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Probing Quantum Entanglement in Nuclear Decays

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Abstract

Entanglement is one of the most striking features of quantum mechanics, and while it has been extensively probed in photonic and atomic systems, its manifestation in nuclear systems – where the strong interaction dominates – remains largely unexplored. This thesis investigates quantum correlations in two distinct nuclear contexts: entangled proton pairs produced in two-proton radioactivity, and entangled photon pairs emitted in nuclear gamma decays.

The first part addresses the feasibility of an experiment to measure spin correlations between the two protons emitted in the decay of ${}^6\text{Be}$, produced via the ${}^7\text{Be}({}^3\text{He}, {}^4\text{He}){}^6\text{Be}$ reaction. Cross sections are computed with the FRESKO and TWOFNR codes and validated against experimental data on the mirror and neighboring nuclei ${}^7\text{Li}$ and ${}^9\text{Be}$, in the absence of direct measurements. A GEANT4 simulation of a Time Projection Chamber coupled to a graphite scatterer is then used to model the spin-orbit scattering that reveals the proton spin correlation, from which the beam energy and intensity required to observe a violation of the Bell-CHSH inequality are derived.

The second part exploits data collected with the AGATA gamma-ray tracking array to search for polarization entanglement between photon pairs from a ${}^{214}\text{Po}$ cascade decay and from ${}^{22}\text{Na}$ positron-electron annihilation, using Compton scattering as a polarization analyzer. For ${}^{214}\text{Po}$ the large statistical uncertainties on several transitions prevent a conclusive test, whereas for ${}^{22}\text{Na}$ the measured correlation is found to be significantly incompatible with the non-entangled hypothesis, pointing to specific experimental and analysis improvements for future measurements. These results open prospects both for fundamental studies of quantum correlations in nuclear systems and for applications exploiting entangled nuclear radiation, such as hybrid qubit schemes and other quantum technologies.

Chapter 1

Introduction

The phenomenon of entanglement is a manifestation of quantum mechanics. Entangled states of two particles are those in which we can't speak separately of the state of each particle, they are in a common state in which they are somehow linked. Entanglement was central to historical debates regarding the completeness of quantum mechanics where a group of physicists believed that the theory of quantum mechanics was incomplete and introduced the concept of local hidden variables because it violated seemingly natural assumptions, as shown by Einstein, Podolsky and Rosen [1]. In 1964 Bell showed that a local hidden variable theory has observable physical implications [2] and several experiments proved that there are no hidden variables employing the Bell inequalities [3, 4, 5].

Despite these advancements, experimental investigations into nuclear systems remain sparse compared to the extensive research conducted on entangled electrons and photons. Because the influence of the strong interaction on quantum correlations remains largely unknown, measurements of Bell inequalities in nuclear contexts offer a potential probe into new physics.

The aim of this thesis is to explore the feasibility of an experiment on pairs of entangled protons generated by a two-proton decay. Additionally, an analysis on experimental data of pairs of gamma photons is also made.

Two-proton ($2p$) radioactivity is a rare type of three body decay where two protons are ejected from the mother nucleus. It is one of the most recently discovered decay mode and one of the least known [6].

Two-proton radioactivity was discovered in the decay of ^{45}Fe [7] and in ^6Be [8] and ^{12}O [9]. The $2p$ emission process leads from an initial unbound state to a final state of separated fragments interacting by the Coulomb force. This transition to the final state can be realized in different ways, such as sequential two-body decay via an intermediate resonance, virtual sequential two-body decay via the correlated continuum of an intermediate nucleus, direct decay into the three-particle continuum or its particular diproton (^2He) limit of two sequential two-body emissions [10].

In “prompt” $2p$ emitters the two valence protons are confined inside the Coulomb barrier and the interaction between themselves can compete or dominate the interaction between the proton and the core bringing to the formation of a diproton correlation, illustrated in Fig. 1.1.

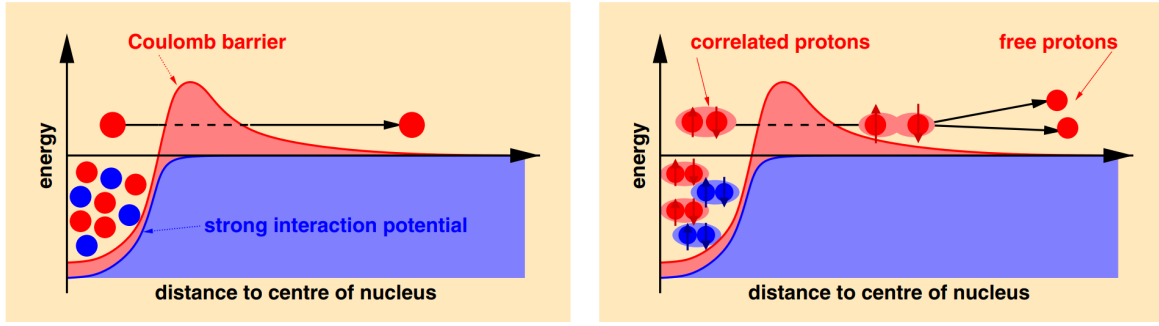


Figure 1.1: (left) Tunneling through the Coulomb barrier in single proton emitters, (right) in $2p$ emitters the tunneling is influenced by the correlations between the protons [10]

While experiments with $2p$ emitters have not yet been done to study the entanglement between the protons, experiments generating a correlation between two protons with a scattering have been made and they have found that Bell's inequalities are violated [11, 12]. However, there remains significant potential to improve these results through new experimental techniques and setups.

An experimental measurement of these systems could provide insights into the effective nuclear interaction in finite systems [13]. In addition, it could be possible to extract information on the three-body decay process depending on if the decay is democratic or sequential [14].

From the point of view of gamma photons, entangled pairs can be produced in a radiative cascade, in a similar way to the optical case, and from the annihilation of particle and antiparticle, in particular the e^+e^- annihilation that produces two 511 keV photons. For the radiative cascade $0^+ \rightarrow 2^+ \rightarrow 0^+$, a ^{214}Po source was analyzed, given the presence of this particular transition that by momentum conservation forces the two photons to have opposite quantum number. A similar analysis was performed on ^{22}Na , a β^+ source, to compare the results with the ^{214}Po source, as ^{22}Na is an extensively studied system and the photon pairs were found to violate the Bell inequalities [15, 16]. The study of gamma photons presents significant technological promise. If pairs of photons produced by these type of sources are found to be entangled it could pave the way for new high-energy quantum computing scenarios that are less susceptible to decoherence.

Chapter 2

Theoretical methods and experimental setups

2.1 Bell-CHSH inequality

The Bell-CHSH (from Clauser, Horne, Shimony and Holt) inequality is a set of mathematical constraints that arise from the study of experiments designed to test whether quantum mechanics can be explained by local hidden variable theories. In particular it provides a criterion to distinguish between quantum predictions and classical explanations of entangled particle correlations [17, 2, 18].

In 1935 Einstein, Podolsky and Rosen showed that entangled states violated two seemingly natural assumptions:

- **Reality principle:** If, without in any way disturbing a system, we can predict with certainty (i.e., with probability equal to unity) the value of a physical quantity, then there exists an element of physical reality corresponding to this physical quantity.
- **Locality Principle:** If two systems are causally disconnected, the result of any measurement performed on one system cannot influence the result of a measurement performed on the second system. Following the theory of relativity, two measurements are disconnected if $(\Delta x)^2 > c^2(\Delta t)^2$ [8, 1].

The CHSH inequality is defined in the following way:

$$S = E(\vec{a}, \vec{b}) - E(\vec{a}, \vec{b}') + E(\vec{a}', \vec{b}) + E(\vec{a}', \vec{b}') \quad (2.1)$$

With $E(\vec{a}, \vec{b})$ and $E(\vec{a}', \vec{b}')$ the correlation coefficient of the measurement done along arbitrary directions $\vec{a}$ and $\vec{b}$ or $\vec{a}'$ and $\vec{b}'$. From the Bell theorem [2] if quantum mechanics can be explained by local hidden variables, or in other words it satisfies the reality and locality principles, then $|S| < 2$. Bell showed that with a suitable entangled system the value of $|S|$ can exceed 2, up to a maximum of $2\sqrt{2}$ which is known as Tsirelson's bound [19].

Quantum entanglement is a uniquely quantum mechanical resource that plays a key role in many of the most interesting applications of quantum computation and quantum information [20].

The formalism of quantum mechanics allows the description of entangled states through vectors in a Hilbert space. The Hilbert space $\mathcal{H}$ associated with a composite system is the tensor product of the Hilbert spaces $\mathcal{H}_i$ associated with the components i . In the case of two components:

$$\mathcal{H} = \mathcal{H}_1 \otimes \mathcal{H}_2 \quad (2.2)$$

A basis for the Hilbert space $\mathcal{H}$ is constructed from the tensor product of the basis of the Hilbert spaces $\mathcal{H}_i$. In the case of two spin 1/2 particles:

$$\{|\uparrow\rangle_1, |\downarrow\rangle_1\} \quad \quad \quad \{|\uparrow\rangle_2, |\downarrow\rangle_2\} \quad (2.3)$$

$$\{|\uparrow\rangle_1 \otimes |\uparrow\rangle_2, |\uparrow\rangle_1 \otimes |\downarrow\rangle_2, |\downarrow\rangle_1 \otimes |\uparrow\rangle_2, |\downarrow\rangle_1 \otimes |\downarrow\rangle_2\} \quad (2.4)$$

The most general state in the space $\mathcal{H}$ is:

$$|\Psi\rangle = \sum_{i,j} c_{ij} |ij\rangle \quad (2.5)$$

Where the notation is simplified defining $|i\rangle_1 \otimes |j\rangle_2 \equiv |ij\rangle$. A state in the space $\mathcal{H}$ is called entangled if it cannot be written as a tensor product of a state $|\alpha\rangle_1$ and a state $|\beta\rangle_2$.

The Von Neumann entropy is a tool to quantify the entanglement between two systems A and B, defined in the following way:

$$S(\rho) = -Tr(\rho \ln(\rho)) \quad (2.6)$$

where ρ is the density matrix operator:

$$\rho = \sum_i p_i |\psi_i\rangle \langle \psi_i| \quad (2.7)$$

where $|\psi_i\rangle$ are pure states, p_i are the probabilities to measure the system in the state $|\psi_i\rangle$ and $\ln$ is the logarithm in base e . If the system described by a density operator is pure then $Tr(\rho^2) = 1$, if instead the system is a statistical mixture then $Tr(\rho^2) < 1$. The reduced density matrix is used to describe a subsystem of a composite system, and is defined as

$$\rho_A = Tr_B(\rho_{AB}) \quad (2.8)$$

where ρ_{AB} is the density matrix of the composite system made up of the subsystems A and B. If the composite system is a pure state then its entropy is $S(\rho_{AB}) = 0$. If subsystems A and B are entangled then $S(\rho_A) > 0$. The singlet state $|S = 0, S_z = 0\rangle$ of two particles with spin 1/2 is described by the following density matrix:

$$\rho = |S = 0, S_z = 0\rangle \langle S = 0, S_z = 0| = \frac{1}{2}(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)(\langle\uparrow\downarrow| - \langle\downarrow\uparrow|) = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & -\frac{1}{2} & 0 \\ 0 & -\frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad (2.9)$$

Where the spin states are represented as two dimensional vectors and the tensor product is represented as the Kronecker product, defined in Eq. 2.10.

$$A \otimes_k B = \begin{pmatrix} a_{11}B & \dots & a_{1n}B \\ \vdots & \ddots & \vdots \\ a_{m1}B & \dots & a_{mn}B \end{pmatrix} \quad (2.10)$$

In the following it will be considered $\otimes \equiv \otimes_k$. For a state with the same correlation but not entangled the following density matrix is defined:

$$\rho_c = \frac{1}{2}|\uparrow\downarrow\rangle \langle\uparrow\downarrow| + \frac{1}{2}|\downarrow\uparrow\rangle \langle\downarrow\uparrow| = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad (2.11)$$

Physically the system will have a probability of 50% of being in the state $|\uparrow\downarrow\rangle$ or in the state $|\downarrow\uparrow\rangle$, contrary to the entangled system where the two states are in superposition and the wave function will collapse into one of the two states at the moment of the measurement.

2.2 Spin-Orbit interaction

The spin-orbit interaction is a fundamental component of nuclear physics. It is not only responsible for the nuclear shell structure and the emergence of “magic numbers”, but it is also essential for describing the dynamics of nucleon-nucleus scattering [21, 22]. Unlike central forces, this interaction couples a nucleon’s orbital angular momentum $\vec{l}$ to its intrinsic spin $\vec{s}$. The spin-orbit potential generally takes the following form:

$$H_{ls} \propto V_{so}(r) \vec{s} \cdot \vec{l} \quad (2.12)$$

From a phenomenological perspective, the radial term $V_{so}(r)$ is typically modeled as the derivative of the central nuclear potential. Since this derivative peaks at the nuclear radius, the spin-orbit interaction is primarily a surface effect. This term lifts the energy degeneracy of angular momentum states and, in scattering processes, introduces a dependence on the spin orientation relative to the scattering plane. When using a polarized beam, the differential cross-section loses its azimuthal symmetry.

The reference frame is defined by the directions of the incoming and outgoing particle, defining the scattering plane, and the normal to the plane. The incident momentum is along the z axis, the y axis is along the normal to the scattering plane and x is such that it forms a right-handed coordinate system. ϕ is the angle between the spin projected on the xy plane and the y axis, and not from the x axis, in this way a proton scattering to the left corresponds to $\phi = 0^\circ$ and to the right $\phi = 180^\circ$. In Fig. 2.1 the coordinate system is shown. The spin vector can be written in terms of the angles β , ϕ

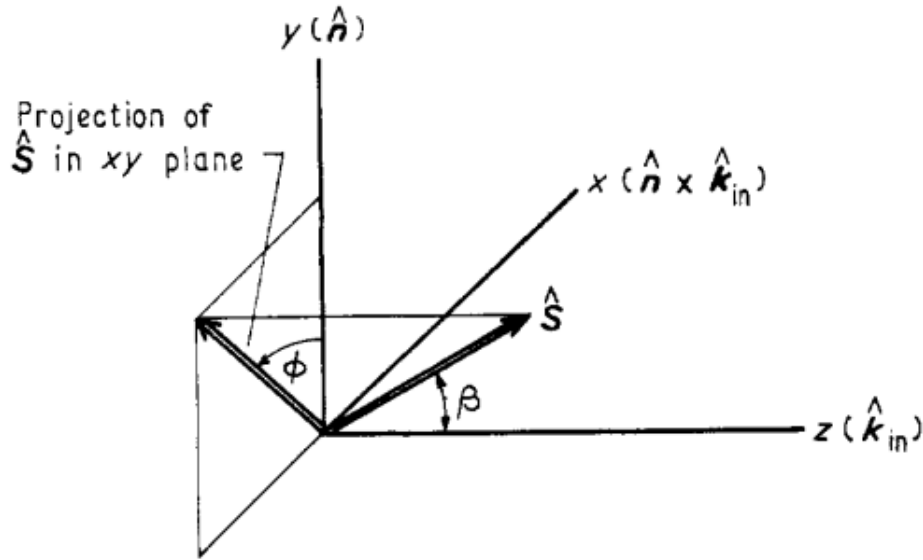


Figure 2.1: Coordinate system for the spin orbit interaction. $\hat{k}_{in}$ is the direction of the incoming particle and $\hat{n}$ is the normal to the scattering plane [23].

and the vector polarization magnitude p_Z :

$$\begin{aligned} p_x &= -p_Z \sin \beta \sin \phi \\ p_y &= p_Z \sin \beta \cos \phi \\ p_z &= p_Z \cos \beta \end{aligned} \quad (2.13)$$

The operator that relates the final spinor state to the initial one is defined as:

$$\chi_f = M \chi_i \quad (2.14)$$

Where χ_j are the spinors of a particle. In the case of the proton there are only two spinors and the are $|\uparrow\rangle$ and $|\downarrow\rangle$. The density matrix of a proton in a generic spin state is described in Sec. 2.7. The density matrix in the final state is related to the initial one with

$$\rho_f = M \rho_i M^\dagger \quad (2.15)$$

For a polarized beam the initial density matrix can be expanded into Eq.2.16.

$$\rho_i = \frac{1}{2} \left(\mathbb{1} + \sum_{j=1}^3 p_j \sigma_j \right) \quad (2.16)$$

Where σ_j are the Pauli matrices. and then the cross section is calculated with Eq. 2.17 [24].

$$\sigma(\theta, \phi) = \text{Tr} M \rho_i M^\dagger \quad (2.17)$$

If the beam is unpolarized the cross section reduces to [23]

$$\sigma_0(\theta) = \frac{1}{2} \text{Tr} M M^\dagger \quad (2.18)$$

Finally,

$$\rho_f = \frac{1}{2} M M^\dagger + \frac{1}{2} \sum_{j=1}^3 p_j M \sigma_j M^\dagger \quad (2.19)$$

that yields

$$\sigma(\theta, \phi) = \sigma_0(\theta) \left(1 + \sum_{j=1}^3 p_j A_j(\theta) \right) \quad (2.20)$$

Given the coordinate system and the interaction the component x and z of the polarization do not contribute to the cross section because the angular momentum is along the y axis and so $l_x \cdot S_x = l_z \cdot S_z = 0$. The cross section can be then expressed as:

$$\sigma(\theta, \phi) = \sigma_0(\theta) (1 + P A_y(\theta) \cos \phi) \quad P A_y = \frac{N_L - N_R}{N_L + N_R} \quad (2.21)$$

Where $P = p_Z \sin \beta$ is the degree of polarization of the beam, $\sigma(\theta)$ is the unpolarized cross section and A_y is the analyzing power, which describes the left-right asymmetry and how effectively the reaction reveals the spin polarization state of the initial state. The degree of polarization P is a number in the range $[-1, 1]$ and is the magnitude of the polarization vector of spin. A theoretical evaluation of the analyzing power A_y is shown in Eq. 2.22

$$A_y(\theta) = \frac{\text{Tr} M \sigma_y M^\dagger}{\text{Tr} M M^\dagger} \quad (2.22)$$

The spin orbit force deflects the incident nucleon based on the alignment of the spin and angular momentum vectors. It can be repulsive or attractive, but if the spin direction is fixed then the net displacement is along one direction as illustrated in Fig.2.2.

While for the measurement of A_y a polarized beam is compulsory, for the measurement of the Bell inequalities it is not needed. When the two protons are measured the direction in which they are deflected is known. If the two directions are opposite relative to the other then it is possible to assume that the spins of the protons were anti-correlated, like in the case of the singlet state. In the case of double spin orbit scattering the cross section is calculated from the starting state

$$\rho_{12} = \frac{1}{2} (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)(\langle\uparrow\downarrow| - \langle\downarrow\uparrow|)$$

Then the cross section is computed, following the formalism given by Wolfenstein [24], with Eq.2.23.

$$\sigma(\Omega_1, \Omega_2) = \text{Tr}[(M_1 \otimes M_2) \rho_{12} (M_1^\dagger \otimes M_2^\dagger)] \quad (2.23)$$

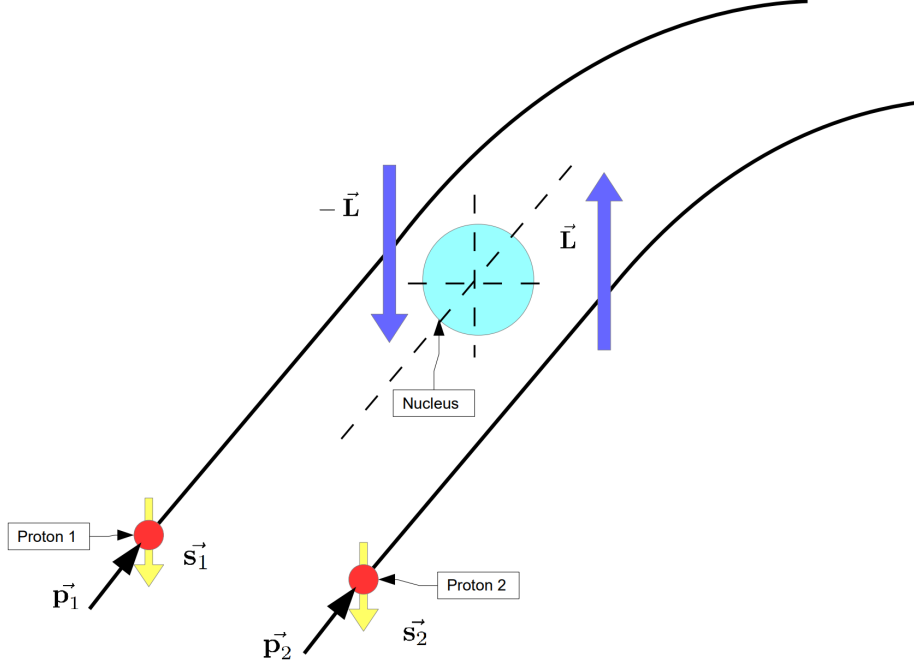


Figure 2.2: Illustration of the deflection of two protons with spin down due to the spin-orbit interaction.

The final result is then:

$$\sigma(\Omega_1, \Omega_2) = \sigma_0(\theta_1)\sigma_0(\theta_2)[1 - A_1(\theta_1)A_2(\theta_2)\cos\Delta\Phi] \quad (2.24)$$

The dimension of this cross section is not right because of the product of the two cross sections σ_0 . However in the calculation of number of events the terms accounting for the density of the target and its thickness must be inserted for each particle, so there is a factor $n_1 t_1 n_2 t_2$ that adjusts the dimension. In the singlet state the spin axis is the same for both particles, so by using the scattering plane of one proton as reference the polarization of the other proton will always be equal to one. Thanks to this the polarization axis in the reaction is not needed and methods to polarize the beam can be avoided. Physically the spin measurement is done by the target nucleon, depending on the position of the incoming proton with respect to the target nucleus which determines $\vec{l}$. Since the two protons are not guaranteed to hit two target nuclei in the exact same position two different scattering planes will be formed at an angle $\Delta\Phi$, same as the one in the cross section. Assuming the two protons have an initial parallel trajectory, the normals of the scattering planes coincide with the directions $\vec{a}$ and $\vec{b}$ of the correlation function $E(\vec{a}, \vec{b})$ in the CHSH inequality in Eq. 4.5. In particular the correlation function for this particular case is:

$$E(\vec{a}, \vec{b}) \equiv C(\Delta\Phi) = \frac{N_{LL} + N_{RR} - N_{LR} - N_{RL}}{N_{LL} + N_{RR} + N_{LR} + N_{RL}} = \frac{N(\Delta\Phi) - N(\Delta\Phi + 180^\circ)}{N(\Delta\Phi) + N(\Delta\Phi + 180^\circ)} = -\bar{A}_1 \bar{A}_2 \cos\Delta\Phi \quad (2.25)$$

Where N_{XY} are the coincidence counts in which one proton is deviated in the X direction and the other in the Y direction and $N(\Delta\Phi)$ is obtained by integrating over the solid angles Eq. 2.24 and multiplying by the luminosity of the experiment. The degree of polarization P is not present in the cross section since the singlet state has spherical symmetry and there is no spin axis before the measurement and the collapse of the wave function. To illustrate this, Fig. 2.3

To compare the previous result the same cross section for a non-entangled state is calculated. The initial density matrix is

$$\rho_{12,c}(\hat{s}) = \frac{1}{2}|\hat{s}_+\hat{s}_-\rangle\langle\hat{s}_+\hat{s}_-| + \frac{1}{2}|\hat{s}_-\hat{s}_+\rangle\langle\hat{s}_-\hat{s}_+| \quad (2.26)$$

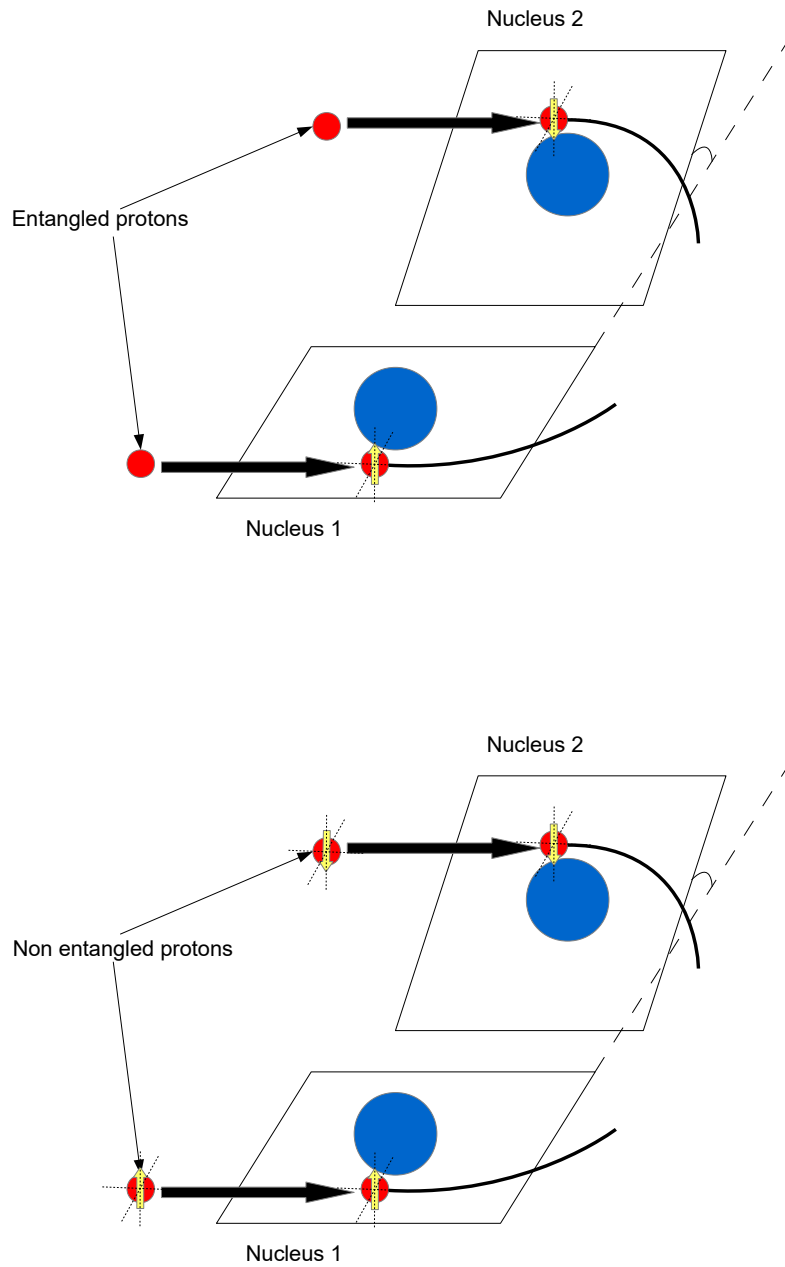


Figure 2.3: Difference in spin axis between entangled and not entangled states. The entangled state prior to the interaction with the target nucleus does not have a defined spin axis (top). In the entangled state the axis is defined even before the interaction (bottom).

