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**"Inflation-at-risk and fiscal policy: A
quantile regression approach"**

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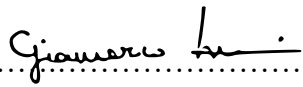
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1. Introduction

Since the COVID-19 crisis, there has been an emerging necessity to study the factors underlying the upside risk of inflation. The challenge has been addressed by several research papers, that discovered an important relationship and trade-off between inflation and an increasing number of variables.

The most commonly recognized relationship between inflation and unemployment is the so-called Phillips Curve. [29] Discovered by A.W. Phillips, the Phillips curve was a concept with great historical significance that played a central role in shaping future macroeconomic policies. His work was based on empirical observations in the United Kingdom during 1861-1957. (Phillips 1958) The intuition was striking, he found an inverse relationship between wages and unemployment. When unemployment is low, wage pressures tend to increase, contributing to higher overall inflation.

This concept gained significant attention and quickly became a widely discussed economic theory. However, it's important to note that the relationship between unemployment and inflation is not always linear and can be influenced by various factors, including expectations and supply shocks.

Additionally, Okun introduced the concept of Okun's Law, which suggests that for every 1% increase in unemployment above the natural rate, GDP falls by approximately 2% below its potential, and inflation falls by about 0.4%. (Ball et al. 2013) [5]

Moreover, recent research, such as Ball et al. (2015), has explored the idea that the Phillips Curve may have become flatter in recent decades, suggesting that the trade-off between unemployment and inflation may not be as strong as in the past. (Plosser et al. 1979) [30] (Del Negro et al. 2020) [13]

Like James Bullard, former president of the St. Louis Federal Reserves said *"If you put it in a murder mystery framework "Who Killed the Phillips Curve?" it was the Fed that killed the Phillips Curve."* (Ratner et al. 2021) [32] He pointed out that the Phillips curve is potentially weakening. The reason behind this is the capacity of the central bank to better target inflation levels, and they might respond more aggressively, weakening the causality between inflation and unemployment.

These studies collectively challenge the theoretical framework and its practical application, highlighting the complex and evolving dynamics between unemployment and

inflation in the economy. Lopez-Salido et al.(2021) clearly described the abundant variability in the tails of inflation outlooks even during times of low and stable inflation. Indeed, they suggest an increasing effort in linking inflation to other financial variables to better understand its tail behavior. [26] Overall, the Phillips curve provides an initial framework for studying the causality between these two variables.

Another concept that emphasizes the importance of inflation is the recently developed Fiscal theory of price level. The relationship between fiscal policy implemented by the government and the price level can be explained through this newly developed theory, a macro-level economic perspective that delineates this specific association. The alternative perspective goes against the paradigm attributing rising prices mainly to money supply and central banking conduct and offers an incomplete portrayal. [17]

According to this theory, adjustments in government spending and tax policies have the capability to considerably influence the general price level throughout the economic system. The key idea underlying the fiscal theory of the price level is that maintaining consistency between public receipts and expenditures, as imposed by the state's budget constraint, serves a primary part in determining price levels.

This theory holds that it is the budget constraint that fundamentally dictates the stability of the price level. Ultimately, the fiscal health of the nation depends upon the government's obligation to fund them through taxation and debt. For example, if the government runs persistent deficits, it will eventually have to monetize its debt by printing money, which will lead to higher inflation. The fiscal theory of the price level has important implications for predicting inflation patterns, it emphasizes the role of government debt and deficits in influencing the inflation rate. In essence, fiscal policy decisions play a crucial role in shaping the dynamics of inflation and unemployment, illustrating the intrinsic link between the Phillips Curve and the Fiscal Theory of the Price Level.

Based on this framework, this thesis investigates the risk factors on inflation using an Augmented Phillips Curve to analyze the influence of fiscal policy variables as risk variables to uncover the implications of inflation in the United States in the period that starts in 1960 until the first quarter of 2023. This study extends the analysis of quantile regression by exploring the effects of these variables on the tails of the inflation distribution. Additionally, I constructed a density forecast from a quantile regression based on the work of Koeker and Bassett. [22]

Quantile regression models the relationship between the dependent variables and one or more variables, conditional on different quantiles of the unconditional distribution of the dependent variables. One of the advantages of quantile regression is the possibility of analyzing the asymmetric effect at different quantiles. This concept helps to shed light on the interpretation and the sources of identification of the inflation tails.

Using the quantile regression approach this study aims to unveil potential nonlinearities and asymmetric effects in the transmission mechanism between these variables. The results provide valuable insights into the differential impact of fiscal policies on inflation.

The econometric strategy for this study is based on a two-step approach that sheds light on the upside risk of inflation. The first step is a quantile regression to estimate the conditional quantile function of the Augmented Phillips Curve as a function of fiscal policy variables. The second step is the fitting process, in which a parametric inverse cumulative function is fitted to a density function.

The method used for this work was inspired by the paper "Vulnerable Growth" by Adrian et al. (2019). [1] According to their argument, the financial sector's amplification mechanisms are responsible for the growth vulnerability dynamics observed.

This study models the distribution of future gross domestic product growth as a function of current financial conditions. The authors found that at lower quantiles, the distribution of future GDP varies with financial and economic conditions. This study analyzes downside and upside entropy, expected shortfall, and long rise as indicators of economic vulnerability.

The authors explain that the vulnerability of GDP growth to downside risks is measured using the relative entropy of the unconditional relative to the conditional predictive distribution. This means that the document analyzes the difference between the predicted GDP growth distribution under normal conditions and the predicted distribution when financial conditions are considered. The measure of vulnerability shows how much GDP growth is at risk of experiencing negative outcomes due to unfavorable financial conditions. The document further states a correlation between growth vulnerability and financial conditions, indicating that when financial conditions worsen, the vulnerability of GDP growth to downside risk increases.

What Adrian and the others found innovative was that their work was then expanded and used in numerous papers. (Loria et al. 2021) [26] (Musso et al. 2021) [14] (Jeasakul et al. 2019) [3] (Mitchel et al. 2023) [19]

The thesis is structured in five sections. The first section is a review of the literature concerning the upside risk of inflation and the implications of fiscal policy's impact on inflation. In addition, there is a review of the literature concerning the Quantile Regression and its application in economics. The second section presents the methodology that drives the structure of the model, such as the description of the data and the quantile regression analysis. The third section explains the second piece of the two-step process and the vulnerability measurement of the conditional distribution of inflation. The fourth section presents a robustness check that includes in the model two more variables to assess the limitations and reliability of the model. The last section

discusses the implication of the findings and the conclusion of the thesis.

2. Literature Review

2.1 Impact of fiscal policy on inflation

Inflation has revived. The US government, with the help of the Federal Reserve, established a new policy after March 2020 that has increased the debt by 30%. This strategy was designed as a fiscal and monetary stimulus to enhance aggregate demand. From the standpoint of fiscal theory, the occurrence appears to be a classic fiscal helicopter drop, where people received checks up to \$3200 for each person. There was a big unexpected deficit, a negative surplus with no adjustment in fiscal policy that would lead to future surpluses to repay debts. [18] This may have brought inflation back, but this is just one of the many hypotheses under analysis.

The impact of fiscal policy on inflation is a complex and multifaceted issue that has been extensively studied in economics. The literature contains various empirical findings addressing how fiscal policies affect inflation. While there is some evidence for a connection when studying scenarios of significant inflation, research outside these situations frequently points to a weak relationship. [20]

Two main categories may be used to categorize empirical research on the relationship between fiscal policy and inflation. The first part frequently takes a longer-term perspective and focuses on determining the extent to which high and persistent deficit levels affect inflation. The second part consists of more recent contributions that stress how inflation is affected by changes in fiscal policy.

Fiscal policy, which involves government spending, taxation, and debt, can influence inflation through various channels. Expansionary fiscal policies, such as increased government spending or tax reductions, can stimulate aggregate demand in the economy. This increase in demand can lead to upward pressure on prices and potentially contribute to inflation. Conversely, contractions in fiscal policies, such as reduced government spending or tax increases, can lower demand, putting downward pressure on prices.

According to research by Blanchard and Fischer in 1993, having modest governmental shortfalls when an economy is functioning beneath its full potential need not inevitably cause inflation and could help incentivize economic expansion. [9]

However, large and persistent deficits, especially when the economy is already near full employment, which can put upward pressure on prices and contribute to inflation. On the other hand, some economists argue that deficits may not be the only primary driver of inflation. Instead, they highlight the importance of monetary policy and the money supply in determining inflationary pressures, but this study does not contribute to the existing literature. Sargent (2013) discusses the relationship between government deficits and price levels in different debt-servicing regimes. He was one of many who explored the concept of "Ricardian Equivalence". [33] The concept was introduced by Barro and Gordon (1983). [6]

Ricardian equivalence is an economic theory that states that the method of financing government spending, whether through taxation or borrowing, does not significantly impact household consumption decisions and, consequently, does not have a direct effect on inflation.

According to this theory, individuals are forward-looking and rational. When the government runs a budget deficit and borrows to finance it, households anticipate that future taxes will increase to repay the debt. Consequently, they reduce their current consumption to save for the expected future tax charge, thus offsetting the stimulative effects of government spending. In essence, Ricardian equivalence suggests that fiscal policy actions do not alter aggregate demand in the economy and therefore have limited direct implications for inflation.

Subsequently, Sargent states that the inflationary consequences of government deficits depend on the government's strategy for servicing the debt and compares the two main debt-servicing regimes: the Ricardian and Friedman regimes.

The Ricardian regime assumes temporary government deficits with offsetting future surpluses, whereas the Friedman regime allows persistent deficits financed by issuing additional base money, concluding that the inflation path can be influenced only by the expected change in base money.

It also mentions debt-servicing regimes intermediate between Ricardo's and Friedman's. In these regimes, interest-bearing government debt is issued but eventually repaid partly by issuing additional base money. Increases in interest-bearing government debt are typically inflationary, signaling prospective increases in base money.

According to the regime that Sargent and Wallace (1975) studied, the deficit path leads to a continuous stream of significant deficits, and eventually, the inflation tax must be used. [34] In closing, the writer determines that the association between governmental shortfalls and the price stage is affected by the prevailing debt-servicing infrastructure. Empirical studies have yielded mixed results on the effect of fiscal deficits on inflation, with some finding a strong correlation, whereas others highlight a weaker or even insignificant relationship.

Perotti (2005) examined the effects of fiscal policy on GDP, inflation, and interest rates in five OECD countries, including the United States. [28] The methodology used in this document is a VAR (Vector Autoregression) model. By synthesizing the empirical results from this investigation with those of prior examinations into fiscal measures enacted by American authorities, similarities and divergences between the respective studies' outcomes are brought into focus and consideration.

This study highlights the importance of considering the price elasticity of government revenues and spending in understanding the effects of fiscal policy.

The estimated effects of government spending shocks on inflation also tend to be small, and there is no evidence that tax cuts have a faster or higher multiplier effect on inflation compared with spending increases. The study finds that under plausible values of price elasticity, government spending typically has small negative effects on inflation in the US at 4 and 12 quarters after the shock.

Based on the information in the research by Mountford et al. (2009), it can be deduced that unanticipated changes in fiscal measures such as spending boosts or tax reductions covered by deficit spending are capable of influencing price levels. [27] For example, in the case of a deficit-financed tax cut fiscal policy scenario, the impulse responses show that the effect on prices is initially negative but subsequently becomes positive following the rise in output.

Jørgensen et al. (2022) suggested that introducing a technology variable can explain the empirical findings regarding the price response to government spending shocks. [20] The authors set up a Structural VAR (Vector Autoregressive) model and used the forecast errors of government spending to identify shocks. They consider alternative specifications of the VAR model and different identification schemes, including the standard Cholesky decomposition. The estimated VAR model is based on quarterly U.S. data and includes linear and quadratic time trends. The authors found that the response of prices to a positive government spending shock is flat or even negative, contradicting the predictions of standard New Keynesian models. This challenges the generally accepted belief that increases in government spending lead to inflation.

Dupor et al. (2015) discussed the expected inflation channel, which suggests that government spending can drive up expected inflation. [15] They used the instrument by Fisher and Peters (2010), which captures the news aspect of government spending shocks. [16] However, it also presents findings that suggest that the expected inflation channel is quantitatively weak.

The authors examine the change in inflation expectations following the announcement of the Recovery Act, which was a large federal spending program. They mention that the political environment during this period allowed them to isolate the timing of news arrival to the period between late 2008 and early 2009.

They also discuss the core and headline inflation rates, noting that there was no substantial deflation during the quarters surrounding the Act's passage. The authors then consider the possibility that the Act reversed an expected deflation that would have occurred in its absence, but they show that neither the election outcome nor the Act's passage was associated with a major change in expected inflation.

Between the 2008 election and the passage of the 2009 Act, 1-year-ahead inflation expectations remained relatively stable. The 5-year-ahead inflation expectations also remained unchanged at around 2.5% during both the economic downturn and the period of the Recovery Act.

In the six months following the Act's adoption, expected inflation rose by only 25 basis points. However, following Obama's election victory, projected inflation dropped by 5 basis points. Overall, the median forecast indicates that the Recovery Act did not avoid a deflationary spiral. The predicted inflation response to the government expenditure shock during the 2009 Recovery Act period, according to the study, was too small.

The evidence presented by these studies and many others accepts the fact that fiscal policy shocks have a major impact on the overall economy while creating a dubious effect on inflation. The consensus on the changes in the price level is different and even with different signs. Recalling the studies of Caldara et al. (2008), Ben Zeev et al. (2017), and many others, they showed that the price increase is tiny but significant. The previously discussed authors are certain of a negative effect on inflation. [10][7]

The study by Bianchi et al. (2022) used a New Keynesian model with a fiscal block and a discrete shock that can push the economy to the zero lower bound (ZLB). The model allows for the possibility of a change in agents' beliefs about the exit strategy from the zero lower bound.[8]

The analysis focuses on the perceived credibility of the fiscal framework, the strength of the central bank's anti-inflation commitment, and the joint dynamics of the debt-to-GDP ratio, real interest rate, and inflation. It also explores the impact of different policy responses on inflation, output, and fiscal policy. The analysis shows that a more restrictive monetary policy can lead to higher inflation and larger output losses, while a credible fiscal arrangement is necessary to combat fiscal inflation effectively. Increasing the response to inflation through monetary policy results in a smaller initial jump in inflation but generates a larger contraction in real activity. The success of inflation control through a more restrictive monetary policy is temporary, as inflation eventually increases and output losses become larger.

