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"ONLINE SELLING STRATEGIES: THE MATRIX SCHEME CASE"

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TABLE OF CONTENTS

TABLE OF TABLES

TABLE OF FIGURES

INTRODUCTION

The technological era provided new methods to supply and purchase product or services. These new methods are mainly related to the introduction of Internet and all the consequent developments. One of the most important is the so-called e-commerce which permits customers to purchase product and services outside the seller's premise, through the use of websites. This practice permits generally to lower price for the customers and the operating costs for the seller. This is due principally to the drop of transaction costs. Moreover, the classical e-commerce developed in some variants. One of them is the possibility for the customers to create a group of people in order to exploit a sort of bulk price from the supplier. Successively, another form of e-commerce is born, providing a new type of selling strategy. This practice is undertaken by some websites which allow customers to pay a part of the product price, while the remaining is added by other customers. This system creates a list in which the first customer pays a little price and, when other customers replicate the same action, the first receive the product. It means that all customers have to wait new ones in order to receive their product. However, this practice seems to recall some famous pyramidal schemes or scams in which the majority of the participants suffer a loss.

The main objective of this thesis is the analysis of the most important feature of this type of selling strategy in order to understand whether it can consider merely a variation of the pyramidal scheme or a new efficient type of sale. This analysis is conducted with the instrument provided by the game theory environment: a game is built up and then examined through the backward induction technique. Starting from the simplest case, successively some extensions of the same game will be investigated. The aim is to comprehend the customers and the seller behaviours in the basic game and then the behaviours changes in the extensions. By analysing this game, it is possible to analyse the reasons which lead these subjects to sell and purchase with this practice.

The dissertation is divided in five chapters. The first one is dedicated to the detailed explanation of the mechanism of this type of selling strategy and the causes which lead to his launch. Then, the majority of the pyramidal schemes are considered in order to point out the similarities and the differences with respect to the new selling strategy and whether it can be connected with one or some of them.

The central chapters constitute the core of the dissertation. The second chapter comprehends the construction of the game in order to point out the main key issues of this mechanism of sale.

The subjects are made up by the seller on one side and by a number of customers on the other one. The aim of the last ones is to reach a positive utility function while the former has to maximize his profit. With this analysis is possible to highlight the optimal strategy of each of them. In the third and the fourth chapters, the analysis proceeds with some variations of the initial game. These additional cases are characterized by different values of the key parameters in order to find a pattern which allows to collect the best response of the subjects to the changes of some relevant parameters. By comparing the initial game and the relative extensions, it is possible to underline the best case for the customers and the seller.

The last chapter is dedicated to explanation of the results in the various cases. Moreover, the results are compared with the scenario provided by the real world. Similarities and differences are analysed in order to figure out whether the created game is suitable in order to replicate the real world websites dynamics and critical issues. Then, the actual websites are considered in order to discuss about some relevant issues. The aim is to understand whether this type of selling strategies is a genuine and profitable mechanism for both seller and players or, otherwise, if there are some measure that has to be applied in order to make it sustainable.

CHAPTER 1
FEATURES OF NEW MECHANISM OF SALES

1.1 E-commerce era characteristics

Rupert Murdoch once said: "The internet has been the most fundamental change during my lifetime and for hundreds of years" and it's difficult not to share it. In fact, the use of internet has changed radically various actions that people do on a daily basis: from interactions with each other, to the reading of newspaper and also the purchase of goods and services. These are only few examples of the activities that internet has contributed to modify, but a complete list would include almost all the activities that a common person is able to perform every day. Indeed, letters that people used to exchange through traditional mail have become instant messages through the use of internet connection. Newspapers have lost their physical component given by pieces of paper and they have become digital pages available on electronic devices. In the purchase of goods and services from the customers' point of view is changed the purchase location but also the way people buy and sell. Nowadays customers are able to search for, buy and pay product in a place different from the seller's premises through electronic devices with internet connections. This new form has taken the name of e-commerce. It has grown year after year, as demonstrated by several statistical data. An example is the research of the Istat, the Italian national institute of statistics. This institute produces annually a survey on the use of information and communication technologies by Italians citizens and enterprises. The survey[1] published in December 2016 shows that:

- 69.2% of households are internet-provided;
- 50.5% of people aged more than 15 surfed the internet and made online purchases in the months prior to the interview (48.7% in the previous year).

In particular, 28.7% of the people purchased online in the last three months, 12% in the last year and 9.7% before the last 12 months. Moreover, among the people who didn't make a purchase in the last three months, 40.9% of them made online research to get information on products and services and/or sold goods online. Hence, there are 50.5% of internet-accessed people who are actual or potential customers of e-commerce; and 29% of people aged between 16 and 74 made online purchases in the last 12 months. With these figures, Italy ranks 25th among 28 European states. It is well below the average value of the 28 EU countries, that is

[1] Istat. 2016. *Cittadini, imprese e ICT*. http://www.istat.it/it/archivio/194611

55%, and it is far away from the first country in the list, UK, with a percentage of 83%. These are only examples to show the growing impact of e-commerce. There are countries that, even if they have a quite low value of online purchases, have a growing trend, like Italy. Then, there are more advanced countries with an actual greater share of people who make nowadays use of e-commerce, like UK.

These data are significant values and they contribute to explain the positive trend of e-commerce in recent years. One example is the growth of one of the most famous and biggest e-commerce operator: Amazon Inc. Its revenues increased year after year from 74,452 USD millions of 2013 to 135,987 USD millions of 2016, though these values are not entirely attributable to products sales[2]. In fact, one part of the revenues derives from "AWS", i.e. Amazon Web Service, consisting in "sales of compute, storage, database, and other service" as reported on the annual report. Even if Amazon has gained the leadership of this sector with its competitive advantages, all the e-commerce companies can exploit several benefits which lie in their own nature. They can provide to customers easy access to store remotely located, even if a great importance has to be granted to marketing in order to drive traffic as much as possible to the website. Moreover, e-commerce has a lower level of costs related to physical store, like rents and wages of salesman because sales lies in online transactions so physical store are limited to warehouses. Therefore, the main economic strength derives from the low level of transaction costs with respect to a similar business with physical stores. For these reasons, e-commerce enterprises have more possibilities to scale up the business paying particular attention to the logistics function.

However, this kind of business implies some disadvantages and the most important ones are customers-related. In fact, buyers normally want to see, touch, try and obtain information about the products. This last aspect can be partially solved because nowadays people are able to look for information themselves in several ways through online research. Then, customers are subject to: scams because it's not always immediate to certify the authenticity of the e-commerce; problems with refund/assistance because it could be complicated or slow; a sort of "stress" due to the waiting time of the product related to the dispatch of it.

Enterprises that are able to exploit advantages and to overcome disadvantages are then able to survive in the e-commerce market, even if the long-term ability to make profits also depends on the competition.

One of the most important and common benefit that customers can exploit from e-commerce firms are lower level of prices. In fact, due to all the characteristic of this business, firms are

[2] Amazon Inc. financial data as reported on https://it.investing.com/equities/amazon-com-inc-income-statement

able to provide the products at a lower price with respect to a competitive and "classical" firm. Thanks to this, e-commerce customers are always looking for the best price on the market and for this reason new online selling strategies can be performed in order to provide the best prices. Indeed, with the development of the e-commerce sector, entrepreneurs and sellers developed new strategy of sales in order to attract a number of customers increasingly greater. Two main examples are the eBay Inc. and Groupon cases. The former allows two types of sales: a classical e-commerce activity where customers can purchase products from a plurality of sellers; and the possibility for the customers to participate to online auctions. This practice could have developed only because of Internet connection. In fact, only with these websites, sellers can reduce his transactions cost and provide auctions easily and at a lower price to the customers, placed everywhere. Otherwise, the seller should gather all customers in one place: this represent a relevant cost both for the seller (for example location cost) and for the customers (for example transportation cost). Groupon instead has a different type of sales in fact it acts as intermediary between the seller and the customers. The seller with the Groupon website is able to sell coupons that allow the customers to exploit discount on his goods or services. He is able to provide a lower price for goods and services because, thanks to Groupon website, he can reach a greater number of customers, exploiting economies of scale. Generally, Groupon developed a type of the group purchase technique. In fact, with this method a group of people is able to purchase goods or service at a discounted price. Basically, this is possible because a customers' group purchases a significant amount of goods directly from the producer, so avoiding the intermediation cost. Normally in the group purchases, the customers contact directly the seller, but this implies that the customers have to spend time and effort in the research of other people and of the seller. In this situation of difficulty for the customers, Groupon set his activity. In fact, it is the intermediary that connect the seller with customers. In both cases (with or without Groupon), the seller should be able to provide a kind of bulk price, but Groupon make the purchase group more easy and convenient.

Some of these new types of sales through e-commerce share an essential aspect: the customers cooperation. This last one can be voluntary or not and among this type of e-commerce, a peculiar one stands out: the so-called matrix scheme.

1.2 The matrix schemes case: a genuine cooperation among customers?

In the cluster of pyramid scheme there is a peculiar one that has to be analysed: the matrix scheme. The business model of this one is characterized by a company that sells products to customers and gives them the opportunity to be rewarded with another product. Customers buy low price products and they are added to a waiting list for a product of higher value. It is not a purchase that imply the possibility to win a price, like in a lottery, because all customers on waiting list should receive the high value product. Nonetheless, the reward is not certain because of the presence of the waiting list which creates the pyramid structure, involving uncertainty. Matrix scheme functions as follows: the first customer on the waiting list receives the high value product (in a chronological order) when a predetermined number of new customers joins the list. The prices of the low value product and of the high value product are interconnected with the number of required customers who have to join the list in order to reward one customer. An example of matrix scheme is the following. The high value product has a price X while the low value product has a price $X/_n$, where n is the number of customers required in order to provide the high value product to a customer on the waiting list. In order to give another high value product to a second customer, the company require n more sold low value products. In this way, the pyramid is build up and the more n is great:

- the more people are needed to provide high value products to a little number of customers,
- the more decrease the probability to obtain the high value product for a new customer.

The characteristic of this type of sales is that the customers buy a product of low value in order to have the opportunity to win a high value product. Following this idea and exploiting the customers cooperation, online store created a new selling strategy in order to provide the best price to customers. This mechanism is provided by new-born websites that typically sell technological products. These websites are not the producers, but they purchase products from several companies and only act as intermediaries for the customers. The reason can be found in the fact that nowadays most people are able to look for products information by themselves, without a salesman aid, for example looking directly at the producer's website or from online newspaper articles or blogs. It means that customers are able to collect information about the product at a certain moment and then, in a second one, to choose where to buy it at the best price. With this new practice, customers are able to purchase a product with a discounted price generally up to a third of the "advised" price, i.e. the price imposed by the producer company to end users. This discount is possible because one product is dispatched only if a determined number N of customers pays the product price, creating a waiting list. In this case, the term "group purchase" would not be correct. In fact, in a group purchase, all the customers pay the same price to buy a product and all the members receive the good at the same time. The time aspect is lost in the case of this new selling strategy: in fact, customers receive the good on a chronological basis, because they have to wait for new customers orders. So, if the first customer pays the price, he has to wait that the specified number N of new customers performs the payment. In this case, the price assumes the characteristic of a reservation fee, but without the certainty of the delivery of the product. In this way, the website collects the necessary amount of money to buy and then to sell the product to the first customer. In the same way, the second customer has to wait other *N* new customers in order to receive the product. The website is able to make profit because the cost of buying the product is lower than the value of the *N* payments received from customers. These last ones gain in the sense that they are able to save money because they can buy a product paying only a portion (on average one third) and relying on other customers who made their payment. It seems to be a win-win strategy. However, customers have to consider the waiting time. In fact, since each shipped product require *N* customers, the waiting list become gradually longer and so the time that new customers, at the end of the list, have to wait. So, this is the first downside and the second one is correlated to it. As the list become longer day by day, a new customer is discouraged to add his order to the list for one basic motive: he does not know how many other customers will join to the list. So, he has two possible future scenarios:

- a sufficient number of customers will join to the list, allowing him to receive the product but, at the time he joins the list, the exact waiting time is unknown;
- not enough customers will join the list and he has to redeem the product at a total price higher than the "advised" price (if the website provides this practice).

Due to the characteristics of the mechanism used by these new websites, it can be considered a matrix scheme. But, a crucial point has to be explained. At a first glance, the matrix scheme could differ from this new selling strategy. In fact, matrix scheme customers accept the low value product price and by buying it they can be considered "satisfied", while the high value product is only a possible extra. Instead, in this new selling strategy there is only one product and the customers are "satisfied" only when they find the required number of new members. However, the price of the low value product and the relation between the two kinds of product have to be analysed. In fact, if the product that the customers purchase has very little value, it could be said that the high value product is not only the reward, but it is the real sold product. It means that if the low value product would not be sold without the high value product, this last one is the real product that customers desired and want to buy. Given that, the matrix scheme shows a structure that is quite equal to the new selling strategy. If matrix scheme customers purchase the low value products only to join the list in order to obtain the high value one, this last one can be considered in the same way as the price (or fee) that customers pay in the websites which use this new strategy. Indeed, the low value product of the matrix scheme can be considered as a simple fee that the customers pay in order to join the waiting list, which is exactly the same system provided by the new considered websites. The low value product in this way can be ignored and these two schemes coincide. However, due to lack of the presence of the low value product (even if it can be ignored), this new selling strategy can be considered as a peculiar variant of the classical matrix scheme.

The table below shows an example of a waiting list. Orders are recorded chronologically and each order, but the first, is assigned to an older one, as the "referral code" column shows. In this table, the assumption is that each order needs three new orders to be completed. The first order has no referral code because there is no older order to be associated with. The second, third and fourth has the same referral code and it coincide with the first order. It means that these three orders payment are used to dispatch the product of the first order. So, order A has a 3/3 state because there are three orders after it, as well as order B. It means that they can receive the product. Instead, order C has only one order assigned to it and has to wait two further customers in order to be completed and receive the product. All other orders have a 0/3 state and have to wait three new customers each.

Table 1 - Example of waiting list

Place	***Date***	***Order***	***Referral code***	***State***
1	1st January	A	-	3/3
2	2nd January	B	A	3/3
3	3rd January	C	A	1/3
4	4th January	D	A	0/3
5	5th January	E	B	0/3
6	6th January	F	B	0/3
7	7th January	G	B	0/3
8	8th January	H	C	0/3

Figures 1 and 2 illustrate two examples of the websites that use this strategy. In the first screen, there are three product which are examples of what these websites sell. Moreover, on the left of this screen, there is a list of all categories of products. These are mainly technological goods as TVs, videogames, smartphone, PCs but also fashion accessories and gift vouchers. Products prices are already at the discounted value as it can be seen by the prepaid MasterCard with a value of 100€ but a price of 49€ for the customer. In the second screen, there are three examples of notebooks and in this case the website illustrates the discounted price and the gross one. Notebooks profile recall one of the concept introduced in previous paragraph. In fact, customers need only the product code contained in the product name in order to look for characteristics and information autonomously, without a salesman aid.