Where $\hat{s}$ is the spin axis and the subscript is the eigenvalue of the state. Since the singlet state is invariant and is rotationally invariant the inclusion of the spin axis is needed to describe all the initial states. Simplifying the form of the density matrix I obtain:

$$|\hat{s}_{\pm}\rangle\langle\hat{s}_{\pm}| = \frac{\mathbb{1} \pm \vec{\sigma} \cdot \hat{s}}{2} \quad \rho_{12,c}(\hat{s}) = \frac{1}{4} [\mathbb{1} \otimes \mathbb{1} - (\vec{\sigma}_1 \cdot \hat{s}) \otimes (\vec{\sigma}_2 \cdot \hat{s})] \quad (2.27)$$

Where $\vec{\sigma}$ represents the three Pauli matrices. The average over all values of $\hat{s}$ is then taken:

$$\rho_{12,c} = \frac{1}{4\pi} \int d\Omega \rho_{12,c}(\hat{s}) = \frac{1}{4} \mathbb{1} \otimes \mathbb{1} - \frac{1}{4\pi} \int d\Omega \left(\sum_{i,j}^3 (\hat{s}_i \hat{s}_j) \vec{\sigma}_1 \otimes \vec{\sigma}_2 \right) \quad (2.28)$$

The terms with $i \neq j$ are equal to zero so only the diagonal terms remain. The average of the remaining terms $\langle \hat{s}_x^2 \rangle = \langle \hat{s}_y^2 \rangle = \langle \hat{s}_z^2 \rangle = \frac{1}{3}$. Finally the density matrix assumes the form of Eq. 2.29.

$$\rho_{12,c} = \frac{1}{4} \left[\mathbb{1} \otimes \mathbb{1} - \frac{1}{3} (\sigma_{x,1} \otimes \sigma_{x,2}) - \frac{1}{3} (\sigma_{y,1} \otimes \sigma_{y,2}) - \frac{1}{3} (\sigma_{z,1} \otimes \sigma_{z,2}) \right] \quad (2.29)$$

Then applying Eq. 2.23 to the non entangled density matrix the result yields:

$$\begin{aligned} \sigma_C(\Omega_1, \Omega_2) &= \sigma_0(\theta_1) \sigma_0(\theta_2) \left[1 - \frac{1}{3} (p_{x1} p_{x2} A_1(\theta_1) A_2(\theta_2) + p_{y1} p_{y2} A_1(\theta_1) A_2(\theta_2)) \right] = \\ &= \sigma_0(\theta_1) \sigma_0(\theta_2) \left[1 - \frac{1}{3} P^2 A_1(\theta_1) A_2(\theta_2) (\sin \phi_1 \sin \phi_2 + \cos \phi_1 \cos \phi_2) \right] \end{aligned}$$

The component along the z axis does not contribute to the cross section because a spin completely aligned with the direction of the incoming particle cannot interact with the angular momentum. However, unlike the case for single scattering, the component along the x axis must be considered. The reason consists in the definition of the frame of reference of the particles. Once it is chosen for one particle then it must be the same so for the other particle the xz plane may not coincide with the scattering plane, adding the x component to the cross section. The z component remains always null because it is assumed that the protons travel along the z axis. P^2 can be extracted since, considering two particles have the spin oriented on the same axis but different direction, $\sin \beta = \sin(\pi - \beta)$. Finally simplifying the trigonometric expression:

$$\sigma_C(\Omega_1, \Omega_2) = \sigma_0(\theta_1) \sigma_0(\theta_2) \left(1 - \frac{1}{3} P^2 A_1(\theta_1) A_2(\theta_2) \cos \Delta\Phi \right) \quad (2.30)$$

The asymmetric component of Eq. 2.30 is a third of the entangled one, so comparing experimental data to these theoretical estimation should reveal if the sampled state is entangled or not. In this case the degree of polarization P reappears since for the non entangled state the spin axis is defined since the formation of the pair.

2.3 Time Projection Chamber

The Time Projection Chamber (TPC) concept was invented by David Nygren [25]. It consists of a square or cylindrical field cage filled with a gaseous or liquid material. When a charged particles pass through the detector they produce tracks of ionization electrons that drift towards a position-sensitive amplification stage thanks to an electric field. The amplification stage provides only a 2D projection of the trajectories, however the third dimension can be calculated from the arrival times of the drifted detectors. Typically the starting time of the measurement is given by a second trigger detector. The TPC can also work in the presence of a magnetic field, in that case it is possible to reconstruct the momentum and the charge of the particle from a helix fit to the particle trajectory. From the measurement of the energy deposit through ionization (dE/dx) and, together with the information of charge and momentum, the particle can be identified. Typically the spatial resolution of a TPC is of about $100 \mu\text{m}$ [26].

Active targets consist of a TPC-like detector in which the gas is also the target. In the context of nuclear physics active targets are very useful in experiments with radioactive nuclear beams, which typically have lower intensities than stable beams, thanks to the fact that the vertex interaction can be reconstructed for each event and the low energy recoil particles do not need to escape a layer of inert target material. This allows to increase the thickness of the target without impacting the determination of the energy of the particles before and after the reaction and the energy and angular distributions. The large solid angle coverage of the detector further contributes to the increase in luminosity [27]. A scheme of an active target is shown in Fig. 2.4

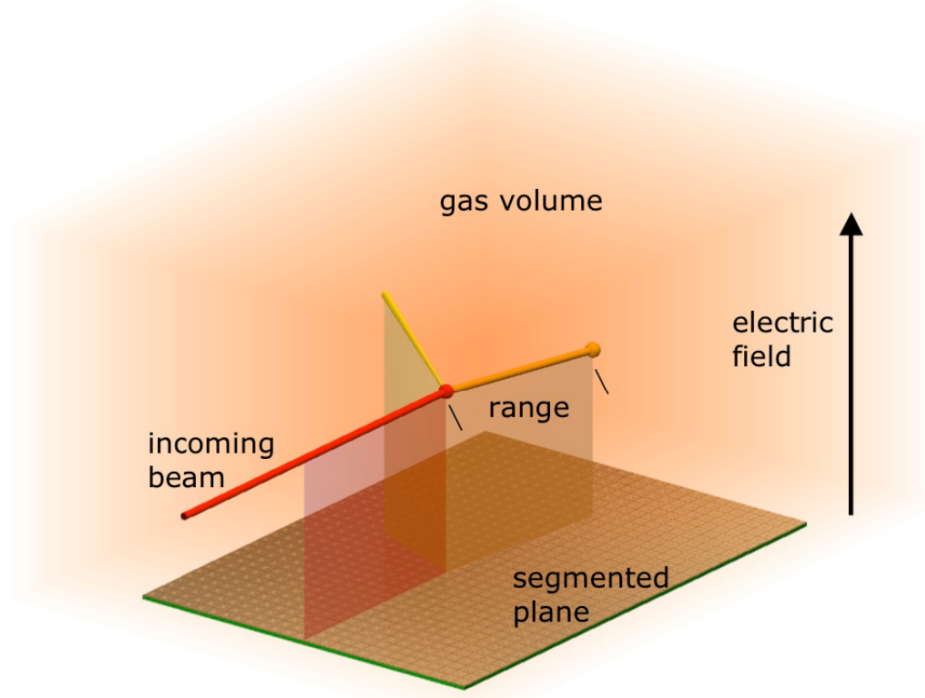


Figure 2.4: Scheme of a TPC used as an active target [28].

The advantage of using a TPC for an experiment measuring the spin correlation of protons relies on the reconstruction of the tracks. Using the TPC as an active target means that the trajectory of the protons can be measured from the moment they are generated until they exit the detector, allowing to have an angular coverage of 2π and reconstruct the scattering plane for each event. Thanks to the spatial resolution a more precise determination of the scattering and azimuthal angle can be made. One disadvantage of a TPC is that the target material must be used respecting the properties of the TPC, for example most of them cannot sustain a high pressure gas (> 1 atm).

One example of a TPC used as an active target is ACTAR [29], constructed at GANIL. The chamber, in Fig. 2.5, has an internal volume of $606 \text{ mm} \times 606 \text{ mm} \times 335 \text{ mm}$ and is equipped with a field cage attached to the cathode surrounding the active volume. The anode is segmented into 128×128 square pads with a side edge of 2 mm. The range of pressures allowed are from a few tens of mbar up to one bar. The performance of ACTAR was measured through the study of the reactions $^1\text{H}(^{18}\text{O}, ^{18}\text{O})^1\text{H}$ and $^1\text{H}(^{18}\text{O}, ^{15}\text{N})^4\text{He}$ at a beam energy of 57.6 MeV. The FWHM of the energy spectrum for the (p,p) channel was measured at 38 ± 4 , while for the (p, α) channel was 54 ± 9 .

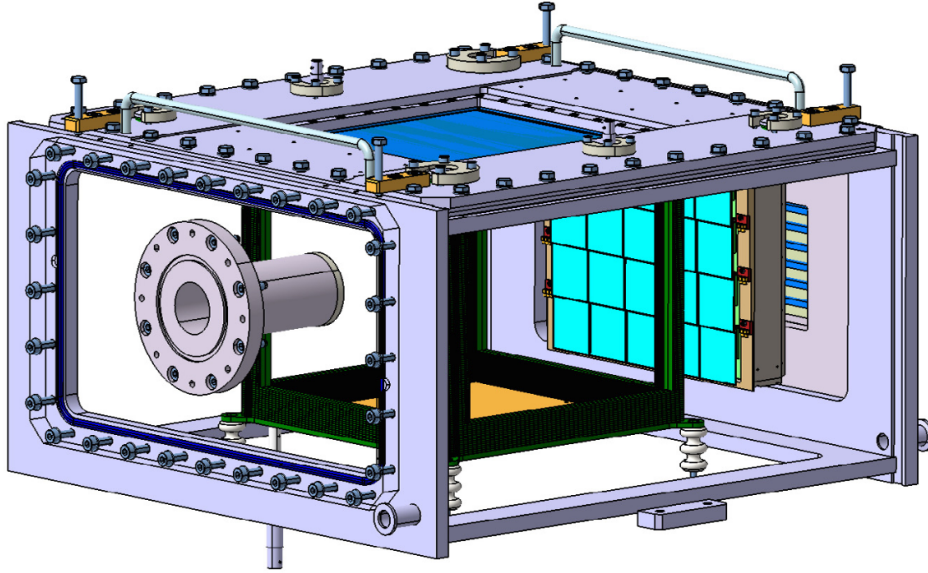
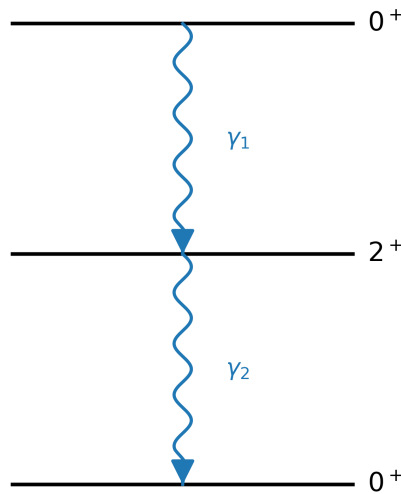


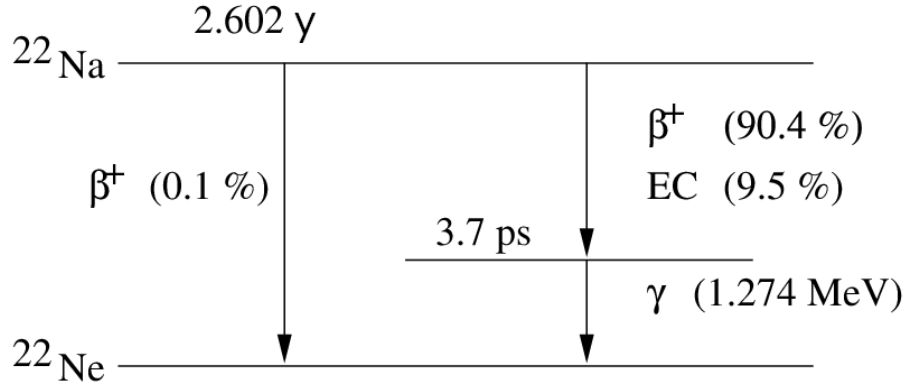
Figure 2.5: 3D CAD drawing of ACTAR TPC [29].

In particular, ACTAR was used to measure not only the single proton decay of ^{54m}Ni [30], but also the two proton decay of ^{48}Ni [31] (with very low statistics), suggesting that a TPC could be successfully used to measure also two proton decay. The work of this thesis will focalize on understanding if it possible to extract information on the correlation of the two protons.

2.4 Polarization measurements of gamma photons

The gamma photons produced in a cascade decay of the type $0^+ \rightarrow 2^+ \rightarrow 0^+$, shown in Fig. 2.6 must have opposite quantum numbers due to the conservation of the total momentum. In a similar way, in nuclei that decay through β^+ emit two gamma photons produced in the e^+e^- annihilation, due to the formation of para-positronium, and so must have opposite polarization. An example is ^{22}Na , with the decay scheme shown in Fig. 2.7

Figure 2.6: Cascade decay of two photons in ^{214}Po .


 Figure 2.7: ^{22}Na decay scheme [32].

The measurement of the polarization of a gamma photon is made by employing Compton scattering, since there is no way to measure directly the polarization, like using a polarizing filter for optical photons. The polarization is extracted from the distribution of azimuthal angles measured between a defined plane in the laboratory and the vector of the scattered photon. In the case of studying the correlation of two gamma photons from the azimuthal modulation curve it is possible to establish if the correlation is either classical or quantum. The distribution is obtained by the difference of the azimuthal angles of the pairs of photons. The typical setup for the measurement in the case of annihilation photons consists of a source in the center of the setup, two solid scatterers for the Compton scattering in opposite direction with respect to the source and two detectors that can be moved to cover the range of angles desired for the experiment. The extraction of the azimuthal angle is derived from the Klein-Nishina cross section formula for Compton Scattering, Eq. 2.31.

$$\sigma_C(\theta, \phi_c) = \frac{r_0^2}{2} \left(\frac{E'_\gamma}{E_\gamma} \right)^2 \left(\frac{E_\gamma}{E'_\gamma} + \frac{E'_\gamma}{E_\gamma} - 2 \sin^2 \theta \cos^2 \phi_c \right) \quad (2.31)$$

Where r_0 is the classical electron radius, E'_γ is the scattered photons energy, θ the scattering angle and ϕ_c is the angle between the scattering plane and the plane defined by the momentum and electric field vectors of the incident gamma photon. However the polarization of the gamma photon in the laboratory reference frame, where $\phi_c \neq \Phi$, is described by the Stokes parameters P_1 and P_2 . In the case that $P_1 \equiv P = 1$ and $P_2 = 0$, which means that the photon is linearly polarized along the horizontal or vertical direction, then Eq. 2.31 holds for $\phi_c = \Phi$:

$$\sigma_C(\theta, \Phi) = \frac{r_0^2}{4} \left(\frac{E'_\gamma}{E_\gamma} \right)^2 \left[\frac{E_\gamma}{E'_\gamma} + \frac{E'_\gamma}{E_\gamma} - 2 \sin^2 \theta (1 + P \cos 2\Phi) \right] \quad (2.32)$$

In particular an ensemble of partially polarized gamma photons can be described as composed of a fraction $|P|$ of linearly polarized photons and a fraction $1 - |P|$ of unpolarized ones. In this case, then, it follows that $P_1 \equiv P \in [-1, 1]$ [33]. A simplified form of the cross section after averaging on the scattering angle θ is the following one:

$$\sigma(\Phi) = \sigma_0(1 + \bar{A}P \cos 2\Phi) \quad (2.33)$$

Where σ_0 is the unpolarized cross section and $\bar{A}$ is the integrated analyzing power. For the measurement of the correlation of the two gamma photons both azimuthal angles are measured, and to populate the azimuthal modulation distribution their difference is taken. A scheme of the interaction is shown in Fig. 2.8. The cross section of the double Compton scattering event is calculated starting from an entangled state:

$$|\Psi\rangle = \frac{1}{\sqrt{2}} (|xy\rangle - |yx\rangle) \quad (2.34)$$

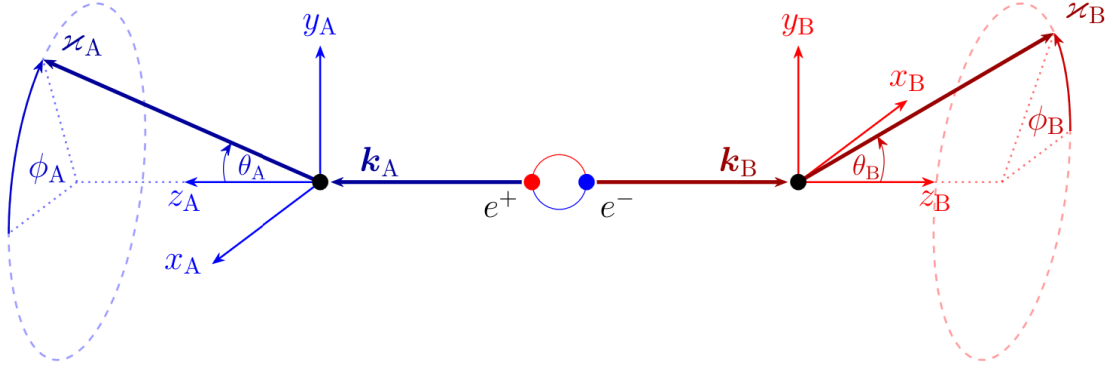


Figure 2.8: Scheme of correlation measurement of two gamma photons through Compton scattering [34].

Where x and y denote the direction of the polarization of each photon, assuming they are traveling along the z axis. The differential cross section of two annihilation photons, assuming an experiment where the beam is collimated, is the following one [35]:

$$\sigma(\Omega_1, \Omega_2) = \frac{r_0^2}{16} \left(\frac{E_1 E_2}{E_0^2} \right)^2 [\varepsilon_1 \varepsilon_2 - \varepsilon_1 \sin^2 \theta_2 - \varepsilon_2 \sin^2 \theta_1 + 2 \sin^2 \theta_1 \sin^2 \theta_2 \sin^2(\phi_1 - \phi_2)] \quad (2.35)$$

Where E_0 is the energy of the incoming photons, E_i the energy of the scattered ones and $\varepsilon_i = \frac{E_i}{E_0} + \frac{E_0}{E_i}$. A more compact form is reported in Eq. 2.36.

$$\sigma(\Omega_1, \Omega_2) = \sigma_0(\theta_1) \sigma_0(\theta_2) [1 - A_1(\theta_1) A_2(\theta_2) \cos 2\Delta\Phi] \quad (2.36)$$

Regarding the cross section of the correlated but no entangled photons the initial state is the following one:

$$\rho_{12} = \frac{1}{2} |x'y'\rangle \langle x'y'| + \frac{1}{2} |y'x'\rangle \langle y'x'| \quad (2.37)$$

Where x' and y' are arbitrary directions orthogonal to the direction of the incoming photon. More specifically:

$$\begin{aligned} |x'\rangle &= \cos \beta |x\rangle + \sin \beta |y\rangle \\ |y'\rangle &= -\sin \beta |x\rangle + \cos \beta |y\rangle \end{aligned}$$

The cross section computed from this state, after averaging over all possible rotations, is then [36]:

$$\sigma(\Omega_1, \Omega_2) = \sigma_0(\theta_1) \sigma_0(\theta_2) \left[1 - \frac{1}{2} A_1(\theta_1) A_2(\theta_2) \cos 2\Delta\Phi \right] \quad (2.38)$$

Comparing this cross section and the entangled one the azimuthal asymmetry is halved. Integrating over the scattering angles and the targets thicknesses and densities the resulting distribution is the following one for the entangled state:

$$N(\Delta\Phi) = N_0(1 + \mu \cos 2\Delta\Phi) \quad C(\Delta\Phi) = \mu \cos 2\Delta\Phi = \frac{N(\Delta\Phi) - N(\Delta\Phi + 90^\circ)}{N(\Delta\Phi) + N(\Delta\Phi + 90^\circ)} \quad (2.39)$$

$C(\Delta\Phi)$ is the correlation function of the two photons, defined in a similar way for protons in Sec. 2.2. From the parameter μ , which is the integrated product of the analyzing powers, it is possible to reconstruct the Bell inequalities [18].

$$S(\Delta\Phi) = |2C(\Delta\Phi) + C(-\Delta\Phi) - C(3\Delta\Phi)| = |3\mu \cos 2\Delta\Phi - \mu \cos 6\Delta\Phi| \quad (2.40)$$

The classical limit $S(\Phi) < 2$ is broken if $\mu > \frac{\sqrt{2}}{2}$. However the parameter μ depends on the analyzing power of the detectors, so the photons could be entangled but the measurement of the Bell inequalities is less than two like a classical state. For the case of classical correlation the factor μ is replaced by $\frac{1}{2}\mu$. Ultimately comparing the cross sections to the experimental data it is possible to establish if the sampled state is entangled or not. These calculations assume that the photons are always emitted back to back so for the case of photons emitted in a cascade decay, since there is no constraint on the momenta, the cross section must be modified to include the different angles of emission, increasing the complexity because the axis z is not shared by the two photons.

Interestingly, the difference of entangled and non entangled states in the cross section is not the same for photons and protons. The polarization of photons is 2D, since polarization along the direction of motion of the photon is not allowed. Instead, the spin of the protons has no limitation and can be oriented in 3D. This extra degree of freedom decreases further the visibility of the asymmetry.

2.4.1 Positronium

Positronium (Ps) is the quasi-stable bound system of an electron and the positron, its antiparticle, and was discovered by Deutsch in 1951 [37]. When a positron interacts with an electron it can either annihilate as a free particle or it can form a bound state with it, resembling a hydrogen atom [38]. From the charge conjugation properties Ps has a charge conjugation eigenvalue of $(-1)^{l+s}$, while for a state of n photons is $(-1)^n$. In the decay of Ps where the electron and the positron annihilate, the number of photons is determined by the state of Ps, in particular from $(-1)^{l+s} = (-1)^n$ the singlet state $S = 0$ decays into two photons and the triplet state ($S = 1$) decays into three (one is not allowed due to linear momentum conservation). The singlet state is called para-positronium (p-Ps) and the triplet state is called ortho- positronium (o-Ps), and they are shown in Fig. 2.9. In vacuum the ratio of production o-Ps:p-Ps is 3:1, reflecting the statistical ratio of triplet and singlet states. A positronium source usually consists in a β^+ emitter like ^{22}Na , which produces positrons that then interact with the electrons of other atoms to form positronium. In this thesis it is studied the correlation between of the photons generated by p-Ps. In the center of mass reference frame the polarization of the photons must be opposite, since in the initial state the total angular momentum is equal to zero. The energy of the photons is equal to 511 keV, so the experimental energy spectrum will be centered at that value. The entanglement between the two photons has shown in various experiment that the value S of the Bell inequalities surpasses the classical limit [15, 16]. In the case of o-Ps, where three photons are emitted in the decay, the energy spectrum will be continuous from 0 to 511 keV, with the possibility of studying the entanglement of the three photons [39].

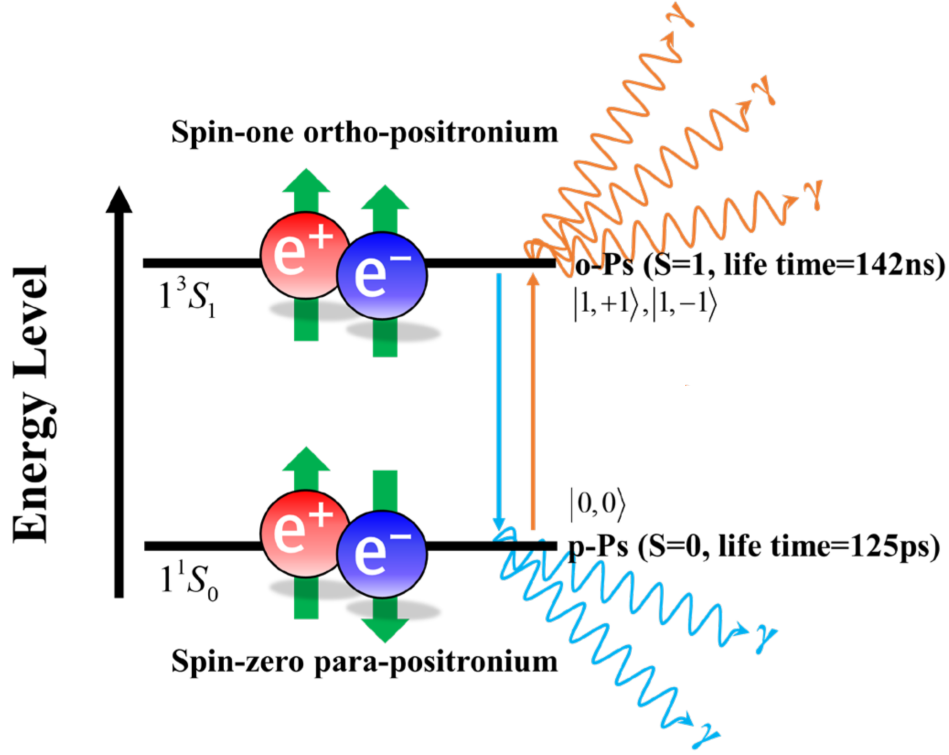


Figure 2.9: Positronium level scheme, the energy gap is 0.845 meV [40].

2.4.2 Cascade decay

In a cascade decay two or more gamma photons are emitted in quick succession. Cascade decays are used to study the multipolarity of a transition since it is connected to the angular correlation of the two photons. The parameters $a_2, a_4, \dots$ of Eq. 2.41 describe the angular correlation.

$$W(\theta) = 1 + a_2 \cos^2 \theta + a_4 \cos^4 \theta \quad (2.41)$$

Where $W(\theta)$ is the relative probability of the second photon of being emitted at an angle θ with respect to the first one.