The author also talks about the importance of fiscal stagflation and how agents expect the increase in fiscal restriction to contribute to future inflation. Policymakers need a credible fiscal arrangement to address fiscal inflation effectively. A more restrictive

monetary policy unsupported by a credible fiscal arrangement accelerates inflationary dynamics and further slows down economic activity. The fiscal component of inflation is likely to persist unless the necessary fiscal backing is reinstated.

2.2 Quantile Regression and Economics

Quantile regression is a statistical technique developed by Koenker et al. (1978) that has several advantages when used in economics. [22] When compared with conventional ordinary least squares (OLS) regression, one of its main advantages is its capacity to offer a more comprehensive understanding of the relationship between variables.

Quantile regression enables economists to investigate how various quantiles of the dependent variable respond to changes in the independent variables, in contrast to OLS, which focuses on estimating the conditional mean of the dependent variable. This is especially helpful when analyzing economic data that has heterogeneous effects or non-normal distributions because it may not be true that the effects are constant across the distribution. [11]

Researchers obtain insights into how economic issues affect distinct parts of a population or distribution by studying various quantiles, making it a significant tool for policy analysis, risk assessment, and discovering potential disparities within an economic setting. The advantages of using quantile regression are numerous. For example, it allows the capture of nonlinear relationships between variables by focusing on different quantiles of the conditional distribution of the dependent variable.

Unlike classical linear regression, which characterizes only the effect of explanatory variables on the conditional mean of the dependent variable, quantile regression provides a much richer set of results. This study examines the impact of changes in the explanatory variables on different quantiles of the conditional distribution, providing insights into the nature of the relationship not captured by the mean.

It determines whether the parameters of the equations are different when evaluated in the lower or upper quantiles of the conditional distributions of the variables.

Given the nature of this study, the objective is to use the power of quantile regression to better understand the downside risk of inflation. Traditional inflation forecasting models often focus on point estimates, such as the mean or median inflation rate.

However, these models may fail to capture the variability and tail risks associated with inflation, which can have significant economic implications. Quantile regression enables economists to examine how various factors, such as monetary policy changes, fiscal measures, or external shocks, affect different quantiles of the inflation distribution.

Lopez-Salido et al. (2021) used the concept of quantile regression and pointed out that it can be particularly valuable when analyzing "inflation at risk" within an economic context. [26] Inflation-at-risk is a financial concept that assesses the possible danger of inflation exceeding a specific threshold or level. In basic terms, it is comparable to value-at-risk (VaR), except that it focuses on inflation rates rather than financial asset values. Inflation-at-risk includes evaluating the likelihood and amount of inflation reaching a certain level within a specific time frame.

The author mentions that efforts have been made to understand why the response of inflation to economic and financial conditions has been muted and why labor market conditions have failed to explain recent inflation outcomes.

The article also emphasizes that the Phillips curve relationships, which have historically been used to explain inflation behavior, appear to be deteriorating. According to the authors, the conditional mean of inflation does not accurately predict inflation in the presence of tail risks.

They demonstrate that the reaction of inflation tails and the median to real and financial shocks offer an understanding of the effects of real and financial shocks on inflation. The authors discover that large downside risks to the inflation forecast have existed in the previous 20 years, mostly due to financial restrictions.

They connect these findings to earlier research that highlights the role of financial conditions in the dynamics of inflation. The authors also highlight that examining the complete predictive distribution of economic growth can provide more insights into the dynamics of inflation. In addition, they demonstrate that tight financial conditions have a considerable impact on the left tail of the conditional distribution of real GDP growth and imply medium-term unemployment risks. Overall, they follow the standard belief that the Phillips curve paradigm should be expanded to account for risks, including the impact of financial circumstances.

In the spirit of Adrian et al. (2019), who studied the term structure of growth-at-risk, it offers considerable potential as a model for measuring the intertemporal trade-offs associated with macroprudential policy. The growth-at-risk framework is a tool used in IMF country surveillance to assess the distribution of future GDP growth and quantify macroeconomic financial risks.[31] The analysis involves generating future growth distributions. This is achieved by fitting a t-skewed distribution to the predicted values of the estimated conditional quantile regressions. The complete distribution of future GDP growth, based on the state of the financial environment, allows for an assessment of the likelihood of future economic activity at any level.

Linnemann et al. (2016) discuss the estimation of nonlinear effects of fiscal policy using quantile regression methods. [24] The focus is on capturing nonlinearities generally, without relying on a specific parametric nonlinear model. They present evidence on

the nonlinear effects of government spending shocks in the United States. The findings suggest that fiscal policy is more effective in a depressed economy than in normal times. The cumulative effect of fiscal spending expansions on unemployment rates is practically zero at certain unemployment rates, slightly negative around the median, and becomes significantly negative in the highest conditional quantiles.

This study confirms that the output effects of fiscal policy changes are systematically larger in recessions than in booms. In addition, the effects of fiscal spending shocks were state-dependent and particularly large during the Great Recession and the associated zero lower bound on nominal interest rates.

Responses to fiscal shocks are highly nonlinear and vary depending on the conditional distribution of the outcome variable. The effects of fiscal spending shocks are significantly different across different quantiles of the output distribution, with larger effects observed at low-output quantiles than at high-output quantiles.

3. Methodology and Empirical findings

3.1 Data and Variables

The data used for this analysis were sourced exclusively from the publicly available economic indicators published on the website of the United States Federal Reserve Bank of St. Louis, commonly known as FRED. The timeline chosen for this framework starts in the first quarter of 1960 and concludes in the first quarter of 2023. The variables are those that apply the best for this scenario and, the time preference is a quarterly time series.

Following the structure of the Phillips Curve, the two important variables that build it are the inflation rate and the unemployment rate, or more precisely the unemployment gap, the difference between the unemployment rate and the non-cyclical rate of unemployment, i.e., the natural rate of unemployment. The FRED defines the natural rate of unemployment (NAIRU) as the rate of unemployment arising from all sources except fluctuations in aggregate demand.

As provided by the Fiscal Theory of Price Level, the key determinant of the long-run price level is the government's budget constraint. The theory assumes that the government must satisfy an intertemporal solvency constraint, ensuring that the present value of its primary surpluses is sufficient to cover the present value of its debt. [17] The fiscal dominance of the budget constraint continues because, following the Fiscal Theory, if fiscal policy is not sustainable, it can lead to inflation. For example, suppose the budget deficit becomes too large. In that case, it can lead to inflation as the government may resort to monetizing its debt to meet its obligations, causing an increase in the money supply and, consequently, inflation.

The standard model is then composed of inflation, the unemployment gap, and the budget deficit over the gross domestic product. The series presented has been then adjusted to seasonality. Seasonal adjustment is a statistical technique employed in time series analysis to remove the periodic and repetitive patterns, known as

seasonality. This process helps analysts and researchers to better understand the underlying trends and variations within the data. Seasonal adjustment is particularly important when dealing with economic, financial, and social data, where certain events or factors recur at specific times of the year, such as holidays. This adjustment is essential for obtaining a clearer picture of the true underlying dynamics of a time series and is a fundamental tool for making informed decisions and forecasts in various fields.

For this reason, I employed the X-13 ARIMA SEATS toolkit built by the US Census Bureau on the budget deficit over the GDP series. The seasonal adjustment is provided in the Appendix A.

Figure 3.1 shows the time series of inflation, unemployment gap, and budget deficit over GDP adjusted. The time series already shows patterns of non-linearity between the variables and different responses of the variables in the same time framework, as well as opposite variations of inflation and budget deficit.

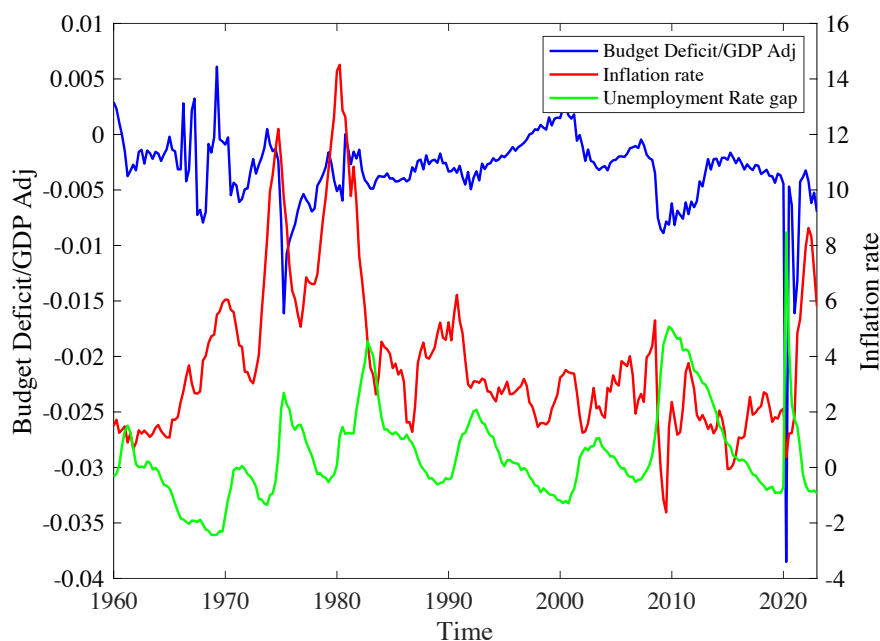


Figure 3.1: Raw Data

Figure 3.1 shows that during the volatile decade of the 1970s, countries confronted inflated increases in consumer prices, as well as low sitting rises in GDP and rampant unemployment, in a state known as stagflation. Several converging actions, including rising petroleum prices as a result of OPEC actions, an excess of monetary growth, and disruptions in the provision, had all contributed to the level of inflation.

Due to the economy's high inflation in the late 1970s, which continued throughout

Carter's administration, Federal Reserve Chairman Paul Volcker imposed extreme monetary adjustments in an aggressive effort to reduce inflation by significantly raising interest rates. This policy also succeeded in reducing inflation.

The "Great Inflation" of the late 1960s and early 1980s was characterized by sustained inflation as a result of a collaboration of loose monetary policy, rising oil prices, and a related increase in wages and consumer prices.

The United States experienced a brief period of moderate inflation in the early 1990s, which was influenced by factors such as the economic downturn following the savings and loan crisis, as well as the consequences of the Gulf War.

Following the devastation of the 2008 financial crisis, there were concerns about the potential rise from urging fiscal strategy actions such as reduced loan fees and quantitative easing. However, inflation remained relatively low in the years following the crisis. However, the recent COVID-19 pandemic has profoundly changed the scenario with increasing inflation starting in 2020. This new framework is still concerning policymakers and central banks for the possible economic downturn that this environment can bring.

In addition, the model is expanded for a robustness check with two more variables, the change in Federal government expenditure over GDP and the change in government debt over GDP.

The budget deficit is closely linked to the accumulation of government debt and spending as many researchers have shown. According to the fiscal theory, government debt dynamics play a crucial role in determining the price level. In this theory, government debt is seen as a promise to tax in the future to pay off current obligations. If markets believe that the government will, at some point, raise taxes significantly to service its debt, this expectation can lead to current inflation as people try to spend their money before taxes increase.

In other words, government debt can indirectly impact inflation through the expectations and behaviors of economic agents, rather than solely by increasing the money supply. Bianchi et al. (2022) suggest that only when the public debt can be successfully stabilized by credible future fiscal plans can the monetary authorities fully control inflation. [8] When the fiscal authority is not seen as entirely responsible for covering current fiscal imbalances, the private sector expects inflation to grow in order to guarantee the national debt's sustainability. As a result, a substantial budget imbalance paired with deteriorating fiscal status may cause inflation to deviate from the monetary authority's long-run goal.

Figure 3.2 shows that America's debt obligation and spending have changed dramatically in combination with changes in financial scenery. The nation's debt level was relatively low in the early 1960s, sitting at 290 billion.

Nonetheless, causes such as the Vietnam War and President Lyndon B. Johnson's implementation contributed significantly to the significant augmentation observed at the time. Because of escalating economic challenges paired with rising living costs and energy issues, the debt witnessed substantial increases during the 1970s.

During the late twentieth century, politicians responded with a series of responsive measures, such as tax cuts and increased defense spending, culminating in a significantly increased national debt. The nation witnessed a transition in the late 1990s when the rising economy and fiscal restraint demonstrated by the Clinton administration led to budgetary excesses and a decrease in liabilities owed.

However, a breaking point occurred in the early 2000s, pushed by tax cuts, increased defense spending, and the consequences of the 9/11 terrorist attacks. The 2008 financial crisis and its associated recession boosted debt accumulation. Responses to the COVID-19 epidemic have resulted in enormous spending in recent years, creating a considerable increase in the national debt, which will top 28 trillion in 2021.

