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Dipartimento di Fisica e Astronomia "G. Galilei"

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Flavour physics and supersymmetry after the first run of the LHC

Relatore
Prof. Fabio Zwirner

Correlatore
Dott. Paride Paradisi

Laureando
Gabriele Spada

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Abstract

In this thesis we study some selected phenomenological aspects related to flavour physics in supersymmetric scenarios, revisited in the light of the LHC results after the 8 TeV run. Specifically, the lack of evidence for new coloured particles up to the TeV scale and the discovery of a scalar boson with a mass of about 125 GeV and similar properties (up to the current experimental uncertainties) to those of the Higgs boson of the Standard Model provide non-trivial information on the mass spectrum of supersymmetric models. In this context, we explore virtual effects of new particles on low energy observables, which could provide crucial information about the presence and properties of new particles that so far escaped the direct search at the LHC. We find that flavour observables and the Higgs boson mass are highly complementary in probing supersymmetric scenarios.

Contents

Introduction

The Standard Model (SM) of strong and electroweak interactions recently marked the last of a long series of successes with the discovery of the Higgs boson at LHC. Still, valid reasons coming from both the experimental and the theoretical side push us to consider the SM as an effective low energy version, although extremely accurate, of a more fundamental underlying theory. On the experimental side, dark matter and the observation of neutrino oscillations are the most striking evidences for New Physics (NP) beyond the SM. On the theoretical side, the SM does not contain a quantum description of the fourth force in nature – gravity – which requires that, at the latest for energies of the order of the Planck scale $M_P \sim 10^{19}$ GeV, the SM has to be replaced by a new theory. Furthermore, the Higgs mass parameter in the SM is not protected by any symmetry from receiving quadratically divergent quantum corrections and should be naturally of the order of the scale at which NP enters. This is known as the *hierarchy problem* and it strongly suggests that new dynamics should be present at the TeV scale. Among the many possible extensions of the SM, the most popular is still the Minimal Supersymmetric Standard Model (MSSM). Theories with TeV scale Supersymmetry (SUSY) are in fact able to address the gauge hierarchy problem and provide also a dark matter candidate. However, the MSSM introduces many new sources of flavour violation and one would expect too large contributions to flavour transitions unless some protection mechanism is at work. In this perspective, low-energy flavour physics provide a complementary tool to the direct searches for new particles, which can be used to investigate the symmetry properties of the new degrees of freedom. In this work we analyse the implications of the LHC measurements (specifically the presence of a 125 GeV SM-like Higgs boson and the absence of coloured supersymmetric particles up to the TeV scale) on flavour observables.

The material is organized as follows. In the first chapter, we review the SM focusing on its shortcomings and analysing the motivations for new physics. In Chapter 2 we briefly introduce SUSY as a potential solution to some of the SM problems, in particular to the hierarchy problem. This material is well-known by the specialists, but not treated in the last-year courses. We then analyse in detail the building blocks of the MSSM focusing on its particle content and highlighting the peculiar properties of its lightest Higgs boson. In particular the predicted mass is bounded from above and large quantum correction are needed in order to reproduce the measured Higgs boson mass. This fact greatly restricts the parameter space of the theory, leading to two possible scenarios, either with a high SUSY scale or with large mixing between the top squarks.

Chapter 3 is devoted to flavour physics. The great accuracy of the Cabibbo-Kobayashi-Maskawa (CKM) description of flavour violating processes leaves little room for NP effects, telling us that the flavour structure of the extensions of the SM should be highly non-generic if the scale of NP is taken at the TeV scale. This is the so-called *flavour problem.* Within the MSSM the squark and slepton sectors introduce new sources of flavour violation through the off diagonal entries of their mass matrices. These new sources of flavour violation are then transmitted to the visible sector at the loop-level through fermion-sfermion-gaugino interactions. We study these new flavour violating interactions in detail, both in the mass eigenstate basis and in the super-CKM basis through the mass insertion approximation.

In Chapter 4 we discuss some selected low energy observables. The underlying processes are described most efficiently by an effective low energy theory, where the effects of heavy particles are encoded in the Wilson coefficients of higher dimensional local operators. We explicitly calculate the most relevant MSSM contributions to the Wilson coefficients for the selected $\Delta F = 2$, $\Delta F = 1$ and $\Delta F = 0$ transitions. In particular, we focus on meson anti-meson mixing as well as the processes $b \to s\gamma$, $\mu \to e\gamma$ and the electron Electric Dipole Moment (EDM). We perform from scratch the analytical evaluation of the relevant loop diagrams, many details of which are collected in the Appendix. We explain the determination of the relative sign between interfering diagrams and revisit the cancellation of divergences. We check our results with the literature and expand them in the mass insertion approximation.

Chapter 5 contains our numerical analysis. We consider two scenarios which can accommodate the measured Higgs mass of 125 GeV. The first one is a Split-SUSY scenario, in which the supersymmetric scalar particles are taken to be heavy while the gauginos are kept at the TeV scale. In this case we assume a simple $U(1)$ flavour model which sets the magnitude of the mass insertions. The second one is a Partial Compositeness scenario, where the hierarchy among the SM fermion masses is explained by the mixing with heavy resonances of a strongly coupled sector. In this case the presence of large A-terms – the trilinear soft SUSY breaking parameters – is predicted. Furthermore they are taken to be the only additional source of CP-violation. With this setup we analyse the interplay between flavour physics and the high energy physics in a suitable 2-dimensional parameter space. We see that the requirement that flavour observables do not exceed the present experimental bounds greatly constrains the parameter space compatible with the observed Higgs mass, generally increasing the scale $\tilde{m}$ of SUSY-breaking mass terms.

Chapter 1

The Standard Model of Particles

The Standard Model (SM) of elementary particle physics successfully describes three of the four known forces in nature – the electromagnetic, the weak and the strong force – in a consistent quantum field theoretical framework. There are however strong motivations to look for theories beyond it. In this chapter we briefly outline the structure of the SM, the spontaneous symmetry breaking of its gauge group and the Cabibbo-Kobayashi-Maskawa matrix. We then focus on its shortcomings and argue that it should be considered as an effective low energy theory. In the following we will largely omit derivations, which can be found in textbooks (see for example [1, 2]).

1.1 SM Lagrangian and field content

The SM is a renormalizable gauge theory that describes the strong, weak and electromagnetic interactions and provides an extremely successful description of basically all experimental data in particle physics. It is based on the invariance under the Poincaré group $\mathcal{P}$ and the gauge group $\mathcal{G} = SU(3) \times SU(2) \times U(1)_Y$. The latter is spontaneously broken to $SU(3) \times U(1)_{\rm EM}$ by the vacuum expectation value of a single Higgs doublet. Since the SM is a local gauge theory, the form of the allowed interactions is fully defined once the transformations of the fundamental fields are given. The SM Lagrangian reads

$$\mathcal{L}_{\rm SM} = \mathcal{L}_{\rm gauge} + \mathcal{L}_{\rm Dirac} + \mathcal{L}_{\rm Yukawa} + \mathcal{L}_{\rm Higgs}\,. \tag{1.1}$$

The first piece, $\mathcal{L}_{\rm gauge}$, is the Yang-Mills Lagrangian for the gauge group $\mathcal{G}$. For each generator of the group $\mathcal{G}$ the theory predicts the existence of a gauge boson: there are eight gluons g_μ^A $(A = 1, \ldots, 8)$, which mediate the strong interaction, and four vector bosons, W_μ^a which mediate the electroweak interactions. The electric charge operator is defined as

Names		gauge group	gauge coupling
Gluons	g_μ^A	$SU(3)$	g_3
W bosons	W_μ^a	$SU(2)$	g_2
B boson	B_μ	$U(1)_Y$	g_1

Table 1.1: Gauge fields of the Standard Model. The index $A = 1, \ldots, 8$ refers to the colour group $SU(3)$ while the index $a = 1, 2, 3$ refers to the weak isospin $SU(2)$.

$Q = T^3 + Y$, where T^3 is the third generator of the $SU(2)$ group ($T^3 = \sigma^3/2$ for doublets, $T^3 = 0$ for singlets) and Y is the hypercharge generator In the charge eigenstate basis weak gauge bosons form two charged and two neutral bosons

$$W^\pm_\mu = \frac{W^1_\mu \mp W^2_\mu}{\sqrt{2}}\,, \qquad \begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta_W & -\sin\theta_W \\ \sin\theta_W & \cos\theta_W \end{pmatrix} \begin{pmatrix} W^3_\mu \\ B_\mu \end{pmatrix}. \tag{1.2}$$

The fields $W^\pm_\mu$ and Z_μ are the charged and neutral vector bosons which mediate the weak force, while A_μ is the field describing the electromagnetic interaction. The parameter θ_W is called is called electroweak mixing angle and satisfies the following relations with the gauge couplings g_1, g_2 and the electric charge e

$$\sin\theta_W = \frac{g_1}{\sqrt{g_1^2+g_2^2}}\,, \qquad \cos\theta_W = \frac{g_2}{\sqrt{g_1^2+g_2^2}}\,, \qquad g_2\sin\theta_W = g_1\cos\theta_W = e\,. \tag{1.3}$$

The second piece in (1.1), $\mathcal{L}_{\rm Dirac}$, contains the fermion kinetic terms and the couplings between fermions and the gauge fields. The fermion content of SM consists of quarks (triplets under the $SU(3)$ group) and leptons (singlets under $SU(3)$), each coming in three different families or flavours (see Tab. 1.2). Note that the SM is a chiral theory, that is the left and right handed components of the spinors $\psi_{L,R} = P_{L,R}\,\psi$, where $P_{L,R} = (1\mp\gamma_5)/2$, transform differently under the electroweak gauge group.

Names		Families			$SU(3)$, $SU(2)$, $U(1)_Y$
quarks	Q	$(u_L \quad d_L)$	$(c_L \quad s_L)$	$(t_L \quad b_L)$	$(\mathbf{3},\ \mathbf{2},\ \frac{1}{6})$
	u_R	u_R	c_R	t_R	$(\mathbf{3},\ \mathbf{1},\ \frac{2}{3})$
	d_R	d_R	s_R	b_R	$(\mathbf{3},\ \mathbf{1},\ -\frac{1}{3})$
leptons	L	$(\nu_{eL} \quad e_L)$	$(\nu_{\mu L} \quad \mu_L)$	$(\nu_{\tau L} \quad \tau_L)$	$(\mathbf{1},\ \mathbf{2},\ -\frac{1}{2})$
	e_R	e_R	μ_R	τ_R	$(\mathbf{1},\ \mathbf{1},\ -1)$
Higgs	H		$(\phi^+ \quad \phi^0)$		$(\mathbf{1},\ \mathbf{2},\ \frac{1}{2})$

Table 1.2: Lepton, quark and Higgs fields of the SM. Quarks and leptons comes in three generations of flavour (also called families) often collectively called with the name of the first one.

The third piece of the SM Lagrangian is $\mathcal{L}_{\rm Yukawa}$, which features the Yukawa couplings between fermions and the scalar Higgs field

$$-\,\mathcal{L}_{\rm Yukawa} = Y_e \overline{L} H\, e_R + Y_u \overline{Q} H^c u_R + Y_d \overline{Q} H d_R + \text{h.c.}\,, \tag{1.4}$$

where $H^c = i\sigma^2 H^*$ is the conjugate of the Higgs field and Y_u, Y_d, Y_e are arbitrary complex 3×3 matrices in flavour space. In the above expression the family indices have been omitted, the first term for example reads $Y_e^{IJ}\overline{L}^I H\, e^J_R$, where the summation over the indices $I, J = 1, 2, 3$ is understood. The Yukawa terms we just outlined will provide the mass terms for the SM fermions after the Higgs field gets a non-vanishing vacuum expectation value (VEV).

The last term in Eq. (1.1) specifies the kinetic part, the gauge couplings and the potential for the Higgs field H. Its explicit form reads

$$\mathcal{L}_{\rm Higgs} = (\mathcal{D}_\mu H)^\dagger(\mathcal{D}^\mu H) - V(H)\,, \quad \text{with} \quad V(H) = -\mu_h^2 H^\dagger H + \lambda_h (H^\dagger H)^2 \tag{1.5}$$

and $\mathcal{D}_\mu H$ being the covariant derivative of the Higgs field. We can now proceed with the spontaneous symmetry breaking of the electroweak group in following section.

1.2 Higgs mechanism and spontaneous symmetry breaking

In the Lagrangian (1.1) no explicit mass term appears, neither for the fermions nor for the gauge bosons. In fact such terms are not allowed since they would break gauge invariance. Spinor matter fields and the $W^\pm$ and Z gauge boson fields gain mass through the mechanism of symmetry breaking, which is achieved thanks to the potential in Eq. (1.5). For $\lambda_h > 0$ and $\mu_h^2 > 0$, the classical energy density is a minimum for a constant Higgs field $H(x) = \phi$ such that

$$\phi^\dagger \phi = \frac{\mu_h^2}{2\lambda_h} \equiv \frac{v^2}{2} \,. \tag{1.6}$$

Choosing for the ground state $\langle H \rangle_0$ a particular value, compatible with Eq. (1.6) but not invariant under $SU(2) \times U(1)_Y$ gauge transformations, leads to the spontaneous symmetry breaking. Without loss of generality we can choose

$$\langle H \rangle_0 = \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} . \tag{1.7}$$

As a result, the neutral and the two charged Goldstone degrees of freedom mix with the gauge fields corresponding to the broken generators of $SU(2) \times U(1)_Y$ and become the longitudinal components of the Z and W physical gauge bosons, respectively. The fourth generator remains unbroken since it is the one associated to the conserved $U(1)_{\mathrm{EM}}$ gauge symmetry, and its corresponding gauge field, the photon, remains massless. From the initial four degrees of freedom of the Higgs field, three are absorbed by the $W^\pm$ and Z gauge bosons that become massive. At tree level one finds

$$m_W^2 = \frac{v^2 g_2^2}{4} \,, \qquad m_Z^2 = \frac{v^2 \left(g_1^2 + g_2^2\right)}{4} \,, \qquad \rho \equiv \frac{m_W}{m_Z \cos\theta_W} = 1 \,. \tag{1.8}$$

The last degree of freedom of the Higgs field, h, is the physical Higgs boson, which is a CP-even, spin 0 massive field. Its mass is given by $m_h = \sqrt{2\lambda_h} v$. As one can see it depends on two parameters, the Higgs VEV – which is fixed by the Fermi constant G_F through the relation $v = (\sqrt{2} G_F)^{-1/2} \approx 246\mathrm{GeV}$ – and the quartic coupling λ_h which is a free parameter of the SM.

The fermions of the SM acquire mass thanks to the Yukawa interactions in Eq. (1.4). Once the Higgs gets the VEV and the fermion fields are rotated to the mass eigenstate basis each fermion f (except neutrinos) gets a mass $m_f = y_f v/\sqrt{2}$, where y_f is the corresponding eigenvalue of the Yukawa matrix. In the next Section we give some details on how this rotation is performed. The last remarkable fact we want to stress is that the Higgs boson couplings to the fundamental particles are set by their masses, as one can see in the following table.

g_{hVV}^{SM}	$=$	$2im_V^2/v$
g_{hhVV}^{SM}	$=$	$2im_V^2/v^2$
g_{hff}^{SM}	$=$	$-im_f/v$
g_{hhh}^{SM}	$=$	$-6iv\lambda_h$
g_{hhhh}^{SM}	$=$	$-6i\lambda_h$

Table 1.3: Tree level couplings of the SM Higgs boson given as the constant appearing in the Feynman rules. We denoted with $V = W^\pm \,, Z$ the weak gauge boson.

1.3 The Yukawa sector

As we have seen, the matter fields in the SM come in three generations or flavours. Concerning their gauge interactions, the three flavours are identical copies and behave in exactly the same way. They can be only distinguished by their Yukawa couplings to the Higgs field. Let us rephrase this concept in terms of internal symmetries of the theory. In the absence of $\mathcal{L}_{\text{Yukawa}}$, the SM has a large $U(3)^5$ global symmetry corresponding to the unitary rotations in flavour space of the fermion fields. The Yukawa interactions break this symmetry down to a smaller group which only preserve four $U(1)$ charges,[1] namely the baryon number (B) and lepton flavour numbers (L_e , L_μ and L_τ). These are the four *accidental* symmetries of the SM, that is symmetries which are not imposed on the action but present in the theory because of the field content and the requirement of renormalizability.

We now turn our attention to the diagonalization of the Yukawa matrices, which is needed to write the fermions in the mass eigenstate basis. This can proceed with the biunitary transformations

$$Y_e = V_{EL}\, y_e\, V_{ER}^\dagger\,, \qquad Y_u = V_{UL}\, y_u\, V_{UR}^\dagger\,, \qquad Y_d = V_{DL}\, y_d\, V_{DR}^\dagger\,, \tag{1.9}$$

where we indicated with y_e, y_u, y_d the diagonal Yukawa matrices and V_{EL}, V_{ER}, V_{UL}, V_{UR}, V_{DL}, V_{DR} are unitary matrices. In the lepton sector we are free to choose the two matrices necessary to diagonalize Y_e without modifying the rest of the Lagrangian. In fact it is possible to change the lepton basis according to

$$L \to V_{EL}\, L\,, \quad e_R \to V_{ER}\, e_R\,, \tag{1.10}$$

and this is just enough to put the first term of Eq. (1.4) in the diagonal form. The case of the quark sector is more subtle since we can freely choose only three of the four unitary matrices necessary to diagonalize both Y_u and Y_d. The unitary transformations

$$Q \to V_{UL}\, Q\,, \quad u_R \to V_{UR}\, u_R\,, \quad d_R \to V_{DR}\, d_R\,, \tag{1.11}$$

yield a diagonal Yukawa interaction for the up-type quarks. However the Yukawa matrix for the down-type quarks remains non-diagonal. It explicitly reads $V_{UL}^\dagger V_{DL}\, y_d$. The non trivial unitary matrix $V \equiv V_{UL}^\dagger V_{DL}$ is the Cabibbo Kobayashi Maskawa (CKM) matrix [3, 4]. To go to the mass-diagonal basis (i.e. to remove V) the field u_L has to be rotated independently of its $SU(2)$-partner d_L. The only term that feels this rotation in the SM Lagrangian is the charged current interaction of quarks with the charged gauge bosons $W_\mu^\pm$, which in the physical basis reads

$$\mathcal{L}_{\text{ch.c.}} = \frac{g_2}{\sqrt{2}}\big(\overline{u}_L \gamma^\mu V d_L W_\mu^+ + \text{h.c.}\big). \tag{1.12}$$

A careful count shows that V depends on three real angles and one single complex phase. It is important to notice that within the SM, all the flavour and CP violation are induced by the CKM matrix, which parametrizes the misalignment between the up and down quark mass eigenstates in flavour space. The weak eigenstates (d', s', b') are connected to the mass eigenstate (d, s, b) by the CKM rotation

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix} = V \begin{pmatrix} d \\ s \\ b \end{pmatrix}. \tag{1.13}$$

[1] Of course, the gauged $U(1)_Y$ also remains a good symmetry.