In particular states, like those in Fig. 2.6, entanglement can emerge from the superposition of all channels with the proper ΔM , the transferred magnetic quantum number. In Fig. 2.10 it is shown an example. In this case photons with $\Delta M = \pm 1$ are allowed, since a transition with $\Delta M = 0$ is forbidden. A photon with $\Delta M = 1$ has a right-handed circular polarization, while with $\Delta M = -1$ it is a left-handed circular polarization. The wave state is written in the following way:

$$|\Psi\rangle = \frac{|LR\rangle + |RL\rangle}{\sqrt{2}} \quad (2.42)$$

Where $|R\rangle$ and $|L\rangle$ indicate a right/left-handed circular polarization for the first and second photon. It is possible with a base change to write the circular polarization into a linear polarization base:

$$|R\rangle = \frac{|x\rangle + i|y\rangle}{\sqrt{2}} \quad |L\rangle = \frac{|x\rangle - i|y\rangle}{\sqrt{2}} \quad (2.43)$$

Where $|x\rangle$ and $|y\rangle$ are, respectively, a horizontal and vertical linear polarization. Changing the basis:

$$|\Psi\rangle = \frac{|xx\rangle + |yy\rangle}{\sqrt{2}} \quad (2.44)$$

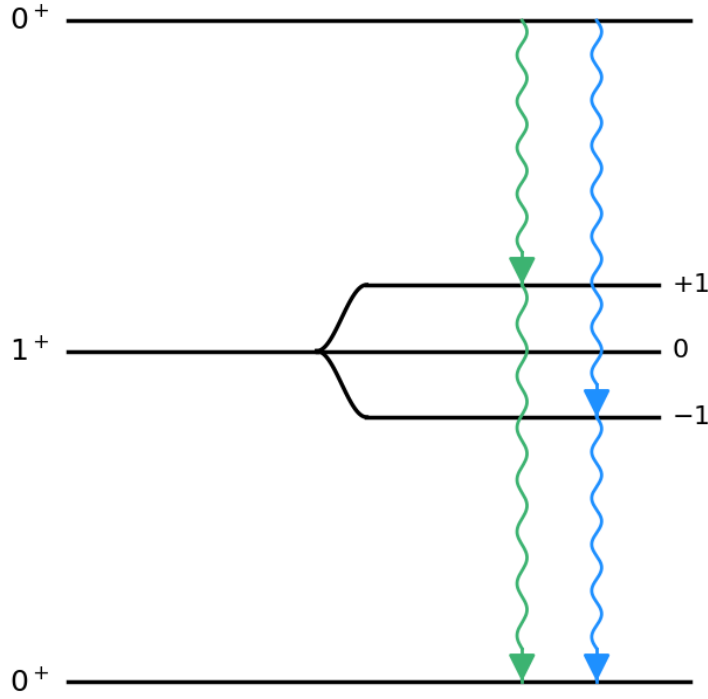


Figure 2.10: Example of decay channels in a generic nucleus with transition $0^+ \rightarrow 1^+ \rightarrow 0^+$.

Which is an entangled state in the linear polarization. To find the relationship with other states, it is necessary to describe the density matrix including the angle Θ between them. The state for a $0^+ \rightarrow 2^+ \rightarrow 0^+$ is:

$$|\Psi(\Theta)\rangle = \frac{\cos \Theta |xx\rangle - |yy\rangle}{\sqrt{1 + \cos^2 \Theta}} \quad (2.45)$$

Where it was considered a frame of reference in which z is the direction of propagation of the first photon, y is the normal to the plane formed by the two first hits of the photons and the source and the second photon is contained in the xz plane. Then the density matrix is:

$$\rho(\Theta) = \frac{1}{1 + \cos^2 \Theta} \begin{pmatrix} \cos^2 \Theta & 0 & 0 & -\cos \Theta \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -\cos \Theta & 0 & 0 & 1 \end{pmatrix} \quad (2.46)$$

The density matrix for two Compton polarimeter [36]s is:

$$\rho_e = \frac{r_0^4}{4} \left(\frac{E(\theta_1)}{E_1} \right)^2 \left(\frac{E(\theta_2)}{E_2} \right)^2 \begin{pmatrix} t_{11}^{(1)} t_{11}^{(2)} & t_{12}^{(2)} t_{11}^{(1)} e^{-2i\phi_2} & t_{12}^{(1)} t_{11}^{(2)} e^{-2i\phi_1} & t_{12}^{(1)} t_{12}^{(2)} e^{-2i(\phi_1 - \phi_2)} \\ t_{12}^{(2)} t_{11}^{(1)} e^{2i\phi_2} & t_{11}^{(1)} t_{11}^{(2)} & t_{12}^{(1)} t_{12}^{(2)} e^{-2i(\phi_1 + \phi_2)} & t_{12}^{(1)} t_{11}^{(2)} e^{-2i\phi_1} \\ t_{12}^{(1)} t_{11}^{(2)} e^{2i\phi_1} & t_{12}^{(1)} t_{12}^{(2)} e^{2i(\phi_1 + \phi_2)} & t_{11}^{(1)} t_{11}^{(2)} & t_{12}^{(2)} t_{11}^{(1)} e^{-2i\phi_2} \\ t_{12}^{(1)} t_{12}^{(2)} e^{2i(\phi_1 - \phi_2)} & t_{12}^{(1)} t_{11}^{(2)} e^{2i\phi_1} & t_{12}^{(2)} t_{11}^{(1)} e^{2i\phi_2} & t_{11}^{(1)} t_{11}^{(2)} \end{pmatrix} \quad (2.47)$$

Where:

$$t_{11}^{(i)} = 1 + \cos^2 \theta_i + [E_i - E(\theta_i)] (1 - \cos \theta_i) \quad t_{12}^{(i)} = \sin^2 \theta_i$$

$$E(\theta_i) = \frac{E_i}{1 + \frac{E_i}{m_e c^2} (1 - \cos \theta_i)}$$

With E_i and $E(\theta_i)$ the initial and scattered photon energy. The cross section is calculated with $\frac{1}{4} \text{Tr}[\rho(\Theta)\rho_e]$ [36]. The result is:

$$\sigma(\Omega_1, \Omega_2) = \frac{r_0^4}{16(1 + \cos^2 \Theta)} \left(\frac{E(\theta_1)}{E_1} \right)^2 \left(\frac{E(\theta_2)}{E_2} \right)^2 \cdot \left(t_{11}^{(1)} t_{11}^{(2)} - \cos \Theta t_{12}^{(1)} t_{12}^{(2)} e^{2i(\phi_1 - \phi_2)} - \cos \Theta t_{12}^{(1)} t_{12}^{(2)} e^{-2i(\phi_1 - \phi_2)} + \cos^2 \Theta t_{11}^{(1)} t_{11}^{(2)} \right)$$

Which simplifies to:

$$\sigma(\Omega_1, \Omega_2) = \frac{r_0^4}{16} \left(\frac{E(\theta_1)}{E_1} \right)^2 \left(\frac{E(\theta_2)}{E_2} \right)^2 \left(t_{11}^{(1)} t_{11}^{(2)} - \frac{2 \cos \Theta}{1 + \cos^2 \Theta} t_{12}^{(1)} t_{12}^{(2)} \cos 2\Delta\Phi \right) \quad (2.48)$$

That can be rewritten in a similar form to Eq. 2.36.

$$\sigma(\Omega_1, \Omega_2) = \sigma_0 \left[1 - \frac{2 \cos \Theta}{1 + \cos^2 \Theta} A_1 A_2 \cos 2\Delta\Phi \right] \quad (2.49)$$

A modulation on the analyzing powers is present, and it is dependent on the type of initial state. In fact, the value of Θ is determined by the angular correlation of the photons $W(\Theta)$. It is necessary to use $W(\Theta)$ as a weight in the averaging. If the relative phase of the initial state is positive then the sign of the second term becomes positive.

2.4.3 AGATA

AGATA [41], the Advanced GAMMA Tracking Array, is a highly sensitive gamma ray spectrometer employing segmented high-purity germanium crystals, shown in Fig. 2.12. It is one of the only two detectors in the world to use the gamma ray energy tracking technique. This allows to reconstruct the trajectory of the photon inside the detector allowing the removal of the Compton shields typically found in other spectrometers, resulting in a gain in efficiency without compromising spectral quality. Furthermore, with the availability of the photon direction of emission measured with high precision allows a good Doppler energy correction and hence achieve an energy resolution of the order of $\sim 2\%$ at 1 MeV even if the photons are emitted from fast moving nuclei. AGATA is composed of clusters, which is a group of three hexagonal and tapered n-type germanium crystals (hexagonal in the front and circular in the rear). Each cluster is cooled with a dewar containing liquid nitrogen due to the small energy gap in the germanium crystal, and the temperature is always monitored to keep it in the ideal range. Inside each cluster are also installed charge sensitive preamplifiers, designed to fulfill the requirements for the Pulse Shape Analysis (PSA) and high counting rate capability. Each crystal has a diameter of 8 cm and is 9 cm long, and are segmented in 6 layers and 6 slices for a total of 36 segments, shown in Fig. 2.11, in addition to an internal common core that collects all the electrons. The 37 signals coming from the crystal are analyzed with PSA algorithms to determine the positions of the photon interactions by comparing the signals with a calculated reference basis and with a set of algorithms, like grid search, genetic algorithms and others, optimized for different types of events. This is possible thanks to the difference in drift velocities inside the crystal for electrons and holes, making the signal distance dependent from the core. Another set of algorithm is used to reconstruct the trajectory of the photons inside the detector, extracting information from position, energy and time of a hit in the crystal. Gamma photons interact mainly via photoelectric effect, Compton scattering and pair production. Photoelectric absorption releases all the energy in the detector in one hit, so the event contributes to the photopeak in the energy spectrum. In Compton scattering, described by Eq. 2.31, the photon does not get absorbed but scatters from an electron releasing only a part of the

total energy. After the first scattering the photon can exit from the detector, so the measured energy will end up in the Compton continuum of the spectrum, or can scatter multiple times until it exits again or gets absorbed with a final photoelectric interaction. Finally in pair production the gamma ray passing through the detector can generate a e^-e^+ pair. The process has a threshold, in particular $E_\gamma > 2m_e c^2$ where m_e is the mass of the electron. The clusters can be used individually, for example in the experiment for measuring the ^{22}Na photons two pairs of clusters are mounted on opposite ends of a source facing each other, as shown in Fig. 2.13.

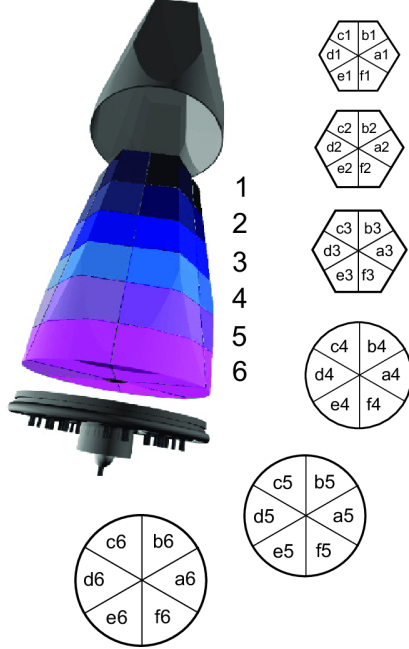


Figure 2.11: Segmentation in one germanium crystal of AGATA [41].

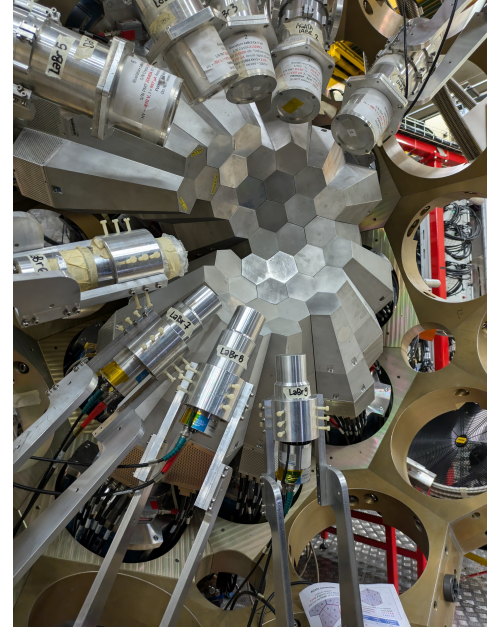


Figure 2.12: AGATA spectrometer with LaBr_3 scintillators used as ancillary detectors.

The advantage of using clusters of AGATA as polarimeters comes from the tracking capability. Having the possibility to reconstruct each vertex point inside the detector allows to reconstruct accurately the angles of the Compton scattering needed for the correlation analysis, in contrast to other experiment where a solid scatterer is used and the detector must be positioned manually. The availability of the measurement of the angle allows to calculate event by event the analyzing power from Eq. 2.50.

$$A(E_\gamma, \theta) = \frac{\sin^2 \theta}{\frac{E_\gamma}{E'_\gamma} + \frac{E_\gamma}{E'_\gamma} - \sin^2 \theta} \quad (2.50)$$

It is possible to then select from the data events with scattering angle comprised in an arbitrary range to maximize the value of the analyzing power. Another experiment possible with AGATA is the measurement of the azimuthal distribution after the photons scatter multiple times. Recent experiment measure the correlation of the photons after one of the two hits a so called “pre-scatterer”, having a total of three Compton scatterings. With the technique of gamma tracking it should be possible to measure what happens to the correlation when both photons undergo multiple Compton scattering.

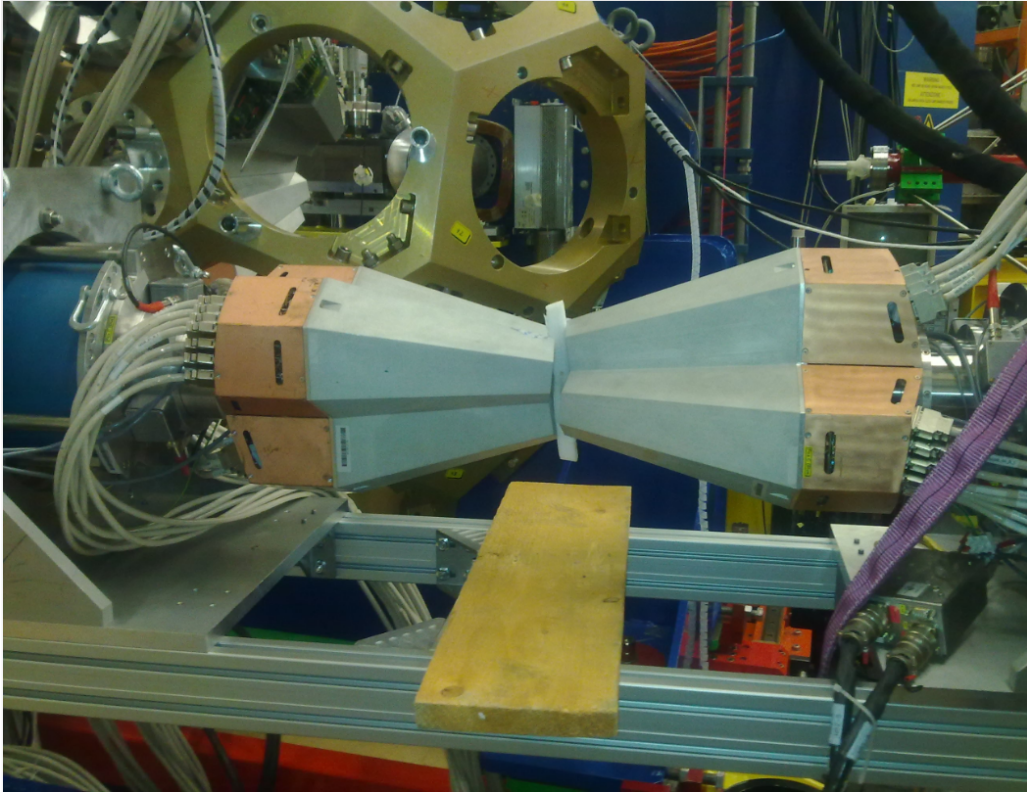


Figure 2.13: AGATA configuration for measuring annihilation photons coming from ^{22}Na [42].

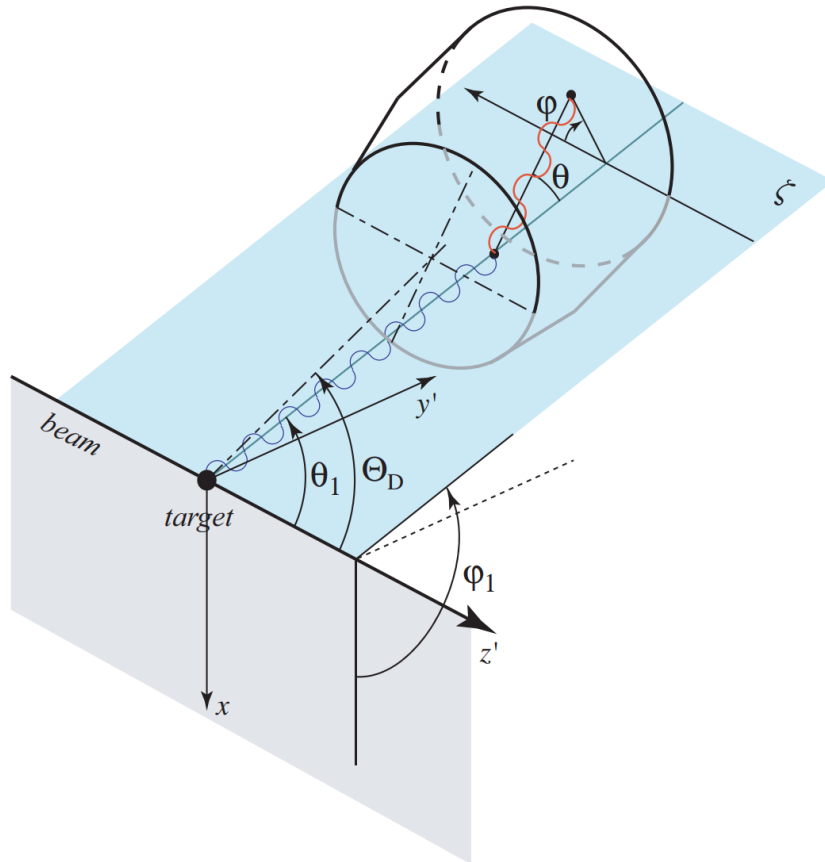


Figure 2.14: Compton scattering in one AGATA cluster [33].

Chapter 3

Cross sections calculations

Calculations of the cross sections were made in order to study the reaction rate and the angular distributions of the transfer reaction ${}^3\text{He}({}^7\text{Be}, {}^6\text{Be}){}^4\text{He}$, leading to the $2p$ decay process from ${}^6\text{Be}$. Two computational frameworks were employed for these calculations: Fresco [43] and TWOFNR [44]. Fresco is a program developed to perform coupled-reaction channels calculations in nuclear physics, supplemented by Sfresco, which is used to provide Chi-squared searches of potential and coupling parameters. TWOFNR is a direct reactions code that evolved from the code TWOSTP, authored by M. Toyama and M. Igarashi, used to calculate the scattering differential cross section using the general form of the distorted wave Born approximation up to the second order. TWOFNR is a finite-range generalisation of this code and it was used because it is less configuration-intensive and does not require a unique input script for every reaction, like Fresco. The Fresco input structure is divided into several distinct components. First, integration parameters are defined, followed by the properties of the target and projectile in both the initial and final states. Subsequently, the necessary potentials are specified. For this specific reaction, five distinct potentials are required: two for the initial and final states, two describing the interaction between the transferred neutron, the projectile core, and the target, and one for the core-core interaction. Finally, the single-particle overlaps and spectroscopic factors are included. The values of the potentials were obtained from global fits of experimental data. In particular were used “A global nucleon optical model potential” [45], “Global ${}^3\text{He}$ optical model potential below 270 MeV” [46] and “Global optical model potential for alpha projectile” [47] to better the properties the range of validity of these fits. some modification of the parameters were made to better match the result of experimental data.

In TWOFNR, instead, two separate codes are used. The first, called FRONT, is responsible for the input file generation, once executed it asks the user all the necessary information for the reaction, with the advantage of having the possibility to use internal values for the potentials derived from global fits of experimental data. Fig. 3.1 shows the comparison of the results from both codes at a laboratory energy of 20 MeV, while Tab. 3.1 shows the values of the potentials used for the inverse reaction, derived from experimental data. I chose to calculate the cross section for the inverse reaction as it was easier finding experimental data for similar reactions.

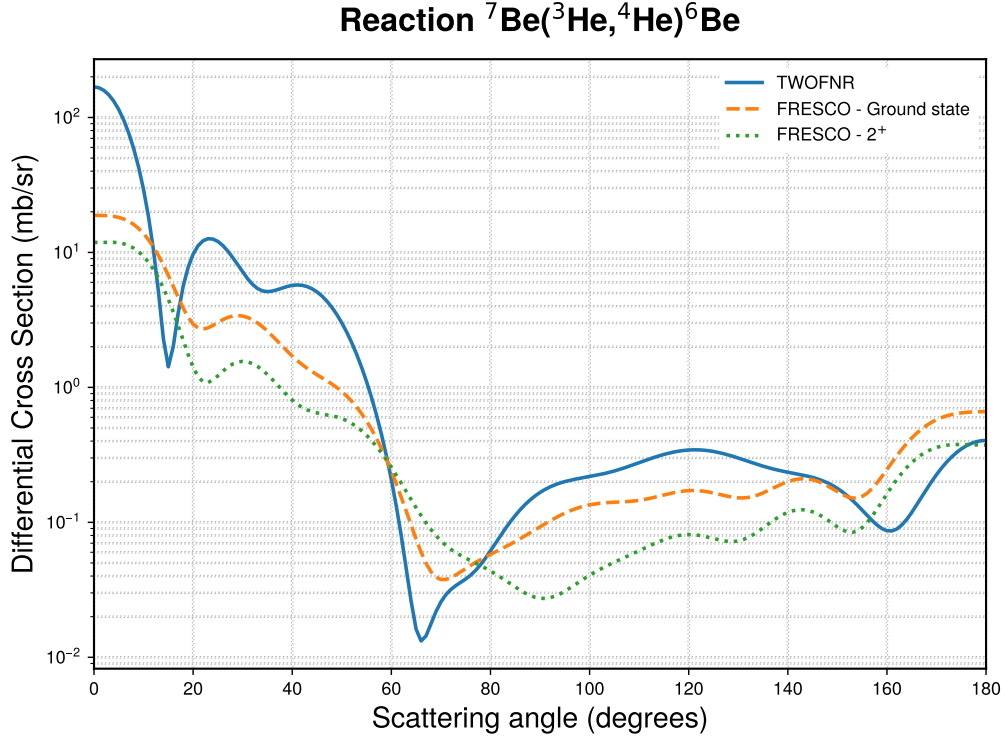


Figure 3.1: Comparison of the cross sections calculations between FRESKO and TWOFNR at $E_{lab} = 20$ MeV.

While the agreement is not exact, the general trends are consistent, with discrepancies attributed to the specific potentials used and the inherent algorithmic differences between the two codes. For the 2^+ state some care was needed for the cross section calculation. This state has a very short lifetime so that the energy width is so large it cannot be approximated by a single discrete state ($\Gamma=1.16$ MeV). To include the width an energy range was defined from the ground state to 3.5 MeV and it was divided in 100 bins, and for each energy bin the cross section was computed. Then, all the cross sections were convoluted in the following way [48]:

$$\frac{d\sigma}{d\Omega} = \int_{E_{min}}^{E_{max}} \left(\frac{d\sigma}{d\Omega dE} \right)_{DWBA} \rho(E) dE \quad \rho(E) = \frac{1}{N} \frac{\Gamma/2\pi}{(E - E_0)^2 + (\Gamma/2)^2} \quad (3.1)$$

Where N is a normalization constant. Fig. 3.2 shows the width of the ground state and the 2^+ state.

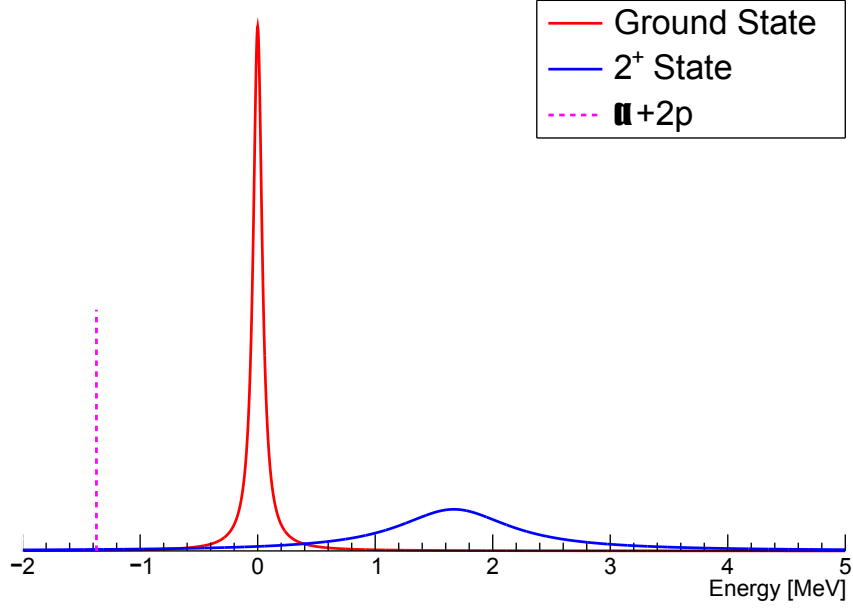


Figure 3.2: Width of the first two energy levels of ${}^6\text{Be}$.

While the comparison between the two programs provides information about the correctness of the Fresco code, there is no way to know if the calculations are right or wrong because experimental data of this reaction are not available at the time of this work. For this purpose the calculations were compared with experimental data of the same reaction but, instead of ${}^7\text{Be}$, ${}^7\text{Li}$ [49] and ${}^9\text{Be}$ [50] were used. ${}^7\text{Li}$ because it is the mirror nucleus of ${}^7\text{Be}$, so it should behave in a similar way because of the small difference between protons and neutrons with respect to the strong force, while ${}^9\text{Be}$ was used because the two extra paired neutrons should have a negligible contribution to the distribution of the differential cross section, with the only difference being the integrated cross section. The choice of experimental data was also driven by the energy set in the experiment, where experimental data taken at energies closest to 20 MeV was chosen to match the condition of the hypothetical experiment.

Starting from the comparison with ${}^9\text{Be}$, in order to compare the differential cross section it is necessary to change the spectroscopic amplitudes of the Fresco code in order to match the experimental points. In Fig. 3.3 the result is shown. The value of the error was not reported in the original article [50].