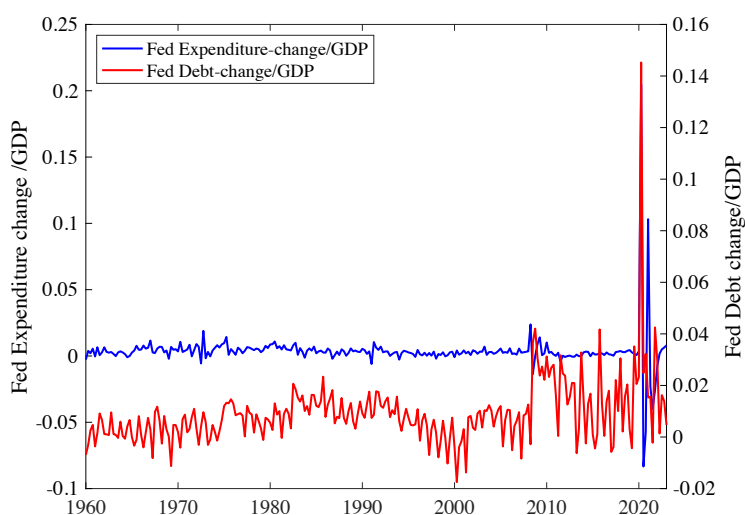


Figure 3.2: Government Expenditure change and Total Debt over GDP change

3.2 Quantile Regression analysis

Quantile regression, as presented by Koenker and Bassett (1978), models the relationship between the dependent variables and one or more variables, conditional on different quantiles of the unconditional distribution of the dependent variables. As presented before the main advantage of quantile regression is the possibility of analyzing the asymmetric effect and it helps to shed light on the sources of identification of the dependent variable tail. Koenker explains that just as the sample mean is obtained by minimizing the sum of squared residuals, the median can be obtained by minimizing the sum of absolute residuals. [21]

The linear absolute value function used in this minimization process ensures that the number of positive and negative residuals is equal, guaranteeing that there are an equal number of observations above and below the median. This symmetry property of the absolute value function helps in finding the median. For what concerns the other quantiles the quantile regression puts different weights on the errors in the function of the quantile under analysis.

Denote y_{t+h} as the inflation rate between t and $t+h$ and by x_t a vector that contains the control variables, including a constant. The quantile function $\hat{Q}_{y_{t+h}|x_t}(\cdot)$ for quantile $\tau \in (0, 1)$ is $\hat{Q}_\tau(y_t) = F^{-1}(\tau)$, where $F(\cdot)^{-1}$ is the conditional inverse cumulative distribution function (CDF) of y_t . The forecast horizon h is arbitrarily chosen to be equal to one and four to provide a short and medium-term view of this framework.

$$\hat{Q}_{y_{t+h}|x_t}(\tau|x_t) = x_t \hat{\beta}_\tau$$

The estimated quantile parameter $\hat{\beta}_\tau$ of the conditional quantile function is estimated by the quantile regression estimator which minimizes the sum of weighted absolute residuals,

$$\underset{\beta_\tau \in \mathbb{R}^k}{\operatorname{argmin}} \sum_{t=1}^{T-h} \left(\tau \cdot \mathbf{1}_{(y_{t+h} \geq x_t \beta)} |y_{t+h} - x_t \beta_\tau| + (1 - \tau) \cdot \mathbf{1}_{(y_{t+h} < x_t \beta)} |y_{t+h} - x_t \beta_\tau| \right)$$

where $y_{t+h} - x_t \beta_\tau$ are the residuals, and $\mathbf{1}$ is an indicator function that takes value one if the condition is satisfied. The absolute values of positive residuals are weighted by τ , while the absolute values of negative residuals are weighted by $1 - \tau$.

The estimated univariate quantile regression for the model shows different behavior of the variables at different quantiles, the fifth, the fiftieth, the OLS regression line, and the ninety-fifth quantile. Figure 3.3 shows that at different quantiles the inflation is behaving differently but with a strong upward trend.

Moreover, the relationship between inflation and itself shows an estimated positive slope, especially for the upper quantile both at one and four quarters ahead.

The estimated slope for the median and OLS does not vary much, indicating that at the median, the values of a classical regression are well-behaving.

Figure 3.3 also shows an interesting behavior of the quantile regression. Looking at the band 0-2% current quarter inflation rate, the projected inflation one quarter ahead is smaller than the one four quarters at high quantiles. The estimated slope at lower quantiles is similar between the two. This finding is also consistent with the literature, especially with Loria et al. (2021) who pointed out the possible creation of upside risks to the inflation outlook. [26]

Interestingly, the relationship between inflation and the unemployment rate gap behaves differently. Looking at figure 3.4 at different quantiles, the estimated slope is drastically divergent. The pattern is similar at both one-quarter and four-quarters ahead but with a higher slope in the latter. At ninety-fifth quantiles the higher the unemployment rate gap, the lower the estimated inflation rate. At lower quantiles, the estimated slope is less steep but still indicates a negative relationship between variables.

The quantile regression lines' spacing also demonstrates the right-skewed conditional distribution of inflation. The lower quantiles' narrower spacing indicates a higher density and a shorter lower tail, whereas the upper quantiles' wider spacing indicates a lower density and a longer upper tail.

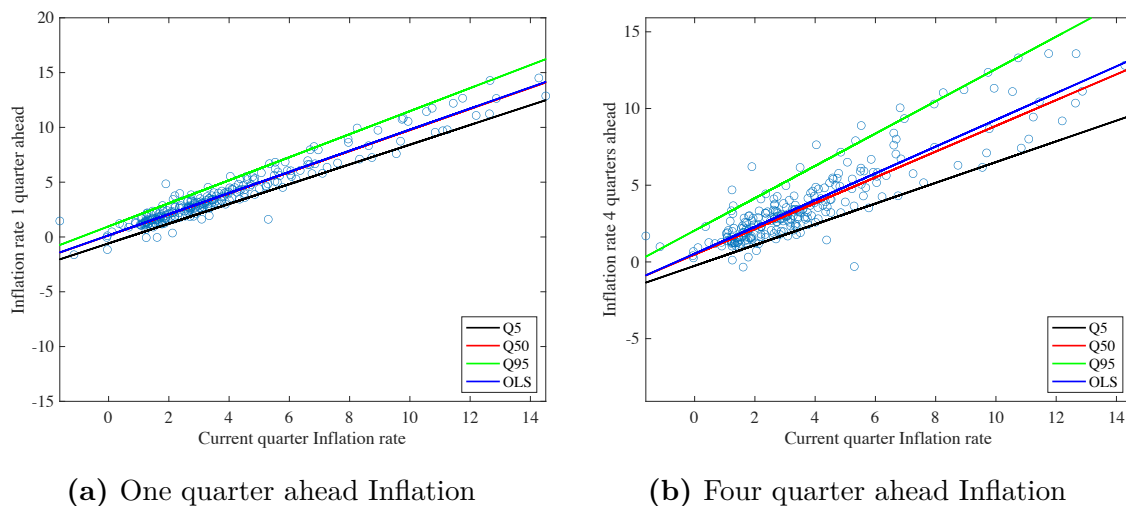


Figure 3.3: Univariate quantile regression of Inflation on current Inflation

Figure 3.5 shows a different pattern from the one above, indicating that quantile regression is better at examining the relationship between inflation and budget deficit than OLS. The latter is lacking in indicating a different slope between different quantiles.

The estimated slope at the upper quantiles shows a positive relationship, whereas the

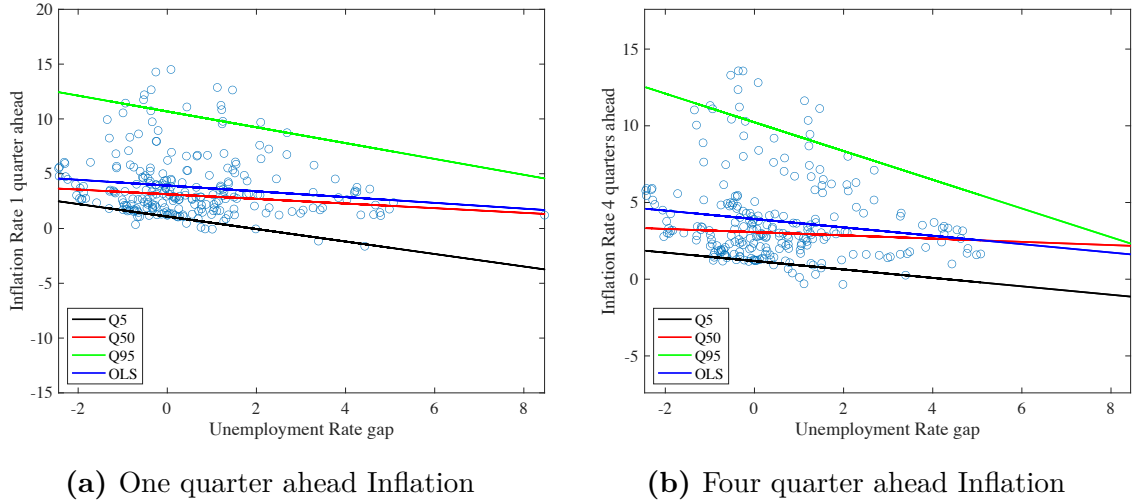


Figure 3.4: Univariate quantile regression of Inflation on current Unemployment rate gap

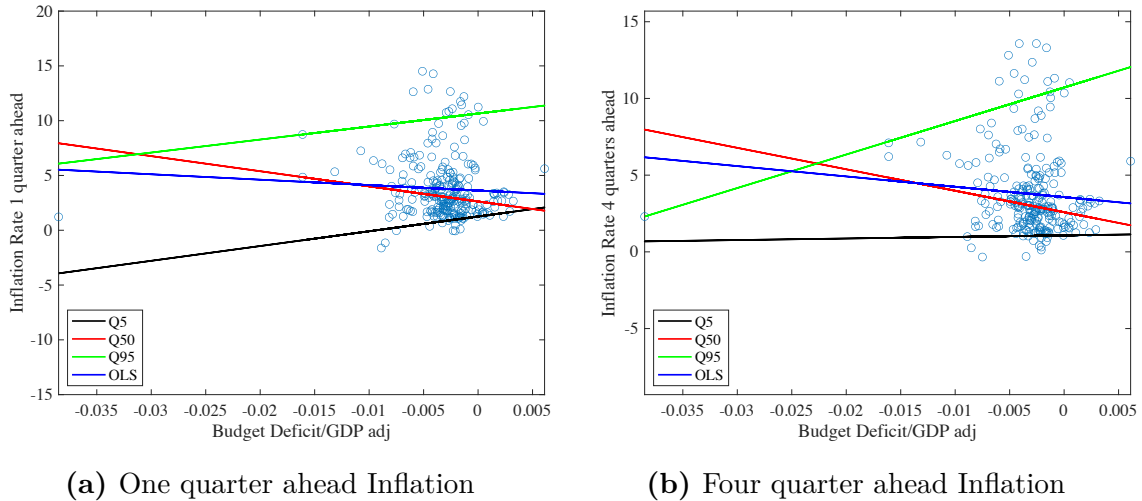


Figure 3.5: Univariate quantile regression of Inflation on current Budget deficit over GDP adjusted

median as the OLS estimated slope is negative.

Interestingly, the lower quantiles have a positive relationship with the variables.

As indicated above, the quantile regression lines' spacing also demonstrates the right-skewed conditional distribution of inflation. The lower quantile spacing is narrower, with respect to the upper, indicating a lower density and a lower left tail. In comparison, the wider spacing of the upper quantiles has a lower density and a longer upper tail.

Figure 3.5 shows that a deficit in the current quarter would produce a positive relationship with the inflation rate just at lower and upper quantiles at one quarter ahead, hence increasing the possibilities of upside inflation risks. In the case of a small surplus, the estimated slope clearly shows a negative relationship conditional at the median at

both periods.

I analyzed the estimated coefficient at different quantile levels, which shows an intricate response between variables. These figures show confidence bounds at 95% confidence level computed using a 1000 bootstrapped sample. The confidence bounds are useful because they test the null hypothesis that the generated data can be modeled with a linear VAR model with four lags. Following Adrian et al. (2019), the VAR is constructed with Gaussian innovations and a constant that uses all data.

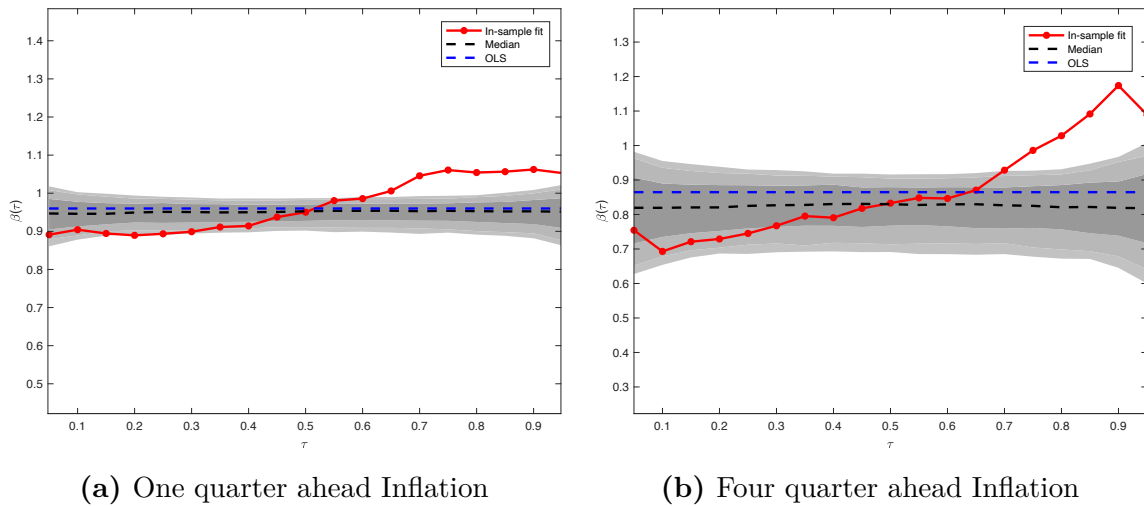


Figure 3.6: Estimated Quantile regression coefficients of inflation on inflation

Figure 3.6 shows the coefficients of the quantile regression with respect to the linear regression and the median. The estimated coefficients are stable throughout all quantiles in figure 3.6a. Figure 3.6b shows that estimated coefficients vary considerably from lower to upper quantiles, especially the latter falling outside the confidence bounds, rejecting the hypothesis of the true generating process as a linear model.

