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# "EFFICIENCY OF PUBLIC EXPENDITURE ON EDUCATION IN EU: A NON-PARAMETRIC APPROACH"


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## 1. Introduction

The public sector of a country must be able to provide high-quality goods and services in a cost effective way in order to deliver the maximum possible value for money to taxpayers. Biases are generated in the allocation of resources given the fact that these resources are mostly collected through distortionary taxes. It is also crucial for countries to use public expenditure in the most efficient and effective way in order to foster long-term growth.

At a time in which the Member States of the European Union must deal with increased pressures on public finances to maintain the fiscal discipline requested by the Stability and Growth Pact, the debate has shifted from how to cut public expenditure to how to obtain the most from the limited resources of the public budgets.

Governments achieve their objectives by adjusting the composition of public expenditures. Moreover, governments can try to improve the efficiency of expenditure, in terms of "output" or "performance", by increasing the output per unitary amount of spending or by obtaining the same level of output with lower spending. A growing academic literature has been focusing on the key issue of the public sector efficiency, not only from the point of view of its determinants, but also from the one of the definition of a proper way to assess it.

In 1957, M.J. Farrell proposed a first investigation on how to measure efficiency and highlighted the relevance of the question for policy makers. His seminal paper is the precursor of many other studies on the subject. The initial target of these analyses shifted over time from the private sector (in terms of industrial productivity) to applications to the public spending in terms of efficiency. Measures of public expenditure efficiency, though, present some intrinsic problems. In the public sector, outputs are often not sold on the market, which implies that prices are not available or cannot be quantified. Thus, there is no common agreement in the scientific literature on the one methodology to adopt to assess efficiency in the public sector.

Two main branches have developed since Farrell's precursor paper, finding broad application in the academic literature: the parametric and the non-parametric techniques. Both methodologies are based on frontier analysis to evaluate the public sector efficiency.

In practice, these techniques calculate (or estimate) the efficiency frontiers (also called "production possibility frontiers"), i.e. the set of optimal input consuming-output producing decision making units (or DMUs, which can be firms, hospitals, national governments...) that operate at ''best practice'' and that are said to be "technically efficient".

The most used parametric technique is the Stochastic Frontier Analysis (SFA), an econometric based method to estimate efficiency frontiers. This is opposed to the most

common non-parametric approaches, Data Envelopement Analysis (DEA) and Free Disposal Hull (FDH), which construct the efficiency frontier using input/output data, and are based on mathematical programming. Non-parametric techniques calculate efficiency scores directly on the basis of the distance from the frontier and are therefore primarily data-driven.
The parametric approach, instead, assumes a specific form of the production function, that is to say, of the relationship between input and output. The pros and the limits of both approaches are presented in the text, focusing in particular on the non-parametric techniques, which are adopted to conduct the empirical analysis.

Among the different functions of public expenditure[1], spending on education has particular relevance, in terms of both short-term recovery and long-term economic growth (EC, 2015). The importance of investing in education is aknowledged by the European Commission, as this function of the public sector has a fundamental role in Europe 2020 (the ten-year strategy adopted in 2010 aiming at relaunching EU economies), in particular in terms of productivity increases, social mobility, and prevention of structural unemployment and of social exclusion. Despite this, by looking at the data, one can observe that there has been an overall decrease in recent years of public spending on education in the EU as a whole.
The sector on which this dissertation focuses is indeed the public expenditure on education, and this is the main object of the empirical analysis presented in the text.
A descriptive cross-country assessment of education spending is initially provided.
In general, the share of the public budget allocated to education in the European Union substantially differs across Member States, according to countries' history, characteristics and policy orientations. This heterogeneity has recently motivated numerous empirical studies on the subject, involving cross-country comparisons and efficiency analysis. An overview of the main scientific publications on the efficiency of the education systems is presented[2].
Most of this literature analyses a sample of countries and defines within them the efficiency frontier (through one of the frontier analysis techniques) in order to find the best practice countries.
An empirical analysis of the efficiency of expenditure on education in the European Union is presented in Chapter 4, using the non-parametric methodologies.
The focus is put on the European Union countries in order to be able to compare the results obtained with the ones of previous empirical studies. In fact, most of the empirical literature

[1] The main functions of government expenditure on public services identified in the literature are education, healthcare and investments in human capital, R&D (innovation) and infrastructure.
[2] Frontier analysis techniques have been broadly applied in the other functions of expenditure as well.

that employs the non-parametric techniques to assess the efficiency of public spending on education includes (some or all of) the EU Member States, together with other OECD countries or developing countries.

In this analysis, output indicators are compared to the input indicator for each Member State and then a comparison is made across countries. Since each non-parametric technique follows a specific methodology to define the efficiency frontier in a sample (i.e. efficiency can be assigned to different DMUs depending on the technique applied), both the FDH and the DEA are explored in the empirical part of the text. Moreover, Data Envelopement Anlysis provides different outcomes in a constant returns to scale (CRS) framework and in a variable returns to scale (VRS) one, thus both methodologies are applied in the analysis.

The input oriented approach (input minimization by holding fixed the level of output) is adopted to calculate the frontiers on the sample of DMUs (corresponding to the EU Member States). This approach is preferred to the output oriented approach (output maximisation with a fixed level of input) as governments are assumed to have more control over inputs than over outcomes.

Input is at a first stage measured by the public expenditure on education. Output of the education sector is captured using the most approaprate indicators selected referring to the economic literature examined in the survey: five output indicators are identified for the efficiency analysis, as they allow for international comparaisons.

They are:

- the *OECD-PISA test scores* (of 2012), i.e. a by country average of results of the most recent assessment of the level of competences that school systems provide;
- the *educational attainment* and the *youth educational attainment*, i.e. indicators of the population having completed at least upper secondary education in each country;
- the rate of *early school leavers* (the 18- to 24-year-olds who fail to reach the upper secondary education level);
- the *quality of the educational system* output indicator, that averages the answers provided to the World Economic Forum (WEF)'s by each country on how well the education system meets the needs of competitiveness.

On the input side, three input indicators are considered: *expenditure on education* (calculated as the ten-year-average of public expenditure on education over the years 2002-2011, as a share of GDP for each country), the *teachers per student ratio* (as the inverse of the pupil per teacher ratio) and the *schooling hours per year ratio* (as a by country average of instruction time).

Using the public expenditure on education as the input variable and the above introduced five output indicators, one input-one output DEA and FDH frontiers are calculated, with the graphical results and the numerical scores presented in the text.
As a further exercise, the DEA methodology is extended to the two input-one output framework. Taking the PISA scores as the output variable and the teacher per student ratio and the teaching hours per year as the inputs, the DEA frontier is defined. This extention is particularly interesting as it allows assessing the performance of countries in terms of PISA scores not only from the point of view of financial inputs (expenditure on education), but also from the one of "quantitatively measured'' (i.e. non-financial) indicators of input.
The empirical analysis is concluded with a robustness check of the one input-one output methodology: the non-parametric efficiency frontiers are re-calculated using a modified input[3]: the private expenditure share is added to the public component in order to obtain the total expenditure on education in % of GDP as the input variable. In fact, the results obtained by the comparison of the output with the sole public expenditure could be misleading, as the output variables selected are influenced by the private financing as well.
This robustness test allows assessing how the performance of Member States changes when the private sector is included in the input.

The results of the one input-one output analysis define Czech Republic, Finland, Bulgaria and Greece as the efficient countries across most of the output indicators.
According to the VRS DEA technique, Czech Republic is efficient in terms of PISA score and educational attainement in both the public and the total expenditure on education input frameworks. Czech Republic is also assigned efficiency in the youth educational attainement and early school leavers outputs, but only if considering the sole public expenditure as the input variable (it becomes inefficient if considering total expenditure, yet it is still assigned efficiency by the FDH methodology in both types of expenditure).
As for Czech Republic, in two cases Finland is assigned efficiency (in both the public and the total expenditure frameworks): in PISA scores and in the quality of the educational system.
Bulgaria and Greece represent a rather surprising result, as they are found efficient in more than one output indicator. In particular, Bulgaria is assigned efficiency by the VRS DEA methodology with respect to educational attainment, early school leavers and the quality of educational system survey for both functions of expenditure on education (public and total).
The Education and Training Monitor (EC, 2015) presents Bulgaria as a country that is still improving its performances in education. At the same time, Bulgaria is among the lowest

[3] The output indicators of the previous one input-one output assessment are kept unchanged.

spending countries on education, which might lead the data driven non-parametric approaches to consider it as a sort of "origin'' of the efficiency frontier, and therefore efficient[4].
Greece, as Bulgaria, shows a low education spending (both public and total). DEA considers it efficient in educational attainment, youth educational attainment, early school leavers and in the survey on quality of education. Nevertheless, the country becomes inefficient in all of these outputs as soon as the share of private expenditure on education is considered in the input variable. Such result illustrates the importance of conducting this robustness check, as it rules out the efficiency assigned to Greece in the public spending case.
Germany is considered efficient by the DEA technique with respect to PISA and to the quality of the educational system but, again, inefficient in the total education input case. In the PISA framework, Germany is FDH efficient in both the public and the total expenditure scenarios.
Croatia is DEA efficient (in both the public and the total expenditure frameworks) with respect to youth educational attainment. This is coherent with the fact that it is also efficient with respect to early leavers (in the total expenditure case), according to the same technique.
In the two input-one output exercise, the efficient countries identified are Estonia, Finland, Poland and Sweden.
The results of Finland and Sweden do not surprise, as these countries show a high level of output (Finland leads the PISA score ranking among the European countries) and a below-average bundle of inputs (Sweden).
Estonia is found efficient as it has above average PISA scores and it is contextually the country with the lowest average number of teaching hours per year in the sample.
Poland, finally, shows above average PISA results and a rather low input consumption package with respect to the two variables considered.

Most of the efficiencies assigned to countries in this analysis are corroborated by previous empirical studies (as seen in the text). These results are meant to give a broad picture of the efficiency in the education sector in EU. By providing the observed efficiencies of DMUs, DEA may help to identify possible benchmarks for relatively less efficient countries.
These techniques have the merit of highlighting shortcomings in Member States and thereby triggering follow-up discussions and in-depth analysis aimed at improving performance. However, one should refrain from drawing strong conclusions on the basis of a non-parametric assessment only: the results obtained with these techniques are data-driven and should not be interpreted as the only authoritative argument for policy guidance. Concrete

[4] Romania shows a similar level of expenditure on education, but given the fact that it performs worse than Bulgaria in almost every output indicator considered, it is outclassed by its peer.

policy recommendations should be based on more in-depth analysis focussing on specific expenditure areas and / or specific countries, taking into account more peculiar aspects that have to be abstracted from this general assessment, which still represents a valuable starting point for a broader discussion.
Just as the limitations must be recognized, so must be the potential benefits of a DEA and FDH assessment (in conjunction with other measures), to increase the understanding of public sector performance and to identify potential ways to improve it.

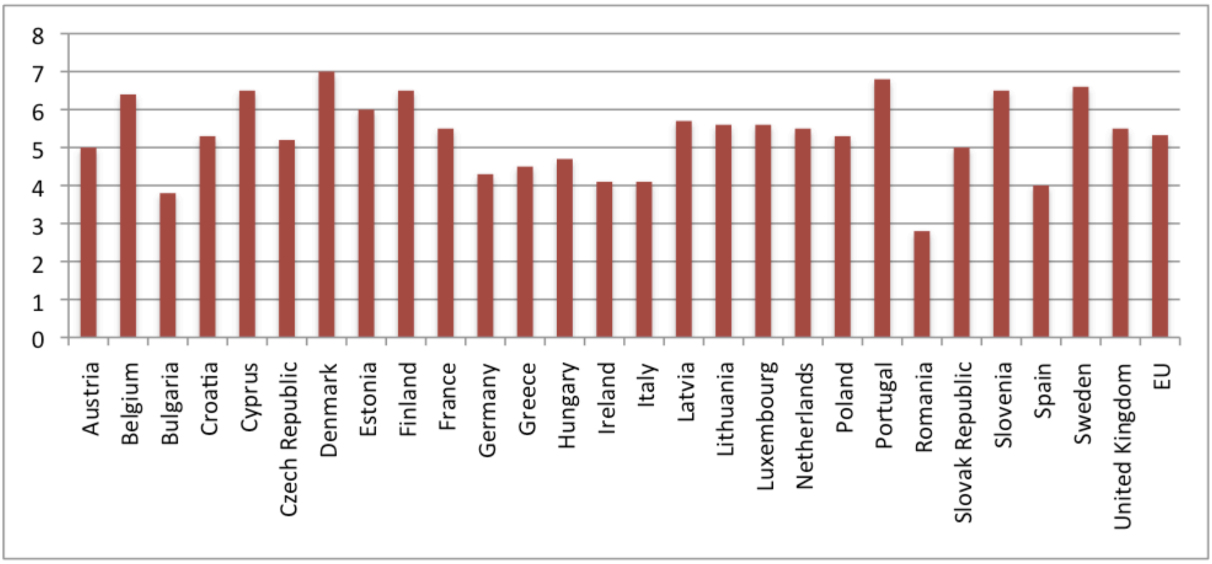
This dissertation is structured as follows: Chapter 2 presents a descriptive analysis of the education sector and of the education expenditure function. In section 2.1 and 2.2 the issues of ageing and the private sector in education expenditure are addressed.
Chapter 3 details the theoretical framework of the methodology that will be applied in the empirical analysis, and in particular it focuses on the DEA and the FDH set-up.
Chapter 4, after a description of the input and output variables used in the models, presents the results and the commentaries of the analysis carried out.
The Annex presents the numerical results described graphically in Chapter 4 and the tables that summarize the input variables selected and the main results obtained through the implementation of the models.

## 2. Public expenditure on education in the European Union

The public provision of education has always been one of the hot topics of analysis of the social sciences. From a macroeconomic point of view, education plays a fundamental role in the long-term growth of a country. Therefore, its patterns have been studied for policy reasons for decades. It is interesting to notice that the amount of public money allocated to education is strongly differentiated within European countries[5], given their characteristics and policy orientations in this field.

Figure 1 shows a graphical representation of the expenditure on education as a percentage of GDP for each Member State in 2013.

**Figure 1 : Public expenditure on education as a share of GDP, 2013**




Source : Author's elaborations of Eurostat data

In 2013, the Member States whose expenditure on education was above the EU average (in % of their GDP) were Belgium (BE), Cyprus (CY), Denmark (DK), Estonia (EE), Finland (FI), France (FR), Latvia (LV), Lithuania (LT), Luxembourg (LU), Netherlands (NL), Portugal (PT), Slovenia (SI), Sweden (SE) and the United Kingdom (UK).

5 Table A14 in the Annex shows the ISO Country Codes that will be used in the rest of the text.

The expenditure on education of Bulgaria (BG), Germany (DE), Greece (EL), Hungary (HU), Ireland (IE), Italy (IT), Romania (RO), Slovakia (SK), and Spain (ES) was recorded as below the average in the European Union, and Croatia (HR), Czech Republic (CZ) and Poland (PL) present on-the-average expenditure.

As one can notice, Scandinavian countries (Finland (FI), Sweden and Denmark) show high expenditures, while a lower expenditure on education corresponds to countries of Southern Europe (with the exception of Portugal). DK and RO are respectively the countries with the highest and the lowest expenditure on education (in % of GDP) in EU.

Table 1 reports the numbers for public expenditure on education in each country as a percentage of GDP and as a share of a country's total public expenditure, over the period 2010-2013.

**Table 1: Public expenditure on education**[6]

| | As a share of GDP | | | | As a share of total public expenditure | | | | Year-on-year real growth * | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 2010 | 2011 | 2012 | 2013 | 2010 | 2011 | 2012 | 2013 | 2010 | 2011 | 2012 | 2013 |
| EU | 5.3 | 5.1 | 5.0 | 5.0 | 10.6 | 10.5 | 10.3 | 10.3 | 2.0 | -1.2 | -1.5 | -0.5 |
| Belgium | 6.1 | 6.3 | 6.3 | 6.4 | 11.7 | 11.7 | 11.5 | 11.8 | 5.0 | 2.5 | 0.7 | 1.6 |
| Bulgaria | 3.7 | 3.5 | 3.4 | 3.8 | 10.0 | 10.0 | 9.7 | 9.8 | -9.1 | 0.3 | -3.1 | 3.3 |
| Czech Republic | 5.1 | 5.1 | 5.0 | 5.2 | 11.8 | 12.1 | 11.5 | 12.3 | 2.0 | 2.3 | -1.6 | 3.6 |
| Denmark | 7.2 | 6.9 | 7.1 | 7.0 | 12.6 | 12.1 | 12.0 | 12.3 | 6.3 | -2.9 | 2.4 | -0.7 |
| Germany | 4.4 | 4.3 | 4.3 | 4.3 | 9.3 | 9.7 | 9.6 | 9.7 | 4.8 | 1.7 | -1.4 | 0.2 |
| Estonia | 6.6 | 6.3 | 6.3 | 6.0 | 16.4 | 16.6 | 15.9 | 15.4 | -4.2 | 2.8 | 4.5 | -4.3 |
| Ireland | 4.9 | 4.7 | 4.4 | 4.1 | (7.5) | 10.1 | 10.4 | 10.2 | 2.5 | -2.8 | -6.4 | -4.3 |
| Greece | 4.0 | 4.4 | 4.4 | 4.5 | 7.6 | 8.2 | 8.2 | 7.6 | -1.2 | 5.4 | -4.3 | 0.3 |
| Spain | 4.5 | 4.4 | 4.1 | 4.0 | 9.8 | 9.6 | 8.7 | 9.1 | -1.2 | -2.1 | -6.1 | -3.8 |
| France | 5.6 | 5.5 | 5.5 | 5.5 | 10.0 | 9.8 | 9.7 | 9.6 | 1.2 | -0.8 | 0.6 | 1.3 |
| Croatia | 5.0 | 5.0 | 5.0 | 5.3 | 10.6 | 10.5 | 10.7 | 11.2 | -1.0 | 1.1 | -1.0 | 5.8 |
| Italy | 4.4 | 4.1 | 4.1 | 4.1 | 8.8 | 8.3 | 8.0 | 8.0 | -3.1 | -4.5 | -1.4 | -0.4 |
| Cyprus | 6.8 | 6.6 | 6.1 | 6.5 | 16.0 | 15.4 | 14.5 | 15.7 | 4.4 | -4.3 | -6.9 | 4.7 |
| Latvia | 6.2 | 5.9 | 5.7 | 5.7 | 14.1 | 15.2 | 15.5 | 15.7 | -11.7 | -2.5 | 1.9 | 2.6 |
| Lithuania | 6.4 | 6.1 | 5.8 | 5.6 | 15.2 | 14.3 | 16.1 | 15.7 | -8.6 | 2.6 | 1.6 | 0.5 |
| Luxembourg | 5.2 | 5.1 | 5.4 | 5.6 | 11.8 | 11.8 | 12.2 | 12.7 | 1.2 | -0.2 | 6.6 | 6.8 |
| Hungary | 5.6 | 5.1 | 4.7 | 4.7 | 11.2 | 10.3 | 9.7 | 9.5 | 6.0 | -4.5 | -6.1 | 4.6 |
| Malta | 5.6 | 5.7 | 5.8 | 5.9 | 13.7 | 13.9 | 13.7 | 13.9 | 8.3 | 4.2 | 4.7 | 3.5 |
| Netherlands | 5.7 | 5.6 | 5.6 | 5.5 | 11.8 | 11.8 | 11.8 | 11.8 | 1.4 | -0.7 | -3.0 | -0.4 |
| Austria | 5.1 | 5.0 | 5.0 | 5.0 | 9.7 | 9.8 | 9.7 | 9.8 | 1.4 | 0.7 | -0.6 | 1.3 |
| Poland | 5.6 | 5.5 | 5.4 | 5.3 | 12.1 | 12.5 | 12.6 | 12.5 | 3.5 | 2.0 | -0.7 | -1.2 |
| Portugal | 7.7 | 7.3 | 6.5 | 6.8 | 14.9 | 14.5 | 13.4 | 13.5 | 7.0 | -5.9 | -6.7 | -0.6 |
| Romania | 3.3 | 4.1 | 3.0 | 2.8 | 8.4 | 10.5 | 8.3 | 8.1 | -11.3 | 33.2 | -27.0 | -5.8 |
| Slovenia | 6.6 | 6.6 | 6.4 | 6.5 | 13.5 | 13.2 | 13.3 | (10.9) | -0.4 | -2.9 | -4.1 | 0.9 |
| Slovakia | 4.9 | 4.6 | 4.4 | 5.0 | 11.7 | 11.3 | 11.1 | 12.2 | 9.9 | -4.7 | -2.1 | 13.3 |
| Finland | 6.6 | 6.5 | 6.4 | 6.5 | 12.0 | 11.9 | 11.5 | 11.2 | 2.4 | -0.7 | -3.0 | -0.8 |
| Sweden | 6.5 | 6.5 | 6.5 | 6.6 | 12.5 | 12.6 | 12.5 | 12.4 | 0.5 | -0.1 | -0.5 | 0.7 |
| United Kingdom | 6.6 | 6.0 | 5.8 | 5.5 | 13.5 | 12.9 | 12.3 | 12.0 | 2.4 | -4.8 | -1.3 | -3.0 |

Source: European Commission (2015): elaboration of Eurostat's general government finance and national accounts statistics.

6 "()" = total public expenditure includes one-off significant expenditure in support of the financial sector; * = real growth is computed as the change over the previous year of total expenditure of general government on education, valued at constant prices using the implicit deflator for the final consumption of the general government.

For the European Union as a whole, public expenditure on education started to decline in 2011 in real terms. Four Member States (IE, ES, IT and RO) record a level of education expenditure below the EU average during the whole period considered, both when measured as a share of GDP and as a share of total public expenditure.

Consecutive drops in the two following years, over the time span 2011-2013, caused a total fall of 3.2%. Eleven countries (Denmark, Estonia, Ireland, Italy, Netherlands, Poland, Portugal, Romania, Finland and the United Kingdom) show the highest drop in 2013. According to the Education and Training Monitor (2015), the persistent negative trend of these countries is not due to one-off cuts across Europe, but rather to consecutive reductions in education expenditure in these Member States: Italy has been recording a decrease of the budget allocated to education since 2008, Spain has been decreasing it for four years, Ireland, Netherlands, Finland, UK and Portugal for three years and Romania for two. However, positive trends are also recorded in the report, as shown in the figures for Belgium, Lithuania, Luxembourg and Malta.

By pointing the attention to the share of expenditure on education over the total public expenditure, data show that the EU average educational expenditure was equal to 10.3% of total public expenditure in 2013. Moreover, the table indicates the by country spending choice of public authorities on education in comparison to other policy areas that is, in a way, the relative importance of education on the policy agenda of each Member State (EC 2015).