Figure 1 - Website 1 example

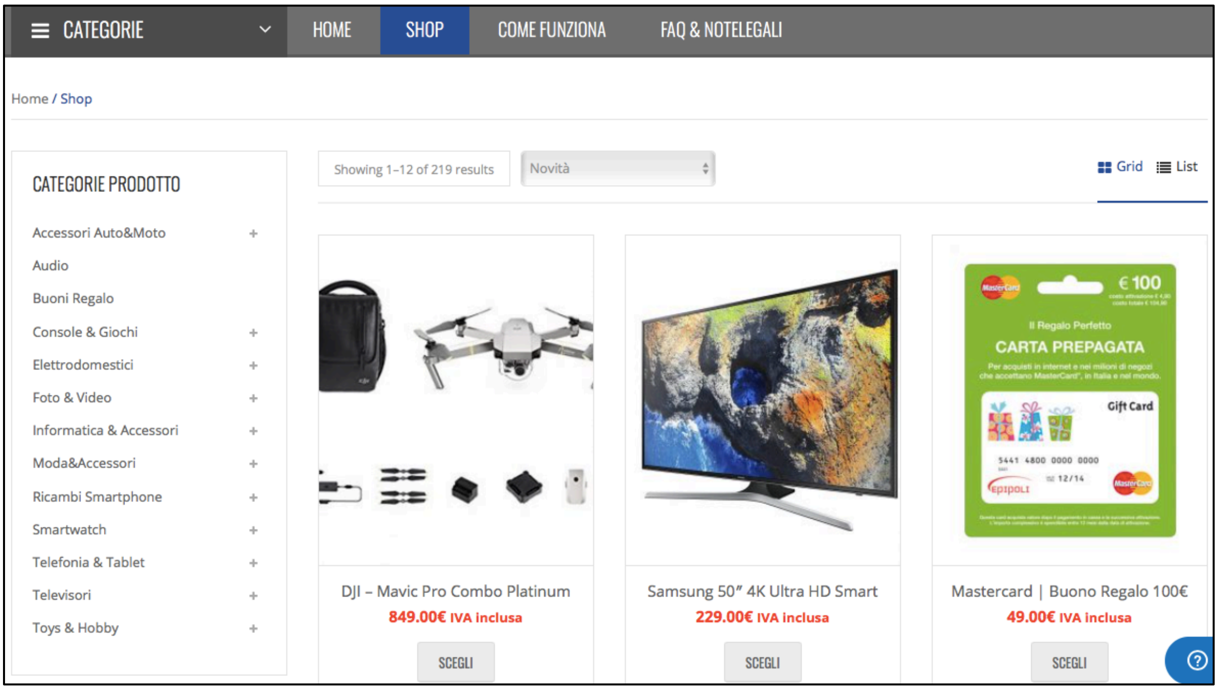


Figure 2 - Website 2 example

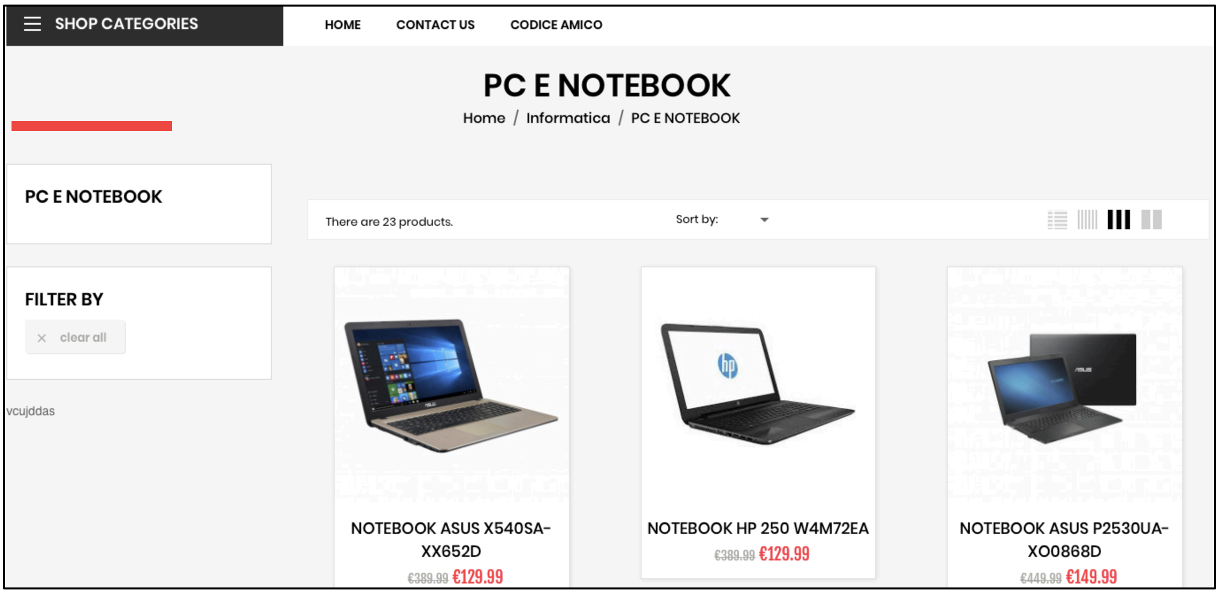
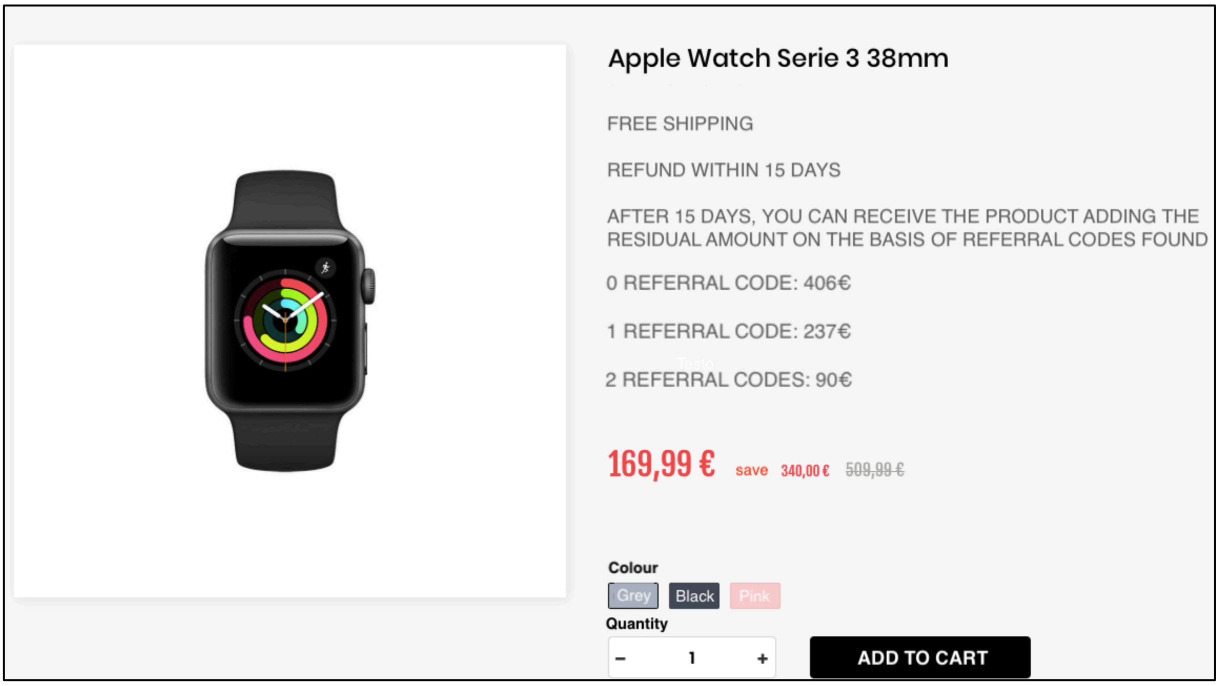


Figure 3 shows the screen of a product taken as example. In the picture, there are some information that are considerably important for customers. In fact, it is written that a customer has the possibility to obtain the purchase refund if it is requested within 15 days from the order payment, following the Italian "costumer code" law. Then, if these days are spent, a customer has two possibilities:

- to wait three referral codes to obtain the product at the price initially paid;
- to not wait and to obtain the product paying the residual amount on the basis of referral codes found.

So, in this second case, the website shows for each product three amounts a customer is obliged to pay if he wants to obtain the products before having found three referral codes. With a simple calculation, it easy to show that if a customer wants to redeem the product with no referral code it will pay a higher price than the gross one indicated by the website. In the case of the screen with no referral code, a customer should add 406€ to the 170€ already paid with a total of 576€. This amount is higher than the 510€, which is the full (or gross) price of the website. The reason is the website profit. If the price to redeem the product with no referral code was the difference between gross price and discounted one, the website's profits would decrease. In fact, since the price paid by the customer is assigned to another older order, a customer with no referral code has to add the amount that the website needs in order to buy the product entirely. In this case, website needs 406€ in order to buy and then sell the product, so the difference between 510€ and 170€ (340€) is not sufficient.

Figure 3 - Product profile



It's useful to consider also a numerical example. Assuming that a new customer joins an existing list, which already has three waiting clients on it. So, there are three old customers, called subject Alfa, Beta and Gamma, and one new customer, called Delta. All these individuals paid a price of 300€ for the "chance" to obtain a product, with an "advised price" of 900€ and a cost of 800€ for the website. Before Delta joins the list, the website collected the price paid by Alfa, Beta and Gamma. However, the fee paid by the very first client of the list can be considered entirely as a profit for the website. So, the website collected 300€ from Alfa that are full profit, and a total of 600€ by Beta and Gamma. This last amount of money will be used by the website in order to purchase first and sell after the product to Alfa, so his order can be considered "completed". But Alfa's order lack one of the three further orders. So, when subject Delta joined the list, the website collected another fee and with this last one it is able to ship the first product to Alfa. So now the situation is the following:

- subject Alfa paid 300€ and received the product;
- subject Beta paid 300€ and he has to wait three other customers, after Delta;
- subject Gamma paid 300€ and he has to wait three further customers, after the three ones that subject Beta has to wait;
- subject Delta paid 300€ and he has to wait three further customers, after the three ones that subject Gamma has to wait;
- website collect 300€ for each of the four players, with a total cost of 800€, gaining a profit of 400€.

In this case, the website gains 300€ from the first player, and 100€ for each product that is able to dispatch. The scheme shows that actually one subsequent order has the task to cover partially the cost of the item for one of previous order. The portion that this order is able to cover depends on the number of orders that website require in order to dispatch one product. In this case each order covers one third of the cost of an item.

The Italian situation shows the presence of few websites which used this selling strategy, with three different mechanism. The example explained above is clear but is limited in the sense that these kinds of websites sell several products with various prices and it has to be specified in order to distinguish between different methodologies. In the first mechanism, there is the creation of as many waiting lists as many the product types. So, if there are three different products, X, Y and Z, each of this product has a different list with a total of three list. The second system can be considered a variation of the first one because orders of one list could "help" another list of a product with a lower price (in order to preserve the profit for the website). So, if X, Y and Z prices are such that $X < Y < Z$ and the price of Y is slightly bigger than the X one, website could set that the order of Y will be used to dispatch product X. It means that a new order on the Y list can be used to dispatch a product of list X or Y. Normally, websites rely on the chronological order so new orders will be used to ship previous ones on the basis of the older order made. In the third mechanism, there is a unique list and it works as a "compensation" system. It means that if there are, as before, three different products, they will join the same unique list. The prices paid by new customers will be used to dispatch previous orders whatever is the product. Even in this case a numerical example is helpful. Suppose that there are three products: product X with a price of 100€, Y with a price of 300€, Z with a price of 500€. The first order is for product X, the second one for Y, the third one for Z and each product requires three times the price in order to be dispatched. So, the first customer makes an order of product X paying 100€ and he needs 300€ in order to obtain the product. The second customer makes a product Y order paying 300€. This amount is used in order to dispatch the first order because the required amount match the paid amount. However, the second customer needs 900€ in order to have the product. The third customer makes a product Z order paying 500€ and he will need 1500€ to obtain his product. The amount he paid is used to cover partially the required amount of the second customer, that is 900€. So, at the end of day, the first customer obtained his product, the second one collected 500€ but he needs 400€ more so he has to wait for new customers, as well as the third customer. Actually, some websites use this third mechanism combined with the first one, creating a sort of fourth mechanism. It means that a new order payment is used to cover partially the older order, whichever is, and partially the older order of the same product.

The correlation between advantages and downside lies in the trade-off between numbers of customers and waiting time. The third mechanism has the benefit to attract a larger number of people: when a customer has to decide whether to join the list or not he has to consider a unique list and he knows that all possible future orders can help his order. But, by doing so, the waiting list become quickly bigger, consequently boosting the time that new customers have to wait, discouraging them. It means that each new customer increases the waiting time for all the following ones. For this reason, the fourth method has been created in order to decrease this time. Instead, the first and second mechanism can rely on a smaller effect of a new customer because there are several lists each for a product. So, in this case a new customer can rely only on future orders of the same product. However, the benefit is that the list takes more time, with respect to third mechanism, to become long enough to discourage new customers.

Whichever is the mechanism used, the website will have a proportion of about 1:3 between completed orders and waiting customers. This is due to the fact that approximately customers are able to purchase products at a price equal to one third of the "advised" price.

For this reason, it seems that these websites apply an up to date form of a matrix (and pyramid) scheme. It is characterized by the fact that among all the customers only a minority can succeed to normally gain profits or, in this case, obtain products, at the expense of all the other customers that represent the majority. In the next paragraph the most common forms of pyramid scheme will be analysed in order to highlight characteristics, similarities and differences with respect to the mechanism applied by the websites in analysis.

1.3 Comparison with other similar schemes

The relation between one dispatched product and a predefined number of paid prices leads to presume that this mechanism is nothing but a pyramid scheme, as explained above. So, in this paragraph some type of schemes will be discussed in order to understand whether these new selling strategies[3] can be equated to one of them. These schemes are: pyramid scheme, Ponzi scheme, multi-level marketing and referral programs.

1.3.1 Pyramid scheme

The mechanism of pyramid scheme is based on the promise to customers or investors, by paying an upfront fee, to gain large profits if they are able to recruit other people to join this system. This means that the source of revenues is the recruitment of people itself, and not the real return on the investment or the revenues from the sale of goods and services. A probability model of a pyramid scheme is explained by J. L. Gastwirth (1977). His model provides the following results:

- the majority of the participants have less than 10% probability to recover the initial investment when a small profit is achieved as soon as three people are recruited;
- on average, about the 50% of the participants will not be able to recruit new members and will lose all the investment;
- on average, about the 12.5% of the participants will recruit three or more members;
- less than 1% of the participants can expect to recruit six or more new members.

To better understand how this mechanism works and how it differentiates from the other ones, it is useful to present a numerical example. Suppose that there are four types of subject: one promoter, the principal investor, few successful investor and several victims. An example of this type of scheme is illustrated as follows: each investor has to pay 500€ to the promoter and has to enlarge the pyramid by recruiting three more members, who in turn have to do the same.

[3] In the following pages, the terms "scheme in analysis" is used in order to refer to the new selling strategy.

The revenues accruing to the promoter derive from all members payments of 500€ while participants earnings are the result of two sources: a payment of 150€ received from the promoter for each player he's able to recruit at the first level and an additional payment of 30€ for each new member that join the scheme in the next three levels.

Table 2 - Pyramid scheme example

Promoter					
Principal investor	Level 0				
Successful investors	Level 1	150€ x 3 = 450€	1 x	1 x	1 x
	Level 2	30€ x 9 = 270€	3 x	3 x	3 x
Victims	Level 3	30€ x 27 = 810€	9 x	9 x	9 x
	Level 4	30€ x 81 = 2430€	27 x	27 x	27 x
	Level 0 total gain: 3960€		Members: 121		

This table represents the construction of the pyramid and for calculation purpose it is assumed that the scheme collapses after level 4. Level 0 investor revenues derive from the commission that he receives for each new recruitment: 150€ for each member at level 1 and 30€ for each member from level 2 to 4. It seems a good investment because he can gain 3960€ by paying a 500€ fee. Each of the three level 1 investors gains 150€ for the three members at the level immediately below and 30€ for each member at level 3 and 4: each of them gains 1530€ with a 500€ fee. Proceeding with the calculation it is clear that this scheme results in a loss for the level 3 investors: in fact, they can rely on only three 150€ commissions so they obtain 450€ by paying 500€, resulting in a 50€ loss. This situation can be enlarged to the level 4 investors because they lose the entire 500€ they "invested". Part of their losses contribute to generate promoter profits. He gains: 500€ from the first investor, 350€ from the three investors at level 1 (500€ minus 150€ commission), 320€ from the nine ones at level 2 (500€ minus 150€ and 30€ commissions), 290€ from the twenty-seven members at level 3 (500€ minus 150€, 30€ and 30€), 260€ from the new eighty-one investors at level 4 (500€ minus 150€, 30€, 30€ and 30€). The total sum of these value lead to a result of 33,320€ net profit for the promoter. This example assumed that the pyramid collapses after level 4. In reality, however, this is unknown for the majority of investors. A level 4 investor may think to be at the top (or close to it) of the pyramid when he joins the scheme but, after the scheme collapse, he realizes that he is actually at the bottom, as level 3 investors. So, among the 121 members joining the scheme, there are 108 who suffer losses.

Pyramid scheme create an ethical issued, in fact they are largely considered unethical, as explained by D. Koehn (2001). She sustains that pyramid schemes are unethical for two reasons: they are fraudulent and recruitment-centred businesses.

The first point derives from the fact that they typically promised high rate of return with a small investment. In fact, the system implies that the early adopter of this scheme can make a great investment, but at the expenses of those who comes later, who will make little gain or even lose money because there are no other people to recruit.

The second point is related to the fact that is not in the public interest to have business who are recruitment-centred because of two motives. First, a company should focus on the marketing of "non-harmful products" to customers. Otherwise it would mean that a company is concentrated on making money "per se", but this is not a public interest. Second, a company should be concentrate its effort on growing through developing the market for its product and not on the growth "per se". Then, it is necessary to evaluate whether a company aims at:

- making profits with the development, the sales and the advertising of a non-harmful products,
- growing thanks to the enlargement of the customers base by offering new products.

Several aspects of a company tell that a company and its agents are recruitment-centred schemes. Even if they are not thorough, they are:

- the focus on growth through recruitment of new adherent,
- the request of considerable upfront fees,
- the pressure on recruits to purchase for consumption.

1.3.2 Ponzi scheme

Pyramid and Ponzi schemes share some characteristics, but it is not possible to consider them identical. In a Ponzi scheme, there is one schemer and several victims. The schemer is the individual who creates and manages the scheme while all the investors are the victims: the latter do not know that profits are generated by new investor payments; instead they believe that the schemer is a capable investor. The schemer acts as the principal actor because he manages all payments from and to investors. This scheme provides quite high return on investment, so first investors that achieve this gain are incentivized to invest more money into the project. In this way, the scheme's life is extended: the schemer and early investor reach their profit. However, this kind of scheme persists if fund's money output rate is lower than the one of money deposited by new investors. When the output rate is larger than the input one, the scheme starts to collapse.

Copious is the doctrine about this type of scheme because of his seniority, as explained by U. Bhattacharya (2001). His study is focused on the promoter sales of certificates with a price P to a mass n_0 of citizens, promising a return of r. At maturity, the promoter redeems the certificates, but he sells a new batch of them at the same conditions to a mass of citizen equal to

$$n_1 = (1+g)n_0,$$

where $g > r$. The last round is the round L. The variables introduced so far are endogenous so decided by the promoter. The promoter revenues are equal to $n_0 P$ in the first round, plus a portion t of the next rounds revenues. The revenues collected in one round are used to pay the return to previous round citizens, then $(1-t)Pn_i = (1-t)Pn_i(1+g)$ must be equal to $P(1+r)n_{i-1}$. It means that t is equal to $t = \frac{g-r}{1+g}$. So, the promoter expected revenues are:

$$\pi = n_0 P + \left[\frac{g-r}{1+g}\right]\left[(1-\theta)^i n_i P + \cdots + (1-\theta)^L n_L P\right],$$

where $(1-\theta)^i$ is the probability of survival of the Ponzi scheme till round L. In fact, θ is the probability that the regulator may intervene to stop this practice.

The cost that the promoter faces are the direct marketing expenses and the penalty imposed by the regulator if the scheme collapse. The marketing cost are a distinctive feature of the Ponzi scheme. In fact, the promoter bears only an initial amount of this type of cost: after the first amount of successful payment to investors, information about this scheme spread among investors (word of mouth) without or with negligible further marketing expenses by the promoter. When a circumstance of this type happens, i.e. marketing costs do not increase as the number of rounds, the system has to be recognized as a Ponzi scheme. The marketing theory on the effectiveness of marketing expenses, Rao and Miller (1975) provides strong evidence of this reasoning.

The participation constraint of the promoter is given by the comparison between the choice to run away and the termination in round L. The promoter always faces the desire to run away with the money before the last round. In this case, if the promoter terminates in a round i, he gets revenues equal to tPn_i. While if he terminates in the next round $i+1$, he gains revenues of $(1-\theta)tPn_{i+1}$, equal to $(1-\theta)tP(1+g)n_i$. To ensure that it is optimal for the promoter to wait until the last round (i.e. Ponzi scheme is a subgame-perfect equilibrium), the latter expected revenues should be greater than the former:

$$(1-\theta)tP(1+g)n_i > tPn_i.$$

This inequality gives the lower bound of the promoter participation constraint on g: $g \geq \frac{\theta}{1-\theta}$.

The participation of citizens is solved backwards. If in the last round, citizens know that there will be no bailout from the government in the case of the scheme collapse, they will not participate, repeating this choice in all previous round and none of the citizens would participate. Then, the promoter in the last should in involve at least a sufficient number of people that obliged the government to implement the bailout, i.e. $n_L \geq n^*$ where n^* is the critical mass of citizens. This means that Ponzi schemes do not exist if the probability of a bailout is zero. However, this does not imply that Ponzi schemes necessarily exist of the bailout probability is positive. In fact, if the probability of bailout exist, citizens has to evaluate whether to participate or not. If the they participate in the last round, they gain:

$$-P + \alpha(1/n_L - 1)\beta,$$

where α are the government asset and β is the probability of bailout. If they do not participate they bear a loss in any case equal to

$$-\alpha\beta,$$

which is the expected loss due to the redistribution of the government assets. Citizens participate if the former is greater than the former:

$$-P + \alpha(1/n_L - 1)\beta \geq -\alpha\beta.$$

This inequality solution provides a condition for the price:

$$P \leq \frac{\alpha}{n_L}\beta.$$

This explanation provides an essential point: if citizens participate, they lose, but they lose less than what they would if they did not participate. It means that is rational for a citizen to participate in the last round, whether the loss deriving by not participating is greater than the one deriving by the participation. For this reason, governments are interested in stop this scheme to born and/or spread.

Therefore, a Ponzi scheme may exist if:

- the assets (α) of the state can be used for the bailout (β),
- the probability of early termination of the scheme by the regulator is low (θ),
- there is an inexpensive access to citizens through mass media,
- there are no severe penalties on the promoter.

However, no government has ever compensated of the entire loss the citizens who were victims of Ponzi schemes. The money that participating citizens lose are equal to P, which is equal to $\frac{\alpha}{n_L}\beta$, while the net bailout is $\alpha(1/n_L - 1)$. Therefore, the portion of money lost which is rescue by the bailout is:

$$\delta = \frac{\alpha(1/n^{*}-1)}{\alpha\beta/n^{*}} = \frac{1-n^{*}}{\beta}.$$

A partial bailout implies that $\delta < 1$, so $\beta < 1 - n^{*}$. Then, a Ponzi scheme may exist even under partial bailout if the conditions just listed hold and if the probability of bailout is higher than $(1 - n^{*})$, where n^{*} is the critical fraction of citizens that are required to be involved in order that a possibility of bailout exists.

This elaboration demonstrates that:

- citizens believe that they could achieve a higher expected return without taking the corresponding risk,
- a promoter exploits the belief of a bailout of the citizens in order to convince them to participate the scheme.