The negative coefficient in figure 3.7 for lower quantiles suggests that, at the lower end of the inflation distribution, an increase in the unemployment gap is associated with a decrease in inflation. This could be indicative of a situation where economic downturns and higher unemployment lead to reduced consumer demand and spending, putting downward pressure on prices. The evidence so far shows that the Phillips Curve exhibits a non-linear relationship across quantiles. This refers to the asymmetries in the response of inflation to low versus high rates of unemployment or economic activity. [12] These nonlinearities were already present in the original estimated regressions by Phillips and Lipsey. [25]

Conversely, the positive coefficient for higher quantiles indicates that, at the upper end of the inflation distribution, an increase in the unemployment gap is linked to an

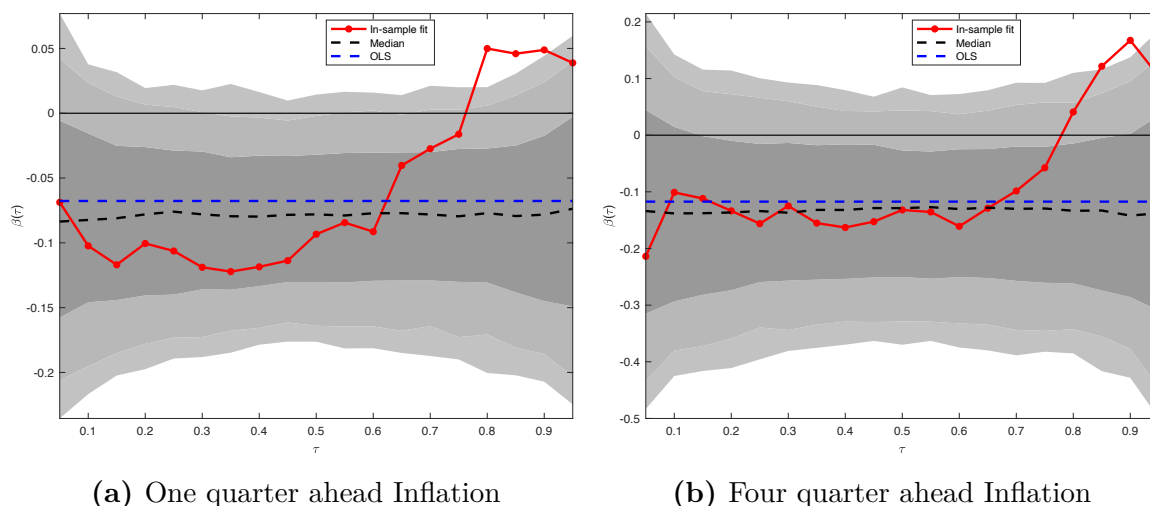


Figure 3.7: Estimated Quantile regression coefficients of inflation on Unemployment gap

increase in inflation. This might be reflective of conditions where inflation is being driven by factors such as cost-push pressures or supply-side constraints, and higher unemployment exacerbates those factors.

Another piece of information that arises especially in figure 3.7a is that at high quantiles the estimated coefficient falls outside the bands being an indicator of the non-linear relationship of the variables.

The negative coefficient for the unemployment gap at lower quantiles might be interpreted as the Fiscal theory of price level suggests that periods where fiscal policies aiming at reducing unemployment lead to lower levels of inflation due to increased demand without immediate inflationary pressures.

The positive coefficient for the unemployment gap at higher quantiles could be consistent with the idea that during periods of high inflation, fiscal measures targeting unemployment may contribute to inflationary pressures, especially if such policies lead to increased government spending or expansionary fiscal policies. The behavior of the non-linear relationship between inflation and other predictive variables is even clearer when looking at figure 3.8, where there is evidence of inflationary behavior of the budget deficits at high quantiles.

This may be one of the core findings of this study, showing the relationship between inflation and the budget deficit over GDP. Figures 3.8a and 3.8b are clear indicators of slope change at all quantiles, especially at the upper quantiles.

Importantly, confronting figure 3.6 and 3.8 is possible to assert that most of the information on future inflation change arises from the predictive power of the deficit variable. The negative estimated slope at high quantiles is also consistent with the literature on this matter, whereas the findings on the lower quantiles four quarters

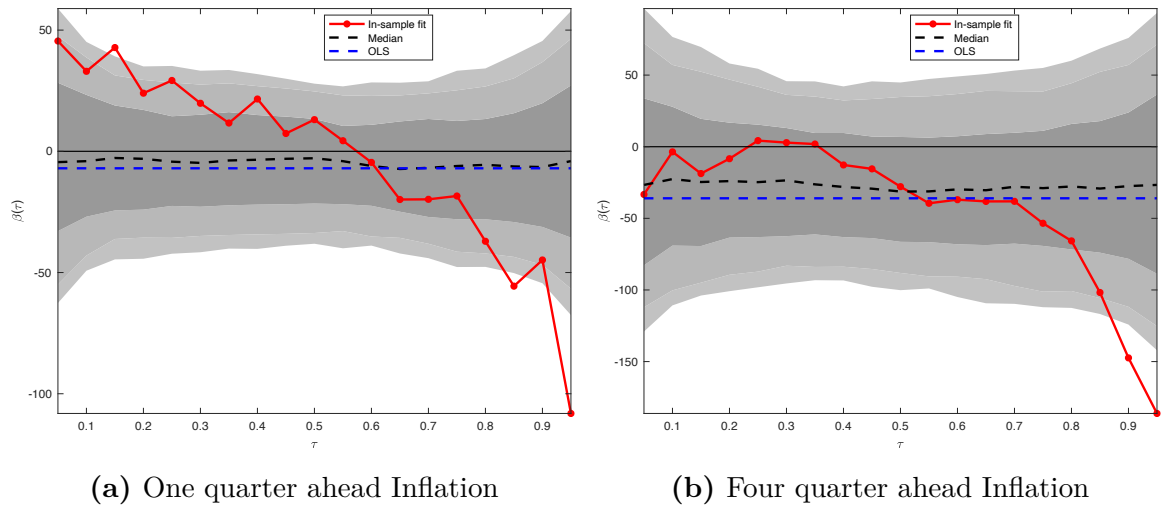


Figure 3.8: Estimated Quantile regression coefficients of inflation on budget deficit over GDP

ahead are consistent with the literature regarding the small significance of the impact. The basic linear regression and median estimation fail to represent the significance of this relationship, and it is worth noting that while OLS estimates are consistent and negative, the coefficient at the upper quantiles is drastically lower, thereby substantially increasing the odds of downside inflation risks.

The positive coefficient for lower quantiles in figure 3.8a suggests that, at the lower end of the inflation distribution, an increase in the budget deficit is associated with an increase in inflation. This could be due to specific conditions prevalent in the lower percentiles, such as economic fragility, low investor confidence, or other factors that amplify the impact of budget deficits on inflation.

Conversely, the negative coefficient for higher quantiles indicates that, at the upper end of the inflation distribution, an increase in the budget deficit is linked to a decrease in inflation. This might be reflective of situations where higher levels of inflation are associated with different economic dynamics, such as excess demand, where an increased budget deficit could alleviate inflationary pressures.

The positive coefficient for lower quantiles may be consistent with the idea that during economic downturns or periods of low inflation, expansionary fiscal policies, reflected in a higher budget deficit, have an inflationary impact. Fiscal measures intended to increase economic activity through stimulating demand during periods of downturn productive capacity are prone to generating price pressures that can lead to inflationary risks.

The negative coefficient for higher quantiles could be interpreted in the same context of the Fiscal theory of price level as reflecting the role of fiscal policies in controlling inflation during economic expansions. In this scenario, contractionary fiscal policies that

project a lower budget deficit might be implemented to reduce inflationary pressures. This may be good information for predicting the tail outcomes of inflation, given that the estimated coefficient displays extreme values at high quantiles that may affect the tail behavior of the inflation distribution conditional on the variable of fiscal stance.

4. Measures of Vulnerability

4.1 Conditional Inflation Distribution

Following the two-step process from Adrian et al. (2019), the first step is the quantile regression which has provided a clear understanding of the Augmented Phillips curve that has shown non-linear behavior throughout its framework.[1] This section describes the econometric specification that allows to linking of the fiscal condition with risks to the inflation outlook. This framework allows to study the core determinants of inflation at risk. In this case, Inflation risks are then quantified by how much the tails of the inflation distribution change as fiscal circumstances change.

Sticking to the two-step process, the second process is to smooth the quantile function using the skewed t -distribution of Azzalini and Capitanio (2003) in order to recover a probability density function. This fitting process is done by minimizing the square distance between the estimated quantile function and the quantile function of the implied distribution.[4]

Mitchel et al. (2023) clarify that this second step contrasts with the non-parametric model of the first part because it imposes a global density to a precise quantile, while they argue that it would be a better model if they would assume just a local uniformity between the quantile forecast. [19]

The skewed t -distribution is characterized by employing four parameters that adjust the shape of the probability density function $t(\cdot)$ and the conditional density function $T(\cdot)$.

$$f(y; \mu; \sigma; \alpha; \nu) = \frac{2}{\sigma} t\left(\frac{y - \mu}{\sigma}; \nu\right) T\left(\alpha \frac{y - \mu}{\sigma} \sqrt{\frac{\nu + 1}{\nu + \left(\frac{y - \mu}{\sigma}\right)^2}}; \nu + 1\right)$$

The parameters on the left side of the equation describe the location μ , the scale σ , the fatness ν , and the shape α . The α parameter also describes the shape or the skewness effect of the Conditional density function on the probability density function. When the parameter $\alpha = 0$, the skewed t -distribution identify as a Student t distribution. Another particularity of this case is if $\alpha = 0$ and $\nu = \infty$ in which case the distribution transforms itself into a Gaussian with mean μ and standard deviation σ , while the

other interesting case is when $\nu = \infty$ and $\alpha \neq 0$ where the distribution transform into a skewed normal. The fitting process is identified by matching the 0.05, 0.25, 0.75, and 0.95 quantiles and choosing the four parameters of the given skewed t -distribution that minimize the squared difference between the estimated quantile function of the chapter above $\hat{Q}_{y_{t+h}|x_t}(\tau)$ and the quantile function of the new skewed distribution $F^{-1}(\tau; \mu; \sigma, \alpha, \nu)$. This allows the production of an estimated conditional distribution of inflation visible in figure 4.1.

$$\{\hat{\mu}_{t+h}; \hat{\sigma}_{t+h}; \hat{\alpha}_{t+h}; \hat{\nu}_{t+h};\} = \underset{\mu, \sigma, \alpha, \nu}{\operatorname{argmin}} \sum_{\tau} \left(\hat{Q}_{y_{t+h}|x_t}(\tau|x_t) - F^{-1}(\tau; \mu; \sigma, \alpha, \nu) \right)^2$$

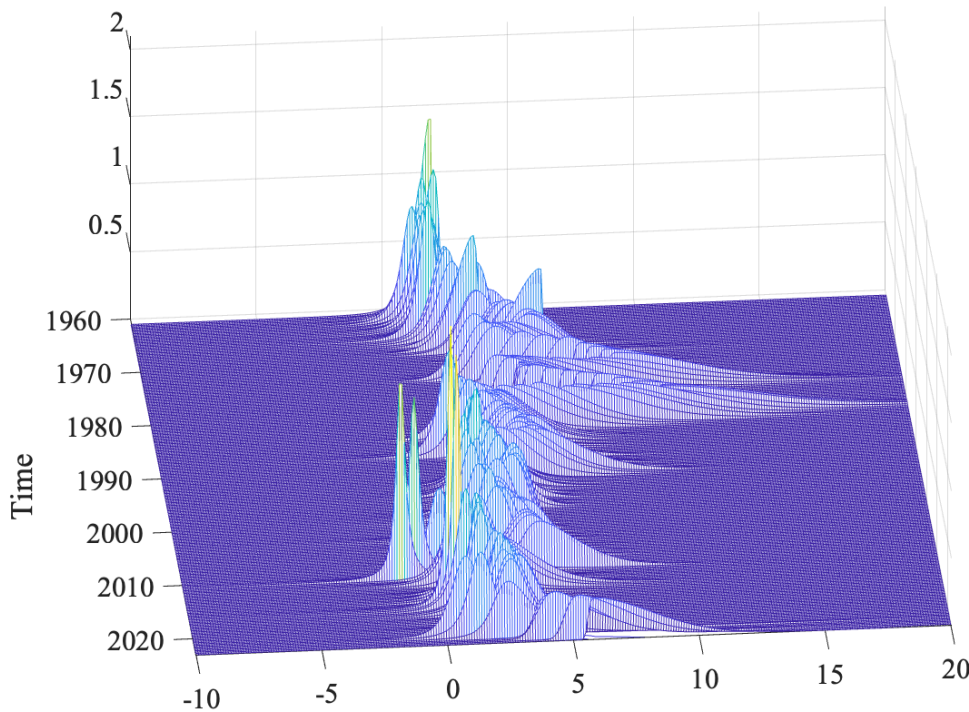


Figure 4.1: One-year-ahead predictive distribution of Inflation based on quantile regression with inflation, unemployment gap, and budget deficit over GDP adjusted.

The full distribution of future inflation based on the fiscal environment allows for an estimate of the possibility of potential economic activity and the estimate of the possible upside risk to inflation. A feature of this figure is that the distribution evolves and the location parameter changes over time. Another feature that strikes is the particular shape of the distribution during times of crisis. The figure shows the right-skewed distributions during times of recession and the symmetric curves during periods of growth. It is also clearly visible the spikes of the distribution, especially during the financial crisis and the OPEC crisis.

Interestingly, the distribution shows the same behavior as that analyzed in the quantile regression in the previous chapter. The variables generally showed a pattern where they suggested a narrower left tail and a longer and wider right tail. Moreover, the left tail of the distribution is characterized by stability, whereas the right tail evolution shows a pattern of volatility associated with time of economic recession.