An overall decrease in the percentage of public expenditure on education is detectable from the data, and it is worth highlighting that such trend might be an indicator of two possible scenarios. On the one hand, Member States might be cutting expenditure on education with the objective of improving efficiency (by reducing the input and still obtaining the same outcome level). Given the control on public budgets, EU countries are in fact required to provide their educational services by minimizing the amount of public expenditure devoted to them. On the other hand, such adjustments to education expenditure levels may be a reaction to (or an anticipation of) a change in demographics, as lower resources are needed for a shrinking school-age population. This second hypothesis is discussed in the next section.

### 2.1 Demographic ageing and educational expenditure

The link between changes in expenditure and in the school-age population decrease is not clear-cut, but supporting evidence of this relationship is provided by the Eurostat data represented graphically in Figure 2. This represents a first insight into the question of the intergenerational distribution of public funds. Figure 2 shows the decomposition of expenditure by age group recipient. Three categories of expenditure are considered: expenditure targeted to under 25-year-olds, expenditure targeted to over 65-year-olds and untargeted expenditure. Expenditure specifically targeted to recipients under 25 years old is composed of expenditure on education as well as the share of health-care and long-term care expenditure targeted to this age group. Expenditure specifically targeted to recipients over 65 years old is composed of old-age and survivors' pensions as well as the share of health-care and long-term care expenditure targeted to this age group.

**Figure 2 : Composition of public expenditure by age group of recipients[7]**


Source: Author's elaboration of Eurostat data

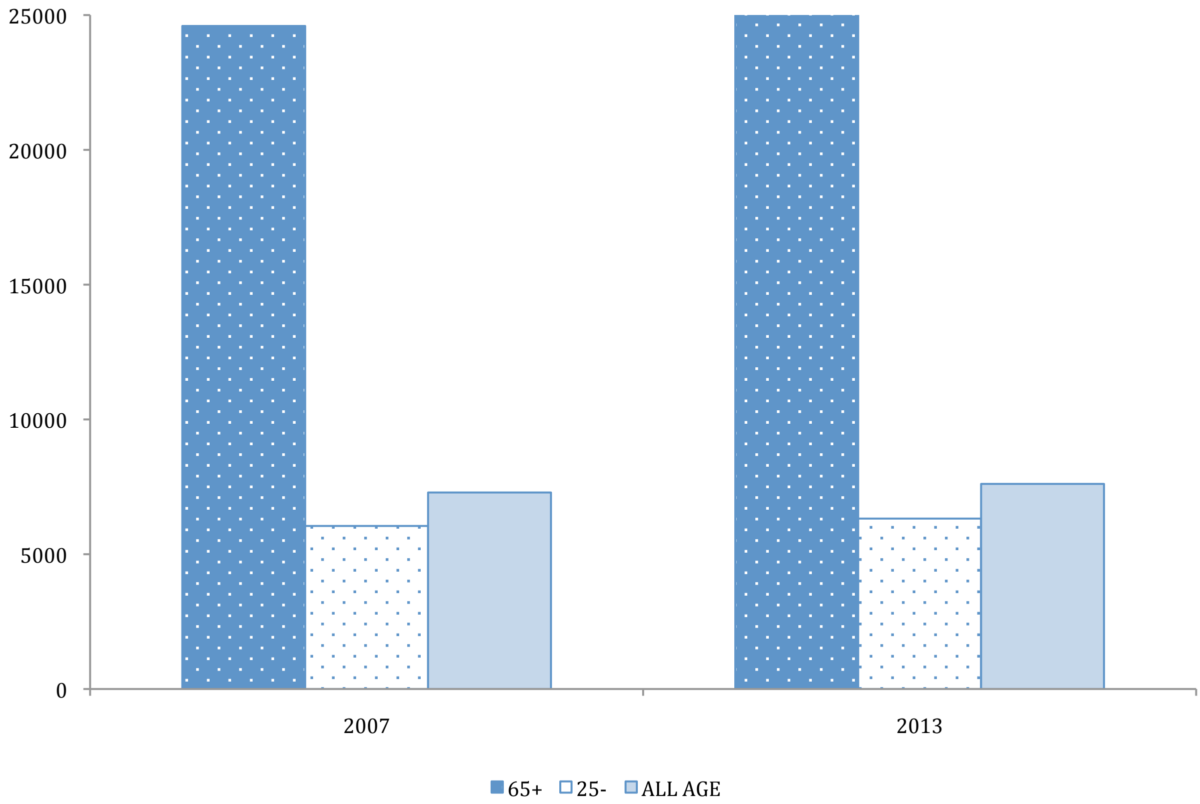
[7] Shares in total expenditure for Euro Area countries. 65+: old-age and survivors' pensions, health and long-term care targeted to the age group 65+. 25-: education, health and long-term care targeted to the age group 25-.

The remaining part of expenditure is considered as not targeted towards a specific age group (indicated as All Age in the graph above), and the breakdown of the age of recipients of health-care and long-term care expenditure is based on the datasets of the Ageing Reports of 2007, 2010 and 2013. Moreover, the graph above assumes that all expenditure on education is targeted to the age group under 25 (which can be considered as a realistic approximation, as the share of expenditure on education not identifiable by level is very small). In a similar way, it is assumed that all survivors pensions are targeted to the age group over 65 years old. The graph shows that the percentage of the total public expenditure targeted to the over-65 group is about 34%, that is to say, close to three times the share of expenditure targeted to the age group under 25 (i.e. about 12% of total expenditure). Moreover, the percentage of the expenditure targeting the 65+ shows an increasing trend throughout the years, as it passed from a 32,8% of total expenditure in 2007 to a 33,9% of total expenditure in 2013, while the one targeting the group under 25 years old shows a slightly decreasing trend, as it decreased of 0.6% from 2007 to 2013.

Furthermore, by looking at the decomposition of expenditure per capita in the graph below, it is possible to see that a person over 65 years old receives four times more public funds than a person under the age of 25 (Figure 3). As in the case of the age decomposition considering total expenditure (Figure 2 above), the per-capita decomposition changes over time, showing an increase in expenditure targeted to the over 65 that exceeds the increase in per-capita expenditure targeted to the under 25 in the 2007-2013 period.

No direct causality can be inferred from the graphs presented in this section, as the relationship between the composition of public expenditure and the demographical change in Europe requires further in-depth analysis. Nevertheless, data show that public expenditure composition and the ageing process tend to be correlated. Ageing could therefore be one of the components influencing Member States to overall reduce the expenditure on education: as the median voter, given the demographic structure, is shifting more and more to the 65+ side of the spectrum, the policy maker is pushed to favor policy decisions more oriented to this recipient (and, consequently, a higher share of the total budget allocation) in order to satisfy the median voter, at the expenses of the share allocated to education, given the limited budget for the different public expenditure functions.

**Figure 3 : Expenditure per capita – composition by age group of recipients**[8]




Source: Author's elaboration of Eurostat data

This interpretation is corroborated by OECD (2013), in which it is argued that the decreasing trend of expenditure on education and the increasing trend of expenditure on health will continue as the transition toward a "greying population" continues, and that ageing societies increase demands on health care and social services, resulting in a shift of expenditure towards the elderly. For instance, among the EU countries considered in the report, the OECD takes UK as an example of a country in which older people are more likely to vote than the young, arguing that such greater civic participation from ageing populations could influence how governments choose to spend their resources.

What follows from the above discussion is that, as resources for education are more and more limited, efficiency increases are required to maintain the same outcome level while reducing the input (the resources invested).

The analysis in the empirical chapter provides an insight with regard to the countries of the

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8 Real annual expenditure per person in 2010 euro. 65+: old-age and survivors' pensions, health and long-term care targeted to the age group 65+. 25-: education, health and long-term care targeted to the age group 25-.

European Union.

## 2.2 Public and private expenditure on education

National governments are not the only body to invest a part of the total budget on the education sector. In fact, enterprises, students and their families also make choices on the financial resources to be set aside for education.

When considering the educational outcome it is necessary to make the distinction between the public-funded institutions and the private-funded ones.

In order to confront figures of public and private systems, UNESCO (2006) distinguishes between public and private education according to who exerts the « ultimate control » over an institution: whether it's a public agency or a private entity. With this definition, all educational institutions that belong to a private owner (whether individual or collective) are defined as private, in opposition to public ones (belonging to a State or to a municipality).

A second definition is provided by European data gathering agencies such as Eurydice[9], which does not distinguish schools by institutional status but rather by funding origin: state or municipal versus household, business or NGO.

Sticking to this second definition, Table 2 below presents the amounts corresponding to the two systems in Europe for years 2006 and 2011.

In 2011, public expenditure on education was estimated as 5.3% of GDP at the EU aggregate level. Private funds instead have been estimated as 0.7% of GDP for the same year.

It can be observed that the highest public spending on education (relative to GDP) was in Denmark, and it corresponded to 8.8% of its GDP in 2011, followed by Malta (8.0%), Cyprus (7.9%) Sweden and Finland (both with 6.8%), and Belgium (6.6% of GDP).

On the other side of the spectrum, Romania and Luxembourg are the Member States that invested the least in public education (with 3.1% and 3.2% respectively).[10]

On the private side, Cyprus is again the country that presents the highest share invested in education (1.7% of GDP in 2011), followed by the United Kingdom (1.6%), Malta (1.3%), and Netherlands (1.1%). The lowest private expenditure percentage corresponds to Romania with 0.1% of GDP, followed by Finland and Sweden (0.2% of GDP in 2011).

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[9] Network and data provider of education systems information and policies by the European Commission

[10] Luxemburg's percentage is taken from 2007 and is not shown in the table.

**Table 2 : Public and private expenditure on education (years 2006 and 2011)**

| | Public expenditure (% of GDP) | | Private expenditure (% of GDP) | | Expenditure on public and private educational institutions per pupil/student (PPS for full-time equivalents) | |
|---|---|---|---|---|---|---|
| | 2006 | 2011 | 2006 | 2011 | 2006 | 2011 |
| EU-28 | 4.9 | 5.3 | 0.7 | 0.7 | 5 930 | 6 846 |
| Belgium | 6.0 | 6.6 | 0.3 | 0.3 | 6 961 | 8 235 |
| Bulgaria | 4.0 | 3.8 | 0.6 | 0.7 | 2 127 | 2 713 |
| Czech Republic | 4.4 | 4.5 | 0.5 | 0.6 | 4 410 | 5 032 |
| Denmark | 8.0 | 8.8 | 0.6 | 0.4 | 8 386 | 9 665 |
| Germany | 4.4 | 5.0 | 0.7 | 0.7 | 6 463 | 8 042 |
| Estonia | 4.7 | 5.2 | 0.3 | 0.3 | 3 175 | 4 426 |
| Ireland (2) | 4.7 | 6.2 | 0.3 | 0.5 | 6 023 | : |
| Greece (3) | 4.1 | : | 0.3 | : | 4 479 | : |
| Spain | 4.3 | 4.8 | 0.5 | 0.9 | 6 158 | 6 689 |
| France | 5.6 | 5.7 | 0.5 | 0.7 | 6 481 | 7 368 |
| Croatia | 4.0 | 4.2 | 0.4 | 0.4 | : | 3 902 |
| Italy | 4.7 | 4.3 | 0.4 | 0.5 | 6 436 | 6 107 |
| Cyprus | 7.0 | 7.9 | 1.2 | 1.7 | 7 134 | 9 519 |
| Latvia | 5.1 | 5.0 | 0.7 | 0.6 | 3 093 | 3 988 |
| Lithuania | 4.8 | 5.2 | 0.5 | 0.7 | 2 750 | 4 044 |
| Luxembourg (4) | 3.4 | : | : | : | : | : |
| Hungary | 5.4 | 4.7 | 0.5 | : | 3 987 | : |
| Malta | 6.5 | 8.0 | 0.4 | 1.3 | 6 176 | 9 435 |
| Netherlands | 5.5 | 5.9 | 0.9 | 1.1 | 7 591 | 8 591 |
| Austria | 5.4 | 5.8 | 0.6 | 0.5 | 8 617 | 9 449 |
| Poland | 5.3 | 4.9 | 0.5 | 0.7 | 3 035 | 4 641 |
| Portugal (5) | 5.1 | 5.3 | 0.4 | 0.4 | 5 007 | 5 302 |
| Romania (3) | 3.5 | 3.1 | 0.4 | 0.1 | 1 437 | 2 075 |
| Slovenia | 5.7 | 5.7 | 0.8 | 0.7 | 6 297 | 6 782 |
| Slovakia | 3.8 | 4.1 | 0.6 | 0.6 | 2 931 | 4 262 |
| Finland | 6.2 | 6.8 | 0.2 | 0.2 | 6 388 | 7 716 |
| Sweden | 6.8 | 6.8 | 0.2 | 0.2 | 7 381 | 8 571 |
| United Kingdom | 5.4 | 6.0 | 1.4 | 1.6 | 7 912 | 7 914 |
| Iceland | 7.6 | 7.4 | 0.8 | 0.7 | 7 903 | 7 493 |
| Liechtenstein | 2.1 | 2.5 | : | 0.7 | 7 647 | : |
| Norway | 6.5 | 6.7 | : | 0.1 | 9 273 | 10 377 |
| Switzerland | 5.3 | 5.3 | 0.5 | 0.5 | : | : |
| Turkey | 2.9 | 4.1 | : | 0.4 | : | 2 552 |
| Japan | 3.5 | 3.8 | 1.7 | 1.6 | 7 209 | 7 956 |
| United States | 5.2 | 5.1 | 2.3 | 2.2 | 10 798 | 11 308 |

(1) Refer to the internet metadata file (http://ec.europa.eu/eurostat/cache/metadata/EN/educ_esms.htm).
(2) Expenditure on public and private educational institutions per pupil/student: data for 2005 instead of 2006.
(3) Data for 2005 instead of 2006.
(4) Excludes tertiary education.
(5) Expenditure on public and private educational institutions per pupil/student: data for 2009 instead of 2011.
*Source:* Eurostat (online data codes: educ_figdp, tps00068 and tps00067), UNESCO, OECD

Source: Eurostat

## 3. The measurement of efficiency in the public sector

This chapter describes the main empirical techniques to conduct efficiency analysis. An application of such techniques will be proposed in the empirical exercise of Paragraph 4, after having presented a review of the recent literature on the application of the methodologies in the context of public spending on education.

### 3.1 Efficiency analysis: a general overview

The economic literature refers to different concepts of efficiency. *Technical efficiency* refers to the process of conversion of physical inputs (e.g. services of employees and machines) into outputs and to the concept of best practice. Technical efficiency means that, given an output level and given the current technology, there is no waste of inputs whatsoever in producing such output. If operating at best practice, the producing entity is then said to be technically efficient at 100%. Such percentage decreases if operating below best practice, as technical efficiency is expressed as a precentage of best practice (Steering Committee for the Review of Commonwealth/State Service Provision, 1997). Technical efficiency is affected by managerial practices and decisions and by the scale and the size of the operations, but not by prices and costs.

In addition to technical efficiency, *allocative efficiency* can be considered if information on prices of inputs is available and if it is possible to assume a cost minimizing (or profit maximizing) behaviour of the producer. Then, allocative efficiency in input selection is attaiend by selecting that mix of inputs that produces a given quantity of output at minimum cost, given the prevailing input prices (Coelli & al., 2005). The combination of both technical and allocative efficiency provides the *overall economic efficiency*.

This dissertation presents an analysis of the efficiency of public expenditure on education. In light of the above definitions, only technical efficiency is considered in the analysis, given the difficulties in identifying the allocative efficiency for the public sector, characterized by unknown or non-existing prices.

The measurement of economic efficiency has been linked to the use of frontier functions through the last sixty years.

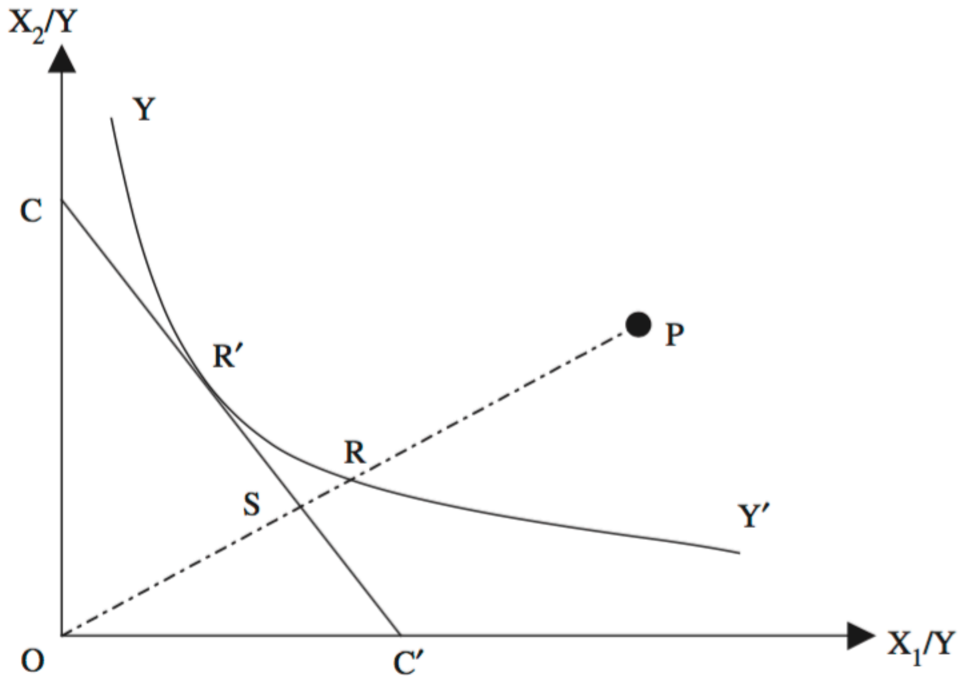
Modern literature on efficiency measurement refers to Farrell's seminal paper "The Measurement of Productive Efficiency'' (1957), in which the author introduces a method to

decompose the overall efficiency of units of production into allocative and technical efficiency[11].
Farrell's analysis is developed primarily in terms of efficiency of firms (as productive units), drawing upon the work of Debreu (1951) and Koopmans (1951). His methodology and its further developments have been generalized over years and have been widely applied to the public sector as well (HM Treasury, 2000).
Without loss of generality, and according to the vast majority of literature, from now on the productive unit will be called "decision making unit'' (DMU), defining with this term not only profit generating enterprises, but rather any decision making entity including national governments.
Farrell's seminal work shows how the inefficiency of a DMU can be identified when it obtains a lower quantity of output with respect to the maximum obtainable, given a determined group of inputs (the converse of the above described technical efficiency).
Farrell's analysis focuses also on allocative efficiency measurement, that here is not further detailed since the DMUs considered are national central governments.
The two input-one output case in Figure 4 is used as an example in order to present the analysis of technical efficiency through the usage of frontiers, originally introduced by Farrell.

Under the assumption of constant returns to scale (CRS), the unit isoquant YY' in Figure 4 is composed of all the minimum combinations of inputs (reported on the x- and y-axis) needed to produce one unit of output.
Any point belonging to this isoquant represents a technically efficient input combination, while the points above the curve (e.g. point P) are considered inefficient as there exists a lower input package that can be used to obtain the same output.
The distance between the inefficient point and the frontier, along the straight line passing through the origin and that point (as constant returns to scale are assumed in this example), measures the technical inefficiency of production in that point. Such measure represents the amount of inputs to be reduced without a decrease of the output.

[11] The terminology employed in the field of efficiency measurement has evolved over time and is not homogeneous in all pieces of literature. In Farrell's original paper, the author used "price efficiency" instead of "allocative efficiency" and "overall efficiency" instead of "economic efficiency". The terminology used in this dissertation conforms with the one that has been used most often in recent literature.

**Figure 4 : CRS isoquant**




Source: Murillo-Zamorano (2004)

The technical inefficiency level associated to point P is then calculated as the ratio RP/OP (or conversely, the technical efficiency of the producing unit in P is given by 1- (RP/OP), or OR/OP).

The analysis presented by Farrell follows an input-oriented scheme, that is to say, the input minimization while holding constant the output level. The output-oriented approach is instead the output maximization while maintaining the inputs constant.

Under the CRS assumption, Färe and Lovell (1978) show that input and output orientation schemes provide the same results. Under non-constant returns to scale, instead, such equivalence does not hold, as shown in Forsund and Hjalmarsson (1979).

In the empirical analysis in the present dissertation, it will be adopted an input-oriented scheme, as it will be explained in Chapter 4.

Following Farrell's analysis, different techniques have been developed through years in order to define efficiency frontiers.

The most recurrent methods used in economic literature to calculate or estimate the efficiency frontiers can be distinguished in parametric and non-parametric techniques.

Such distinction is made on the basis of whether a functional form of the production function is assumed a priori (parametric approach) or not (non-parametric approach). In the latter case, the frontier is calculated directly from the sample of observations.
The most used parametric technique is the Stochastic Frontier Analysis (SFA) and the most common non-parametric ones are the Free Disposal Hull (FDH) and the Data Envelopment Analysis (DEA).
A brief description of parametric and non-parametric techniques is provided in the following paragraphs.

### 3.2 Parametric techniques: Stochastic Frontier Analysis (SFA)

Stochastic Frontier Analysis identifies the relationship between output and input(s) by using statistical methods. It was originally introduced by Aigner et al. (1977) and it is an econometric model to estimate the efficiency frontiers.
This technique allows for two types of deviation from input-output relationship.
The first one is statistical « noise » (random variations in the data caused by different errors such as inaccurate measurement of output), and the second one is inefficiency.
Statistical noise is assumed to be zero on average if and only if it is uncorrelated with the inputs and with the inefficiency measure. The measure of inefficiency is one-sided: it is negative the more the producing unit is inefficient, and, by definition, it is equal to zero only in the case of full efficiency.
One major shortcoming of this approach derives from the fact that the overall deviation of DMUs from the estimated frontier can be attributed to either the noise or to actual inefficiency. Such conclusion strongly depends on the choices made about the joint distributions of the two components.
Moreover, Stochastic Frontier Analysis requires an important amount of information, about both the shape of the production technology (in order to be able to assume an a-priori specific form of the function) and the distributions of the two types of deviation, in order to yield a useful answer (Chote, Emmerson, and Simpson, 2003).
Efficiency rankings and efficiency scores can vary significantly depending on choices made about any of these elements, which can be very arbitrary. In fact, economic theory and data are often not very informative about the shape of the frontiers and the distributions of the two deviations.
Stone (2002) remarks that SFA has a theoretically imaginative approach that raises the evergreen question of realism. In his paper, he also points out the delicacy and lack of

robustness of the assumptions underlying any method for the separation of the contribution of inefficiency and noise to the deviation of each unit.