1.4 Motivations for New Physics

The Standard Model provides a remarkably successful description of presently known phenomena, being superbly confirmed by experimental data and represents the success of the gauge theory approach to particle physics. Still, there are several *observational* (data-driven) and *theoretical* reasons to go beyond it (see, e.g., ref. [5]).

Observational reasons for new physics. All the experimental particle physics results of these last years have marked one success after the other of the SM. The observational difficulties mainly originate from astroparticle physics as unexplained phenomena.

- *Neutrino masses and mixing.* The discovery of flavour conversion of solar, atmospheric, reactor, and accelerator neutrinos has established conclusively that neutrinos have nonzero mass and they mix among themselves much like quarks (see, e.g., ref. [6]), while in the SM neutrinos are massless “by construction”.

- *Dark Matter.* There exists an impressive evidence that most of the matter in the Universe does not emit radiation, moreover such dark matter (DM) has to be provided by particles other than the usual baryons. Since the SM does not provide any viable non-baryonic DM candidate we are pushed to introduce new particles in addition to those of the SM.

- *Baryogenesis.* We have strong evidence that the Universe is vastly matter-antimatter asymmetric: for some reason no sizeable amount of primordial antimatter has survived. It is appealing to have a dynamical mechanism to give rise to such large baryon-antibaryon asymmetry starting from a symmetric situation. In the SM it is not possible to have such an efficient mechanism for baryogenesis. Hence a dynamical baryogenesis calls for the presence of new particles and interactions beyond the SM.

- *Inflation.* Several serious cosmological problems (flatness, causality, age of the Universe, ...) are beautifully solved if the early Universe underwent some period of exponential expansion (inflation). If minimally coupled to gravity, the SM with its Higgs doublet does not succeed to originate such an inflationary stage. Again some extensions of the SM, where in particular new scalar fields are introduced, are able to produce a temporary inflation of the early Universe.

Theoretical reasons for new physics. We state here three fundamental questions that do not find any satisfactory answer within the SM: the flavour problem, the unification of the fundamental interactions and the gauge hierarchy problem.

- *Flavour Problem.* Within the SM all the masses and mixings of fermions are just free (unpredicted) parameters showing a strongly hierarchical pattern. Even leaving aside neutrinos, the fermion masses span at least five orders of magnitude from the electron mass to the top quark mass. If one has in mind the usual Higgs mechanism to give rise to fermion masses, it is puzzling to insert Yukawa couplings (which are free parameters of the theory) ranging from $\mathcal{O}(1)$ to $\mathcal{O}(10^{-6})$ or so without any justification whatsoever. Saying it concisely, we can state that a “Flavour Theory” is completely missing in the SM.

- *Unification of forces.* It would be extremely appealing to reach a unified description of all the forces existing in Nature. The running of the three SM gauge couplings in the absence of NP gives reasons to hope that this could actually be the case. Therefore we cannot say that the SM represents a true unification of fundamental interactions, even leaving aside the problem that gravity is not considered at all by the model.

- *Gauge hierarchy problem.* Fermion and vector boson masses are "protected" by symmetries in the SM (i.e., their mass can arise only when we break certain symmetries). On the contrary the Higgs scalar mass does not enjoy such a symmetry protection. We would expect such mass to naturally jump to some higher scale where new physics sets in (this new energy scale could be some grand unification scale or the Planck mass, for instance). The only way to keep the Higgs mass at the electroweak scale is to perform incredibly accurate fine tunings of the parameters of the scalar sector.

The above considerations lead us to go beyond the SM. On the other hand, the clear success of the SM predictions up to energies of the order of the electroweak scale is telling us that the new physics must accurately reproduce the SM in the low-energy limit. Indeed, it may even well be the case that we have a "tower" of underlying theories which show up at different energy scales.

1.5 Neutrino masses

Experiments spanning half a century have shown that neutrino are not massless, but their masses are indeed extraordinarily light compared to other SM fermions (for a recent review, see e.g. [7]). As we pointed out before, in the minimal SM neutrinos are massless "by construction". Neutrinos do not have the right-handed counterparts, therefore they can not have a Dirac mass term. But if we view the SM as an effective theory there is still the possibility to introduce the Majorana mass terms by the dimension five Weinberg operator

$$O_\nu = \frac{\lambda_{ij}}{\Lambda}(H^c L_i)^T (H^c L_j) \tag{1.14}$$

where Λ can be considered the cut-off of the Standard Model effective theory (see Sec. 1.6), and λ_{ij} is a 3×3 matrix in flavour space. This operator is lepton number violating. After the Higgs field gets its VEV, Eq. (1.14) produce the following neutrino mass matrix

$$(m_v)_{ij} = \lambda_{ij}\frac{v^2}{\Lambda}\,. \tag{1.15}$$

The absolute value of neutrino masses has not been measured but we have the differences between the squared masses of the various neutrino. Such differences range from about 10^{-5} to $10^{-2}\,\mathrm{eV}^2$. It is reasonable therefore to suppose that the largest neutrino mass in the theory should be around $0.1\,\mathrm{eV}$. If we assume $\lambda \sim 1$ we see from (1.15) that the scale of the cutoff Λ should be

$$\Lambda \simeq \frac{(246\mathrm{GeV})^2}{0.1\,\mathrm{eV}} \simeq 10^{15}\,\mathrm{GeV}\,. \tag{1.16}$$

The operator given by (1.14) is the only gauge-invariant, Lorentz-invariant operator that one can write down at the next higher dimension ($d = 5$) in the theory. Thus, it is a satisfactory approach to neutrino physics, leading to an indication of new physics beyond the Standard Model at the scale Λ.

The see-saw mechanism

Dimension five operators lead to nonrenormalizable theories but as long as we accept that the SM is an effective theory, nothing forbids to introduce them. A more fundamental theory should explain how to get such operators in the low-energy limit. In our case one can get the operator (1.14) introducing heavy right-handed (RH) neutrinos N_{Ri} (i is the family index) which belong to the $(\mathbf{1},\, \mathbf{1}, 0)$ representation of the gauge group. Since RH neutrinos are siglets under the isospin group we can introduce in the Lagrangian explicit Majorana mass terms and the Yukawa terms as

$$\Delta\mathcal{L}_N = -(\nu^c)^T y_\nu (H^c L) + \frac{1}{2}(\nu^c)^T M \nu^c + \text{h.c.} \tag{1.17}$$

where the charge conjugated field of ν, ν^c, is given by $\nu^c = C(\overline{\nu})^T$ where $C = i\gamma_2\gamma_0$. The resulting 6×6 mass matrix in the $\{\nu_L, \nu^c\}$ basis is

$$m_\nu = \begin{pmatrix} 0 & m_D \\ m_D^T & M \end{pmatrix}, \tag{1.18}$$

where M is the matrix of Majorana masses with values M_{ij} taken straight from (1.17), and m_D are the neutrino Dirac mass matrices taken from the Yukawa interaction with the Higgs boson

Consistently with effective field theory ideas (as we will see in Sec. 1.6) we assume the Majorana mass matrix M to be of the order of the scale Λ. This is reasonable since RH neutrino Majorana masses are $SU(3)\times SU(2)\times U(1)_Y$ invariant, hence unprotected and naturally of the order of the cut-off of the low-energy theory. In that limit, the see-saw matrix of (1.18) has three heavy eigenvalues, and three light eigenvalues that, to leading order and good approximation, are eigenvalues of the 3×3 matrix

$$m_\nu^{\text{light}} = -m_D^T M^{-1} m_D \sim y^2 \frac{v^2}{M}\,, \tag{1.19}$$

which is parametrically of the same form as Eq. (1.15). This is expected since the light eigenvalues can be evaluated from the operators left over after integrating out the heavy right-handed neutrinos in the effective theory. That operator is simply (1.14), where schematically Λ can be associated with the scale M and λ can be associated with y^2.

Neutrino oscillations

Massive neutrinos generally mix and provide an explanation of neutrino oscillations (for recent reviews, see e.g. [8, 9]). Just as in the case for quarks, the neutrino mass matrix will be non-diagonal and complex. One needs to transform it into a diagonal form by unitary rotations. Thus the mass eigenstates $\nu = (\nu_1,\, \nu_2,\, \nu_3)$ are different from gauge eigenstates $\nu' \equiv (\nu_e,\, \nu_\mu,\, \nu_\tau)$ and are related to them by

$$\nu' = U\nu\,, \tag{1.20}$$

where U is a unitary 3×3 mixing matrix which is commonly referred to as the Pontecorvo–Maki–Nakagawa–Sakata (PMNS) matrix. The PMNS matrix can be parametrized like the CKM matrix for quark mixing angles. Given the definition of U and the transformation properties of the effective light neutrino mass matrix m_ν

$$\nu'^T m_\nu \nu' = \nu^T U^T m_\nu U \nu\,, \tag{1.21}$$

$$U^T m_\nu U = \text{diag}(m_1,\, m_2,\, m_3) \equiv m_{diag}\,, \tag{1.22}$$

we obtain the general form of m_ν (i.e. of the light ν mass matrix in the basis where the charged lepton mass is a diagonal matrix):

$$m_\nu = U^* m_{diag} U^\dagger\,, \tag{1.23}$$

where the matrix U can be parameterized in terms of three mixing angles θ_{12}, θ_{23} and θ_{13} ($0 \leq \theta_{ij} \leq \pi/2$) and one phase φ ($0 \leq \varphi \leq 2\pi$), exactly as for the quark mixing matrix V_{CKM}. The following definition of mixing angles can be adopted [9]

$$U \;=\; \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{i\varphi} \\ 0 & 1 & 0 \\ -s_{13}e^{-i\varphi} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}\,, \tag{1.24}$$

where $s_{ij} \equiv \sin\theta_{ij}$, $c_{ij} \equiv \cos\theta_{ij}$. In addition, if ν are Majorana particles, we have the relative phases among the Majorana masses m_1, m_2 and m_3. If we choose m_3 real and positive, these phases are carried by $m_{1,2} \equiv |m_{1,2}|e^{i\phi_{1,2}}$. Thus, in general, 9 parameters are added to the SM when non-vanishing neutrino masses are included: 3 eigenvalues, 3 mixing angles and 3 CP violating phases.

A summary on oscillation parameters is given in Table 1.4.

Quantity	Value
Δm^2_{sun}	$7.54^{+0.26}_{-0.22} \cdot 10^{-5}$ eV^2
Δm^2_{atm}	$2.43^{+0.06}_{-0.10} \cdot 10^{-3}$ eV^2
$\sin^2\theta_{12}$	$0.307^{+0.018}_{-0.016}$
$\sin^2\theta_{23}$	$0.386^{+0.024}_{-0.021}$
$\sin^2\theta_{13}$	0.0241 ± 0.025

Table 1.4: Fits to neutrino oscillation data from Ref. [10].

Where the squared mass differences are parametrized in terms of the ν mass eigenvalues by

$$\Delta m^2_{sun} \equiv |\Delta m^2_{12}|, \qquad \Delta m^2_{atm} \equiv |\Delta m^2_{23}| \quad . \tag{1.25}$$

where $\Delta m^2_{12} = |m_2|^2 - |m_1|^2 > 0$ (positive by the definition of $m_{1,2}$) and $\Delta m^2_{23} = m_3^2 - |m_2|^2$.

1.6 Gauge hierarchy problem

We now focus on the theoretical issue of the SM known as the *hierarchy problem*, showing the "bad behaviour" of the scalar sector of the SM which arises with quadratic divergences in the quantum correction of the two-point function. We then compare it with the mild logarithmic correction to the electron two-point function, explaining the different behaviour in terms of internal symmetries.

The two fundamental concepts that enter this description are effective theories and symmetries. In the modern point of view, a given theory (e.g. the Standard Model) is always the effective theory of a more complete underlying theory and its description is valid up to the scale Λ at which new physics enters. This scale acts as a cut-off on loop momenta in calculations. Of course, this doesn't mean that there can be no momenta above Λ – rather, the cut-off in the effective theory is the scale of short-range physics that has been omitted from the effective theory.

Let us consider the SM Higgs field with squared mass $m_h^2 = 2\mu_h^2$ and a matter fermion field f with a Yukawa coupling to the Higgs field via the term

$$\mathcal{L}_{\bar{f}fh} = -\frac{\lambda_f}{\sqrt{2}} h\bar{f}f + \text{h.c.}\ , \tag{1.26}$$

We want to compute the one loop $f - \bar{f}$ contribution to the scalar two-point function (inverse propagator) as illustrated in Fig. 1.1, performing the calculation at zero external momentum for simplicity.

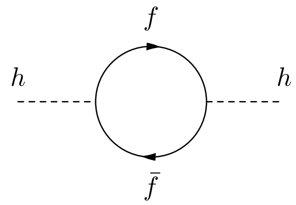


Figure 1.1: Fermionic one-loop contribution to the Higgs scalar two-point function

$$\begin{aligned}
\Pi^f_{\phi\phi}(0) &= -\int \frac{\mathrm{d}^4k}{(2\pi)^4} \,\mathrm{Tr}\left[\left(i\frac{\lambda_f}{\sqrt{2}}\right)\frac{i}{\not{k} - m_f}\left(i\frac{\lambda_f}{\sqrt{2}}\right)\frac{i}{\not{k} - m_f}\right] \\
&= -2\lambda_f^2 \int \frac{\mathrm{d}^4k}{(2\pi)^4} \frac{k^2 + m_f^2}{(k^2 - m_f^2)^2} \\
&= -2\lambda_f^2 \int \frac{\mathrm{d}^4k}{(2\pi)^4} \left[\frac{1}{k^2 - m_f^2} + \frac{2m_f^2}{(k^2 - m_f^2)^2}\right].
\end{aligned} \tag{1.27}$$

Note that the first term in the last line of Eq. (1.27) is quadratically divergent. In quantum field theory, the renormalization procedure allows us to deal with contributions that diverge when the cut-off is sent to infinity. However the cut-offs that we consider here are physical and thus cannot be sent to arbitrary values. Evaluating the integral of the first term inside the square brackets up to the scale Λ, we obtain the following correction to the Higgs mass,[2]

$$\delta m_h^2(f) = -2\frac{\lambda_f^2}{16\pi^2}\left[\Lambda^2 - m_f^2 \log\left(\frac{\Lambda^2 + m_f^2}{m_f^2}\right) + \dots\right], \tag{1.28}$$

where the ellipses stand for the second part of the integral (1.27) and represent terms proportional to m_f^2 which grow at most logarithmically with Λ. The physical (observable) mass is the renormalized quantity

$$(m_h^2)_R = m_h^2 + \delta m_h^2\,. \tag{1.29}$$

If we were to replace the divergence Λ^2 by the Planck mass M_{Pl}^2, the resulting "correction" would be some 30 orders of magnitude *larger* than the physical SM Higgs mass. In fact

[2]Since we performed the integral at zero external momentum it will not be the on-shell (pole) mass, but it is easy to see that the difference between these two quantities can at most involve logarithmic divergences.

we could cancel these large correction with a bare mass of the same order and opposite sign. However, these two contributions should cancel with a precision of one part in 10^{26} and even then we should worry about the two loop contribution and so on. This is the so-called *hierarchy problem.* Note also that the correction (1.28) is itself independent of m_h. This is related to the fact that setting $m_h = 0$ does not increase the symmetry group of the SM.

Symmetries are in fact the second fundamental concept and play a key role in the description of the fundamental interactions. If a parameter of the theory is equal to zero because of an exact quantum symmetry, it will remain zero even after we have included all quantum corrections. This is why a small parameter is not necessarily problematic, if it is "protected" by a symmetry.

Let us see how it works in Quantum Electro Dynamics (QED), the best understood ingredient of the SM. We compute the electron self-energy correction (Fig. 1.2) at zero external momentum and take the cut-off at the Planck scale. We get the following correction

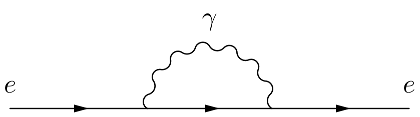


Figure 1.2: The electron self-energy in QED.

to the electron mass:

$$\delta m_e \simeq 2\frac{\alpha_{\rm em}}{\pi} m_e \log \frac{M_{Pl}}{m_e} \simeq 0.24\, m_e\,, \tag{1.30}$$

which is quite modest. At a deeper level, the fact that this correction is quite benign can again be understood from a symmetry: In the limit $m_e \to 0$, the model becomes invariant under chiral rotations

$$\psi_e \to \exp(i\beta\gamma_5)\psi_e\,, \quad (\beta \in \mathbb{R}) \tag{1.31}$$

If this symmetry were exact, the correction of Eq. (1.30) would have to vanish. In reality the symmetry is broken by the electron mass, so the correction must itself be proportional to m_e.

The gauge hierarchy problem is often expressed as the *naturalness problem* of the SM. The naturalness criterion is a powerful guiding principle for physicists as they try to construct new theories (for a recent pedagogical review, see [11]) and we now present this concept.