Another core finding from this study is that the asymmetry that arises from this figure indicates that the upside risk to inflation is more volatile than the downside risk to inflation. This volatility is correlated with both by an increase in the budget deficit and an increment in the unemployment gap. The associated times when the conditional distribution does not show right tail volatility, but the distribution produces high spikes may not estimate the effectively realized value. This concern has been described by many studies, even by some IMF working papers that show the predictive power of this model may not be so strong. [3]

The results that arise from this study imply that when inflation is already elevated, factors, such as budget deficit and unemployment gap, appear to exercise a more significant influence on the overall inflationary distribution. The evidence for this statement comes from the study of the conditional distribution during periods of recession. While many recessions can affect inflation, especially in the short period, the goal of a credible fiscal framework is to bring low and stable inflation. For this reason, it is interesting to employ this framework for tracking risk evidence during and after the COVID-19 pandemic.

Figure 4.2 shows the different fitted conditional probability densities of inflation for three different quarters that describe the framework before COVID-19 hit, during the crisis, and the last available sample of the first quarter of 2023.

Comparing the findings enables to assert the evolution of the upside risk to inflation throughout the crisis. Starting with the second quarter of 2020, the United States was marked by an unprecedented economic disruption due to the COVID-19 pandemic. As the virus spread rapidly, businesses shuttered, unemployment soared, and economic activity plummeted. In response to the crisis, the government implemented some fiscal measures to mitigate the severe impacts, including the CARES Act, a historic 2 trillion dollar relief package. This legislation aimed to provide financial aid to individuals, support struggling businesses, and strengthen the healthcare systems. However, despite these efforts, the economic fallout was substantial, with GDP contracting sharply. The fiscal response during this period highlighted the urgency of addressing both the public health crisis and the resulting economic challenges. Through decisive intervention, the Federal Reserve pursued an array of vigorous monetary policy strategies, drastically reducing interest rates and enacting diverse measures intended to restore equilibrium in turbulent financial sectors.

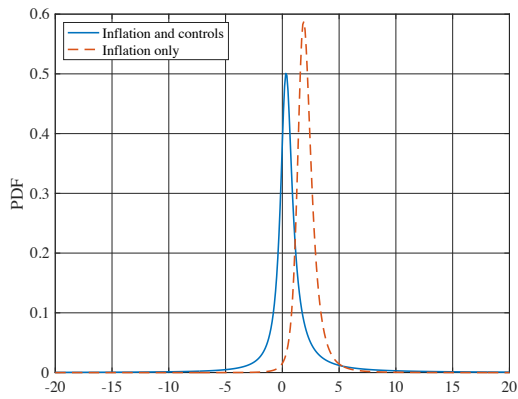
Figure 4.2a and 4.2b show the evolution of the distribution that concentrates on the left side for the one quarter ahead and on the right side for the four quarters ahead. Interestingly, the distributions of inflation conditional on the parameters of fiscal stance at the start of 2020 are different from the distribution where these parameters are not considered. The latter presents less variance and smaller skewness than the former. Indeed, the selected sample shows that the conditional distribution shifts to the left because of the realization of fiscal conditions, confirming that there is a higher likelihood of negative realizations.

Having these characteristics, the conditional distribution is less accurate in this period, considering that the realized inflation rate was located exactly where the unconditional distribution is. This result explains that in this particular situation, the power of predictability is less accurate when the parameters of interest are considered.

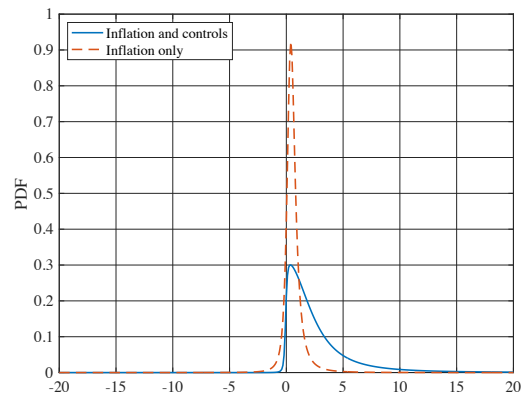
The story behind figure 4.2c and 4.2d is different and changes the point of focus. These figures talk about the center part of the COVID-19 pandemic when the US government experienced a complex fiscal landscape where the unemployment gap registered a positive sign and the repercussions were already ongoing. The Biden administration implemented various stimulus measures, including the American Rescue Plan Act, aimed at providing relief to individuals. These initiatives contributed to a gradual economic recovery. However, concerns about inflation and rising government debt remain as massive spending fights with the need for fiscal responsibility. The Federal Reserve continued its accommodating monetary policy to support the rebound.

Overall, the fiscal situation reflected a delicate balancing act between stimulating economic recovery and addressing the challenges posed by the pandemic-induced downturn. Figures 4.2c and 4.2d show the building up of upside risk to inflation given that the conditional distribution is characterized by higher variance, and positive skewness, and a location parameter far off the unconditional distribution. This can be shown in Appendix B where the figure displayed explains the pattern of the parameters over time. Indeed, the realized inflation that these figures estimate lies below the conditional density function, increasing the predictability power of the model. Therefore, the conditional distributions, in this time sample, shift to the right during the recession, thereby realizing a higher likelihood of looking at positive realizations of inflation.

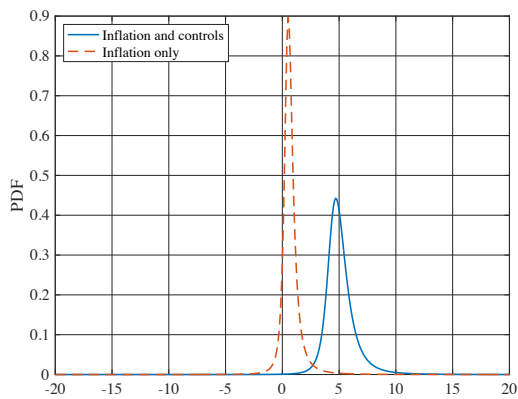
The last row of figure 4.2 shows the probability density function of the last quarter in the sample. The situation concerning this figure is similar to that above, in fact the the probability densities are characterized by a higher mean, positive skewness, and higher variance. The combination of these realizations results in the probability density function having a higher likelihood of looking at positive realizations of inflation.



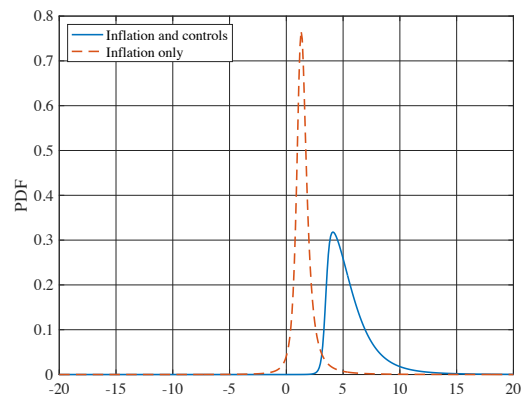
(a) One quarter ahead 2020:Q2



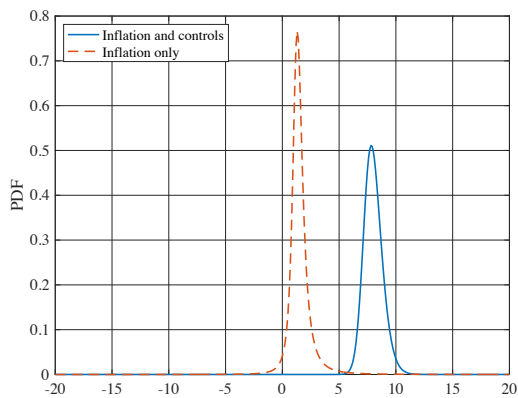
(b) 1 year ahead 2020:Q2



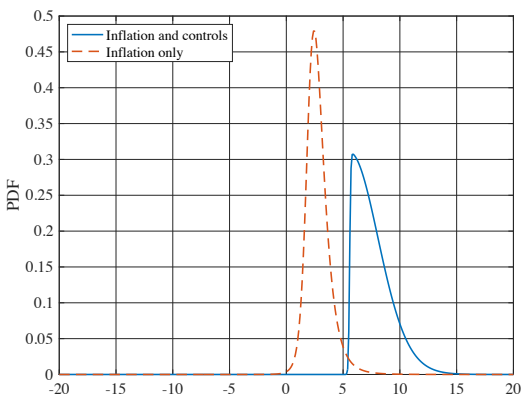
(c) One quarter ahead 2021:Q2



(d) 1 year ahead 2021:Q2



(e) One quarter ahead 2023:Q1



(f) 1 year ahead 2023:Q1

Figure 4.2: Probability densities of selected time periods of Inflation

Combining the findings in the probability densities figures it is possible to assess that during the COVID-19 pandemic, there is evidence from the conditional distribution of a building up of upside risk to inflation.

4.2 Vulnerability

Policymakers are now more than ever interested in the assessment of inflation vulnerability, especially they are interested in quantifying the vulnerability when an unexpected shock surprises the system. Following the work of Adrian et al. (2019), the measures of vulnerability under analysis are upside entropy and expected longrise. [1] The concept of upside entropy was introduced by Kullback and Leibler. [23] They studied the divergence also known as relative entropy as a measure of how one conditional probability distribution diverges from an unconditional probability distribution. This divergence is a measure of the non-symmetric difference between the two distributions.[2] In information theory, the concept of information content refers to the amount of surprise or uncertainty associated with an event. If the information content of the unemployment gap and budget deficit is high, it implies that these factors provide valuable and perhaps unexpected information about inflation behavior.

Both instruments are used to shed light on the right tail behavior of the distribution but with different meanings. they focus on the measure of upside vulnerability as the extra probability mass assigned by the conditional density to extreme right and left tail realization, relative to the likelihood of these occurrences under the unconditional density. It is then possible to compare the probability assigned to the same outcome in the case of the unconditional distribution and in the case of the conditional distribution. This allows to asses whether the conditional distribution of inflation implies more or less vulnerability than the unconditional.

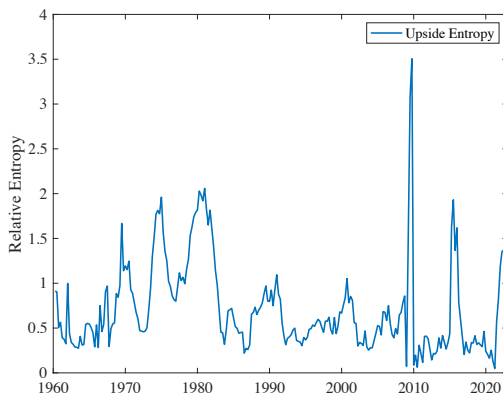
Denote with \hat{g}_{t+h} the unconditional density computed by matching the estimated skewed t-distribution of inflation by $\hat{f}_{y_{t+h}|x_t} = (y; \hat{\mu}_{t+h}; \hat{\sigma}_{t+h}; \hat{\alpha}_{t+h}; \hat{\nu}_{t+h};)$ with the empirical unconditional distribution of inflation.

$$\mathcal{L}_t^U \left(\hat{f}_{y_{t+h}|x_t}; \hat{g}_{t+h} \right) = - \int_{\hat{F}_{y_{t+h}|x_t}^{-1}(0.5|x_t)}^{\infty} \left(\log \hat{g}_{y_{t+h}}(y) - \log \hat{f}_{y_{t+h}|x_t}(y|x_t) \right) \hat{f}_{y_{t+h}|x_t}(y|x_t) dy$$

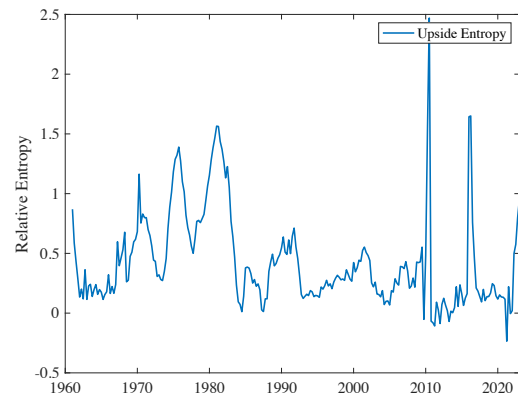
where $\hat{F}_{y_{t+h}|x_t}(y|x_t)$ is the cumulative distribution associated with $\hat{f}_{y_{t+h}|x_t}(y|x_t)$ and $\hat{F}_{y_{t+h}|x_t}^{-1}(0.5|x_t)$ is the conditional median. \mathcal{L}_t^U measures the upside entropy, which measures the separation between the conditional and unconditional density when is above the median. This instrument should be a signal that quantifies the excess of the right-tailed behavior of the conditional distribution compared to the right-tailed behavior of

the unconditional. Graphically, when the curve is high, the conditional density gives a higher probability of more extreme right-tail events than the unconditional density. Moreover, it appears to have a larger right skewness in the conditional density function. The Longrise LR_{t+h} instead is a concept closer in specific to the Value at risk statistic, because for a target likelihood π that an increase will occur it evaluates the entity's potential for increase. The measure is calculated as the integral of the inverse cumulative distribution function $\hat{F}_{y_{t+h}|x_t}^{-1}$ in the right part of the tail, specifically from the 95th percentile to the 100th percentile.

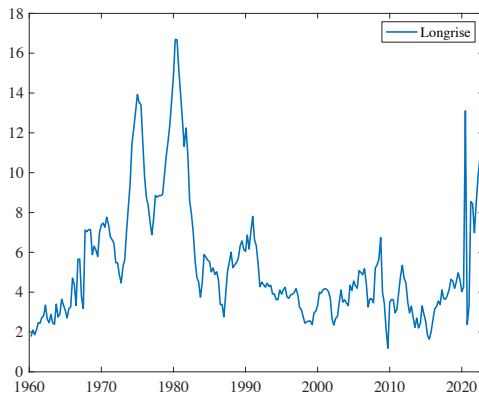
$$LR_{t+h} = \frac{1}{\pi} \int_{1-\pi}^1 \hat{F}_{y_{t+h}|x_t}^{-1}(\tau|x_t) d\tau$$



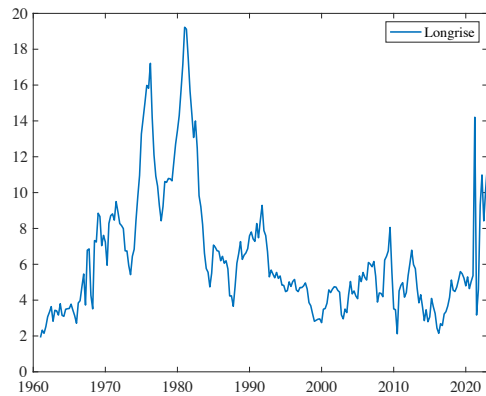
(a) One quarter ahead Upside Entropy



(b) One year ahead Upside Entropy



(c) 95% Expected Longrise one quarter ahead



(d) 95% Expected Longrise one year ahead

Figure 4.3: Upside Entropy and Expected Longrise

Graphically, the expected longrise curve shows a high value if both the conditional and unconditional distributions are positively skewed.