As noted in Chote et al. (2003), failure in measuring the inputs accurately can further complicate the task of correctly identifying DMU's efficiency scores.

Other shortcomings of the parametric approach are identified by Tauchmann (2011), such as input endogeneity issues.

For the above described limitations of the parametric approach of efficiency frontier estimation, the empirical analysis in this dissertation will be carried out by using the non-parametric techniques. The two main methodologies are introduced in the following paragraph.

### 3.3 Non-parametric techniques: Free Disposal Hull and DEA

Farrell's seminal paper was the starting point for the development of non-parametric measurement of efficiency. It is shown how efficiency can be measured as the maximum radial reduction in observed inputs, holding constant the observed output (Flavin, Murphy, Ruggiero, 2012).

Boles (1966, 1971) presented an extension of Farrell's measure and introduced linear programming for efficiency estimation. The later work of Afriat (1972) further extended the methodology by providing a preliminary variable return to scale formulation for the non-parametric frontier, and Charnes, Cooper, and Rhodes (1978) provided the constant return to scale formulation, and named the technique Data Envelopment Analysis (DEA).

Färe, Grosskopf, and Logan (1983) and Banker, Charnes, and Cooper (1984) extended non-parametric efficiency measurement to variable returns to scale.

FDH[12] and DEA approaches have become popular in evaluating technical efficiency of governmental authorities because of their non-parametric characterisation, and because they do not require input price data, which are often difficult to measure accurately in the public sector (Ruggiero, 1996).

Non-parametric techniques are deterministic, therefore the efficiency score of a DMU calculated with these methodologies refers to relative efficiency (relative to the other DMUs in the sample).

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[12] FDH methodology was initially proposed in an efficiency study of post office operations in Deprins, Simar, and Tulkens (1984). Tulkens (1993) presents an overview of the methodology. Advanced issues such as shifts in the production possibility frontier and technological change can be found in Tulkens and Vanden Eeckhaut (1995).

### 3.3.1 Free Disposal Hull (FDH)

Following Gupta and Verhoeven (2001)'s approach, the FDH methodology is briefly described through a general example.

The first step is to establish the production possibility frontier. The frontier consists in the combination of the best practice DMUs in the sample, that is to say, the best-observed production results within the observations available.

The second step is the calculation of relative inefficiencies of DMUs that find themselves below the frontier, using distances from the frontier itself.

A remarkable advantage of the FDH analysis is that it is a methodology that imposes only weak restrictions on the production technology, while allowing for comparison of efficiency scores among DMUs (Pang, Herrera 2005).

The only assumption it makes is free disposability of inputs and/or outputs. This assumption guarantees continuity in the FDH frontier for any sample of production results.

The underlying idea is that a DMU is relatively efficient if no other DMU generates a higher amount of output with equal or lower input. For this reason, a DMU that is not on the efficiency frontier is relatively inefficient.

In figure 5, this is represented in the one-input one-output case.

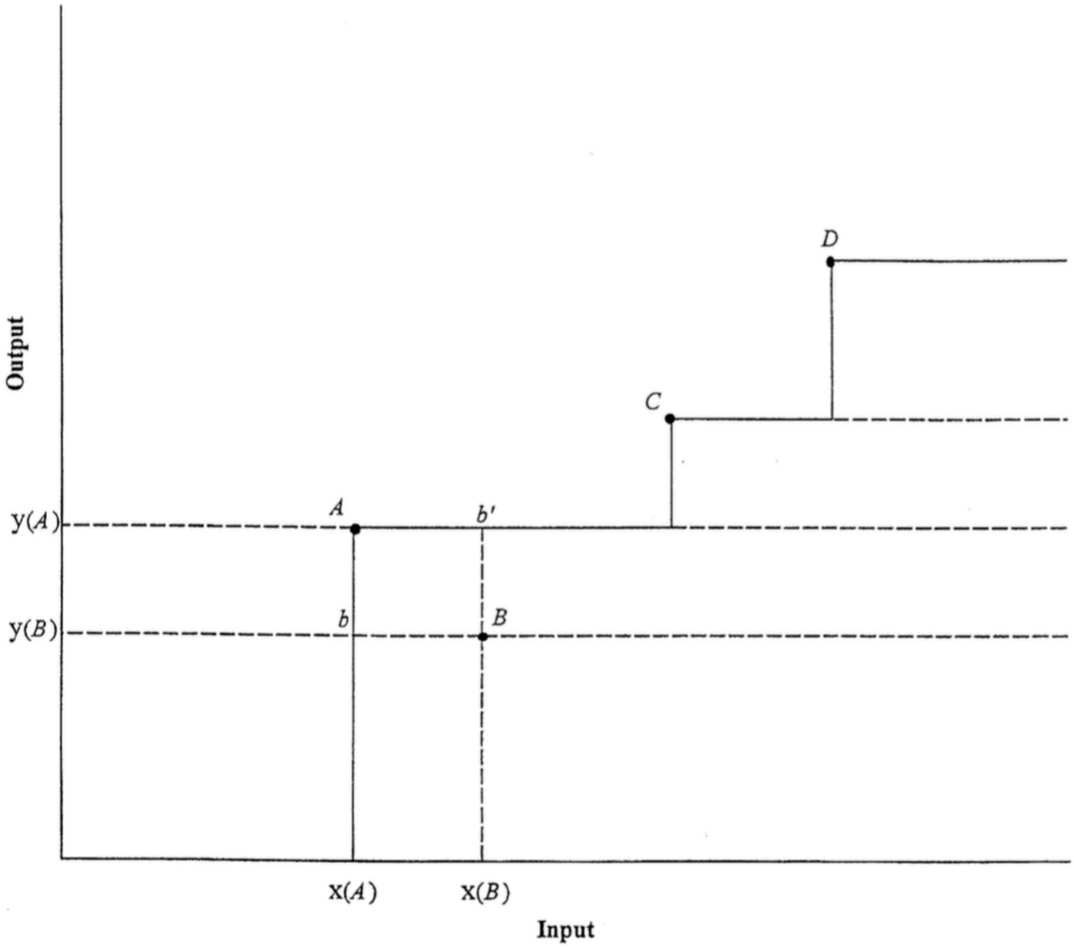
Producer B uses more input to produce less output than producer A, and therefore is relatively inefficient with respect to A.

Producers A, C, and D are relatively efficient as no other DMU in the sample produces a higher output using lower input. The step-like line connecting them is the production possibility frontier.

If a DMU is on the frontier (that is, relatively efficient) and no other DMUs are inefficient with respect to it, such DMU is called "independently efficient'' (C and D are examples of independently efficient DMUs).

Efficiency scores can be calculated on the base of the distance from the frontier, in order to rank DMUs.

**Figure 5 : FDH production possibility frontier**




Source : Gupta, and Verhoeven (2001)

Following an input-oriented approach, for DMU B efficiency is given by the segment bB, calculated as the quotient of inputs used by A over inputs used by B, i.e. x(A)/x(B). This quotient is then the efficiency score.[13]

DMUs that lay on the efficiency frontier have a score of 1 while the ones that lay below it have a non-negative score lower than 1.

Such score corresponds to an indication of output loss relative to the most efficient producer with an equal or lower level of input.

This analysis can be extended to a multiple input/multiple output case. In this context,

[13] Conversely, in the output oriented approach b'B represents the efficiency score of B and is calculated as y(B)/y(A).

efficiency scores can be calculated for each input and output.

The overall input (output) efficiency score is then defined as the score of the input (output) that is closest to the production possibility frontier. This input (output) efficiency score indicates by how much the efficiency of the use of inputs (production of outputs) should increase for the production result to become relatively efficient, assuming that the efficiency in all inputs (outputs) is increased by the same percentage (Gupta and Verhoeven, 2001).

The step-by-step procedure for the calculation of the input efficiency score in the case of multiple inputs is presented, as it will be put in practice in Paragraph 4.

The first step is the selection of a DMU (for the sake of the example, called A). All producers that are more efficient than A are then identified. In the case in which no more efficient DMUs are available in the sample, A is assigned an efficiency score of 1.

For every pair of DMUs (that is to say, A and a more efficient one with respect to A), scores are calculated for each input by dividing that DMU's input use by A's use.

As a result, an MxN matrix is obtained, where M is the number of inputs and N is the number of DMUs who are more efficient than A.

The next step is to select for every more efficient DMU the input that brings A the closest to the production possibility frontier, i.e. from each column of the matrix, the largest score (one for each more efficient DMU), is taken, yielding a N dimentional vector of scores. Then, the score relative to the most efficient DMU is selected — that is, the smallest score in the 1xN vector. The result is the input efficiency score.

Mathematically, $f(x;y)$ denotes the production set, and x and y are inputs and outputs.

$x_m(n)$ denotes the use of the $m^{th}$ type of input by producer $n$, therefore the input efficiency score of A is

$$\min_{n \in N} \max_{m \in M} \frac{x_m(n)}{x_m(A)}.$$

It is important to remark that in the case of multiple inputs (outputs), the FDH methodology does not rely on some weighting of inputs (outputs) in order to obtain the indicator of efficiency. The calculation of the input efficiency score involves the selection of the quotient of inputs that most accurately captures the distance to the production possibility frontier, which differs substantially from a weighted average of input quotients.

### 3.3.2 Data Envelopment Analysis (DEA)

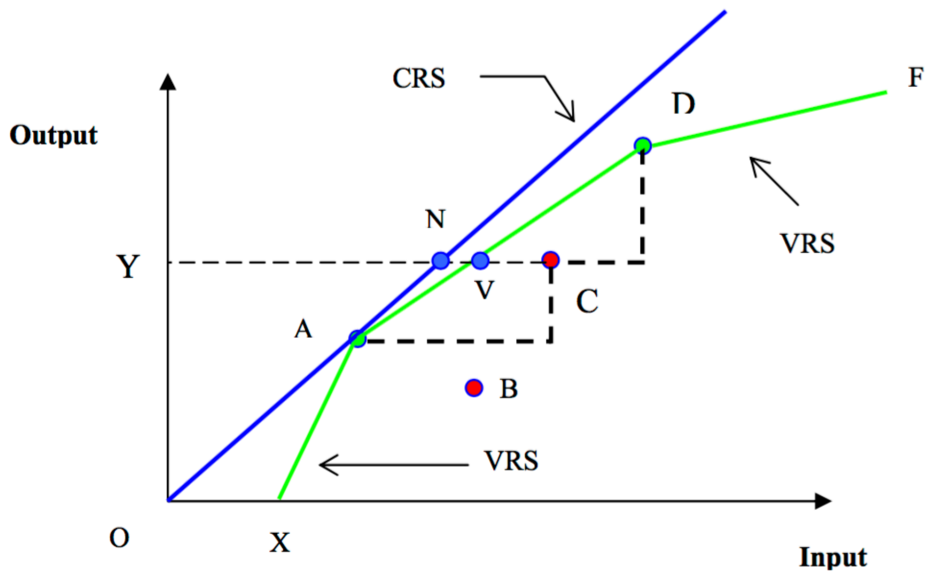
With a similar approach as the one used to explain the theoretical basis of the FDH technique, the focus is now put on Data Envelopment Analysis.

The DEA methodology is more demanding of assumptions with respect to the Free Disposal Hull, as it assumes convexity of the production set in order to have feasibility of linear combinations of the input-output bundles.

DEA constructs an envelope around the input-output observations of the sample.

Following Herrera and Pang (2005), it is shown the single input-single output case in the following figure, which can be generalized in the multiple-input multiple-output case (an application of the two-input one-output methodology will be presented in the empirical chapter of this dissertation).

**Figure 6 : DEA production possibility frontier**




Source: Herrera and Pang (2005)

In opposition to the step-like FDH frontier, DEA frontier is a piecewise line connecting all the efficient DMUs. Such piecewise linearity in the frontier implies that a DMU is not ranked only against other real DMUs in the sample (called peers in the literature) but also with virtual DMUs, which employ a weighted collection of the inputs of the peers in order to yield a virtual output[14]. To show it with an example, in the one input-one output framework DMU C of Figure 6 is not only compared to its efficient peers A and D, but also to the virtual DMU V, which employs efficiently a set of input calculated as a linear combination of the inputs of A and D.

In the example, the FDH frontier is represented as the dotted line, while the DEA frontier is the line connecting points A, D, and F. As it is possible to see, DMU C is considered efficient in the FDH case but inefficient in the DEA scenario, as convexity of the production functions allows for the computation of the more efficient DMU V. In general, the DEA technique assigns efficiency to less Decision Making Units than the FDH methodology.

As in the FDH case, the technical efficiency score (TE) of DMU C is defined by TE = YV/YC.[15]

Banker, Charnes and Cooper (1984) extend the DEA methodology to account for different returns to scale scenarios. Two types of return to scale can be in fact assumed when calculating the DEA frontier. In the case of constant returns to scale (CRS), the frontier is the ray connecting the origin and the efficient DMU. Such DMU is determined by the highest achevable ratio of output to input in the sample, i.e. DMU A in Figure 6.

In the variable returns to scale (VRS) case, instead, the frontier passes through the (multiple) efficient DMUs and reflects, segment by segment, local scale economies.

In the figure, segment XA reflects locally increasing returns to scale (IRS), that is to say, for the DMUs belonging to this segment an increase in the input would result in a more than proportional increase in the output.

Segments AD and DF of the frontier reflect instead the decreasing return to scale part of the frontier.

The scale efficiency component is represented by the distance between the CRS frontier and the VRS efficiency for each DMU. Thus, the application of the sole CRS DEA methodology is appropriate only in the case in which it holds the assumption that all DMUs are operating at

[14] The weak essentiality assumption of the production function holds, as the production of positive output is impossible without hte use of at least one input. Such assumption is usually replaced by the strong essentiality assumption in the multiple input framework, where every input is essential for production.

[15] Following an input oriented approach.

the optimal scale (else, the technical efficiency score captures also the scale inefficiency). By conducting both a CRS and a VRS DEA upon the same sample of DMUs, then, it is possible to decompose the efficiency scores into two components, one due to scale inefficiency and one due to "pure" technical inefficiency.

Therefore, scale efficiency scores can be computed, under the VRS assumption, by dividing the distance of a DMU's projection on the CRS frontier from the output axis by the distance between the DMU and the output axis. In Figure 6, for DMU V, the scale efficiency score is given by the ratio YN/YV. As this ratio is lower than 1, DMU V can be considered as scale inefficient, because it could use more units of input to obtain a higher and more then proportional level of output. DMU A instead is considered by the methodology both scale and technically efficient, as it lays on both the VRS DEA frontier and the CRS DEA frontier.

It is important to remark that the scale efficiency scores represent a technical computation and should not be considered as a precise measure from which to infer policy indications without further in-depth studies.

More in general, all non-parametric approaches find their major limitation in the sensitivity of the results to sampling variability, to quality of data and to presence of outliers (Herrera and Pang, 2005).

Another limitation of these approaches is the inadequate treatment of dynamics: these models do not take into accont the time lag between input consumption and output production.

For its deterministic nature and underlying assumptions, DEA is not a proper way to determine the direct impact of inputs on outputs. On the other hand, it is an instrument to empirically identify, in a pool of DMUs (that in the empirical exercise in this dissertation correspond to EU countries), which ones are the best performers compared to those that could have a margin of improvement in the efficiency of output production through input transformation (recalling that the efficiency of a DMU is in fact relative to the performance of the other DMUs in the sample).

Efficiency scores produced through the non-parametric techniques should therefore be handled with care and should be accompanied by further country-specific investigations in order to be able to make policy decisions from the results obtained. Nevertheless, these scores represent a useful tool for policy guidance as they provide a first insight in the topic of efficiency. In particular, these scores can be used to identify possible peers or role models for DMUs (which gives it an edge over other measures such as total factor productiviy indices). These methodologies in fact provide a set of potential role models that a DMU can look to, in the first instance, for ways of improving its operations. This makes DEA a potentially usefool tool for benchmarking and policy guidance.

### 3.3.3. Partial frontier efficiency analysis

This paragraph introduces an extension to the non-parametric approach. Given the vulnerability to outliers and measurement errors to which this methodology is subjected, partial frontier approaches have been developed in order to generalize the FDH technique by allowing for super-efficient observations to be located beyond the estimated production possibility frontier (Tauchmann, 2011). These approaches are the so called order-m (Cazals et al., 2002) and order-α (Aragon et al, 2005) efficiencies.

With the application of these methodologies, few abnormal observations (which could be the results of measurement error) will not entirely shape the estimated frontier.

In fact, the partial frontier approaches allow scores to exceed the value of one (in the input-oriented framework).

Following Tauchmann (2011)'s approach, the theoretical framework of these two methodologies is introduced.

**Order-m efficiency**

Order-m is a generalization of the FDH methodology which adds a layer of randomness to the computation of the efficiency score.

With this technique, a DMU is not benchmarked by the best performing peer in the sample, but rather by the expected best performance in a sub-sample of m peers.

Tauchmann (2011) provides an analytical explaination of the technique.

The FDH methodology is first recalled, using Tauchmann's notation:

To a sample of DMUs $i = 1,\dots,N$ corresponds a set of inputs of production $x_{i1}, \dots, x_{iK}$ and a set of outputs $y_{i1}, \dots, y_{iL}$. The (input-oriented) FDH efficiency is estimated by comparing each DMU $i = 1,\dots,N$ with all other DMUs $j = 1,\dots,N$ that produce at least as much of any output as DMU $i$.

The set of peer DMUs in the sample that satisfy the condition

$$y_{lj} \geq y_{li} \forall l$$

is denoted as $B_i$

FDH efficiency score $\hat{\theta}_i^{\mathrm{FDH}}$ is calculated as

$$\hat{\theta}_i^{\mathrm{FDH}} = \min_{j \in B_i} \left\{ \max_{k=1,\ldots,K} \left\{ \frac{x_{kj}}{x_{ki}} \right\} \right\}$$

where among the peer DMUs, the DMU that exibits the minimum input consumption serves as reference to DMU $i$ and as its own reference, meaning that a DMU in the set that exhibits abnormally little – possibly misreported – input consumption renders all its peers inefficient. In order to address such sensitiveness of FDH to outliers and measurement error, four steps are identified in the procedure to compute order-m efficiency (Dario and Simar, 2007):

1. From $B_i$ a sample of $m$ peer DMUs is randomly drawn with replacement.

2. Pseudo FDH efficiency $\hat{\theta}_{mi}^{\widetilde{\mathrm{FDH}}_d}$ is calculated using this artificial reference sample.

3. Steps 2 and 3 are repeated $D$ times.

4. Order-m efficiency is calculated as the average of pseudo FDH scores:

$$\hat{\theta}_{mi}^{\mathrm{OM}} = \frac{1}{D} \sum_{d=1}^{D} \hat{\theta}_{mi}^{\widetilde{\mathrm{FDH}}_d}.$$

Because of random re-sampling, in each replication a DMU may or may not be available as its own peer. Order-*m* efficiency scores may therefore exceed the value of 1. That is, order-*m* allows to spot the possibly super-efficient DMUs that are located beyond the estimated efficiency frontier (in the standard FDH methodology, instead, a DMU is always available as

its own peer, which rules out the possibility for relative input consumption to exceed unity). When calculating order-*m* efficiency, it is therefore necessary to choose the value for parameters *D* and *m*. The choice of *D* is a pure matter of accuracy (i.e. computing time), while for the choice of *m* one should consider that the smaller its value, the larger the share of super-efficient DMUs. For $m \to \infty$, order-*m* coincides with FDH.
If an observation remains above the frontier (i.e. maintains an attributed efficiency score higher than one) as m increases, then it may be an outlier (Budunenko, 2010).

**Order-α efficiency**

As order-m, order-α generalizes FDH. This methodology, though, differs from the previous as it uses as benchmark not the minimum input consumption among the available peers but the (100-α)th percentile :

$$\hat{\theta}_{\alpha i}^{\mathrm{OA}} = \mathrm{P}_{\substack{(100-\alpha) \\ j \in B_i}} \left\{ \max_{k=1,\dots,K} \left\{ \frac{x_{kj}}{x_{ki}} \right\} \right\}$$

For α = 100, order-α coincides with FDH, while for α < 100 some DMUs will be classified as super-efficient and obtain a score above 1.
Order-α , in contrast to order-m, does not require a re-sampling procedure.
If an observation remains outside the frontier (i.e., it is attributed, by the partial frontier methodology, a score above 1) as α increases, then it may be an outlier (Budunenko, 2010).

The FDH results presented in Chapter 4 have been tested for order-m and order-α partial frontier techniques and, according to Budunenko's criteria, do not present potentially super-efficient DMUs.

## 4. Data, methodology and empirical analysis

In this chapter, after a review of the relevant literature on frontier analysis targeting the public expenditure on education, an empirical application is presented.
The analysis follows an input-oriented scheme and applies non-parametric methods in order to identify the technical efficiency frontier in the context of expenditure on education in EU.

## 4.1. Empirical literature

There is abundant literature measuring public sector spending efficiency using national governments as decision-making units and conducting non-parametric frontier analysis. In this paragraph, an overview of the main papers focusing on the educational sector is provided; The methodologies applied, the variables taken into account and the main conclusions that the authors derived are highlighted for each study.

Gupta, and Verhoeven (2001) apply the input-oriented FDH approach to assess the efficiency of the expenditure on education and health in 37 countries in Africa, both towards each other and towards Asian countries and Western Hemisphere ones. The time horizon of the observations concerns the years between 1984 and 1995. The authors apply the single input-single output approach, and in each time period they construct efficiency frontiers for each of the several output indicators. The main results of their analysis show that, on average, African countries are inefficient in providing both health and education services relative to Asian and Western Hemisphere countries. Furthermore, concerning the output variable of the educational attainment, the authors document a negative relationship between the efficiency scores and the level of public spending, which leads them to conclude that a higher educational attainment output requires efficiency improvement more than an increase in the budget allocated to the education sector.

Afonso, Schuknecht and Tanzi (2003) adopt a non-parametric approach to measure efficiency of public spending. Their analysis is based on the construction of composite indicators of public sector performance for 23 OECD countries. The variables considered for such indicators capture the quality of different functions of the public expenditure: administration, education (measured as educational attainment) and infrastructure. In their analysis, the performance indicator is the output variable and the total expenditure is the input. Thanks to this aggregation, they carry out a single input-single output FDH and rank the efficiency scores of the expenditure of the countries in their sample. The results of their analysis show that countries with small public sectors exhibit the higher overall performance.