The Naturalness criterion. *Let us consider a theory valid up to a maximum energy Λ and make all its parameters dimensionless by measuring them in units of Λ. The naturalness criterion states that one such parameter is allowed to be much smaller than unity only if setting it to zero increases the symmetry of the theory [12]. If this does not happen, the theory is unnatural.*

This criterion states that in an effective theory, all operators should have their dimensionality set by the cut-off of the theory (for a recent pedagogical review, see [13]). Therefore an operator $O^{(d)}$ with mass dimension d should appear in the effective Lagrangian as

$$\mathcal{L}_{\rm eff|d} = c\Lambda^{4-d}O^{(d)} \tag{1.32}$$

where Λ is the cut-off of the theory and c is expected to be of order $\mathcal{O}(1)$ in value. Irrelevant ($d > 4$) operators are suppressed and marginal ($d = 4$) operators cause no harm since they are scale-independent and perturbative quantum corrections introduce only a mild logarithmic dependence.

The SM is almost exclusively a theory of $d = 4$ marginal operators with its kinetic terms, gauge interaction terms, and Yukawa interaction terms. What is potentially problematic is the existence of any $d < 4$ relevant operators. In that case, the coefficients should be large, set by the cut-off of the theory. When we consider the SM extended with neutrino masses there are only two gauge-invariant, Lorentz-invariant relevant $d < 4$ operators.

- The $d = 3$ right-handed neutrino Majorana mass interaction terms $N_R^T i\sigma^2 N_R$. In this case one expects that the coefficient should be $M_R \sim \Lambda$. This expectation is nicely met in the see-saw mechanism which can naturally explain the smallness of the left-handed neutrinos.

- The $d = 2$ Higgs boson mass operator $|\Phi|^2$. This time the coefficient should be $\mu^2 \sim \Lambda^2$ which constitutes a potential disaster since this term fixes the scale of the Higgs VEV and of all related masses.

The naturalness problem arises because the coefficient μ is not suppressed by any symmetry. Since empirically the Higgs mass is light (and, by naturalness, it should be of $o(\Lambda)$) we would expect that Λ, i.e. some form of new physics, should appear near the TeV scale.

Chapter 2

The minimal supersymmetric standard model

This chapter is devoted to supersymmetry (SUSY) and the Minimal Supersymmetric extension of the Standard Model (MSSM). Everything discussed here can be found in textbooks (e.g. [14, 15]) or lecture notes (e.g. [16, 17, 18, 19]) on the subject. We begin introducing SUSY, presenting the main motivations, the SUSY algebra and explain briefly how supersymmetric Lagrangians can be constructed. We continue introducing the concept of soft SUSY breaking, which will play a crucial role in describing flavour phenomena we are interested in. We then present the MSSM building blocks and we discuss the minimization of its scalar potential. We end analysing the MSSM spectrum.

2.1 Motivations for Supersymmetry

Supersymmetry (SUSY) is a generalization of the space-time symmetries of quantum field theory that transforms fermions into bosons and vice versa. This non-trivial extension of the Poincaré symmetry provides an interesting framework which addresses the hierarchy problem. Moreover the Minimal Supersymmetric extension of the SM (MSSM) can account for the gauge coupling unification (for a review, see e.g. [16]) and contains a candidate for dark matter, namely the lightest supersymmetric particle.

The SUSY way to the hierarchy problem. One of the main motivations for SUSY comes from the systematic cancellation of the quadratically divergent contributions to the Higgs mass. To see how this cancellation is achieved, let us suppose there exists a scalar field $\tilde{f}$ with mass $m_{\tilde{f}}$ that couples to the scalar Higgs field through the term

$$\mathcal{L}_{\tilde{f}h} = \frac{1}{2}\tilde{\lambda}_f h^2 \left|\tilde{f}\right|^2 . \tag{2.1}$$

This new field gives a quadratically divergent one-loop contribution to the Higgs two-point function. In fact, evaluating the first diagram in Fig. 2.1 at zero external momentum we get

$$\Pi^{\tilde{f}}_{\phi\phi}(0) = -\tilde{\lambda}_f \int \frac{\mathrm{d}^4 k}{(2\pi)^4}\frac{1}{k^2 - m^2_{\tilde{f}}}\,, \qquad \delta m_h^2(\tilde{f}) = -\frac{\tilde{\lambda}_f}{16\pi^2}\left[\Lambda^2 - m^2_{\tilde{f}}\log\left(\frac{\Lambda^2 + m^2_{\tilde{f}}}{m^2_{\tilde{f}}}\right)\right]. \tag{2.2}$$

The correction to Higgs mass depends quadratically on the cut-off scale Λ. Comparing this contribution to the one due to the fermion loop Eq. (1.28) we see that the quadratic divergence can cancel if the couplings $\tilde{\lambda}_f$ and λ_f satisfy the relation $\tilde{\lambda}_f = -\lambda_f^2$ and provided that there exists two scalar fields $\tilde{f}$ for each Dirac fermion f. This last requirement imply the matching of the fermion and boson degrees of freedom.

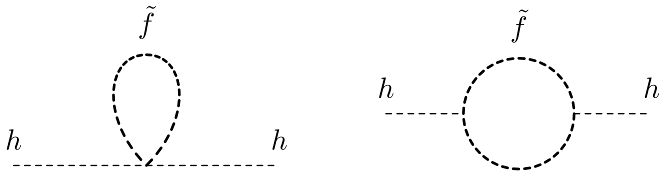


Figure 2.1: One-loop quantum corrections to the Higgs two-point function due to a scalar field $\tilde{f}$.

SUSY is the kind of symmetry which guarantees these relations between fermion and boson fields, leading to a quadratic-divergence free theory. Actually in a theory with exact SUSY also the second diagram in Fig. 2.1 is generated and the relation $m_{\tilde{f}} = m_f$ is ensured, leading to a vanishing total correction to the Higgs mass. As no superpartner of a SM particle has been found yet, these new particles are required to be heavier then their SM partners. Thus quantum corrections to the Higgs mass cannot cancel exactly, but in the case of softly broken SUSY, they do not reintroduce quadratic divergences.

2.2 The SUSY algebra

Supersymmetry is an extension of spacetime symmetry. Therefore it is useful to recall briefly some basics about the Poincaré symmetry. The algebra reads

$$\begin{aligned}
[P^\mu, P^\nu] &= 0 \\
[P^\mu, J^{\rho\sigma}] &= i(g^{\mu\rho} P^\sigma - g^{\mu\sigma} P^\rho) \\
[J^{\mu\nu}, J^{\rho\sigma}] &= i(g^{\nu\rho} J^{\mu\sigma} - g^{\mu\rho} J^{\nu\sigma} - g^{\nu\sigma} J^{\mu\rho} + g^{\mu\sigma} J^{\nu\rho})
\end{aligned} \tag{2.3}$$

where P_μ are the generators for translations and $J_{\mu\nu}$ the ones for (proper, orthochronous) Lorentz transformations. The irreducible representations of this algebra are characterized by their mass and their spin, which can either be half integer (fermions) or integer (bosons). Both the generators for translations and for Lorentz transformations can be represented as differential operators acting on fields living on spacetime. For scalar fields such operators read

$$\begin{aligned}
P_\mu &= i\partial_\mu \\
J^{\mu\nu} &= i(x^\mu \partial^\nu - x^\nu \partial^\mu)
\end{aligned} \tag{2.4}$$

Up to now there is no evidence that Poincaré symmetry is violated in nature and most of the models for *new physics* respect this symmetry. A possible way to go beyond the SM is to look for new symmetries which improve the symmetry structure of the theory $\mathcal{P} \times \mathcal{G}$ (where $\mathcal{P}$ is the Poincaré group and $\mathcal{G}$ is the internal gauge group). As long as $\mathcal{G}$ is a compact Lie group, we can construct a consistent relativistic quantum field theory preserving invariance under local transformations of $\mathcal{G}$. The generators of both $\mathcal{P}$ and $\mathcal{G}$ are bosonic, in the sense that they satisfy the commutation relations of a Lie algebra.

Coleman and Mandula found a *no-go theorem* [20], which roughly states that any meaningful extension of the Poincaré symmetry, that is based on Lie Algebras, has to be a direct product of the Poincaré symmetry and an internal symmetry. This means that the possible extensions based on a Lie Algebra can only have trivial commutation relations between the generators of Poincaré transformations and the new symmetry generators.

But then Wess and Zumino discovered models with a symmetry that connected bosons and fermions and which therefore was a nontrivial extension of the Poincaré symmetry such that the generators Q_i satisfy the nontrivial relation

$$[Q_i, J_{\mu\nu}] \neq 0\,. \tag{2.5}$$

Haag, Lopuszanski and Sohnius finally systematically investigated this so-called supersymmetry and classified all of its possible realizations [21]. The reason why supersymmetry can circumvent the Coleman-Mandula theorem is that it is not based on a Lie Algebra but on a graded Lie Algebra, which means that one also allows for anticommutation relations among the group generators. This is in fact the only possibility to construct a non trivial extension of the Poincaré symmetry. The simplest version of this supersymmetry that does not involve additional central charges, extends the original Poincaré algebra by the following relations

$$\begin{aligned}
\left\{\hat{Q}_\alpha, \hat{Q}^\dagger_{\dot\beta}\right\} &= 2i(\sigma^\mu)_{\alpha\dot\beta}\partial_\mu = 2(\sigma^\mu)_{\alpha\dot\beta}\,\hat{P}_\mu\,, \\
\left\{\hat{Q}_\alpha, \hat{Q}_\beta\right\} &= 0\,, \qquad \left\{\hat{Q}^\dagger_{\dot\alpha}, \hat{Q}^\dagger_{\dot\beta}\right\} = 0\,, \\
\left[\hat{Q}_\alpha, \hat{P}_\mu\right] &= 0 \qquad \left[\hat{Q}^\dagger_{\dot\alpha}, \hat{P}_\mu\right] = 0\,. \\
\left[\hat{Q}_\alpha, J^{\mu\nu}\right] &= (\sigma^{\mu\nu})_\alpha{}^\beta\,\hat{Q}_\beta \\
\left[\hat{Q}^\dagger_{\dot\alpha}, J^{\mu\nu}\right] &= (\overline{\sigma}^{\mu\nu})_{\dot\alpha}{}^{\dot\beta}\,\hat{Q}^\dagger_{\dot\beta}
\end{aligned} \tag{2.6}$$

where the SUSY generators $\hat{Q}_a$ and $\hat{Q}^\dagger_{\dot a}$ are two component Weyl spinors. Sigma matrices σ^μ, $\overline{\sigma}^\mu$, $\sigma^{\mu\nu}$, $\overline{\sigma}^{\mu\nu}$ are defined in Appendix A.2.

2.3 Superspace and superfields

To classify the representations of supersymmetry and build supersymmetric Lagrangians, it is convenient to use the superfield formalism. This is achieved enlarging the usual Minkowski space by four fermionic coordinates θ^α and $\theta^\dagger_{\dot\alpha}$ which are constant complex anticommuting two-component spinors with dimension $(\text{mass})^{-1/2}$.
A possible choice for the supersymmetry charges is

$$\begin{aligned}
\hat{Q}_\alpha &= i\frac{\partial}{\partial\theta^\alpha} - (\sigma^\mu\theta^\dagger)_\alpha\partial_\mu, \qquad & \hat{Q}^\alpha &= -i\frac{\partial}{\partial\theta_\alpha} + (\theta^\dagger\overline{\sigma}^\mu)^\alpha\partial_\mu, \\
\hat{Q}^{\dagger\dot\alpha} &= i\frac{\partial}{\partial\theta^\dagger_{\dot\alpha}} - (\overline{\sigma}^\mu\theta)^{\dot\alpha}\partial_\mu, \qquad & \hat{Q}^\dagger_{\dot\alpha} &= -i\frac{\partial}{\partial\theta^{\dagger\dot\alpha}} + (\theta\sigma^\mu)_{\dot\alpha}\partial_\mu.
\end{aligned} \tag{2.7}$$

Note that the hatted objects $\hat{Q}_\alpha$, $\hat{Q}^\dagger_{\dot\alpha}$, $\hat{P}^\mu$ are differential operators acting on functions in superspace. A general superfield $S(x,\theta,\theta^\dagger)$ is defined by its expansion in powers of θ and $\theta^\dagger$. Since there are two independent components of θ^α and likewise for $\theta^\dagger_{\dot\alpha}$, the expansion always terminates, which each term containing at most two θ's and two $\theta^\dagger$'s. Therefore one can write $S(x,\theta,\theta^\dagger)$ as

$$S(x,\theta,\theta^\dagger) = a + \theta\xi + \theta^\dagger\chi^\dagger + \theta\theta b + \theta^\dagger\theta^\dagger c + \theta\sigma^\mu\theta^\dagger v_\mu + \theta^\dagger\theta^\dagger\theta\eta + \theta\theta\theta^\dagger\zeta^\dagger + \theta\theta\theta^\dagger\theta^\dagger d\,. \tag{2.8}$$

Expression (2.8) features four Weyl spinors ξ, $\chi^\dagger$, η and $\zeta^\dagger$, which amounts to 16 real fermionic degrees of freedom. In addition there are four complex scalar fields: a, b, c and

d as well as one complex vector field v_μ. In total 16 real bosonic degrees of freedom. In fact it can be proven very generally, that in a supersymmetric theory the number of fermionic and bosonic degrees of freedom has always to be the same.
A finite (pure) SUSY transformation can be written in the following way[1]

$$G(\epsilon, \epsilon^\dagger) = \exp\left[-\frac{i}{\sqrt{2}}\left(\epsilon^a \hat{Q}_a + \epsilon^\dagger_{\dot{a}} \hat{Q}^{\dagger\dot{a}}\right)\right] \tag{2.9}$$

where we introduced the anticommuting Weyl spinors ϵ and $\epsilon^\dagger$ to parameterize the transformation. The infinitesimal version for any superfield S is given by

$$\sqrt{2}\,\delta_\epsilon S = -i\left(\epsilon\hat{Q} + \epsilon^\dagger \hat{Q}^\dagger\right) S \;=\; \left(\epsilon^\alpha \frac{\partial}{\partial\theta^\alpha} + \epsilon^\dagger_{\dot{\alpha}} \frac{\partial}{\partial\theta^\dagger_{\dot{\alpha}}} + i\left[\epsilon\sigma^\mu\theta^\dagger + \epsilon^\dagger\overline{\sigma}^\mu\theta\right]\partial_\mu\right) S \tag{2.10}$$

$$\sqrt{2}\,\delta_\epsilon S = S(x^\mu + i\epsilon\sigma^\mu\theta^\dagger + i\epsilon^\dagger\overline{\sigma}^\mu\theta,\, \theta+\epsilon,\, \theta^\dagger + \epsilon^\dagger) - S(x^\mu,\, \theta,\, \theta^\dagger). \tag{2.11}$$

Equation (2.11) shows that a supersymmetry transformation can be viewed as a translation in superspace. Since $\hat{Q}$, $\hat{Q}^\dagger$ are linear differential operators, the product or linear combination of any superfields satisfying eq. (2.10) is again a superfield with the same transformation law.
It can be shown that such a general object as (2.8) is not an irreducible representation of the SUSY algebra. In order to get irreducible superfields, one has to impose some restrictions. These restrictions make use of covariant derivatives, which are defined in the following way

$$D_\alpha = \frac{\partial}{\partial\theta^\alpha} - i(\sigma^\mu\theta^\dagger)_\alpha\partial_\mu, \qquad D^\alpha \;=\; -\frac{\partial}{\partial\theta_\alpha} + i(\theta^\dagger\overline{\sigma}^\mu)^\alpha\partial_\mu, \tag{2.12}$$

$$D^{\dagger\dot{\alpha}} = \frac{\partial}{\partial\theta^\dagger_{\dot{\alpha}}} - i(\overline{\sigma}^\mu\theta)^{\dot{\alpha}}\partial_\mu, \qquad D^\dagger_{\dot{\alpha}} \;=\; -\frac{\partial}{\partial\theta^{\dagger\dot{\alpha}}} + i(\theta\sigma^\mu)_{\dot{\alpha}}\partial_\mu. \tag{2.13}$$

Using the definition eq. (2.10), it follows that

$$\delta_\epsilon(D_\alpha S) = D_\alpha(\delta_\epsilon S), \qquad \delta_\epsilon\left(D^\dagger_{\dot{\alpha}} S\right) = D^\dagger_{\dot{\alpha}}(\delta_\epsilon S), \tag{2.14}$$

which means that covariant derivatives commute with an infinitesimal SUSY transformation. Thus the derivatives D_α and $D^\dagger_{\dot{\alpha}}$ are supersymmetric covariant; acting on superfields, they return superfields. This makes them useful for defining constraints on superfields in a covariant way.

Chiral superfields

Using (2.12) we can now define a so-called *chiral superfield* by requiring that its covariant derivative vanishes

$$D^\dagger_{\dot{\alpha}}\Phi = 0\,. \tag{2.15}$$

It can be shown that this restriction indeed leads to an irreducible representation of the SUSY algebra. (The so-called chiral multiplet with superspin $Y = 0$.) Its complex conjugate Φ^* is called *antichiral* and satisfies

$$D_\alpha\Phi^* = 0\,. \tag{2.16}$$

[1] The factor of $\sqrt{2}$ is a convention, not universally chosen in the literature.