Starting with the first row of figure 4.3, it is possible to appreciate the information provided by the variables. Figures 4.3a and 4.3b show different spikes throughout the

time series, especially located in times of recession. Between 2010 and 2020, the function created noticeable spikes, showing a strong difference between the conditional and the unconditional. A reason behind this behavior is the sharp decrease in the unemployment gap and the increase in the budget deficit.

This volatility confirms the information provided by the budget deficit and unemployment gap, which would have otherwise created a more stable curve. In other words, the conditional distribution behaves differently than the unconditional distribution and this shows movement in the right tail that explains the upside risk to inflation.

The second row of figure 4.3 shows the expected longrise of inflation that displays in both figures extreme volatility with spikes of 16%. This volatility is proof of the information content of the conditioning variable that changes inflation expectations throughout the series.

When comparing vulnerability metrics, the most evident divergence occurred during the Great Recession, when the upside entropy showed a large increase while the predicted longrise did not. Such differences arise from the information content of the variable of fiscal stance but the vulnerability concerning an upside risk to inflation during the period was already strong. Contrarily, the expected longrise displays less variability during that time, and this substantial change encourages the use of this last metric for understanding the upside risk to inflation.

5. Robustness checks

The results that have been derived so far are based on a model based on an Augmented Phillips curve built by the inflation rate, unemployment gap, and budget deficit. Given that the primary goal of this study is to shed light on the upside risk to inflation caused by fiscal policy, it is possible that the only use of budget deficit and unemployment gap could lack of explanatory power.

Testing for robustness also means looking at the model considered and assess if this particular framework changes the results if the model is expanded. The model is expanded for a robustness check with two variables: the change in government expenditure over GDP and the change in government debt over GDP.

This chapter goes through the methodology that has been used in the previous part, with the main purpose of increasing the explanatory power of the model and looking at potential differences brought about by these new variables. The robustness check developed in this chapter is constructed to examine how changes in key parameters affect the results and, this should help to identify whether the findings are highly dependent on specific assumptions.

The standard model, as seen in Chapter 3, has produced very interesting causality between the predictive variables. Continuing on the analysis of the univariate quantile regression, it is important to analyze how government expenditure and debt change throughout the quantiles.

Figure 5.1 shows the different estimated slopes for both one quarter and four quarters ahead. Interestingly, the upper quantile at both periods behaves in the same way, creating a wider spacing between lines, indicating a lower density and a longer upper tail. In comparison, the narrower space for the lower quantiles shows a high density and a short lower tail. This is caused by the concentrations in the data and by the presence of few outliers. The increase in government expenditure shows a positive relationship just at the upper quantile, while the estimated slope for the lower quantile and OLS are similar.

Concerning the debt level figure 5.2 clearly shows a negative relationship at all quantiles. However, the spacing between the different slope lines is different from that in the previous case, where a narrower spacing of the upper quantiles indicates a high density

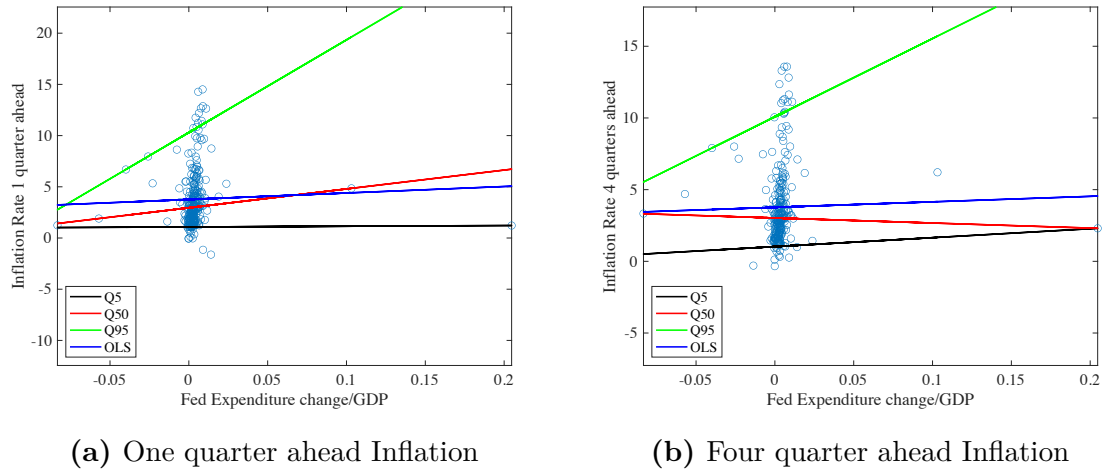


Figure 5.1: Univariate quantile regression of Inflation on current government Expenditure change over GDP

and a short upper tail. The lower quantiles are wider, indicating, a lower density and a longer left tail. The different estimated quantile slopes at both periods show a negative relationship, so an increase in the debt level should decrease the inflation conditional at all quantiles. The estimated coefficient for both the change in government expenditure and debt level shed light on their impact on inflation.

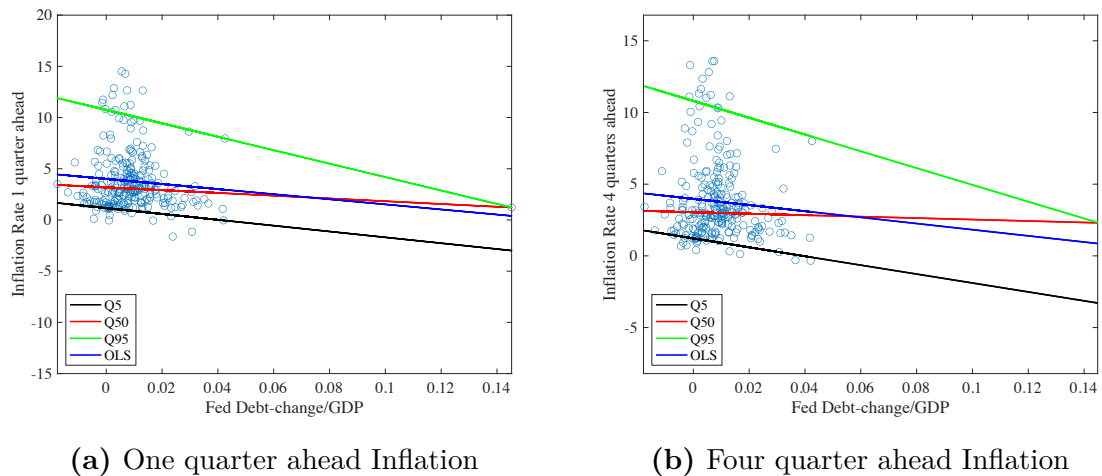


Figure 5.2: Univariate quantile regression of Inflation on current government Debt change over GDP

Figure 5.3 shows the estimated coefficient at all quantiles for the change in government expenditure. Interestingly, both figures show positive estimated coefficients with a decreasing trend with respect to the conditional quantile, but figure 5.4a shows an increasing coefficient in the last decile while figure 5.4b is decreasing.

The positive coefficient for all quantiles suggests that irrespective of the inflation level

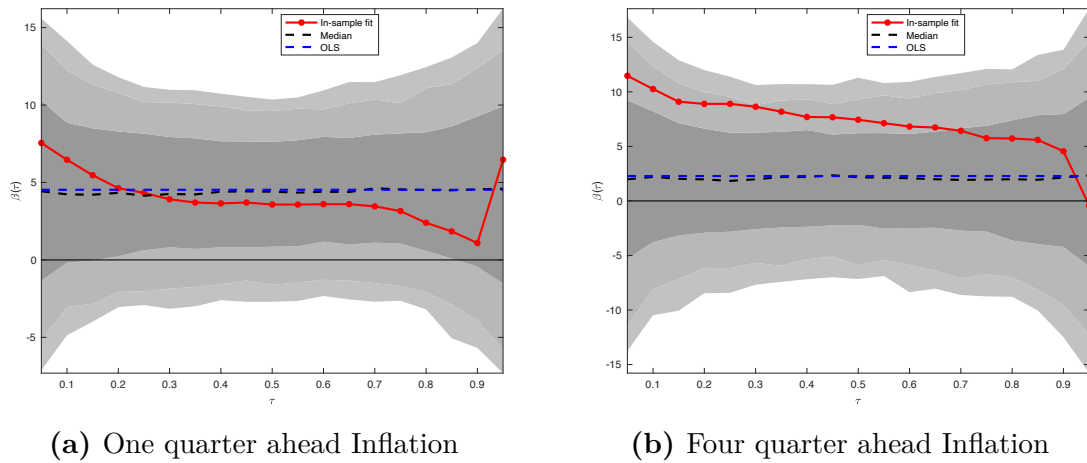


Figure 5.3: Estimated Quantile regression coefficients of inflation on Government Expenditure change over GDP

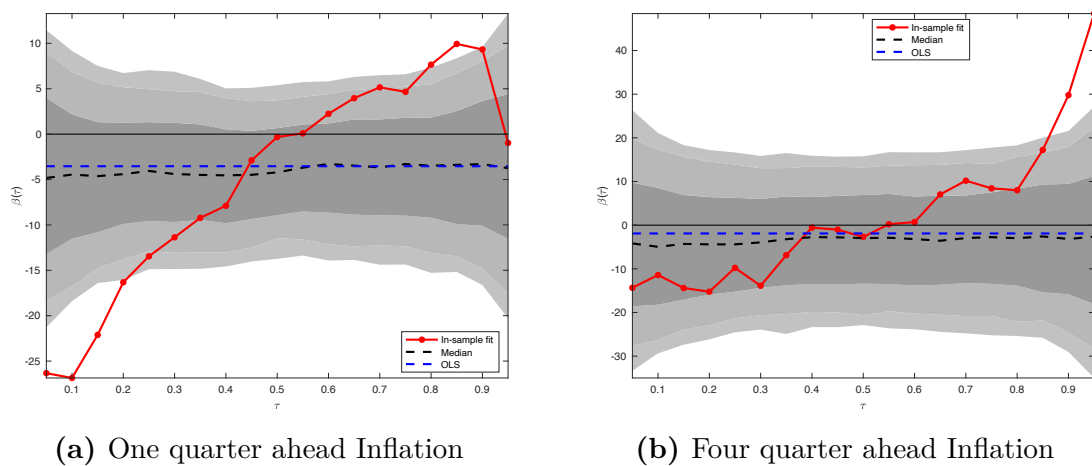


Figure 5.4: Estimated Quantile regression coefficients of inflation on government Debt change over GDP

conditional on its quantile, an increase in government expenditure is associated with an increase in inflation. This is indicative of an overall expansionary fiscal policy, where higher government spending stimulates economic activity and contributes to upward pressure on prices across the entire inflation spectrum.

Figure 5.4 shows the estimated coefficients for the Debt level and the behavior of the curve changes drastically at all quantiles. It is worth noting that at lower quantiles in both periods the estimated coefficients are negative and increasing until they reach the median level where the impact is insignificant.

Confronting the univariate quantile regression and the estimated coefficient regarding the debt, it strikes that the coefficient is increasing across quantiles but when it reaches the 95th quantile it shows a sharp drop causing a decrease in inflation when the debt change increase. The relationship between inflation and Debt becomes no more

predictable with a linear model at high quantiles for the four-period panel, because it falls outside the boundaries of the confidence bounds determining the non-linearity of the relationship.

The negative coefficient for lower quantiles might suggest that, at the lower end of the inflation distribution, an increase in the debt coefficient could imply that higher indebtedness is related to an increase in inflation. This could be indicative of situations where fiscal consolidation, often accompanied by reducing debt or controlling its growth, is implemented during economic downturns or periods of low inflation to address economic challenges.

The positive coefficient for higher quantiles implies that at the upper end of the inflation distribution, an increase in debt is linked to an increase in inflation. This might be reflective of situations where fiscal expansion, often associated with higher debt levels, is implemented during economic expansions to stimulate economic activity and address inflationary pressures.

Considering the impact of these two variables on inflation, the two-step procedure of

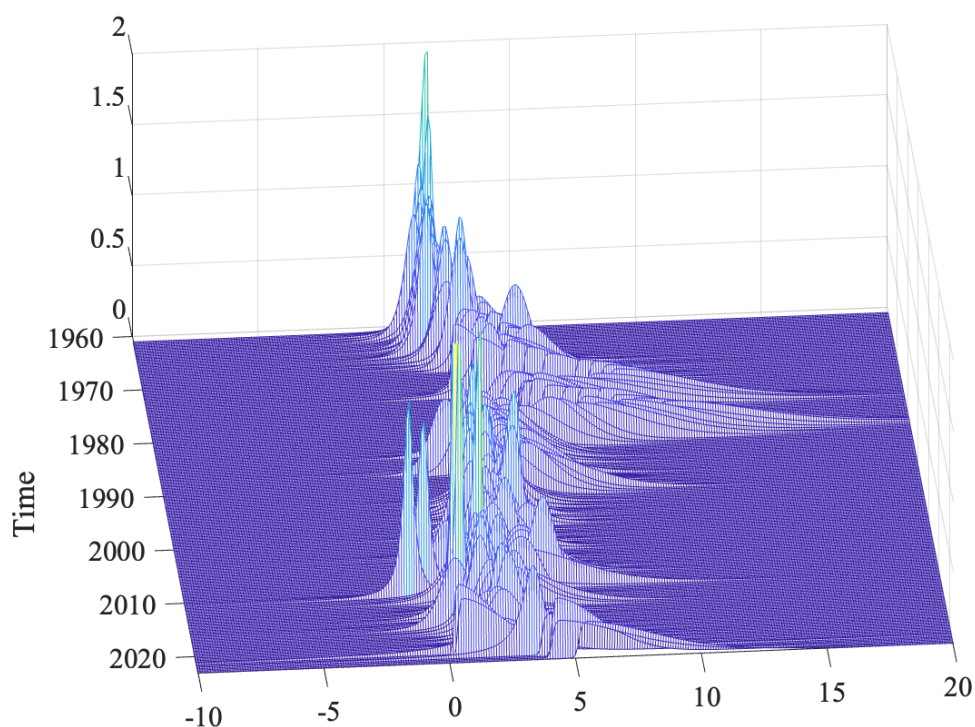


Figure 5.5: One-year-ahead predictive distribution of Inflation based on quantile regression with inflation, unemployment gap, budget deficit over GDP adjusted, change in government expenditure over GDP, and change in debt over GDP.