The table 3 illustrates an extremely simplified example of the Ponzi scheme functioning. Differently from the pyramid scheme, in this case it is better to consider the input and output of money (chronologically) rather than the number of investor that deposit money. The output column is calculated assuming a 10% interest rate. The residual is the difference between the output and the input in the next period. Then, it is easy to calculate that this scheme works until there are enough input of money to compensate the output level of the previous period.

Table 3 - A simple example of a Ponzi scheme

Time	Input	Output	Residual
T	100 €	110 €	90 €
T+1	200 €	220 €	170 €
T+2	300 €	330 €	- 60 €
T+3	100 €	110 €	. . .

The investor in time T deposits an amount of 100€ and he expects to gain a total of 110€ because of the 10% interest rate. This amount can only be compensated with new deposits of investors. In the example, it is assumed that it is equal to 200€, higher than the output of previous period that the schemer has to pay. The difference between input and output is the residual amount in the schemer current account. The investment in time T+1 results in an output of 220€, considering the interest rate. It can be paid only with a new investment in the next period, T+2. Similarly, the input is greater than the output of the previous round so the schemer is able to repay it with a residual in his favour of 170€. This value is equal to the first residual of 90€ plus the second residual of 80€ (300€ - 220€). This example starts to collapse in period T+3. In fact, the schemer in this period has a current account equal to 270€, that is the value of the input in time T+3 and the residual amount after the output of time T+1. This value (270€) is not sufficient to compensate the output of time T+2 equal to 330€. So, the schemer is in a default situation because there is not enough input to pay the output to investors.

Whether the amount of input is larger than the amount of output, the schemer is able to repay the investment with the interest rate. However, this mechanism requires a level of input always greater than the one of the previous period. Normally, the schemer capable of persuade old investors to deposit again their money is essential for the scheme life because he should only pay the interest rate and not the entire amount. This scheme collapses for several reasons. The most common are the escape of the promoter or if a great number of investors in few time require their deposit back which is greater than the amount that the promoter owns.

1.3.3 Multi-level marketing

The so-called multi-level marketing is a third type of scheme and it relies on the goods sale. In this case, there are three types of subjects:

- the parent company which produce the good,
- the salesmen,
- non-participant customers.

The business model is build up as follows: the parent company provides products to the salesmen who have two tasks: to sell them to non-participant customers and simultaneously to undertake promotional activities. In this way, the parent company decrease its marketing costs after the initial period. Additionally, salesman can build their "downline" of distributors. It means that a distributor can convince other people to become distributor as well, by expanding the organization. It is a possibility for the distributor, but it is also an obliged step in order to increase his income. In fact, a distributor's revenues derive from:

- his actual sales,
- a percentage of the sales of his downline.

However, salesman cannot rely on a monthly or periodic certain wage because all distributor's revenues are made up of commissions paid by the parent company. The first are commissions paid on the basis of the salesman direct sales while the second are commissions derived from the sales of the distributors that constitute the downline. Then, the company is able to make large profit because of the goods sale provided by the salesmen and because these last ones are remunerated only on a commission basis. The possibility of the distributor to build his own downline of salesman shapes the pyramid scheme of this business model. In fact, each salesman has to recruit new agents in order to increment his revenues because in this way he can exploit revenues made by someone else effort. This mechanism guarantees sure revenues to the company and to the salesmen at the very top of the pyramid; on the other hand, the scenario is negative for all other members. In this case the pyramid concept has to be considered differently than before, in fact each salesman considers the pyramid that he is able to build, with himself at the summit. So, if in the classic pyramid scheme and in the Ponzi one, the scheme collapses once and for everyone, the multi-level marketing can collapse continuously several times and it involves different members in different periods. Then, the relation between salesman and customers has to be considered. In fact, theoretically customers should be merely end-user, but in the practice, consumers are new members recruited by salesmen positioned above in the structure. At the end of the day, the majority of participants are unable to make profits because of two reasons: they become real seller and have to sustain operating expenses and then each person of the pyramid can quit the system at any time, cutting other salesmen revenues. This statement is demonstrated by studies on MLM companies. The result is that about 60% to 90% of participants quit the scheme and that the 99% of workforce are unable to make profits. The majority of people are convinced by the value of the profits made by few members at the top, believing that at a certain point in the future they will be at the summit of the pyramid gaining the same profits. Then, they discovered that this option will be not verified so they quit the scheme but in the meanwhile they contributed to company profits by consuming products and by recruiting more workforce. However, there are some feature that split the multi-level marketing company in legal or illegal. The legal multi-level marketing plan is characterized by:

- business focus on the sales of product to end-user,
- the profits derived primarily by the sales of the distributor,
- generally, products are typically used in everyday life with a good quality-price ratio,
- there is low or none start-up cost,
- the support system is characterized by a low pressure and by the desire to answer all possible questions with all the required information,
- the company provides detailed information about the job and it does not promise easy money,
- it is easy and without cost to quit the company,
- the business is sustainable.

So, the profits of distributor are based on the time and effort committed to the sales activities.

Opposite is the case of an illegal MLM plan because the business focus is on the recruitment of new members. They are also obliged to pay an initial fee which will be used to pay older distributor. The support system is focused on a constantly recruiting activity and the company promises large profits with low effort. Then, distributors are discouraged to quit the company because of the high-pressure tactic on keeping participants within the scheme. At the end, this system fate is the collapse, causing a loss of money for the majority of the salesman.
Multi-level marketing is not considered illegal in advance, but it depends on the methodologies that the specific company uses in order to develop it.

1.3.4 Referral programs

One peculiarity of the variant of the matrix scheme in analysis (and also of other systems) is the recruitment and this aspect reminds the so-called referral programs. This kind of programs can be considered as a marketing strategy. In fact, the aim of it is to use the most ancient advertisement form in order to increase revenues: the word of mouth. The mechanism is simple: a customer has the opportunity to obtain discounts, prizes and rewards if he is able to lead a new client to purchase the company product. This is called referral program because a new client can buy the product with the reference of an older customer. It is a form of decentralized advertisement because not only the company promote the product but also customers are involved in the marketing strategy and they have to look for new customers in order to obtain rewards. A customer in a referral program has two options: to look for a new customer or not. It means that he has the possibility to obtain a reward if he finds another customer to activate the referral program but if it does not happen the customer has no negative effect. So, the recruitment is an extra option for the actual customers that can decide to exploit it or not.
An example of a referral program can be the following. A customer has a valuation V of a product that costs P. His utility function is given by: $U = V - P$. In order to exploit the benefit of this program, a person has to be already a customer. It means that a customer has accepted the price and obtained the product. In this way, the customer utility is positive or at least zero. By adhering to the program, a customer has a possibility to gain a benefit α if he is able to "bring" a new customer to the company. Then, this new customer paid the price (if he considers it equivalent or lower than the valuation) and give the old customer the possibility to gain a positive utility. In fact, by considering α, the new utility is $U = (V - P) + \alpha$. Since the difference between V and P must be positive (otherwise a rational customer would not have paid), the final utility is at least equal to α. For ease, it can be assumed that the effort on the

research of a single new customer (relative, friend, colleague) is very low, or null, compared to the benefit. In the case the cost is higher than the benefit, a customer would not accept this program but normally is not. For this reason, normally this program implies that a customer should convince only another one and the company tries to make this practice easy. In fact, the new customer has only to report the "referral code" on the form (for this reason the name referral program). Generally, this program implies also a benefit for the new customer in order to convince him, or otherwise, he has the same possibility to exploit the benefit using the same practice with another new customer. In this way, the company has the cost of providing the benefit but not the advertising cost because all the process is possible with the word-of-mouth principle.

1.3.5 Differences among schemes

Pyramid and Ponzi schemes are similar because in both older investors' profits are generated by fees paid by new investors, and not by business or financial activities. However, the strategy and actors are different. In the pyramid case, actors know that commissions derive from the recruitment of new investors, while in the Ponzi one, they do not because they believe that earnings derive from the ability of a capable investor. Then, since the schemer manages all the process it could be said that the scheme is centralized in his figure while the pyramid scheme is characterized by the decentralization. In fact, each investor has the duty to recruit new ones in order to gain commissions. A third difference lies on the possible duration that is general longer for a Ponzi scheme. In fact, in this case the schemer can rely on payments of actual investor by persuading them, shifting forward the collapse. In the pyramid scheme, this is not possible because actual investors require necessarily new ones. The scheme in analysis (i.e. the new selling strategy) share characteristics of both pyramid and Ponzi schemes but it differs in other aspects. As these two cases, the crucial activity derives from the recruitment of new members even if the subject designated to do it is different. For this aspect, the scheme in analysis is more similar to the pyramid case where all members have to recruit other ones. Conversely, instead, happens in the Ponzi scheme where only the creator has the task of looking for new investors because only he knows that new members are necessary to satisfy older ones' contracts. Then, also the purpose is different. In fact, pyramid and Ponzi schemes are both considered financial "activities" because the investors purpose is to make profits. This aspect is not the same of the scheme in analysis. Indeed, its purpose is to allow people to purchase consumption goods (at a discounted price). A further element is the number of times a member

can act within the scheme. The scheme in analysis allows people to made multiple purchases, as it occurs in the Ponzi scheme where actual members can add money to the investment, but differently from the pyramid scheme where exclusively new members are needed.
So, considered these characteristics, the scheme in analysis could look more similar to a pyramid scheme than a Ponzi scheme. In fact, the only aspect similar to this last one is the recruitment, while all other aspects are different. However, an important trait distinguishes the scheme in analysis and the pyramid (and Ponzi) scheme. In fact, none of these two are based on a goods sale. Even if in the Ponzi scheme investors are buying a financial product because they invest money in order to make future profit, when the scheme is revealed or collapse they discover that no financial product has ever existed.
The MLM shares the actual sale of goods with the scheme in analysis but they differ for the purpose and for one aspect of the pyramid structure. In fact, the MLM purpose from the point of view of participants is to make profits, while in the scheme in analysis it is the purchase of discounted products. Then, the pyramid concept is slightly different. In fact, in the scheme in analysis a member has to consider only the level immediately below him and when he obtains the required number of people he has no more work to do. In the MLM a member has to consider not only the few recruited members but he has to continuously monitor and manage the entire pyramid of members that he recruited. In fact, a salesman can gain if his recruited members sell and convince other members to join the pyramid. But, he bears the risk of losing revenues if recruited members quit. So, on one hand, in the matrix scheme variant, a member is satisfied when he finds the required members, on the other hand, in the MLM, a member has to be continuously active in order to gain and to delay "his" pyramid collapse. Given that, there are more differences that similarities between these two systems, so it is difficult to consider the scheme in analysis as a multi-level marketing.
Referral program mechanism seems to be quite equal to the scheme in analysis: there is the same decentralized marketing activity by customers and they obtain discounts only if they are able to lead to purchase new clients. However, there is an important aspect that distinguishes the referral program with respect to the scheme in analysis. In the former, a customer already obtained the product, i.e. he has already purchased the product at a certain price and he is obtaining additional gains out of it thank to the referral program, in terms of rewards or discounts on future products. In the latter, a customer has to look for new members in order to obtain the product, without them he has nothing. Then, the situation is different because, in the matrix scheme variant, the recruitment is preparatory in order to obtain the product. A customer has no option but to recruit a certain number of people otherwise he is not able to catch the

desired product. Due to this important difference between referral programs and the scheme in analysis it can be said that they are not the same thing.

Concluding, this new selling strategy implies the creation of a waiting list which determine the structure of a pyramid scheme. However, due to the mechanism, nature or purpose, the selling strategy cannot overlap perfectly to none of the schemes explained above. The case that best describes the functioning of this new selling strategy remains the matrix scheme, as discussed in the paragraph 1.2. Then, this new practice can be considered as a variant of the pure matrix scheme.

1.4 Legal issue

In several states, these various types of pyramidal sales or schemes are considered illegal. The reason is the social damage caused by this type of mechanism. In fact, history shows that this "company" caused a damage to several citizens/investors/customers. Then, due to the focus of the company on the recruitment of other members, the aim is not to offer products or services to the public but only to collect revenues with upfront fees. All the activity described above are considered illegal because they caused an economic damage for a significant number of citizens, who normally lost money. The most famous and recent case is Madoff investment scandal. Madoff managed a Ponzi scheme for several years. It collapsed alongside with the financial crisis in 2008 because major investors withdrew the money they had in the fund, but its current account was not sufficient. Madoff caused a shortage of 65 billion dollar, owned by citizens and financial institution.

In Italy, the pyramid sales are expressly disciplined by the law no. 173/2005. This clause forbids the promotion or realization of all the sales structure in which the main economic incentive of the participants is based on the mere recruitment of new subjects, rather than on their capacity to sell or promote.

Conversely, as explained in multi-level marketing paragraph, this scheme can be considered legal in some states (it is legal in all 50 U.S. states) if some features are respected. The key point is the focus of the company: whether it is the recruitment itself of new members, it is considered an unethical business. In fact, all customers, once they made their investment (except for the Ponzi scheme), want to shift the possible cost of the collapse to new members by recruiting them, creating a social problem.

Even a company implementing the matrix scheme has been stopped by the UK Office of fair trading in 2005. The reason which explain this decision is given by the OFT Chairman, Vickers J.: "These schemes are unsustainable and will eventually collapse to the detriment of many people. They can also undermine consumer confidence in e-commerce. The OFT's targeting of mass-marketed scams is an important part of its work of making markets work well for consumers"[4]. Therefore, the aim is to provide a genuine e-commerce, which does not cause economic or social losses for some people. In the previous paragraph, the selling strategy in analysis in considered like a matrix scheme, or a variant of it, due to the similar functioning concept. However, few differences exist and therefore it cannot be said whether also this new selling strategy should be considered illegal as well as the classical matrix scheme.

[4] Office of fair trading. 2005. *Press release 161/05*. https://web.archive.org/web/20070314233701/http://www.oft.gov.uk/news/press/2005/161-05

CHAPTER 2
A MODEL WITH THREE ROUNDS

2.1 The Model

The realistic situation analysis of the sale with this peculiar variant of the matrix scheme requires a basic model in order to get an insight of this purchase system's dynamics. Therefore, in this and in the next chapters several variants of the same game will be analysed. These variants differ in the values of the parameters of the game. The aim is to replicate the real functioning and to comprehend the variables that can influence the game result. For this purpose, it is necessary to introduce some assumptions and to generalize some peculiarities that will shape the cases analysed further on.

In the following games, the main parameters are:

- a number *N* of rational and equal players/customers,
- a number *R* of rounds,
- a variable *M* which indicates the required number of payments which the customers have to perform in order to dispatch a product,
- a product price *P,*
- a product valuation *V*.

Games have a number *R* of rounds. In each of them, one among *N* players is extracted. In each round, all players have the same possibility to be extracted, that is equal to $1/_N$. This probability can have different explanation. In fact, it can be considered the probability that players have to gain access to the website before or faster than other players; or the probability to have knowledge of this method of purchase before other players. At the end of the day, it can look like a lottery in which all the participants have the same chance to be extracted. Once a player is selected, he has to decide whether to enter or not the game. If he does not, he does not pay and he receives nothing. If he participates, he pays a price *P* and has the opportunity to obtain the product with a value *V*. When the selected player takes his decision, the game moves to the next round with another "extraction". The main "feature" of this type of purchase is the following: players don't have the guarantee to get the product because they know that, in order to get one, a number *M* of prices has to be paid throughout *R* rounds, excluding the payment of the first player. It means that once a player pays the price, he is added to a waiting list that requires a number *M* of further payments in order to dispatch a product. For this reason, this parameter will be named "dispatch coefficient". Generally, this parameter is helpful to understand the portion of the product value of an older order that a subsequent one has to cover, as explained in paragraph 1.2. Then, the price P is the amount of money that a website asks in order to be added on the waiting list, while the valuation V is the amount of money that the product is worth from the customers point of view. Each player has a utility function as follows: $u(y) = V - yP$, where y is the number of payments made by the player, at the end of all R rounds. This utility function assumed the "unit demand" concept. It means that if a player receives his product, he has no interest to enter the game again. All the equilibrium resulting from the analysis of the following cases are subgame perfect equilibrium.

In the following games, changes in variables are applied in order to understand the effect they have on the equilibrium from the players point of view. Then, the optimal seller's profit is computed. In order to understand better the following paragraphs, some explanations are useful. The sentence "pay the price" is a synonym of "enter/participate the game". The terms "receive", "obtain" and "win" (a product) are used with the same sense in order to explain that a player has been satisfied with the product dispatch and he has no more interest in participating the game.

2.2 Equilibrium analysis

In the analysis of the following case, only the range in which $V > P$ will be considered. The reason is that the range in which $V \leq P$ is trivial, whichever are the number of players, rounds and the dispatch coefficient required by the seller.

If $V < P$, players should pay a price which is greater than the value they attribute to the product, consequently none of the players is incentivized to enter the game. In fact, they would obtain for sure a negative utility equal to the difference between the value and price paid. For each value of P such that $V < P$, all players will ignore the game, then choosing the "no" decision. As consequence, the seller is not able to achieve any positive profits.

If $V = P$, a player utility is apparently zero, so this case should provide a solution in which players enter the game, differently from previous case. However, without complex calculations, it is simple to understand that this game is not profitable for customers and the reason is straightforward. If the dispatch coefficient is equal to 1, it means a product needs a further payment by a customer in order to be shipped, and so on. The case equilibrium can be computed with certainty because all players will act in the same way in all rounds, independently by the state of the game. Using the backward induction technique, the starting point is round III, which

one player is extracted in. His decision is based on what happened in previous two rounds and there are two possibilities: he paid the price previously, so he is on the waiting list or he did not. The statement "being in the waiting list" means that a player has paid one price, but he needs another one to have the product dispatched. So, a player in this state paid the price in round I and nobody paid the price in round II or he paid the price only in round II. If a player extracted in round III is already on the waiting list, he should pay another time the price in order to have the price. But since $V = P$, his utility function is $U = V - 2P = -P$. This result is equal to the utility he gains if he decides to not pay the price in round III because he does not receive product, but he pay one price: $U = 0 - P = -P$. Opposite is the case in which he is not on the waiting list. He has no incentive to pay the price because he has no possibility to receive the product due to the lack of further round. Then the utility is $U = 0 - P = -P$. If he does not pay the price, there is no variation in player wealth because he has a utility of zero. If he is on the waiting list, his utility is for sure –P, whichever his decision is. If he is not on the waiting list, his best strategy is to not enter the game, otherwise he would obtain a negative utility.
Given that, in the second round, the same exact reasoning of round III can be done. Since $V = P$ and $M = 1$, the marginal utility to enter the game is always *–P*. It means that does not matter the round, the player and the decision taken by other players in the past. Any player that decide to pay the price in any round obtains for sure a utility equal to *–P* whether he receive the product or not. For this reason, the solution of this case is equal to the previous one. All players decide to not enter the game. This behaviour is the same one of the first player extracted. Then, also in this case, none of the players will decide to pay the price and seller is unable to make profit.

Only the case with a valuation *V* such that $V > P$ should provide a positive solution both for players and the seller. In fact, only with this relationship between *V* and *P*, a player has incentive to enter the game and to pay the price. This case is called A in order to be compared with successive cases further on. It is characterized by *N=3* players who have the opportunity to enter the game in *R=3* rounds. In each round, players have to decide whether enter the game and pay a price *P*, or not to enter and paying nothing. In this game the dispatch coefficient is equal to *1*. It means that a product requires one payment in order to be shipped, excluding the first payment. If two among three players pay the price, only the player that paid the price first obtain the product. An example is useful to explain this mechanism: a player in round I, who has paid the price, wins the product if a player in round II pays the same price. In turn, the player of round II wins the product if a player in round III pays the same price, and so on. Each player has a probability of $^1/_3$ to enter the game and has a utility function: $u(y) = V - yP$.