To solve these constraints, we define the new variables

$$y^\mu \equiv x^\mu - i\theta\sigma^\mu\theta^\dagger \,, \qquad y^{\mu *} \equiv x^\mu + i\theta\sigma^\mu\theta^\dagger \,, \tag{2.17}$$

which satisfy

$$D_\alpha y^\mu = 0\,, \qquad D^\dagger_{\dot\alpha} y^{\mu *} = 0\,. \tag{2.18}$$

As a consequence, the chiral superfield constraint eq. (2.15) is solved by any function of y^μ and θ only and not $\theta^\dagger$. Therefore, one can expand:

$$\Phi \;=\; \phi(y) + \sqrt{2}\theta\psi(y) + \theta\theta F(y), \tag{2.19}$$

and similarly

$$\Phi^* \;=\; \phi^*(y^*) + \sqrt{2}\theta^\dagger\psi^\dagger(y^*) + \theta^\dagger\theta^\dagger F^*(y^*), \tag{2.20}$$

where the factors of $\sqrt{2}$ are conventional. Rewriting the chiral superfields in terms of the original coordinates $x, \theta, \theta^\dagger$, by expanding in a power series in the anticommuting coordinates, gives

$$\begin{aligned}\Phi = \phi(x) - i\theta\sigma^\mu\theta^\dagger\partial_\mu\phi(x) - \frac{1}{4}\theta\theta\theta^\dagger\theta^\dagger\partial_\mu\partial^\mu\phi(x) + \sqrt{2}\theta\psi(x)\\ - \frac{i}{\sqrt{2}}\theta\theta\theta^\dagger\overline{\sigma}^\mu\partial_\mu\psi(x) + \theta\theta F(x),\end{aligned} \tag{2.21}$$

$$\begin{aligned}\Phi^* = \phi^*(x) + i\theta\sigma^\mu\theta^\dagger\partial_\mu\phi^*(x) - \frac{1}{4}\theta\theta\theta^\dagger\theta^\dagger\partial_\mu\partial^\mu\phi^*(x) + \sqrt{2}\theta^\dagger\psi^\dagger(x)\\ - \frac{i}{\sqrt{2}}\theta^\dagger\theta^\dagger\theta\sigma^\mu\partial_\mu\psi^\dagger(x) + \theta^\dagger\theta^\dagger F^*(x).\end{aligned} \tag{2.22}$$

Concerning the degrees of freedom, the chiral superfield consists of

- one left chiral Weyl spinor ψ that describes a two component fermion,
- one complex scalar field ϕ, the sfermion (scalar SUSY partner of the fermion)
- and one additional complex scalar field F , that is an "auxiliary" field, which will not describe any dynamical degrees of freedom in the end.

One way to construct a chiral or antichiral superfield is

$$\Phi \;=\; D^\dagger D^\dagger S \;\equiv\; D^\dagger_{\dot\alpha} D^{\dagger\dot\alpha} S\,, \qquad\qquad \Phi^* \;=\; DDS^* \;\equiv\; D^\alpha D_\alpha S^*\,, \tag{2.23}$$

where S is any general superfield. This follows immediately from the fact that acting three consecutive times with the anticommuting two-component derivative $D^\dagger$ always produces a vanishing result, and similarly for D. The converse is also true; for every chiral superfield Φ, one can find a superfield S such that eq. (2.23) is true. We can now obtain the supersymmetry transformation laws for the component fields of Φ using $\sqrt{2}i\delta_\epsilon\Phi = \epsilon\hat{Q} + \epsilon^\dagger\hat{Q}^\dagger$. The results are

$$\begin{aligned}\delta_\epsilon\phi &= \epsilon\psi,\\ \delta_\epsilon\psi_\alpha &= -i(\sigma^\mu\epsilon^\dagger)_\alpha\partial_\mu\phi + \epsilon_\alpha F,\\ \delta_\epsilon F &= -i\epsilon^\dagger\overline{\sigma}^\mu\partial_\mu\psi,\end{aligned} \tag{2.24}$$

We see that the $\theta\theta$ component of a chiral superfield transforms as a total derivative, Therefore it is a candidate for a term to appear in a supersymmetric Lagrangian. From now on we will refer to this term as the F-term.

Vector superfields

In addition to the chiral superfield, we now define a *vector superfield* (an irreducible representation of the SUSY algebra with superspin $Y = 1/2$) by imposing the following SUSY invariant condition to a general superfield

$$V = V^* \,. \tag{2.25}$$

The component expansion of the vector superfield is

$$\begin{aligned} V(x,\theta,\theta^\dagger) &= a + \theta\xi + \theta^\dagger\xi^\dagger + \theta\theta b + \theta^\dagger\theta^\dagger b^* + \theta\sigma^\mu\theta^\dagger A_\mu + \theta^\dagger\theta^\dagger\theta\left(\lambda - \frac{i}{2}\sigma^\mu\partial_\mu\xi^\dagger\right) \\ &+ \theta\theta\theta^\dagger\left(\lambda^\dagger - \frac{i}{2}\overline{\sigma}^\mu\partial_\mu\xi\right) + \theta\theta\theta^\dagger\theta^\dagger\left(\frac{1}{2}D - \frac{1}{4}\partial_\mu\partial^\mu a\right). \end{aligned} \tag{2.26}$$

The vector multiplet contains 8 bosonic degrees of freedom (the real scalar fields a and D, the complex scalar field b and the four real components of A_μ) and 8 fermionic ones (the two-component spinors ξ and λ).
The supersymmetry transformations of the vector field components are:

$$\sqrt{2}\,\delta_\epsilon a = \epsilon\xi + \epsilon^\dagger\xi^\dagger \tag{2.27}$$
$$\sqrt{2}\,\delta_\epsilon\xi_\alpha = 2\epsilon_\alpha b + (\sigma^\mu\epsilon^\dagger)_\alpha(A_\mu - i\partial_\mu a), \tag{2.28}$$
$$\sqrt{2}\,\delta_\epsilon b = \epsilon^\dagger\lambda^\dagger - i\epsilon^\dagger\overline{\sigma}^\mu\partial_\mu\xi, \tag{2.29}$$
$$\sqrt{2}\,\delta_\epsilon A^\mu = -i\epsilon\partial^\mu\xi + i\epsilon^\dagger\partial^\mu\xi^\dagger + \epsilon\sigma^\mu\lambda^\dagger - \epsilon^\dagger\overline{\sigma}^\mu\lambda, \tag{2.30}$$
$$\sqrt{2}\,\delta_\epsilon\lambda_\alpha = \epsilon_\alpha D - \frac{i}{2}(\sigma^\mu\overline{\sigma}^\nu\epsilon)_\alpha(\partial_\mu A_\nu - \partial_\nu A_\mu), \tag{2.31}$$
$$\sqrt{2}\,\delta_\epsilon D = -i\epsilon\sigma^\mu\partial_\mu\lambda^\dagger - i\epsilon^\dagger\overline{\sigma}^\mu\partial_\mu\lambda\,. \tag{2.32}$$

A superfield cannot be both chiral and real at the same time, unless it is identically constant however, if Φ is a chiral superfield, then $\Phi + \Phi^*$ and $i(\Phi - \Phi^*)$ and $\Phi\Phi^*$ are all real (vector) superfields. In eq. (2.26) we used peculiar combinations of fields to make apparent that the components a, b and ξ can be transformed to zero by a $U(1)$ gauge transformation of the form

$$V \to V + i(\Omega^* - \Omega), \tag{2.33}$$

where Ω is a chiral superfield, $\Omega = \phi + \sqrt{2}\theta\psi - \theta\theta F$ in the basis of eq. (2.17). Under this transformation

$$\begin{aligned} a &\to a + i(\phi^* - \phi), \\ \xi_\alpha &\to \xi_\alpha - i\sqrt{2}\psi_\alpha, \\ b &\to b - iF, \\ A_\mu &\to A_\mu - \partial_\mu(\phi + \phi^*), \\ \lambda_\alpha &\to \lambda_\alpha, \\ D &\to D. \end{aligned} \tag{2.34}$$

From (2.34) we see that there is a special gauge in which a, b and ξ are set to zero. This gauge is the so-called Wess-Zumino gauge. It breaks supersymmetry and leaves a residual transformation $A_\mu \to A_\mu - \partial_\mu\alpha$ which is the usual gauge ambiguity. In the Wess-Zumino gauge, (2.26) reduces to

$$V_{\mathrm{WZ\,gauge}} \;=\; \theta\sigma^\mu\theta^\dagger A_\mu + \theta^\dagger\theta^\dagger\theta\lambda + \theta\theta\theta^\dagger\lambda^\dagger + \frac{1}{2}\theta\theta\theta^\dagger\theta^\dagger D. \tag{2.35}$$

This vector superfield contains

- one vector field A_μ that will describe a gauge boson in the end,
- one Weyl spinor λ, the fermionic partner of the gauge boson, also called gaugino
- and again one auxiliary scalar field D, which is real.

From eq. (2.32) we see that the $\theta\theta\theta^\dagger\theta^\dagger$ component of the vector superfield transforms under SUSY transformations as a total derivative $\sqrt{2}\,\delta_\epsilon D = -i\partial_\mu\big(\epsilon\sigma^\mu\lambda^\dagger + \epsilon^\dagger\overline{\sigma}^\mu\lambda\big)$. Thus the $\theta\theta\theta^\dagger\theta^\dagger$ component of a vector superfield is another candidate for a term in a supersymmetric Lagrangian. We will refer to this component as the D-term.

2.4 A first supersymmetric Lagrangian

With the above pieces of information, we are now able to construct a first Lagrangian that is invariant under SUSY. Given a set of chiral superfields $\{\Phi_i\}$, it is clear from their definition (2.15) that both products and linear combinations of these chiral superfields again give chiral superfields. From this fact we conclude that the following expression is an admissible term for a supersymmetric Lagrangian

$$\mathcal{L}_W(x) = [W(\Phi)]_F + \text{h.c.}\ , \tag{2.36}$$

where $[\ldots]_F$ indicates that we only take the $\theta\theta$ component. The quantity $W(\Phi)$ is called *superpotential* and it can be written most generally as

$$W = L^i\Phi_i + \frac{1}{2}M^{ij}\Phi_i\Phi_j + \frac{1}{6}y^{ijk}\Phi_i\Phi_j\Phi_k\,. \tag{2.37}$$

with complex numbers L^i, M^{ij} and y^{ijk}. Higher powers of the chiral superfields are in principle allowed by supersymmetry, but they would give rise to higher dimensional (and therefore nonrenormalizable) terms in the Lagrangian. The parameter L^i is only allowed if Φ_i is a gauge singlet. Since there are not such chiral supermultiplets in the MSSM with the minimal field content we will omit the L^i parameters from now on. Without loss of generality, M^{ij} and y^{ijk} can be chosen to be totally symmetric in their indices, which leads to the following explicit form of $\mathcal{L}_W$ once one plugs in the general expression (2.21) for the chiral superfields.

$$\mathcal{L}_W(x) = L^iF_i - M^{ij}\left(\frac{1}{2}\psi_i\psi_j - \phi_iF_j\right) - \frac{1}{2}y^{ijk}(\phi_i\psi_j\psi_k - \phi_i\phi_jF_k) + \text{h.c.}\,. \tag{2.38}$$

$\mathcal{L}_W$ provides only interaction terms. To introduce a kinetic Lagrangian for the fields in (2.36), we note that the expression $\Phi^*\Phi$ is real and therefore a vector superfield. Thus we are allowed to add the following term to the Lagrangian

$$\mathcal{L}_{K_\Phi}(x) = \left[\Phi^{*i}\Phi_i\right]_D, \tag{2.39}$$

where $[\ldots]_D$ means that we only take the $\theta\theta\theta^\dagger\theta^\dagger$ component. Using again the explicit form of the chiral superfields (2.21), we obtain

$$\mathcal{L}_{K_\Phi}(x) = \partial^\mu\phi^{*i}\partial_\mu\phi_i + i\psi^{\dagger i}\overline{\sigma}^\mu\partial_\mu\psi_i + F^{*i}F_i + \ldots \tag{2.40}$$

The ... indicates a total derivative part, which may be dropped since this is destined to be integrated $\int d^4x$. These are exactly the kinetic terms we were looking for. By putting together (2.36) and (2.40) we can write the most general renormalizable supersymmetric Lagrangian for chiral multiplets. It reads

$$\mathcal{L}(x) = \left[\Phi^{*i}\Phi_i\right]_D + ([W(\Phi_i)]_F + \text{h.c.}) \tag{2.41}$$

This expression can be seen as a first step towards a supersymmetric version of the Standard Model. One main ingredient of the Standard Model is the concept of local gauge invariance. Thus, in order to construct a supersymmetric version of the Standard Model, we now have to *supersymmetrize* gauge theories.

2.5 Supersymmetric gauge theories

Gauge transformations for chiral superfields can be defined in complete analogy to the conventional ones. A general gauge symmetry realized on chiral superfields Φ_i in a representation R with matrix generators T^a reads:

$$\Phi_i \to \Phi'_i = \left[e^{ig\Lambda^a T^a}\right]_i^{\ j} \Phi_j \,, \qquad \Phi^{*i} \to \Phi'^{*i} = \Phi^{*j}\left[e^{-ig\Lambda^a T^a}\right]_j^{\ i} . \tag{2.42}$$

The gauge couplings for the irreducible components of the Lie algebra are g_a. Λ^a are the supergauge transformation parameters. To be consistent with supersymmetry, Φ' has to be a chiral superfield, as Φ is. Therefore also the parameters Λ^a have to be chiral superfields, which in general means that $\Lambda^{a*} \neq \Lambda^a$. If one now looks at the kinetic Lagrangian (2.36), one sees that it is not invariant under the *super gauge transformation* (2.42).

$$\Phi^*\Phi \;\to\; \Phi^* e^{-ig\Lambda^\dagger}\, e^{ig\Lambda}\Phi \;\neq\; \Phi^*\Phi \,, \tag{2.43}$$

where $\Lambda = \Lambda^a T^a$ and we omitted the summed index over the chiral superfields. To make the kinetic term invariant under the gauge transformations (2.42), it is convenient to define the matrix-valued vector superfield $V = V^a T^a$ which transforms in the following way

$$e^{2gV} \;\to\; e^{2gV'} = e^{ig\Lambda^\dagger} e^{2gV} e^{-ig\Lambda} . \tag{2.44}$$

Now it is possible to construct a term for the Lagrangian, which is obviously invariant under both SUSY and gauge transformations

$$\mathcal{L}_\Phi = \left[\Phi^*(e^{2gV})\Phi\right]_D . \tag{2.45}$$

Writing it in terms of component fields one obtains

$$\begin{aligned}\mathcal{L}_\Phi(x) = \mathcal{D}_\mu\phi^{*i}\mathcal{D}^\mu\phi_i + \frac{i}{2}\psi^{\dagger i}\overline{\sigma}^\mu(\mathcal{D}_\mu\psi)_i + \frac{i}{2}\psi_i\sigma^\mu(\mathcal{D}_\mu\psi^\dagger)_i + F^{*i}F_i \\ - \sqrt{2}g(\phi^* T^a\psi)\lambda^a - \sqrt{2}g\lambda^{a\dagger}(\psi^\dagger T^a\phi) + g(\phi^* T^a\phi)D^a \,,\end{aligned} \tag{2.46}$$

with the gauge covariant derivatives

$$\mathcal{D}_\mu\phi_i = \partial_\mu\phi_i + igA^a_\mu(T^a\phi)_i \,, \qquad \mathcal{D}_\mu\psi_i = \partial_\mu\psi_i + igA^a_\mu(T^a\psi)_i . \tag{2.47}$$

This is the gauge invariant version of Eq. (2.40), which now contains not only the kinetic terms but also gauge interactions. Now it is necessary to make kinetic terms and self-interactions for the vector supermultiplets, to this end we define a chiral *field-strength superfield*

$$\mathcal{W}_\alpha \;=\; -\frac{1}{8g}D^\dagger D^\dagger\left(e^{-2gV}D_\alpha e^{2gV}\right) , \tag{2.48}$$

that is a chiral superfield since it is constructed with two antichiral covariant derivatives. The super field strength can be shown to have the following explicit form in the Wess-Zumino gauge, once one plugs in the explicit expression (2.35) for the gauge field V

$$(\mathcal{W}_\alpha)_{\text{WZ gauge}} = \lambda_\alpha + \theta_\alpha D + \frac{i}{2}(\sigma^\mu\overline{\sigma}^\nu\theta)_\alpha F_{\mu\nu} + i\theta\theta(\sigma^\mu\mathcal{D}_\mu\lambda^\dagger)_\alpha \,, \tag{2.49}$$

with the gauge field strength defined as

$$F_{\mu\nu} = T^a F^a_{\mu\nu}\,, \qquad F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - g f^{abc} A^b_\mu A^c_\nu\,, \tag{2.50}$$

and the gauge covariant derivative in the adjoint representation

$$\mathcal{D}_\mu \lambda^\dagger = T^a (\mathcal{D}_\mu \lambda^\dagger)^a\,, \qquad (\mathcal{D}_\mu \lambda^\dagger)^a = \partial_\mu \lambda^{\dagger a} + g f^{abc} A^b_\mu \lambda^c\,. \tag{2.51}$$

One can easily prove that the super field strength has the following transformation properties under gauge transformations

$$\mathcal{W}_\alpha \to \mathcal{W}'_\alpha = e^{ig\Lambda}\,\mathcal{W}_\alpha\, e^{-ig\Lambda}\,. \tag{2.52}$$

Then the following term for the Lagrangian

$$\mathcal{L}_V(x) = \frac{1}{2}(\mathrm{Tr}\,[\mathcal{W}^\alpha \mathcal{W}_\alpha]_F + \text{h.c.}\,)\,, \tag{2.53}$$

is automatically invariant under both supersymmetry and supergauge transformations. The explicit form of Eq. (2.53) reads

$$\mathcal{L}_V(x) = \frac{i}{2}\lambda^a \sigma^\mu \left(\mathcal{D}_\mu \lambda^\dagger\right)^a + \frac{i}{2}\lambda^{\dagger a}\overline{\sigma}^\mu (\mathcal{D}_\mu \lambda)^a + \frac{1}{2}D^a D^a - \frac{1}{4}F^{a\mu\nu}F^a_{\mu\nu}\,, \tag{2.54}$$

which contains the needed kinetic terms for both the gauge bosons and the gauginos. Now we have all ingredients to construct a Lagrangian for a supersymmetric gauge theory. We have to add up (2.53), (2.45) and a gauge invariant superpotential term (2.36) to get the following most general Lagrangian containing a multiplet of chiral superfields Φ and a super gauge field V, that is invariant under both global SUSY and local gauge transformations.

$$\mathcal{L} = \mathcal{L}_\Phi + \mathcal{L}_W + \mathcal{L}_V = \left[\Phi^*\, e^{2gV}\,\Phi\right]_D + ([W(\Phi)]_F + \text{h.c.}\,) + \frac{1}{2}(\mathrm{Tr}\,[\mathcal{W}^\alpha \mathcal{W}_\alpha]_F + \text{h.c.}\,) \tag{2.55}$$