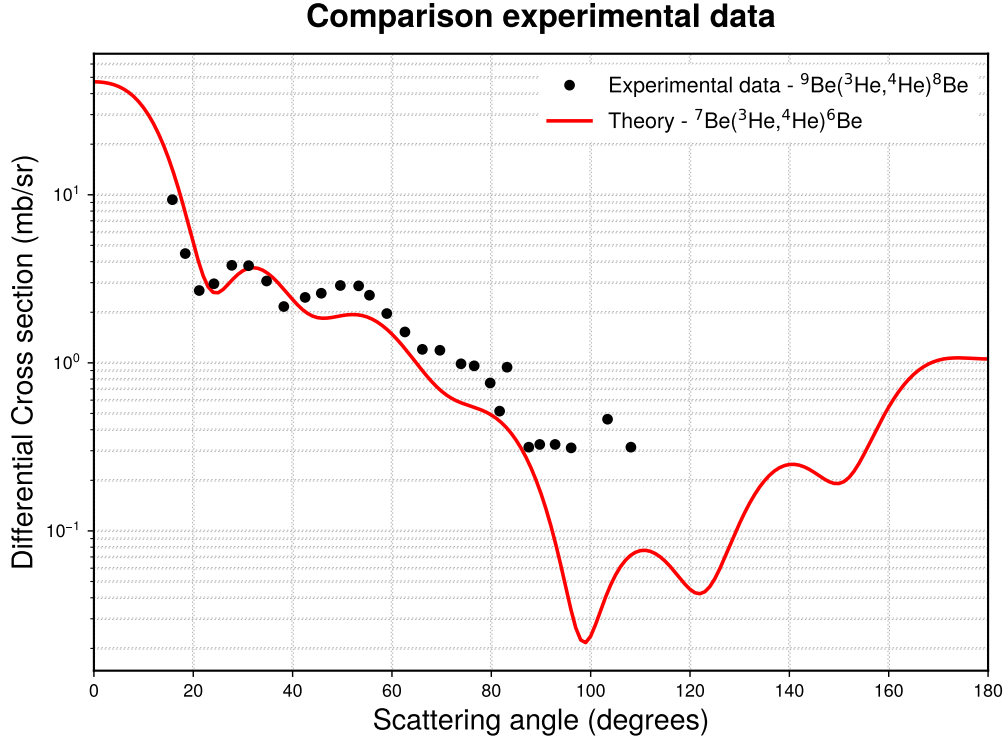


Figure 3.3: Comparison between the differential cross sections of Fresco calculations and ${}^9\text{Be}({}^3\text{He}, {}^4\text{He}){}^8\text{Be}$ experimental points at $E_{lab} = 32.8$ MeV, errors of the experimental points were not reported [50].

For higher scattering angles the structure of the nucleus is an important contribution to the cross section, so due to the two extra neutrons in ${}^9\text{Be}$ the agreement between theory and experimental data is lost. The potential parameters used were the same as the ones used for the comparison between Fresco and TWOFNR, shown in Tab. 3.1. The reason for it was to try not to adapt the potentials in the code to the experimental points, even if there is a large gap in energy, as a way to validate the modification to the parameters to adapt to the experiment. In the case of ${}^7\text{Li}$, the results are shown in Fig. 3.4. The uncertainty on the experimental points in the article was reported as “The net relative errors are shown as error bars in the figures displaying cross sections. When no error bars are shown, they are contained within the spot size.” [49]. Since it was not possible to extract the exact value from the figures, no errors were assigned.

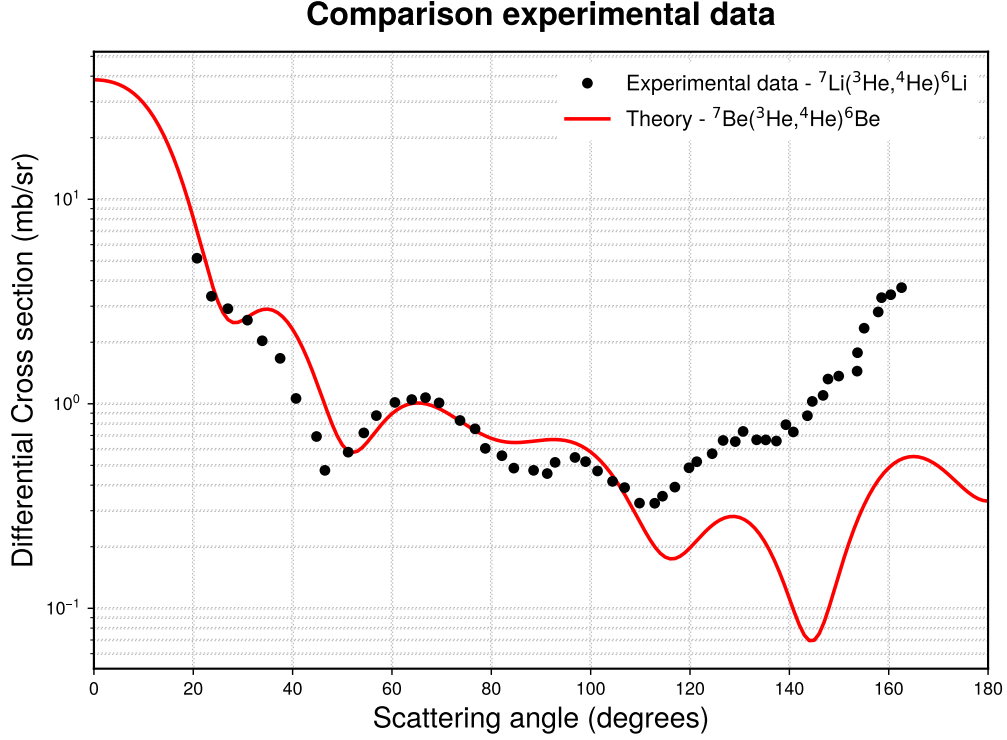


Figure 3.4: Comparison between differential cross section of Fresco calculations and ${}^7\text{Li}({}^3\text{He}, {}^4\text{He}){}^6\text{Li}$ experimental points $E_{lab} = 18$ MeV, errors of the experimental points are not present because it was not possible to extract them.

For the same reason as in the case of ${}^9\text{Be}$, a loss of agreement is observed at higher scattering angles. The parameters of the potentials are reported in Tab. 3.1, and they were used as reference for the other reactions since it has the smallest gap in energy with respect to the other experimental points. The optimization consisted in matching the peaks and minimums of the differential cross section in order to reproduce the same trend.

Channel	r_C	r_V	a_V	V	r_W	a_W	W_V	r_S	a_S	W_S	r_{SO}	a_{SO}	V_{SO}
${}^3\text{He}+{}^7\text{Be}$	1.289	1.187	0.72	112.3	1.424	0.82	1.751	1.211	0.76	20.42	0.752	0.86	2.068
${}^4\text{He}+{}^6\text{Be}$	1.35	1.342	0.658	154.1	-	-	-	1.293	0.636	10.216	-	-	-
$n+{}^6\text{Be}$	1.31	1.581	0.677	43.9	1.10	0.677	1.744	1.31	0.522	5.086	0.829	0.59	5.365
$n+{}^3\text{He}$	1.32	1.09	0.677	43.7	1.03	0.677	1.839	1.32	0.52	5.084	0.737	0.59	5.356
${}^3\text{He}+{}^6\text{Be}$	1.289	1.188	0.72	107.4	1.425	0.813	1.751	1.213	0.749	21.332	0.753	0.862	2.067

Table 3.1: Potential parameters used for the Fresco code for the reaction ${}^3\text{He}({}^7\text{Be}, {}^6\text{Be}){}^4\text{He}$ for the comparison with ${}^7\text{Li}({}^3\text{He}, {}^4\text{He}){}^6\text{Li}$. V and W are the real and imaginary part of the potential in MeV, r and a the radius and diffuseness in fm. $R_x = r_x A^{\frac{1}{3}}$, where $x = V, W, S, SO, C$.

Finally, having established that the Fresco input file was prepared correctly and the resulting cross sections reproduce well the available experimental data, the integrated cross sections were calculated. The results are shown in Tab 3.2 for both ground state and first excited state.

ground state	2^+
9.3 mb	3.01 mb

Table 3.2: Integrated cross section of the reaction ${}^3\text{He}({}^7\text{Be}, {}^6\text{Be}){}^4\text{He}$ for the ground state and first excited state at 20 MeV.

The differential cross section in the laboratory reference frame is shown in Fig. 3.5. To convert the

coordinates the following formulas were used:

$$E_r = \frac{m_A}{m_A + m_B} E_{LAB} \quad \rho = \left(\frac{m_A m_C}{m_B m_D} \frac{E_r}{E_r + Q} \right)^{\frac{1}{2}}$$

$$\theta_{LAB} = \arctan \left(\frac{\sin \theta_{cm}}{\rho + \cos \theta_{cm}} \right) \quad \left(\frac{d\sigma}{d\Omega} \right)_{LAB} = \frac{(1 + \rho^2 + \rho \cos \theta_{cm})^{\frac{3}{2}}}{|1 + \rho \cos \theta_{cm}|} \left(\frac{d\sigma}{d\Omega} \right)_{cm}$$

Where E_r is the total available energy in the center of mass reference frame, ρ is the kinematic factor. For the subscript the notation $A + B \rightarrow C + D$ or $B(A,C)D$. The reaction has a maximum angle of

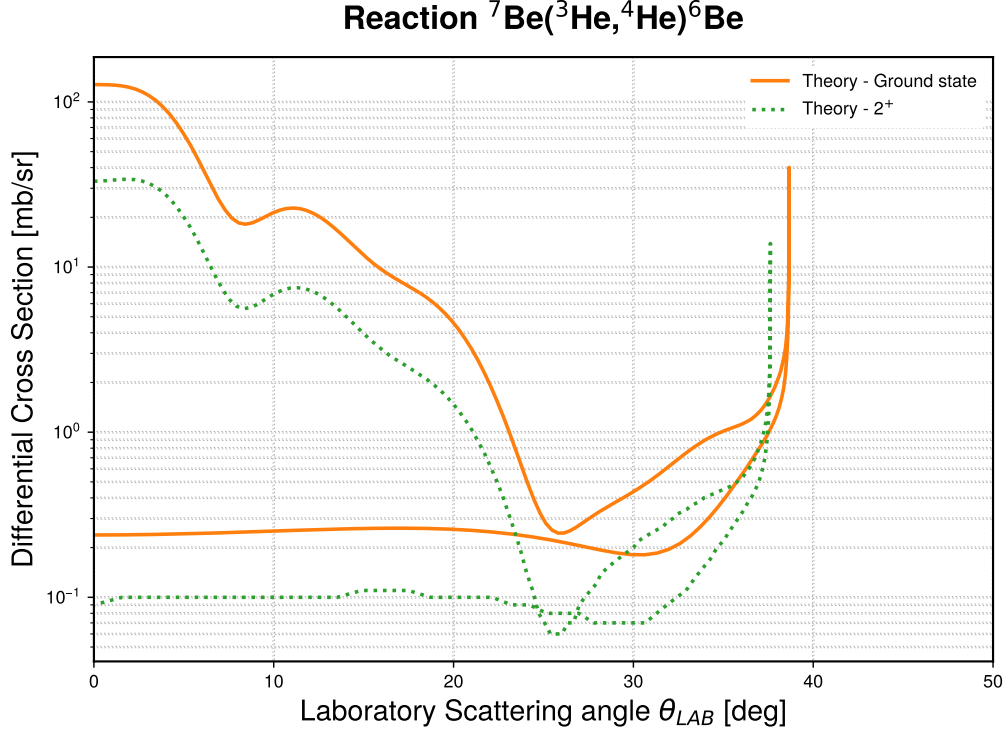


Figure 3.5: Differential cross section in the laboratory reference frame for ground state and first excited state.

emission, which is typical for inverse kinematics reactions, and for the 2^+ state the value is lower, due to the excitation energy necessary to produce the state.

Chapter 4

Simulations

Various simulations were developed in order to study the feasibility of an experiment aimed at the study of the bell inequalities with two protons coming from the decay of ${}^6\text{Be}$. ${}^6\text{Be}$ was chosen as it could potentially be produced via the reaction ${}^7\text{Be}({}^3\text{He}, {}^4\text{He}){}^6\text{Be}$ at the LNL. The first two programs are developed in a generic way with the idea of using the code already written to simplify the development of the following programs, allowing to trust the results of more complex simulations.

4.1 ${}^6\text{Be}$ decay

The first program generates the momenta of the particles produced in the decay, given the energy of the mother nucleus. In this way the properties that a detector needs to have relative to spatial or angular resolution can be determined. The ROOT framework [51] was used in this program for making the plots and the TGenPhaseSpace class was used for the generation of the momenta, assuming a flat cross section. The code is based on the GENBOD function that uses the Raubold and Lynch method [52] for the simulation of multi-body particle decays via Monte Carlo integration. The n-body phase space integral is defined in Eq. 4.1. It can be rewritten, after some calculations, into Eq. 4.2

$$R_n = \int^{4n} \delta^4 \left(\mathbf{P} - \sum_j^n \mathbf{p}_j \right) \prod_{i=1}^n \delta(\mathbf{p}_i^2 - m_i^2) d^4 \mathbf{p}_i \quad (4.1)$$

Where $\mathbf{P}$ is the total four-momentum of the n-body system, $\mathbf{p}_i$ the four-momentum of the i-th particle and m_i the mass of the particle.

$$R_n = \frac{1}{m_1} \int \int \prod_{i=1}^{n-1} \{2M_i R_2(M_{i+1}; M_i, m_{i+1})\} dM_{n-1} \dots dM_2 \quad (4.2)$$

Where the n-body system is divided into $n - 2$ two body cases and M_k is the invariant mass of the first k particles. Fig. 4.1 shows the scheme of the decay for the calculation. All the M_k values are chosen generating $n - 2$ random numbers ($0 \leq r_2 \leq \dots \leq r_{n-1} \leq 1$) and are computed with Eq. 4.3

$$M_k = r_k \left(M_n - \sum_{i=+1}^n m_i \right) + \sum_{i=1}^j m_i \quad (4.3)$$

Finally, the momenta of each particle in the center of mass reference frame is found with Eq. 4.4.

$$\mathbf{p}_k^* = \frac{\sqrt{\lambda(M_k^2, M_{k-1}^2, m_k^2)}}{2M_k} \quad \lambda(a, b, c) = a^2 + b^2 + c^2 - 2ab - 2ac - 2bc \quad (4.4)$$

Then a Lorentz boost is applied to each momentum.

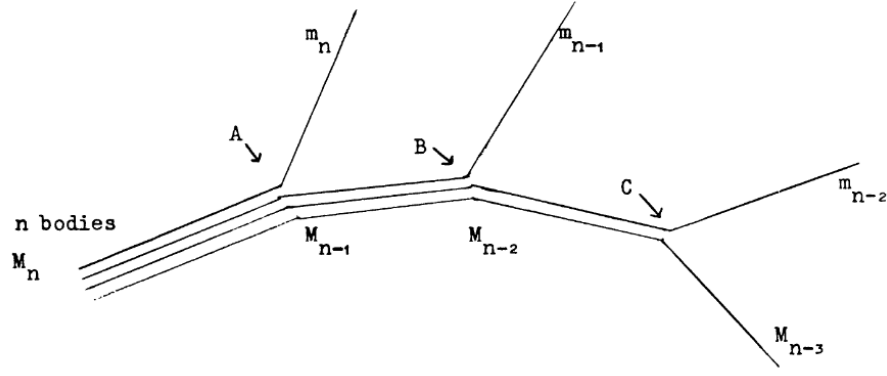


Figure 4.1: Sequential decay for the phase space integration.

In Fig. 4.2 the momenta generated in a single decay event are shown, with an initial energy of 40 MeV.

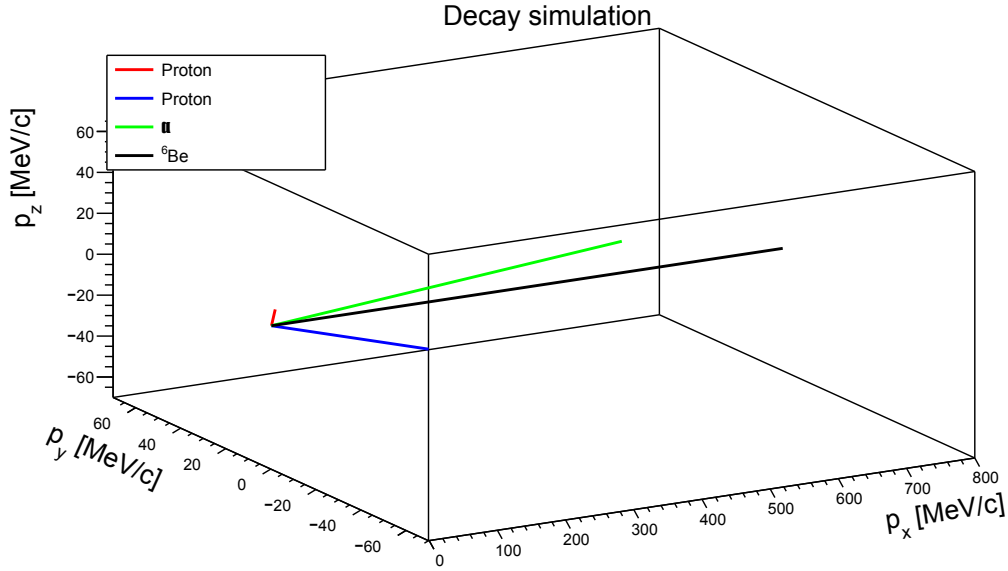


Figure 4.2: Particle momenta from a single simulation event. In black the initial momentum of the ${}^6\text{Be}$, in red and blue the momenta of the protons and in green the momentum of the α particle. The sum of the protons and α particle momenta was verified to be the same as the initial momentum of ${}^6\text{Be}$

To check the correctness of the simulation the conservation of momentum and energy was verified event by event. From the momenta of the decaying particles it is possible to compute their relative position in a hypothetical detector, with the objective of determining the minimum position/angular resolution required in order to study the decay. I analyzed the two different cases where the ${}^6\text{Be}$ was in the 0^+ or 2^+ state. starting from the 0^+ state two histograms recording the y and z coordinates of each particle after having traveled 20 cm along the x axis were made and shown in Fig 4.3. In this way the lateral spread can be analyzed. From the simulations 90% of the protons are produced in a cone of radius of 6.3 cm, while the radius of α particles is 2.0 cm. Due to the relative small aperture of the cones a detector with high position resolution is needed, like a Time Projection Chamber. Fig. 4.4 shows instead the momentum spread in the directions orthogonal to the beam.

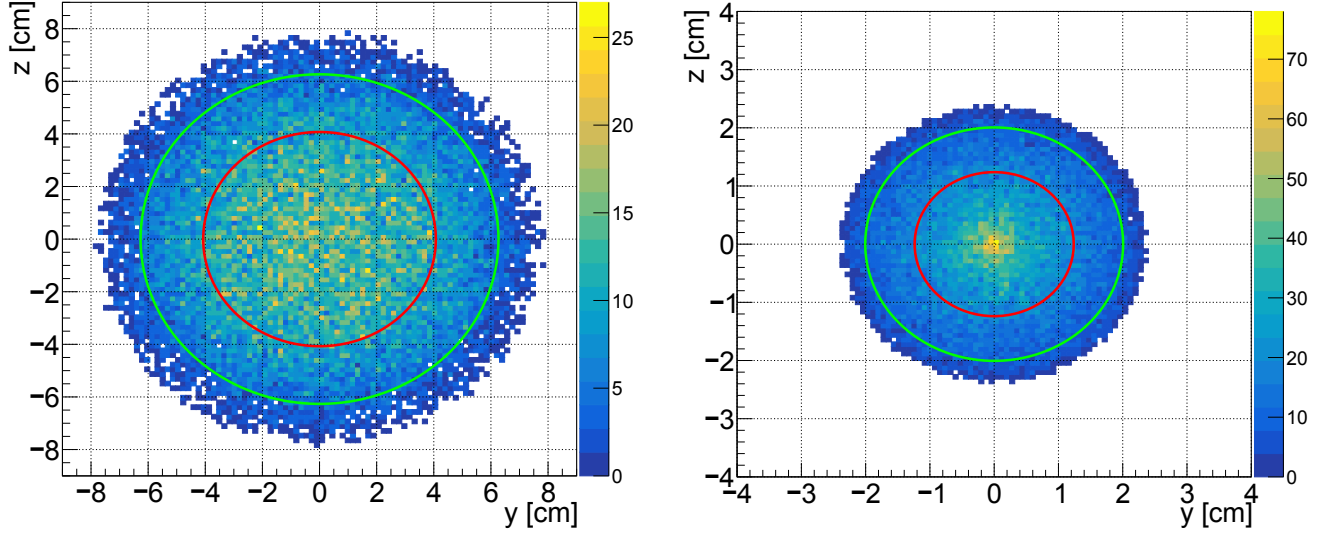


Figure 4.3: Position spread for ${}^6\text{Be}$ in the 0^+ state of (left) protons and (right) α particles. The red circle delimits 50% of the data while the green one 90%.

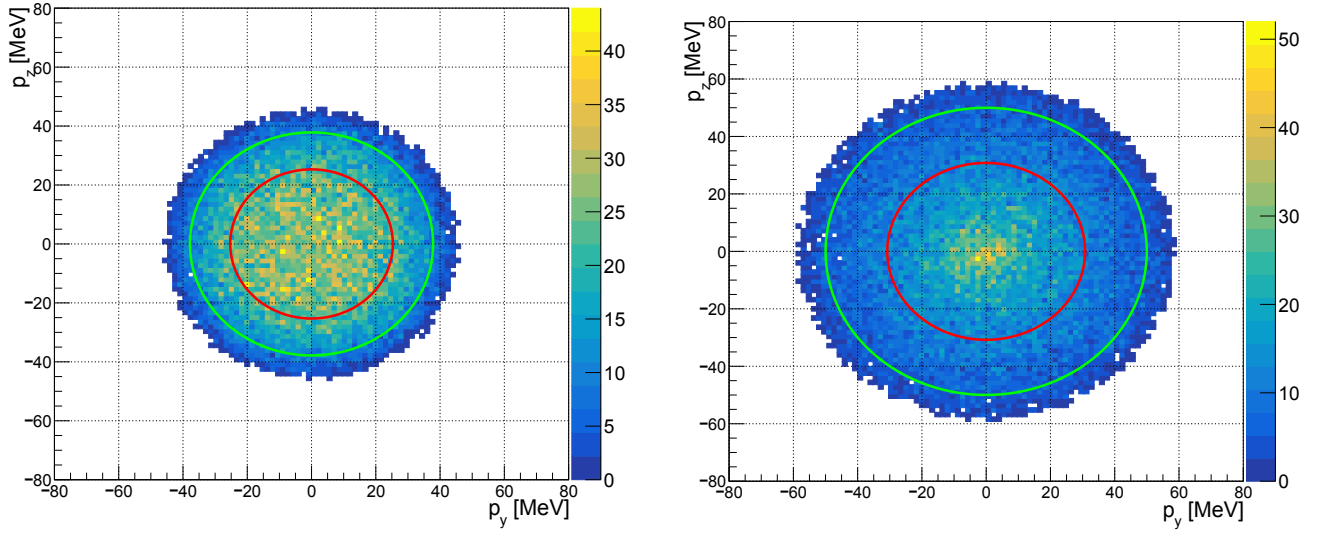


Figure 4.4: Momentum spread for ${}^6\text{Be}$ in the 0^+ state of (left) protons and (right) α particles. The red circle delimits 50% of the data while the green one 90%.

Finally, the Dalitz plot of the reaction was computed from the calculations of the invariant masses $M_{p_2\alpha}^2 = (\mathbf{P}_p + \mathbf{P}_\alpha)^2$ and $M_{p_1p_2}^2 = (\mathbf{P}_{p_1} + \mathbf{P}_{p_2})^2$, where $\mathbf{P}_p$ and $\mathbf{P}_\alpha$ are the four-momenta of the particles. It is shown in Fig. 4.5.

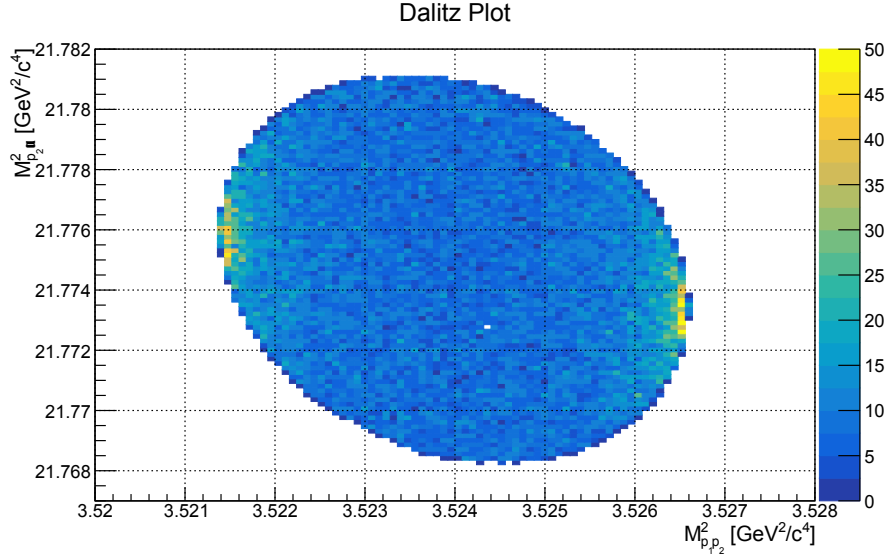


Figure 4.5: Dalitz plot of the three body decay for ${}^6\text{Be}$ in the 0^+ state.

For the 2^+ state the effect of the centrifugal barrier was added to determine its effect on the phase space, by assigning a weight on the event depending on the relative momentum between the particles. In the 2^+ state two configuration were considered, based on the relative angular momentum between the two protons (l), the relative angular momentum between the α particle and the two protons (L) and the total spin of the protons (S). The two states considered are the (1,1,1) and (0,2,0) in the notation (l,L,S). The state (2,0,0) is equivalent to (0,2,0) from the point of view of the weights, so it is already included in the calculations. The Position spread is shown in Fig. 4.6, the momentum spread in Fig. 4.7 and the Dalitz plot in Fig 4.8.

