $|y_3|/m$ (see (2.30), (2.32), (3.88)). Then in the left hand side of the equation above we can approximate $|y|^4/m^2 \simeq |y_3|^4/m^2$. In this way we see that the dominant term of (3.92) is the one which receives corrections from the RGE, so neglecting the y^4 and exploiting $y_t \simeq 0.98$ we get

$$|y_3| < 2.5 \times 10^{-1} \cdot \left(\frac{m}{\text{TeV}}\right) . \tag{3.93}$$

This is a new and very interesting result, which improves the bound previously obtained from $\bar{c}_g$ (3.86) and from direct searches of one order of magnitude. Even if we haven't been able to obtain direct informations about $|y_3|^2/m$, the relation above indirectly provides the constraint $|y_3|^2/m(\text{TeV}) < |y_3|/m(\text{TeV}) < 2.5 \times 10^{-1}$ since now we known that $|y_3|/m(\text{TeV})$ must be smaller that unity while $m(\text{TeV}) > 1.3$.

The only operators left to be discussed are the pure gauge ones, which don't contain any y and thus can provide a complementary, independent bound on m. In fact in our basis it is evident that at 1-loop the RGE can't include $\hat{O}_{Hu}$, so we can directly look at the coefficients of (3.40).

The major constraints on these operators come from electroweak precisions tests and LEP2 data. Indeed gauge boson self energies are modified by these operators, so their coefficients can be linked to the precision parameters. The relations ([18]) for $\hat{O}_{2W}, \hat{O}_{2B}$ are very simple

$$W = \bar{c}_{2W}, \quad Y = \bar{c}_{2B} \tag{3.94}$$

and for these parameters we can use the data from [24]: $W < 1.2 \times 10^{-3}, Y < 2.5 \times 10^{-3}$ (at 2σ). Plugging these in the relation above with $\bar{c}_{2W}, \bar{c}_{2B}$ from the 1-loop computation the strongest bound comes from O_{2B}:

$$m > 2.0\, m_W \to m \gtrsim 0.16 \text{ TeV}. \tag{3.95}$$

This is the best value for m that we can get from the EFT. The coefficients $\bar{c}_{3W}, \bar{c}_{3g}$ have similar constraints ([21]), but from the 1-loop calculation they get an additional $\sim 10^{-1}$ suppression factor which makes the corresponding bound weaker.

This result is extremely important: it exists even when the Yukawa interaction is turned off, $y_i = 0$, meaning this bound is always present and any mass greater than that respects EWPT constraints. That being so, with the current experimental precision the best bound on the mass of Ψ is still the one from direct searches (2.8).

Chapter 4

Conclusions

In this thesis we have derived the effective theory at 1-loop for a Standard Model extension with a vector-like fermion (X, T) and, thanks to the constraints coming from low-energy experiments, we obtained some new bounds on the coupling constants of this model.

The first part of the work has been devoted to a general analysis of the possible fermionic SM extensions. Here we focused our attention on models with the presence of an interaction involving the new fermion, a SM fermion and the Higgs doublet. We saw that chiral extension are experimentally ruled out, exploring in detail the particular case of a fourth generation of quarks and leptons. Then we moved toward vector-like extensions, for which the new fermion can be of Majorana or Dirac type. The introduction of Majorana fermions leads to the type-I and type-III see-saw mechanism, which needs masses too big to produce relevant effects at the EW scale. Dirac fermions are interesting possibilities predicted in many SM UV completions. We classified these particles by looking at the allowed terms in the lagrangian having fixed their charges under the SM gauge group, dividing them in the two categories of vector-like leptons and vector-like quarks. VLL mainly have constraints coming from flavor mixing processes, while VLQ have strong constraints also from LHC. For this reason we decided to focus on the latter.

In the second part, then, we picked a picked the particular $(X, T) \sim (\mathbf{3}_c, \mathbf{2}_W, 7/6_Y)$ VLQ and we studied this model in detail. The choice has been dictated from the fact that the bounds on the up-quark sector are the weakest, together with the realization that the number of parameters that this fermion introduces is minimal.
For this model we reviewed the constraints already present in literature, that bound only the mixing with the u, c quarks, and the ones coming from high energy experiments, which essentially set the mass to be $m \gtrsim 1.3$ TeV. For this reason we decided to adopt an effective field theory approach and look for bounds coming from low-energy experiments. We introduced the concept of EFT, and we derived the tree-level effective theory for the model of interest thanks to which we obtained the bounds (2.30), (2.32): $|y_{1,2}| < 4.4 \times 10^{-1}\,(m/\text{ TeV})$, $|y_1 y_2|\sqrt{|\sin(2(\phi_1 - \phi_2))|} < 5.2 \times 10^{-4}$. To go on further we needed the EFT at 1-loop , so we decided what terms were worth to be derived using considerations

from $\hbar$-counting, spurion analysis and the tree-level bounds, confronting the order-of-magnitude estimate with the data present in literature. We saw that the most interesting operators are the two Higgs, four Higgs, pure gauge and four fermions one.