Looking at the explicit expressions of the different terms (2.46), (2.38) and (2.54), one sees that they still involve the auxiliary fields F and D. But these fields have dimension two and no derivatives of them appear in the Lagrangian. This means they are no dynamical degrees of freedom and they can be eliminated completely from the Lagrangian by applying the corresponding *equations of motion*, that read

$$F^{*i} = -\frac{\delta W}{\delta \phi^i} = -\left(M^{ij}\phi_j + \frac{1}{2}y^{ijk}\phi_j\phi_k\right) \tag{2.56}$$

$$D^a = -g(\phi^* T^a \phi) \tag{2.57}$$

Finally we obtain the following explicit result for the Lagrangian

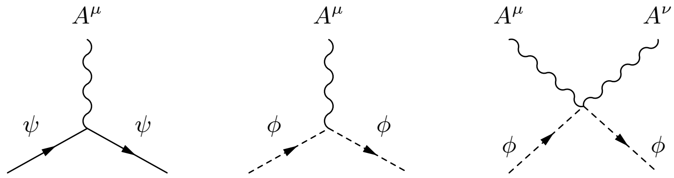
$$\begin{aligned}
\mathcal{L} = {} & \mathcal{D}_\mu \phi^{*i}\mathcal{D}^\mu \phi_i + \frac{i}{2}\psi^{\dagger i}\overline{\sigma}^\mu (\mathcal{D}_\mu \psi)_i + \frac{i}{2}\psi_i \sigma^\mu (\mathcal{D}_\mu \psi^\dagger)_i - \frac{1}{2}M^{ij}(\psi_i\psi_j + \text{h.c.}\,) \\
& - \frac{1}{4}F^{a\mu\nu}F^a_{\mu\nu} + \frac{i}{2}\lambda^a\sigma^\mu\left(\mathcal{D}_\mu\lambda^\dagger\right)^a + \frac{i}{2}\lambda^{\dagger a}\overline{\sigma}^\mu(\mathcal{D}_\mu\lambda)^a \\
& - \sqrt{2}g(\phi^* T^a \psi)\lambda^a - \sqrt{2}g\lambda^{a\dagger}(\psi^\dagger T^a \phi) \\
& - \frac{1}{2}y^{ijk}(\phi_i\psi_j\psi_k + \text{h.c.}\,) - \mathcal{V}(\phi,\phi^*)
\end{aligned} \tag{2.58}$$

where the scalar potential is the following sum of positive terms

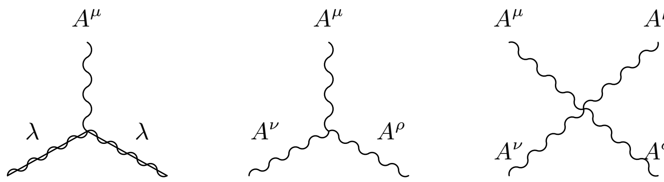
$$\mathcal{V}(\phi,\phi^*) = F^{*i}F_i + \frac{1}{2}\sum_a D^a D^a = \sum_i \left|\frac{\delta W}{\delta\phi^i}\right|^2 + \frac{1}{2}\sum_a g^2(\phi^* T^a \phi)^2. \tag{2.59}$$

The Lagrangian in eq. (2.58) contains the following terms:

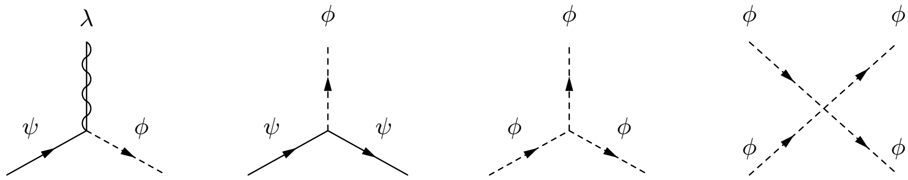
- In the first line we have kinetic terms for the fermions and the sfermions as well as their gauge interactions with the gauge bosons, contained in the covariant derivatives (the diagrams corresponding to these interactions have been depicted in Fig. 2.2a). In addition, there are also explicit mass terms for the fermions coming from the superpotential. These terms are of course only admissible if they respect gauge invariance. (They will particularly be absent for the SM fermions in the MSSM.)
- In the second line there are the kinetic terms for the gauge bosons and the gauginos and their corresponding gauge interactions (Fig. 2.2b).
- The third line contains the *supersymmetrized version* of the gauge interaction, namely an interaction between the fermions, the sfermions and the gauginos (the first diagram in Fig. 2.2c). It arises from the term (2.45) and guarantees the SUSY invariance of the full Lagrangian.
- Finally in the fourth line we find the potential for the sfermions and Yukawa couplings between the fermions and sfermions (the last three diagrams in Fig. 2.2c). The constants that appear in the terms coming from the superpotential cannot be arbitrary, but have to be chosen such that gauge symmetry is preserved.



(a) Gauge interactions of the fermions and sfermions.



(b) Interactions of the gauginos and the gauge bosons.



(c) Fermion-gaugino-sfermion interaction, Yukawa couplings of fermions with sfermions, and sfermion self interactions.

Figure 2.2: SUSY interactions

2.6 Soft supersymmetry breaking

Supersymmetric theories have special properties with respect to quantum corrections, which go under the name of non-renormalization theorems [17]. In particular, it can be shown that renormalizable theories with exact supersymmetry are free of quadratic divergences, and therefore potential candidates to address the naturalness problem of the scalar mass terms. However, looking back at the complete SUSY algebra (2.3) and (2.6) it can be shown that, as in the case of pure Poincaré symmetry, P^2 is still a Casimir

operator. Thus, if supersymmetry is exact, all particles of an irreducible representation must have the same mass. In our case this means that the fields ψ and ϕ as well as A^μ and λ have the same mass. But this is something which is obviously not realized in nature. Not a single supersymmetric partner of a Standard Model particle has been observed yet, which means that supersymmetry has to be broken.
There is a vast literature on the problem of spontaneous supersymmetry breaking in realistic supersymmetric extensions of the SM, which typically involves complicated *hidden sectors* and often the coupling to gravity. In the rest of this thesis we will follow a simpler phenomenological approach, introducing into the Lagrangian terms that break explicitly supersymmetry, but nevertheless do not reintroduce field-dependent quadratic divergences that would make scalar masses unnatural. These new terms are called *soft-breaking* terms.

$$\mathcal{L}_{\text{soft}} = -\left(\frac{1}{2}M_a\,\lambda^a\lambda^a + \frac{1}{6}a^{ijk}\phi_i\phi_j\phi_k + \frac{1}{2}b^{ij}\phi_i\phi_j + t^i\phi_i\right) + \text{h.c.} - (m^2)^i_j\phi^{j*}\phi_i \qquad (2.60)$$

Of course, the (in general complex) parameters M_a, a^{ijk}, b^{ij}, t^i and $(m^2)^i_j$ have to be such that gauge invariance is not violated. As expression (2.60) contains only sfermions and gauginos, but not their partners the fermions and gauge bosons, it is obvious that the introduction of $\mathcal{L}_{\text{soft}}$ explicitly breaks SUSY. In fact $\mathcal{L}_{\text{soft}}$ provides additional mass terms for the sfermions and gauginos, which can make these particles heavy enough to be unobserved until now.

2.7 The MSSM Lagrangian

With the supersymmetric Lagrangian for gauge theories (2.58) and the soft SUSY breaking Lagrangian (2.60) we are ready to construct the MSSM. The only thing we have to do is to specify the gauge group and the particle content and to write down the most general supersymmetric Lagrangian that can be built out of these particles. As suggested by its name, the Minimal Supersymmetric extension of the Standard Model (MSSM) make use of the lowest possible number of superparticles and new interactions, and can reproduce the SM when appropriate limits are taken. In the MSSM we take the SM gauge group

$$\mathcal{G} = SU(3) \times SU(2) \times U(1)_Y\,. \qquad (2.61)$$

This completely specifies the content of the model in terms of vector superfields. There are three vector superfields V_1, V_2 and V_3, which contain the following particles

$SU(3)$: The super gauge field for the strong interaction is V_3. It contains the *gluons* g^A_μ and their superpartners, the *gluinos* $\widetilde{g}^A$. The corresponding gauge coupling is g_3.

$SU(2)$: The super gauge field for the weak isospin is called V_2. The corresponding particles are the *vector bosons* W^i_μ and the *winos* $\widetilde{W}^i$. Their gauge coupling is g_2.

$U(1)_Y$: For the hypercharge there is the supergauge field V_1, which contains the *vector boson* B_μ and the *bino* $\widetilde{B}^0$. The corresponding gauge coupling is called g_1.

Here $A = 1,\ldots,8$ and $i = 1,2,3$ are indices in the adjoint representation of the corresponding symmetry group. The next step in the MSSM construction is the identification of the chiral multiplets and of their gauge transformation properties. The MSSM chiral superfields and the corresponding gauge quantum numbers are listed in Table 2.2, where we have written only one lepton and quark generation. The supersymmetric partners of

Names	spin 1/2	spin 1	$SU(3)$, $SU(2)$, $U(1)_Y$
gluino, gluon	$\widetilde{g}$	g_μ	$(\mathbf{8}, \mathbf{1}, 0)$
winos, W bosons	$\widetilde{W}^\pm \quad \widetilde{W}^0$	$W^\pm_\mu \; W^0_\mu$	$(\mathbf{1}, \mathbf{3}, 0)$
bino, B boson	$\widetilde{B}^0$	B^0_μ	$(\mathbf{1}, \mathbf{1}, 0)$

Table 2.1: Gauge supermultiplets in the Minimal Supersymmetric Standard Model.

Names		spin 0	spin 1/2	$SU(3)_C$, $SU(2)_L$, $U(1)_Y$
squarks, quarks	Q	$(\widetilde{u}_L \;\; \widetilde{d}_L)$	$(u_L \;\; d_L)$	$(\mathbf{3}, \mathbf{2}, +\frac{1}{6})$
($\times$3 families)	$\overline{u}$	$\widetilde{u}^*_R$	$u^\dagger_R$	$(\overline{\mathbf{3}}, \mathbf{1}, -\frac{2}{3})$
	$\overline{d}$	$\widetilde{d}^*_R$	$d^\dagger_R$	$(\overline{\mathbf{3}}, \mathbf{1}, +\frac{1}{3})$
sleptons, leptons	L	$(\widetilde{\nu} \;\; \widetilde{e}_L)$	$(\nu \;\; e_L)$	$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$
($\times$3 families)	$\overline{e}$	$\widetilde{e}^*_R$	$e^\dagger_R$	$(\mathbf{1}, \mathbf{1}, +1)$
Higgs, higgsinos	H_u	$(H_u^+ \;\; H_u^0)$	$(\widetilde{H}_u^+ \;\; \widetilde{H}_u^0)$	$(\mathbf{1}, \mathbf{2}, +\frac{1}{2})$
	H_d	$(H_d^0 \;\; H_d^-)$	$(\widetilde{H}_d^0 \;\; \widetilde{H}_d^-)$	$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$

Table 2.2: Chiral supermultiplets in the Minimal Supersymmetric Standard Model. The spin-0 fields are complex scalars, and the spin-1/2 fields are left-handed two-component Weyl fermions.

quarks and leptons, described by one complex scalar field for each chiral fermion, are called *squarks* and *sleptons*.

The construction of the Higgs sector of the MSSM is slightly more involved, as it turns out that one needs at least two Higgs doublets. We recall that in the SM a complex field H and its charge conjugate H^c, which has the same $SU(2)$ transformation but opposite hypercharge, are needed to give mass both to the down-type and to the up-type quarks when the neutral component of the Higgs acquire a VEV. This cannot be reproduced in the MSSM, since the superpotential has to be a holomorphic function of the superfields. Therefore one is forced to explicitly introduce a second Higgs doublet, that is responsible for the mass terms of the up quarks. Furthermore, two Higgs doublets are needed if we want to construct a supersymmetric version of the SM free from chiral anomalies. In fact, the superpartner of the Higgs boson is a fermion (not present in the SM) which contributes to triangle gauge anomalies. Since SM fermions already cancel the gauge anomalies by themselves, the addition of another fermion charged under $SU(2) \times U(1)_Y$ introduces an uncompensated contribution. A second fermion that is the vector complement of the first cancel the anomalies.

Now that we defined the field content of the MSSM, we are ready to write the complete MSSM Lagrangian

$$\mathcal{L}_{\text{MSSM}} = \mathcal{L}_\Phi + \mathcal{L}_V + \mathcal{L}_W + \mathcal{L}_{\text{soft}} \,. \tag{2.62}$$

The first three terms are obtained as in the general example (2.55), we need only to write down the explicit expression for the superpotential and the soft breaking terms. The superpotential for the MSSM reads

$$W_{\text{MSSM}} = \overline{u} Y_u Q H_u - \overline{d} Y_d Q H_d - \overline{e} Y_e L H_d + \mu H_u H_d \,. \tag{2.63}$$

The dimensionless Yukawa coupling parameters Y_u, Y_d, Y_e are 3×3 matrices in family space. All of the gauge and family indices in Eq. (2.63) are suppressed. For example the first term $\overline{u}Y_uQH_u$ stands for $\overline{u}^{aI}\,(Y_u)^{IJ}\,Q^J_{\alpha a}\,(H_u)_\beta\,\epsilon^{\alpha\beta}$, where $I, J = 1, 2, 3$ are flavour indices, and $a = 1, 2, 3$ is a colour index which is lowered (raised) in the $\mathbf{3}$ ($\overline{\mathbf{3}}$) representation of $SU(3)_C$.

A priori, on the basis of gauge invariance and renormalizability requirements, in the superpotential (2.63) we could also have written terms of the form

$$W_{\Delta\mathrm{L}=1} = \frac{1}{2}\lambda^{ijk}L_iL_j\overline{e}_k + \lambda'^{ijk}L_iQ_j\overline{d}_k + \mu'^i L_iH_u \tag{2.64}$$

$$W_{\Delta\mathrm{B}=1} = \frac{1}{2}\lambda''^{ijk}\overline{u}_i\overline{d}_j\overline{d}_k \tag{2.65}$$

These operators violate respectively the lepton (L) and baryon (B) number. We recall that in the SM the conservation of these quantum number is an accidental symmetry of the renormalizable interactions, i.e. is automatically present in the action, once gauge and Poincaré symmetries are imposed. The existence of the operators (2.64) and (2.65) in the MSSM Lagrangian is somehow disturbing, since the corresponding L and B violating processes have not been experimentally observed.

In general, the presence of such terms can be forbidden by requiring the preservation of an additional global symmetry, the so called *R-parity*

$$R = (-1)^{L+3B+2S} \tag{2.66}$$

with B the baryon number, L the lepton number and S the spin of a particle. We observe that all SM particles have R-parity $+1$, while their SUSY partners all have R-parity -1. The symmetry principle to be enforced is that a term in the Lagrangian is allowed only if the product of R for all the fields in it yields $+1$. Hence, as long as R-parity is exactly conserved, supersymmetric particles are always produced in pairs. R-parity conservation has another important consequence for Dark Matter Physics, since it provides a natural particle candidate for explaining the Dark Matter: the lightest SUSY particle (LSP) that, due R-parity, is stable. More specifically, the lightest neutralino is usually the most popular candidate for (Cold) Dark Matter in the MSSM and other SUSY models.

Finally we quote the soft SUSY breaking terms

$$\begin{aligned}
\mathcal{L}^{\mathrm{MSSM}}_{\mathrm{soft}} &= -\frac{1}{2}\Big(M_3\widetilde{g}\widetilde{g} + M_2\widetilde{W}\widetilde{W} + M_1\widetilde{B}\widetilde{B} + \text{h.c.}\Big)\\
&\quad + \Big(\widetilde{u}^*_R\,A_u\,\widetilde{Q}H_u - \widetilde{d}^*_R\,A_d\,\widetilde{Q}H_d - \widetilde{e}^*_R\,A_e\,\widetilde{L}H_d + \text{h.c.}\Big)\\
&\quad - \widetilde{Q}^\dagger\,(\widetilde{m}^2_Q)\,\widetilde{Q} - \widetilde{L}^\dagger\,(\widetilde{m}^2_L)\,\widetilde{L} - \widetilde{u}^*_R\,(\widetilde{m}^2_u)\,\widetilde{u}_R - \widetilde{d}^*_R\,(\widetilde{m}^2_d)\,\widetilde{d}_R - \widetilde{e}^*_R\,(\widetilde{m}^2_e)\,\widetilde{e}_R\\
&\quad - m^2_{H_u}H^*_uH_u - m^2_{H_d}H^*_dH_d - (bH_uH_d + \text{h.c.})\,.
\end{aligned} \tag{2.67}$$

In eq. (2.67), M_3, M_2, and M_1 are the gluino, wino, and bino mass terms. Here, and from now on, we suppress the adjoint representation gauge indices on the wino and gluino fields, and the gauge indices on all of the chiral supermultiplet fields. Each of A_u, A_d, A_e is a complex 3×3 matrix in family space, with dimensions of [mass]. They are in one-to-one correspondence with the Yukawa couplings of the superpotential. Each of $\widetilde{m}^2_Q$, $\widetilde{m}^2_u$, $\widetilde{m}^2_d$, $\widetilde{m}^2_L$, $\widetilde{m}^2_e$ is a 3×3 matrix in family space that can have complex entries, but they must be hermitian so that the Lagrangian is real. Finally, in the last line of eq. (2.67) we have supersymmetry-breaking contributions to the Higgs potential.

Unlike the SUSY-preserving part of the Lagrangian, Eq. (2.67) introduces $\mathcal{O}(100)$ new parameters in the theory that cannot be rotated away with field redefinitions. Fortunately, there is already good experimental evidence that some powerful organizing principle must govern the soft supersymmetry breaking Lagrangian. This is because most of the new parameters imply flavour mixing or CP violating processes of the types that are severely restricted by experiment [16]. For example the squared soft matrix $\widetilde{m}_e^2$ is constrained by experimental bounds on the process $\mu \to e\gamma$, and there are also important experimental constraints on the squark squared-mass matrices, the strongest of these coming from the neutral kaon system.