Afonso and St. Aubyn (2004) apply both DEA and FDH in order to address the efficiency of public spending in education and health. Their sample comprehends 23 OECD countries. The analysis carried-out details the results by comparing input-oriented and output-oriented approaches in the efficiency measurements. An interesting result obtained by the authors, as they reported in earlier drafts of the paper, concerns the inclusion of Mexico as one of the countries that lays on the efficiency frontier. The non-parametric models consider in fact Mexico as a benchmark country because of its low spending on education (and low educational attainment outcome). It then becomes, for these reasons, a sort of « origin » of the efficiency frontier. In the empirical chapter this topic will be discussed by providing similar examples for other low spending-low performance countries in Europe.

Herrera and Pang (2005) conduct non-parametric analysis on a sample of 140 developing countries over the years 1996-2002 and estimate the efficiency of public expenditure in education and health. For the analysis of education, the input used by the authors is the gross primary enrollment and the output is completion rates. The authors remark a positive relationship between the level of economic development and expenditure, and suggest that it can be explained by the Balassa-Samuelson effect (price levels being higher in wealthier countries than in poorer). By the application of this principle to factor prices of non-tradable goods and services such as education, the authors conclude that the price of education will be higher in countries with a higher GDP per capita. By conducting further analysis, the authors also conclude that rich countries tend to be less efficient, and they identify urbanization as a factor associated with higher efficiency. Moreover, they show that the level of expenditure, the ratio between government wage bill and total expenditure, and income inequality are negatively correlated with efficiency.

Afonso and St. Aubyn (2006) use a semi-parametric procedure in order to investigate the cross-country efficiency of secondary education provision. They compare the efficiency of education spending of 25 OECD states with the PISA results. The two methodologies that the authors apply place Finland and Sweden on the efficiency frontier. Moreover, in a previous version of the paper (2005), Hungary would belong to the efficiency frontier as well. This was due to reasons linked to the application of the methodology. In fact, this DMU would be considered a benchmark by the FDH methodology but it was not assigned efficiency by the DEA approach (in the most recent version of the paper). Furthermore, the authors' investigation comes to the conclusion that most European Member States are inefficient in the education system, and that substantial efficiency gains could be achieved by reducing the input (this is particularly true, according to the authors, for Austria, Germany, Italy and Portugal).

Clements (2002) applies the non-parametric approach (FDH technique) in a sample of

European countries. He uses the total primary and secondary expenditures per student as the input and the percentage of the population finishing secondary school at the normal graduation age as the output. Clements' investigation shows that the most efficient countries in Europe are Hungary, Norway, Ireland, Finland and Greece.

Afonso, Schuknecht and Tanzi (2010) conduct a study on a pool of 24 EU States and other emerging states. In their analysis, they first compute public sector performance indicators for the DMUs in their sample. Then, they apply the DEA methodology and conclude that, on average, countries could use 45 percent less of resources in order to obtain the same outcomes, and obtain an additional third of the fully efficient output.

Wilson (2005) applies FDH and DEA to conduct an analysis on the efficiency on education in a sample comprehending some former Soviet states, Latin American and East Asian countries. The author concludes that, in line with the results on the efficiency of the public sector as a whole, the sector of education shows both output inefficiencies and input inefficiencies. Analyzing the FDH technique results, he finds that average output could be improved of 3.3% while still holding fixed.

In his article, Aristovnik (2013) uses public expenditure on education as the input variable in order to measure technical efficiency of the latest entrants of the EU, in comparison to selected EU Members and OECD countries. The author applies the non-parametric methodology (DEA) and calculates relative efficiency scores following the output-oriented approach, in the variable returns to scale framework. The results of his study show that, among the new EU member states, Hungary, Estonia and Slovenia are the benchmark countries in the field of primary, secondary and tertiary education respectively. Moreover, the author’s empirical analysis indicates that new EU Member States show relatively high efficiency measures in tertiary education. In the sample of the paper, the most efficient countries in the educational sector are Finland, Japan and Korea.

The OECD itself has extensively analyzed the efficiency of spending in the education sector. OECD (2007) investigates the linkage between performance and institutions in primary and secondary education, while earlier OECD papers assessed technical and cost-efficiency. The input indicators used are the teaching resources and the socio-economic background of parents, in order to measure technical efficiency. According to the analysis, it is shown that countries such as Greece, Hungary, Ireland, and Sweden, show inefficiencies, as they could obtain a higher output (of 6%) using the same input.

Agasisti (2014) investigates the importance for European Countries of the minimization of the amount of public money devoted to educational services while, at the same time, maintaining a high level of efficiency. In his empirical study, he compares the spending efficiency during the period 2006-2009 of a sample of 20 European States. The author uses OECD-PISA test scores as the output and expenditure per student as the input, and applies the DEA procedure. Further analysis is carried out (Malmaquist indexes calculations) in order to investigate the change in efficiency over the years considered. The author's conclusion is that, on average, efficiency remained stable over time.

### 4.2. Description of the main indicators of input and output

Based on the literature specific on the sector of education and inspired by previous empirical papers, five different output indicators have been selected for the empirical analysis: the OECD PISA scores, educational attainment, youth educational attainment, quality of educational system and early school leavers.[16]

<u>*OECD-PISA score*</u>: launched in 2000 by the OECD, the Programme for International Student Assessment (PISA) constitutes an important source for studying the competences acquired by the students during their schooling years. The test is administered to 15 year olds in 65 countries, for a total of 510,000 participating students in 2012, regardless of grade, achievement, and socio-economic status (although home-schoolers are not considered). PISA tests critical thinking in math, science, and reading in a two hours written test, in part multiple choice, in part full answer.

The PISA data, collected every three years, is useful at several levels. It reveals common patterns among high performing school systems, and it is also used for benchmarking (as in the analysis presented in this dissertation). As a high ranking on PISA has been proven to correlate to economic success, researchers have concluded that PISA is one indicator of whether school systems are preparing students for the global knowledge economy of the 21st century (asiasociety.org). The following table reports the detail of the last available PISA data (2012).

[16] It is worth to notice that the assessment of the efficiency of public spending on education carries a number of difficulties with regards to the timely availability of data as well as to the measurement of both input and output variables, which was an important component in the selection of the input and output indicators.

Table 3 reveals mixed results across Member States. EE, FI, PL, NL are the best performers in all the subjects of the test, and among OECD countries as well. Above all, Finland shows the highest average score, and this country obtained the highest result in science. It also outperforms most of the EU Members in the other two subjects. Moreover, Finland shows one of the lowest spreads between well- and poor-performing students (OECD, 2014).

Germany performs above the average in mathematics (as it ranks between 6th and 10th among OECD countries), reading (ranks between 9th and 15th among OECD countries) and science (between 5th and 10th among OECD countries) (OECD, 2012).

Ireland also shows above average results in all categories, with a particularly high performance in science and reading.

In general, data shows that countries with an education expenditure above the average do not necessarily perform better then lower spending countries (examples of low spending, well performing countries are Czech Republic and Netherlands, and a clear example of high spending-average performing country is Denmark).

Analysis of results reveal that ten Member States (BG, CZ, DE, EE, IE, HR, LV, AT, PL and RO) have achieved significant progress in diminishing their share of low achievers across all three basic skills since the earlier PISA test of 2009 (European Commission, 2013).

**Table 3 : OECD-PISA scores by subject (EU Member States)**

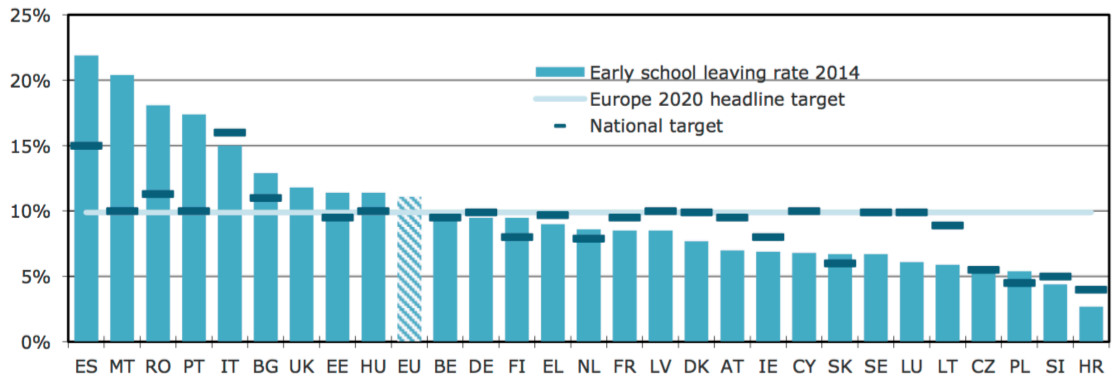
| | Mathematics | Reading | Science |
|---|---|---|---|
| | Mean score | Mean score | Mean score |
| OECD average | 494 | 496 | 501 |
| Austria | 506 | 490 | 506 |
| Belgium | 515 | 509 | 505 |
| Bulgaria | 439 | 436 | 446 |
| Croatia | 471 | 485 | 491 |
| Cyprus | 440 | 449 | 438 |
| Czech Republic | 499 | 493 | 508 |
| Denmark | 500 | 496 | 498 |
| Estonia | 521 | 516 | 541 |
| Finland | 519 | 524 | 545 |
| France | 495 | 505 | 499 |
| Germany | 514 | 508 | 524 |
| Greece | 453 | 477 | 467 |
| Hungary | 477 | 488 | 494 |
| Ireland | 501 | 523 | 522 |
| Italy | 485 | 490 | 494 |
| Latvia | 491 | 489 | 502 |
| Lithuania | 479 | 477 | 496 |
| Luxembourg | 490 | 488 | 491 |
| Netherlands | 523 | 511 | 522 |
| Poland | 518 | 518 | 526 |
| Portugal | 487 | 488 | 489 |
| Romania | 445 | 438 | 439 |
| Slovak Republic | 482 | 463 | 471 |
| Slovenia | 501 | 481 | 514 |
| Spain | 484 | 488 | 496 |
| Sweden | 478 | 483 | 485 |
| United Kingdom | 494 | 499 | 514 |

Source: OECD

*Early school leavers*: The EU regards upper secondary education attainment as a prerequisite for economic development and for lowering chances of poverty and social exclusion (EC, 2015). Therefore, those 18 to 24 year-olds who fail to reach this level of education are considered early school leavers. The latest available data (2014) show that the average early school leaving rate in the European Union stands at 11.1%.

Nineteen Member States (HR, SI, PL, CZ, LT, LU, SE, SK, CY, IE, AT, DK, LV, FR, NL, EL, FI, DE, BE) are currently below the threshold of 10% of early school leaving, as it is seen in Figure 7 below. The figure also shows the headline target of the European Commission (that is to say, the share of early leavers from the education system targeted by the Commission on the framework of Europe 2020) and the national target for each Member State.

**Figure 7: Early school leavers in EU**[17]




Source: European Commission elaboration of Eurostat (LFS, 2014) data

[17] The indicator covers the share of the population aged 18-24 having attained ISCED level 0 to 2 and not receiving any formal or non-formal education or training in the four weeks preceding the survey. National targets follow different definitions of the indicator in some countries (see Table 2.1.1). Data for HR have low reliability due to the small sample size.

*Educational attainment*: The International standard classification of education (ISCED) defines the level of educational attainment of an individual as the highest ISCED level successfully completed. According to this definition, educational attainment levels are generally grouped into three categories:

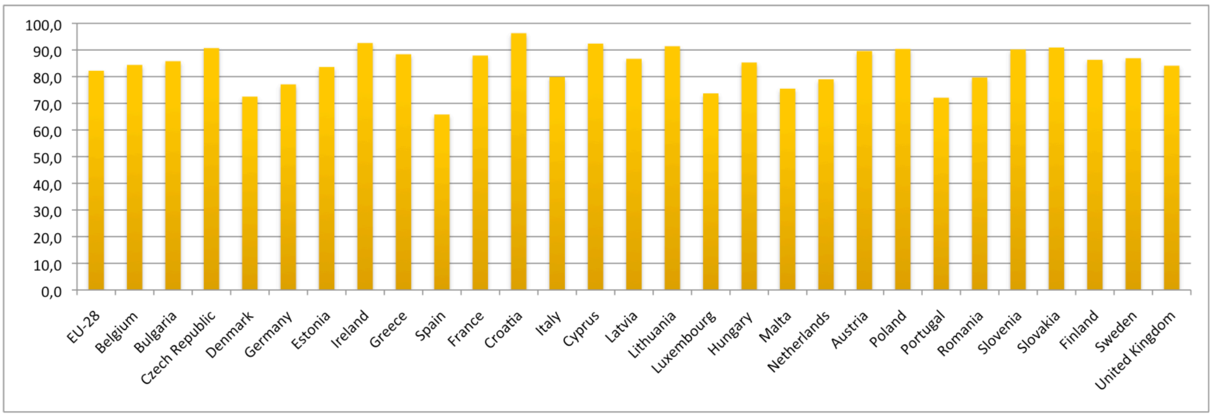
-less than primary, primary and lower secondary education

- lower secondary, upper secondary and post-secondary education

-tertiary education

Eurostat also provides the data for Youth Educational Attainment, that is to say, the percentage of a country's population aged between 20–24 years old that corresponds to each of the three ISCED categories. The following figure shows a decomposition of the population aged 20–24 having completed at least upper secondary education (ISCED 3) for each EU country in 2014.

**Figure 8: Population aged 20–24 having completed at least upper secondary education, 2014**




Source: Author's elaborations of Eurostat data

*Quality of educational system*: This indicator is the result of an Executive Opinion Survey conducted by the World Economic Forum. The respondents were asked to answer to the question on how well the education system in their country met the needs of a competitive economy. At a country perspective, the results are obtained by averaging the scores on a 1 - 7 scale of a large sample group that in each country answered to the survey.

The efficiency of education expenditure is assessed using as the input variable the ten-year-average (2002-2011) public expenditure on education as a share of GDP.[18]

The use of total (i.e. the sum of public and private) education spending as input variable is also considered as a robustness check. The use of the sole public expenditure could in fact blurry the picture regarding the efficiency of the educational system by under-estimating the amount of the input that countries employ in order to obtain a certain amount of output for each output variable considered. The amount of private financing can differ across the EU economies in the sample, as seen in Table 2 in the descriptive Chapter 2. Performance is in fact influenced by both public and private input. Hence, ignoring private financing may distort the cross-country comparison of efficiency.

### 4.3. FDH and DEA analysis: Public expenditure on education

In the present paragraph an application to the 28 EU countries of the non-parametric methodologies described in Paragraph 3.3 is presented.

28 DMUs are a sufficient number to carry out a meaningful analysis, according to the many rules of thumb present in the literature. According to Golany and Roll (1989), the number of DMUs should be of at least twice the number of inputs and outputs considered. According to Bowlin (1998) there must be a number of DMUs equal to at least three times the number of input and output variables, and according to Dyson et al. (2001) it is raccomanded a number of DMUs at least equal to a total of two times the product of the number of input and output variables.

The empirical analysis consists, at a first stage, in conducting one input-one output non-parametric analysis for each of the five output indicators, using public expenditure on education as the input. This means that for each indicator, the FDH and DEA efficiency scores are computed and the efficiency frontiers are determined. Performance of Member States can be assessed on the base of their coordinates in the input – output plane, according to the respective scores.

In all the analysis carried-out, the input oriented approach has been adopted, motivated by the fact that governments presumably have more direct control over inputs (e.g. public expenditure) than over performance levels and outputs.

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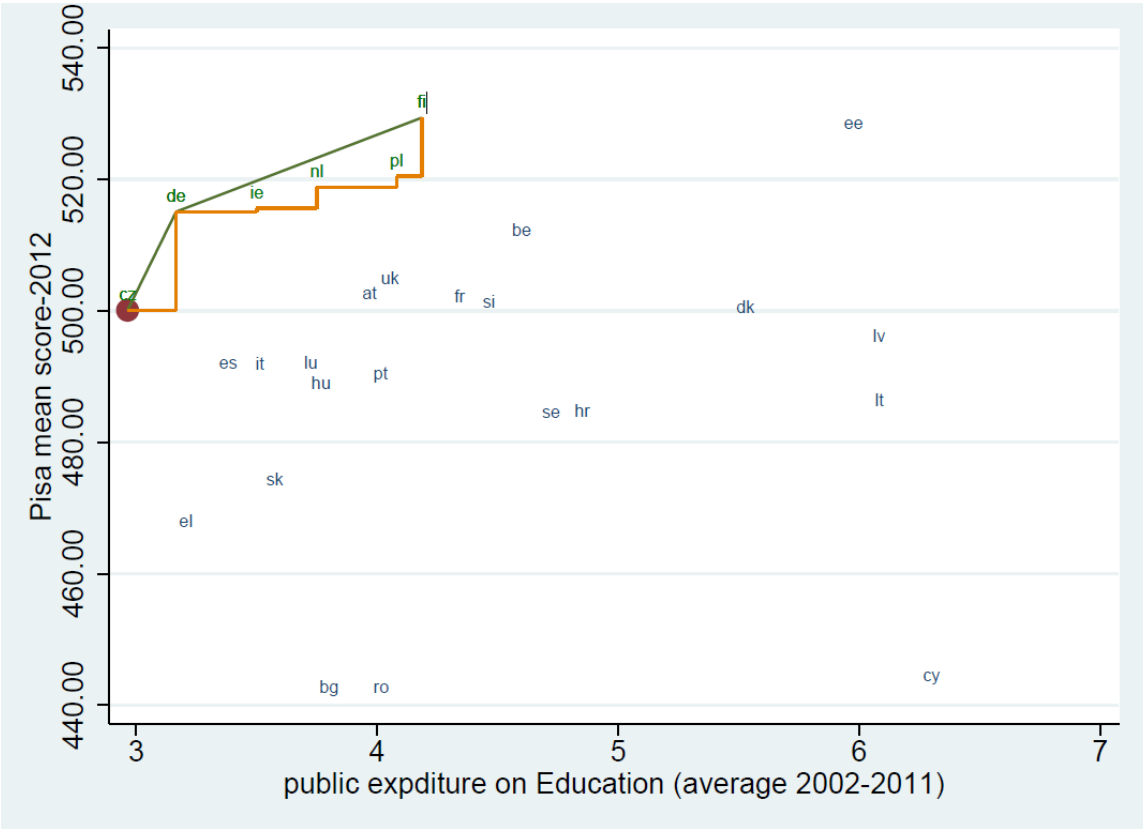
18 This choice is mainly guided by the data at disposal.

#### 4.3.1 DEA and FDH frontiers: public expenditure and PISA

When analyzing the relationship between PISA scores (as the output) and educational expenditure (the input), it is worth to notice that only the expenditure on primary and secondary education has been taken into account in this first analysis (i.e. expenditure addressed to pupils in the 6-15 age range). In fact, as PISA tests are taken by 15 year-old students, the respondants benefit of only these two shares. The PISA score for each country is an average of the scores in each component of the test (reading, math and science) at the national level.

**Figure 9: DEA and FDH for PISA-score and Public Expenditure on education**




Source: Author's elaboration of OECD data

Figure 9 above illustrates the efficiency frontiers for the PISA score output framework, that is, a graphic representation of the performance of Member States in this output indicator in the input / output plane.[19]

Representing PISA scores of Member States as a function of public expenditure on primary and secondary education, one can see that in the variable returns to scale case (VRS) the efficiency frontier is determined by Czech Republic, Germany and Finland. The graph illustrates well the difference between performance and efficiency: for example, Estonia is among the best-performing Member States according to PISA score, however, FI achieves a very similar PISA score with about 30% less expenditure. Similarly, CZ's PISA performance is slightly above the average, however, given its low spending on education, it turns out to be on the efficiency frontier.

CZ is also, according to the DEA technique, the efficient DMU in the constant returns to scale context (CRS). Graphically, this is shown by representing CZ as a red dot, in order to indicate its position on the straight line passing through the origin (which has been omitted in the figure for the sake of clarity). The efficiency of Czech Republic and of Finland is corroborated by the results obtained by Agasisti (2014) in his VRS DEA analysis (that uses PISA scores as output and expenditure as input on a sample of European countries). Besides, Finland is found efficient in most of the previous literature and empirical analyses of the education sector (for example, in Clements (2002), Afonoso and St. Aubyn (2006), and in Aristovnik, (2013)).

Finally, by looking at the FDH frontier it is easy to see that the Member States that are assigned full technical efficiency increase in number with this methodology with respect to the ones identified by the DEA technique. In fact, besides Czech Republic, Germany and Finland, also Ireland, Netherlands and Poland lay on the FDH efficiency frontier. This is not surprising as these three countries have obtained among the highest PISA results in EU. These countries are efficient according to the FDH (and not to the DEA framework) because the FDH methodology does not require the assumption of convexity of the production function (which is instead a crucial assumption of the DEA methodology, in order to compute the virtual DMUs that do not correspond to any observation in the sample) so that, by considering only existing DMUs, FDH tends to assign efficiency to a higher number of DMUs.

In Table A3 in the Annex the numerical results obtained by applying the three methodologies are reported. These numbers explain analytically what Figure 9 shows visually: the DMUs that lay on the efficiency frontier are assigned a score of one, while all the other Member States are assigned a score between 0 and 1. The lower the score assigned, the less efficient the DMU (according to the approach). Moreover, the VRS DEA methodology determines the

[19] Malta was excluded from the sample because of unavailability of data.

returns to scale segment in which each observation is located. CZ is the only DMU obtaining both technical and scale efficiency, as it lays on both the CRS and the VRS DEA frontiers: it obtains a unitary score under the VRS assumption (i.e., technical efficiency) and it also lays on the CRS frontier, therefore the difference between the VRS and the CRS scores is 0 (i.e. it is scale efficient).

In the last column of the table a ranking is assigned to each Member State, relative to its VRS DEA technical efficiency score.

As previously remarked, scale scores (and numerical scores in general), represent a first useful overview of countries' relative performances. Nevertheless, the assessment should be complemented with more in-depth, country specific follow-up analysis, as such numerical results should not be interpreted as exact measures of how much to decrease the input in order to obtain the same level of output and increased efficiency (in the case of technical efficiency scores) nor of how much to increase/decrease the size of the DMU in order to attain the optimal scale (in the case of the scale scores). Scale scores, for example, concern in fact the dimension of a DMU (i.e. the size of the input-output bundle). This measure might be accurate in the context of firms, but not so appropriate when considering national governments, which cannot modify their own size in the way in which other types of DMUs could do.

Later in the text, an extention of this first non-parametric exercise is reported, that is to say, a two inputs-one output model using PISA scores as the output and contextually multiple input variables at a time.

### 4.3.2 DEA and FDH frontiers: public expenditure and educational attainment

The second step of this analysis of the efficiency of public expenditure on education puts the focus on the performance on educational attainment of the EU Member States (educational attainment being measured as the percentage of each country's population having completed at least upper secondary education)[20].