In order to analyse this case equilibrium, it is necessary the backward induction technique, starting from the last round.

Round III

A player extracted in this round has incentive to enter the game only if:

- he has already paid the price in one of previous two rounds,
- by paying he is able to receive the product.

Otherwise he will not enter the game since he will achieve a negative utility equal to the price. The analysis of next rounds will determine the optimal relations between V and P that lead players to enter the game.

Round II

When a player is extracted in round II, he faces three states of the game:

- he paid the price in round I,
- one of other players paid the price in round I,
- nobody paid the price in round I.

In the first state of the game, it is assumed that he already paid the price in the first round. Then, if he pays the price again in round II, he is able to obtain the product by paying two times the price with a utility of $U = V - 2P$. Otherwise, if he decides to not pay the price in round II, his payoff depends on what will happen in next round and on players behaviour. But, this is already explained above. In fact, it is said the in round III, only a player that entered the game in round I or II will decide to enter the game, otherwise he will not. Then, if the player in round II decide to not pay the price in round II, he expects to gain a utility of $U = -P + {}^{1}/_{3}(V - P)$. The first addend is the result of the payment that this player made in round I. The second one is the probability to be extracted in round III and to pay the price. In order to estimate the optimal relationship between V and P that incentivize this player to enter the game in round II, these two utilities have to be compared. The resulting inequality is $V - 2P > -P + {}^{1}/_{3}(V - P)$. By solving this inequality, the result is $V > P$. Then, for all values of V such that $V > P$, the utility that this player obtains in round II by entering the game is greater than the utility to not enter the game. In this case, all values of $V > P$ are considered, then a player who is on the waiting list and who is extracted in round II will decide to pay the price in this round in any case.

In the second state of the game, a player is extracted in round II but a different one paid the price in round I. If the player extracted in round II decides to pay the price in round II he bears an outflow of P and has a possibility of $^1/_3$ to be extracted again in next round in which he pays the price and obtains the products. Then, the utility of the decision to enter the game in round II is $U = -P + {^1/_3}(V - P)$. In the case he decides to not pay the price in round II, his utility is given by his behaviour in round III, already explained above. So, since he did not pay the price in round I or II, he continues to not enter the game with a final utility equal to zero. In order that this player decides to pay the price in round II, the utility of entering the game has to be greater than the utility of not doing it. This inequality is $-P + {^1/_3}(V - P) > 0$ with a result of $V > 4P$. For all values that satisfy this inequality, a player that is extracted for the first time in round II decides to enter the game.

The third state of the game can be considered equal to the second one. In fact, when a player is extracted the first time in round II, he does not consider what happened in the round I. If in the first round another player pay the price or if nobody pays the price, the player in round II has to consider only his utility starting from this round. Then, the utility to enter the game is $U = -P + {^1/_3}(V - P)$, while the utility to not do it is zero. As the second state of the game, for all value of V such that $V > 4P$, this player enters the game.

In order to summarise the result, two cases have to be considered: $V > 4P$ and $V < 4P$.

If $V > 4P$, the player extracted in round II enter the game in any case because with such value of V, the expected utility of entering the game is always greater than the utility to not enter it.

If $V < 4P$, with this inequality, a player who is extracted for the first time in round II prefers to not enter the game because otherwise he would obtain a lower utility. Only the player who paid the price in round I, enter the game again in the second or third round, in order to receive the good.

Round I

The behaviour of a player in round I is analysed on the basis of the relationship between V and P just found. The first case is the one in which $V > 4P$. A player extracted in round I has to consider the expectations to obtain the product or not. If he pays the price in round I, a player has with certainty an outflow of P. In the second round, there are two options: he is extracted or not. His extraction has a probability of $^1/_3$ and since $V > 4P$, it has been shows that the player enters the game also in the second round. If another player is extracted, this time with probability $^2/_3$, the behaviour is the same. In fact, all players in round II in the case of $V > 4P$ will enter the game. It means that a player who enters the game in round I will obtain with certainty the product. However, with $^1/_3$ probability he pays the price twice, while with the residual $^2/_3$ he pays the price once. Then, the utility of a player that is extracted in round I is:

$$U = -P + {}^{1}/_{3}(V - P) + {}^{2}/_{3}V = {}^{1}/_{3}(3V - 4P).$$

This result has to be compared with the utility that the player expects to gain if he does not pay the price in round I. This utility has been already calculated above when the players behaviour in round II was analysed. In fact, it has been said that with a value of V such that $V > 4P$, whoever is the player extracted, he pays the price with an expected utility of $U = -P + {}^{1}/_{3}(V - P)$. However, this utility occurs only with a utility of ${}^{1}/_{3}$ because when player is in round I he has only ${}^{1}/_{3}$ probability to gain this utility. With the remaining probability, he is unable to achieve the product. So, the complete utility is:

$$U = {}^{1}/_{3}[-P + {}^{1}/_{3}(V - P)] + {}^{2}/_{3}(0) = {}^{1}/_{9}(V - 4P).$$

The two utilities, one resulting from the player decision to enter the game in round I and the other one resulting from the decision to not do it, have to be compared in order to select the best strategy for the player. The first is ${}^{1}/_{3}(3V - 4P)$ while the second is ${}^{1}/_{9}(V - 4P)$. It is clear that the latter is greater than the former and therefore the best strategy for the player is to enter the game in round I, if $V > 4P$.

Different is the case in which $V < 4P$ because players change behaviour in next rounds. If a player is extracted in round I and he decides to enter the game, he pays with certainty the price P. In the second round, this same player has a probability of ${}^{1}/_{3}$ to be extracted again and it has been shows that with $V < 4P$ a player will enter the game only if he paid the price in round I. So, this player obtains the product with valuation V by paying the second price. With the remaining probability of ${}^{2}/_{3}$, the player is not extracted and the other two players will not enter the game because they would obtain a lower payoff if they pay the price in round II. In this case, in the third round, the player selected in round I, has another probability of ${}^{1}/_{3}$ to be extracted and to obtain the product by paying the price. The sum of this expected utility is:

$$U = -P + {}^{1}/_{3}(V - P) + {}^{2}/_{3}[{}^{1}/_{3}(V - P)] = {}^{1}/_{9}(5V - 14P).$$

This result has to be compared with the utility that this player achieves if he does not pay the price in round I. Since, $V < 4P$, if a player does not enter the game in round I, he will not enter the game neither in round II nor in round III. So, the utility of the player who does not enter the game in round I with $V < 4P$ is equal to zero. The utility of the decision to pay the price in round I is greater than the utility of not doing it if ${}^{1}/_{9}(5V - 14P) > 0$. The result of this inequality is $V > {}^{14}/_{5}P$.

The recap of the result of the analysis of the round I is the following:

- if $V > 4P$, all players would pay the price in round I and II;
- if ${}^{14}/_{5}P < V < 4P$, all players would pay the price in round I, but in round II only the player selected in round I will pay;

- if $V < {}^{14}/_{5}P$, all players would not pay the price in round I, and consequently also in the next rounds.

The following table reports the numerical recap just explained.

Table 4 - Case A players expected utility

$^{V}/_{P}$ **ratio**	**Expected utility**
$V > 4P$	$^{1}/_{3}(3V - 4P)$
$^{14}/_{5}P < V < 4P$	$^{1}/_{9}(5V - 14P)$
$V < {}^{14}/_{5}P$	0

2.3 Seller's profit

The construction of previous games is an essential tool for the seller, who has to establish the optimal $^{V}/_{P}$ ratio in order to make profit. On the basis of the game set-up, he has to find the value of P that maximize his profits, considering his costs and players decision in the game.

The three relationships between V and P reported in the last table are used in order to compute the seller's profit and to find the one that maximize it. The first one to be analysed is the case in which $V > 4P$. As illustrated in the last recap, with these inequality, whoever player is extracted in the first or second round will enter the game. This means that the seller obtains with certainty two prices and has to bear the cost to dispatch the product considering round I and II. In the third one, only the two players not selected in round I will pay the price, but only if by doing it they obtain the product. So, the player selected in the first round obtains the product and then is not interested in entering the game in round III. If a second player is selected in round II, he will pay the price also in round III and this situation happens with probability $^{1}/_{9}$. The same can be said for the third player. In these two cases the seller cashes in another price but on the other hand he has to dispatch a second product. Then, the total profit of the seller is:

$$\pi = P + (P - C) + (^{1}/_{9} + {}^{1}/_{9})(P - C) = {}^{1}/_{9}(20P - 11C).$$

This is the result considering the range $V > 4P$. Since the seller aim is to maximize his profit he has to select the value in this range that maximize the result just found. This point is the minimum one in which players behave as explained above and it is the maximum value of the price for the seller. In this case, it is $P^* = {}^1/_4 V$. In fact, the more V is greater than $4P$, the more the seller cashes in smaller prices, decreasing the profit. So, by substituting the optimal price into the profit, the result is $\pi = {}^1/_9(5V - 11C)$. This profit is positive for all values of V such that $V > {}^{11}/_5 C$.

The second range to be analysed is the one in which ${}^{14}/_5\, P < V < 4P$. With these values, it has been shown that a player pays the price with certainty in round I and in one of the other two rounds if he is selected. Players who have not been selected in round I will not enter the game in round II or III. Then the profit and the relative cost of the seller depends only on the possibility that the first player is extracted again. In round I, the seller cash in one price for sure. In the second round, he receives another price only if the same player is selected again in this round, with probability ${}^1/_3$. With the remaining probability of ${}^2/_3$, a different player than the one who paid the price in round I is extracted in round II. This player will not pay the price; however, the first player has the possibility to be selected in round III with probability ${}^1/_3$, in which he will pay the price. Then, the resulting profit is:

$$\pi = P + {}^1/_3(P - C) + {}^2/_3({}^1/_3)(P - C) = {}^1/_9(14P - 5C).$$

This is the profit that the seller expects in the range ${}^{14}/_5\, P < V < 4P$. In order to maximize the profit, the seller has to choose the highest price in this range that is the point in which ${}^{14}/_5 P = V$, then $P^* = {}^5/_{14} V$. With this price, the profit is $\pi = {}^5/_9(V - C)$ and it is positive for all values of V greater than C.

The last case is the one in which $V < {}^{14}/_5 P$. However, for these values, players have no incentive to pay the price because they would obtain a negative payoff. Then, the seller is not able to make profit.

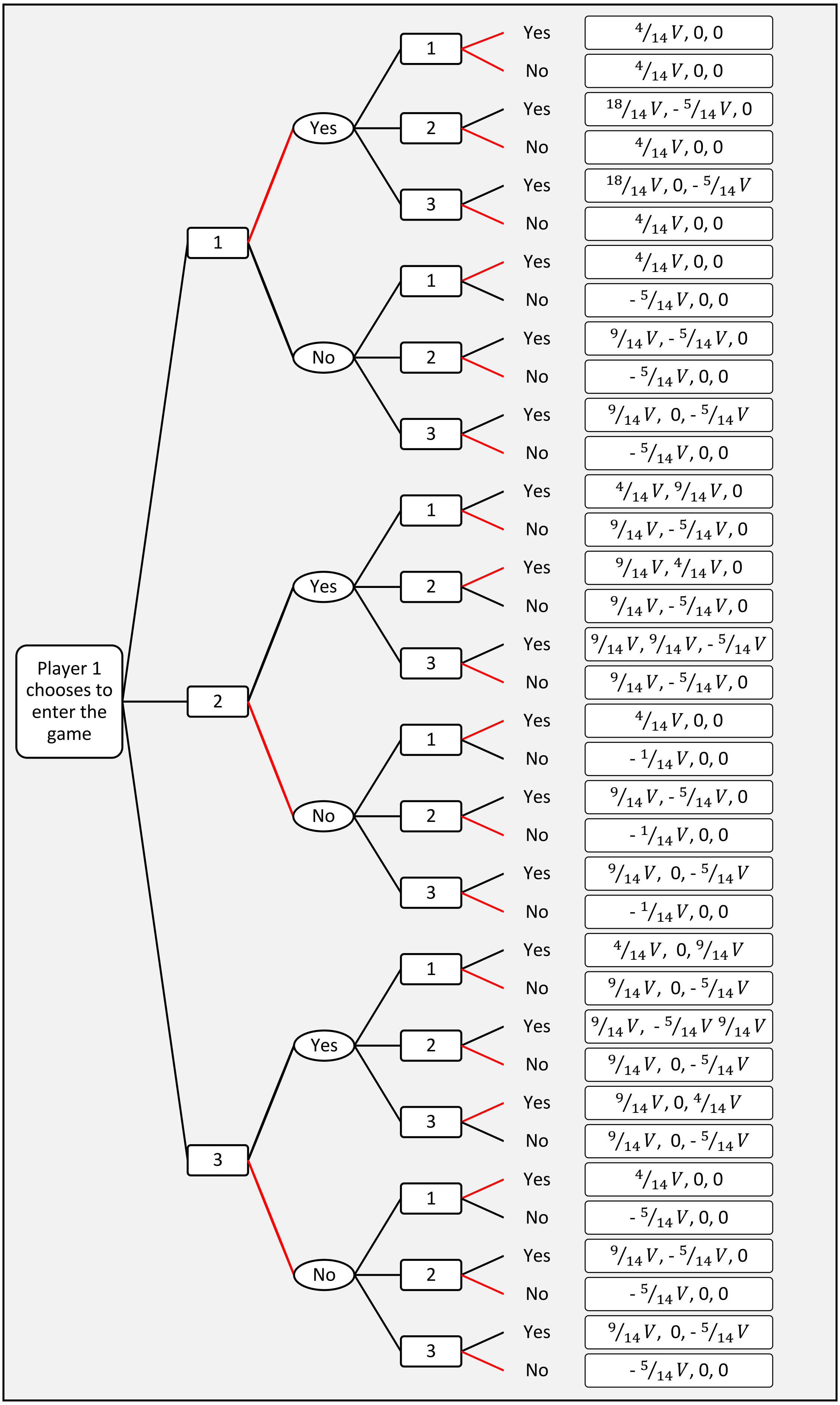
The table below shows the summary of the seller's expected profit. The profit resulting from the behaviour of the players in the range ${}^{14}/_5 P < V < 4P$ is always greater than the other one. Therefore, the seller should implement the highest value of P, i.e. $P^* = {}^5/_{14} V$.

Table 5 - Case A seller's profit

Interval	P^*	**Expected profit**	$\pi > 0$ **if**
$V > 4P$	$P^* = {}^1/_4 V$	${}^1/_9(5V - 11C)$	$V > {}^{11}/_5\, C$
${}^{14}/_5\, < V < 4P$	$P^* = {}^5/_{14} V$	${}^5/_9(V - C)$	$V > C$
$V < {}^{14}/_5 P$	-	0	-

The next figure shows the “yes” scenario, the one in which the first player extracted pays the price. Red lines represent the best choices for the player if he is extracted. For the first player, it is considered the “unit demand” assumptions in fact he is not interest in entering the third round if he has already received the product. For this reason, the payoff in this state of the game are equal. His best decision is to pay the fee in the first and in the second round, as already explained. Conversely, for the other two players, the best decision is to not pay the fee, in any rounds. Otherwise, their expected utility would be negative.

Figure 4 - Representation of the "yes" scenario with $P^* = {}^{5}/_{14}V$



2.4 Cooperation among players

Since the optimal price chosen by the seller implies that only the first extracted player will eventually pay the fee in other rounds, this player bears the risk to pay and to not receive the product if he is not selected again. Then, two of the three players could cooperate in order to achieve a better payoff for both of them:

- the first player could have more probability to receive the product,
- the second player (who would not enter the game otherwise) could achieve a positive payoff.

It seems a positive solution for two of the three players involved. However, it has to be considered that benefits will be split as well as costs. This cooperation can be considered as a binding contract between these two players in which both players are obliged to pay the price if the payment is useful to receive one product. Then, a contract is required to bind the players to behave as reported in the agreement. This is the only procedure that leads the players to follow a predefined behaviour. The contract could specify that these two players will pay the necessary fees in order to obtain the product and then they will split cost (the fees paid) and the benefits (the use of the product). For ease it can be assumed that:

- this type of product or service can be share or split somehow,
- the unit demand concept is already valid so the two players who sign the contract need only one product,
- the contract is proposed only after the first player is extracted so it is signed between round I and II.

Otherwise, players will face a kind of a classical prisoner dilemma in a particular situation. In fact, if one of the two bound players is extracted in two rounds, then obtaining the product, his best option would be to not respect the contract and to not share the product. In this way, he would obtain a higher payoff, because he does not share the product. Then, without a binding contract and if the same player is extracted in two rounds, this player will prefer to ignore the agreement.

The analysis of the cooperation effect will be conducted in two steps:

- firstly, it is established whether the cooperation leads to a higher payoff than the previous game (with the tool of the backward induction),
- second, it is determined the best pricing policy for the seller.

In order to perform the analysis, the employed tool is the backward induction, as the previous case, in order to establish whether the cooperation is an optimal strategy. However, for this one, it has to be considered that two of the three players have a different probability of achieving a positive payoff. This analysis considers only the range that maximized the profit for the seller in the previous case: ${}^{14}/_{5}P < V < 4P$ with the optimal price of $P^* = {}^{5}/_{14}V$.

Round III

In the third round, any player who is on the waiting list will be interested in paying the fee in order to receive the product, as before.

Round II

In the second round, the two type of players has to be analysed separately. The player who is not in the contract will behave as explained above in this chapter. So, he will enter the game only if he is on the waiting list, otherwise he will not because of the range considered. The two players of the contract will behave as the third player if by paying the fee they obtain one product. But, if none of them was extracted in round I, they will enter the game only if $V > {}^{5}/_{2}P$. Since the interval ${}^{14}/_{5}\,P < V < 4P$ is included in $V > {}^{5}/_{2}P$, with $P = {}^{5}/_{14}V$, the two players of the contract will pay the price for sure, whether they are on the waiting list or not.

Round I

In the first round, the player outside the contract will enter the game if selected and the utility is already computed in the second paragraph of this chapter. Conversely, the utility of the two players of the contract has to be determined. If they enter the game in the first round, they have ${}^{2}/_{3}$ probability to be extracted again in the second round, in which they will pay the price. If with ${}^{1}/_{3}$ probability none of them is extracted, the third player will not pay in round II, but they have a further ${}^{2}/_{3}$ probability to be extracted in the third round. They have ${}^{2}/_{3}$ probability because both of them can rely on their own payment as well as on the payment of the other player who signed the contract. Then, these two players have a total utility of:

$$U = -P + {}^{2}/_{3}(V - P) + {}^{1}/_{3}[{}^{2}/_{3}(V - P)] = {}^{1}/_{9}(8V - 17P).$$

However, this utility has to be split into two with a result equal to:

$$U = {}^{1}/_{18}(8V - 17P).$$

This utility has to be compared with the one computed above in the second paragraph, without the binding contract. Then, the solution of the inequality

$${}^{1}/_{18}(8V - 17P) > {}^{1}/_{9}(5V - 14P)$$

is that the first utility is greater than the second one if $V < {}^{11}/_{2}P$. Since the considered interval ${}^{14}/_{5}P < V < 4P$ is a sub interval of $V < {}^{11}/_{2}P$, for all the values of the first range, the cooperation between two players is an optimal strategy. In fact, the players obtain for sure a better payoff with the contract than without it.