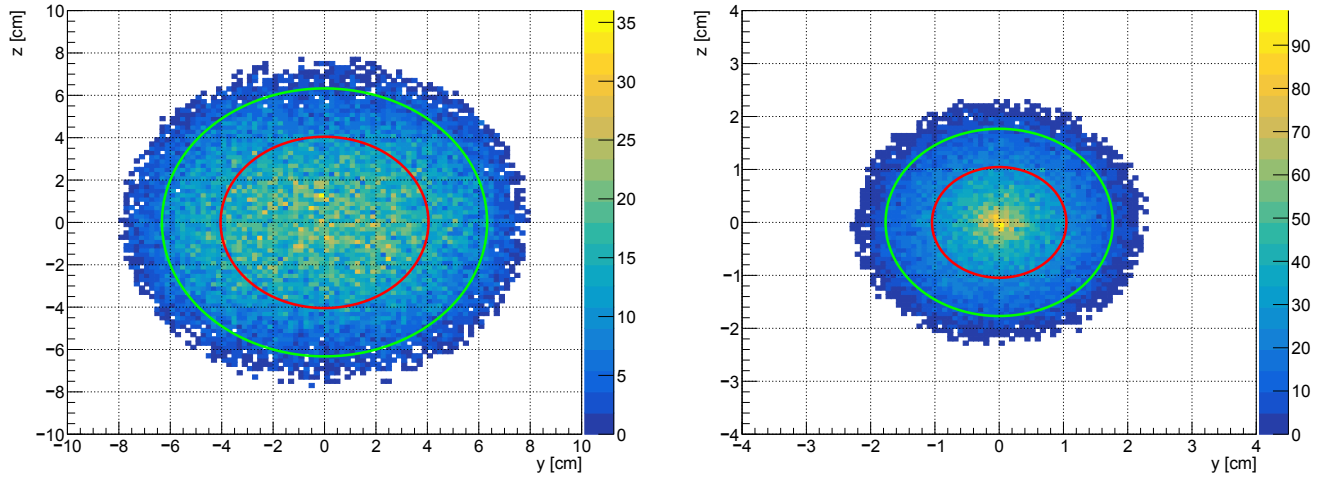


Figure 4.6: Position spread for ${}^6\text{Be}$ in the 2^+ state of (left) protons and (right) α particles. The red circle delimits 50% of the data while the green one 90%.

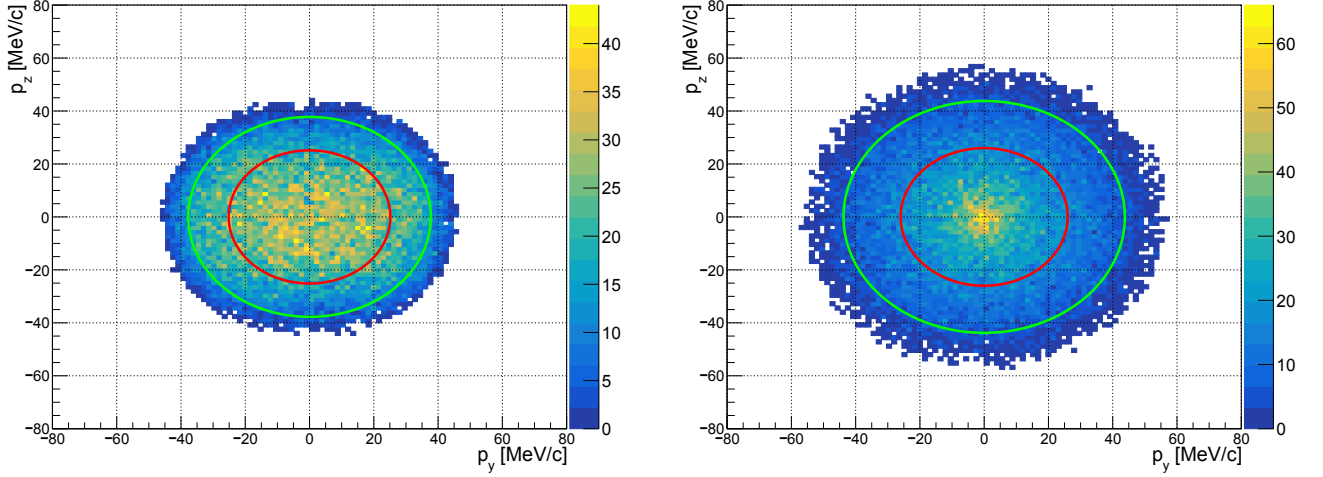


Figure 4.7: Momentum spread for ${}^6\text{Be}$ in the 2^+ state of (left) protons and (right) α particles. The red circle delimits 50% of the data while the green one 90%.

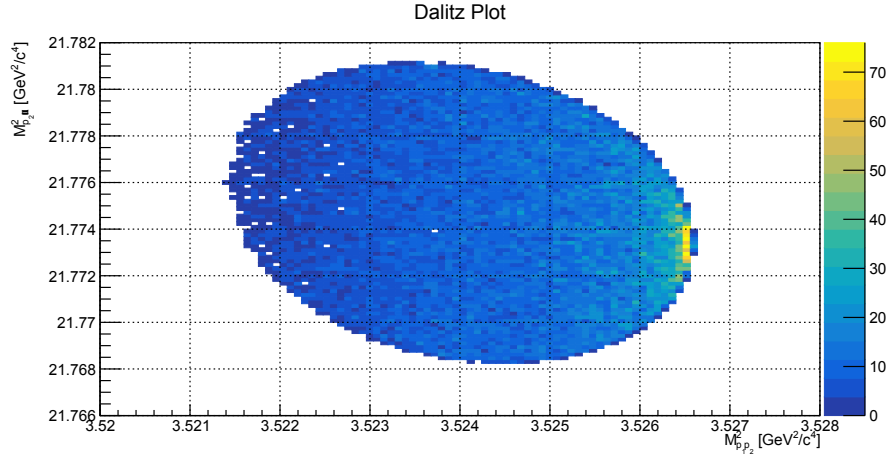


Figure 4.8: Dalitz plot of the three body decay for ${}^6\text{Be}$ in the 2^+ state.

The Dalitz plot is radically different from the one computed in the ground state. There is a concentration of events on the right, where the invariant mass $M_{p_1p_2}$ is higher. This is due to the centrifugal barrier making it harder for the events with low relative momentum to be generated. Also the radius of the cone delimiting 90% of the events slightly increased for the protons and decreased for the α particles, 6.3 cm for the former and 1.8 for the latter.

One thing to keep in consideration is the ability to separate the two α particles produced in the reaction, one produced together with ${}^6\text{Be}$ and the other emitted in the decay. The lifetime of ${}^6\text{Be}$ is very short so it is possible to measure the energy of each particle to reconstruct the event. The energy of the first α particle can be reconstructed from two-body kinematics, so combining the information of both trajectory and energy it is possible to from which process originates the alpha particle. Moreover, The last α particle in the laboratory frame is emitted at a very narrow angle, while the first one does not have this restriction, as can be seen in Chapter 3.

Finally, while the production of protons with relative angular momentum between them might introduce complication in the correlation measurement, it does not deteriorate the sensitivity of the measurement because the spin-orbit interaction is only sensitive to the spin of the incoming particle, in the case where the target nucleus is spinless, and the relative angular momentum between the incoming particle and the target nucleus.

4.2 Bell inequality simulation

The second program simulates the measurements in a Bell experiment for a given state and compares the results with the theoretical calculation. In this way different spin configuration can be studied in order to understand how they can affect the measurement in an experiment. Several states were taken into account, in particular the singlet state, the three triplet states $|S = 1, S_z = +1\rangle$, $|S = 1, S_z = 0\rangle$, $|S = 1, S_z = -1\rangle$ and a statistical mixture of $|\uparrow\downarrow\rangle$ and $|\downarrow\uparrow\rangle$. The code employs the vector notation for spin states, and the tensor product is represented as the Kronecker product. For the vector operations the Eigen template library [53] was used. In Fig. 4.9 the Bell inequality for the different states, computed from the following Eq. 4.5 and a Monte Carlo simulation, are compared.

$$\begin{aligned}
 S_1(\Phi) &= -\langle A_1 B_1 \rangle + \langle A_2 B_1 \rangle + \langle A_1 B_2 \rangle + \langle A_2 B_2 \rangle & \hat{A}_1 &= \hat{\sigma}_z \\
 S_2(\Phi) &= \langle A_1 B_1 \rangle - \langle A_2 B_1 \rangle + \langle A_1 B_2 \rangle + \langle A_2 B_2 \rangle & \hat{A}_2 &= \hat{\sigma}_{z,1} \cos 2\Phi + \hat{\sigma}_{x,1} \sin 2\Phi \\
 S_3(\Phi) &= \langle A_1 B_1 \rangle + \langle A_2 B_1 \rangle - \langle A_1 B_2 \rangle + \langle A_2 B_2 \rangle & \hat{B}_1 &= \hat{\sigma}_{z,2} \cos \Phi + \hat{\sigma}_{x,2} \sin \Phi \\
 S_4(\Phi) &= \langle A_1 B_1 \rangle + \langle A_2 B_1 \rangle + \langle A_1 B_2 \rangle - \langle A_2 B_2 \rangle & \hat{B}_2 &= \hat{\sigma}_{z,2} \cos \Phi - \hat{\sigma}_{x,2} \sin \Phi \\
 S(\Phi) &= \max\{|S_1(\Phi)|, |S_2(\Phi)|, |S_3(\Phi)|, |S_4(\Phi)|\}
 \end{aligned} \tag{4.5}$$

Where:

$$\langle A_i B_j \rangle = \langle \Psi | \hat{A}_i \otimes \hat{B}_j | \Psi \rangle \tag{4.6}$$

With $|\Psi\rangle$ a generic spin state $|\psi_1\rangle \otimes |\psi_2\rangle$.

The simulation of a measurement of an entangled state is articulated in the following steps:

1. First, the state vector is generated, representing the spin state of two protons as a dimension 4 column vector with Eigen;
2. Either A_1 or A_2 , written in their matrix representation also in Eigen, is randomly chosen and its eigenvalues and eigenvectors are computed.
3. The state gets projected onto the eigenvectors to find the probabilities of measuring each eigenvalue and based on that the result of the measurement is randomly chosen, calculating the state after the measurement in the following way:

$$|\psi'\rangle = \sum_i |\lambda_i\rangle \langle \lambda_i | \psi \rangle \tag{4.7}$$

Where $|\psi'\rangle$ and $|\psi\rangle$ are the new and old state, $|\lambda_i\rangle$ are the eigenvectors with eigenvalue λ relative to the used operator.

4. The same operations are made with B_1 and B_2 , except calculating the final state, and the product of the two measurement, namely the eigenvalues chosen randomly, is saved in one of four variables depending on which operators were used, normalizing at the end for the number of states generated. In this way the $S_i(\Phi)$ of Eq. 4.5 are obtained and used to compute $S(\Phi)$.

The error of the simulated events points is given by $\sigma = \frac{2}{\sqrt{N}}$. For each point in the figures ten thousand events were generated, for a total of 180 thousand events. From the figures the limit of 2 is only broken for the singlet state and the $|S = 1, S_z = 0\rangle$ state. For the other two the limit is not surpassed, as expected. The code will be used later as a basis for analyzing the protons. The problem of the $|S = 1, S_z = 0\rangle$ state is that it is not rotational invariant, so that depending on the orientation that it is produced in a detector could measure a $|S = 1, S_z = 1\rangle$ or $|S = 1, S_z = -1\rangle$ state, which are not entangled.

An analysis was made to understand the effect of a non pure singlet state to the Bell inequalities. Event by event the singlet state was replaced by one of the three triplet states with a certain probability, to simulate the situation where in the decay both singlet and triplet states are allowed to be emitted.

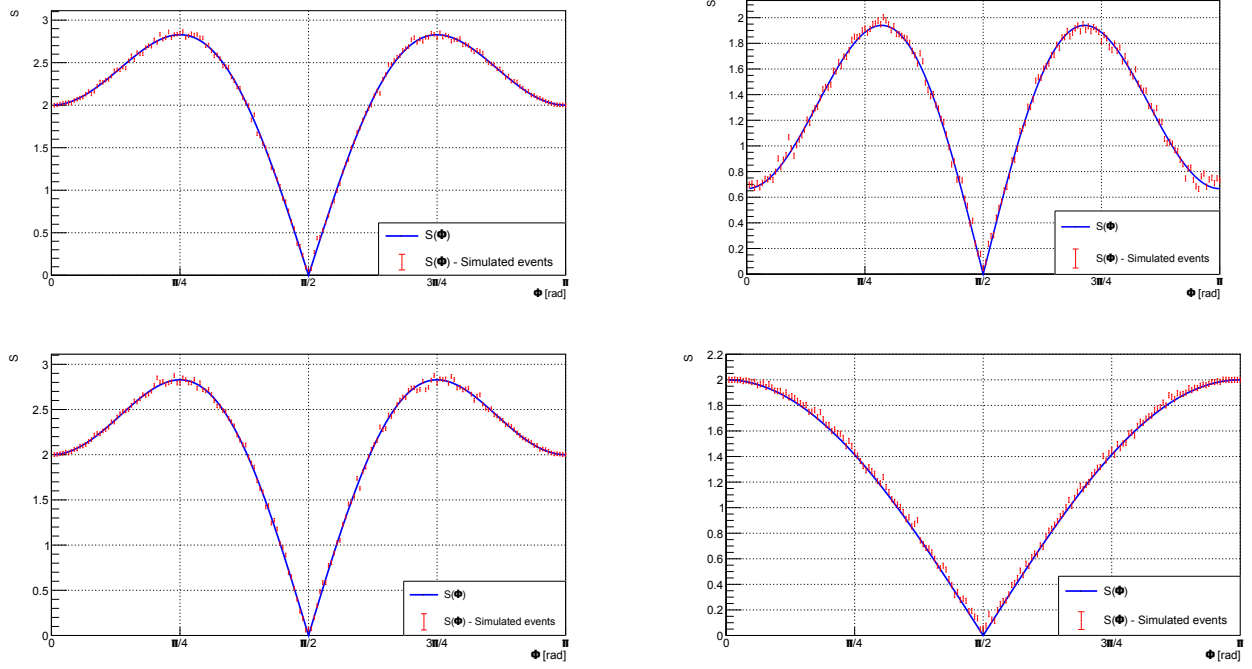


Figure 4.9: Comparison of computed and simulated Bell inequalities for (top left) spin singlet state, (top right) spherical spin triplet state, (bottom left) the $|S = 1, S_z = 0\rangle$ state, (bottom right) mixed state of 50% $|\uparrow\downarrow\rangle$ and 50% $|\downarrow\uparrow\rangle$

Fig. 4.10 shows the different Bell inequalities obtained with this interference, for eight different fractions of triplet states inserted. Curiously, the curve for 100% triplet states fraction is higher than the one with 60% and it appears to be the same as the one with 50%. This is because the triplet states consists also of $|S = 1, S_z = 0\rangle$, which as shown before behaves in the same way as the singlet state. When the spin is not polarized, and so it is not possible to predict which of the three triplet states will be detected, the overall correlation reduces to $\frac{1}{3}$. In a similar way, at a fraction of 50%, the average correlation is also $-\frac{1}{3}$, and due to the absolute value present in the calculation of $S(\Phi)$ the two curve end up being the same. It is to be noted that, following these results, a good knowledge on the final state of the daughters particle is required. One possibility is to execute theoretical calculations on the decay of ${}^6\text{Be}$ to find the fraction of triplet states produced, in this way a correction on the Bell inequalities can be made. Otherwise, the experimental setup must be capable of measuring the relative angular momentum between the particles to discern the spin state of the protons, rejecting events where a triplet state may have been produced.

As explained in section 4.1 the introduction of the coupling between spin and angular momenta does not introduce significant complications, because the spin-orbit interaction is sensible only to the spin of the incoming proton and the relative angular momentum that is developed between the proton and the target nucleus, independently from the second proton. Notably from the measurement of the Bell inequalities it could be possible to extract the fraction of triplet states and compare it to theoretical predictions.

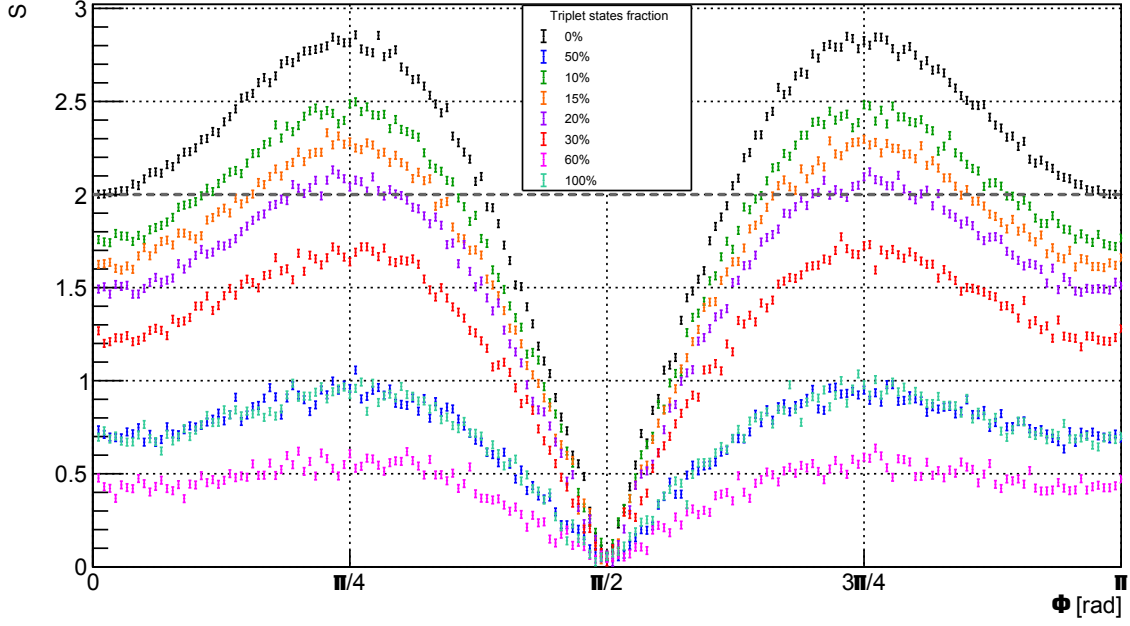


Figure 4.10: Bell inequalities obtained from

4.3 GEANT4 simulation

The third program is written with the toolkit GEANT4 [54]. The objective of this simulation is to study the event rate in function of energy and target parameters. The spin orbit interaction is implemented as a modification of the hadron elastic interaction, as it is not directly implemented in GEANT4. Different scenarios were simulated, in particular a gas chamber filled with ^3He and a graphite block. The gas chamber is a 50 cm cube filled with ^3He at 200 mbar, with the choice of using only the helium or a mixture of other gases, in particular CO_2 and butane, to increase the overall density. To implement the spin orbit interaction a custom class was written. Firstly, from the standard hadronic process included in GEANT4, “G4HadronElasticProcess”, the scattering angle and final momentum are extracted. The scattering angle can be used to compute the analyzing power, from a model or by a fit on experimental data. I chose to leave it constant as a fine description can be added later and, above all, it is dependent on the experimental setup. Then the azimuthal angle is chosen randomly based on the cross section, Eq 2.21, with the acceptance-rejection method. Starting from a randomly generate angle another value r is generated and compared with the value obtained from $p = 1 + A_y P \cos \Phi_{\text{random}}$. r is generated between 0 and the maximum value of the distribution, and if it is greater than p then the angle is rejected and another one gets randomly generated. In this way it is possible to generate azimuthal angles with the same distribution of the cross section. The degree of polarization P of a proton was always set at $+1$ or -1 . Finally the azimuthal angle is applied to the momentum and the initial momentum and vertex coordinates are saved on a ROOT file.

The number of events simulated was chosen looking at the cross section calculated in chapter 3 and the maximum intensity of the ^7Be beam produced by the EXOTIC facility at the LNL, assuming an experiment run of one week. In the developing of the program I ignored uncertainties from the detectors, as the objective was to check if the statistics was high enough to perform the experiment. For the calculation of the number of events I assumed that the beam hits a target of ^3He at 200 mbar, 20 C° and 25 cm long. Using the ideal gas law the total number of event is calculated with Eq. 4.8

$$N = \Phi \frac{P}{k_B T} L \sigma \quad (4.8)$$

Where Φ is the integrated flux of particles, equal to 10^6 particles per second times the seconds in a

week, P and T the pressure of the gas, k_B the Boltzmann constant and L the thickness of the target. The result is $N = 6.8 \cdot 10^5$. While the cross section was calculated with a beam energy of 40 MeV for the ^7Be , the same number of events was kept even for the other energies. The reason for this is to have a comparison of the fraction of wanted events to see at which energy the setup is more sensible to the double spin orbit scattering process. The properties of the chamber were chosen to mimic a Time Projection Chamber (TPC). A picture of the chamber with 40 decay events is shown in Fig. 4.11.

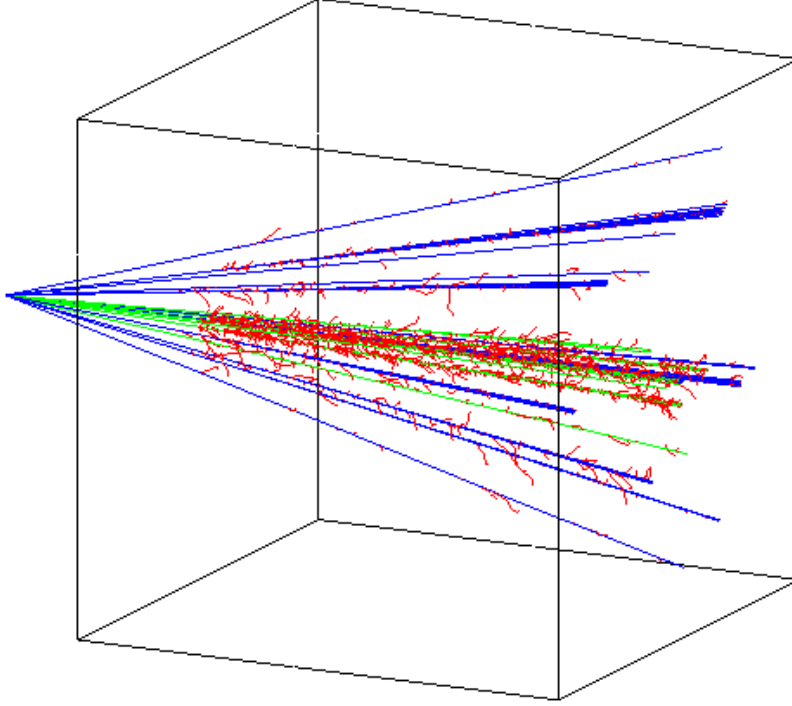
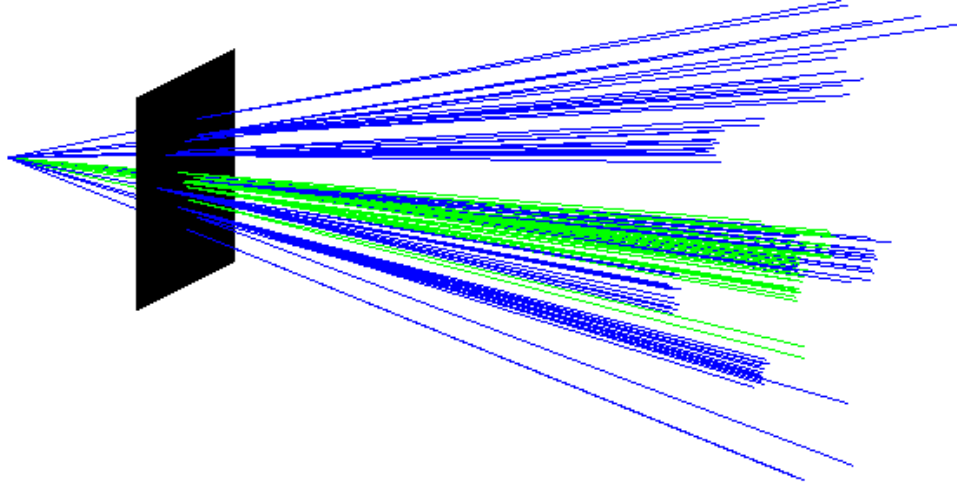


Figure 4.11: Picture of the ^3He gas chamber built with GEANT4. The blue lines represent the trajectory of the protons, in green the one of the α particles and in red the one of the electrons coming from the ionization of the gas.

The dimension and parameters of the chamber were chosen to represent a generic setup of a TPC. As such the maximum pressure considered was 200 bar, and its size was limited to a cube with sides long 50 cm. The choice to mimic a TPC and ^3He as the gas comes from the fact that it can be used as an active target for the ^7Be beam. The number of events calculated used a target 25 cm thick in order to have all the decay events in the first half of the chamber, in order to separate better the position of the particles in the final state. While in a real experiment all the events are generated inside the chamber, it was chosen to put the starting position outside the chamber in order to maximize the number of spin orbits events. Initially the only gas considered was ^3He but after running some simulations it appeared that the density was too low to observe any events. Stable gases with the highest density possible were added as a 50% mixture such as carbon dioxide (CO_2) and butane (C_4H_{10}), even though the latter one is flammable. The temperature was always considered ambient temperature, no cryogenic scenarios were examined. However in the end the total number of events is still too low (< 10) to conduce an experiment. Unfortunately, while a setup consisting of a TPC provides many advantages, such as the 3D reconstruction of the trajectory starting from the production of the protons and 2π coverage, it alone is not capable of measuring the desired effect. The graphite block is a square prism of side 50 cm and with variable thickness. The block must be thin enough to let the protons through without stopping them, but thick enough to maximize elastic scattering. It is shown in Fig. 4.12 Due to the fact that the protons are produced in a three body decay their energy will be in a certain range, so the

	^3He	CO_2	C_4H_{10}
40 MeV	2	46	122
80 MeV	1	65	134
160 MeV	1	34	67

Table 4.1: Number of simulated events interacting via spin orbit with different energies and materials.

Figure 4.12: Picture of the graphite block built with GEANT4. The blue lines represent the trajectory of the protons, in green the one of the α particles.

protons produced with minimum energy could be stopped. I chose to select the maximum thickness allowed of the graphite block for each energy to not stop any proton, in order to not lose any event. The increase in thickness, allowed by the higher beam energy, suggests that higher energies are better for this type of experiment.

As a result of the larger thickness at higher energy the number of events is an order of magnitude higher than the result obtained from the gas chamber.

From the simulation of the gas chamber it is evident that it is not a suitable alone to measure the double spin-orbit scattering required. The average number of events does not exceed the thousands. This is because a gas mixture in the typical situation of a TPC has not enough density to produce a significant number of events. The pressure chosen was one of the maximum values of pressures that are able to be sustained by a TPC, so increasing the pressure might bring to a higher number of events but then it would be experimentally difficult to realize. A better solution is the use of the graphite block. A solid block can be easily inserted in a hypothetical TPC to significantly increase the counting rate, due to higher density of the material.

In summary, the results of the simulation indicate the following:

	Events
40 MeV (200 μm)	10
80 MeV (1 mm)	127
160 MeV (3 mm)	1284

Table 4.2: Number of simulated events interacting via spin orbit with different energies and thickness of graphite.

- The use of only a TPC filled with a mixture of ^3He and other gases is not realistic due to the low rate coming from the requirement of having a double hadron elastic scattering of the protons;
- The presence of a solid scatterer, in this case a sheet of carbon, increases significantly the number of events at higher energy;

With these result a TPC with a sheet of carbon inserted inside as a scatterer should be suitable for this type of measurement.