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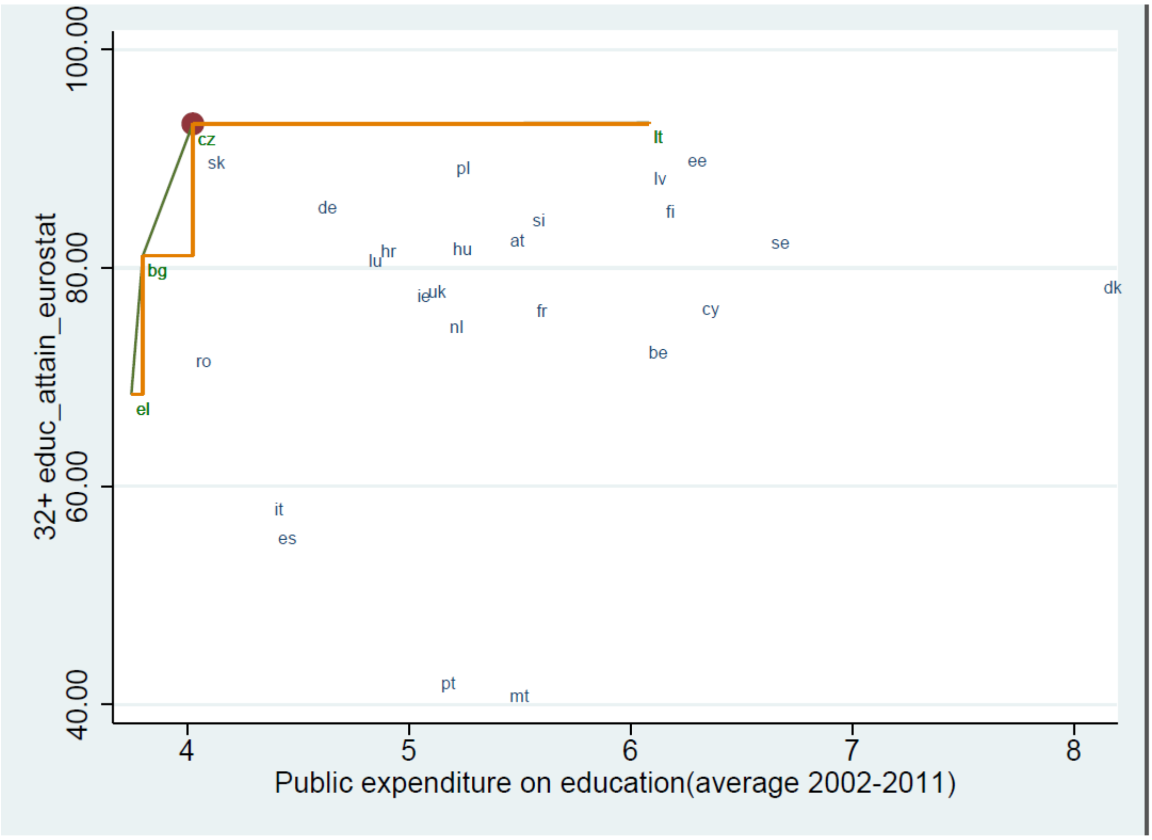
[20] "Statistics on the level of the educational attainment of the population are based on the EU Labour Force Survey (EU-LFS). The EU-LFS covers the total population usually residing in Member States, except for persons living in collective or institutional households. For data on educational attainment based on the EU Labour Force Survey (EU-LFS) the International Standard Classification of Education 2011 (ISCED 2011) is applied as from 2014. Up to 2013 ISCED 1997 is used.'' (EUROSTAT website)

**Figure 10: DEA and FDH for educational attainment and Public Expenditure on education**




Source: Author's elaboration of OECD and EUROSTAT data

In this exercise and in the ones that follow (where the other output indicators are used in order to assess the efficiency of public expenditure on education), all levels of the public expenditure on education are considered (recalling that in the PISA output case, only primary and secondary education expenditure were taken into account).

The efficiency frontiers defined according to the non-parametric methodologies are reported in Figure 10.

In this case Bulgaria, Czech Republic, Greece, and Lithuania are the efficient countries identified by the DEA methodology (with CZ, as in the PISA case, being the CRS efficient Member State). With educational attainment as the output variable, DEA and FDH propose the same results in terms of countries laying on the respective efficiency frontiers.

The low efficiency score of Denmark is driven by the fact that, despite the fact that its public expenditure on education is the highest among the EU Member States, DK performs slightly below the EU average in terms of educational attainment (OECD, 2013). This inefficiency is reflected by the high distance of the DMU from the frontier.

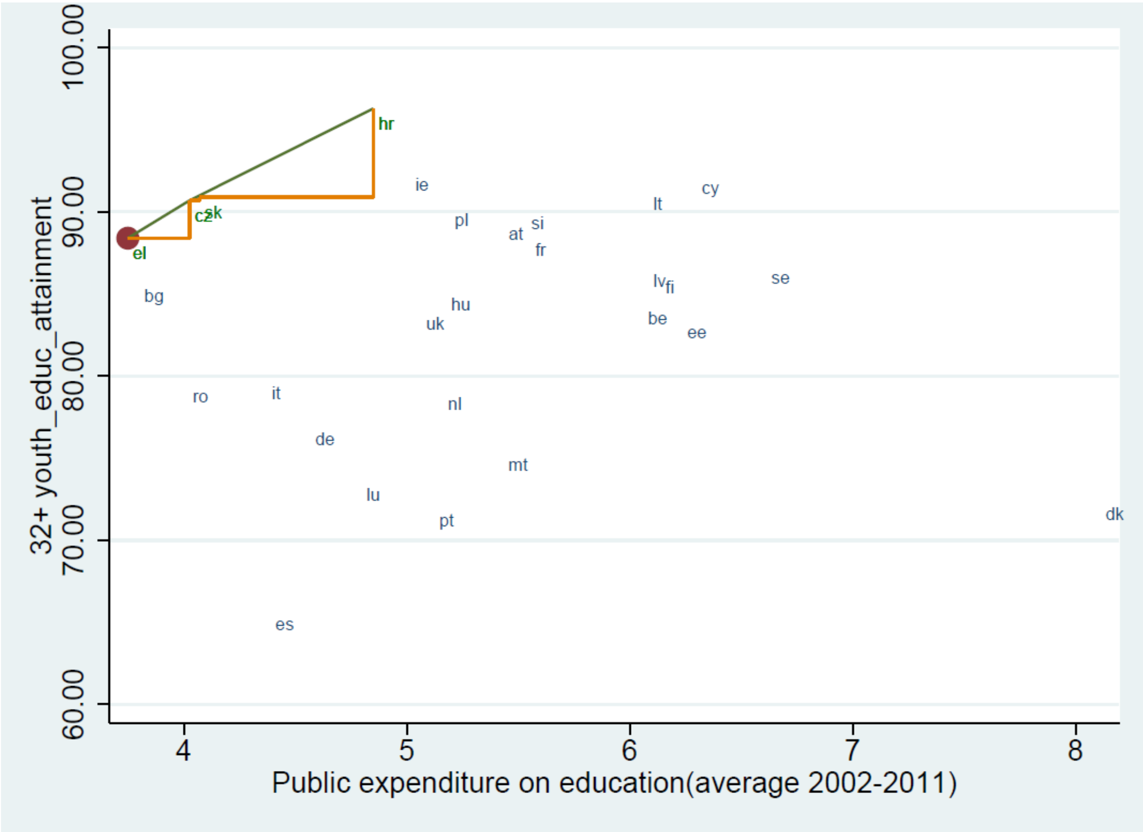
Table A4 in the Annex further details the efficiency scores according to each methodology and the scale scores corresponding to the variable return to scale DEA methodology.

In this exercise, Lithuania represents a peculiar case worth highlighting. This Member State lays in fact on the efficiency frontier (it is assigned a unitary technical efficiency score), but it presents the same output level (in terms of educational attainment) as the one of Czech Republic. CZ, then, outperforms LT in the sense that it obtains the same level of output using a lower amount of expenditure. In this special case, LT reaches the frontier with respect to its peers, but a further reduction of the input could in principle be achieved without a reduction of the output, by taking CZ as the benchmark. Such extra input reduction possibility is known in DEA studies as "slack". When conducting a non-parametric assessment, it is necessary to check for the presence of slacks, as an application of the methodology without a critical interpretation of the results could be misleading (and the case of Lithuania is the clear representation of this). In fact, as pointed out in Chapter 3, the efficiency scores obtained are data driven. Still, they can provide a useful first insight for further analysis. Concerning scale scores, for instance, VRS DEA places BG and EL in the locally increasing returns to scale segment of the frontier and LT in the decreasing one. Such results suggest the need of further investigations on the effects of an increase in expenditure in these countries (in order to assess if this could lead to a more than proportional increase of educational attainment for BG and EL and a less than proportional increase for LT).

### 4.3.3 DEA and FDH frontiers: public expenditure and youth educational attainment

The next exercise consists in comparing the results obtained in the analysis of the comprehensive (regardless of age tranches) educational attainment with the ones obtained using youth educational attainment as the output variable, which focuses specifically on the 20-24 years old population.

**Figure 11: DEA and FDH for youth educational attainment and Public Expenditure on education**




Source: Author's elaboration of OECD and EUROSTAT data

Figure 11 shows graphically the results obtained by the non-parametric analysis. As in the educational attainment case, Greece and Czech Republic are on the DEA efficiency frontier, but Bulgaria and Lithuania are not considered efficient. Croatia, instead, becomes efficient.

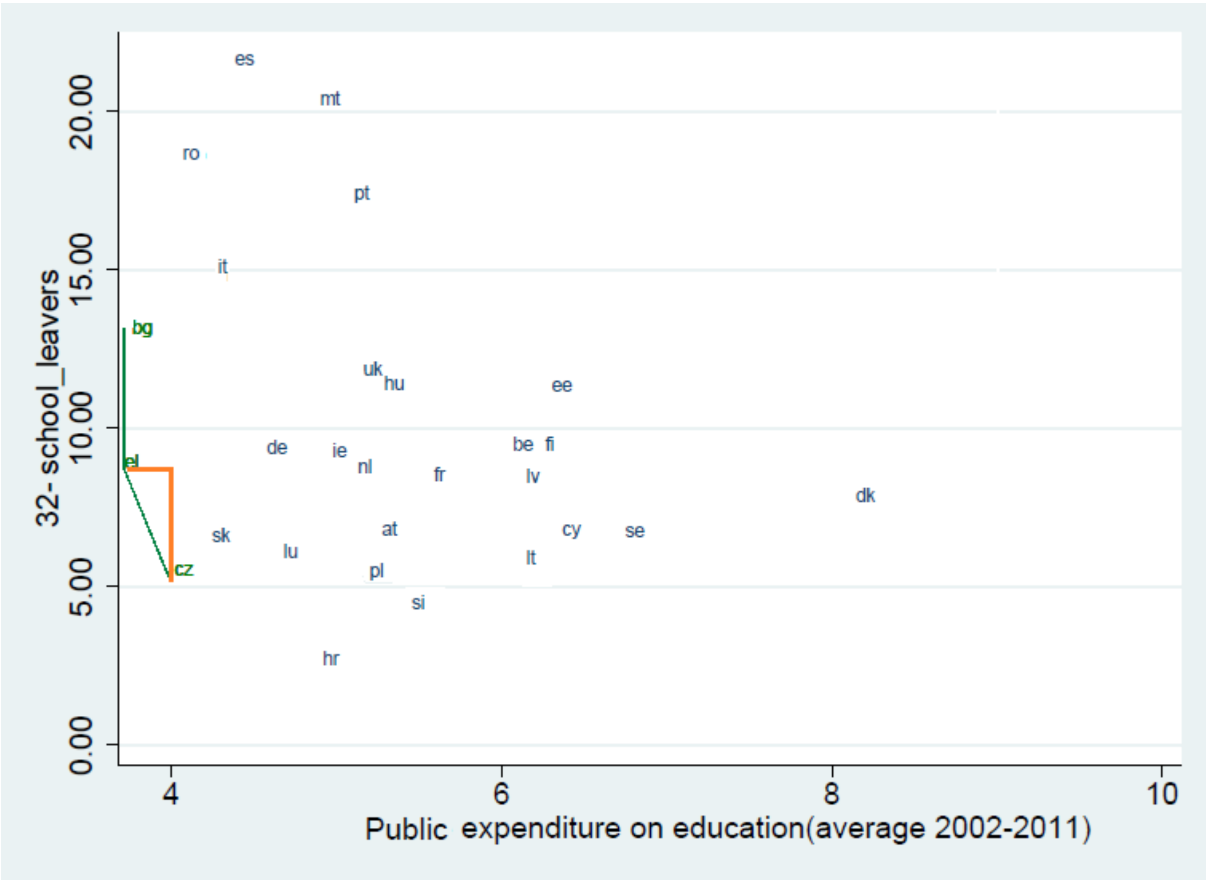
The efficient DMUs identified by the variable returns to scale DEA correspond to the ones identified by the FDH procedure, with the exception of Slovakia, which is found efficient only by the latter methodology.

Numerical scores for the three methodologies and returns to scale are shown in Table A5 in the Annex.

By comparing the DEA efficiency scores in the two output scenarios, it is possible to remark that the frontiers slightly change because of the switch from educational attainment to youth educational attainment. In particular, BG and LT pass from being efficient to being inefficient and HR passes from being inefficient (with a score of 0,790432 in the educational attainment scenario) to being on the efficiency frontier. Also, IE and CY's efficiency scores are remarkably higher when using the youth educational attainment as the output variable with respect to using educational attainment data.

### 4.3.4 DEA and FDH frontiers: public expenditure and early school leavers

**Figure 12: DEA and FDH for early school leavers and Public Expenditure on education**




Source: Author's elaboration of OECD and EUROSTAT data

The Figure above provides a graphical representation of the frontier on which the efficient EU Member States lay in the early school leavers output case.

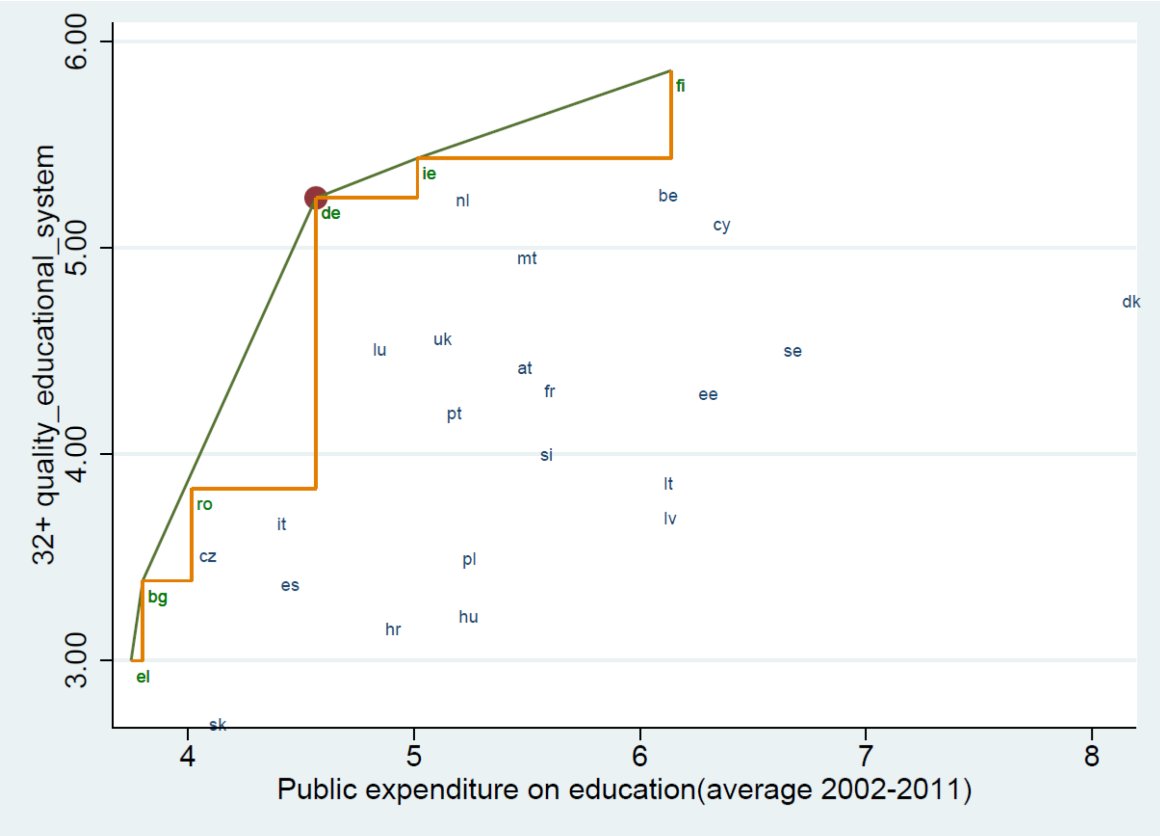
It is interesting to notice that the frontiers in this framework are differently shaped than the ones in the previous cases. The early school leavers are in fact a "negative'' output, that is to say, given the level of public expenditure, a country that obtains a higher result on this

indicator is performing worse.
According to the VRS DEA technique, the efficient countries defined are Bulgaria, Greece and Czech Republic. These countries happen to be also the efficient DMUs according to the FDH methodology.
As one would expect, the early school leavers results are strongly correlated with the ones obtained in the educational attainment and youth educational attainment scenarios. In fact, two out of three countries identified as the most efficient ones in this former output variable case (Czech Republic and Greece) are also on the DEA and FDH efficiency frontiers when considering the two latter output indicators.
Bulgaria, on the other hand, is DEA and FDH efficient in the educational attainment scenario and in the school leavers one, but inefficient in the youth educational attainment case, yet it lays very close to the efficiency frontier (it is assigned a VRS DEA efficiency score of approximately 0.986 in the youth educational attainment scenario). Such results of BG are corroborated by the Directorate General for Education and Culture (DG EAC)'s overview on the achievements of Member States. In fact, Bulgaria is at present improving its performance with regards to basic skills and teriary educational attainment (it is efficient with respect to secondary educational attainment in this analysis), and it is still in need of improvement in terms of access to education for disadvantaged children (it is in fact inefficient in terms of youth educational attainment), in particolar Roma, as integrarion in the general education system is an ongoing challenge in this country (EC 2015).

#### 4.3.5 DEA and FDH frontiers: public expenditure and quality of education system

The next output indicator that has been analyzed is the quality of the educational system, the Executive Opinion Survey conducted by the World Economic Forum. The output variable taken into consideration corresponds to the average of results of the survey for each of the EU Member States, and the efficiency frontier according to DEA and FDH is reported in Figure 13.
The DEA methodology in this exercise identifies five efficient countries: Bulgaria, Finland, Germany, Ireland and Greece.

**Figure 13: DEA and FDH for quality of education and Public Expenditure on education**




Source: Author's elaboration of OECD and WEF data

Germany is the scale efficient country, as it lays on both the CRS and VRS frontiers (in fact, the difference between its VRS efficiency score and the CRS one, that is its scale inefficiency, is zero).

The FDH methodology, instead, identifies six efficient countries: the five already defined by DEA, that is to say, BG, DE, EL, FI, IE, and also Romania. As the non-parametric techniques are data driven, they are particularly sensitive to DMUs employing a low level of input. In the present case, it is possible to notice that BG, EL and RO obtain a score of 1 but stay in the lower part of the y-axis representing the output level, that is to say, are poor performers. Given the fact that they also employ a low level of the input (public spending on education), they are shown as efficent DMUs (belonging to the FDH frontier), but in order to derive conclusions on their real efficiency in the sector analyzed, it is necessary to conduct further country specific analyses. In particular, it is necessary to disentangle the real objective of their below-average expenditure, in order to understend if the objective is to allocate fewer resources to try to improve their outcome in education or if inefficiency sources are driving their poor performance, and in this second case, the unitary score assigned by the non-parametric methodology is purely technical, and these low-spending countries do not represent real optimum points. The presence of these low spending-low performing countries on the efficiency frontier can be interpreted in a similar way as the presence of Mexico in Afonso and St. Aubyn (2004)'s frontier.

In Table A7 in the Annex the numerical results derived for the quality of educational system output indicator are presented.

### 4.3.6 Multiple input-output analysis with quantitatively measured inputs

The last non-parametric methodology that has been applied is a multiple input-output specification of the DEA technique. Such methodology is not often employed in the non-parametric studies surveyed, but Afonso and Aubyn (2003) provide a first attempt, by applying a multiple-input approach to a sample of OECD countries. This analysis follows their approach in terms of variables selection (i.e. the two inputs and the output).

As in the authors' model, the teaching hours per year ratio and the number of teachers per student ratio have been chosen as the variables of input. The results of the OECD-PISA test are again the output variable. This selection allows to analyze the (in)efficiencies of European countries not only from the point of view of the monetary amounts invested in education, but also from the one of real inputs, such as the number of schooling hours that students are required to attain, and the dimension of classes. The use of these inputs also presents a shortcoming: in fact, many observations had to be dropped because of the non homogeneous availability of the data for all the EU countries, i.e. BG, CY, DK, IT, LT, LU, LV, MT, NL, PL, RO. Such reduced dataset (from the 28 original countries to a sub-sample of 17 DMUs) is still in line with the minimum DMU number of the various rules of thumb for DMU selection reported in Paragraph 4.3.

The teacher per student ratio is the inverse of the average number of pupils per teacher in primary education, based on headcounts of both pupils and teachers. The original Worldbank index divides the total number of pupils enrolled at the specified level of education by the number of teachers at the same level. Naturally, one can expect education performance to increase with the number of teachers per student (Afonso and Aubyn, 2003).[21]

The teacher per student ratio is reported on the x-axis of Figure 14 below. The y-axis, instead, represents the second input used for this analysis: the teaching hours per year. In order to obtain a readable graph, the OECD data compulsory instruction time[22] in public institutions (available for the year 2006) has been rescaled: it was devided for 200, an indicative number of school days per year (in the European educational system the number of days of school is usually lower but, as reported in EC Eurydice (2015), in some countries even 200 days a year are devoted to school time). A similar logic has been applied in order to obtain the teacher-pupil ratio data. In any case, as such rescalings were applied to all the observations in the sample, the results are not affected by these manipulations.