Adrian et al (2019) follows the fitting process using to smooth the quantile function using the skewed t-distribution. The quantile model is now extended so the fitting

process produces a predictive distribution of inflation that shows the impact of these new variables. The distribution of future inflation in figure 5.5 is characterized by a feature similar to that seen before. The similarities can be summarized in the same location parameter that evolves over time, and the right-skewed distribution during times of recession. Another feature is that during time of growth, the distribution takes a symmetric form.

Concerning the differences, the figure shows higher spikes, and symmetric distribution in the first decade. The shape evolves and moves to the right, creating a larger right skewness. Going forward in time during the financial crisis, the distribution shows higher spikes with smaller variances. The shape is more symmetric and it ends with right-skewed distributions during the COVID-19 pandemic.

The left part of the tail has much more stability and a narrower tail than the right one. Moreover, it shows a right-skewed distribution pattern throughout time, with implications of upside risk given the contribution of the new variables.

In summary, the differences from figure 4.1 are significant only for small periods, while the main behavior of the conditional distribution is similar, concluding that upside risk to inflation varies more strongly than downside risk.

The fitted conditional probability densities in figure 5.6 also share similarities with those in the previous chapter. Comparing the evolution of the density function through time and the increase in specification allows to assert the evolution of the upside risk to inflation. The differences can be seen already in figure 5.6b, where the conditional probability density function has a more symmetric shape and the location parameter moved to the right. Another difference is observed in figure 5.6d because the shape of the function presents stronger right skewness.

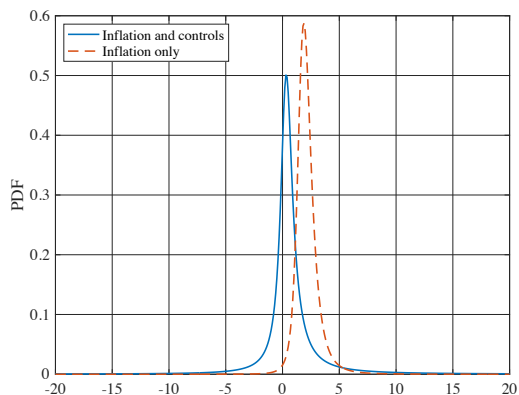
Interestingly, 5.6e presents a left skewness, whereas the model in the previous chapter had a symmetric shape in the same time sample. This result is brought about by a strong decrease in debt considering also that the debt change estimated slope is decreasing at all quantiles. Overall, the shape of the probability density function remains similar, but this framework presents a higher variance at all time periods.

Concluding the robustness check, it is interesting to examine the produced differences in the right tail behavior and those instruments that measure this change.

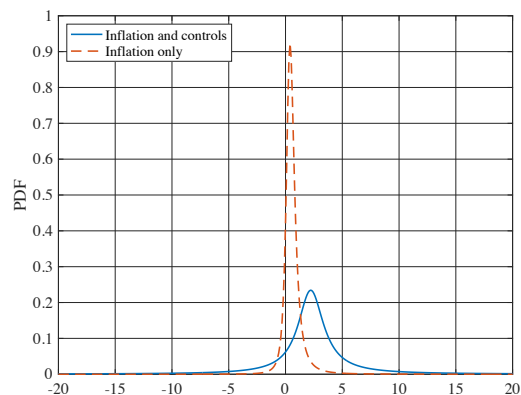
Figure 5.7 shows in two separate rows the upside entropy and the 95% expected longrise. The most noticeable differences shown in figure 5.7a and 5.7d. The former has a higher upside entropy during half part of the sample, increasing the explanatory power of the conditional distribution for this specific period. The latter instead, shows a higher expected longrise that touches the 25%.

The robustness check has produced an estimated model that presents various implications, especially since the new parameter increases the dependence on fiscal conditions

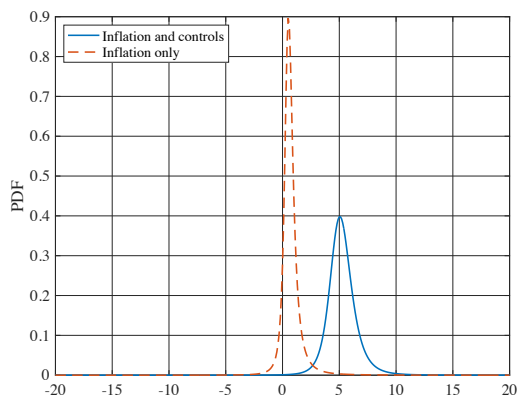
over a few time periods. Other evidence suggests that inflation vulnerability arises at high frequencies given the changes in upside entropy and longrise.



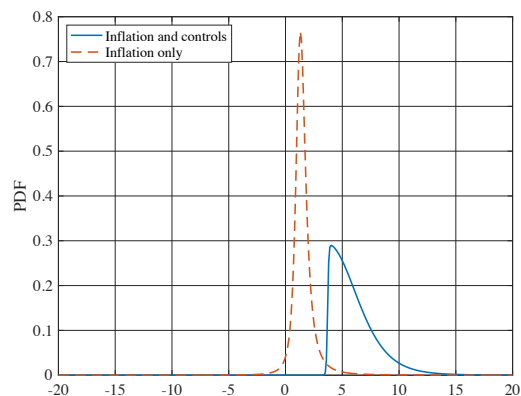
(a) One quarter ahead 2020:Q2



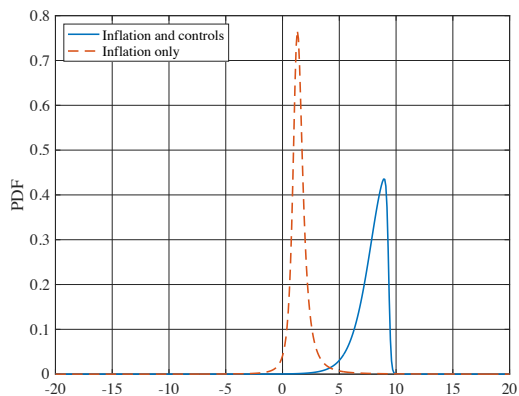
(b) 1 year ahead 2020:Q2



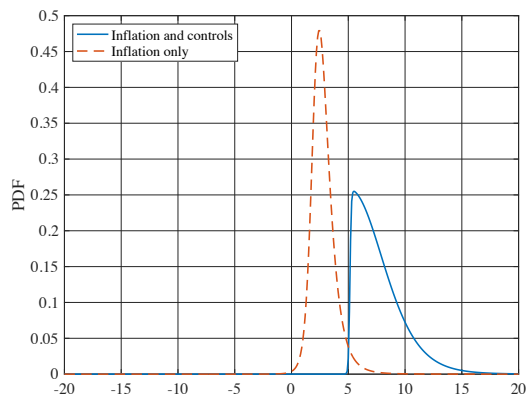
(c) One quarter ahead 2021:Q2



(d) 1 year ahead 2021:Q2



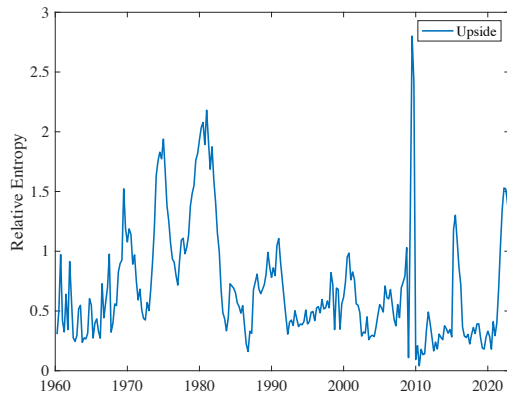
(e) One quarter ahead 2023:Q1



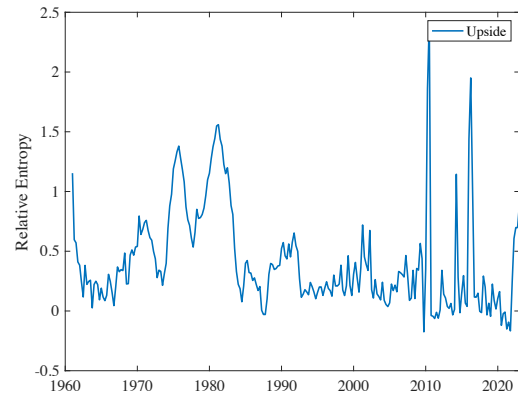
(f) 1 year ahead 2023:Q1

Figure 5.6: Probability densities of selected time periods of Inflation for Robustness check

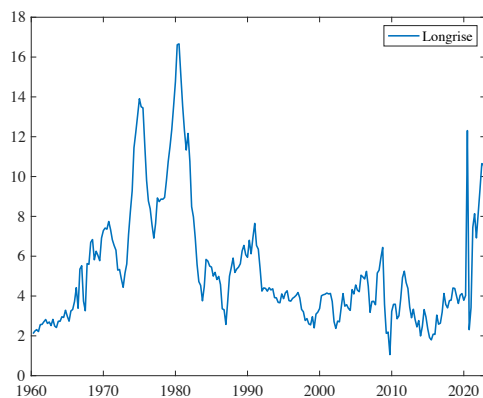
The information content of this model changes the framework in absolute terms and exhibits variation in the tail behavior. Understanding the upside risks to inflation, as influenced by the changes in government expenditure and debt, can inform about the already strong explanatory power of the previous model. Nevertheless, the expected long rise does not show important changes during the COVID-19 pandemic with respect to the previous model. Overall, the results exhibit strong similarities across time, increasing the already strong reliability of the model.



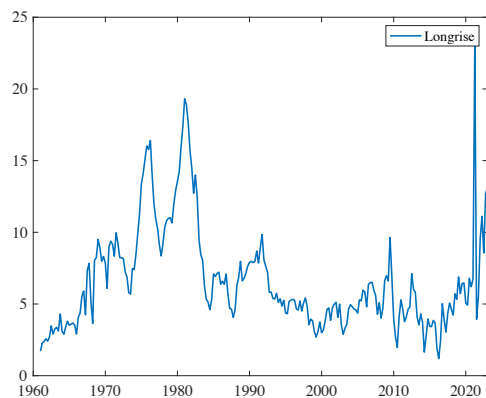
(a) One quarter ahead Upside Entropy



(b) One year ahead Upside Entropy



(c) 95% Expected Longrise one quarter ahead



(d) 95% Expected Longrise one year ahead

Figure 5.7: Upside Entropy and Expected Longrise for Robustness check

6. Conclusions

The COVID-19 crisis has revamped the academic interest in the upside risk to inflation. This thesis explored the intricacies of the Augmented Phillips curve, introducing the budget deficit as a new explanatory variable alongside unemployment and inflation. Through a quantitative approach, utilizing quantile regression analysis has allowed us to offer a new empirical perspective on these conditional effects on inflation, specifically the positive estimated coefficient at high quantiles of inflation for unemployment, and the negative estimated coefficient for the budget deficit.

Furthermore, the study has contributed by constructing a predicted distribution of inflation based on the quantile regression results. Additionally, the predicted inflation distribution offers a practical framework for assessing inflation risk in an economy, with particular attention to tail events during economic downturns. Notably, it has highlighted the increased susceptibility to inflationary pressures during times of recession, underscoring the importance of understanding and managing fiscal deficits in such economic conditions. The empirical investigation points also to the higher likelihood of looking at positive and higher realization of inflation during the COVID-19 pandemic. Moreover, the conditional probability density function shifted notably to the right of the unconditional distribution. This shift is indicative of the amplified vulnerability and upside risk to inflation during the pandemic.

In this context, understanding changes in the expected longrise to inflation offers an empirical perspective on the vulnerability of inflation, especially during economically challenging periods. By exploring the expected longrise, it is possible to gain insights into the vulnerability and volatility of the conditional distribution of inflation, identifying scenarios where the information content of the conditioning variables points to a potential increase in inflation.

In conclusion, it confirms the thesis's fundamental point that budget deficits, unemployment, and inflation are intrinsically linked during periods of high inflation and that understanding their dynamics can considerably contribute to prudent economic decision-making.

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Appendix A. Seasonal Adjustment

X-13 ARIMA SEATS is a powerful and widely-used software program developed by the U.S. Census Bureau for performing seasonal adjustment and time series analysis.[35] It is a successor to the earlier X-12 ARIMA program and incorporates more advanced features and methodologies. X-13 ARIMA SEATS is designed to help researchers, economists, and statisticians analyze and decompose time series data into its various components, including trend, seasonal, and irregular components.

The program employs sophisticated seasonal adjustment techniques based on the Autoregressive Integrated Moving Average (ARIMA) and Seasonal Decomposition of Time Series (SEATS) methods. Seasonal adjustments to economic data help guarantee policymakers and researchers possess a steadier foundation for decisions and projections, as their work accounts for fluctuations ensuring a more dependable reading of trends. The enhancements of this new version let users recognize and correct any flaws in seasonal and calendar impact adjustments acquired from the specified software.