Seller's profit

Because of the players cooperation, the seller's profit changes. In fact, the new profit is $\pi = {}^{1}/_{27}(56P - 29C)$. By substituting the optimal price $P^* = {}^{5}/_{14}V$, the profit becomes $\pi = {}^{1}/_{27}(20V - 29C)$. This result has to be compared with the best profit that the seller can achieve without the players cooperation. Then, by comparing it with ${}^{5}/_{9}(V - C)$, there is no one which prevails on the other one. In fact, the profit in the case of cooperation is greater than the other one if $V > {}^{14}/_{5}C$. Then, even if players cooperation, the seller could achieve a greater profit, for some values of *V*.
However, the seller should verify whether there are better pricing strategies in case of players cooperation. In fact, because of symmetric information, the seller know that two players can cooperate after round I and then he can anticipate these players move by changing the price in advance (before round I). In the round II analysis, it has been individuated a critical value, $V = {}^{5}/_{2}P$. This value is necessary to analyse the seller's profit in the case he considers (or anticipates) the presence of the contract. If $V > {}^{5}/_{2}P$ the players (of the contract) will enter the game in round II even if no one pays the price in round I. If $V < {}^{5}/_{2}P$, the players (of the contract) will pay the price in round II or III only if they paid previously in round I and only if $V > {}^{17}/_{8}P$. In both cases, the best decision for the two players is to enter the game gaining an expected utility of ${}^{1}/_{9}(8V - 17P)$, which has to be split into two.
Then, two critical intervals of values emerge:

- $V > {}^{5}/_{2}P$
- ${}^{17}/_{8}P < V < {}^{5}/_{2}P$.

In the first case, the seller sets $P^* = {}^{2}/_{5}V$ and he gains a profit of $\pi = {}^{1}/_{27}({}^{88}/_{5}V - 20C)$. In the second case, the seller sets $P^* = {}^{8}/_{17}V$ and he gains a profit of $\pi = {}^{1}/_{27}(16V - 16C)$. By comparing the two profits, the seller cannot choose definitely one price because the first is greater than the second if $V > {}^{5}/_{2}C$. Then, none of the prices prevails on the other one. These two expected profits have to be compared also with the one computed before, i.e. with the presence of the contract and with $P^* = {}^{5}/_{14}\,V$. At the end of the day, the seller's profit depends on the relationship between *V* and *C*:

- if $C < {}^{12}/_{45}V$, the higher profit is given by $P^* = {}^{5}/_{14}V$,
- if ${}^{12}/_{45}V < C < {}^{2}/_{5}V$, the higher profit is given by $P^* = {}^{2}/_{5}V$,
- if $C > {}^{2}/_{5}V$, the higher profit is given by $P^* = {}^{8}/_{17}V$.

The contract is an optimal strategy for the two players who signs it. However, the presence of the contract does not determine with certainty a lower or higher payoff for the seller because it remains subject to the values of V and C.

A second type of cooperation can be analysed. In fact, when the first player is extracted, he could convince both the two other players to cooperate and not only one. This type of contract is equal to the previous one, but it involves all three players: it obliges the player selected in the second round to pay the fee. Then, players will split benefits and costs. It means that after round II, all players with certainty gain a part of the product, paying a portion of the price. The result is a certain utility equal to:

$$U = {}^{1}/_{3}(V - 2P).$$

Since the price chosen by the seller is $P^* = {}^{5}/_{14}V$, the utility is:

$$U = {}^{1}/_{3}[V - 2({}^{5}/_{14}V)] = {}^{2}/_{21}V.$$

At the end of the day, this utility is the same whether the players sign the contract before or after round I. However, conceptually the contract is signed after round I. This utility has to be compared with the payoff resulting from the contract between two players and the payoff without any contract. The following table summarizes the payoff of the three cases.

Table 6 - Comparison of expected utilities with and without cooperation

Case	**Expected utility function**	**Payoff with $P^* = {}^{5}/_{14}V$**
No cooperation	$U = {}^{1}/_{9}(5V - 14P)$	0
Cooperation between 2 players	$U = {}^{1}/_{18}(8V - 17P)$	${}^{5}/_{28}V$
Cooperation among 3 players	$U = {}^{1}/_{3}(V - 2P)$	${}^{2}/_{21}V$

The result is that:

- a cooperation among the three players is better than to not cooperate,
- a cooperation among exclusively two players is better than the cooperation of all the three players.

It means that a cooperation among all players will not be implemented if two of the three players are already bound by a two-players contract.

The possible extensions of this case are two: on one hand, the possibility for the player excluded by the contract (for ease he is called "the third player"), to pay a bigger part of the fee than the other two players; on the other hand, the sign of 3-players contract with the aim to receive two product (and not only one). In the former, the other two players could be incentivized to include also the third player in the contract because they would pay a lower price, gaining a higher payoff. This seems an optimal strategy for the third player. In fact, since it is assumed that the contract is sign between round I and II, with the optimal price without cooperation ($P^* = {}^{5}/_{14}V$), he would not enter the game because someone else paid in round I. However, he can propose this type of contract in order to convince the other two players to sign a 3-players contract. In this way, the third player can achieve a positive payoff and the other two players are incentivized to sign the 3-players contract because they gain a better payoff than the 2-players contract one. So, a 3-players contract with a transfer from one player to the other two is an optimal strategy for each player. The seller's profit, in this case, is given with certainty by the difference of the two fees he cashes in and the cost of one product. Then, he should ask the highest possible price. This last one coincides with the highest price that the third player is able to pay, considering the transfers to the other two players. In fact, it is sufficient that the third player is incentivized to enter the game by cooperating in order to create the precondition for (and also to make mandatory) the 3-players contract. Then the optimal price is the one which make zero (or slightly positive) the utility function of the third player, after the two transfers.

The cooperation among players is feasible if it is achievable with a really low cost or better costly. This is a strong simplifying assumption because whether cooperation requires some costs for the players, their best strategy depends on the extent of these cost. Then, when players decide to cooperate they should account for these cost as well as the seller. If the latter anticipates the cooperation among players, he has to pay particular attention on the choice of the optimal price. In fact, he has to consider all the possible costs for the players. If he overestimates these cost, he could set a lower price in order to encourage player to enter the game cooperating and:

- he "lose" a part of the payoff because he could have set a higher price,
- players do not cooperate, modifying the expected profit.

If he underestimates these cost, he could set a higher price and this action could discourage player to enter the game with cooperation or not, gaining no profit.

In the second extensions, the three players sign a contract (after round I) in order to receive two products, meaning that they will pay in both round II and III. With this strategy, the utility of all the three players is:

$$U = {}^{1}/_{3}(2V - 3P).$$

Since it is assumed that the contract is signed after round I, the seller has already set the optimal price equal to $P^* = {}^{5}/_{14}V$ which is the best price in the case of no cooperation. With this price the utility is:

$$U = {}^{1}/_{3}[2V - 3({}^{5}/_{14}V)] = {}^{13}/_{42}V.$$

This value is greater than both the 2-players contract and the 3-players contract (the first one analysed). Then, after round I, the first player should propose this type of contract to the other two players. However, in this case the seller has to possibility to retaliate. In fact, he can set the price equal to the maximum amount which convince the 3 players to behave in this way. This price is the result of the following inequality:

$${}^{1}/_{3}(2V - 3P) > 0.$$

The result is $P^* = {}^{2}/_{3}V$. With this price, the payoff of the three players is equal to zero. Then, for the first player extracted is better to choose a 2-players contract cooperating with one of the other two players. In this way, he has the possibility to achieve a positive payoff, which is better than gain for sure a zero payoff. Then, the only contract among all players that could work is the one which implies a transfer from one player to the other two ones.

CHAPTER 3
ADDING ONE MORE ROUND

3.1 Equilibrium analysis

In this chapter, the effect which an introduction of a new round has on the game is examined. It is considered only the case in which $V > P$. In fact, it has been explained in chapter 2 that for all values of V such that $V \leq P$, all players decide to not pay the price. This strategy is always the best one for all players, then the seller is unable to make any profit. The only case that provide a positive outcome for both players and seller is the one in which V P. This game has the same characteristics of the case A, except for the increased number of rounds. Then: $N=3$, $R=4$, $M=1$, "unit demand" concept and the utility function equal to $u(y) = V - yP$.
In order to analyse this case, the backward induction technique is needed, as before. Then, it is necessary to analyse this case form the last round, that now is round IV.

Round IV

The analysis of this round is not different by the one made for the last round in the case A. When a player is extracted in the last round, the only relevant aspect is whether he is on the waiting list. To be on the list means that a player paid the price once and he have not receive the product yet. If he is on the waiting list and he is extracted, his decision is to pay the price in order to get the product. Otherwise, so if he is not on the waiting list, he has no incentive to pay the price because of two different reasons:

- he has already been satisfied by receiving the product in a previous period
- or he has never paid the price and then he will not do it in the last round for sure.

Round III

In the round III, a player has to analyse not only his current situations but also the possible future situations which could happen. A player extracted in this round faces different states of the game:

- he already received the product,
- he is on the waiting list,
- he is not on the waiting list.

The first state of the game implies that this player paid the price in round I and someone else in round II. The first player received the product thanks to the second payment and he is no more interesting in entering the game. This is the concept of "unit demand" introduced in chapter 2. The meaning is that when a player obtains a product he has no interest anymore in participating the game, even if this could result in a second positive payoff for him (i.e. he gets another product). Therefore, his best strategy is to not pay the price. The utility gained by the first player is equal to $U = V - P$ that is positive since $V > P$.

In the second state of the game, the player extracted in round III is the same that paid the price in round II, but not in round I. In fact, if the same player paid the price in both round I and II, he has no interest in entering the game again in the next round. Then, if this kind of player is extracted in round III and he decides to pay the price, he has a payoff equal to $U = V - 2P$ because he pays the price twice and he obtains the product. If his decision is to not do it, his utility depends on the probability to be extracted again or not in round IV. In fact, it has been explained that only a player that is on the waiting list would pay the price in the last round and this circumstance happens with probability ${}^{1}/_{3}$. Then, the utility of the player is

$$U = -P + {}^{1}/_{3}(V - P) = {}^{1}/_{3}(V - 4P).$$

The best strategy for the player is to pay the price if selected in round III, because he would obtain a greater utility. In fact: $V - 2P$ is greater than ${}^{1}/_{3}(V - 4P)$ for all values of V such that $V > P$, that is the case in analysis.

The third state of the game implies that this player is extracted for the first time in round III. There are two histories that lead to this state: the same player paid in round I and II receiving the product or none of the players entered in the first two rounds. These two situations are equal for a new player extracted in round III. In fact, if a player is extracted for the first time in round III, the only aspect that matter is that by paying in the third and in the fourth round he has some probability to receive the product. Then, if the player extracted in round III is not on the waiting list, he expects to gain a utility of ${}^{1}/_{3}(V - 4P)$ if he pays the price. Otherwise, if he does not pay the price in round III, he gains a zero utility. In fact, the player who is not in the waiting list in round IV does not enter the game. The utility of the decision to pay the price in round III is greater than the one to not do it if ${}^{1}/_{3}(V - 4P) > 0$ and the result is $V > 4P$. For all these values, the player who is extracted in round III and who is not on waiting list is incentivized to pay the price because he obtains a greater payoff. This analysis shows that:

- if $V > 4P$, the best strategy for all players is to enter the game whether they are on the waiting list or not,
- if $V < 4P$, only a player who is on the waiting list would pay, otherwise he would not.

These two ranges of values are necessary to analyse the players behaviour in the second round.

Round II

Considering first $V > 4P$, a player extracted in second round behaves differently if he is on the waiting list or not. If he is on the waiting list and he decides to pay the price he obtains with certainty a payoff equal to $U = V - 2P$. In fact, by paying the second time the price, he obtains the product. If he decides to not pay the price in this round the utility depends on next rounds extractions. From the explanation of round III, the final utility can be predicted. In fact, if he is extracted in the third round, since he is on the waiting list, he will pay with certainty. Moreover, also players who are not on the waiting list will pay the price in round III if they are extracted. This is a peculiar circumstance that reveals only in this case and that differentiate it from the case with 3 rounds. Then, the utility of the player is composed by three factors:

- the price paid in previous round, P,
- the fact that he is extracted in round III and pays the price, as explained in round III analysis,
- the fact that he is not extracted in round III and someone else pays the price.

The last point derives from the fact that in round III, whoever is the player extracted, he will pay the price because "to pay the price" is an optimal strategy for all players in this specific round. The utility is:

$$U = -P + {}^{1}/_{3}(V - P) + {}^{2}/_{3}(V) = V - {}^{4}/_{3}P.$$

By comparing the utility that he obtains by paying the price in round II to the one obtained by not paying the price, the best strategy for a player extracted in round II, who is on the waiting list, is to not pay the price. In fact, $V - {}^{4}/_{3}P > V - 2P$. This result is quite interesting. The explanation is that a player extracted in round II knows that if another player is extracted in round III, he will pay the price. Then, player extracted in round II obtains one product by paying only one price. This is due because of:

- the high difference between the valuation and the price that he should pay,
- the fact that players are extracted in a round "far" from the last one.

For this reason, it is more convenient to rely on the payment of another player in round III, than paying the second price to receive the product.

The second range is such that $V < 4P$. In this case, if the selected player is on the waiting list and he decides to pay the price, he gains the same payoff than the one in the range $V > 4P$: $U = V - 2P$. If the player decides to not pay the price in this round he bears the risk to not receive the product. In fact, since $V < 4P$, it has been explained that in round III only a player who is on the waiting list will decide to pay the price. It means this player can obtain the good only if he personally pays the price twice, because none of other players will pay. Then, the utility is:

$$U = -P + {}^{1}/_{3}(V - P) + {}^{2}/_{3}[{}^{1}/_{3}(V - P)] = {}^{1}/_{9}(5V - 14P).$$

The first addend is the price paid in round I; the second one is the probability to be extracted in round III in which he pays the price; the last one is the probability to be extracted in round IV and not in round III, in which he pays the price. This utility has to be compared with the one resulting from his decision to pay the price in round II. In this case this last one is greater. Then if a player on the waiting list is extracted in round II, he will decide to pay the price, receiving the product.

A different case is the one in which a player is extracted in round II but he is not on the waiting list, always considering $V < 4P$. If a player decides to pay the price in round II, he gains the same utility as the player who is on the waiting list but does not pay the price in round II. In fact, the utility to enter the game in round II is:

$$U = -P + {}^{1}/_{3}(V - P) + {}^{2}/_{3}[{}^{1}/_{3}(V - P)] = {}^{1}/_{9}(5V - 14P).$$

If the player, who is not on the waiting list, decide to not enter the game neither in round II, his utility will be zero. In fact, a player, who is not on the waiting list, will enter the game in round III only if $V > 4P$, but in this case the opposite is considered, i.e. $V < 4P$. In order that this type of player chooses to enter the game in round II, the respective utility should be greater than zero, ${}^{1}/_{9}(5V - 14P) > 0$. This inequality is satisfied for all values of V such that $V > {}^{14}/_{5}\,P$.

The analysis of the round II shows multiple results. In fact:

- if $V > 4P$, a player decides to pay the price if he is not on waiting list, otherwise he choose to not pay;
- if ${}^{14}/_{5}\,P < V < 4P$, a player decides to pay the price in any case;
- if $V < {}^{14}/_{5}\,P$, a player decides to enter the game if he is already on the waiting list, otherwise he does not.

These three ranges of values will be used in order to analyse the behaviour of a player extracted in round I.

Round I

When a player is extracted in the first round he has to compute his utility considering the behaviour of other players in the next rounds. All these behaviours have been explained so far. Then, for each ranges of values, the first player extracted knows how players in the next round will reacts to the decision he takes in the first round.

The first range considered is $V > 4P$. If the player chooses to pay the price in the first round has a certain cash out equal to P. In the second round, he faces two possibilities. The first occurs with probability ${}^{2}/_{3}$ and it is in the case in which a different player is extracted. This player will pay the prices for sure and because of this, the player extracted in round I obtains the product with valuation V by paying only one P. With the remaining probability of $^1/_3$, the same player is extracted again in round II. In this case, this player will not pay the price because he gains an expected higher utility. Then, with probability $^1/_3$, it has to be considered the behaviour of players in round III. Whoever player extracted in round III will take the decision to pay the price. With a probability of $^1/_3$ is the same player extracted in round I, so he pays the price for the second time and he receives the product. With a probability of $^2/_3$, another player is selected. This player pays the price and allows the first player to get the product. The following utility function are the result of all these situations:

$$U = -P + {}^1/_3[{}^1/_3(V - P) + {}^2/_3(V - P)] + +{}^2/_3(V) = V - {}^{10}/_9P.$$

On the other hand, if a player does not pay the price in round I, he bears the risk to be selected in the following rounds or not. With $^1/_3$ probability he is selected in round II and since he is not on the waiting list, his best strategy is to pay the price. Then, with another probability $^1/_3$, he is extracted in round III, in which he will pay the price the second time in order to receive the product. But, with a probability of $^2/_3$, a different player is extracted, who will pay the price. In this way, the first player receives the product by paying only one price. In round II, he has a $^2/_3$ probability to not be extracted. In this circumstance, he remains only the last two rounds to enter the game. If he is extracted in round III, his best strategy is to pay the price. Then, he will pay the price also in round IV if he is extracted, with probability $^1/_3$. With the residual probability, he is not extracted and he receives nothing even if he paid one price. The utility function is:

$$U = {}^1/_3[-P + {}^1/_3(V - P) + {}^2/_3(V)] + {}^2/_3\{{}^1/_3[-P + {}^1/_3(V - P)]\} = {}^1/_{27}(11V - 20P).$$

Then, the two results that a player expects in round I whether he enters the game or not are: $V - {}^{10}/_9\, P$ if he pays the price in round I and $^1/_{27}(11V - 20P)$ if he does not. The first utility is greater than the second one for all values of V such that $V > {}^{10}/_{16}P$. Since, in this case, the range $V > 4P$ is considered, the valuation V will be in any case higher than $^{10}/_{16}P$. Then, the decision to pay the price for a player extracted in round I is always his optimal strategy.

The second relevant range is $^{14}/_5P < V < 4P$. For these values of V, players in next rounds act differently than the case in which $V > 4P$. Then, the expected utilities are different. If a player decides to pay the price in round I has an immediate outflow of P. Then, in the second round, whoever player is extracted, he will enter the game, if he is in the waiting list or not, as described above in the analysis of this round. With $^1/_3$ probability, this player is the same of the round I and he will pay the second time the price to obtain the product. With the residual probability

($^2/_3$), a different player is extracted and also this second player will pay the price, allowing the first player to receive the product. Therefore, the utility of a player selected in round I is:

$$U = -P + {}^1/_3(V - P) + {}^2/_3(V) = V - {}^4/_3P.$$

If a player decides to not enter the game in the first round he has $^1/_3$ probability to be extracted in the next round and a probability of $^2/_3$ to not be selected. In the first case, he will pay the price in round II. Successively, he could be selected also in round III, in which he enters the game by paying the second time the price and receiving the product. If he is not selected in round III, with a probability of $^2/_3$, he has the last possibility to pay the price in round IV, in which he will pay the second price to gain the product. In the second case, another player is selected in round II and with a $^1/_3$ probability the first player is selected in round III. But in this round, his best strategy is to not enter the game since he is not on the waiting list and $V < 4P$. In fact, the values of V are not high enough to permit the player to achieve a positive expected utility by paying the price in round III. Then, he gains a zero payoff. The utility of this case is:

$$U = {}^1/_3\{-P + {}^1/_3(V - P) + {}^2/_3[{}^1/_3(V - P)]\} + {}^2/_3\{{}^1/_3(0) + {}^2/_3[{}^1/_3(0)]\} = {}^1/_{27}(5V - 14P).$$

The utility to enter the game in round I is greater than the utility to not enter for all values of V such that $V > {}^{14}/_5P$. So, the decision to pay the price in round I is the best strategy for a player selected in round I if ${}^{14}/_5P < V < 4P$.