2.8 Minimization of the Higgs potential

In the MSSM, the description of electroweak symmetry breaking is slightly complicated by the fact that there are two complex Higgs doublets, $H_u = (H_u^+, H_u^0)$ and $H_d = (H_d^0, H_d^-)$ rather than just one in the ordinary Standard Model. Making the self-consistent assumptions that the charged components of the Higgs doublets are zero at the minimum of the potential $\langle H_u^+ \rangle = 0$ and $\langle H_d^- \rangle = 0$, we are left to consider the following scalar potential for the Higgs fields

$$\begin{aligned} V_H = &(|\mu|^2 + m_{H_u}^2)|H_u^0|^2 + (|\mu|^2 + m_{H_d}^2)|H_d^0|^2 - (b\, H_u^0 H_d^0 + \text{h.c.}) \\ &+ \frac{1}{8}(g^2 + g'^2)\big(|H_u^0|^2 - |H_d^0|^2\big)^2 , \end{aligned} \tag{2.68}$$

where it is not restrictive to assume that b is real and positive. One remarkable feature of the Higgs potential in the MSSM is that the Higgs self-couplings are given in terms of the electroweak gauge couplings, in contrast to the SM potential in Eq. (1.5), where the self-couplings were given by the unknown parameter λ_h.

By requiring that V_H is bounded from below, the quadratic part of the potential must be positive along the so-called *D-flat* direction[2] $|H_u^0| = |H_d^0|$, leading to the condition

$$2b < 2|\mu|^2 + m_{H_u}^2 + m_{H_d}^2 . \tag{2.69}$$

Note that the b-term always favors electroweak symmetry breaking. Requiring that one linear combination of H_u^0 and H_d^0 has a negative squared mass near $H_u^0 = H_d^0 = 0$ gives

$$b^2 > (|\mu|^2 + m_{H_u}^2)(|\mu|^2 + m_{H_d}^2) \tag{2.70}$$

If this inequality is not satisfied, then $H_u^0 = H_d^0 = 0$ will be a stable minimum of the potential (or there will be no stable minimum at all), and electroweak symmetry breaking will not occur. The VEVs of the Higgs fields are commonly chosen as

$$\langle H_u \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_u \end{pmatrix} , \qquad \langle H_d \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} v_d \\ 0 \end{pmatrix} , \tag{2.71}$$

and are related to the known mass of the Z^0 boson and the electroweak gauge couplings

$$v_u^2 + v_d^2 = v^2 = 4m_Z^2/(g^2 + g'^2) \approx (246\ \text{GeV})^2 . \tag{2.72}$$

The ratio of the VEVs is traditionally written as $\tan\beta \equiv v_u/v_d$. The conditions $\partial V/\partial H_u^0 = \partial V/\partial H_d^0 = 0$ under which the potential eq. (2.68) will have a minimum now read

$$m_{H_u}^2 + |\mu|^2 - b\cot\beta - (m_Z^2/2)\cos(2\beta) \;=\; 0 , \tag{2.73}$$

$$m_{H_d}^2 + |\mu|^2 - b\tan\beta + (m_Z^2/2)\cos(2\beta) \;=\; 0 . \tag{2.74}$$

[2] *D*-flat directions owe their name to the fact that along them the part of the scalar potential coming from *D*-terms vanishes.

These conditions allow us to eliminate two of the Lagrangian parameters b and $|\mu|$ in favor of $\tan\beta$. Taking $|\mu|^2$, b, $m^2_{H_u}$ and $m^2_{H_d}$ as input parameters, and m^2_Z and $\tan\beta$ as output parameters obtained by solving these two equations, one obtains the tree-level relations

$$\sin(2\beta) = \frac{2b}{m^2_{H_u} + m^2_{H_d} + 2|\mu|^2}\,, \tag{2.75}$$

$$m^2_Z = \frac{|m^2_{H_d} - m^2_{H_u}|}{\sqrt{1-\sin^2(2\beta)}} - m^2_{H_u} - m^2_{H_d} - 2|\mu|^2\,. \tag{2.76}$$

2.9 Physical spectrum of the MSSM

So far we considered fields in the gauge eigenstate basis. We want now to switch to the mass eigenstate basis in order to get the physical spectrum of the theory. The electroweak symmetry breaking procedure outlined in the previous section leads to the usual mass terms for the quarks, leptons and electroweak gauge bosons. On the other hand also many additional mass terms for Higgs particles, squarks, sleptons, gauginos and higgsinos are introduced. We now briefly explain how these masses are diagonalized in order to obtain the physical particle content.

Particle			**Lorentz type**
Photon	γ		vector
Weak gauge bosons	$W^\pm$, Z		vectors
Gluons	g^A	$A = 1,\ldots,8$	vectors
Charged Higgs	$H^\pm$		scalars
Scalar Higgs	h^0, H^0		scalars
Pseudoscalar Higgs	A^0		scalar
Charginos	χ_i	$i = 1,2$	Dirac spinors
Neutralinos	χ^0_i	$i = 1,\ldots,4$	Majorana spinors
Gluinos	$\widetilde{g}^A$	$A = 1,\ldots,8$	Majorana spinors
Neutrinos	ν^I	$I = 1,2,3$	
Sneutrinos	$\widetilde{\nu}^I$	$I = 1,2,3$	scalars
Electrons	e^I	$I = 1,2,3$	Dirac spinors
Selectrons	$\widetilde{L}_i$	$i = 1,\ldots,6$	scalars
Quarks	u^I, d^I	$I = 1,2,3$	Dirac spinors
Squarks	$\widetilde{U}_i$, $\widetilde{D}_i$	$i = 1,\ldots,6$	scalars

Table 2.3: The MSSM particle spectrum.

During the diagonalization procedure one usually also switches to a notation with four component spinors, that is more suitable when one wants to derive Feynman rules in the end. The so obtained particle spectrum of the MSSM is summarized in Tab. 2.3.

The Higgs sector

The Higgs scalar fields in the MSSM consist of two complex $SU(2)_L$-doublets, or eight real, scalar degrees of freedom. When the electroweak symmetry is broken, three of them are the would-be Nambu-Goldstone bosons G^0, $G^\pm$, which become the longitudinal modes of the Z^0 and $W^\pm$ massive vector bosons. The remaining five mass eigenstates are

- two CP-even neutral scalars h^0 and H^0,
- one CP-odd neutral scalar A^0,
- two charged scalars H^+ and its conjugate $H^- = (H^+)^*$.

By convention, h^0 is chosen to be lighter than H^0. The gauge eigenstate fields can be expressed in terms of the mass eigenstate fields in the following way

$$\begin{pmatrix} H_u^0 \\ H_d^0 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} v_u \\ v_d \end{pmatrix} + \frac{1}{\sqrt{2}} R_\alpha \begin{pmatrix} h^0 \\ H^0 \end{pmatrix} + \frac{1}{\sqrt{2}} R_\beta \begin{pmatrix} G^0 \\ A^0 \end{pmatrix}, \qquad \begin{pmatrix} H_u^+ \\ H_d^{-*} \end{pmatrix} = R_\beta \begin{pmatrix} G^+ \\ H^+ \end{pmatrix}, \tag{2.77}$$

with the orthogonal rotation matrices defined as

$$R_\alpha = \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix}, \qquad R_\beta = \begin{pmatrix} \sin\beta & \cos\beta \\ -\cos\beta & \sin\beta \end{pmatrix}. \tag{2.78}$$

The tree-level masses are then the eigenvalues

$$m_{A^0}^2 = 2b/\sin(2\beta) \;=\; 2|\mu|^2 + m_{H_u}^2 + m_{H_d}^2 , \tag{2.79}$$

$$m_{h^0,H^0}^2 = \frac{1}{2}\Big(m_{A^0}^2 + m_Z^2 \mp \sqrt{(m_{A^0}^2 - m_Z^2)^2 + 4 m_Z^2 m_{A^0}^2 \sin^2(2\beta)}\Big), \tag{2.80}$$

$$m_{H^\pm}^2 = m_{A^0}^2 + m_W^2 . \tag{2.81}$$

and $m_{G^0}^2 = m_{G^\pm}^2 = 0$. The mixing angle α is determined, at tree-level, by

$$\frac{\sin 2\alpha}{\sin 2\beta} = -\left(\frac{m_{H^0}^2 + m_{h^0}^2}{m_{H^0}^2 - m_{h^0}^2}\right), \qquad \frac{\tan 2\alpha}{\tan 2\beta} = \left(\frac{m_{A^0}^2 + m_Z^2}{m_{A^0}^2 - m_Z^2}\right), \qquad \Big(-\frac{\pi}{2} \leqslant \alpha \leqslant 0\Big) \tag{2.82}$$

The masses of A^0, H^0 and $H^\pm$ can in principle be arbitrarily large since they all grow with $b/\sin(2\beta)$. In contrast, the mass of h^0 is bounded above. From eq. (2.80), one finds at tree-level

$$m_{h^0} \;<\; m_Z |\cos(2\beta)| . \tag{2.83}$$

However, this tree-level expression for the Higgs mass is subject to quantum corrections that can have relatively drastic effects [22, 23, 24]. The qualitative behaviour of these radiative corrections can be most easily seen in the large top-squark mass limit, where both the splitting of the two diagonal entries and the off-diagonal entries of the top-squark squared-mass matrix are small in comparison to the geometrical mean of the stop mass eigenvalues, $M_{\rm S}^2 = m_{\tilde{t}_1} m_{\tilde{t}_2}$. In this case, the predicted upper bound for m_h, in the limit $m_A^2 \gg m_Z^2$, is approximately given by

$$m_h^2 \lesssim m_Z^2 \cos^2 2\beta + \frac{3 G_F}{\sqrt{2}\pi^2} m_t^4 \left[\log \frac{M_{\rm S}^2}{m_t^2} + \frac{X_t^2}{M_{\rm S}^2}\left(1 - \frac{X_t^2}{12 M_{\rm S}^2}\right)\right], \tag{2.84}$$

where $X_t = A_t - \mu \cot\beta$ with A_t the stop mixing parameter. The upper limit for m_h is obtained for large $\tan\beta$ and $X_t = \sqrt{6} M_{\rm S}$. This value of X_t defines the so-called *maximal mixing* scenario.

A particle very similar to the SM Higgs boson has been discovered at the LHC and its mass has already been measured very precisely by CMS and ATLAS collaborations [25, 26]. The combined value for the mass is [27]

$$M_h = 125.66 \pm 0.34\,\text{GeV}, \tag{2.85}$$

this result is indeed in the allowed range for the loop-corrected MSSM lightest Higgs. On the other hand the measured mass is close to its upper limit. Therefore large loop corrections are needed and, since these are governed by the stop mass, it turns out that heavy stops are needed. This constitutes a problem for naturalness, since heavy stops contribute to worsen the fine-tuning situation of the electroweak scale.

Tree level couplings of the scalar Higgs fields An interesting feature of the MSSM is that the the tree-level couplings of the neutral Higgs bosons are equal to the SM couplings times a factor which can be computed in terms of the mixing angles α and β, as one can see plugging the expansion (2.77) for the neutral Higgs scalars in the Yukawa terms. As an example, the term containing the top quark couplings reads

$$\mathcal{L}_{\text{Yukawa},\,tt} = -\frac{m_t}{v} t_R^\dagger t_L \left[\frac{\cos\alpha}{\sin\beta}\, h^0 + \frac{\sin\alpha}{\sin\beta}\, H^0 + i\cot\beta\, A^0 + \ldots\right] + \text{ h.c.}\,, \tag{2.86}$$

from which we can directly see the multiplicative factors. These tree-level factors for the coupling of the neutral Higgs bosons with the vector bosons and fermions are collected in Table 2.4.

$$\begin{aligned}
g_{hVV} &= \sin(\beta-\alpha)\, g^{\text{SM}}_{hVV} \\
g_{HVV} &= \cos(\beta-\alpha)\, g^{\text{SM}}_{hVV} \\
g_{hAZ} &= \sin(\beta-\alpha)\frac{g'}{2\cos\theta_W} \\
g_{h\bar{b}b}\,, g_{h\tau^+\tau^-} &= -\frac{\sin\alpha}{\cos\beta}\, g^{\text{SM}}_{h\bar{b}b,\,h\tau^+\tau^-} \\
g_{H\bar{b}b}\,, g_{H\tau^+\tau^-} &= \frac{\cos\alpha}{\cos\beta}\, g^{\text{SM}}_{h\bar{b}b,\,h\tau^+\tau^-} \\
g_{h\bar{t}t} &= \frac{\cos\alpha}{\sin\beta}\, g^{\text{SM}}_{h\bar{t}t} \\
g_{H\bar{t}t} &= \frac{\sin\alpha}{\sin\beta}\, g^{\text{SM}}_{H\bar{t}t} \\
g_{A\bar{b}b}\,, g_{A\tau^+\tau^-} &= -i\gamma_5 \tan\beta\, g^{\text{SM}}_{h\bar{b}b,\,h\tau^+\tau^-} \\
g_{A\bar{t}t} &= -i\gamma_5 \cot\beta\, g^{\text{SM}}_{h\bar{t}t}
\end{aligned}$$

Table 2.4: Tree-level neutral Higgs boson couplings with vector-boson ($V = W^\pm\,, Z$) and fermion pairs expressed in terms of the corresponding couplings of the SM Higgs boson. Fermions are here described as four-component Dirac spinors which combine the left and right Weyl spinors.

An interesting case is the so-called *decoupling limit*: when $m_{A^0} \gg m_Z$ the lightest Higgs scalar h^0 behaves in a way nearly indistinguishable from the SM Higgs boson. In such limit the mixing angle α is $\alpha \sim (\beta - \pi/2)$ and h^0 has the same couplings to quarks and leptons and electroweak gauge bosons as would the physical Higgs boson of the ordinary Standard Model without supersymmetry. In the SM the tree level couplings of the Higgs are in proportion to masses and, as a consequence, are very hierarchical. The combined results from ATLAS and CMS show that the measured Higgs couplings are in reasonable agreement (at about a 20% accuracy) with the sharp predictions of the SM (see Fig. 2.3), hence reducing the parameter space of the MSSM.

Gauge bosons, leptons and quarks

The electroweak symmetry breaking of the MSSM, as outlined in Sec. 2.8, leaves the eight gluons and the photon massless, while the $W^\pm_\mu$ and Z_μ gauge bosons become massive.

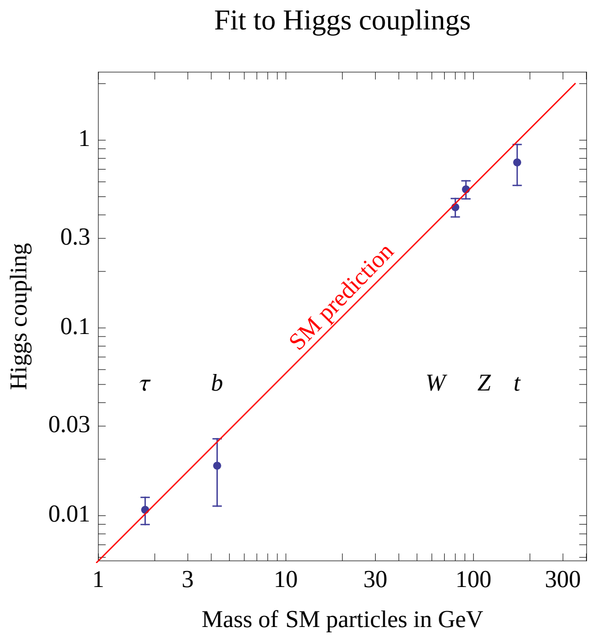


Figure 2.3: The predicted couplings of the SM Higgs compared with the ATLAS and CMS data as combined in [27].

Their tree level masses are

$$m_W = \frac{e}{2s_W}(v_u^2 + v_d^2)^{\frac{1}{2}}\,, \qquad m_Z = \frac{e}{2s_W c_W}(v_u^2 + v_d^2)^{\frac{1}{2}}\,. \tag{2.87}$$

As concerns the quarks and the leptons, the situation is similar to the SM, with the masses arising form the the Yukawa couplings in the superpotential (2.63). The only difference is that we now have two Higgs doublets H_u and H_d (instead of just one) which acquire non-vanishing VEVs and separately provide mass terms for the up quarks and for the down quarks and the leptons respectively. One can then proceed in the diagonalization of the Yukawa matrices Y_e, Y_u, Y_d in order to obtain the mass eigenstate basis. The following simultaneous rotations of quark and squark fields define the so-called *super-CKM basis* [28]

$$\begin{aligned} {Q_1}^I &\to V_{UL}^{IJ}\, {Q_1}^J\,, & \overline{u}^I &\to V_{UR}^{IJ}\, \overline{u}^J\,, \\ {Q_2}^I &\to V_{DL}^{IJ}\, {Q_2}^J\,, & \overline{d}^I &\to V_{DR}^{IJ}\, \overline{d}^J\,, \\ L^I &\to V_{EL}^{IJ}\, L^J\,, & \overline{e}^I &\to V_{ER}^{IJ}\, \overline{e}^J\,. \end{aligned} \tag{2.88}$$

Here the unitary matrices V_{UL}, V_{UR}, V_{DL}, V_{DR}, V_{EL} and V_{ER} are chosen so that they satisfy the following relations

$$\begin{aligned} y_u^I &= (V_{UR}^*)^{IJ}\, Y_u^{JK}\, (V_{UL})^{KM} = \mathrm{diag}(y_u, y_c, y_t)\,, \\ y_d^I &= (V_{DR}^*)^{IJ}\, Y_d^{JK}\, (V_{DL})^{KM} = \mathrm{diag}(y_d, y_s, y_b)\,, \\ y_e^I &= (V_{ER}^*)^{IJ}\, Y_e^{JK}\, (V_{EL})^{KM} = \mathrm{diag}(y_e, y_\mu, y_\tau)\,. \end{aligned} \tag{2.89}$$

Note the fact that the up and down components of the doublet Q_α are transformed independently, therefore this transformation does not belong to the flavour symmetry group G_F and the CKM matrix $V = V_{UL}^\dagger V_{DL}$ will appear in a number of interaction vertexes. In this basis the masses for leptons and quarks are

$$m_\nu^I = 0\,, \qquad m_e^I = \frac{v\cos\beta}{\sqrt{2}} y_e^I\,, \qquad m_u^I = \frac{v\sin\beta}{\sqrt{2}} y_u^I\,, \qquad m_d^I = \frac{v\cos\beta}{\sqrt{2}} y_d^I\,, \tag{2.90}$$

We see that the MSSM neutrinos are predicted to be massless. One can implement a supersymmetric version of the see-saw mechanism of Sec. 1.5, see for example [5, 6].