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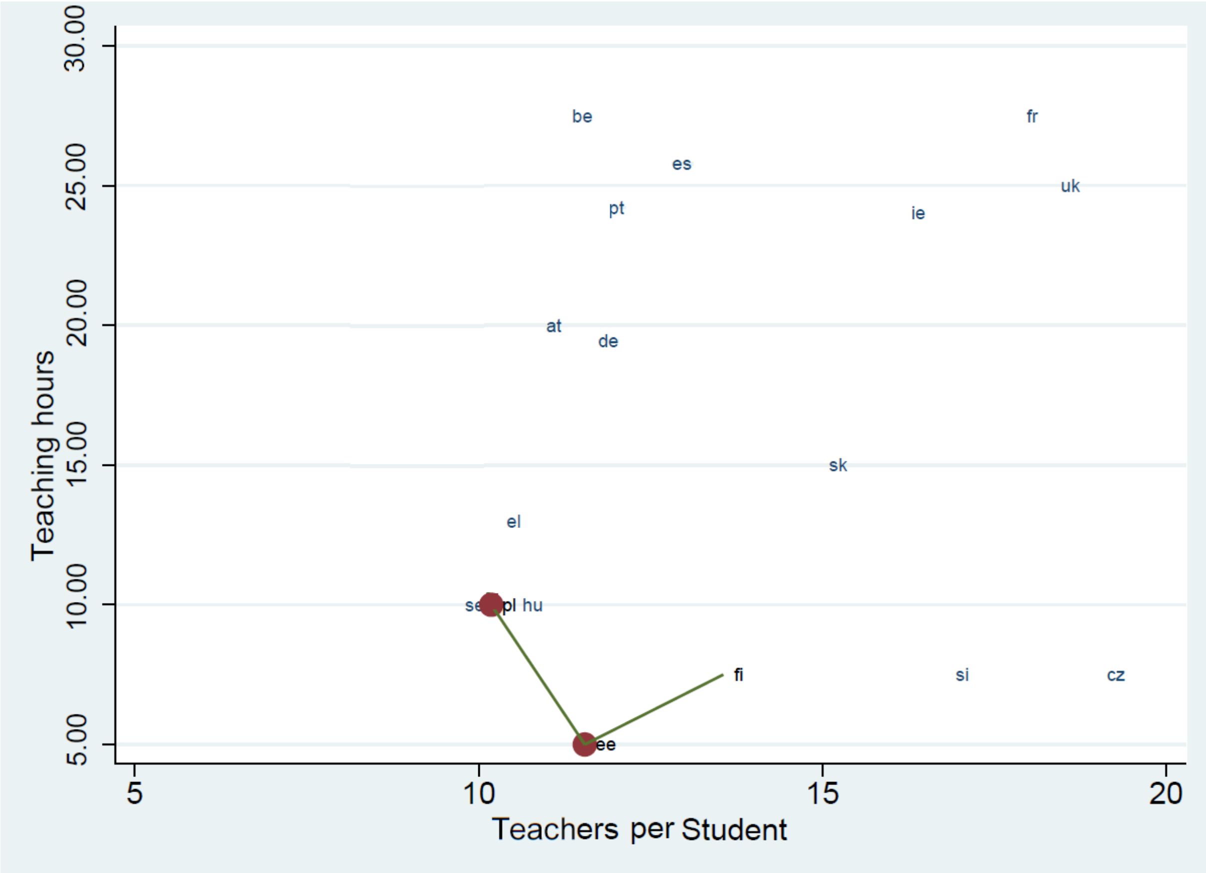
[21] In computing and interpreting this indicator, one should take into account the existence of practices that may affect the precision and meaningfulness of the teacher-pupil ratio such as part-time teaching, multi-grade classes (i.e. classes in which there are pupils from two or more grade levels) and school-shifts (a double-shift system refers to a school system in which the same buildings, equipment and facilities are used by two entirely separate groups of students during a school day. Usually, the first group attends school from early morning until mid-day and the second group from mid-day to late afternoon. Depending on the system, the two groups may or may not be taught by the same teachers. Sometimes this model is extended to triple-shift systems, that cater for three groups of pupils e.g. from 6.30 a.m. to 10.55 a.m., from 11.00 a.m. to 3.25 p.m., and from 3.30 p.m. to 7.55 p.m., and even to quadruple-shifts, as reported by Bray, 2008).

As noted in the Worldbank database, when feasible, the number of part-time teachers was converted to 'full-time equivalent' teachers, and a double-shift teacher is counted twice.

[22] The average number of hours per year of intended instruction time in public institutions for 7 to 14 years old pupils. The indicator is provided by the OECD and refers to the year 2006. The OECD refers to intended instruction time as the number of hours per year for which students receive instruction in both the compulsory and non-compulsory parts of the curriculum. For countries that have no formal policy on instruction time, the number of hours was estimated from survey data (OECD Glossary of Statistical Terms).

**Figure 14: Two input-one output DEA**




Source: Author's elaboration of OECD and Worldbank data

The Figure shows the DEA efficiency frontier on which the efficient countries lay. The DMUs identified as benchmarks by this methodology are Estonia, Finland, Poland and Sweden. In Table A13 in the Annex, the numerical results calculated on the basis of the methodology are presented.

Finland and Poland are efficient in both the two input-one output and in the one input-one output exercises carried out using PISA scores as the output.

By substituting the financial input (public expenditure on education) with the quantitative inputs, Czech Republic is no loger on the efficiency frontier.

Moreover, the unitary efficiency scores assigned to Finland and Sweden are in line with the results of Afonso and Aubyn (2003), as the two countries are assigned efficiency in their two input-one output DEA. It is not possible to compare the results of the two other efficient countries of this exercise, Estonia and Poland, with the authors' results: EE was not considered in Afonso and Aubyn's analysis and PL was dropped by the authors because of inavailability of data. The authors have indeed assigned technical efficiency to other non-EU OECD countries (Korea, Japan and Mexico), that are not considered in the sample used for the present analysis.

Estonia is assigned an efficiency score of 1 because it is the country that has the minimum teaching hours per year in the sample of EU countries analyzed. A similar reasoning is suitable for Poland, whose PISA scores are above the OECD average in all of the three sections of the test. Both of these countries belong to the high performing countries in the educational sector, relative to their PISA mean score.

As in the PISA-public expenditure model, UK is not on the frontier and maintains (approximately) the same level of technical efficiency assigned in the previous exercise.

### 4.3.7 A robustness check: public expenditure vs total expenditure

This section reports the robustness check conducted for the one input-one output non-parametric analysis. Using the same pool of countries used in the previous assessment, the input variable is changed in order to include private funds allocated to the education system in each country: public expenditure on education has in fact been replaced with total expenditure on education. Total expenditure is obtained by the summation of the private expenditure recorded for each country (OECD data) to the original public expenditure input. As in the previous case, the 10 year average in % of GDP is taken (for the years 2002-2011).

Although the literature reviewed does not refer to this specific issue, this robustness check seems necessary in order to confirm the results obtained when using the sole public expenditure as the input indicator. In the education sector in EU, in fact, there is no homogeneous source of financing among Member States.

In Europe, on average, the private funds are overcome by public expenditure on education (in 2013 private financing corresponded to 0.7% of GDP at the EU aggregate level, while public funds corresponded to 5.3% of EU GDP ), but as it was previously shown in Table 2, there are some cases in which private expenditure can reach a considerable share of the total expenditure on education of a country (remarkably, Cyprus and the United Kingdom, in which private educational expenditure in 2011 was up to 1,7% and 1,6% of the national GDP respectively, and above 20% of the respective public funds allocated to the education sector).

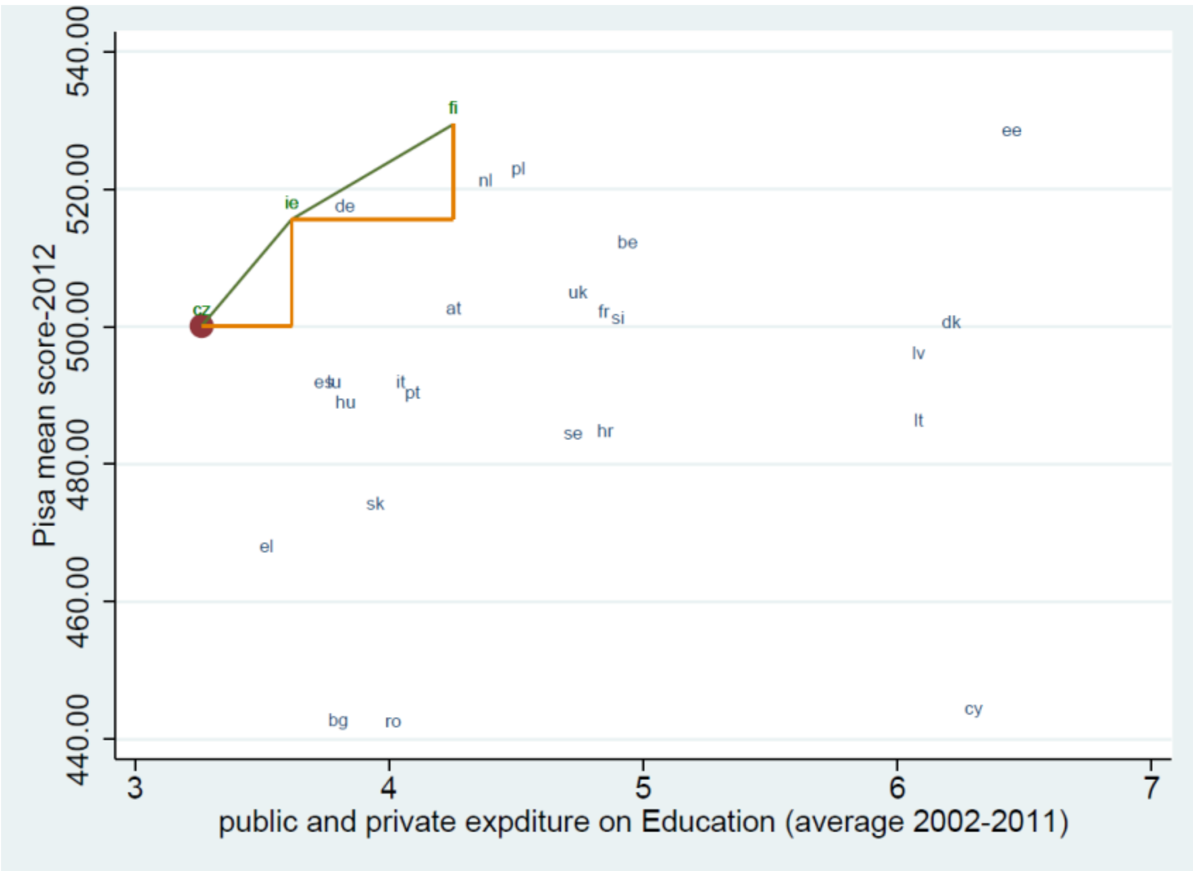
This further exercise helps to address the main drawback of the non-parametric methodology previously employed to determine the efficiency of the education sector in the Euorpean Union. In fact, the output indicators used in the previous one input-one output analysis are influenced by all sources of expenditure on education, and not only by the public share that governments allocate in each country.

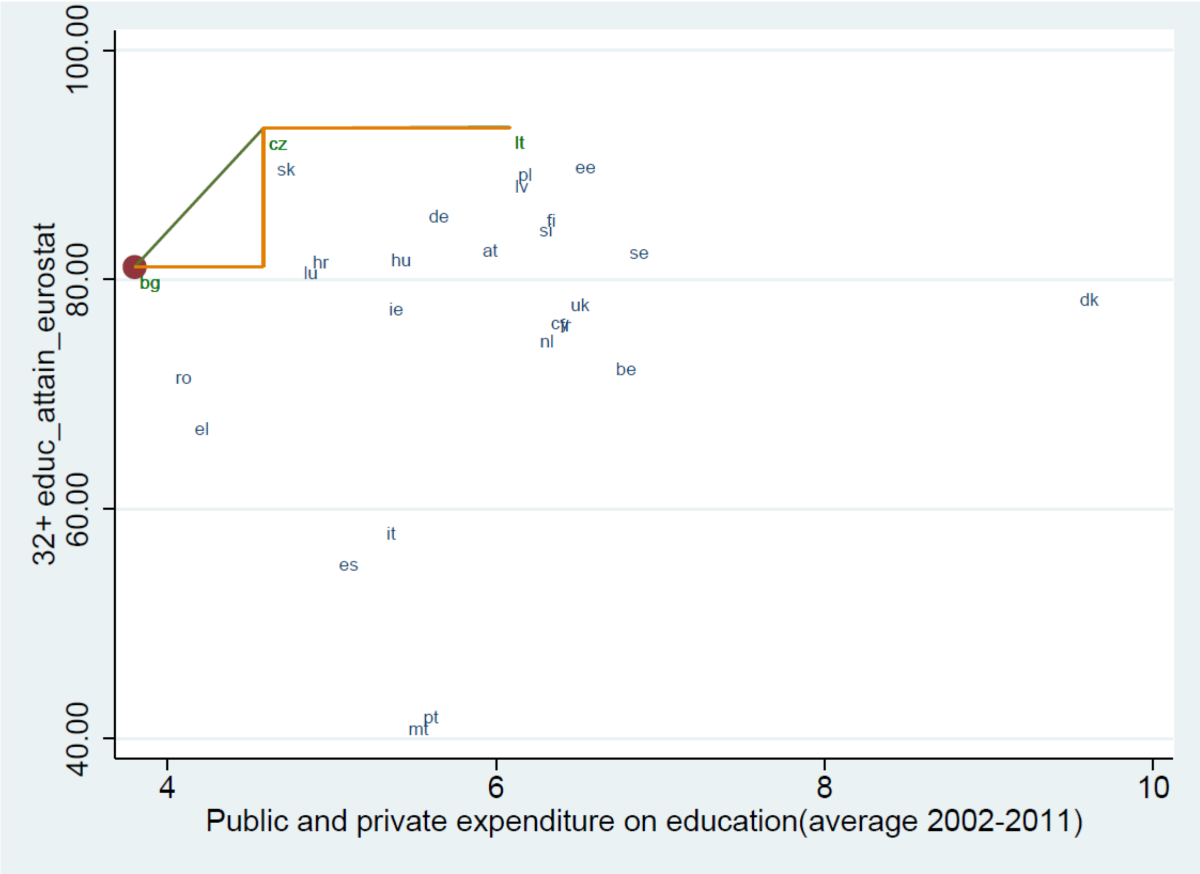
For these reasons, it is necessary to assess if the results previously obtained change significantly in light of the differences of the public/private funding of the education sector across European Member States.

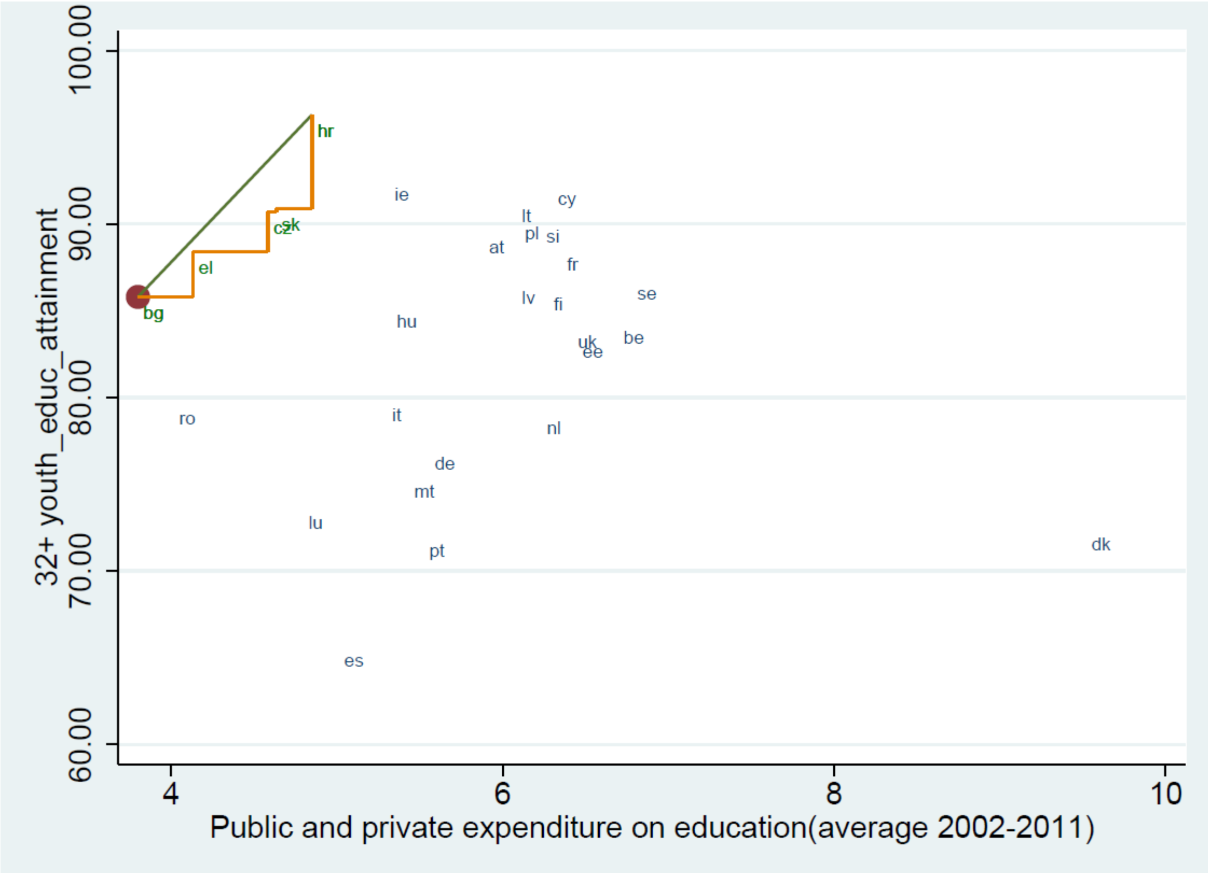
The approach followed in this paragraph is the same as the one used earlier in this section, and the methodologies applied are again the one input-one output DEA and FDH.

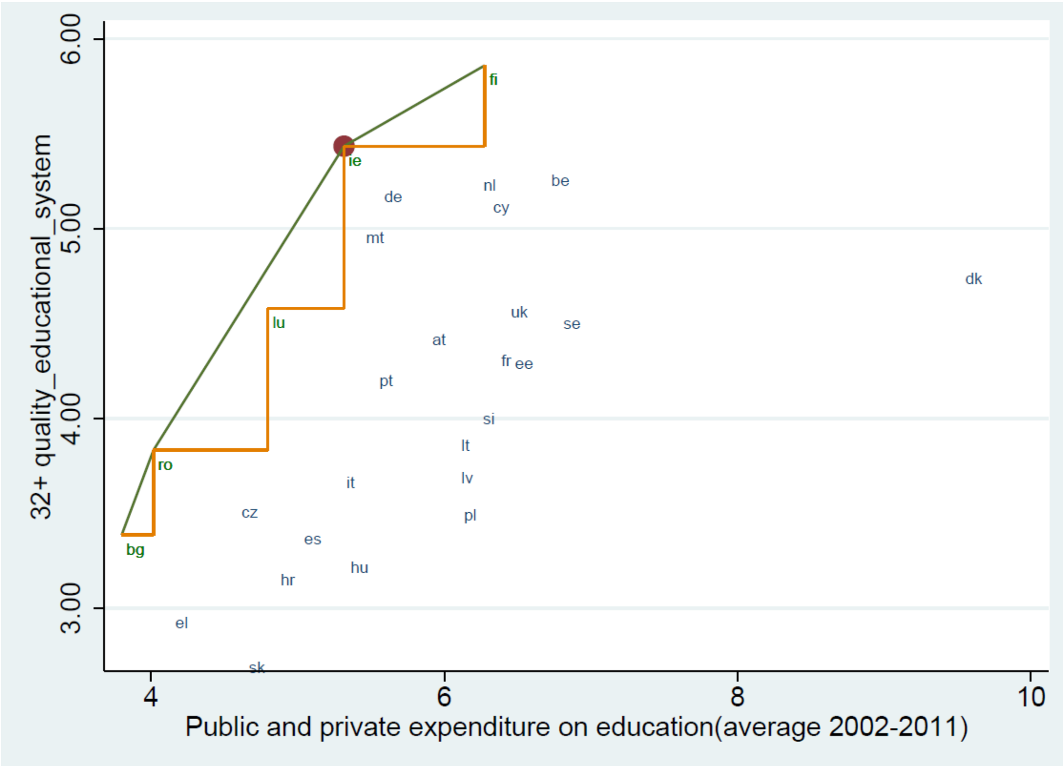
The graphs below (obtained by using both the non-parametric methodologies) show the results of this new implementation. Referring to Figure 15, a commentary of the main results of the DEA re-application compared to the analysis using public expenditure is provided. Tables A8-A12 in the Annex show the numerical results.