4.4 Bell inequality with spin orbit

The fourth program I developed simulates the spin orbit interaction and implements the correlation measurements to compute the bell inequality, analyzing the data set obtained from the simulation of the graphite block with a beam energy of 160 MeV. This code is separate from the GEANT4 simulation because there is no direct implementation to include an entangled state, and therefore the introduction of a singlet state must be written by the user. Instead of writing a new class for GEANT4 the momenta of the protons that interacted in the simulation were saved and taken as input in this program. The program uses the same definition of the spin orbit interaction inserted in GEANT4 and applies it to the saved particles. The Analyzing power can be changed freely so it is possible to analyze the effect on the Bell inequalities. The analysis strategy is based on the measurement of the azimuthal angle of the scattered protons. Since a TPC is used the trajectory of the protons is considered as known, and as such it is possible to reconstruct the scattering planes of the interactions and therefore the azimuthal angle for every event. The histogram obtained in this way then contains the information on the entanglement. The yield for the double spin orbit scattering, derived from the cross section in Sec. 2.2 is shown in Eq. 4.9.

$$N(\Delta\Phi) = N_0(1 + \nu \cos \Delta\Phi) \quad (4.9)$$

Where N_0 is the yield with no spin polarization and ν is the wanted parameter. The relationship between this parameter and the correlation is shown in the following equation:

$$C(0) = \frac{N(0) - N(\pi)}{N(0) + N(\pi)} = \nu \quad (4.10)$$

Where $C(\Delta\Phi)$ is the correlation function. The parameter ν depends on both the product of the analyzing power for the protons and the correlation between them if the initial state is not pure. The expected yield for an entangled state is:

$$N(\Delta\Phi) = N_0(1 - A^2 \cos \Delta\Phi) \quad (4.11)$$

The program simulates the spin-orbit scattering and builds the correlation histogram. In the program it was assumed that the analyzing power was the same for both protons and was constant with respect to the scattering angle θ , so that $A_1 A_2 \equiv A^2$. The analysis was made with different analyzing powers, in particular $A = 0.2$, $A = 0.5$ and $A = 0.9$ to study the effect on the distribution. The results are shown in Fig. 4.13. Once obtained the histogram Eq. 4.9 is used as a fitting function to calculate the parameter ν . In this way it is possible to analyze the value of the parameter acting on the analyzing power and on the initial state. In Tab. 4.3 the results are shown.

Analyzing power	N_0	ν	$\chi^2/NDOF$
$A = 0.9$	12.0 ± 0.3	-0.41 ± 0.04	109/98
$A = 0.5$	11.9 ± 0.3	-0.20 ± 0.04	100/98
$A = 0.2$	12.0 ± 0.3	0.00 ± 0.04	108/98

Table 4.3: Results of the fits on the histograms of Fig. 4.13.

As shown by the results, the setup with analyzing power equal to 0.2 does not show any signal, while the other do. However, no systematic errors were included so the final error in the measurement could

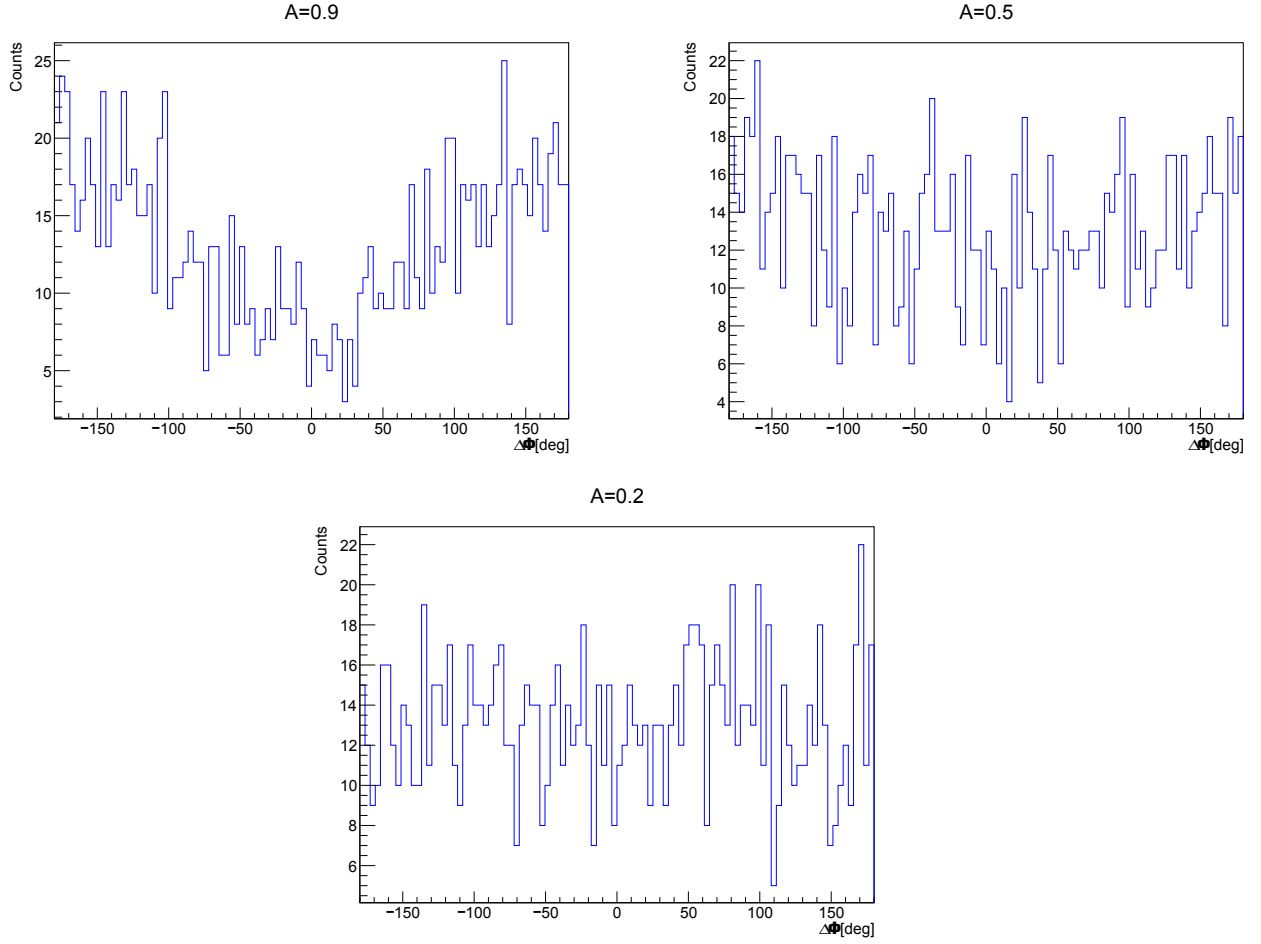


Figure 4.13: Histograms obtained from the spin orbit interaction simulation for different Analyzing powers.

hide the signal. The statistics is barely enough as an average analyzing power of 0.9 is unrealistic for the majority of experimental setups, so the beam intensity or the pressure of the ^3He must be increased. However the number of events simulated were based on the beam intensity of the EXOTIC facility, which cannot produce ^7Be ions with an energy of 160 MeV. In fragmentation facilities both the beam energy and intensity could be higher so more statistics can be collected. In the end the result of the simulation are compatible with the theoretical calculations for an entangled state, in fact the parameter ν is always compatible with the square of the analyzing power for the three cases. As a comparison the same distribution were simulated in the case the initial state was not an entangled state, which from the cross section calculated in Sec. 2.2 should have a value ν equal to one third of the parameters of Tab. 4.3. In Fig. 4.14 the same data are used but the spins are classically correlated, and the analyzing power was set at 0.9.

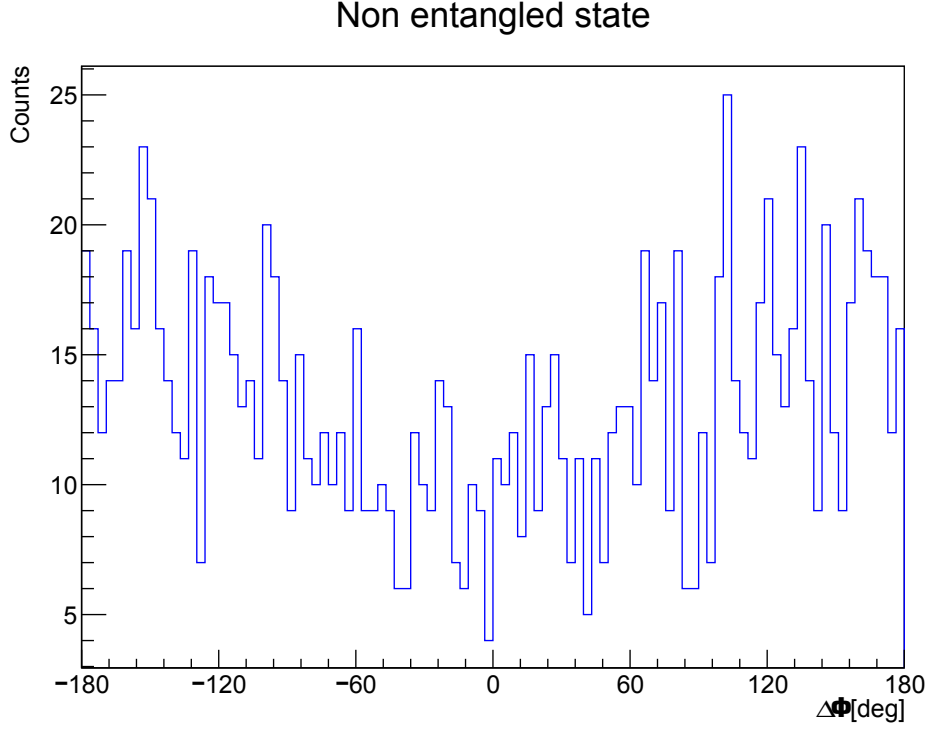


Figure 4.14: Simulated azimuthal correlation distribution for two classically correlated protons ($A=0.9$).

The result of the fit gives a value of $N_0 = 11.9 \pm 0.3$ and $\nu = 0.29 \pm 0.04$. In particular, the value of ν is compatible with the square of the analyzing power divided by three, which is 0.27. With this simulation the cross sections calculated in Sec. 2.2 for the system of two particles are verified. To generate the entangled and not entangled state in the simulation two different pieces of code were used. For the entangled state, since it is spherically symmetric, it is chosen at random if the spin of one proton is up or down, and consequently the other will have opposite orientation along the y axis. For the case of the not entangled state firstly an axis is randomly chosen then, similarly for the entangled state, the spin of the protons is orientated at random with one proton having always the opposite orientation of spin to the other. However in this case the spin must be projected on the plane orthogonal to the direction of the incoming particle.

To consider the effect of the spatial resolution to the determination of $\Delta\Phi$ the propagation of errors was computed. The errors are mainly due to the determination of the initial trajectory and the trajectory after scattering. The measurement of the direction of the particle momentum is done through a fit of all the activated pads of the TPC, and is influenced by:

- Number of active pads N_{points} measuring the track and its length L . It is assumed as error:

$$\sigma_{pad} \approx \frac{\sigma_{point}}{L\sqrt{N_{points}}} \quad (4.12)$$

Where σ_{point} is the error coming from the geometry of the pads, like dimension and pitch. Due to the relative high energy of the protons and α particles the length of the track is very long, usually longer than the dimensions of the chamber, as shown in the GEANT4 simulation in Fig. 4.11, where all the particles pass through the chamber. In these cases the error is almost negligible, however in some scenarios the length of the track may be reduced. For the case of the incoming trajectory it depends on where ${}^6\text{Be}$ is produced, if for some events the decay happens too close to the graphite scatterer the error can increase considerably. For the outgoing momentum, and especially lower energy protons, the energy loss inside the scatterer may be so high that the track stops prematurely before exiting the detector. In addition to this the width of the charge cloud

must also be included. The width for a diffusion coefficient D_T and a drift path z is given by [55]:

$$\sigma_{D_T} = \sqrt{\frac{2D_T(0)z}{v_d}} \sqrt{\frac{1}{1 + \omega^2\tau^2}} \quad (4.13)$$

Where v_D is the drift velocity, $\tau = \langle \frac{\lambda}{v} \rangle$ is the collision time and λ the mean free path for electron scattering, ω is the cyclotron frequency $\frac{eB}{m_e}$ if a magnetic field is present and $D_T(0)$ is the diffusion coefficient in absence of a magnetic field. The choice of gas can reduce the value of σ_{D_T} .

Depending on the dimension of the pads and the width of the charge cloud the error σ_{point} may change. In the case where only one pad is active per point:

$$\sigma_{point} = \frac{d}{\sqrt{12}} \quad (4.14)$$

Assuming a uniform distribution. However, if the electrons contributing to the signal arrive on several pads the position can be averaged from the N_{pad} active pads, reducing σ_{point} of a factor $\sqrt{N_{pad}}$. The situation is illustrated in Fig. 4.15

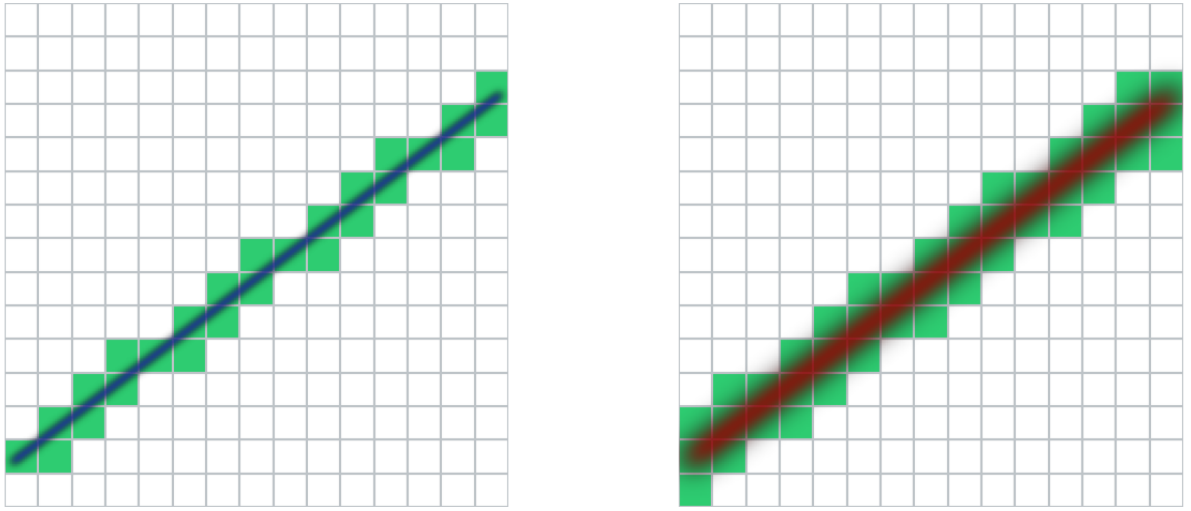


Figure 4.15: Illustration of the charge cloud measured by pads of arbitrary size of a TPC for (left) small and (right) large cloud widths.

Combining all errors the final result is:

$$\sigma_{geom} = \frac{\sqrt{\sigma_{point}^2 + \sigma_{D_T}^2}}{L\sqrt{N_{points}}} \quad (4.15)$$

- Drift time and time sampling. The drift time depends on the gas in the TPC and the electric field, while the time sampling depends on the electronics of the ADC. While σ_{pad} represents the error in the plane parallel tot the readout pads, σ_z is the error on the third axis. The charge cloud after a drift distance z has the following width [55]:

$$\sigma_{D_L} = \sqrt{\frac{2D_L}{v_D}} \sqrt{z} \quad (4.16)$$

Where D_L is the longitudinal diffusion constant and v_D the drift velocity. If the drift time is measured as the average time of arrival of the N electrons of a charge cloud the position resolution is defined in the following way:

$$\sigma_z = \frac{\sigma_{D_L}}{\sqrt{N}} \quad (4.17)$$

- The protons and the α particles are scattered in the Coulomb fields of the target nuclei, described by the Rutherford cross section [55]:

$$\frac{d\sigma}{d\Omega} = z^2 Z^2 \alpha^2 \hbar^2 \frac{1}{\beta^2 p^2} \frac{1}{4 \sin^4(\frac{\theta}{2})} \quad (4.18)$$

Where θ is the scattering angle, z , β and p charge, velocity and momentum of the incoming particle, and Z the charge of target nucleus. In Fig. 4.16 the scheme of the scattering is shown. If

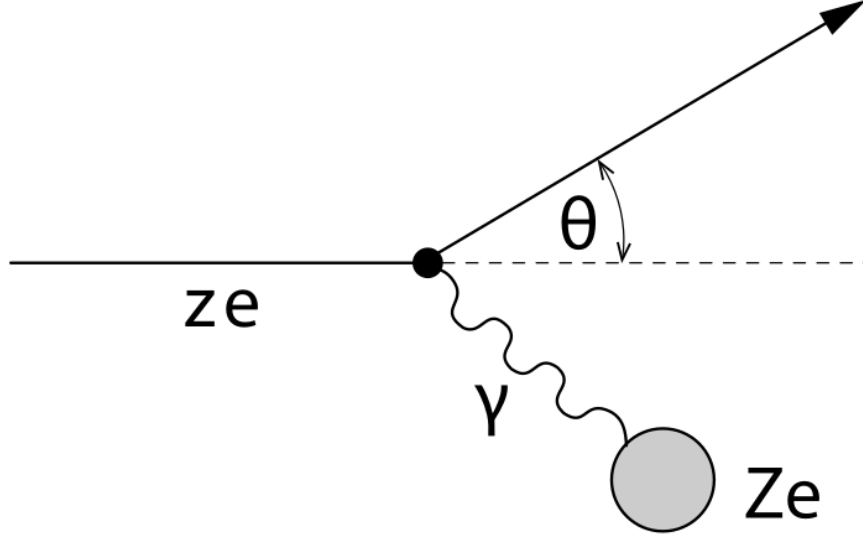


Figure 4.16: Rutherford scattering scheme [55].

the number of scattering events exceeds 20 in the medium it is classified as multiple scattering [55]. While an exact determination of the final direction can not be predicted, the standard deviation θ_0 can be obtained from the so called “Highland formula” [56], Eq. 4.19.

$$\theta_0 = \frac{13.6 \text{ MeV}/c}{p\beta} z \sqrt{\frac{x}{X_0}} \left(1 + 0.038 \ln \frac{x}{X_0} \right) \quad (4.19)$$

Where x is the thickness of the medium and X_0 is the radiation length of the material. In Fig. 4.17 is shown the illustration of the process. The radiation length for ^3He is 67 g/cm^2 , while for graphite is 42.7 g/cm^2 . The uncertainty due to multiple scattering is the combined effect of the scattering on the gas and on the graphite scatterer. The problem for the scatterer is the fact that inside it the measurement of the position is not possible so, unlike scattering in the gas where the direction can be potentially corrected, the direction of the scattered particle has an intrinsic error that cannot be corrected or minimized, given that the statistics has a larger priority for the choice of beam energy and graphite thickness.

Notably, even if the protons scatter multiple times the spin state is conserved, since Coulomb scattering alone does not produce an observable dependent on the initial spin state.

Considered all the potential source of errors, the final uncertainty on the azimuthal angle Φ for one event is:

$$\sigma_\Phi = \frac{1}{\sin \theta_{\text{scattering}}} \sqrt{\sigma_{geom}^2 + \sigma_z^2 + \theta_0^2} \quad (4.20)$$

In the assumption that the error σ_Φ belongs to a gaussian distribution, its effect on the cross section for a single scattering is obtained from a convolution:

$$\sigma_{exp}(\theta, \Phi) = \int_{-\pi}^{\pi} \sigma_0(\theta) [1 + A_y(\theta) \cos(\Phi + \delta\Phi)] \frac{1}{\sigma_\Phi \sqrt{2\pi}} e^{-\frac{\delta\Phi^2}{2\sigma_\Phi^2}} d(\delta\Phi) \quad (4.21)$$

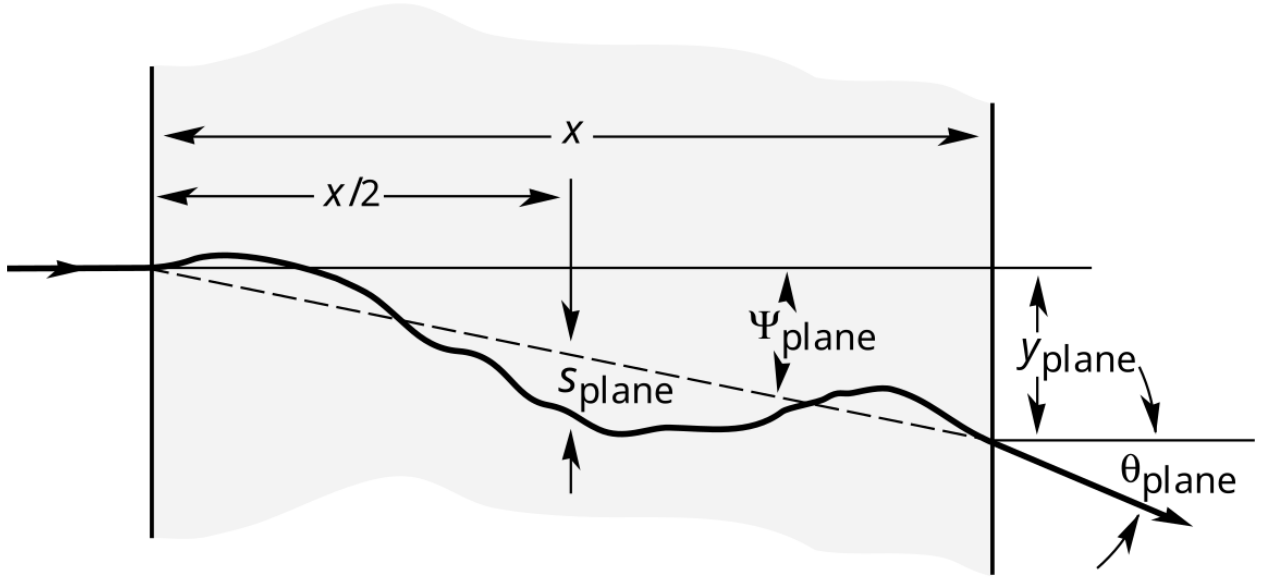


Figure 4.17: Illustration of multiple scattering [26].

Writing $\cos(\Phi + \delta\phi) = \cos \Phi \cos \delta\Phi - \sin \Phi \sin \delta\Phi$ it is possible to see that the part with the sine is equal to zero since it is an odd function integrated over a symmetrical interval. It is possible to widen the domain of integration to infinity to simplify the calculation of the gaussian. The final result, integrating the cosine, is:

$$\sigma_{exp}(\theta, \Phi) = \sigma_0(\theta) \left[1 + A_y(\theta) e^{\frac{-\sigma_\Phi^2}{2}} \cos(\Phi) \right] \quad (4.22)$$

The effect of the uncertainties that affect the determination of the azimuthal angle is a modulation on the analyzing power. The higher the uncertainties, the smaller the visibility of the signal.

Chapter 5

Gamma correlations

For the measurement of the correlation in polarization of two gamma photons two experimental sets of data were analyzed. One from a ^{214}Po , for the case of a cascade decay, and one from ^{22}Na , for the case of photons generated in an annihilation. Both sources were measured with AGATA, although in two different configuration. For the ^{214}Po the clusters have a solid angle coverage of about 1π [41] and are in the configuration described in [41], although on a different beam axis, while for the ^{22}Na only two clusters were used at 3 different positions in opposite directions with respect to the source, shown in Fig. 2.13. The three positions were 309 cm (long), 24.5 cm (medium) and 1 cm (small), measured from the front of the cluster and named P_l , P_m and P_s .

5.1 ^{214}Po

The analysis on the experimental data of ^{214}Po was made on several energy levels and in different ranges of the scattering angle θ , because angles closer to 90° have a larger analyzing power. The level scheme of ^{214}Po is shown in Fig. 5.1. The pair of photons analyzed were the ones were one of the two correspond to the decay of the 2^+ state, at 609.3 keV. The scattering angles were selected in the range $[60^\circ, 90^\circ]$. In Fig. 5.2 some examples of azimuthal correlation distributions obtained from experimental data are shown. To correct for geometrical asymmetries and azimuthal distortions a normalization procedure of the experimental data, collected in histograms, is done. The corrected data are obtained by dividing the histogram of events by a reference histogram of uncorrelated ones. The reference histogram is built by mixing the events in the original histogram, in particular one of the two photons in coincidence is swapped with another measured in a different event, to destroy the correlation between the original pair. Fig. 5.3 shows two example histograms before division.

The error on the points in the corrected histograms is shown in Eq. 5.1.

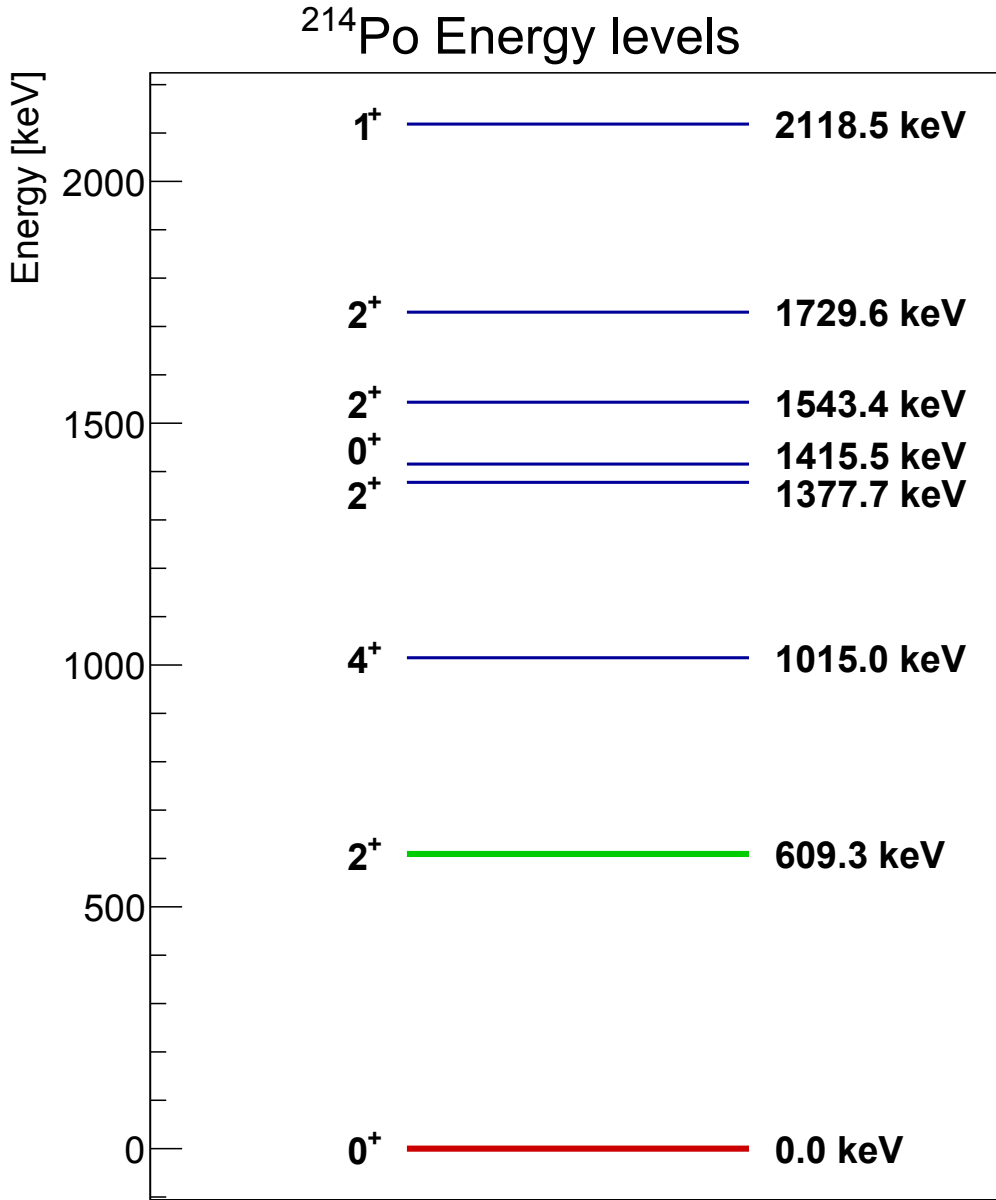
$$\sigma_f = f \sqrt{\left(\frac{\sigma_{A_n}}{A_n}\right)^2 + \left(\frac{\sigma_{B_n}}{B_n}\right)^2} \quad (5.1)$$

Where A_n and B_n are the counts in the n-th bin, σ_{A_n} and σ_{B_n} their respective error, which assuming a poissonian distribution $\sigma_{A_n} = \sqrt{A_n}$ and f is the ratio $\frac{A_n}{B_n}$. From these distributions the parameter μ was extracted trough a fit with Eq 2.39, reported here for clarity:

$$N(\Delta\Phi) = N_0 (1 + \mu \cos 2\Delta\Phi) \quad (5.2)$$

In Tab. 5.1 the results are shown for each photon pair.