Charginos

Four 2-component spinors, namely the charged higgsinos ($\widetilde{H}_u^+$ and $\widetilde{H}_d^-$) and winos ($\widetilde{W}^+$ and $\widetilde{W}^-$), combine to give two 4-component Dirac fermions χ_1, χ_2 corresponding to two physical *charginos*. In the gauge-eigenstate basis $\psi^\pm = (\widetilde{W}^+, \widetilde{H}_u^+, \widetilde{W}^-, \widetilde{H}_d^-)$, the chargino mass Lagrangian reads

$$\mathcal{L}^C_{mass} = -\frac{1}{2}(\psi^\pm)^T \begin{pmatrix} 0 & \mathcal{M}_C^T \\ \mathcal{M}_C & 0 \end{pmatrix} \psi^\pm + \text{h.c.} \tag{2.91}$$

The chargino mixing matrices Z_+ and Z_- are defined by requiring that they diagonalize the 2×2 chargino mass matrix $\mathcal{M}_C$ as

$$(Z_-)^T \begin{pmatrix} M_2 & \sqrt{2}m_W s_\beta \\ \sqrt{2}m_W c_\beta & \mu \end{pmatrix} Z_+ = \begin{pmatrix} m_{\chi_1} & 0 \\ 0 & m_{\chi_2} \end{pmatrix}, \tag{2.92}$$

where $s_\beta = \sin\beta$, $c_\beta = \cos\beta$. The unitary matrices Z_-, Z_+ are not uniquely specified – by changing their relative phases and the ordering of the eigenvalues it is possible to choose M_{χ_i} to be positive and $M_{\chi_2} > M_{\chi_1}$. The fields χ_i are related to the initial spinors as

$$\widetilde{H}_u^+ = Z_+^{2i}\kappa_i^+\,, \quad \widetilde{H}_d^- = Z_-^{2i}\kappa_i^-\,, \quad \widetilde{W}^\pm = iZ_\pm^{1i}\kappa_i^\pm \quad \text{where} \quad \chi_i = \begin{pmatrix} \kappa_i^+ \\ \bar{\kappa}_i^- \end{pmatrix}. \tag{2.93}$$

Neutralinos

Four 2-component spinors, namely the neutral higgsinos ($\widetilde{H}_u^0$ and $\widetilde{H}_d^0$) and the neutral gauginos ($\widetilde{B}$, $\widetilde{W}^0$), combine into four Majorana fermions χ_i^0 ($i = 1, 2, 3, 4$) called *neutralinos*. The lightest neutralino mass eigenstate, χ_1^0, is the favourite candidate for being the LSP in the MSSM spectrum. In the gauge-eigenstate basis $\psi^0 = (\widetilde{B}, \widetilde{W}^0, \widetilde{H}_d^0, \widetilde{H}_u^0)$, the neutralino mass Lagrangian is

$$\mathcal{L}^N_{\text{mass}} = -\frac{1}{2}(\psi^0)^T \mathcal{M}_N\, \psi^0 + \text{h.c.}\,, \tag{2.94}$$

The neutralino mass matrix $\mathcal{M}_N$ is diagonalized with the unitary matrix Z_N as

$$Z_N^T \begin{pmatrix} M_1 & 0 & -c_\beta\, s_W\, m_Z & s_\beta\, s_W\, m_Z \\ 0 & M_2 & c_\beta\, c_W\, m_Z & -s_\beta\, c_W\, m_Z \\ -c_\beta\, s_W\, m_Z & c_\beta\, c_W\, m_Z & 0 & -\mu \\ s_\beta\, s_W\, m_Z & -s_\beta\, c_W\, m_Z & -\mu & 0 \end{pmatrix} Z_N = \begin{pmatrix} m_{\chi_1^0} & & 0 \\ & \ddots & \\ 0 & & m_{\chi_4^0} \end{pmatrix}. \tag{2.95}$$

Here we used $s_W = \sin\theta_W$, and $c_W = \cos\theta_W$, θ_W being the weak angle. The entries M_1 and M_2 in the mass matrix $\mathcal{M}_N$ come directly from the MSSM soft Lagrangian (2.67), while the entries $-\mu$ are the supersymmetric higgsino mass terms coming from the superpotential (2.63). The terms proportional to m_Z are the result of Higgs-higgsino-gaugino couplings with the Higgs scalars replaced by their VEVs. The fields χ_i^0 are related to the initial spinors as

$$\begin{gathered} \widetilde{B} = iZ_N^{1i}\kappa_i^0\,, \quad \widetilde{W}^0 = iZ_N^{2i}\kappa_i^0\,, \quad \widetilde{H}_d^0 = Z_N^{3i}\kappa_i^0\,, \quad \widetilde{H}_u^0 = Z_N^{4i}\kappa_i^0\,, \\ \text{where} \quad \chi_i^0 = \begin{pmatrix} \kappa_i^0 \\ \bar{\kappa}_i^0 \end{pmatrix}. \end{gathered} \tag{2.96}$$

Gluinos

The gluinos $\widetilde{g}$ are the only MSSM fermions in the adjoint representation of the $SU(3)$ colour group, therefore they cannot mix with any other particle. Their mass is directly given by the soft parameter M_3. In this regard, gluinos are unique among all of the MSSM spartlcles.

Sleptons

The mass eigenstates for the three neutral sneutrinos are obtained by diagonalization of the mass matrix $\mathcal{M}_\nu$

$$\mathcal{M}_\nu^2 = \frac{e^2(v_d^2 - v_u^2)}{8s_W^2 c_W^2}\hat{1} + \widetilde{m}_L^2\,, \qquad Z_\nu^\dagger \mathcal{M}_\nu^2 Z_\nu = \mathrm{diag}(m^2_{\widetilde{\nu}_1}, m^2_{\widetilde{\nu}_2}, m^2_{\widetilde{\nu}_3})\,. \tag{2.97}$$

For the charged sleptons one needs to diagonalize the 6×6 mass matrix $\mathcal{M}_E^2$, which in the basis $(\widetilde{e}_L^I, \widetilde{e}_R^I)$ is defined as

$$\mathcal{M}_E^2 = \begin{pmatrix} (\mathcal{M}_L^2)_{LL} & (\mathcal{M}_L^2)_{LR} \\ (\mathcal{M}_L^2)_{LR}^\dagger & (\mathcal{M}_L^2)_{RR} \end{pmatrix}\,, \qquad Z_L^\dagger \mathcal{M}_E^2 Z_L = \mathrm{diag}(m^2_{\widetilde{L}_1}, \ldots, m^2_{\widetilde{L}_6})\,, \tag{2.98}$$

$$(\mathcal{M}_L^2)_{LL} = \frac{e^2(v_d^2 - v_u^2)(1-2c_W^2)}{8s_W^2 c_W^2}\hat{1} + \frac{v_d^2 Y_e^2}{2} + (\widetilde{m}_L^2)^T\,, \tag{2.99}$$

$$(\mathcal{M}_L^2)_{RR} = -\frac{e^2(v_d^2 - v_u^2)}{4c_W^2}\hat{1} + \frac{v_d^2 Y_e^2}{2} + \widetilde{m}_e^2\,, \tag{2.100}$$

$$(\mathcal{M}_L^2)_{LR} = -\frac{1}{\sqrt{2}}\left(v_u \mu^* Y_e + v_d A_e\right)\,. \tag{2.101}$$

Squarks

Turning to the squarks, we have six mass-eigenstates $\widetilde{U}_i$ $(i = 1, \ldots, 6)$ for the up-type squarks, which are obtained by diagonalizing the corresponding mass matrix in the $(\widetilde{u}_L{}^I \;\; \widetilde{u}_R{}^J)$ basis

$$Z_U^T \begin{pmatrix} (\mathcal{M}_U^2)_{LL} & (\mathcal{M}_U^2)_{LR} \\ (\mathcal{M}_U^2)_{LR}^\dagger & (\mathcal{M}_U^2)_{RR} \end{pmatrix} Z_U^* = \mathrm{diag}(m^2_{\widetilde{U}_1}, \ldots, m^2_{\widetilde{U}_6})\,. \tag{2.102}$$

Where the 3×3 entries of the above mass matrix, in the super-CKM basis, read

$$(\mathcal{M}_U^2)_{LL} = -\frac{e^2(v_d^2 - v_u^2)(1-4c_W^2)}{24s_W^2 c_W^2}\hat{1} + \frac{v_u^2 Y_u^2}{2} + (V\widetilde{m}_Q^2 V^\dagger)^T\,, \tag{2.103}$$

$$(\mathcal{M}_U^2)_{RR} = \frac{e^2(v_d^2 - v_u^2)}{6c_W^2}\hat{1} + \frac{v_u^2 Y_u^2}{2} + \widetilde{m}_u^2\,, \tag{2.104}$$

$$(\mathcal{M}_U^2)_{LR} = -\frac{1}{\sqrt{2}}\left(v_d \mu^* Y_u + v_u A_u\right)\,. \tag{2.105}$$

The situation is similar for the down-type squarks, where six mass-eigenstates $\widetilde{D}_i$ $(i = 1, \ldots, 6)$ are obtained as

$$Z_D^\dagger \begin{pmatrix} (\mathcal{M}_D^2)_{LL} & (\mathcal{M}_D^2)_{LR} \\ (\mathcal{M}_D^2)_{LR}^\dagger & (\mathcal{M}_D^2)_{RR} \end{pmatrix} Z_D = \mathrm{diag}(m^2_{\widetilde{D}_1}, \ldots, m^2_{\widetilde{D}_6})\,. \tag{2.106}$$

$$\left(\mathcal{M}_D^2\right)_{LL} = -\frac{e^2(v_d^2 - v_u^2)(1+2c_W^2)}{24 s_W^2 c_W^2}\hat{1} + \frac{v_d^2 Y_d^2}{2} + (\widetilde{m}_Q^2)^T \tag{2.107}$$

$$\left(\mathcal{M}_D^2\right)_{RR} = -\frac{e^2(v_d^2 - v_u^2)}{12 c_W^2}\hat{1} + \frac{v_d^2 Y_d^2}{2} + \widetilde{m}_d^2 \tag{2.108}$$

$$\left(\mathcal{M}_D^2\right)_{LR} = -\frac{1}{\sqrt{2}}\left(v_u \mu^* Y_d + v_d A_d\right) \tag{2.109}$$

Note that the yukawa matrices are diagonal in the super-CKM basis. The terms proportional to the identity matrix in flavour space come from D-term quartic interactions of the form (sfermion)^2(Higgs)^2 after the neutral Higgs scalars get VEVs. Flavour off diagonal terms are introduced by the mass matrices $\widetilde{m}_Q^2$, $\widetilde{m}_u^2$, $\widetilde{m}_d^2$ and by the trilinear coupling A_u, A_d. These are soft SUSY breaking terms and in the MSSM with general flavour mixing they are completely free parameters.

Chapter 3

Flavour Physics

The term *flavour physics* refers to interactions that distinguish between flavours (i.e. the generations of fields which share the same quantum charges) [29]. Within the SM, flavour physics is confined to the weak and Yukawa interactions and is completely governed by the CKM matrix. In this Chapter we first review the basics of flavour physics in the SM, highlighting the good agreement of the CKM description with the experimental data. Then we outline how this agreement with the SM predictions constitute a problem for NP if the energy scale of the new degrees of freedom is taken at the TeV scale. Finally we concentrate on the flavour structure of the MSSM and its new sources of flavour violation.

3.1 Flavour physics in the Standard Model

The source of all flavour physics within the SM are the Yukawa interactions. If we turn off the Yukawa couplings the theory acquires a large global symmetry G_{fl} which can be decomposed as

$$G_{\mathrm{fl}} = SU(3)_q^3 \times SU(3)_\ell^2 \times U(1)^5 \,, \quad \text{with} \quad \begin{array}{l} SU(3)_q^3 = SU(3)_Q \times SU(3)_u \times SU(3)_d \,, \\ SU(3)_\ell^2 = SU(3)_L \times SU(3)_e \,. \end{array} \tag{3.1}$$

This symmetry corresponds to the independent unitary rotations in flavour space of the five fermion fields. Since the Yukawa interactions break this symmetry, such transformations do not leave $\mathcal{L}_{\mathrm{SM}}$ invariant. Instead, they correspond to a change of the interaction basis. In the first chapter we saw that the mass and the interaction eigenstates are not the same but connected by the CKM matrix V. Within the SM, all flavour and CP violating processes are regulated by the V matrix.

A very convenient parametrization of the CKM matrix is given by the approximate Wolfenstein parametrization [30] where the mixing parameters are (λ, A, ρ, η) with $\lambda \simeq |V_{us}| \sim 0.22$ playing the role of a small expansion parameter and η representing the CP violating phase

$$V = \begin{pmatrix} 1-\frac{1}{2}\lambda^2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\frac{1}{2}\lambda^2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}\big(\lambda^4\big) \,. \tag{3.2}$$

The precision tests designed to measure up to a very good degree of accuracy the elements of this matrix and its unitarity are of great importance. Direct and indirect information on the smallest matrix elements of the CKM matrix is neatly summarized in terms of the *unitarity triangle*, one of six such triangles that correspond to the unitarity condition applied to two different rows or columns of the CKM matrix. Unitarity applied to the first and third columns yields $V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0$. This relation can be presented as a triangle in the complex plane as shown in Fig. 3.1, with the sides given by

$$\overrightarrow{AB} \equiv -\frac{V_{tb}^* V_{td}}{V_{cb}^* V_{cd}} = 1 - \bar{\rho} - i\bar{\eta}\,, \quad \overrightarrow{CA} \equiv -\frac{V_{ub}^* V_{ud}}{V_{cb}^* V_{cd}} = \bar{\rho} + i\bar{\eta}\,, \quad \overrightarrow{CB} \equiv 1\,.$$

Here $\bar{\rho} = \rho(1-\lambda^2/2)$ and $\bar{\eta} = \eta(1-\lambda^2/2)$ and we included a $\mathcal{O}(\lambda^5)$ correction in the V_{td} element of Eq. (3.2). One can assume that flavour and CP violation processes are fully

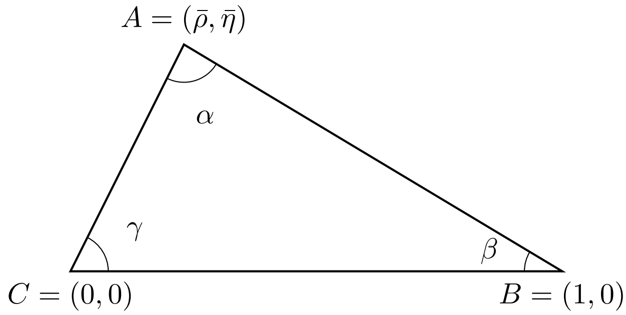


Figure 3.1: Representation in the complex plane of the rescaled triangle formed by the CKM matrix elements $V_{ud}V_{ub}^*$, $V_{td}V_{tb}^*$, and $V_{cd}V_{cb}^*$.

described by the SM, and check the consistency of the various measurements with this assumption. The sides $\overline{AB}$ and $\overline{AC}$ as well the angles α, β and γ of the unitarity triangle are accessible in many flavour changing observables and using the available experimental information on these observables allows to overconstrain the $\bar{\rho} - \bar{\eta}$ plane.

The values of λ and A are known rather accurately [31, 32] from, respectively, $K \to \pi \ell \nu$ and $b \to c\ell\nu$ decays:

$$\lambda = 0.22457^{+0.00186}_{-0.00014}\,, \qquad A = 0.823^{+0.012}_{-0.033}\,. \tag{3.3}$$

Then, one can express all the relevant observables as a function of the two remaining parameters, $\bar{\rho}$ and $\bar{\eta}$, and check whether there is a range in the $\bar{\rho}-\bar{\eta}$ plane that is consistent with all measurements. The principal observables are the following [29]:

- The rates of inclusive and exclusive charmless semileptonic B decays depend on $|V_{ub}|^2 \propto \rho^2 + \eta^2$;
- The CP asymmetry in $B \to \psi K_S$, $S_{B\to\psi K} = \sin 2\beta = \frac{2\eta(1-\rho)}{(1-\rho)^2+\eta^2}$;
- The rates of various $B \to DK$ decays depend on the phase γ, where $e^{i\gamma} = \frac{\rho+i\eta}{\sqrt{\rho^2+\eta^2}}$;
- The rates of various $B \to \pi\pi, \rho\pi, \rho\rho$ decays depend on the phase $\alpha = \pi - \beta - \gamma$;
- The ratio between the mass splittings in the neutral B and B_s systems is sensitive to $|V_{td}/V_{ts}|^2 = \lambda^2[(1-\rho)^2 + \eta^2]$;
- The CP violation in $K \to \pi\pi$ decays, ϵ_K, depends in a complicated way on ρ and η.

The resulting constraints are shown in Fig. 3.2 and lead to the following values of $\bar{\rho}$ and $\bar{\eta}$

$$\bar{\rho} = 0.1289^{+0.0176}_{-0.0094}\,, \qquad \bar{\eta} = 0.348^{+0.012}_{-0.012}\,. \tag{3.4}$$

Global fits to the data on flavour changing processes [31, 32] all lead consistently to a single solution and there is no doubt anymore that the CKM matrix is the main source of flavour and CP violation in the processes entering the unitarity triangle analysis. Correspondingly, new physics effects to such processes can only be small corrections and are strongly bounded by existing data.