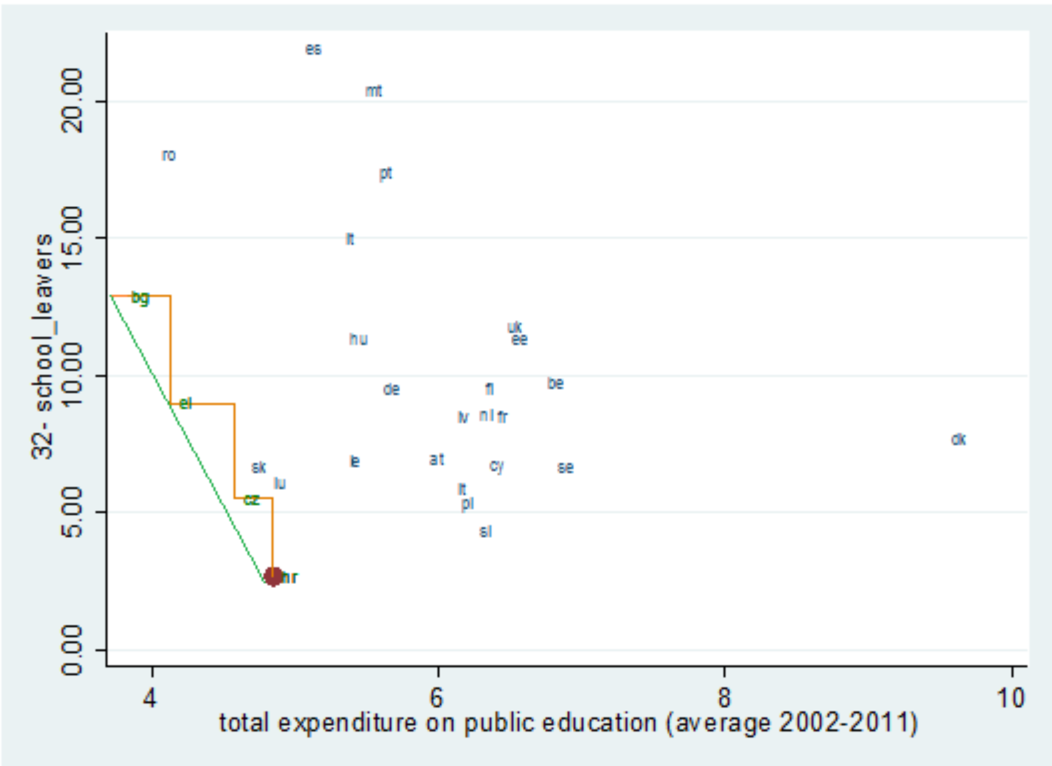
**Figure 15: A robustness check: Private + Public expenditure on education vs all output variables**




32+ educ_attain_eurostat
Public and private expenditure on education(average 2002-2011)
40.00
60.00
80.00
100.00
4
6
8
10
cz
lt
sk
pl
lv
ee
de
fi
si
at
se
hr
lu
hu
bg
ie
uk
dk
cy
fr
nl
be
ro
el
it
es
pt
mt


32+ youth_educ_attainment
Public and private expenditure on education(average 2002-2011)
60.00
70.00
80.00
90.00
100.00
4
6
8
10
hr
ie
cy
lt
pl
si
cz
sk
at
fr
el
lv
fi
se
bg
hu
uk
ee
be
ro
it
nl
de
mt
lu
pt
dk
es




Source: Author’s elaboration of OECD, EUROSTAT and WEF data

The first one input-one output model is revisited using PISA test scores as the output variable and total education as the input. From the upper panel of Figure 15, it is possible to see that the efficiency frontier calculated in this model is similar to the one that uses the public expenditure on education as the input, with one exception. On the one hand, in fact, Czech Republic and Finland are still identified as benchmark countries, but the methodology replaces Germany with Ireland on the frontier. Germany, in fact, disappears from the DEA frontier (although it is still assigned a high efficiency score, as shown in Table A8) when adding its private funding on education share (corresponding to 0,7% of its GDP in 2011). Therefore, the private input for Germany corresponds to a non negligible part of the total input devolved to education. Ireland was inefficient in the public expenditure case and now replaces Germany in becoming efficient. This could be due to the fact that the country has a lower share of private expenditure with respect to Germany (corresponding to about the 8% of the public funds in education in 2011).

As it could be expected, the performance of UK decreases when using total expenditure, as private spending represents an important share of the total input used in this country in the educational sector.

The frontiers calculated by relating the total expenditure to the educational attainment and to youth educational attainments show little changes as well. The main differences with the public expenditure case is the exit of Greece from both the educational attainment and the youth educational attainment frontiers and the exit of Czech Republic from the youth educational attainment DEA frontier. Furthermore, Bulgaria enters in the youth educational attainment efficiency frontier. The DEA methodology assigns efficiency, as in the previous scenario, to Bulgaria and Lithuania when using the educational attainment as the output variable and to Croatia when using youth educational attainment. It is worth noticing that in this second case, CZ is no longer considered efficient in its performance on youth educational attainment according to the DEA methodology, yet it is still technically efficient according to the FDH approach even when considering the private expenditure (which corresponded to 0,6% of its GDP in 2011).

An interesting remark concerns the performance of Finland. The country is not considered efficient in any of the educational attainment cases, but the efficiency score assigned by the methodology is virtually unchanged by considering public or total expenditure, given the fact that private expenditure in Finland is one of the lowest in Europe. This is a recurring feature of FI in all of the robustness analyses (considering the different outputs) that have been carried-out.

As in the PISA output case, UK's performance deteriorates with the substitution of the input.

Concerning early school leavers, the FDH frontier slightly changes when adding the private component to the public expenditure on education input, as Croatia is now assigned efficiency (in addition to the previously efficient countries: Bulgaria, Greece and Czech Republic).
According to the DEA methodology, instead, only Bulgaria (apart from the newly efficient Croatia) passes the robustness test and is attributed an efficiency score of 1.

Finally, the previous DEA frontier estimated using as the output variable the results of the survey on the quality of the educational system is compared with the one computed with the total expenditure on education as the input.
Bulgaria, Finland and Ireland are the efficient countries defined by both frameworks. Germany and Greece were considered efficient in the public expenditure case but exit the frontier when switching to total expenditure (such result is obtained for the two countries by both the FDH and DEA methodologies).
Romania was inefficient in the public expenditure case and becomes efficient in the total expenditure one. In fact, as most countries tend to shift away from the frontier (adding the private component of expenditure implies an augmentation of the input, represented on the x-axis), Romania, as the country whose private sector spends less on education in EU, maintains virtually the same position on the plane, eventually laying on the DEA frontier. As discussed before, low spending countries such as Romania should be carefully treated in deriving conclusions when applying the non-parametric methodologies, as their presence on the frontier and unit efficiency score could be resulting just by the fact that they employ the lowest inputs.

In conclusion, the robustness check shows that high performing countries (such as FI and CZ), that are assigned efficiency when using the public expenditure as the input, are also assigned efficiency when the input is replaced in order to include the private sector. More in general, Bulgaria, Czech Republic and Finland tend to maintain their position on the frontier, while other countries such as Germany may be considered efficient in the public expenditure framework but become inefficient in the total expenditure one. Remarkably, Greece drops its benchmark status in all cases in which it was previously considered efficient (as its results were boosted by the exclusion of the private component of its expenditure on education).
An expected general deterioration of the efficiency scores among DMUs (for example, Italy

and the United Kingdom) is verified when the input increases adding the private expenditure component, as can be seen in the numerical scores tables in the Annex.

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**ANNEX**

**Table A1 : Input and output variables**

| Variable | Destination | Unit of measurement | Source |
|---|---|---|---|
| Education expenditure by sources and level of education | Input | 10 years average (2002-2011) in % GDP. | OECD |
| Students per Teacher Ratio | Input | Average number of pupils per teacher at a given level of education. | Worldbank |
| Hours of schooling days | Input | Average number of hours per year. | OECD |
| PISA total score | Output | Average of 2012 test results by subject. | OECD |
| Educational attainment | Output | % of population with upper secondary education attainment. | EUROSTAT |
| Youth educational attainment | Output | % of 20- to 24-year-olds who have successfully completed at least an upper secondary education level. | EUROSTAT |
| Early school leavers | Output | % of the population aged 18–24 having attained at most lower secondary education and not being involved in further education or training. | EUROSTAT |
| Quality of the educational system survey | Output | Average of answers to the survey on a 1-7 scale. | WEF |

**Table A2 : Efficienct countries**

| PISA | | | |
|---|---|---|---|
| TOTAL | | PUBLIC | |
| DEA | FDH | DEA | FDH |
| CZ | CZ | CZ | CZ |
| FI | FI | DE | DE |
| IE | IE | FI | FI |
| | | | IE |
| | | | NL |
| | | | PL |
| EDUCATIONAL ATTAINMENT | | | |
| TOTAL | | PUBLIC | |
| DEA | FDH | DEA | FDH |
| BG | BG | BG | BG |
| CZ | CZ | CZ | CZ |
| LT | LT | EL | EL |
| | | LT | LT |

| YOUTH EDUCATIONAL ATTAINMENT | | | |
|---|---|---|---|
| TOTAL | | PUBLIC | |
| DEA | FDH | DEA | FDH |
| BG | BG | CZ | CZ |
| HR | CZ | EL | EL |
| | EL | HR | HR |
| | HR | | SK |
| | SK | | |
| SCHOOL LEAVERS | | | |
| TOTAL | | PUBLIC | |
| DEA | FDH | DEA | FDH |
| BG | BG | BG | BG |
| HR | CZ | CZ | CZ |
| | EL | EL | EL |
| | HR | | |

| QUALITY OF EDUCATIONAL SYSTEM | | | |
| --- | --- | --- | --- |
| TOTAL | | PUBLIC | |
| DEA | FDH | DEA | FDH |
| BG | BG | BG | BG |
| FI | FI | DE | DE |
| IE | IE | EL | EL |
| RO | LU | FI | FI |
| | RO | IE | IE |
| | | | RO |
| MULTIPLE INPUTS | | | |
| DEA | | | |
| EE | | | |
| FI | | | |
| PL | | | |
| SE | | | |

Source: Author's elaboration of OECD, EUROSTAT, WEF, Worldbank data

**Table A3 : DEA and FDH scores for PISA vs Public Expenditure on Education**

| DMU | CRS DEA | VRS DEA | Scale | RTS | FDH Score | Rank |
|---|---|---|---|---|---|---|
| AT | 0.747305 | 0.747683 | 0.999493 | drs | .797824 | 14 |
| BE | 0.656929 | 0.672779 | 0.976440 | drs | .6882693 | 19 |
| BG | 0.686845 | 0.779953 | 0.880624 | irs | .7799532 | 13 |
| CY | 0.415887 | 0.470448 | 0.884023 | irs | .470448 | 27 |
| CZ | 1 | 1 | 1 | - | 1 | 1 |
| DE | 0.964387 | 1 | 0.964387 | drs | 1 | 1 |
| DK | 0.534099 | 0.536136 | 0.996201 | irs | .5361356 | 24 |
| EE | 0.521741 | 0.660923 | 0.789413 | drs | .7002915 | 21 |
| EL | 0.860705 | 0.924482 | 0.931014 | irs | .9244815 | 4 |
| ES | 0.857995 | 0.876396 | 0.979004 | irs | .8763957 | 7 |
| FI | 0.749708 | 1 | 0.749708 | drs | 1 | 1 |
| FR | 0.682164 | 0.682574 | 0.999400 | irs | .6825738 | 18 |
| HR | 0.589469 | 0.611097 | 0.964607 | irs | .6110974 | 23 |
| HU | 0.765618 | 0.786859 | 0.973005 | irs | .7868586 | 12 |
| IE | 0.872905 | 0.914444 | 0.954574 | drs | 1 | 6 |
| IT | 0.825901 | 0.843786 | 0.978804 | irs | .8437856 | 9 |
| LT | 0.471421 | 0.487204 | 0.967606 | irs | .4872037 | 26 |
| LU | 0.779177 | 0.795887 | 0.979004 | irs | .795887 | 11 |
| LV | 0.481066 | 0.487204 | 0.987402 | irs | .4872037 | 25 |
| NL | 0.820019 | 0.914587 | 0.896600 | drs | 1 | 5 |
| PL | 0.756089 | 0.870236 | 0.868832 | drs | 1 | 8 |
| PT | 0.720498 | 0.738362 | 0.975805 | irs | .7383624 | 16 |
| RO | 0.649648 | 0.737881 | 0.880424 | irs | .7378811 | 17 |
| SE | 0.605125 | 0.627719 | 0.964007 | irs | .6277189 | 22 |
| SI | 0.662409 | 0.664002 | 0.997600 | irs | .6640024 | 20 |
| SK | 0.782340 | 0.829092 | 0.943611 | irs | .8290917 | 10 |
| UK | 0.734752 | 0.739202 | 0.993980 | drs | .7809889 | 15 |

Source: Author's elaboration of OECD data

Notes on the table:
- The "DMU" column reports each Member State's respective ISO 3166 alpha-2 code.
- The "CRS DEA" and "VRS DEA" columns show the scores assigned by the DEA methodology (to each DMU) in the constant returns to scale framework and in the variable returns to scale scenario respectively.
- The "Scale" column presents the scale efficiency score corresponding to each DMU
- The "RTS" comumn reports "irs" if the DMU is located in the increasing returns to scale segment, "drs" if the DMU if the DMU is located in the decreasing returns to scale one, and "-" for the CRS DEA efficient DMU.
- "FDH Score" shows the efficiency scores assigned to each DMU by the FDH technique.
-The "Rank" comumn is the VRS DEA score ranking attributed to each DMU from the most efficient ones (i.e. with a unitary efficiency score) to the less efficient DMUs.

**Table A4 : DEA and FDH scores for Educational Attainment vs Public Expenditure on Education**

| DMU | CRS DEA | VRS DEA | Scale | RTS | FDH Score | Rank |
|---|---|---|---|---|---|---|
| AT | 0.666863 | 0.708844 | 0.940775 | irs | .740782 | 18 |
| BE | 0.524697 | 0.622131 | 0.843387 | irs | .6271552 | 25 |
| BG | 0.921884 | 1 | 0.921884 | irs | 1 | 1 |
| CY | 0.532060 | 0.600920 | 0.885409 | irs | .6031746 | 26 |
| CZ | 1 | 1 | 1 | - | 1 | 1 |
| DE | 0.821966 | 0.855810 | 0.960455 | irs | .8815566 | 8 |
| DK | 0.423879 | 0.467711 | 0.906286 | irs | .4684612 | 28 |
| EE | 0.631730 | 0.639598 | 0.987699 | irs | .6455833 | 23 |
| EL | 0.788213 | 1 | 0.788213 | irs | 1 | 1 |
| ES | 0.557148 | 0.854214 | 0.652235 | irs | .8542136 | 9 |
| FI | 0.608701 | 0.635478 | 0.957863 | irs | .6558492 | 24 |
| FR | 0.601992 | 0.681513 | 0.883317 | irs | .684217 | 21 |
| HR | 0.738333 | 0.790432 | 0.934088 | irs | .8300675 | 11 |
| HU | 0.693576 | 0.741451 | 0.935430 | irs | .7778729 | 15 |
| IE | 0.678705 | 0.755842 | 0.897946 | irs | .7577037 | 13 |
| IT | 0.585919 | 0.857423 | 0.683348 | irs | .8574231 | 7 |
| LT | 0.662490 | 1 | 0.662490 | drs | 1 | 1 |
| LU | 0.738582 | 0.795874 | 0.928013 | irs | .839461 | 10 |
| LV | 0.635507 | 0.650428 | 0.977060 | irs | .6617798 | 22 |
| MT | 0.335494 | 0.689899 | 0.486295 | irs | .6898988 | 20 |
| NL | 0.635919 | 0.732083 | 0.868644 | irs | .736093 | 16 |
| PL | 0.752799 | 0.765554 | 0.983338 | irs | .7752584 | 12 |
| PT | 0.365133 | 0.731772 | 0.498971 | irs | .7317719 | 17 |
| RO | 0.782897 | 0.937671 | 0.834939 | irs | .9460582 | 6 |
| SE | 0.546736 | 0.581979 | 0.939443 | irs | .6087908 | 27 |
| SI | 0.668763 | 0.702002 | 0.952651 | irs | .7272892 | 19 |
| SK | 0.965688 | 0.978947 | 0.986456 | irs | .9890338 | 5 |
| UK | 0.675666 | 0.748978 | 0.902118 | irs | .7505014 | 14 |

Source: Author’s elaboration of OECD and EUROSTAT data

Notes on the table:
- The “DMU” column reports each Member State’s respective ISO 3166 alpha-2 code.
- The “CRS DEA” and “VRS DEA” columns show the scores assigned by the DEA methodology (to each DMU) in the constant returns to scale framework and in the variable returns to scale scenario respectively.
- The “Scale” column presents the scale efficiency score corresponding to each DMU
- The “RTS” comumn reports “irs” if the DMU is located in the increasing returns to scale segment, “drs” if the DMU if the DMU is located in the decreasing returns to scale one, and “-” for the CRS DEA efficient DMU.
- “FDH Score” shows the efficiency scores assigned to each DMU by the FDH technique.
-The “Rank” comumn is the VRS DEA score ranking attributed to each DMU from the most efficient ones (i.e. with a unitary efficiency score) to the less efficient DMUs.

**Table A5 : DEA and FDH scores for Youth Educational Attainment vs Public Expenditure on Education**

| DMU | CRS DEA | VRS DEA | Scale | RTS | FDH Score | Rank |
|---|---|---|---|---|---|---|
| AT | 0.699106 | 0.716372 | 0.975898 | drs | .740782 | 18 |
| BE | 0.590654 | 0.618647 | 0.954751 | irs | .6186475 | 23 |
| BG | 0.957422 | 0.986434 | 0.970588 | irs | .9864343 | 5 |
| CY | 0.621915 | 0.678734 | 0.916287 | drs | .7698413 | 21 |
| CZ | 0.955326 | 1 | 0.955326 | drs | 1 | 1 |
| DE | 0.715894 | 0.820818 | 0.872172 | irs | .8208179 | 10 |
| DK | 0.378990 | 0.462106 | 0.820136 | irs | .4621062 | 28 |
| EE | 0.568464 | 0.601103 | 0.945701 | irs | .6011031 | 26 |
| EL | 1 | 1 | 1 | - | 1 | 1 |
| ES | 0.635829 | 0.854214 | 0.744344 | irs | .8542136 | 9 |
| FI | 0.596155 | 0.610662 | 0.976244 | irs | .6106616 | 25 |
| FR | 0.676462 | 0.679278 | 0.995855 | drs | .7248788 | 20 |
| HR | 0.841946 | 1 | 0.841946 | drs | 1 | 1 |
| HU | 0.698879 | 0.724278 | 0.964932 | irs | .724278 | 16 |
| IE | 0.782936 | 0.858490 | 0.911992 | drs | .9670692 | 7 |
| IT | 0.774979 | 0.857423 | 0.903846 | irs | .8574231 | 8 |
| LT | 0.637095 | 0.678715 | 0.938678 | drs | .7972602 | 22 |
| LU | 0.651647 | 0.781623 | 0.833710 | irs | .7816227 | 11 |
| LV | 0.604334 | 0.616184 | 0.980769 | irs | .6161836 | 24 |
| MT | 0.589224 | 0.689899 | 0.854072 | irs | .6898988 | 19 |
| NL | 0.650540 | 0.726107 | 0.895928 | irs | .7261075 | 15 |
| PL | 0.738175 | 0.768291 | 0.960801 | drs | .7752584 | 12 |
| PT | 0.596841 | 0.731772 | 0.815611 | irs | .7317719 | 14 |
| RO | 0.841380 | 0.933224 | 0.901584 | irs | .9332243 | 6 |
| SE | 0.557227 | 0.566845 | 0.983032 | irs | .5668455 | 27 |
| SI | 0.690968 | 0.716396 | 0.964506 | drs | .7272892 | 17 |
| SK | 0.946933 | 0.996265 | 0.950483 | drs | 1 | 4 |
| UK | 0.704309 | 0.740320 | 0.951357 | irs | .7403204 | 13 |

Source: Author's elaboration of OECD and EUROSTAT data

Notes on the table:
- The "DMU" column reports each Member State's respective ISO 3166 alpha-2 code.
- The "CRS DEA" and "VRS DEA" columns show the scores assigned by the DEA methodology (to each DMU) in the constant returns to scale framework and in the variable returns to scale scenario respectively.
- The "Scale" column presents the scale efficiency score corresponding to each DMU
- The "RTS" comumn reports "irs" if the DMU is located in the increasing returns to scale segment, "drs" if the DMU if the DMU is located in the decreasing returns to scale one, and "-" for the CRS DEA efficient DMU.
- "FDH Score" shows the efficiency scores assigned to each DMU by the FDH technique.
-The "Rank" comumn is the VRS DEA score ranking attributed to each DMU from the most efficient ones (i.e. with a unitary efficiency score) to the less efficient DMUs.

**Table A6 : DEA and FDH scores for Early School Leavers vs Public Expenditure on Education**

| DMU | DEA Score | FDH Score | Rank |
|---|---|---|---|
| AT | 0.689743 | .6897426 | 18 |
| BE | 0.620393 | .6271552 | 21 |
| BG | 1 | 1 | 1 |
| CY | 0.594992 | .5949921 | 26 |
| CZ | 1 | 1 | 1 |
| DE | 0.822265 | .832106 | 8 |
| DK | 0.462106 | .4621062 | 28 |
| EE | 0.606190 | .6093695 | 25 |
| EL | 1 | 1 | 1 |
| ES | 0.859562 | .8569518 | 7 |
| FI | 0.611738 | .6190595 | 24 |
| FR | 0.674935 | .6749351 | 20 |
| HR | 0.772876 | .7728764 | 12 |
| HU | 0.730407 | .7342384 | 15 |
| IE | 0.747425 | .747425 | 14 |
| IT | 0.889229 | .918775 | 6 |
| LT | 0.616184 | .6161836 | 23 |
| LU | 0.781623 | .7816227 | 9 |
| LV | 0.616184 | .6161836 | 22 |
| MT | 0.780651 | .8076421 | 10 |
| NL | 0.726107 | .7261075 | 16 |
| PL | 0.721844 | .7218435 | 17 |
| PT | 0.778439 | .784133 | 11 |
| RO | 0.937544 | 0.933967 | 4 |
| SE | 0.566845 | .5668455 | 27 |
| SI | 0.677179 | .6771794 | 19 |
| SK | 0.920890 | .92089 | 5 |
| UK | 0.747630 | .7505014 | 13 |

Source: Author's elaboration of OECD and EUROSTAT data

Notes on the table:
- The "DMU" column reports each Member State's respective ISO 3166 alpha-2 code.
- The "DEA Score" column show the scores assigned by the variable returns to scale DEA methodology to each DMU.
- "FDH Score" shows the efficiency scores assigned to each DMU by the FDH technique.
-The "Rank" comumn is the VRS DEA score ranking attributed to each DMU from the most efficient ones (i.e. with a unitary efficiency score) to the less efficient DMUs.