The last range is such that $V < {}^{14}/_5P$. If a player extracted in round I pays the price he has:

- $^1/_3$ probability to be extracted in round II, in which he pays the price again and he receives the product,
- $^2/_3$ probability that another player is extracted in round, who does not pay the price.

In this last case, he has to consider the third round. In this one, he has a probability of $^1/_3$ to receive the product because he is selected and he pays the price, being on the waiting list. In the residual probability, another player is extracted and he does not enter the game. Then, in the round IV, the player extracted in the first round has the last possibility to pay the price and to gain the product, with probability $^1/_3$. The expected utility of the first player is:

$$U = -P + {}^1/_3(V - P) + {}^2/_3\{{}^1/_3(V - P) + {}^2/_3[{}^1/_3(V - P)]\} = {}^1/_{27}(17V - 44P).$$

This result has to be compared with the utility that the player expects if he does not enter the game in the first round. However, since the range $V < {}^{14}/_5P$ is considered, none of the player extracted from round II to IV will decide to enter the game because they are not on the waiting list. Therefore, the utility to not pay the price in round II is zero. A player who is extracted in round I decide to enter the game if: ${}^1/_{27}(17V - 44P) > 0$. The solution is the range of values such that $V > {}^{44}/_{17}P$. The point in which $V = {}^{44}/_{17}P$ is the minimum level that a player requires in order to enter the game in the first round.

This analysis shows that four ranges of values are relevant:

- if $V > 4P$, all players will pay the price in any round if they are on the waiting list, except for the first selected player who does not pay if extracted in round II;
- if ${}^{14}/_{5}P < V < 4P$, all players will pay the price in any rounds in any case, except for a player extracted in round III who is not on the waiting list;
- ${}^{44}/_{17}P < V < {}^{14}/_{5}P$, all players will pay the price in any rounds if they are on the waiting,
- if $V < {}^{44}/_{17}P$, none of the players is interested in entering the game.

For each range of values of the first point, the player expects different utilities because players behaviour modifies in the subsequent rounds. For the last range of values, all players will achieve a utility equal to zero. In fact, none of them is incentivized to pay the price because costs are higher than benefits. The following table shows the best outcome that a player selected in the first round can achieve, on the basis of the ${}^{V}/_{P}$ ratio.

Table 7 - Case B players expected utility

${}^{V}/_{P}$ **ratio**	**Expected utility**
$V > 4P$	$V - {}^{10}/_{9}P$
${}^{14}/_{5}P < V < 4P$	$V - {}^{4}/_{3}P$
${}^{44}/_{17}P < V < {}^{14}/_{5}P$	${}^{1}/_{27}(17V - 44P)$
$V < {}^{44}/_{17}P$	0

3.2 Seller's profit

For each of the four ranges of values illustrated above, the seller achieves different level of profits. As consequence, he should establish the value of P on the basis of the maximum expected profit he can reap.

In the case of the first range, $V > 4P$, the seller cashes in for sure a price in the first round. In the second round, the revenues depend on who is extracted. The breakdown can be divided on the basis of two potential situations: in round II is extracted the same player of round I or not. In the first case, he will not pay the price and the seller has no revenues. Then, in round III, whoever player is selected, he will pay the price and the seller cashes in a price, but he has to bear the cost of the dispatch of one product. At the end in round IV, the seller has the possibility to earn another price and to dispatch another product if the two players who have not been extracted in round I are extracted both in round III and IV. In the second case, in round II a different player than the first one is extracted, who pay the price and then the seller has to dispatch one product. In the following rounds, the first player will not participate anymore because he already received the product. Then, in third round only the two players who have not been extracted in round I will participate for sure, independently they are on the waiting list or not. Conversely, in the last round, the second or the third player will pay the price only if this price let them obtains the product. Therefore, the seller's payoff is:

$$\pi = {}^{1}/_{3}\{{}^{1}/_{3}(P-C) + {}^{2}/_{3}[{}^{1}/_{3}(2P-2C)\} + {}^{2}/_{3}\left[\left({}^{1}/_{3}\cdot{}^{1}/_{3} + {}^{1}/_{3}\cdot 1\right)(2P-2C) + {}^{1}/_{3}\left({}^{1}/_{3}\right)(3P-3C)\right] = {}^{1}/_{27}(56P-29C).$$

Considering that this level of profit is achievable for all values of V such that $V > 4P$, the seller has to choose the maximum value of P. This value lies in the point in which $P^* = {}^{1}/_{4}V$. By substituting the profit becomes $\pi = {}^{1}/_{27}(14V - 29C)$. This profit is positive if $V > {}^{29}/_{14}C$.

In the second case, V is such that ${}^{14}/_{5}P < V < 4P$. As the previous case, in the first round, any extracted player will pay the price and the same situation is replicated in round II. However, seller's payoff depends on who is extracted in the second round. In fact, if the same player is extracted in both first and second round, in the next rounds none of the players will pay the price. Then, the seller gains two prices and he has to dispatch one product. If the players extracted in round I and II are different, the player extracted in the second round will pay the price also the first time is extracted in the last two rounds. In this event, the seller gains a total of three prices and he has to dispatch two products. The total expected profit is:

$$\pi = P + {}^{1}/_{3}(P-C) + {}^{2}/_{3}[{}^{1}/_{3}({}^{1}/_{3}) + {}^{1}/_{3}(1) + {}^{1}/_{3}({}^{1}/_{3})](2P-2C) = {}^{1}/_{27}(52P-25C).$$

The best price that the seller can choose in order to maximize his profits is the lower bound of the range considered: then it is $P^* = {}^{5}/_{14}V$. By substituting the resulting profit is: $\pi = {}^{1}/_{27}({}^{130}/_{7}V - 25C)$ which is positive if $V > {}^{35}/_{26}C$.

The third range to analyse is ${}^{44}/_{17}P < V < {}^{14}/_{5}P$. In this case, only the first player extracted will pay the price in round I and in the first following round in which he is extracted. This round is:

- round II with ${}^{1}/_{3}$ probability
- round III with ${}^{2}/_{9}$ with probability
- round IV with ${}^{4}/_{27}$ probability.

The seller's expected profit is:

$$\pi = P + {}^{1}/_{3}(P-C) + {}^{2}/_{3}({}^{1}/_{3}\cdot 1 + {}^{2}/_{3}\cdot{}^{1}/_{3})(2P-2C) = {}^{1}/_{27}(46P-19C).$$

Considering the range ${}^{44}/_{17}P < V < {}^{14}/_{5}P$, the seller can maximize the profit if $P^* = {}^{17}/_{44}V$. Then, the profit become $\pi = {}^{1}/_{27}({}^{391}/_{22}V - 19C)$ that is positive for all values of V such that $V > {}^{418}/_{391}C$.

In the last range of values $V < {}^{44}/_{17}P$, none of the players will pay the price in any rounds and the seller is unable to make profits.

The table below illustrates the expected profit for each range of values, given the optimal price. The expected profit deriving from the first range is lower than the other two for any value of V. In the case of the second and the third range there is no an absolute optimal price. In fact, if $C > {}^{41}/_{308}V$, the seller should implement a price equal to $P^* = {}^{17}/_{44}V$ (third range), otherwise he should implement a price equal to $P^* = {}^{5}/_{14}V$.

Table 8 - Case B seller's profit

Interval	P^*	Expected profit	$\pi > 0$ if
$\boldsymbol{V > 4P}$	$P^* = {}^{1}/_{4}V$	${}^{1}/_{27}(14V - 29C)$	$V > {}^{29}/_{14}C$
$\boldsymbol{{}^{14}/_{5}P < V < 4P}$	$P^* = {}^{5}/_{14}V$	${}^{1}/_{189}(130V - 175C)$	$V > {}^{35}/_{26}C$
$\boldsymbol{{}^{44}/_{17}P < V < {}^{14}/_{5}P}$	$P^* = {}^{17}/_{44}V$	${}^{1}/_{594}(391V - 418C)$	$V > {}^{418}/_{391}C$
$\boldsymbol{V < {}^{44}/_{17}P}$	-	0	-

3.3 Comparison with case A

The comparison between case A and B is useful for two reasons:

- to establish the effect of a change of the parameter R in the players expected utility,
- to permit the seller to determine which case has the best characteristics in order to maximize his profit.

With respect to the first point, case A and B shows some similarities and some differences. The most important similarity is the behaviours of players in the last two rounds. In fact, players will react equally in these rounds for the same value of V, whether the game has 3 or 4 rounds. On the other hand, differences appear by comparing the first round in case A with the first two ones in case B. The first is straightforward in fact an increase in the parameter R lead to an increase of the expected utility for the first player extracted. This is a quite obvious conclusion because by continuously increasing the parameter R, players have more and more possibility to be extracted, to pay and receive the product. Moreover, case B analysis shows an interesting circumstance. Indeed, the most important difference is the behaviour of players in the second round. It is not possible to make a comparison considering each round of case A and comparing it with the respective one of case B because of the different value of R. However, it is possible to make two groups of rounds. The first group comprehends the last two rounds of both game. In these rounds the player behaviours are identical, as said above. The second group considers the round I of case A and the rounds I and II of case B. In this group there is an important difference in a specific player behaviour. A player that pays the price in round I will choose to not enter the game in round II because, for a sufficient high value of V (in the case $V > 4P$), his best choice is to rely on another player payment in the next round than to pay himself the second amount needed to ship the good. Then, if the first player is extracted again in round II, his best strategy is to wait and not enter in this round because he expects that if a different player is extracted in round III, this last one will pay the price allowing the first player to receive the good. If this event did not happen, the first player will pay the price only in round III. This situation reveals exclusively in round II on a total of four rounds. But, the more the parameter R increases, the more rounds would show an equal circumstance. This is a crucial statement that changes an important aspect of the game. In the case A, the best response of the first player is always to pay the price in order to receive the good. Conversely, this strategy is not always the best one in case B. Generally, if the first player extracted is selected again in a round sufficient far from the last one, this player prefers to wait that a different one pays the price in the next round. Then, with a high value of R and with a sufficient high level of V, this scenario will be repeated in a higher number of rounds. This situation makes more realistic the game because this is what the reality shows. In fact, in the real world, empirics shows that a customer has no incentive to enter the game in the immediate rounds next the one which he paid in. Indeed, he prefers to wait and to rely on another player payment in order to receive the good, gaining the maximum potential utility.

The second point is related to the seller. In fact, the comparison between case A and B is a tool for the seller in order to determine the level of P that maximize his profit. The optimal P value is the one that:

- incentivizes the player to pay the price,
- permits the seller to make profit.

The table below shows a recap of all the relevant optimal prices and expected profits computed for case A and case B. In fact, only these three listed expected profits have to be compared in order to determine the best one for the seller. However, in previous paragraph, it has been shown that case B provide two optimal prices on the basis of the ratio between *V* and *C*. By comparing the profit of case A with the ones of case B the results are:

- case A is better than the first profit of case B if $C > {}^{61}/_{88}V$,
- case A is better than the second profit of case B if $C < {}^{5}/_{14}V$.

Table 9 - Comparison of case A and B expected profit

	Interval	P^*	Expected profit	$\pi > 0$ if
Case A	${}^{14}/_{5}\,P < V < 4P$	$P^* = {}^{5}/_{14}V$	${}^{5}/_{9}(V - C)$	$V > C$
Case B	${}^{14}/_{5}\,P < V < 4P$	$P^* = {}^{5}/_{14}V$	${}^{1}/_{27}({}^{130}/_{7}V - 25C)$	$V > {}^{35}/_{26}\,C$
	${}^{44}/_{17}\,P < V < {}^{14}/_{5}\,P$	$P^* = {}^{17}/_{44}V$	${}^{1}/_{27}({}^{391}/_{22}V - 19C)$	$V > {}^{418}/_{391}\,C$

These two results have to be added to the comparison between the two ranges of case B. Therefore, the complete analysis is:

- if $C < {}^{41}/_{308}V$, the best price is $P^* = {}^{5}/_{14}V$ in case B,
- if ${}^{41}/_{308}V < C < {}^{61}/_{88}V$, the best price is $P^* = {}^{17}/_{44}V$ in case B,
- if $C > {}^{61}/_{88}V$, the best price is $P^* = {}^{5}/_{14}V$ in case A.

CHAPTER 4
THE RELEVANCE OF THE DISPATCH COEFFICIENT

4.1 Equilibrium analysis

The game dynamics has to be analysed also considering the weight that a change of the dispatch coefficient has on the game equilibrium. In this chapter, two cases are considered. The first one (case C) has the same number of rounds and players of the case B but with $M=2$, the second one (case D) has a further round more. Then, these two cases will be compared firstly between each other and secondly with the best cases found so far.

In the analysis of the extensions of the simple case A, variable N will be not considered. In fact, it is straightforward that the more the number of players increase, the more these last one will require a low value of P in order to enter the game. The cause is that the more players has the possibility to participate the game, the more will decrease the probability to be selected again and receive the product, if there is a low number of rounds. So, an increase of the parameter N should be counterbalance by an increase of the parameter R in order to keep quite constant the expected utility to receive one product. However, ceteris paribus, an increase of players will discourage them to enter the game. For this reason, it is more important to concentrate the extensions analysis on parameter R and M.

The two cases which are analysed in the next paragraph consider only a value of V such that $V > P$. In fact, it has been shown in chapter 2 that for all values P greater or equal than V, players has no incentive to participate the game because they would obtain a negative expected payoff.

A value of the dispatch coefficient equal to 2 means that a player is able to receive the product only if in the four rounds there two further payments after his own one and all players that paid before him have already receive the product. This aspect is crucial because in the analysis of this case it does not matter only the fact that a player is or is not on the waiting list, but it matters also that he is the first on the waiting list who has not yet been satisfied with the product dispatch. The backward induction technique is the tool to analyse the game equilibrium, as for previous cases.

Round IV

The first round to be analysed is the fourth one. In this one, a player has incentive to enter the game and pay the price only if by doing it, he can receive the product. It is not sufficient anymore that the last selected player is on the waiting list. The reason is that by considering only that fact that he is on the waiting list, he could pay the price that will be used in order to dispatch a product for another player, then obtaining a negative utility.

Round III

In the round III, two states of the game could show up: the first is the one in which a player can receive the product by paying the price, the second is the opposite so the payment of this round is not sufficient to obtain the product. The former implies two similar circumstances for the selected player:

- he paid in round I and II,
- he paid in round I and someone else in round II.

In both these circumstances, the player will pay the price with certainty. In fact, he gains a positive payoff ($U = V - 3P$ and $U = V - 2P$ respectively) that is greater with respect to the one that he would obtain by not paying the price in this round. The second circumstance provides a better payoff than the first one but in order to analysis the feasibility of this circumstance round I and II have to be considered. However, the key point is that in this first state of the game a player will pay the price absolutely. In the second state of the game, a player is not able to obtain the product by paying in this round. This situation implies three different circumstances for the selected player:

- he paid the price in round II but someone else in round I,
- he paid in round I and no one in round II (or vice versa),
- he never paid.

In the first one, he has no incentive to pay the price. If he pays the price has a utility of *–2P* because he pays the price twice, but he has no possibility to receive the product because he would need another round, after the fourth one. Then, the best strategy is to not pay the price with a utility of *–P* resulting from the fact that he paid in round II. In the second circumstance, he paid once in round I or II (and no one in the remaining) so by paying the price in round III, he has $^1/_3$ probability to be extracted and to obtain the price by paying again the price. If he does not pay the price in this round, he will do the same in the next one. The utility of paying the price is $U = {}^1/_3(V - 7P)$ and the one of not paying is *–P*. Then, a player in this situation will be pay the price only if $V > 4P$. However, by substituting the minimum value $V = 4P$ in the first utility, the player will achieve a negative payoff. It means that, differently from what it has been shown in case A or B, the first utility has not to be only greater than the second one, but it has to be also greater than zero, otherwise even a risk neutral player will not participate. Then, the utility of paying the price is greater than zero if $V > 7P$. The meaning is that:

- if $V > 4P$, the player gains a better utility by paying, but it could be negative,
- if $V > 7P$, the player gains a better and non-negative utility by paying.

These last ranges of values are considered in order to examine the next round.

In the third circumstance, the selected has never paid the price. In this case, it does not matter whether a different player paid the price in round I, II, or whether none of the players paid the price, because this player has no incentive to pay the price. In fact, in order to receive the product, he would need at least a further round after the fourth round. So, the best strategy is to not pay the price. The analysis of all these three circumstance shows that in round II has to be considered two ranges of values: $V > 7P$ and $V < 7P$.

Round II

In the second round, if $V > 7P$, there are three possible states of the game:

- the extracted player paid the price in round I,
- he did not,
- or someone else did it.

If in the first one he pays the price, he has $5/9$ probability to pay the price in round III or IV, with a utility of $U = 1/9(5V - 23P)$. If he does not pay the price in this round, the only possibility to receive the product is to be selected in both game. The resulting utility is $U = 1/9(V - 11P)$. The first utility is always better than the second and also positive, then the player should pay.

In the second state of the game none of player paid in round I. The utility to pay the price is equal to the last found, in which the player does not pay the price, but he did pay in round I. If the player does not pay in second round, his utility will be zero because he has no possibility to receive the product then he will never pay. The "yes" decision is better than the "no" one if $1/9(V - 11P) > 0$, that is $V < 11P$. For all these values of *V*, the player will pay and will obtain a positive expected payoff.

In the third state of the game, a player is selected in round II but a different one paid in round I. For this reason, the best strategy of the one selected in this round II is to not pay the price, because there are no enough rounds to satisfy also a second player.

The second round has to be analysed also considering all values such that $V < 7P$. The three states of the game remain the same.

In the first one, a player is extracted in this round and he also paid in round I. The utility to enter the game remains the same, $U = {}^1/_9(5V - 23P)$. While, the utility to not do it changes. In fact, considering that $V < 7P$, if this player does not pay the price in this round, his best strategy is to not pay the price in next rounds because he would obtain a negative expected utility. Therefore, the utility to pay the price is greater to not do it if $V > {}^{14}/_5P$. However, for these values, a player risk to obtain a negative utility. In fact, the utility to pay the price has to be greater than both –P and zero in order to be a best strategy. Only with a value of V such that $V > {}^{23}/_5P$, the utility to pay the price is greater than zero.

In the second state of the game, the one in which no one paid the price in round I, the best strategy for a selected player is to not pay the price. In fact, since $V < 7P$, he would obtain a negative expected utility by entering the game in round II. The solution of the third state of the game is the same whether $V < 7P$ or $V > 7P$, i.e. the selected player does not enter the game. Round II analysis shows four relevant ranges of values: $V > 11P$, $7P < V < 11P$, ${}^{23}/_5P < V < 7P$ and $V < {}^{23}/_5P$. In order to examine the players' behaviour in the first round exactly these ranges will be crucial alongside the fact that only the first player extracted will enter the game, because of rounds constraint and as shown in round II.

Round I

In the first round, the utility to enter the game is greater than the one to not do it for all values of V such that $V > 11P$ and $7P < V < 11P$. In fact, for all these values, when the same player is selected again in next rounds, he will always pay the price in order to receive the product. In the case of the third range, ${}^{23}/_5P < V < 7P$, the player will not always pay the price. In fact, if he is extracted in round I and II, he will pay the price again in the last two rounds, if extracted; but if he is not selected in the round II, he will not pay in next rounds. The resulting utility is $U = {}^1/_{27}(5V - 37P)$. This result has to be compared with the utility of not paying the price in round I, that is zero. In fact, if $V < 7P$ and if the player does not pay the price in the first round, he will not pay in the second one neither. Then, the utility to enter the game is greater than zero if $V > {}^{37}/_5P$. It means that this is the minimum value that a player needs in order to enter the game in round I. But this value is outside the range considered, which is ${}^{23}/_5P < V < 7P$. Then, for all the values of this interval, the first player extracted will not pay the price because he could gain a positive payoff only with a value greater than the upper bound, which is impossible.