The X-13 ARIMA SEATS model is intended for the construction of regARIMA models using seasonal economic time data. The regARIMA model is one of several calendar variation models that may be used to anticipate data based on seasonal trends with changing period lengths. Several preset regression variable types are provided in X-13 ARIMA-SEATS based on this purpose, including trend constants or overall averages, fixed seasonal effects, trading day effects, holiday effects, and many others.

The conventional seasonal ARIMA model notation $(pdq)(PDQ)s$ are used by X-13 ARIMA SEATS. Non-seasonal autoregressive (AR), differencing, and moving average (MA) are denoted by (pdq) . In the meanwhile, $(PDQ)s$ represents seasonal autoregressive, differencing, and moving averages.

The general multiplicative seasonal ARIMA model describing a time series can be expressed as the nonseasonal and seasonal autoregressive components affecting the differenced series are balanced by the nonseasonal and seasonal moving average pieces and a residual term, where B represents the backward shift operator, s signifies the periodic seasonal period, $\phi(B)$ and $\Phi(B^s)$ symbolize the autoregressive elements, $\theta(B)$ and $\Theta(B^s)$ denote the moving average parts, and the error is represented by a_t with mean 0 and variance σ^2 . The model allows for nonseasonal and seasonal order d and D differencing, respectively.

$$\phi(B)\Phi(B^s)(1-B)^d(1-B^s)^D z_t = \theta(B)\Theta(B^s)a_t$$

The inclusion of a time-varying mean function described by linear regression effects results in a valuable expansion of ARIMA models. The regression equation for a time

series y_t can be written as

$$y_t = \sum_i \beta_i x_{it} + z_t$$

where z_t is assumed to follow an ARIMA model as the one above, addressing the issue of autocorrelation and the need for differencing in time series data. Combining the two equations above defines the general regARIMA model allowed by the X-13 ARIMA SEATS program.

$$\phi(B)\Phi(B^s)(1-B)^d(1-B^s)^D \left(y_t = \sum_i \beta_i x_{it} + z_t \right) = \theta(B)\Theta(B^s)a_t$$

It is worth noting that the regARIMA model in the above equation assumes that the regression variables x_t affect the dependent series y_t only at concurrent time points, the X-13ARIMA-SEATS program, on the other hand, can include lag effects by reading appropriate user-defined lagged regression variables.

A.0.1 Data and Results

The seasonal adjustment has been used on the Budget Deficit/GDP time series and shows a strong seasonal pattern. It contains 253 data points, starting in the first quarter of 1960 and finishing in the first quarter of 2023.

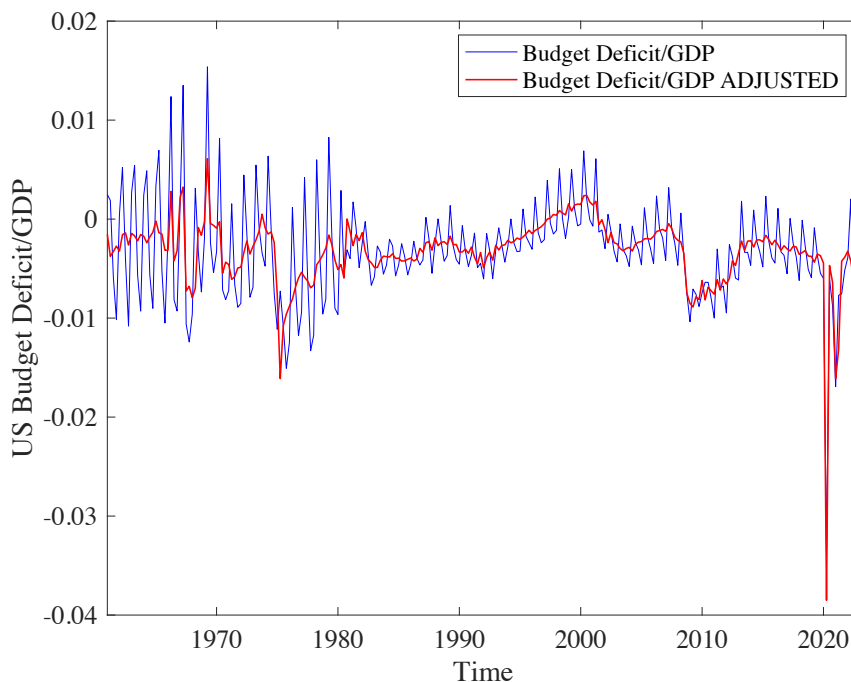
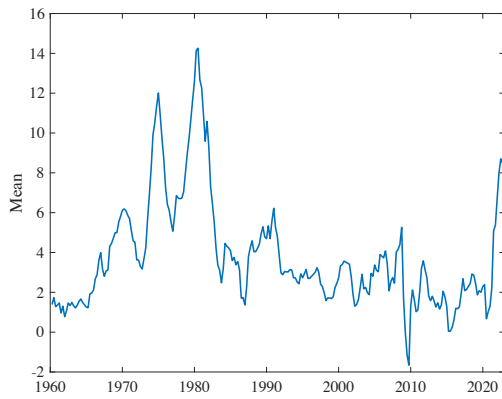


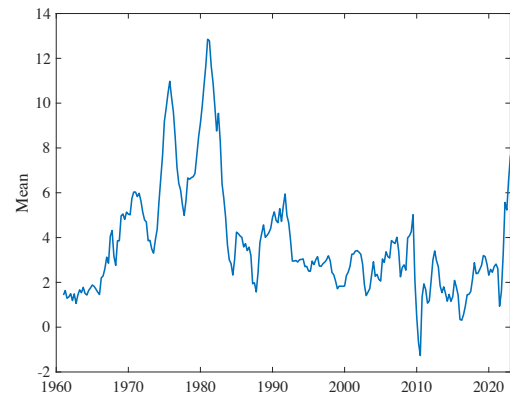
Figure A.1: Budget Deficit/GDP seasonally adjusted

The X-13 ARIMA SEATS program computed the best candidate model as $(0\ 1\ 1)(0\ 1\ 1)$, which is the one with the lowest BIC of -2355.4 and the smallest AIC of -2412.5. The differencing process carried out by the program has estimated the coefficients of the MA non-seasonal 0.65 and MA seasonal 0.23 both significantly different from 0. Furthermore, a diagnostic check shows that the residuals of the series are not correlated with a Box-Ljung test of 22.52 which accepts the hypothesis of no serial correlation. The QS test, a variant of the Ljung-Box test, computed on seasonal lags, where it only considers positive auto-correlations found no seasonality on the final series with a test of 0.3437 which accepts the hypothesis of no seasonality.

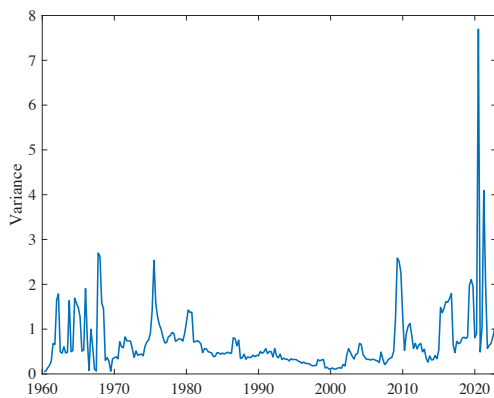
Appendix B. Figures on parameter of predictive distribution



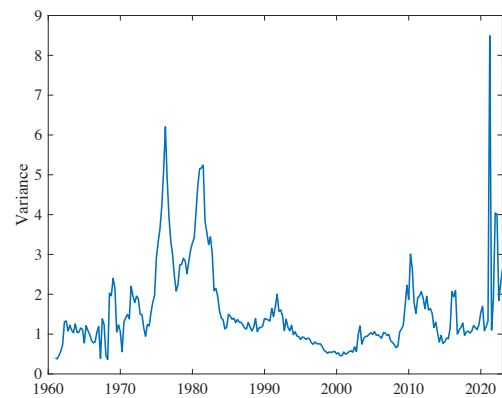
(a) One quarter ahead Mean



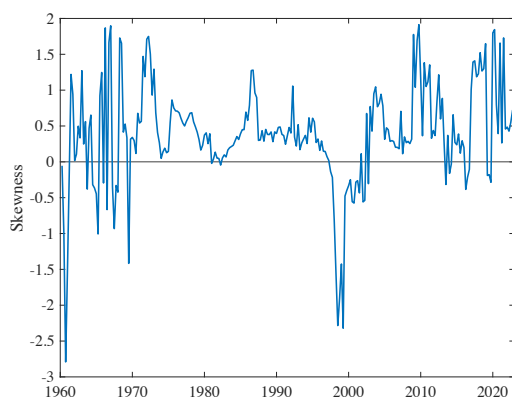
(b) One year ahead Mean



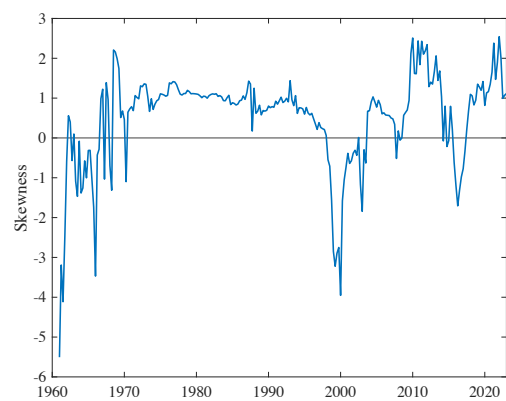
(c) One quarter ahead Variance



(d) One year ahead Variance

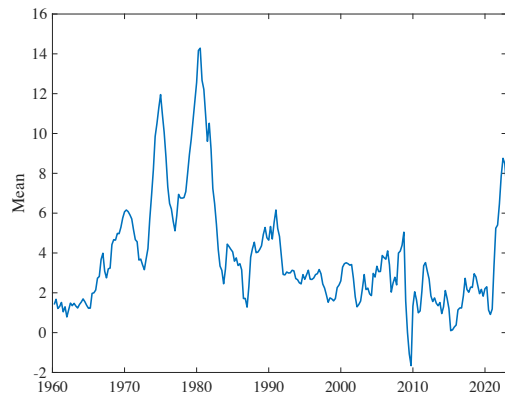


(e) One quarter ahead Skewness

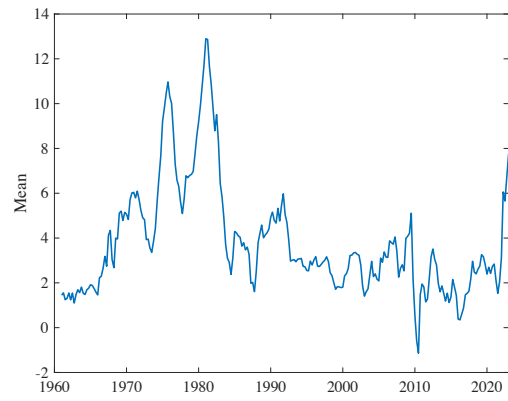


(f) One year ahead Skewness

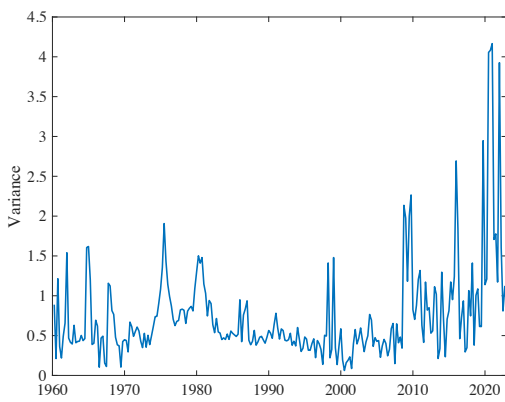
Figure B.1: Mean, Variance, and Skewness of the predictive conditional distribution of inflation



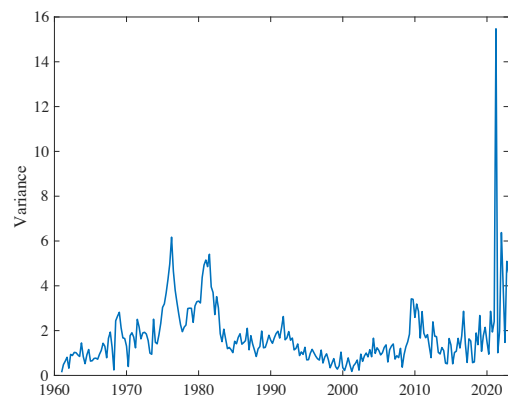
(a) One quarter ahead Mean



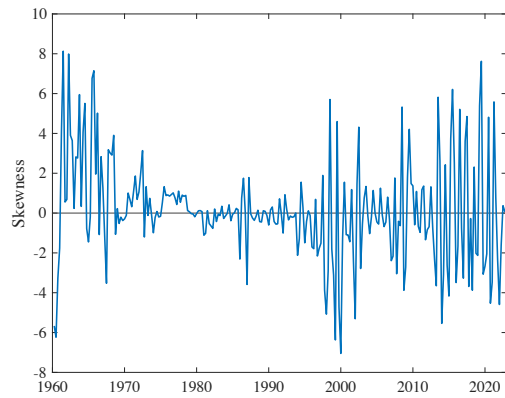
(b) One year ahead Mean



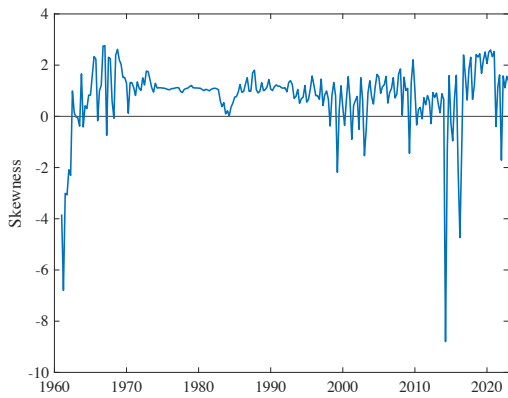
(c) One quarter ahead Variance



(d) One year ahead Variance



(e) One quarter ahead Skewness



(f) One year ahead Skewness

Figure B.2: Mean, Variance, and Skewness of the predictive conditional distribution of inflation for Robustness check