**Table A7 : DEA and FDH scores for Quality of Education System vs Public Expenditure on Education**

| DMU | CRS DEA | VRS DEA | Scale | RTS | FDH Score | Rank |
|---|---|---|---|---|---|---|
| AT | 0.720107 | 0.783279 | 0.919350 | irs | .8403113 | 17 |
| BE | 0.766202 | 0.787173 | 0.973359 | drs | .8277051 | 16 |
| BG | 0.776511 | 1 | 0.776511 | irs | 1 | 1 |
| CY | 0.717510 | 0.721382 | 0.994633 | irs | .7248771 | 23 |
| CZ | 0.774500 | 0.963618 | 0.803742 | irs | .9977245 | 7 |
| DE | 1 | 1 | 1 | - | 1 | 1 |
| DK | 0.516890 | 0.541113 | 0.955235 | irs | .5629826 | 28 |
| EE | 0.609782 | 0.674181 | 0.904478 | irs | .732322 | 24 |
| EL | 0.697071 | 1 | 0.697071 | irs | 1 | 1 |
| ES | 0.683285 | 0.871112 | 0.784382 | irs | .9153358 | 12 |
| FI | 0.831564 | 1 | 0.831564 | drs | 1 | 1 |
| FR | 0.687242 | 0.758205 | 0.906407 | irs | .8222713 | 19 |
| HR | 0.579401 | 0.779080 | 0.743699 | irs | .7835051 | 18 |
| HU | 0.553650 | 0.731721 | 0.756640 | irs | .7342384 | 22 |
| IE | 0.943988 | 1 | 0.943988 | drs | 1 | 1 |
| IT | 0.744671 | 0.902293 | 0.825310 | irs | .918775 | 10 |
| LT | 0.562939 | 0.661611 | 0.850860 | irs | .7506946 | 25 |
| LU | 0.832051 | 0.895219 | 0.929438 | irs | .9522486 | 11 |
| LV | 0.539077 | 0.650290 | 0.828979 | irs | .6602739 | 26 |
| MT | 0.805885 | 0.824077 | 0.977924 | irs | .8405017 | 14 |
| NL | 0.895182 | 0.912902 | 0.980589 | drs | .9714788 | 9 |
| PL | 0.598289 | 0.746033 | 0.801960 | irs | .7734942 | 20 |
| PT | 0.726783 | 0.813356 | 0.893561 | irs | .8915156 | 15 |
| RO | 0.831466 | 0.992006 | 0.838166 | irs | 1 | 6 |
| SE | 0.602989 | 0.649025 | 0.929070 | irs | .6905861 | 27 |
| SI | 0.641036 | 0.737719 | 0.868944 | irs | .8250056 | 21 |
| SK | 0.591327 | 0.920890 | 0.642125 | irs | .92089 | 8 |
| UK | 0.797215 | 0.852247 | 0.935428 | irs | .90193 | 13 |

Source: Author’s elaboration of OECD and WEF data

Notes on the table:
- The “DMU” column reports each Member State’s respective ISO 3166 alpha-2 code.
- The “CRS DEA” and “VRS DEA” columns show the scores assigned by the DEA methodology (to each DMU) in the constant returns to scale framework and in the variable returns to scale scenario respectively.
- The “Scale” column presents the scale efficiency score corresponding to each DMU
- The “RTS” comumn reports “irs” if the DMU is located in the increasing returns to scale segment, “drs” if the DMU if the DMU is located in the decreasing returns to scale one, and “-” for the CRS DEA efficient DMU.
- “FDH Score” shows the efficiency scores assigned to each DMU by the FDH technique.
-The “Rank” comumn is the VRS DEA score ranking attributed to each DMU from the most efficient ones (i.e. with a unitary efficiency score) to the less efficient DMUs.

**Table A8 : DEA and FDH scores for PISA vs Total Expenditure on Education**

| DMU | CRS DEA | VRS DEA | Scale | RTS | FDH Score | Rank |
|---|---|---|---|---|---|---|
| AT | 0.767275 | 0.768043 | 0.999000 | drs | .8502772 | 16 |
| BE | 0.673479 | 0.705575 | 0.954512 | drs | .7324276 | 17 |
| BG | 0.756109 | 0.858606 | 0.880624 | irs | .8586059 | 9 |
| CY | 0.457826 | 0.517889 | 0.884023 | irs | .5178893 | 27 |
| CZ | 1 | 1 | 1 | - | 1 | 1 |
| DE | 0.878212 | 0.942265 | 0.932022 | drs | .9452529 | 4 |
| DK | 0.523215 | 0.525211 | 0.996201 | irs | .5252108 | 26 |
| EE | 0.532144 | 0.635749 | 0.837034 | drs | .6593081 | 23 |
| EL | 0.863532 | 0.927518 | 0.931014 | irs | .9275178 | 5 |
| ES | 0.853417 | 0.871720 | 0.979004 | irs | .8717198 | 6 |
| FI | 0.812188 | 1 | 0.812188 | drs | 1 | 1 |
| FR | 0.673058 | 0.673462 | 0.999400 | irs | .6734618 | 20 |
| HR | 0.648913 | 0.672722 | 0.964607 | irs | .6727222 | 21 |
| HU | 0.829217 | 0.852222 | 0.973005 | irs | .8522223 | 11 |
| IE | 0.929978 | 1 | 0.929978 | drs | 1 | 1 |
| IT | 0.789369 | 0.806463 | 0.978804 | irs | .8064628 | 14 |
| LT | 0.518961 | 0.536335 | 0.967606 | irs | .5363346 | 25 |
| LU | 0.844199 | 0.862304 | 0.979004 | irs | .8623042 | 7 |
| LV | 0.529578 | 0.536335 | 0.987402 | irs | .5363346 | 24 |
| NL | 0.772844 | 0.859554 | 0.899123 | drs | .9710009 | 8 |
| PL | 0.753216 | 0.852350 | 0.883694 | drs | .9432486 | 10 |
| PT | 0.778039 | 0.797330 | 0.975805 | irs | .7973302 | 15 |
| RO | 0.715160 | 0.812291 | 0.880424 | irs | .8122911 | 13 |
| SE | 0.665808 | 0.690667 | 0.964007 | irs | .6906672 | 19 |
| SI | 0.664223 | 0.665820 | 0.997600 | irs | .6658205 | 22 |
| SK | 0.780185 | 0.826808 | 0.943611 | irs | .826808 | 12 |
| UK | 0.691161 | 0.699428 | 0.988179 | drs | .7625758 | 18 |

Source: Author's elaboration of OECD data

Notes on the table:
- The "DMU" column reports each Member State's respective ISO 3166 alpha-2 code.
- The "CRS DEA" and "VRS DEA" columns show the scores assigned by the DEA methodology (to each DMU) in the constant returns to scale framework and in the variable returns to scale scenario respectively.
- The "Scale" column presents the scale efficiency score corresponding to each DMU
- The "RTS" comumn reports "irs" if the DMU is located in the increasing returns to scale segment, "drs" if the DMU if the DMU is located in the decreasing returns to scale one, and "-" for the CRS DEA efficient DMU.
- "FDH Score" shows the efficiency scores assigned to each DMU by the FDH technique.
-The "Rank" comumn is the VRS DEA score ranking attributed to each DMU from the most efficient ones (i.e. with a unitary efficiency score) to the less efficient DMUs.

**Table A9 : DEA and FDH scores for Educational Attainment vs Total Expenditure on Education**

| DMU | CRS DEA | VRS DEA | Scale | RTS | FDH Score | Rank |
|---|---|---|---|---|---|---|
| AT | 0.667982 | 0.676508 | 0.987398 | drs | .7788684 | 19 |
| BE | 0.514978 | 0.567456 | 0.907522 | irs | .5674558 | 27 |
| BG | 1 | 1 | 1 | - | 1 | 1 |
| CY | 0.577144 | 0.603175 | 0.956843 | irs | .6031746 | 23 |
| CZ | 0.952697 | 1 | 0.952697 | drs | 1 | 1 |
| DE | 0.732658 | 0.751359 | 0.975110 | drs | .824788 | 10 |
| DK | 0.391801 | 0.399184 | 0.981504 | irs | .3991837 | 28 |
| EE | 0.662518 | 0.690578 | 0.959367 | drs | .7106637 | 17 |
| EL | 0.775343 | 0.919302 | 0.843403 | irs | .9193025 | 6 |
| ES | 0.529191 | 0.758258 | 0.697904 | irs | .7582577 | 9 |
| FI | 0.645887 | 0.661307 | 0.976682 | drs | .730468 | 20 |
| FR | 0.570645 | 0.597924 | 0.954377 | irs | .5979244 | 24 |
| HR | 0.800895 | 0.807545 | 0.991765 | drs | .9451094 | 7 |
| HU | 0.730885 | 0.737613 | 0.990880 | drs | .8604176 | 11 |
| IE | 0.694552 | 0.714824 | 0.971640 | irs | .7148242 | 14 |
| IT | 0.524200 | 0.716907 | 0.731196 | irs | .7169074 | 13 |
| LT | 0.718626 | 1 | 0.718626 | drs | 1 | 1 |
| LU | 0.801165 | 0.804528 | 0.995820 | drs | .9558049 | 8 |
| LV | 0.689357 | 0.714101 | 0.965350 | drs | .7534982 | 15 |
| MT | 0.363922 | 0.699386 | 0.520345 | irs | .6993865 | 16 |
| NL | 0.571278 | 0.609614 | 0.937115 | irs | .609614 | 22 |
| PL | 0.694979 | 0.722586 | 0.961794 | drs | .7512498 | 12 |
| PT | 0.367247 | 0.687846 | 0.533909 | irs | .6878459 | 18 |
| RO | 0.849236 | 0.946058 | 0.897657 | irs | .9460582 | 5 |
| SE | 0.578565 | 0.585438 | 0.988260 | drs | .6762198 | 26 |
| SI | 0.644487 | 0.657717 | 0.979885 | drs | .7356891 | 21 |
| SK | 0.919981 | 0.958258 | 0.960056 | drs | .989005 | 4 |
| UK | 0.578024 | 0.591891 | 0.976572 | irs | .5918908 | 25 |

Source: Author’s elaboration of OECD and EUROSTAT data

Notes on the table:
- The “DMU” column reports each Member State’s respective ISO 3166 alpha-2 code.
- The “CRS DEA Score” and “VRS DEA Score” columns show the scores assigned by the DEA methodology (to each DMU) in the constant returns to scale framework and in the variable returns to scale scenario respectively.
- The “Scale” column presents the scale efficiency score corresponding to each DMU
- The “RTS” comumn reports “irs” if the DMU is located in the increasing returns to scale segment, “drs” if the DMU if the DMU is located in the decreasing returns to scale one, and “-” for the CRS DEA efficient DMU.
- “FDH Score” shows the efficiency scores assigned to each DMU by the FDH technique.
-The “Rank” comumn is the VRS DEA score ranking attributed to each DMU from the most efficient ones (i.e. with a unitary efficiency score) to the less efficient DMUs.

**Table A10 : DEA and FDH scores for Youth Educational Attainment vs Total Expenditure on Education**

| DMU | CRS DEA | VRS DEA | Scale | RTS | FDH Score | Rank |
|---|---|---|---|---|---|---|
| AT | 0.674287 | 0.710259 | 0.949354 | drs | .7788684 | 13 |
| BE | 0.558197 | 0.567456 | 0.983683 | irs | .5674558 | 27 |
| BG | 1 | 1 | 1 | - | 1 | 1 |
| CY | 0.649573 | 0.707936 | 0.917558 | drs | .7698413 | 14 |
| CZ | 0.876354 | 0.935908 | 0.936367 | drs | 1 | 5 |
| DE | 0.614425 | 0.683758 | 0.898601 | irs | .6837575 | 18 |
| DK | 0.337306 | 0.399184 | 0.844988 | irs | .3991837 | 28 |
| EE | 0.574041 | 0.589147 | 0.974359 | irs | .5891473 | 25 |
| EL | 0.947160 | 0.982202 | 0.964323 | drs | 1 | 3 |
| ES | 0.581508 | 0.758258 | 0.766900 | irs | .7582577 | 9 |
| FI | 0.609094 | 0.613533 | 0.992765 | drs | .6587226 | 22 |
| FR | 0.617437 | 0.641982 | 0.961767 | drs | .7212511 | 20 |
| HR | 0.879389 | 1 | 0.879389 | drs | 1 | 1 |
| HU | 0.709138 | 0.713295 | 0.994172 | irs | .7132948 | 12 |
| IE | 0.771477 | 0.842740 | 0.915439 | drs | .9123414 | 7 |
| IT | 0.667610 | 0.716907 | 0.931235 | irs | .7169074 | 10 |
| LT | 0.665428 | 0.716712 | 0.928445 | drs | .7972602 | 11 |
| LU | 0.680627 | 0.792372 | 0.858974 | irs | .7923717 | 8 |
| LV | 0.631210 | 0.639452 | 0.987111 | drs | .6794907 | 21 |
| MT | 0.615427 | 0.699386 | 0.879953 | irs | .6993865 | 15 |
| NL | 0.562721 | 0.609614 | 0.923077 | irs | .609614 | 23 |
| PL | 0.656183 | 0.698184 | 0.939843 | drs | .7512498 | 16 |
| PT | 0.578015 | 0.687846 | 0.840326 | irs | .6878459 | 17 |
| RO | 0.878798 | 0.946058 | 0.928904 | irs | .9460582 | 4 |
| SE | 0.567780 | 0.576821 | 0.984327 | drs | .6098025 | 26 |
| SI | 0.641170 | 0.680513 | 0.942187 | drs | .7356891 | 19 |
| SK | 0.868630 | 0.929933 | 0.934078 | drs | 1 | 6 |
| UK | 0.580163 | 0.591891 | 0.980186 | irs | .5918908 | 24 |

Source: Author's elaboration of OECD and EUROSTAT data

Notes on the table:
- The "DMU" column reports each Member State's respective ISO 3166 alpha-2 code.
- The "CRS DEA Score" and "VRS DEA Score" columns show the scores assigned by the DEA methodology (to each DMU) in the constant returns to scale framework and in the variable returns to scale scenario respectively.
- The "Scale" column presents the scale efficiency score corresponding to each DMU
- The "RTS" comumn reports "irs" if the DMU is located in the increasing returns to scale segment, "drs" if the DMU if the DMU is located in the decreasing returns to scale one, and "-" for the CRS DEA efficient DMU.
- "FDH Score" shows the efficiency scores assigned to each DMU by the FDH technique.
-The "Rank" comumn is the VRS DEA score ranking attributed to each DMU from the most efficient ones (i.e. with a unitary efficiency score) to the less efficient DMUs.

**Table A11 : DEA and FDH scores for Early School Leavers vs Total Expenditure on Education**

| DMU | CRS DEA | VRS DEA | Scale | RTS | FDH Score | Rank |
|---|---|---|---|---|---|---|
| AT | 0,3178686 | 0,6854956 | 0,4637064 | irs | 0,7788684 | 17 |
| BE | 0,1995391 | 0,580585 | 0,3436862 | irs | 0,6172678 | 27 |
| BG | 0,2671359 | 1 | 0,2671359 | irs | 1 | 1 |
| CY | 0,3056723 | 0,6427507 | 0,475569 | irs | 0,7275842 | 20 |
| CZ | 0,5194203 | 0,9105927 | 0,5704201 | irs | 1 | 5 |
| DE | 0,2480278 | 0,7016565 | 0,353489 | irs | 0,7437786 | 12 |
| DK | 0,1786504 | 0,4189014 | 0,4264736 | irs | 0,4815186 | 28 |
| EE | 0,1780906 | 0,5948173 | 0,2994038 | irs | 0,6408634 | 26 |
| EL | 0,3519961 | 0,9484398 | 0,3711317 | irs | 1 | 3 |
| ES | 0,1193149 | 0,7582577 | 0,157354 | irs | 0,7582577 | 9 |
| FI | 0,2196642 | 0,6214173 | 0,353489 | irs | 0,6587226 | 22 |
| FR | 0,2424093 | 0,6205629 | 0,390628 | irs | 0,7212511 | 23 |
| HR | 1 | 1 | 1 | crs | 1 | 1 |
| HU | 0,2156185 | 0,7201595 | 0,2994038 | irs | 0,7759087 | 10 |
| IE | 0,3570032 | 0,7602885 | 0,4695627 | irs | 0,8622625 | 8 |
| IT | 0,1647 | 0,7169074 | 0,2297368 | irs | 0,7169074 | 11 |
| LT | 0,3648479 | 0,6788647 | 0,5374383 | irs | 0,7534982 | 18 |
| LU | 0,4476319 | 0,8569784 | 0,5223374 | irs | 0,9558049 | 7 |
| LV | 0,2532474 | 0,6483083 | 0,390628 | irs | 0,7534982 | 19 |
| MT | 0,1181433 | 0,6993865 | 0,1689242 | irs | 0,6993865 | 13 |
| NL | 0,2442746 | 0,6319082 | 0,3865666 | irs | 0,7353517 | 21 |
| PL | 0,3974407 | 0,6860611 | 0,579308 | irs | 0,7948813 | 16 |
| PT | 0,1362272 | 0,6878459 | 0,198049 | irs | 0,6878459 | 15 |
| RO | 0,1801197 | 0,9460582 | 0,1903896 | irs | 0,9460582 | 4 |
| SE | 0,2883333 | 0,5985362 | 0,4817307 | irs | 0,6762198 | 24 |
| SI | 0,4776648 | 0,6960702 | 0,6862308 | irs | 0,7784168 | 14 |
| SK | 0,4217017 | 0,8753887 | 0,4817307 | irs | 0,989005 | 6 |
| UK | 0,1728548 | 0,5959265 | 0,2900606 | irs | 0,6438477 | 25 |

Source: Author's elaboration of OECD and EUROSTAT data

Notes on the table:
- The "DMU" column reports each Member State's respective ISO 3166 alpha-2 code.
- The "CRS DEA" and "VRS DEA" columns show the scores assigned by the DEA methodology (to each DMU) in the constant returns to scale framework and in the variable returns to scale scenario respectively.
- The "Scale" column presents the scale efficiency score corresponding to each DMU
- The "RTS" comumn reports "irs" if the DMU is located in the increasing returns to scale segment, "drs" if the DMU if the DMU is located in the decreasing returns to scale one, and "-" for the CRS DEA efficient DMU.
- "FDH Score" shows the efficiency scores assigned to each DMU by the FDH technique.
-The "Rank" comumn is the VRS DEA score ranking attributed to each DMU from the most efficient ones (i.e. with a unitary efficiency score) to the less efficient DMUs.

**Table A12 : DEA and FDH scores for Quality of Education System vs Total Expenditure on Education**

| DMU | CRS DEA | VRS DEA | Scale | RTS | FDH Score | Rank |
|---|---|---|---|---|---|---|
| AT | 0.746683 | 0.773341 | 0.965529 | irs | .8148822 | 16 |
| BE | 0.778456 | 0.781075 | 0.996647 | irs | .7938398 | 15 |
| BG | 0.871929 | 1 | 0.871929 | irs | 1 | 1 |
| CY | 0.805678 | 0.812169 | 0.992008 | irs | .8438083 | 12 |
| CZ | 0.763812 | 0.849335 | 0.899306 | irs | .876278 | 9 |
| DE | 0.922693 | 0.928455 | 0.993795 | irs | .9565394 | 6 |
| DK | 0.494574 | 0.505445 | 0.978492 | irs | .5584362 | 28 |
| EE | 0.661990 | 0.689600 | 0.959962 | irs | .7435238 | 22 |
| EL | 0.709803 | 0.919302 | 0.772110 | irs | .9193025 | 7 |
| ES | 0.671822 | 0.763553 | 0.879863 | irs | .8014916 | 17 |
| FI | 0.913394 | 1 | 0.913394 | drs | 1 | 1 |
| FR | 0.674367 | 0.701960 | 0.960691 | irs | .7546008 | 21 |
| HR | 0.650599 | 0.783505 | 0.830369 | irs | .7835051 | 14 |
| HU | 0.603950 | 0.713295 | 0.846704 | irs | .7132948 | 20 |
| IE | 1 | 1 | 1 | - | 1 | 1 |
| IT | 0.689659 | 0.748936 | 0.920852 | irs | .7577837 | 18 |
| LT | 0.632113 | 0.673265 | 0.938877 | irs | .788339 | 25 |
| LU | 0.934295 | 0.963947 | 0.969239 | irs | 1 | 5 |
| LV | 0.605319 | 0.654749 | 0.924506 | irs | .6602739 | 26 |
| MT | 0.904913 | 0.917423 | 0.986364 | irs | .9784035 | 8 |
| NL | 0.832467 | 0.835931 | 0.995856 | irs | .8528166 | 10 |
| PL | 0.571760 | 0.637044 | 0.897520 | irs | .6583037 | 27 |
| PT | 0.756697 | 0.791689 | 0.955801 | irs | .8680849 | 13 |
| RO | 0.933638 | 1 | 0.933638 | irs | 1 | 1 |
| SE | 0.660533 | 0.681591 | 0.969105 | irs | .7074873 | 23 |
| SI | 0.639492 | 0.675872 | 0.946173 | irs | .7697064 | 24 |
| SK | 0.583150 | 0.819895 | 0.711249 | irs | .819895 | 11 |
| UK | 0.705991 | 0.726764 | 0.971417 | irs | .8280229 | 19 |

Source: Author's elaboration of OECD and WEF data

Notes on the table:
- The "DMU" column reports each Member State's respective ISO 3166 alpha-2 code.
- The "CRS DEA Score" and "VRS DEA Score" columns show the scores assigned by the DEA methodology (to each DMU) in the constant returns to scale framework and in the variable returns to scale scenario respectively.
- The "Scale" column presents the scale efficiency score corresponding to each DMU
- The "RTS" comumn reports "irs" if the DMU is located in the increasing returns to scale segment, "drs" if the DMU if the DMU is located in the decreasing returns to scale one, and "-" for the CRS DEA efficient DMU.
- "FDH Score" shows the efficiency scores assigned to each DMU by the FDH technique.
-The "Rank" comumn is the VRS DEA score ranking attributed to each DMU from the most efficient ones (i.e. with a unitary efficiency score) to the less efficient DMUs.

**Table A13 : Two input-one output DEA**

| DMU | DEA Score | Rank |
|---|---|---|
| AT | 0.901966 | 10 |
| BE | 0.831900 | 14 |
| CZ | 0.965517 | 7 |
| DE | 0.910424 | 9 |
| EE | 1 | 1 |
| EL | 0.940138 | 8 |
| ES | 0.794136 | 13 |
| FI | 1 | 1 |
| FR | 0.756757 | 17 |
| HU | 0.978582 | 5 |
| IE | 0.786757 | 15 |
| PL | 1 | 1 |
| PT | 0.862232 | 12 |
| SE | 1 | 1 |
| SI | 0.965517 | 6 |
| SK | 0.875000 | 11 |
| UK | 0.777778 | 16 |

Source: Author's elaboration of OECD and Worldbank data

Notes on the table:
- The "DMU" column reports each Member State's respective ISO 3166 alpha-2 code.
- The "VRS DEA Score" column shows the scores assigned by the DEA methodology (to each DMU) in the variable returns to scale framework.
-The "Rank" comumn is the VRS DEA score ranking attributed to each DMU from the most efficient ones (i.e. with a unitary efficiency score) to the less efficient DMUs.

### Table A14 : EU Country Codes ISO 3166 alpha-2

| | |
|---|---|
| Austria | AT |
| Belgium | BE |
| Bulgaria | BG |
| Croatia | HR |
| Cyprus | CY |
| Czech Republic | CZ |
| Denmark | DK |
| France | FR |
| Estonia | EE |
| Finland | FI |
| Germany | DE |
| Greece | EL |
| Hungary | HU |
| Ireland | IE |
| Italy | IT |
| Latvia | LV |
| Lithuania | LT |
| Luxembourg | LU |
| Malta | MT |
| Netherlands | NE |
| Poland | PL |
| Portugal | PT |
| Romania | RO |
| Slovenia | SI |
| Slovakia | SK |
| Spain | ES |
| Sweden | SE |
| United Kingdom | UK |

Source : Author's elaboration of Eurostat data