Concluding, this game equilibrium shows that:

- if $V > 7P$, in round I the player extracted will pay the price in this round and in all the next ones until a total of three payments,
- if ${}^{23}/_{5}P < V < 7P$, none of the player will pay the fee,
- if $V < {}^{23}/_{5}P$, none of the player will pay the price in any rounds.

4.2 Seller's profit

Given the three ranges of values just described, the seller's profit can be computed and analysed with respect to case A. Because of the characteristics of the case (*R=4* and *M=2*), the seller has to consider the first player extracted, who is the only interested one in paying the price in other rounds next the first one.

If $V > 7P$, the first player extracted pays the price each time he is extracted until round III. In the last round, he will enter the game only if it means that he will receive the product. The resulting expected profit for the seller is: $\pi = {}^{1}/_{27}(53P - 11C)$. The seller should choose the price that maximize his profit on the basis of the range of values considered. In this case it is $P^{*} = {}^{1}/_{7}V$ and the resulting profit is $\pi = {}^{1}/_{27}({}^{53}/_{7}V - 11C)$. This result is positive if $\mathrm{C} < {}^{53}/_{77}V$.

The second range of values is ${}^{23}/_{5}P < V < 7P$. For these values, the first player extracted will not pay the fee in the first round and consequently none of the players will pay in any rounds. Then, the strategy to set a price in these range of values is dominated by the previous one, which sets $P^{*} = {}^{1}/_{7}V$. In fact, with this price the seller should be able to make profits.

In the last range, $V < {}^{23}/_{5}P$, the reasoning is the same: none of the player is interested in entering the game because of the certain negative expected utility. Then, the seller is unable to dispatch products and gains profits.

Therefore, in this case there is a unique range to consider and a unique profit function equation. The range is $V > 7P$ and the seller expected profits are:

$$\pi = {}^{1}/_{27}(53P - 11C).$$

By substituting the optimal price for the seller, which is the maximum one he can ask, $P^{*} = {}^{1}/_{7}V$, the result is: $\pi = {}^{1}/_{27}({}^{53}/_{7}V - 11C)$. This profit is positive for all values of *C* such that $V > {}^{53}/_{77}C$. So, the best strategy for the seller is to choose the higher feasible price because it provides also the highest profit.

The table below summarizes the characteristics of the optimal game that the seller should implement in order to maximize the profit in each case. In case B, the profit maximization is not unique, but it is provided by two different prices on the basis of the value of the parameter *C*.

Table 10 - Comparison of case A, B and C expected profit

	Interval	P^*	**Expected profit**	$\pi > 0$ **if**
Case A	${}^{14}/_{5}P < V < 4P$	$P^* = {}^{5}/_{14}V$	${}^{5}/_{9}(V - C)$	$V > C$
Case B	${}^{14}/_{5}P < V < 4P$	$P^* = {}^{5}/_{14}V$	${}^{1}/_{27}({}^{130}/_{7}V - 25C)$	$V > {}^{35}/_{26}C$
	${}^{44}/_{17}P < V < {}^{14}/_{5}P$	$P^* = {}^{17}/_{44}V$	${}^{1}/_{27}({}^{391}/_{22}V - 19C)$	$V > {}^{418}/_{391}C$
Case C	$V > 7P$	$P^* = {}^{1}/_{7}V$	${}^{1}/_{27}({}^{53}/_{7}V - 11C)$	$V > {}^{77}/_{53}C$

4.3 Comparison with case A and B

In the comparison between case A and B, it has been shown that the seller has multiple optimal prices in order to maximize the profits:

- if $C < {}^{41}/_{308}V$ [≈ 0.13], the best profit is provided by $P^* = {}^{5}/_{14}V$ of case B,
- if ${}^{41}/_{308}V < C < {}^{61}/_{88}V$ [${}^{61}/_{88} \approx 0.69$], the best price is $P^* = {}^{17}/_{44}V$ of case B,
- if $C > {}^{61}/_{88}V$, the best option is case A.

Case A is the best option until cost equalize the valuation, with a profit of zero. If the costs are greater than the valuation, all the cases provided negative expected profits.

In order to determine whether case C provide a better or worse profit than these cases, it has to be compared with each of them. The results are the following:

- case C is better than A if $C > {}^{13}/_{7}V$ [≈ 1.86],
- case C is better than the first one of case B if $C > {}^{11}/_{14}V$ [≈ 0.79],
- case C is better than the second one of case B If $C > {}^{1571}/_{1232}V$ [≈ 1.28].

However, if $C > {}^{11}/_{14}V$, the expected profits of case C is negative. While if $C < {}^{11}/_{14}V$, the best cases remain case A and B as illustrated above.

Then, the case C is dominated by all the other cases and the best one remains the same illustrated in the comparison between case A and B. By setting a higher value of the dispatch coefficient, the seller should have relied in a situation in which he has more probability to gain prices and dispatch few products. However, empirics displays the opposite result. In fact, the seller gains a lower expected profit than case A and B due to the value of dispatch coefficient which discourage players to enter the game in a higher number of circumstances, causing a lower expected profit. At the end of the day, the option of increasing the parameter *M* lead to a decrease or to nullify the seller's profit. Indeed, this strategy should be considered alongside an increase of the number of rounds in order to counter-balanced the negative effect of a higher value of *M*. The reason is that by increasing the number of rounds, a higher number of players should be incentivized to enter the game. The result should be higher profits for the seller, but it depends on the number of products he has to dispatch. The aim of the next paragraphs is to analyse this strategy in order to understand whether it can lead to a positive and better payoff.

4.4 The effect of one further round if the dispatch coefficient is equal to 2

This is the last case which complete the analysis of this type of game. It is characterized by 3 players that can or not participate the game in 5 rounds. In order to have one product dispatch, the seller requires that two prices have to be paid, except for the payment of the first player on the waiting list. The price is lower than the valuation *V*, because only in this case players would consider entering the game.

Round V

The starting point is always the last round, that in this case is the fifth one. In this round, there is one type of player interested in entering the game. He is the one that can receive the product by paying the price, while all other players will not participate.

Round IV

In round IV, players behaviour has to be split on the basis of one basic circumstances: whether the selected player is the first subject on the waiting list or he is the second one. The former implies that the selected player:

- paid in round I and II, but no one in round III,
- paid in round I and someone else in round II or II (and no one in the residual),
- paid only in round III and no one in the previous two rounds.

The first two states can be considered similar. In both of them, the selected player has the possibility to receiving the product by paying the price. Indeed, this is the best strategy for a player in this state of the game. Obviously, the second circumstance implies a higher expected utility because the player pays twice and not three times as in the second circumstance. But the choice to pay the price is in any case the best one.

In the third state, the players selected in round IV paid the price in only one round in the past three rounds, and no one else paid in the remaining. In this case the only possibility to receive the product is the one in which he pays the price in this round and he is extracted again in round V. In this case the expected utility is $U = {}^1/_3(V - 7P)$. If the player decides to not enter the game in this round, he will not enter in round V neither with a final negative utility equal to the price paid in round III. By comparing $U = {}^1/_3(V - 7P)$ with $U = -P$, the result is that the first is greater than the second for all values of V such that $V > 4P$. However, by substituting $V = 4P$ in the first utility, the player would obtain a negative utility. It means that it is true that the decision to pay in round IV gives a greater expected utility, but for some values of V the player would obtain anyway a negative utility. The values of V which makes positive this expected utility are all the values such that $V > 7P$.

The second circumstance is the one in which the selected player is the second on the waiting list. It implies that this player:

- paid in round II but someone else in round I and III,
- paid in round II, someone else in round I and no one in round III,
- paid in round II and III but someone else in round I.

The first state is equal to last analysed one. In fact, if someone else paid in round I and III and the selected player paid in round II means that with the payment in round III, the first player received the product, quitting the list. Then, the player who paid in round II become the new first one on the list. In this case, his expected utility is equal to the one of a player who paid only in round III and no one in the others rounds. So, the result is that this type of player will obtain a greater expected utility by paying the price if $V > 4P$, but he gains a positive payoff only if $V > 7P$.

In the second state, if a player is the second one on the list, he has no possibility to receive the product because of a lack of rounds. Then, he will choose to not pay the price, with a resulting utility of *–P*, which is better than the utility to pay also in round IV, *–2P*.

In the third state, the player is the second one on the list and he paid already twice the price. As the first point, thanks to the payment in the third round he is the new first one on the list. However, differently from the first point, he will require a higher value of V in order to enter the game, because he paid already twice. The utility to pay the price in round IV is $U = {}^1/_3(V -$

$10P$), while the one to not do it is $U = -2P$. The former is greater if $V > 4P$ but it is positive only if $V > 10P$. So:

- if $V < 4P$, he does not pay the price,
- if $4P < V < 10P$, he does pay the price obtaining a negative expected utility (but better than *–2P)*,
- if $V > 10P$, he does pay the price and he gains a positive expected utility.

If the selected player in round IV is not on the waiting list, his best strategy is to not pay the price in fact he cannot receive the product because of a lack of rounds.

Therefore, round IV analysis shows three significant range of values: $V > 10P$, $7P < V < 10P$ and $V < 7P$. The analysis of the third round are conducted for each of these three ranges.

Round III

In this round, two cases have to be considered:

- if the selected player is on the waiting list and consequently if he is the first or the second,
- if he is not on the list.

If he is the first on the list, it means that he:

- paid in round I and II,
- he paid in round I and someone else in round II,
- he paid in round I and no one else in round II.

The first two points are similar. In fact, the player can receive the product by paying the price in this round. However, in the second case he gains a higher payoff because he pays the price twice and not three times as in the first case. In the third point, the behaviour in the following rounds has to be consider. In the fourth round, a player who paid the price once is incentivized to pay the price when $V > 7P$. Since in this analysis all values of V greater than *10P* are considered, the player will pay the price in the third round.

If the selected player is the second one on the list means that he paid in round II but someone else pain round I. This player only possibility to receive the product is that in all round a payment is performed. Round IV analysis showed that if he is extracted in round III he will pay the price, given that $V > 10P$. The same will occur if the first player is extracted again in round III, but successively, in round IV and V, only the second player will pay the price to win the product. All these behaviour result in an expected utility equal to $U = {}^{1}/_{9}(V - 20P)$. This last one is greater than the utility to not pay the price in round III if $V > 11P$, but these value is not sufficient to guarantee a positive expected utility. The minimum value of V that make non-

negative this utility function is $V = 20P$. For all the values greater than this one, a player who is the second on the list will pay the price in round III.
The last case is the one in which the player who is extracted is not on the list. This time, the only aspect that matters for this player is that if a future payment in round IV will help him directly to receive the product. If this is not so, the player will not pay in round III. Conversely, if he is the case, the player will pay because it means that after the payment in round III, he is the first one on the waiting list. His possibility to receive the product is the one in which he is selected again in round IV and V. Then, the resulting utility is $U = {}^{1}/_{9}(V - 11P)$. This player will enter the game only if this function is greater than zero, so for all values of *V* such that $V > 11P$.
The range in which $7P < V < 10P$ shows an identical behaviour for the player who is in the first position of the list. In fact, for all values of *V* greater than *7P*, a player will pay the price in any case in the third and fourth round. Conversely, some differences reveal for a player who is the second one on the list and for a player who is not in it. In round IV it has been shown that a player who ranks second on the list and paid the price in round II and III, require a minimum value of *V* equal to *10P* in order to pay also in round IV. Given that, since in this range $V < 10P$, if this type of player pays the price in round III, his best strategy in round IV is to not pay the price. Then, his resulting utility is $U = -2\mathrm{P}$ if he pays in round III and $U = -\mathrm{P}$ if he does not. For this reason, the choice to not enter the game is his best response. The last case is the one in which the extracted player is not on the waiting list. His behaviour remains the same illustrated for previous range of values in which he requires a minimum value of $V = 11P$. This value does not fit with the considered range because it is outside it. For this cause, a player who is not on the waiting list will decide to not enter the game, with a utility of zero.
In the case of the last range of values $V < 7P$, it can be forthwith reported that a player who is second on the list or not on it, the decision is to not pay the price in round II, as for previous range of values. The only difference lies in the behaviour of a player who is first on the list, precisely in one specific state of the game. In fact, if this player can receive the product by paying the price in round III, he will do it with certainty. This decision is not immediate in one only state of the game: the one in which he paid in round I and no one else in round II, or vice versa. If he pays the price also in round III, he will pay the price also in round IV or V if extracted because with this last payment he gets the product. If he decides to not pay the price in round III, he will replicate this decision also in next rounds because a player who pays one price and who is extracted in round IV requires a value of *V* greater than *7P*. So, in the case of a payment in round III, he reaches a utility of $U = {}^{1}/_{9}(5V - 23P)$, otherwise he has a utility of *–P*. The first one is greater than the second one if $V > {}^{14}/_{5}P$. However, not all the values of this

range allow the player to have a positive utility. Indeed, this type of player can achieve a non-negative utility only with a minimum value of V equal to $V = {}^{23}/_{5}P$.

This analysis adds some significant range to the ones found in round IV. They are: $V > 20P$, $11P < V < 20P$, $10P < V < 11P$, $7P < V < 10P$, ${}^{23}/_{5}P < V < 7P$, $V < {}^{23}/_{5}$.

For all these values, the players behaviour has to be studied in round II.

Round II

The states of the game are essentially three:

- the player selected in round II is on the list,
- the player selected in round II he is not on the list and someone else paid in round I,
- the player selected in round II he is not on the list and no one else paid in round I.

If $V > 20P$, whoever is the player extracted in this round, he will pay the price.

If $11P < V < 20P$, a player who is the first one the list will pay the price, as a player who is not on the list but no one else paid in round I. The case of a player who is not on the list but in round I someone else paid in round I is different. This type of player gains an expected utility of $U = {}^{1}/_{27}(V - 29P)$ by paying the price, while he has a zero utility by not doing it. So, in order to pay the price, he requires at least a value of V such that $V = 29\text{P}$. However, it is now considered the range such that $11P < V < 20P$, which does not include this new value. For this reason, a player who would be the second on the list by paying the price is incentivized to not do it achieving a utility equal to zero.

The same result just explained in the case of $11P < V < 20P$ are replicated also if $10P < V < 11P$ and $7P < V < 10P$.

In the case of ${}^{23}/_{5}P < V < 7P$, the behaviour of a player who is on the list and of a player who would be the second on the list remain the same: the former one will pay the price while the latter will not do it. Conversely, the behaviour of a player who is not on the list and who would be the first one by paying changes. In fact, the utility of paying the price is $U = {}^{1}/_{27}(5V - 37P)$, while the one of not doing is $\text{U} = 0$. Then, the player is interested in paying the price if $V > {}^{37}/_{5}P$. However, this value is outside the range considered. For this reason, this type of player will not enter the game.

In case of the last range, $V < {}^{23}/_{5}$, if a player is not on the list and he is extracted in round II, he will not pay the price, in any case. If the selected player is already on the list, he will pay gaining a utility of $U = {}^{1}/_{27}(19V - 73P)$. If he does not pay the price, his utility is composed exclusively by the price he paid in round I, *–P*. The utility of paying the price is greater than the one of not paying if $V < {}^{46}/_{19}P$. However, this value is not sufficient to guarantee a positive expected utility. The minimum value that make this utility function non-negative is $V < {}^{73}/_{19}$. Because of this value, in the round I a further range of values has to considered.

Round I

For ease, the analysis of the players behaviour in the first round is synthesized in the following table.

Table 11 - Case D all potential expected utilities of players

Interval	Decision	Expected payoff	Optimal choice
$V > 20P$	To enter	$U = {}^{1}/_{81}(73V - 144P)$	To enter is better
	Not to Enter	$U = {}^{1}/_{81}(7V - 21P)$	
$11P < V < 20P$	To enter	$U = {}^{1}/_{81}(41V - 145P)$	To enter is better
	Not to Enter	$U = {}^{1}/_{81}(7V - 21P)$	
$7P < V < 11P$	To enter	$U = {}^{1}/_{81}(33V - 147P)$	To enter is better
	Not to Enter	$U = {}^{1}/_{81}(7V - 21P)$	
${}^{23}/_{5}P < V < 7P$	To enter	$U = {}^{1}/_{81}(29V - 139P)$	To enter is better if $V > {}^{139}/_{29}P$
	Not to Enter	$U = 0$	
${}^{73}/_{19}P < V < {}^{23}/_{5}P$	To enter	$U = {}^{1}/_{81}(19V - 119P)$	Not to enter is better
	Not to Enter	$U = 0$	
$V < {}^{73}/_{19}$	To enter	$U = -\mathrm{P}$	Not to enter is better
	Not to Enter	$U = 0$	

This table shows that the first player extracted will enter the game if $V > {}^{139}/_{29}P$ and the expected utility changes for some predefined ranges of values.

Each of these ranges of values is characterized by different behaviours that players follow in the subsequent rounds.

If $V > 20P$, in round I and II, all players will pay the price, while in the last three rounds the players who will pay are:

- the first on the list,
- the second on the list if by paying in round III he becomes the "new" first one.

If $11P < V < 20P$, the player extracted in the first round will pay the price in order to receive the product and in round III a new player could pay the price if by doing it he becomes the "new" first one on the list.

If $7P < V < 11P$, only the player extracted in round I will pay the price again in next rounds in order to receive the product.

If $^{139}/_{29}P < V < 7P$, only the player extracted in the first round will pay the price in round from I to III, but he will not to do it in round IV if until this round he paid only once.

If $V < {}^{139}/_{29}P$, none of the player will pay the price in any rounds.

4.4 Seller's profit

The last five ranges of values are essential to compute all the possible profit of the seller in order to analyse the optimal price that maximize it.

If $V > 20P$, the seller expected profit is equal to $\pi = {}^{1}/_{81}(259P - 79C)$. The seller should choose the maximum profit in the considered range and in this case is $P^* = {}^{1}/_{20}V$. With this optimal price, the expected profit become $\pi = {}^{1}/_{81}({}^{259}/_{20}V - 79C)$, which is positive if $V > {}^{1580}/_{259}C$.

If $11P < V < 20P$, the seller profit is $\pi = {}^{1}/_{81}(183P - 39C)$. In this case the price that maximize the profit is the lower bound of the range, i.e. $P^* = {}^{1}/_{11}V$. This is the maximum price that the seller can ask in order that customers behave as described above. The resulting profit is $\pi = {}^{1}/_{81}({}^{183}/_{11}V - 39C)$ that is positive if $V > {}^{143}/_{61}C$.

If $7P < V < 11P$, the expected profit changes again because players modify their behaviour with respect to previous case. Then, the profit is $\pi = {}^{1}/_{81}(171P - 33C)$. As before, the optimal price is the lower bound of the range $P^* = {}^{1}/_{7}V$. By substituting in the profit function, the result is $\pi = {}^{1}/_{81}({}^{171}/_{7}V - 33C)$. This value is greater than zero if $V > {}^{77}/_{57}C$.

If $^{139}/_{29}P < V < 7P$, the expected profit of the seller is equal to $\pi = {}^{1}/_{81}(155P - 29C)$. The maximum price that the profit can ask is $P^* = {}^{29}/_{139}V$. With this value the seller payoff become $\pi = {}^{1}/_{81}({}^{4495}/_{139}V - 29C)$ which is positive if $V > {}^{139}/_{155}C$.

In the case of the last range, $V < {}^{139}/_{29}P$, none of the player will enter the game. Then, the profit of the seller is equal to zero.