Finally, the third part has been devoted to the 1-loop derivation and analysis of the EFT. We introduced the formalism and the tools needed for this computation, and subsequently applied them to the model of interest. In particular we adopted the functional integral approach, exploiting the new Covariant Derivative Expansion technique (explained in Appendix A). In this way all the operators marked as interesting in the second chapter have been evaluated, and when possible we explicitly reported the various steps of the calculation.
Once all these terms had been worked out, the last step to do before the comparison with the experimental data was to evaluate the running of Wilson coefficients. This has been done through the RGE where the entries of the anomalous dimension matrix have been taken from literature. Fortunately we only had to compute the running of $\hat{O}_T$, from which we obtained the relation (3.92). The known bounds on $y_{1,2}/m$ allowed us to approximate that inequality, from which we got an interesting explicit bound on the third component of the new coupling: $|y_3| < 2.5 \times 10^{-1} \cdot (m/\text{TeV})$. From the four fermions operators we have derived the relations (3.88), which give a bound on $y^2_{1,2}/m$. Putting the tree-level and 1-loop results together, the common (qualitative) assumption $y_{1,2} \simeq 0$ is now quantified by the ratio $(y_{1,2}/m(\text{TeV})) \lesssim 10^{-2}$ coming from the analysis above.
The pure gauge operators did not improve the bound on m from high energy experiments, giving however the universal result $m \gtrsim 160$ GeV.

In summary, thanks to EFT approach we've been able to study the space of parameters of the (X, T) model in detail. We determined what are the possible values that y_i can take, and the result that we found is quite astonishing: a coloured particle that induces flavor violation at tree level is allowed already in the TeV range with Yukawa couplings of the same order as the Standard Model ones. Models with such properties are usually strongly forbidden at that energy scale. This means that new physics with quite big couplings could be just around the corner.
The EFT we obtained can be used as a reference for future studies, either to improve the actual bounds with data coming from the next generation of high energy colliders and precision tests or to describe the new particle low-energy interactions in case someday it will actually be found. An interesting extension of this work would be to derive also $\mathcal{L}_{cc}, \mathcal{L}_{dipole}$ at 1-loop and compute the RGE to bring all the terms of $\mathcal{L}^{(1)}_{EFT}$ matched at $\mu = m$ to the scale of interest for the experiments.

Appendix A

Covariant Derivative Expansion

The Covariant Derivative Expansion is a very powerful technique that allows the computation of functional traces in a gauge invariant way. This method has been fully developed in the work [29] as an upgrade of the more traditional Partial Derivative Expansion, for which the covariant derivatives must explicitly be opened and carefully resummed after the trace evaluation. The CDE method allows to keep D_μ intact, making the calculations faster and greatly reducing the probabilities of making mistakes in the resummation process.
In this appendix we will show what is a functional trace and what are the basis of the CDE.

Given a a generic functional $f(\hat{x}, \hat{p})$ the evaluation of its trace begins by formally writing it in the momentum operator basis: $\langle p'| f(\hat{x}, \hat{p}) |p'\rangle$. The trace, then, is simply given by the sum of the diagonal matrix elements ($p = p'$), that for the continuous case is given by

$$\mathrm{tr} f(\hat{x}, \hat{p}) = \int d^4p \, \langle p| f(\hat{x}, \hat{p}) |p\rangle \, . \tag{A.1}$$

For the PDE method one should simply stop there and use $\hat{p}\,|p\rangle = p\,|p\rangle$. However this would require to split the covariant derivative $D_\mu = i\partial_\mu - ig_* V_\mu$, and then to recombine the various vector fields in a gauge invariant expression after the p integral has been computed. This is an annoying and definitely not straightforward process which can be avoided thanks to the CDE.
Indeed, let's insert a complete set of eigenstates of the position operator

$$\begin{aligned} \mathrm{tr} f(\hat{x}, \hat{p}) = & \int d^4x d^4p \, \langle p|x\rangle \, \langle x| f(\hat{x}, \hat{p}) |p\rangle \\ & \int d^4x d^4p \, e^{ipx} f(x, i\partial_x) e^{-ipx}. \end{aligned} \tag{A.2}$$

Then using the BCH formula for the momentum operator

$$e^{ipx} i\partial_x e^{-ipx} = i\partial_x + p \tag{A.3}$$

and flipping the sign of p in the integral, the trace reads

$$\mathrm{tr} f(\hat{x}, \hat{p}) = \int d^4x d^4p\, f(x, i\partial_x - p). \tag{A.4}$$

Since $D_\mu = i\partial_\mu - ig_* V_\mu$ the shift is the same also for the covariant derivative: $iD_\mu \to iD_\mu - p$. In this way the functional can be expanded in term of D_μ, keeping its gauge invariant structure.
The term "expansion" has been used because usually the covariant derivatives are present at the denominators, coming either from the perturbative computation of A^{-1}, B^{-1} or from the non local form of $\mathcal{L}_{UV}$.