The values of μ measured are compared with the values of the averaged values of the analyzing powers, calculated integrating Eq. 2.50 with respect to the scattering angle. In addition to this the factor dependent on the relative angle Θ between the photons is taken into consideration. Unfortunately, some states have a large error that prevents a meaningful comparison of the result with the expected

Figure 5.1: Analyzed energy levels of ^{214}Po .

State	E_γ	μ	$-\langle A_1 A_2 \rangle$	χ^2/n	μ $60^\circ < \theta < 90^\circ$	$-\langle A_1 A_2 \rangle$	χ^2/n
$4^+ \rightarrow 2^+ \rightarrow 0^+$	405	-0.2 ± 0.2	-0.164	16/22	-1 ± 4	-0.425	10.0/22
$2^+ \rightarrow 2^+ \rightarrow 0^+$	768	-0.15 ± 0.02	-0.124	23/22	-0.35 ± 0.08	-0.330	8.0/22
$0^+ \rightarrow 2^+ \rightarrow 0^+$	806	-0.23 ± 0.04	-0.121	19/22	-0.5 ± 0.1	-0.321	16/22
$2^+ \rightarrow 2^+ \rightarrow 0^+$	934	-0.15 ± 0.02	-0.111	40/22	-0.2 ± 0.1	-0.300	16/22
$2^+ \rightarrow 2^+ \rightarrow 0^+$	1120	-0.17 ± 0.01	-0.100	95/22	-0.19 ± 0.04	-0.264	15/22
$1^+ \rightarrow 2^+ \rightarrow 0^+$	1509	-0.19 ± 0.03	-0.082	31/22	-0.4 ± 0.1	-0.214	20/21

Table 5.1: Results of the fit on the azimuthal correlation distribution.

value, like for the 405 keV photon. Only two fits do not pass the χ^2 test, the photons at energy 934 keV and 1120 keV, rejecting the null hypothesis that the data follow the distribution described in Eq. 2.39 with a confidence level of 95%. The gate on the scattering angle reduces the statistics and so the error increases. This is further confirmed by the low values of the χ^2 compared to the number of degrees of freedom.

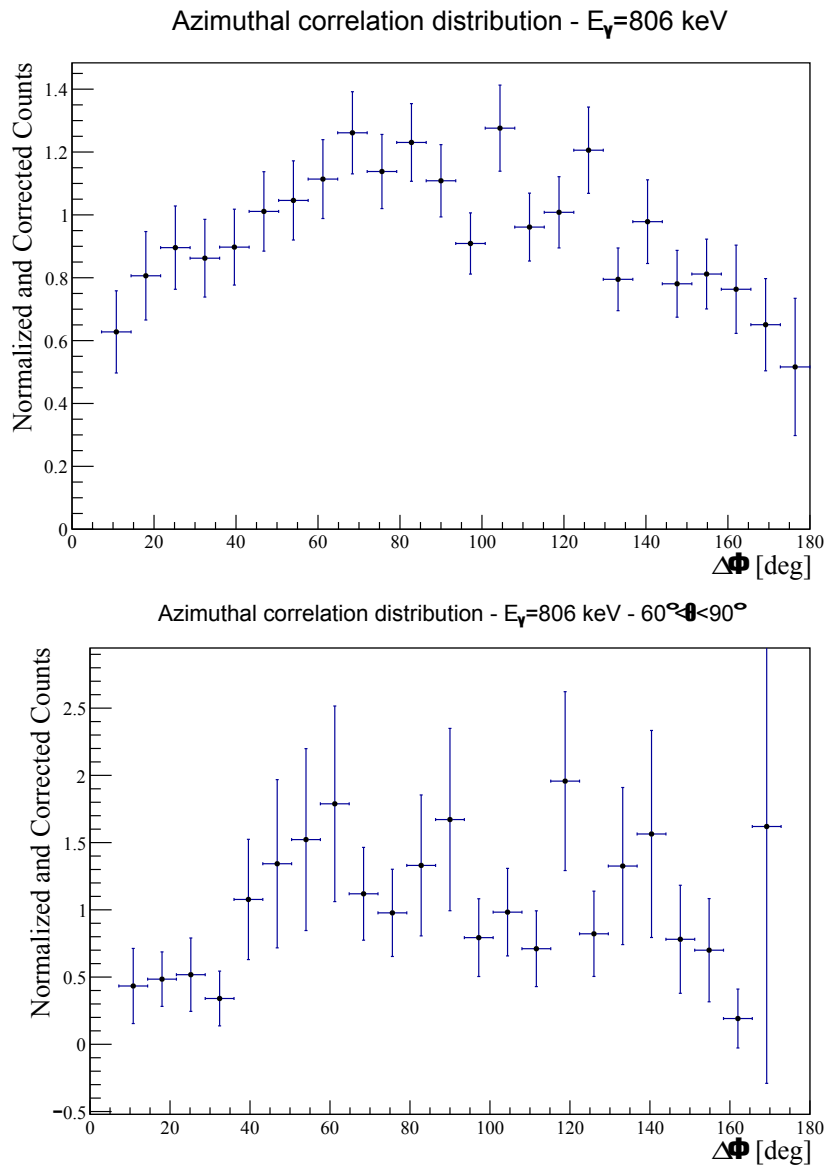


Figure 5.2: Azimuthal correlation distributions for the transition $0^+ \rightarrow 2^+ \rightarrow 0^+$ with energy of the first photon of 806.2 keV.

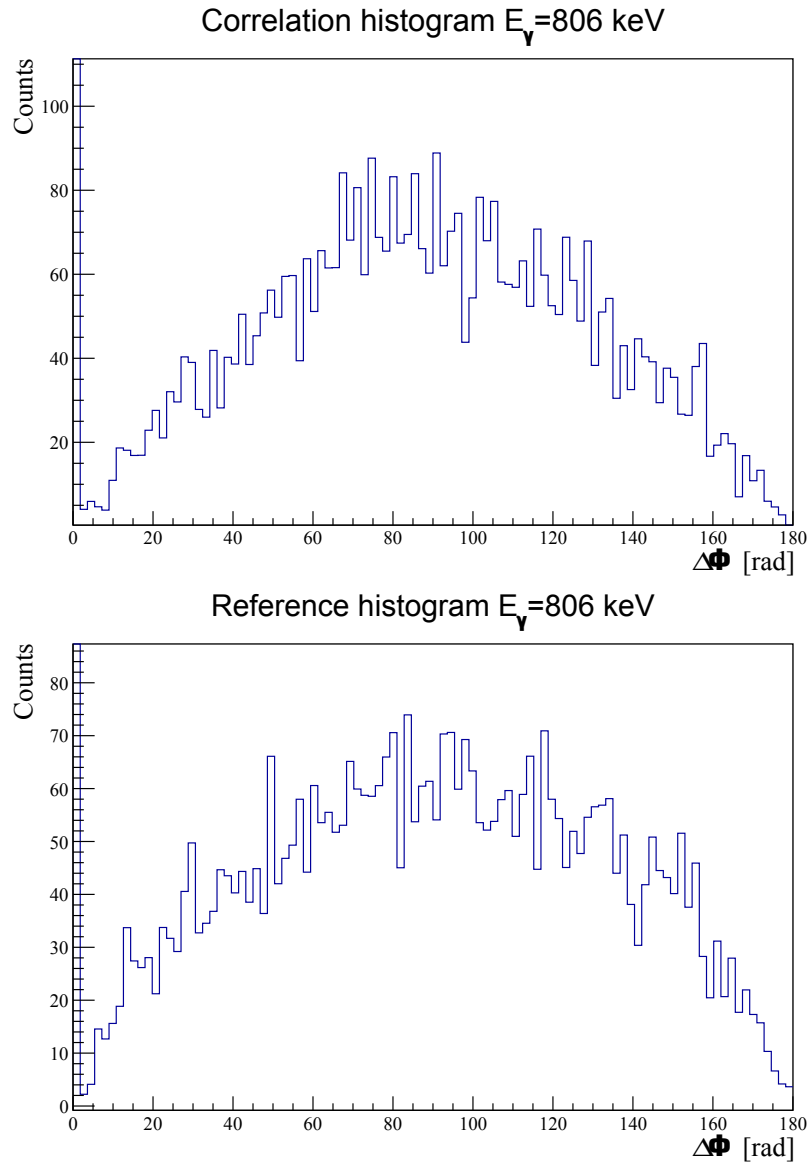


Figure 5.3: Azimuthal correlation distribution non corrected (top) and reference histogram (bottom).

5.2 ^{22}Na

For the case of ^{22}Na , where results are already known [15, 16] and are useful as a comparison to the ^{214}Po data, the study is only on the 511 keV photons. The photons are produced in the annihilation of the positron emitted in the β^+ decay and an electron. The positron and electron, before decaying, can form para or ortho positronium. The state of interest is para-positronium since it decays in two photons and the initial state as zero total spin, so in the ground state the two photons emitted must have opposite polarization due to the conservation of total angular momentum. The analysis proceeds in a similar way to Sec. 5.1 but thanks to the fact that the raw data were available it was possible to study the correlation distribution in function of the product of the two analyzing power. However, the files did not include the exact position and orientation of the clusters, so it was necessary to extract them from the data choosing the position by maximizing the number of events that pass through a virtual sphere of 2 mm assuming that all pair of photons are emitted exactly at 180° to each other. The positions found are in agreement with the ones reported in the logbook, which could not be used because they were not directly related to the position and orientation of the crystals. In Fig. 5.4 is shown two example events that after the optimization are collinear.

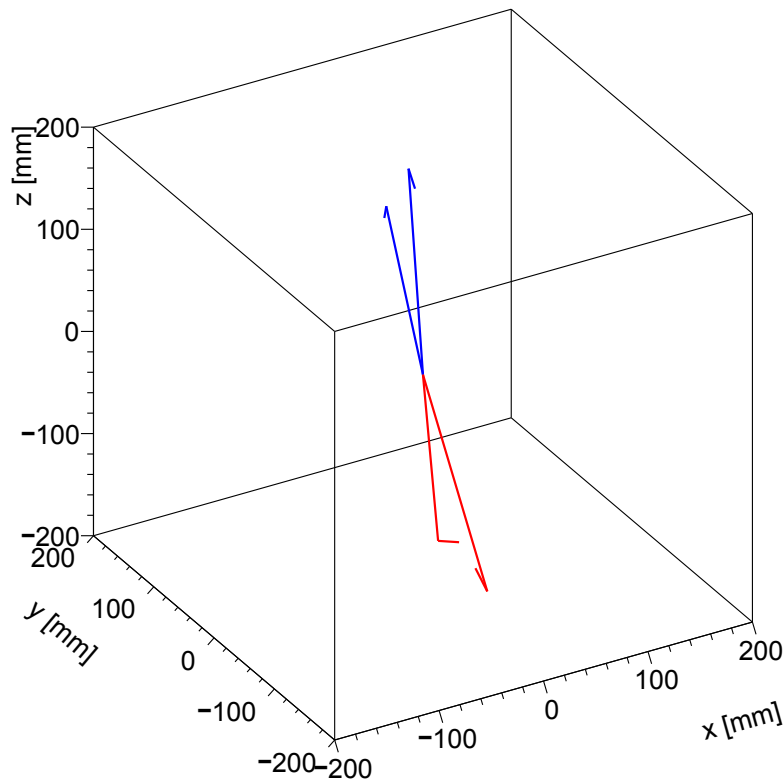
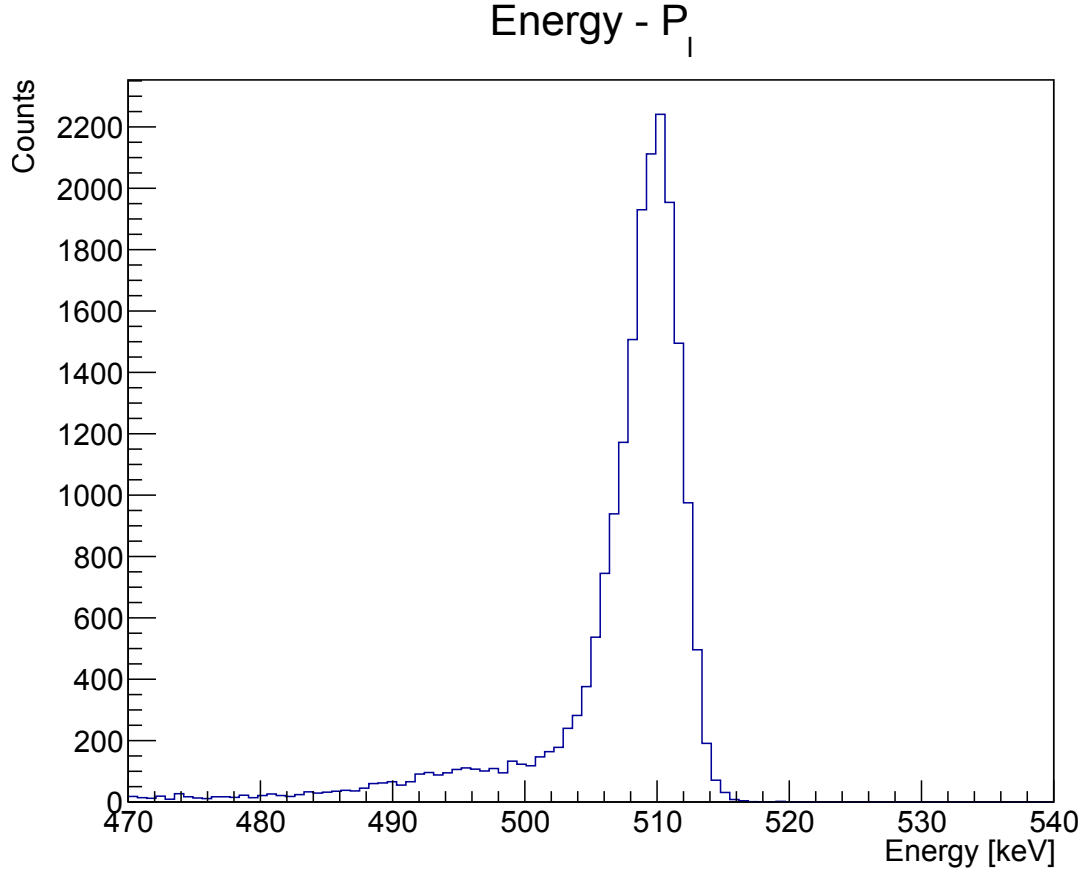


Figure 5.4: Two example events of double Compton scattering to show the collinearity. The different colors represent the hits into the two different clusters.

To obtain the final results several parameters were checked to reduce the background and increase the sensibility. Firstly only events where two hits were registered for cluster were selected, and also with total energy in each cluster in the range [504 keV, 518 keV]. The energy spectrum is shown in Fig. 5.5.


 Figure 5.5: Energy spectrum of the P_l position.

Because of some errors in the tracking algorithm due to wrong position assigned to the two clusters during the experiment a new simpler one was implemented. Given the two positions of the hits the scattering angle is computed in the two different possibilities exchanging the order of the two hits. The energy of the scattered photon is then computed from this angle and compared the energy of the candidate last hit. The right order is chosen based on how close the two values of energy are. Eq. 5.3 are the equations used for these calculations.

$$E'_\gamma = \frac{E_\gamma}{1 + \frac{E_\gamma}{m_e c^2} (1 - \cos \theta)} \quad \cos \theta = 1 + m_e c^2 \left(\frac{E_\gamma - E'_\gamma}{E_\gamma E'_\gamma} \right) \quad (5.3)$$

In Fig. 5.6 is shown the difference of the cosine of the scattering angle measured with the position and with the energy. Only events with $|\Delta\theta| < 0.1$ were selected. To increase the precision on the measurement of the azimuthal angle the events with a short distance between first and second hit, less than 12 mm, were discarded. While the statistics is reduced, the uncertainty on the measurement of the azimuthal angle decreases. In Fig 5.7 the distance between first and second hit is shown. The same procedure of normalization applied in Sec. 5.1 is done for the data with a small difference: a constraint is put on the event mixing in order to have the swapped photon in the same crystal as the original one. This is necessary because of the nature of the annihilation, the two photons are always emitted back to back and given the configuration of the clusters if one photon hits a particular crystal in one cluster then the second photon will always hit the same crystal on the other cluster, assuming there are no interactions beforehand. Hence, mixing randomly photons will produce a wrong reference histogram since there will be included effects that in the original histogram do not happen, outside of random coincidences or other systematics.

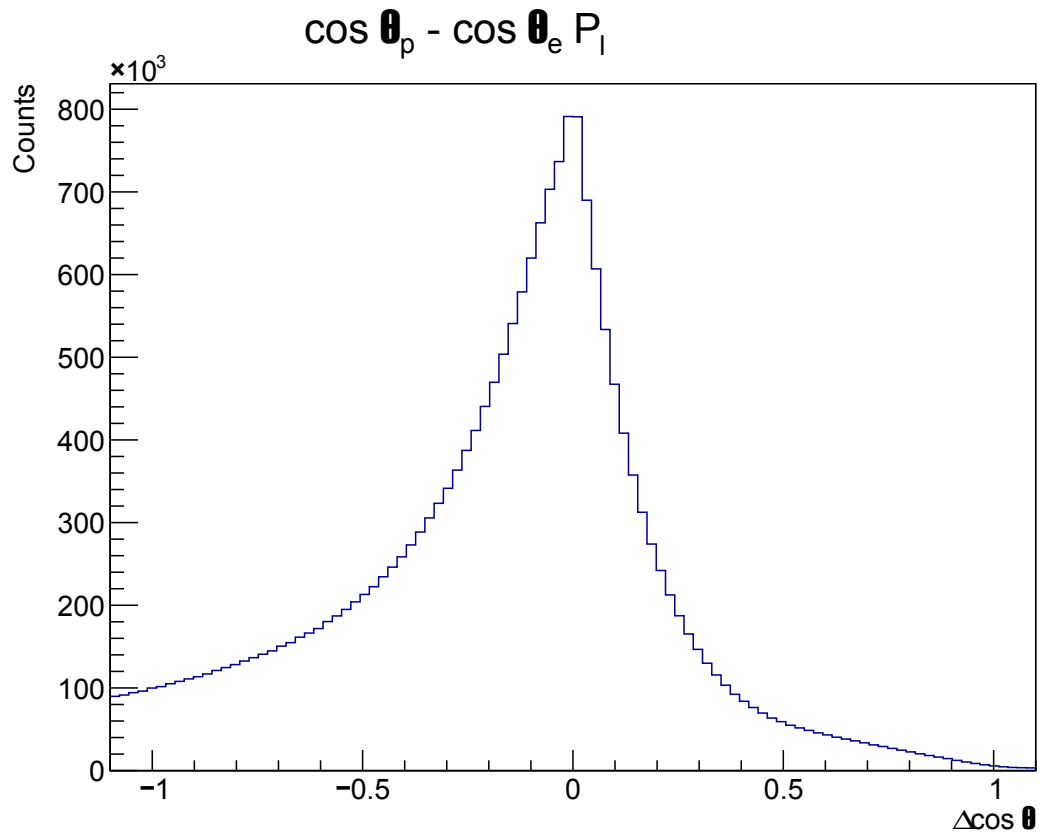


Figure 5.6: Difference of the cosine scattering angles measured with the positions and the energies of the P_l position.

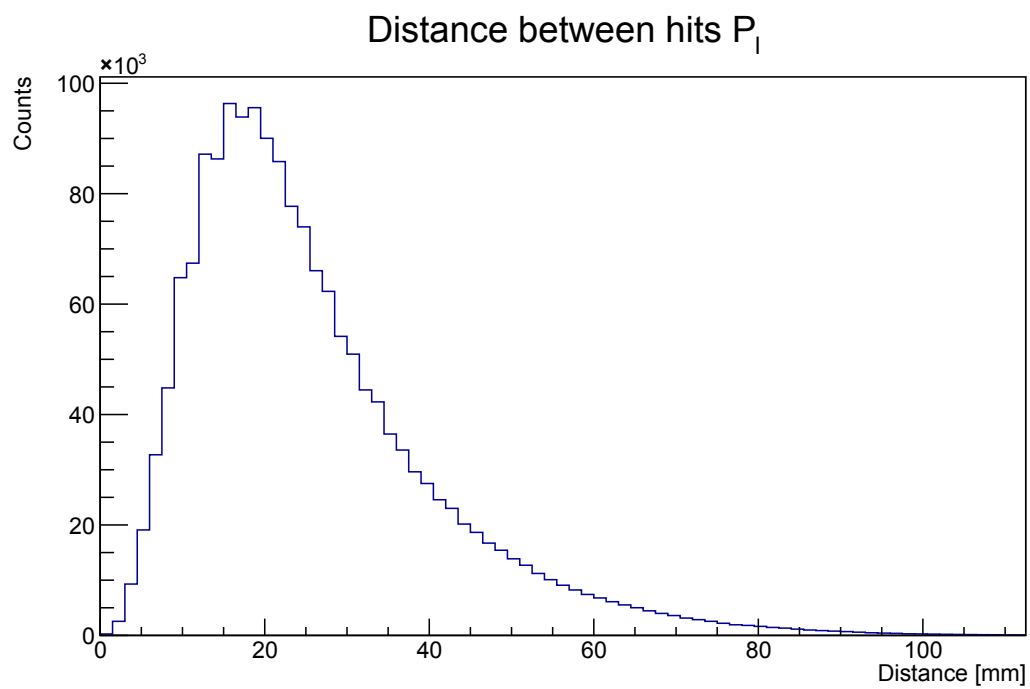


Figure 5.7: Distance between first and second hit of the P_l position.

The final result of $\Delta\Phi$ in function of the product of the analyzing power is shown in Fig 5.8.

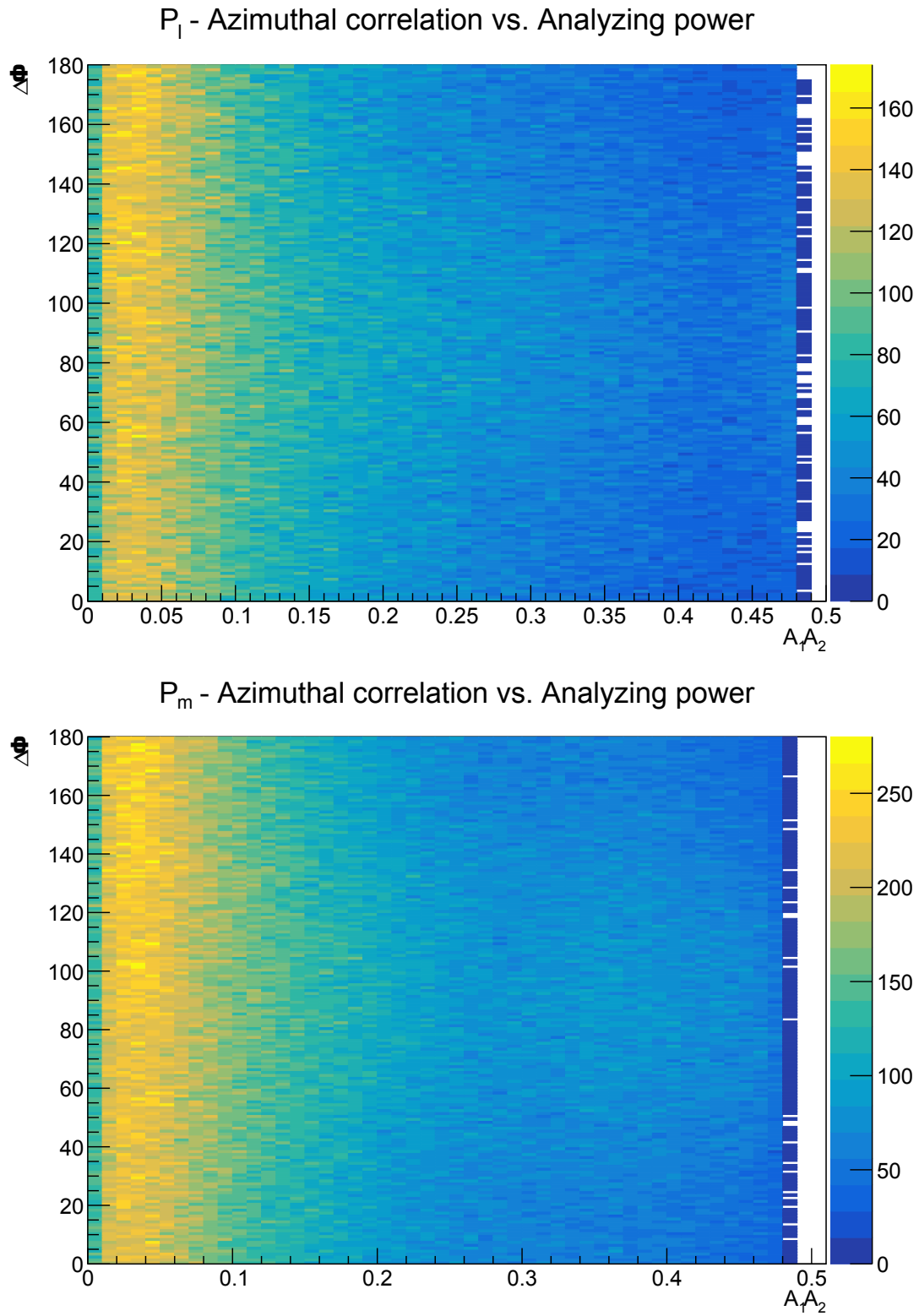


Figure 5.8: Azimuthal correlation distribution in function of the product of the analyzing power for the three positions.