In order to individuate the best profit for the seller is sufficient to compare the last profit function found with the other three ones. The result is that the profit deriving from $P^* = {}^{29}/_{139}V$ is greater than the other ones far any values of C (considering that it must be non-negative). So, the best strategy for the seller is to choose the price which incentivizes at least the first selected player to enter the game, that is $P^* = {}^{29}/_{139}V$.

The table shows all the optimal prices and profit for the best ranges of values in case A, B, C and D.

Table 12 - Expected profit comparison among all cases

	Interval	P^*	Expected profit	$\pi > 0$ if
Case A	${}^{14}/_{5}P < V < 4P$	$P^* = {}^{5}/_{14}V$	${}^{5}/_{9}(V - C)$	$V > C$
Case B	${}^{14}/_{5}P < V < 4P$	$P^* = {}^{5}/_{14}V$	${}^{1}/_{27}({}^{130}/_{7}V - 25C)$	$V > {}^{35}/_{26}C$
	${}^{44}/_{17}P < V < {}^{14}/_{5}P$	$P^* = {}^{17}/_{44}V$	${}^{1}/_{27}({}^{391}/_{22}V - 19C)$	$V > {}^{418}/_{391}C$
Case C	$V > 7P$	$P^* = {}^{1}/_{7}V$	${}^{1}/_{27}({}^{53}/_{7}V - 11C)$	$V > {}^{77}/_{53}C$
Case D	${}^{139}/_{29}P < V < 7P$	$P^* = {}^{29}/_{139}V$	${}^{1}/_{81}({}^{4495}/_{139}V - 29C)$	$V > {}^{139}/_{155}C$

4.5 Comparison with case A, B and C

Case D has to be compared with the other three cases in order to establish if he is the best among all or if there are several optimal prices on the basis of the value of C. However, the comparison between cases A, B and C shows that the last one leads to a lower value of expected profits in any case. Then, case C can be overlooked and the residual relevant cases are A, the two ones of case B (B1 and B2) and D.

By comparing these four cases, the results are the following:

- case D is preferable than case A if $C > {}^{110}/_{139}V$ [$\approx 0.79V$],
- case D is better than case B1 if $C > {}^{22745}/_{44758}V$ [$\approx 0.51V$],
- case D is better than case B2 if $C > {}^{64157}/_{85624}V$ [$\approx 0.75V$],
- case D is positive if $C > {}^{155}/_{139}V$ [$\approx 1.115V$].

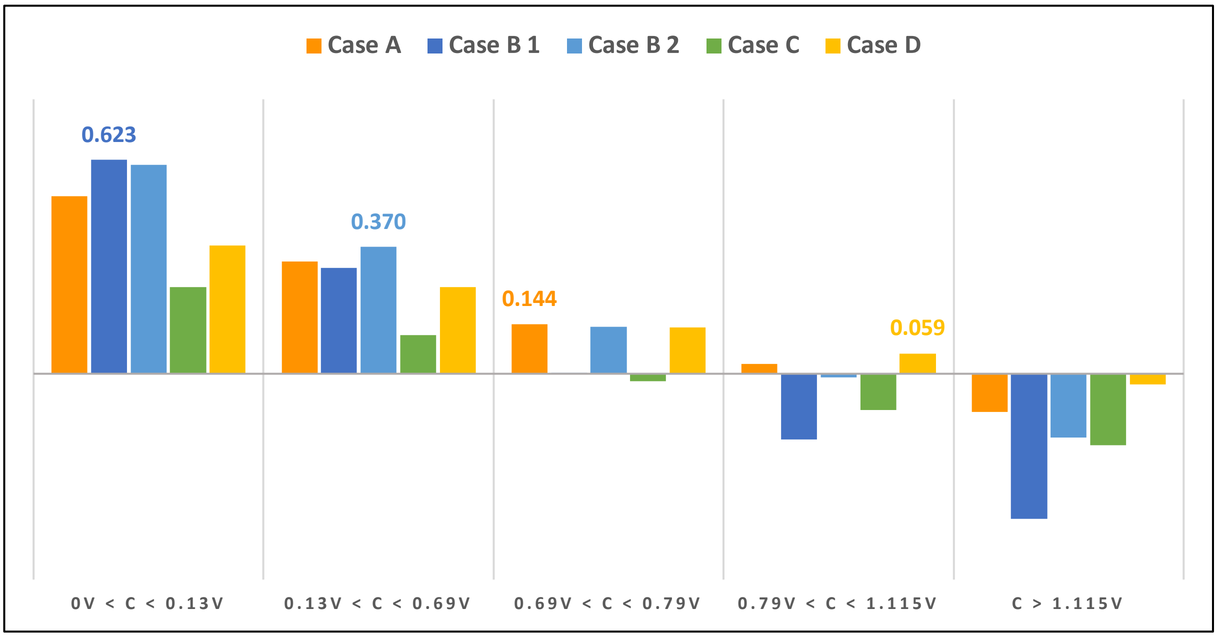
The only relevant points to consider are the first and the last. In fact, if $C < {}^{110}/_{139}V$, the best option between case A and D is the former. Then, the comparison between D and B1/B2 is not relevant because A remains the preferable case. However, case A is not the best option for all values, in fact for values lower than this relevant point the comparison has to be conducted between case A, B1 and B2. This comparison is exactly the one already performed in previous paragraph. Then, the only interval in which case D is better than other cases is ${}^{110}/_{139}V < C < {}^{155}/_{139}V$. For values below than the lower bound, the bests cases are the one illustrated in the previous comparison, While, for values beyond the upper bound, none of the cases provide positive expected profits.

The following table and chart shows the expected profit considering the relevant interval computed so far. Considering the chart, for each range of values, the calculation is made using the median point as the value of the parameter *C*. The calculation confirms what expressed in the comparison among these cases and each of the four interval has a different best case.

Table 13 - Optimal cases for the seller on the basis of parameter C

Interval	**Best case**	$\boldsymbol{P^*}$
$\boldsymbol{C < {}^{41}/_{308}V}$	B 1	$P^* = {}^{5}/_{14}V$
$\boldsymbol{{}^{41}/_{308}V < C < {}^{61}/_{88}V}$	B 2	$P^* = {}^{17}/_{44}V$
$\boldsymbol{{}^{61}/_{88}V < C < {}^{110}/_{139}V}$	A	$P^* = {}^{5}/_{14}V$
$\boldsymbol{{}^{110}/_{139}V < C < {}^{4495}/_{4031}V}$	D	$P^* = {}^{29}/_{139}V$
$\boldsymbol{C > {}^{4495}/_{4031}V}$	-	-

Figure 5 - Expected profit of all cases



CHAPTER 5
RESULTS BREAKDOWN

In the analysis of this game, some assumptions have been considered. These ones allow to make straightforward the solution of the various cases and to collect the key issue. These assumptions are:

- all players have the same valuation,
- all players have the same probability to enter the game,
- valuation and costs are completely exogenous for the seller,
- players are risk neutral,
- the dispatch coefficient is not related to the value of the fee,
- players know the last round of the game,
- the absence of a time coefficient that decrease the valuation over time.

Some of these statements are not valid in all the real situations, nonetheless they allow to conclude some considerations about the subject behaviours in this type of selling strategy. In fact, without these assumptions, the results of the various case could be different. However, all the assumptions just mentioned are necessary in order that the analysis of the various cases reveals the key features of this type of selling strategy. In the first case, the one with 3 players in 3 rounds and a dispatch coefficient equal to 1, the seller optimal choice is to choose the highest price which convinces the first player extracted to pay again the fee when selected. Moreover, it has been shows that a cooperation could be a better strategy for the players. In the case of a 2-players contract, they achieve a higher payoff if they sign the contract after the first round. However, the seller can retaliate by changing the price in advance, i.e. before the game starts. By doing so, the seller cannot ensure a higher level of profit because it depends on the values of V and C, which are exogenous. It means that the seller should choose the optimal price on the basis of these values. If the seller and the players can rely on symmetric information, the former could choose the optimal price knowing the level of the players valuation and the operating costs. Otherwise, he cannot choose with certainty the optimal price. In fact, he can only set a price, but the final value of the profit will depend on the exogenous factors. A second type of cooperation could be achieved among all the three players. In this case, after round I, the player excluded by the 2-players contract could propose a 3-players one, by paying a part of the price (with transfers) charged to the other two players. Then, the third player will obtain a positive payoff and the other two ones will gain a higher payoff than the 2-players contract one. This is an optimal strategy for all the players, but the seller could retaliate

by choosing the highest price that the third player can pay after the transfers to the other two players. This option includes some uncertainty by the seller due to the presence of cooperation costs, relative to drawing up or bargaining of the contract. Whether the seller do not consider them or make an estimation error, this could nullify or lower his profit. Then the “counterattack” is not a best strategy for the seller because it could lead to higher as well as lower profit.
The second case is the one which has a further round than the first one, keeping all other parameters equal. The analysis of this case reveals an essential feature of this type of selling strategies. In fact, it has been shown that a player, who is extracted and who paid in round I, will not pay the second fee if he is selected again in round II. It can be said that it is like he temporized in order to wait that another player pays the second fee in order that he receives the product, maximizing the utility. This type of circumstance happens with a sufficient low value of the price and if the player entered the game in a round “far” enough from the last one. Therefore, it is sufficient to add another round to a game with 3 players, 3 rounds and a dispatch coefficient equal to 1, in order to change some player behaviours.
In the third case, the effect of the dispatch coefficient increase was analysed. This fact leads to a change of players behaviour due mainly to the relationship between the number of rounds and the value of the dispatch coefficient. In fact, with 4 rounds and $M=2$, only the first player extracted will be interested in the game after round I. Moreover, the maximum price which the first player is willing to accept drops considerably. It has been shown that for the seller the strategy to increase the value of M, keeping all other parameters equal, is dominated by all the previous ones because it leads to lower payoff with certainty.
The last case analysis aims to verify whether an increase of the number of rounds is helpful in order to make the previous case an optimal strategy. The result is that this case is the best option for the seller only in some cases, in fact the increase of one round makes this last case the best one only for some values of C and V. Precisely, this case provides the best strategy if the ratio between C and V is high enough. Then, the increase of one round (from four to five) alongside the increase of the dispatch coefficient is:

- necessary in order to make this case a best strategy, for some values,
- not sufficient in order to make this case comparable to the second one.

Indeed, in this case, the behaviour of the first player extracted to “wait” if selected again in the second round does not reveal. This means that the effect of the rounds increase is not sufficient to compensate the opposite effect of an increase of dispatch coefficient.
This statement provides the possibility for the seller to analysed further case with different values of the parameter. Keeping a value of the dispatch coefficient equal to *2*, there will be a determined number of rounds which confirms the behaviour of the player in the second case.

Then, the first player will do not pay in the two next rounds, waiting for other players payments. The same reasoning can be made whether the seller would want to set this parameter equal to *3*. In this case, the number of rounds must have even greater. All these decisions, has to be taken by the seller considering the values of *C* and *V*.

The analysis of all the various cases lead to some issues that are summarized in the following statements:

- in each case, the optimal price chosen by the seller is always the highest one,
- increasing the number of rounds, the effect on the profit is uncertain, ceteris paribus,
- increasing the dispatch coefficient, the optimal price drops, as well as profits,
- increasing both dispatch coefficient and rounds, the price increase with respect to the case having the same value of *M* but less rounds, still the effect on the profits is uncertain,
- with a sufficient high number of rounds (relatively to the other parameters), a player who pays in a round “far” from the last one, will wait that other participants pay the fee instead of paying himself.

The analysis of the various cases is useful in order to understand some features of this selling strategy even if the real world scenario shows some further aspects. In fact, the main difference is that these websites rely on an unknown number of rounds and several (and a growing number of) customers which allow the seller to gain an increasingly higher profit. In fact, with an unknown number of rounds, the behavior of a player according to which he pays once and then he waits is repeated by all the customers through all the rounds. Then, because of the unknown number of rounds, the seller can rely on a value of the dispatch coefficient greater than *1*. In fact, these websites normally apply a value of *M* equal to around 3 even if this can be lower or greater. Because of this, the waiting list becomes longer and longer each time a single customer adds his name on it. So, customers have to take into account also a time factor, which is the more relevant the bigger the number of rounds. The presence of this coefficient of time could lead some customers to the decision to not pay the fee. This parameter can be though as a factor which reduces the valuation over time. It means that when a player pays the fee and he is put on the waiting list, he has to consider that, when he receives the product, the latter actual valuation could be lower than the one assigned at the time of the payment. The more the rounds to wait are, the more the product could lose value. When the time coefficient is so great that the cost (the paid fee) overcome the benefits (the actual valuation at the product delivery), none of the potential customers will want to add his name. This is the moment in which the scheme "collapses" and the majority of the customers paid a price but is unable to receive the product. This is the key issue which allows to compare this scheme with all the other schemes that are considered illegal. This aspect of the game has already been considered in the first chapter but in the case of this type of website the collapse means that not all the player can receive the product with the fee they paid. In fact, all websites provide an option for the customers: if they paid in the past and they do not want to wait more time, they can integrate the fee with another one in order to receive the product. Normally this is not a convenient decision for the customers since the total price they end up to pay is greater than the valuation. However, this option can be an optimal strategy for the customers if two conditions verify:

- the scheme collapsed, i.e. no more customers join the waiting list,
- the utility loss of adding another fee (i.e. $U = V - Q < 0$, where Q is the sum of the initial and the second fee) is lower than the utility loss made up by the price paid in the past (i.e. the case in which the customer paid only the initial fee and does not receive the product).

Then, if $V - Q > -P$, the customer best strategy is to pay the second fee in order to receive the product, even if the total paid amount is greater than the valuation. These websites can use different values of Q and it mainly depends on the business model of the seller.

Three websites of this type have been taken as sample in order to exhibit the satisfied and not satisfied customers. These websites use different mechanisms, as explained in chapter 1, but it is not relevant in order to analyze the percentage of satisfied customers. They show that the percentage of the first type of client is 33%, 29% and 35%, with an average of 32%. It means that more than two over three customers are waiting on the list that additional customers pay their fee. The percentage is due to the dispatch coefficient value. As said before, normally this parameter is equal to *3* and indeed only around a third of the customers can be considered satisfied.

The ability to postpone the mechanism “collapse” depends on the ability of each seller to attract more and more customers. However, since the number of potential customers cannot grow continuously, the seller has to be able to convince old customer to repurchase (which is a similarity with the Ponzi scheme characteristics) or to permit the customers to leave the game, shortening the waiting list. This last fact leads new customers towards the website because their time coefficient is low since the list shortened. However, the option to allow customers to leave the waiting list implies that they can obtain back the fee paid (totally or partially) and this could be harmful for the seller activities. The reason is that the paid fees are used to purchase and dispatch the product to older customers on the waiting list. When a waiting customer ask his money back, conceptually it does not exist anymore and then the seller should refund the customer with his capital. It means that the seller should anticipate the money and then wait for new customers payments that fill the gap. This situation creates a kind of circle:

- old customers ask their fee back,

- the waiting list shorten,
- new customers pay the fee.

This circle could permit the website to continue to be successful and to not collapse but the circle could involve some costs for the seller. Moreover, the number of new customers could be lower than the customers who left. In this case, the seller could have a monetary problem because the new cash in is not sufficient to repay the outflows. In this case, if the seller permits this practice, the sustainability of the mechanism depends on the ability of the seller to have always a positive amount of money to repay the old customer who want to leave.

Concluding, the game is helpful in order to analyse the main characteristics of this type of selling strategy. One of them is the behaviour of the players who pay one time the price and then wait for additional new customers. Then, this system implies the creation of a waiting list and with this practice the customers can reach the maximum level of utility. However, the sustainability of the selling strategy, depends on a greater number of factor, as the seller cost, the time coefficient, or the possibility of the customers to leave the game. Further analysis could be conducted in order estimate these parameters and their effect on players behaviour since they have been excluded in the cases examined in this dissertation. Furthermore, with a deep insight on the costs and on the business model of the sellers (websites), it could be useful to analysis in details the profits also from a point of view of their sustainability over time.

CONCLUSION

The aim of the dissertation is to determine the nature of this type of selling strategy and to analyse whether it can be qualified as a genuine mechanism of sales or otherwise it can be considered as a new scheme included in the illegal pyramidal scheme.

At a first investigation, this type of selling strategy seems to recall the features of the most famous pyramidal or Ponzi schemes which have historically demonstrated their purpose to produce huge profits for the creator (and few early adopter) at the expenses of a plurality of clients/investors. The purpose of this new selling strategy seems to be different and it can be considered the classical purpose of a common company: to provide the goods to the customers at the right price. However, the mechanism used in order to provide this price causes the creation of a waiting list, implying a probability of unsustainability. Besides, from a legal point of view, it could operate in a legislative gap that permit this type of company to carry on its activities even if the creation of the waiting list recalls a pyramidal scheme aspect. One of the feature that could differentiate this mechanism from the classical pyramidal scheme is the possibility for the players to exit the game at any time. The consequence could be a utility loss, but it depends on the various strategies and on the business model that each website/seller adopts.

The game analysis is conducted firstly with a simple case which has the minimum number of rounds, of players and a value of the dispatch coefficient equal to *1*. This case provides the statement according to which the seller provides the highest potential price and only one player is incentivized to pay the fee. In the additional cases, the key point is the establishment of the effect that a change of the number of rounds or of the dispatch coefficient have on the behaviour held by the players. The result of these last cases provides a change in the statement of the first case from the players point of view. In fact, adding more rounds, a great number of players are interested in entering the game because they pay the price in a round "far" from the last one. It simply means that these players have more probability to receive the product they paid. Moreover, the examination of these additional cases confirmed the key issue of this type of selling strategy: the behaviour of the player who prefers to wait rather than pay all the necessary fees and, on the other side, the choice to not enter the game if a player is selected in a round close to the last one. All the analysed cases consider a specified number of rounds, even if the real scenario shows an opposite situation. In fact, the strength of this strategy resides actually on the unknown number of rounds. With this aspect, a greater number of players are induced to pay the fee. However, the behaviour of players found in the cases with a specified number of round can be generalized. In fact, these behaviours can be assimilated to the case with an unknown number of round introducing a time coefficient. This factor leads to gradually decrease the total utility the player expects to gain because this parameter reduces the product valuation of the player. So, in a game with specified number of round, the player chooses to not enter the game if he had to pay in a round close to the last one. While in a game in unknown number of rounds this circumstance can be assimilated to the following one: player has to decide whether to pay or not in a round far from the first one and his time coefficient leads him to choose to not enter. The reason is that he should wait an amount of time such that he would obtain a negative utility. In fact, this is aspect the equate these two circumstances, allowing to compare a game with a specified last round with one with it.

These behaviours of the players are specific of this type of selling strategy which, if it reaches a point of unsustainability, could cause a utility loss for the majority of the participant. Exactly this last one is the main downside of this type of mechanism that could allow to compare this selling mechanism to the most famous pyramidal scheme or scam. However, the seller who provide this type of purchase has the possibility to overcome this downside by allowing customer to leave the waiting list without a utility loss, in order to make this system clearer for the players and for the public. The success rate of this plan depends on the seller ability to manage efficiently and actively all the aspects of the website mechanism and the corresponding costs. As empirics shows, the rate of customers who have not been satisfied yet is on average two third of the total players. This system can be considered successful whether the waiting list continues to be active and player continue to be satisfied. So, even if players should consider the waiting time, the time coefficient decreasing their valuation and that, at any time, the number of unsatisfied players is greater than the number of satisfied one, this system could be genuine if the seller is able to provide a continuous replacement of the waiting list.

Whether the seller is able to provide a sustainable mechanism with continuous substitution of old customers with new one, this scheme could not be considered a fraud or a scam as the Ponzi or the classical pyramid schemes.

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