To see explicitly how CDE works, let's consider a typical case:

$$f(\hat{x}, \hat{p}) = L(x) \frac{1}{-D^2 - m^2 - U(x)} R(x) \tag{A.5}$$

where $L(x), R(x), U(x)$ do not contain any covariant derivative and m is a real number. Applying the formalism developed above, its trace is given by

$$\begin{aligned}
\mathrm{tr} f(\hat{x}, \hat{p}) &= \int d^4x d^4p L(x) \frac{1}{(iD-p)^2 - m^2 - U(x)} R(x) \\
&= \int d^4x d^4p L(x) \frac{1}{p^2 - m^2 - (2ip\cdot D + D^2 + U(x))} R(x) \\
&= \int d^4x d^4p L(x) \frac{1}{p^2 - m^2} \sum_{n=0}^{\infty} \left[\frac{1}{p^2 - m^2} \left(2ip\cdot D + D^2 + U(x) \right) \right]^n R(x)
\end{aligned} \tag{A.6}$$

where the expression $(p^2 - m^2)^{-1}$ has been factorized and the geometric series $\sum_p x^p = (1-x)^{-1}$ has been exploited.
From now on the procedure is mechanical: one just need to perform all the integrals till the desired order $n = n^*$. Note that for $n > 1$ some care is needed if $[D_\mu, U(x)], [m, U(x)] \neq 0$ since the various terms would not commute.

To conclude with the explanation of the CDE technique, let's explicitly compute (A.6) till $n = 2$:

- n=0: this term has no derivatives acting on $R(x)$. Performing the momentum integral in dimensional regularization, this order gives

$$\begin{aligned}
&\int d^4x L(x) R(x) \int d^4p \frac{1}{p^2 - m^2} \\
&= \int d^4x \frac{i}{16\pi^2} m^2 \left[1 + \frac{1}{\epsilon} + \log\left(\frac{\mu^2}{m^2} \right) - \gamma + \log 4\pi \right] L(x) R(x);
\end{aligned} \tag{A.7}$$

- n=1: here $ip \cdot D$ does not contribute since $\int dp\, p^\mu/(p^2-m^2)^m = 0$, so we have

$$\begin{aligned}
&\int d^4x \int d^4p L(x) \frac{D^2+U(x)}{(p^2-m^2)^2} R(x) \\
&= \int d^4x \frac{i}{16\pi^2}\left[1+\frac{1}{\epsilon}+\log\left(\frac{\mu^2}{m^2}\right)-\gamma+\log 4\pi\right]\big(L(x)D^2R(x)+L(x)U(x)R(x)\big)\,;
\end{aligned} \tag{A.8}$$

- n=2: this calculation is less trivial; besides, operators of different dimension will be generated. In a real case one would keep only the operators up to a fixed dimension, however let's pedagogically keep all the terms:

$$\begin{aligned}
&\int d^4x \int d^4p L(x) \frac{(D^2+2ip^\mu D_\mu+U(x))(D^2+2ip^\nu D_\nu+U(x))}{(p^2-m^2)^3} R(x) \\
&= \int d^4x \left[\int d^4p \frac{1}{(p^2-m^2)^3}\big(L(x)D^4R(x)+L(x)D^2U(x)R(x)\right. \\
&\quad \left. +2L(x)U(x)D^2R(x)+L(x)U(x)^2R(x)\big) + \int d^4p L(x)\frac{p^\mu p^\nu}{(p^2-m^2)^3}D_\mu D_\nu R(x)\right] \\
&= \int d^4x \left\{\frac{i}{16\pi^2}\left(-\frac{1}{2m^2}\right)\big[L(x)D^4R(x)+L(x)D^2U(x)R(x)\right. \\
&\quad + 2\,L(x)U(x)D^2R(x)+L(x)U^2(x)R(x)\big] \\
&\quad \left. + \frac{i}{4\cdot 16\pi^2}\left(\frac{1}{\epsilon}+\log\left(\frac{\mu^2}{m^2}\right)-\gamma+\log 4\pi\right)L(x)D^2R(x)\right\}.
\end{aligned} \tag{A.9}$$

In the end one should sum the result of the various orders, keeping only the desired operators. Some care is needed in considering till what order the denominator needs to be expanded: in the example above, dimension four operators are generated in both $n=1$ and $n=2$. If we wanted to keep only these kind of terms, stopping at $n=1$ would have meant to lose some contributions.

One final note: in this example it has been assumed that there were no internal indices to trace on. In a more general case the trace over these degrees of freedom must be performed together with the functional one.

Acknowledgements

I would like to thank Dr. Vecchi for giving me the possibility to work with him. He helped me all throughout this work and every conversation we had was always stimulating. I learned a lot from him during these months.

A huge thanks goes to my family. Especially to my parents, for having always supported and believed in me even in difficult times, and to my sister, one of the craziest people I know.

I would also like to deeply thank all my friends. The Brescia guys, whom I've known for a long time. All the ones here in Padova: particularly Martina, Janka, Nicole. The gang, Jacopo and Lorenzo: you really made the last two years savage, although sometimes you can be *troppo italiani*. Luca and Giovanni, for we started as roommates but we have ended as real friends.

And many thanks to Martina. In the last months you sincerely supported and encouraged me, as well as you helped me in correcting the grammar of this work. I couldn't be happier to have met you.

Finally I'm grateful to all my colleagues in Pisa and Padova, who contributed to make my years of university a wonderful experience.

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