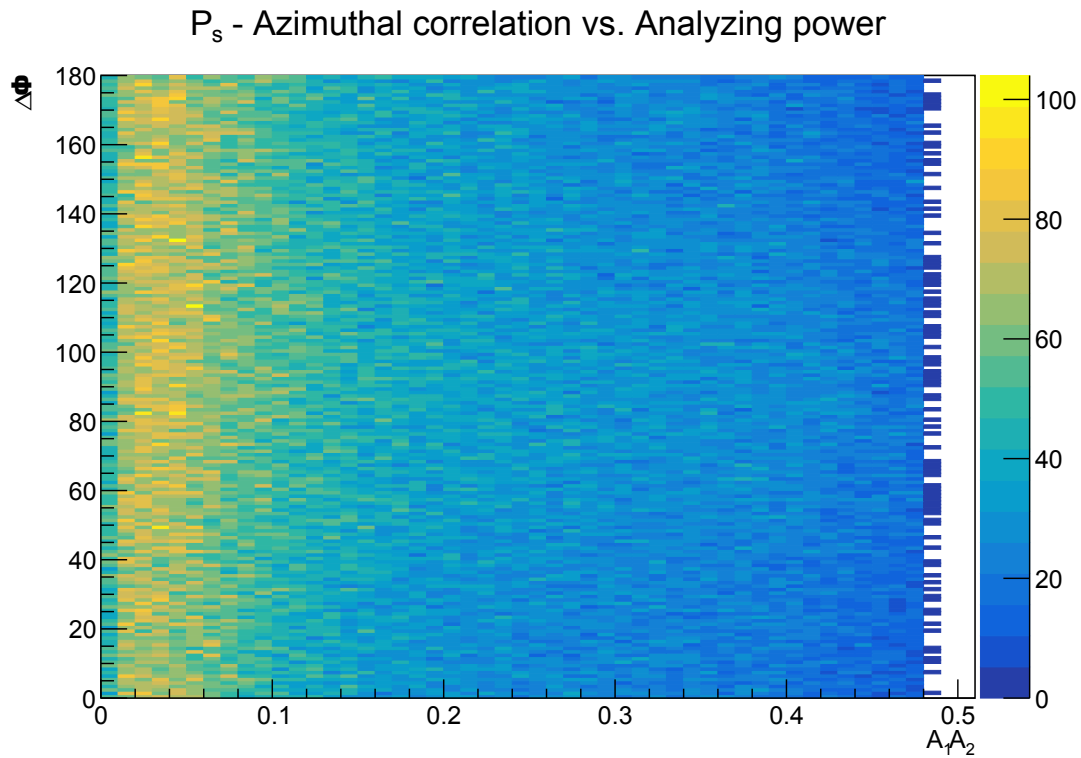


Figure 5.8: Azimuthal correlation distribution in function of the product of the analyzing power for the three positions.

From these results it is possible to take an average of the products of analyzing power in order to compare the result of μ with the values obtained. The azimuthal correlation distributions are obtained projecting on the desired range of product of the analyzing power, shown in Fig 5.9. From the cross section in Sec. 2.4 the measured value of μ should be equal to the average of the analyzing powers, shown in Tab 5.2.

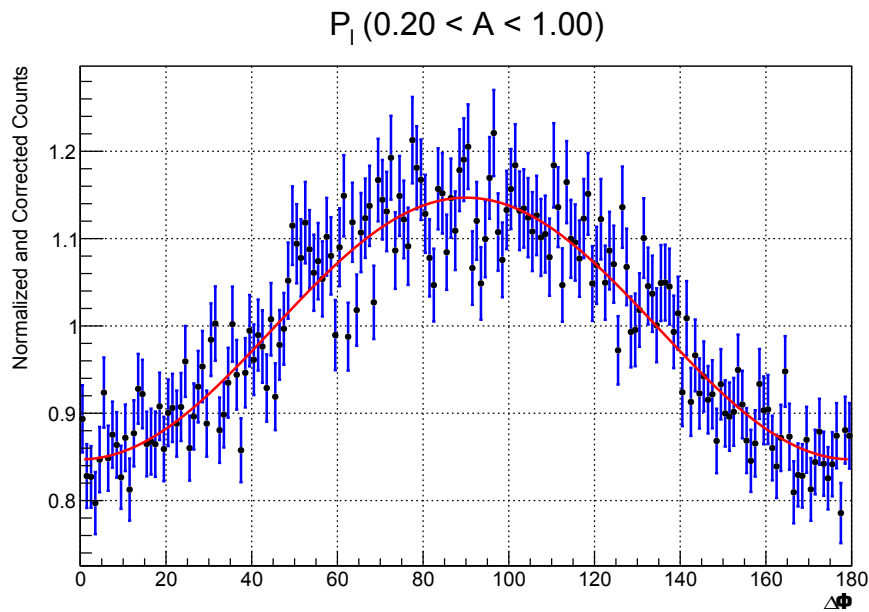


Figure 5.9: Azimuthal correlation distribution for the three positions in different ranges of the analyzing power.

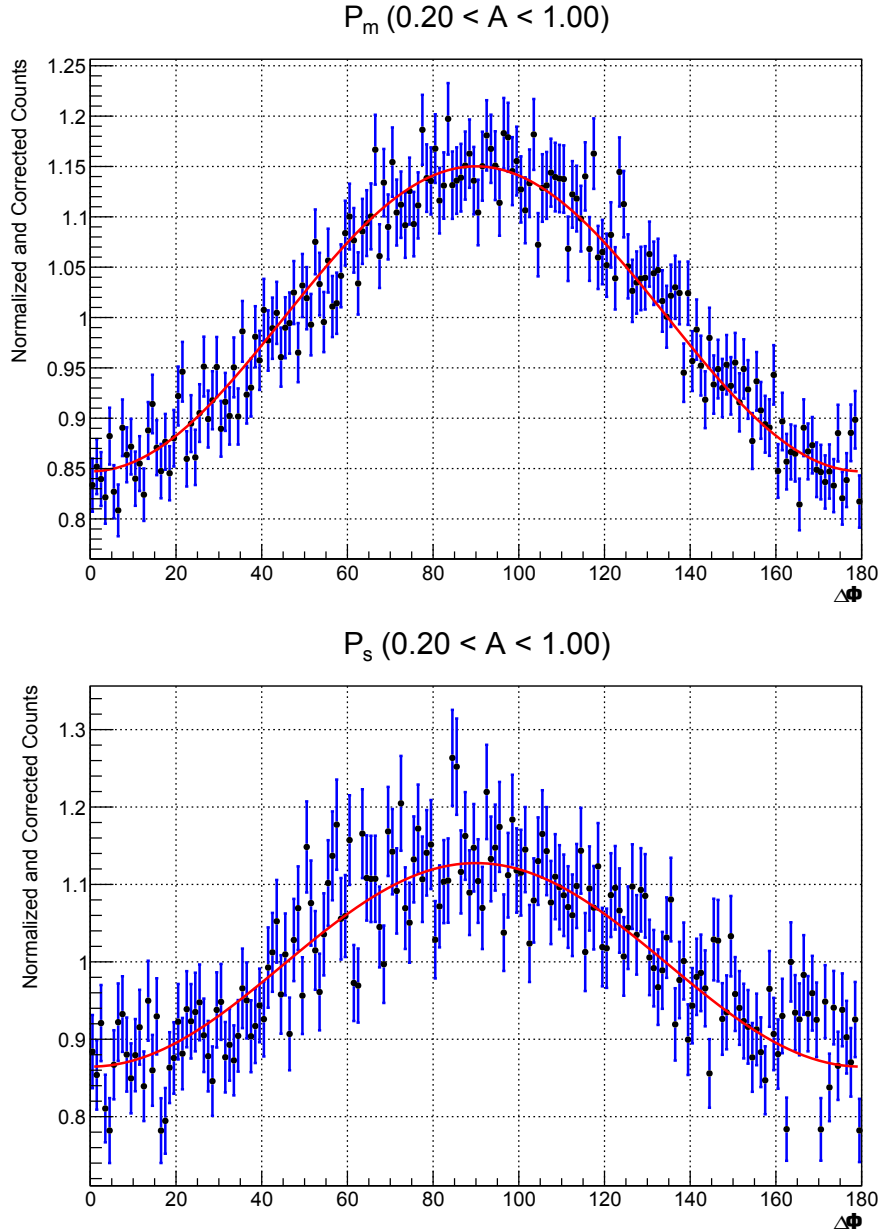


Figure 5.9: Azimuthal correlation distribution for the three positions in different ranges of the analyzing power.

	μ	$-\langle A_1 A_2 \rangle$
P_l	-0.150 ± 0.003	0.3129 ± 0.0001
P_m	-0.141 ± 0.003	0.3023 ± 0.0001
P_s	-0.181 ± 0.007	0.3827 ± 0.0002

Table 5.2: Result of the analysis compared with the averaged product of the analyzing powers.

From the results it is possible to see that the measure values of μ are compatible with half the average of the product of the analyzing power. From this consideration the polarization state of the photons ^{22}Na is not entangled, given the relation from the cross section in Sec. 2.4. However, some considerations must be made to understand if value of μ decreased because of other factors. First of all, the uncertainty on the azimuthal angle can decrease the visibility, especially at lower scattering angles of the Compton scattering. The magnitude of the error depends also on the distance between first and second hit, described in Eq. 5.4

$$\sigma_\Phi = \frac{\sqrt{2}\sigma_r}{d \sin \theta} \quad (5.4)$$

Where σ_r is the spatial resolution of AGATA, quoted with the FWHM is about 5 mm [57]. The attenuation factor and the weighted analyzing power are:

$$D(\theta, \Phi) = e^{-4 \frac{\sigma_r^2}{d^2 \sin^2 \theta}} \quad A_1 A_2 \rightarrow D(\theta_1, \Phi_1) D(\theta_2, \Phi_2) A_1 A_2 \quad (5.5)$$

Where the first one is obtained in a similar way to Eq. 4.21, assuming that the error belongs to a gaussian distribution and computing the convolution with the cross section. Including this correction, the analysis is repeated and the result are shown in Fig. 5.3. Unfortunately the measured values are

	μ	$-\langle A_1 A_2 \rangle$
P_l	-0.207 ± 0.007	-0.2660 ± 0.0002
P_m	-0.204 ± 0.005	-0.2657 ± 0.0001
P_s	-0.175 ± 0.008	-0.2632 ± 0.0002

Table 5.3: Result of the analysis compared with the averaged product of the analyzing powers with the correction for σ_Φ .

still not compatible with the expected result, however they are significantly larger than the half of the product of the analyzing power. In fact, a one tailed Student's t-test to check if the values measured $|\mu|$ were significantly greater than the non entangled state (half of $\langle A_1 A_2 \rangle$), gave for all the positions a value μ is larger than $\langle A_1 A_2 \rangle/2$ with a confidence level of 99.9%.

In this way it is verified that the distribution is not compatible with the non entangled one. This is probably because of the analysis, the process of optimization on the positions of the clusters likely added an ulterior source of error in σ_r . However increasing the error because of this is not reasonable so the FWHM of AGATA was used. Ultimately, a new set of measurement where the position of the clusters is already accounted for in the data is necessary to confirm the results.

The overall error σ_Φ was also checked for the presence of eventual systematic contributions. σ_r is fixed by the experimental setup, the type of detectors and the electronics, and the factor $\sin \theta$ can be acted upon by selecting a range of analyzing powers. The only parameter left is the first-second hit distance d . In Fig. 5.10 it is shown the value of the parameter μ compared with $-\langle A_1 A_2 \rangle$ in function of the cut threshold, removing events with d less than the threshold.

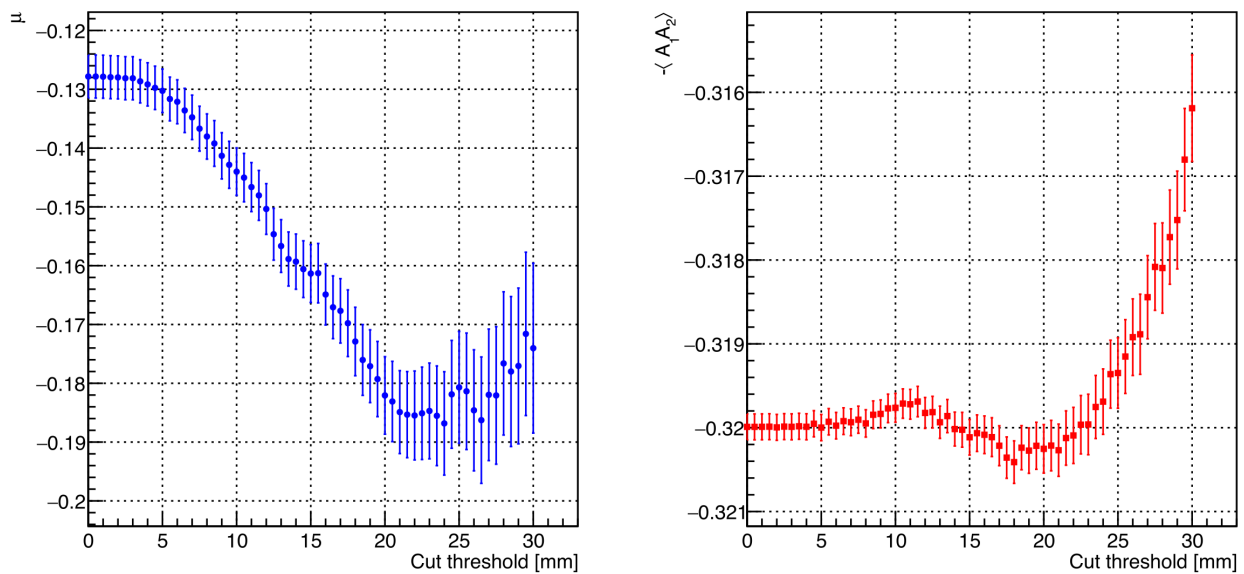


Figure 5.10: Relationship of the value of μ extracted from the data and $-\langle A_1 A_2 \rangle$ with respect to the cut threshold for non corrected (top) and corrected (bottom) data of the P_l position.

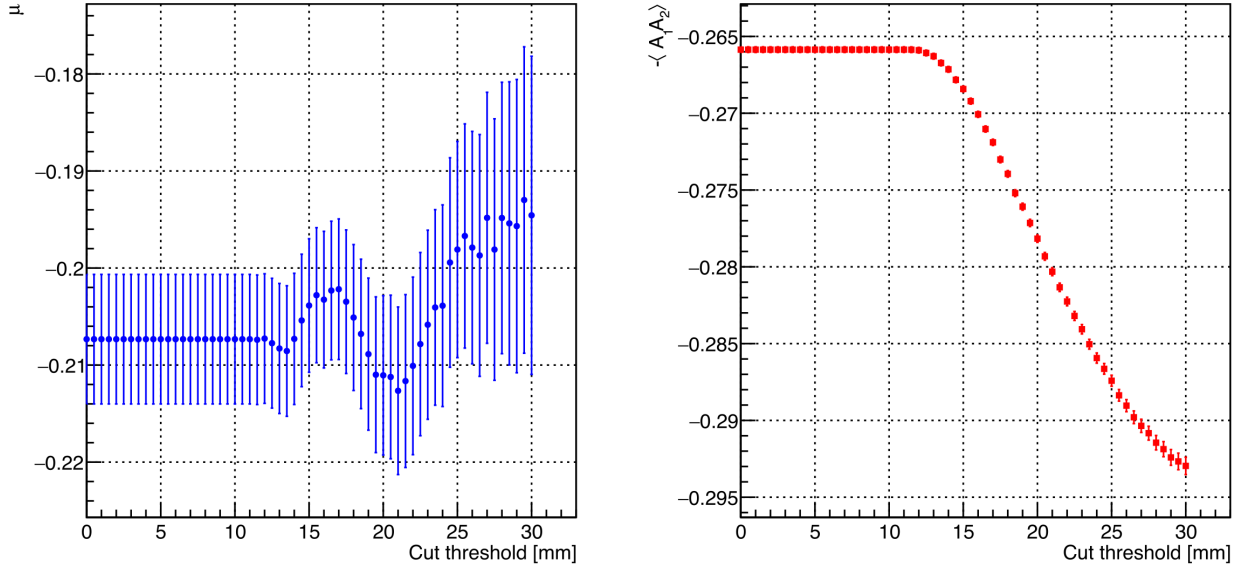


Figure 5.10: Relationship of the value of μ extracted from the data and $-\langle A_1 A_2 \rangle$ with respect to the cut threshold for non corrected (top) and corrected (bottom) data of the P_l position.

For the non corrected data the value of μ decreases by increasing the threshold while $-\langle A_1 A_2 \rangle$ increases. Basically with a higher threshold the determination of $\Delta\Phi$ is better so the value of μ decreases (or increases considering $|\mu|$), while $-\langle A_1 A_2 \rangle$ increases, or the analyzing powers get worse. Considering the diameter of one crystal which is 80 mm, if one photon does not hit the center of the crystal then at a scattering angle near 90° the photon can escape depending on the direction of the scattered photon. This favors detection for small scattering angle that leads to the distribution. In addition a large cut may impact the detection of low energy photons. Considering the linear attenuation coefficient μ of germanium, statistically the intensity of photons traveling through germanium is:

$$I(z) = I_0 e^{-\mu z} \quad (5.6)$$

Where z is the distance of the photon that travels through the medium. The mass attenuation coefficient, defined as $\mu_m = \frac{\mu}{\rho}$ with ρ the density of the material, has an energy dependence which is shown in Fig. 5.11 for germanium.

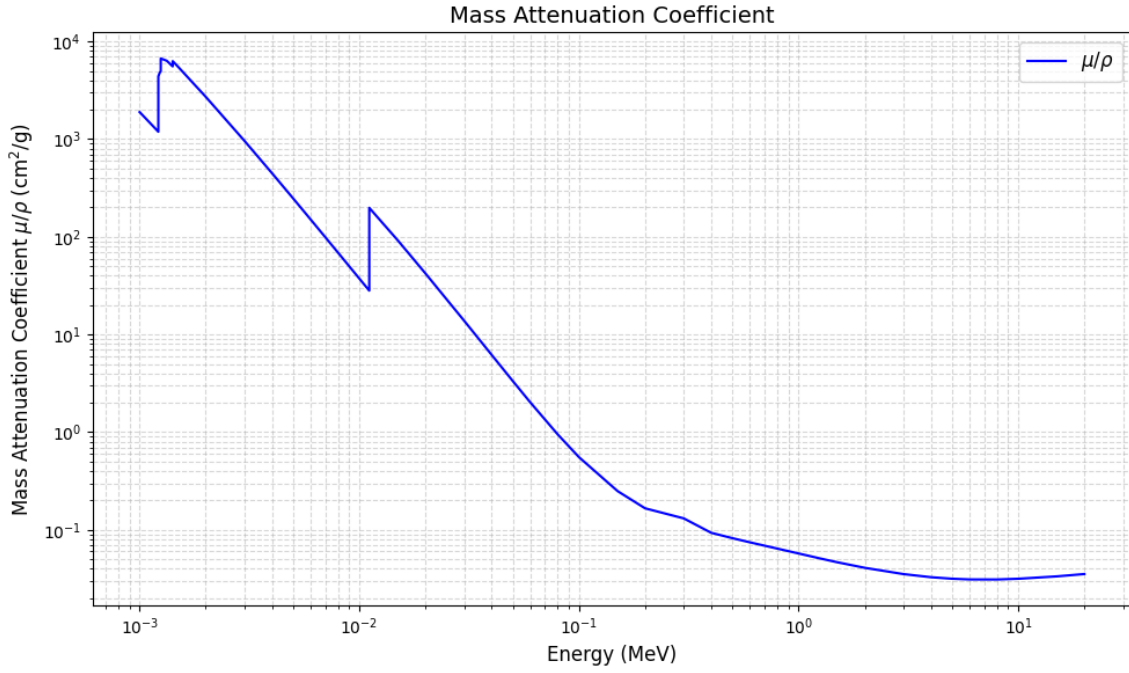


Figure 5.11: Linear attenuation coefficient in function of photon energy for germanium. Data from NIST [58]

Looking at the relationship between attenuation coefficient and energy is clear that if the initial 511 keV photon after scattering the attenuation coefficient is larger, which means that statistically the fraction measured after the cut is reduced. To prove this it is possible to compute the fraction of photons that do not get absorbed with:

$$\frac{I(z)}{I_0} = e^{-\mu z} \quad 100\% \quad (5.7)$$

In Tab. 5.4 the values of this fraction are reported for three scattering angles and for different positions of the cuts.

θ [deg]	E' [keV]	$I(z)/I_0$		
		$z = 10 \text{ mm}$	$z = 20 \text{ mm}$	$z = 30 \text{ mm}$
30°	450.6	(58.5%)	(34.2%)	(20.0%)
90°	255.5	(42.8%)	(18.3%)	(7.8%)
150°	178.3	(28.6%)	(8.2%)	(2.4%)

Table 5.4: Fraction of photon remaining after traveling a distance z , representing the cut threshold for three positions and three scattering angles.

It is possible to see that at large angles and high cuts the photons at large scattering angle are statistically suppressed, with respect to the low scattering angles. It is however to keep in mind that the Compton scattering angle is not isotropic but for high initial energy of the photon it favors low scattering angles. The behavior is shown in Fig. 5.12

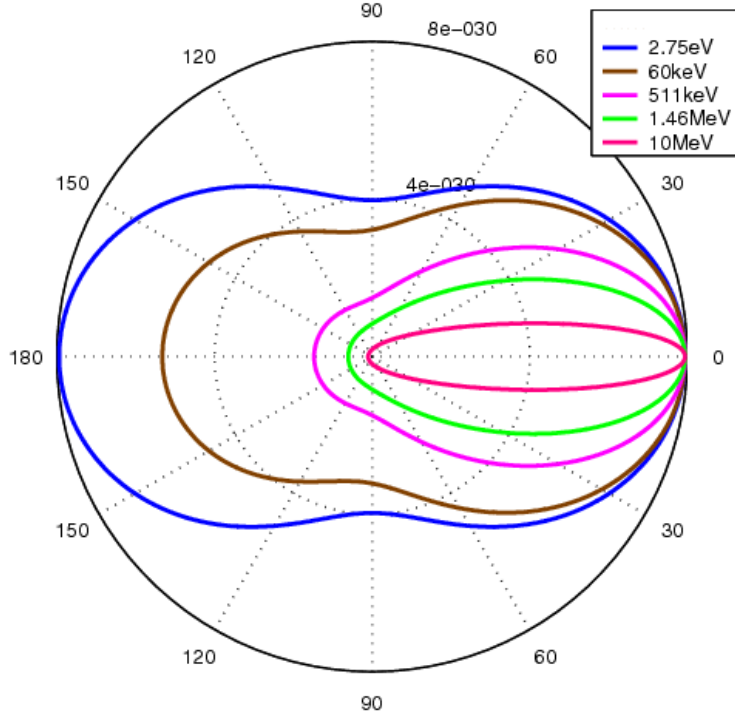


Figure 5.12: Distribution of scattering angle for different photon energies, including 511 keV in magenta [59].

Given the distribution, the impact should be negligible for typical cuts of the order of 10 mm and noticeable only for extreme cuts, however in this range dominates the geometrical effect discussed previously.

For the weighted data the values of μ do not change as much as the non corrected case and given the (large) error could even be considered independent on the cut. This is a consequence of including the effect of the azimuthal error σ_Φ . The value of $-\langle A_1 A_2 \rangle$ decreases because of the fact that the correction due to σ_Φ starts to become negligible because the error becomes smaller the larger d is.

In conclusion, from this analysis there is no evidence of ulterior systematic errors that contribute to the experimental value of μ

Chapter 6

Conclusions

In conclusion, an experiment dedicated to measure spin correlations between two protons from a radioactive nucleus was simulated. The simulations indicate that the best setup is a TPC with a graphite scatterer inside, increasing substantially the number of events going through spin orbit scattering. The GEANT4 simulations indicate the optimal range of energy of the experiment, above 160 MeV, and gives the expected number of events. From the result it was possible to identify beam specifications: high energy and high intensity beam, more than 10^6 pps. A beam with this properties is typically found in fragmentation facilities.

The TPC allows the three dimensional measurement of the vertex of the daughter particles from the ${}^6\text{Be}$ decay in the active volume of the chamber, with the possibility of reconstructing the scattering planes for each pair of scattered protons.

The limitations of this work include the fact that a generic and ideal TPC was used in the simulation and no contamination from other states and other particles produced in the reaction were considered. A more realistic simulation is needed once the facility and the TPC are identified. The analysis on the spin states reveals how the correlations of the singlet state degrades in presence of triplet states, that are not possible to experimentally separate.

The result of such an experiment would permit to understand how the strong force interacts with correlated spin states in the $2p$ decay. In addition, it could improve theoretical three-body models.

For the case of entanglement of polarized gamma photons, the data of two calibration sources were analyzed. The experiments consist in the measurement of the polarization of the pair of photons through Compton scattering events within the AGATA detector. The first experiment used a source of ${}^{214}\text{Po}$. The correlation of photons in a decay cascade, showed that the measured values are in line with the expectations, although for some states the statistical error is too high to be conclusive. For ${}^{22}\text{Na}$ the results are not firmly established, although the factor μ related to the Bell inequalities can be distinguished from the non entangled hypothesis. However, given that the position of the detectors during the experiment was not well determined, it is not possible to establish whether the process of optimization of the clusters position increased significantly the errors of the measurement or if there are other sources of indetermination.

New measurements to collect more statistics with sources similar to ${}^{214}\text{Po}$ are planned. In order to achieve a statistics similar or larger than ${}^{22}\text{Na}$, the measurement would need a minimum of 10 days. These kind of measurements would pave the way to new technologies employing penetrating radiation for quantum communication and quantum imaging. In particular in recent years the promising idea of employing entanglement in PET scans to reduce background noise [60, 61] emerged.

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