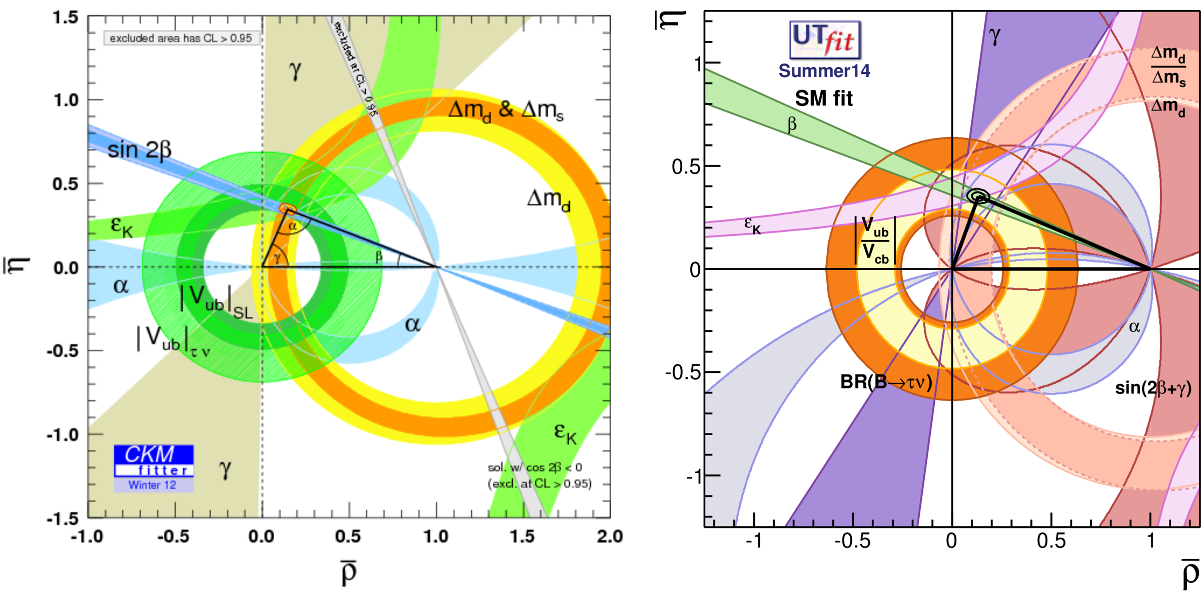


Figure 3.2: Unitarity triangle fits in the $\bar{\rho} - \bar{\eta}$ plane from [31] and [32].

While the identification of tensions and possible inconsistencies in the Unitarity Triangle provides a valuable tool to discern new physics, the main goal of the SM unitarity triangle analysis is to give a determination of the CKM parameters as precise as possible. The global fits seem to be consistent to a large extent and as mentioned above, the CKM picture of flavour and CP violation in the SM appears to be a very good description of the data.

3.2 The SM vs. the NP flavour problem

As already evident from the parametrization (3.2), the measured entries in the CKM matrix show a strongly hierarchical pattern and range over three orders of magnitude

$$\begin{array}{lll} |V_{ud}| \simeq 0.97 & |V_{us}| \simeq 0.22 & |V_{ub}| \simeq 3.6 \cdot 10^{-3} \\ |V_{cd}| \simeq 0.22 & |V_{cs}| \simeq 0.97 & |V_{cb}| \simeq 4.2 \cdot 10^{-2} \\ |V_{td}| \simeq 8.8 \cdot 10^{-3} & |V_{ts}| \simeq 4.1 \cdot 10^{-2} & |V_{tb}| \simeq 1 \,. \end{array}$$

Similarly, also the remaining flavour parameters in the SM, i.e. the fermion masses are strongly hierarchical. Even leaving aside neutrinos, the SM fermion masses range over almost six order of magnitude between the electron mass and the top quark mass. Only the top quark mass has a "natural" value of the order of the electroweak scale, while all the other fermion masses are very small and look unnatural. The lack of a theoretical understanding of the huge hierarchies among the masses and mixing angles is often referred to as the SM flavour problem.

New Physics models typically introduce additional sources of flavour and CP violation with respect to those already contained in the CKM matrix. Employing a generic effective theory approach, the NP effects in flavour observables can be analysed in a model independent way. Under the assumption that the NP degrees of freedom are heavier than the SM fields, they can be integrated out and their effects can be described by higher dimensional operators. The effective Lagrangian contains then the SM Lagrangian and an infinite tower of operators $O^{(d)}$ with dimension $d > 4$, constructed out of SM fields and suppressed by inverse powers of an effective NP scale Λ

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{SM}} + \sum_{i,\, d>4} \frac{c_i^{(d)}}{\Lambda^{d-4}} O_i^{(d)} \,. \tag{3.5}$$

According to the naturalness criterion, the unknown couplings c_i are expected to be ~ 1 in value, unless a protection mechanism is at work. Using this approach, the realistic extensions of the SM are analysed in terms of the coefficients c_i.

Basis for $\mathbf{\Delta F = 2}$ operators The meson-antimeson oscillations are governed in the effective theory by $\Delta F = 2$ operators O_i given, in the case of K^0 mixing[1], by

$$\begin{aligned}
O_1 &= (\bar{s}^\alpha \gamma_\mu P_L d^\alpha) \otimes (\bar{s}^\beta \gamma^\mu P_L d^\beta)\,, & & \\
O_2 &= (\bar{s}^\alpha P_L d^\alpha) \otimes (\bar{s}^\beta P_L d^\beta)\,, & \widetilde{O}_1 &= (\bar{s}^\alpha \gamma_\mu P_R d^\alpha) \otimes (\bar{s}^\beta \gamma^\mu P_R d^\beta)\,, \\
O_3 &= (\bar{s}^\alpha P_L d^\beta) \otimes (\bar{s}^\beta P_L d^\alpha)\,, & \widetilde{O}_2 &= (\bar{s}^\alpha P_R d^\alpha) \otimes (\bar{s}^\beta P_R d^\beta)\,, \\
O_4 &= (\bar{s}^\alpha P_L d^\alpha) \otimes (\bar{s}^\beta P_R d^\beta)\,, & \widetilde{O}_3 &= (\bar{s}^\alpha P_R d^\beta) \otimes (\bar{s}^\beta P_R d^\alpha)\,. \\
O_5 &= (\bar{s}^\alpha P_L d^\beta) \otimes (\bar{s}^\beta P_R d^\alpha)\,, & &
\end{aligned} \tag{3.6}$$

The good agreement between the SM predictions for meson mixing and the experimental data can then be translated into bounds on the combination c_i/Λ^2 (see Tab. 3.1). The general picture that emerges is the following:

- If the coefficients c_i are assumed to be generic, i.e. all of $\mathcal{O}(1)$, then the most stringent bounds on the NP scale Λ, which come from CP violation in $K^0 - \overline{K}^0$ mixing, are at the level of $10^4 - 10^5$ TeV. This scale is far above the scale of NP that one expects from naturalness arguments.

- If on the other hand the NP scale is fixed to 1 TeV as suggested by a natural solution of the hierarchy problem, the couplings that are responsible for CP violation in $K^0 - \overline{K}^0$ mixing are constrained at the level of $10^{-9} - 10^{-11}$. Slightly less stringent bounds arise from $D^0 - \overline{D}^0$, $B_d - \overline{B}_d$ and $B_s - \overline{B}_s$ mixing, where the most stringent constraints on the corresponding couplings are at the level of 10^{-8}, 10^{-7} and 10^{-5} respectively.

This is the so-called *NP flavour problem*: NP with generic flavour structure is forced to be far above the natural TeV scale, while NP at the TeV scale necessarily has to possess a highly non-generic and unnatural flavour structure.

[1] Analogous formulae hold for the B_d, B_s, and D^0 systems, with the appropriate replacements of the quarks involved in the transition.

Operator	Bounds on Λ in TeV ($c_{ij}=1$)		Bounds on c_{ij} ($\Lambda = 1$ TeV)		Observables
	Re	Im	Re	Im	
$(\bar{s}_L\gamma^\mu d_L)^2$	9.8×10^2	1.6×10^4	9.0×10^{-7}	3.4×10^{-9}	Δm_K; ϵ_K
$(\bar{s}_R\, d_L)(\bar{s}_L d_R)$	1.8×10^4	3.2×10^5	6.9×10^{-9}	2.6×10^{-11}	Δm_K; ϵ_K
$(\bar{c}_L\gamma^\mu u_L)^2$	1.2×10^3	2.9×10^3	5.6×10^{-7}	1.0×10^{-7}	Δm_D; $\lvert q/p\rvert, \phi_D$
$(\bar{c}_R\, u_L)(\bar{c}_L u_R)$	6.2×10^3	1.5×10^4	5.7×10^{-8}	1.1×10^{-8}	Δm_D; $\lvert q/p\rvert, \phi_D$
$(\bar{b}_L\gamma^\mu d_L)^2$	5.1×10^2	9.3×10^2	3.3×10^{-6}	1.0×10^{-6}	Δm_{B_d}; $S_{\psi K_S}$
$(\bar{b}_R\, d_L)(\bar{b}_L d_R)$	1.9×10^3	3.6×10^3	5.6×10^{-7}	1.7×10^{-7}	Δm_{B_d}; $S_{\psi K_S}$
$(\bar{b}_L\gamma^\mu s_L)^2$	1.1×10^2		7.6×10^{-5}		Δm_{B_s}
$(\bar{b}_R\, s_L)(\bar{b}_L s_R)$	3.7×10^2		1.3×10^{-5}		Δm_{B_s}

Table 3.1: Bounds on representative dimension-six $\Delta F = 2$ operators. Bounds on Λ are quoted assuming an effective coupling $1/\Lambda^2$, or, alternatively, the bounds on the respective c_{ij}'s assuming $\Lambda = 1$ TeV [29].

Minimal Flavour Violation

An elegant way to avoid the NP flavour problem is provided by the Minimal Flavour Violation (MFV) hypothesis [33]. Under this assumption, flavour violating interactions are linked to the known structure of Yukawa couplings also beyond the SM.

In particular this assumption consists in formally recovering the global symmetry G_{fl} defined in Eq. (3.1) by promoting Y_u, Y_d and Y_e to non-dynamical fields (spurions) with non-trivial transformation properties under G_{fl}

$$Y_e \sim (3,\bar{3})_{\mathrm{SU}(3)^2_\ell}\,, \qquad Y_u \sim (3,\bar{3},1)_{\mathrm{SU}(3)^3_q}\,, \qquad Y_d \sim (3,1,\bar{3})_{\mathrm{SU}(3)^3_q}\,. \tag{3.7}$$

It is easy to check that the Yukawa interactions in Eq. (1.4) are now invariant under the flavour group. An effective theory satisfies the MFV criterion if all higher-dimensional operators, constructed from SM and the Yukawa spurions, are invariant (formally) under the flavour group G_{fl}. Note that using the $SU(3)^3_q \times SU(3)^2_\ell$ symmetry, we can rotate the background values of the spurions to the basis

$$Y_e = y_e\,, \qquad Y_u = V^\dagger y_u\,, \qquad Y_d = y_d\,, \tag{3.8}$$

where y_e, y_u, y_d are diagonal matrices and V is the CKM matrix. Another important fact to keep in mind is that the SM Yukawa couplings for all fermions except the top are small. We now turn again our attention to the $\Delta F = 2$ operators of Eq. (3.6). According to the MFV prescription, the Dirac structures appearing in the operators are modified, at lowest order in the Yukawa spurions, to

$$\begin{aligned}
\bar{s}\gamma_\mu P_L d &\xrightarrow{\text{MFV}} \bar{s}\gamma_\mu P_L (Y_u Y_u^\dagger)_{21}\, d\,,\\
\bar{s}\gamma_\mu P_R d &\xrightarrow{\text{MFV}} \bar{s}\gamma_\mu P_R (Y_d^\dagger Y_u Y_u^\dagger Y_d)_{21}\, d\,,\\
\bar{s}\, P_L d &\xrightarrow{\text{MFV}} \bar{s}\, P_L (Y_d^\dagger Y_u Y_u^\dagger)_{21}\, d\,,\\
\bar{s}\, P_R d &\xrightarrow{\text{MFV}} \bar{s}\, P_R (Y_u^\dagger Y_u Y_d)_{21}\, d\,.
\end{aligned} \tag{3.9}$$

As a result, within the MFV framework the couplings c_i of the FCNC operators are now suppressed by the CKM-matrix elements and a low NP scale at the level of few TeV is still compatible with the flavour data within this minimalistic scenario [29].

Minimal flavour violation is however not a theory of flavour. While it naturally suppresses NP contributions to flavour violating process, it does not provide an explanation for the hierarchical flavour structure that is already present in the SM.

3.3 Flavour physics in the MSSM

We now turn our attention to the flavour structure of the MSSM. Since the gauge interactions are flavour blind, the MSSM gauge sector is invariant under the global group $G_{\rm fl}$ of flavour transformations of the whole chiral supermultiplets – and not only to fermion fields. However this flavour symmetry is broken by two different sources. Looking at the quark sector, $G_{\rm fl}$ is broken by the Yukawa couplings Y_u and Y_d that appear in the superpotential as well as by the soft masses $\widetilde{m}_Q^2$, $\widetilde{m}_u^2$, $\widetilde{m}_d^2$ and the trilinear coupling A_u, A_d in the soft SUSY breaking Lagrangian. All these terms can in principle be generic 3×3 matrices in flavour space. In Sec. 2.9 we saw that in the super-CKM basis (i.e the quark mass eigenstate basis) the squark mass matrices still feature off diagonal terms. The squark mass eigenstates are then obtained with the following unitary transformations of the squarks fields

$$\widetilde{d}_L^I = (Z_D^*)_{Ii}\widetilde{D}_i\,, \quad \widetilde{d}_R^I = (Z_D^*)_{(I+3)i}\widetilde{D}_i\,, \quad \widetilde{u}_L^I = (Z_U)_{Ii}\widetilde{U}_i\,, \quad \widetilde{u}_R^I = (Z_U)_{(I+3)i}\widetilde{U}_i\,. \tag{3.10}$$

After this redefinitions, the unitary 6×6 rotation matrices Z_U and Z_D appear in all vertices involving the up and down squarks respectively, as shown in Fig. 3.3. In particular the gluino interactions are now flavour violating, leading to potentially large flavour changing neutral currents of quarks that are introduced at the loop level through strong interactions.

The disadvantage of this approach is that the unitary matrices Z_U and Z_D are connected

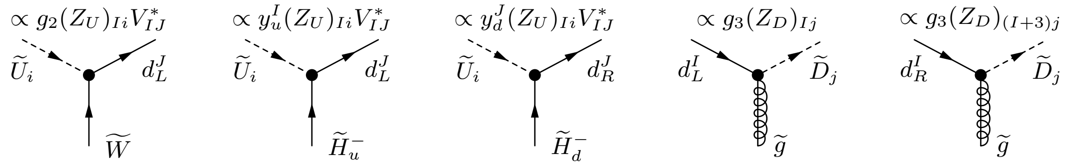


Figure 3.3: Example tree level vertices in the MSSM in the squark mass eigenstate basis. Vertices involving up and down squarks contain the Z_U and Z_D rotation matrices. In particular the *gluino-squark-quark* vertices are flavour violating. For simplicity only vertices with external downs quarks are shown.

to the original flavour changing parameters in the Lagrangian in a highly non trivial way. Therefore it is usually more convenient to work in the super-CKM basis and to treat the off-diagonal entries in the squark mass matrices as small perturbations. This is the so-called mass insertion approximation (MIA) [34].

Mass insertion approximation

Given the large number of unknown parameters involved in flavour changing processes, it is particularly helpful to make use of the MIA. Working in the super-CKM basis, the squark mass matrices are not diagonal and therefore the flavour violation is exhibited by the non-diagonality of the sfermion propagators.

Taking the diagonal elements of the squark mass matrices to be approximately equal to a common sfermion mass $\widetilde{m}$, one writes the off-diagonal entries Δ_{ij}^q in terms of the so-called *mass insertions* $\delta_{ij}^q = \Delta_{ij}^q/\widetilde{m}^2$. With this notation the 6×6 up and down squark mass matrices read

$$\mathcal{M}_u^2 = \mathrm{diag}(\widetilde{m}^2) + \widetilde{m}^2\delta_u\,, \qquad \mathcal{M}_d^2 = \mathrm{diag}(\widetilde{m}^2) + \widetilde{m}^2\delta_d\,, \tag{3.11}$$

where the mass insertions are decomposed according to the "chirality" of the squarks as

$$\delta_q = \begin{pmatrix} \delta_q^{LL} & \delta_q^{LR} \\ \delta_q^{RL} & \delta_q^{RR} \end{pmatrix}\,, \qquad q = u, d\,. \tag{3.12}$$

As the squark masses are hermitian one has the following relations among the mass insertions

$$(\delta_q^{LL}) = (\delta_q^{LL})^\dagger\,, \qquad (\delta_q^{RR}) = (\delta_q^{RR})^\dagger\,, \qquad (\delta_q^{RL}) = (\delta_q^{LR})^\dagger\,. \tag{3.13}$$

Moreover, the left-left blocks of the up and down mass insertions are related to each other

$$(\delta_u^{LL}) = V^\dagger(\delta_d^{LL})V\,. \tag{3.14}$$

The δ's in the squark mass matrices are then treated as perturbations and flavour changing amplitudes arise through mass insertions along squark propagators as shown in Fig. 3.4.

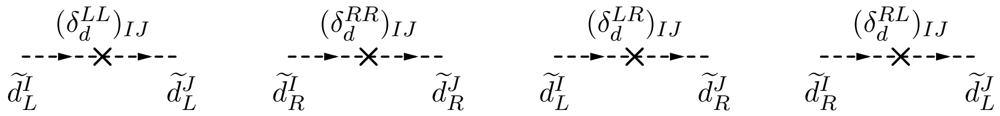


Figure 3.4: The Mass Insertion Approximation in the down squark sector. Off-diagonal entries in the 6×6 squark mass matrices are treated as perturbations and flavour change occurs through mass insertions along squark propagators.

While the Mass Insertion Approximation is only an approximation and breaks down for mass insertions of $\mathcal{O}(1)$, it gives a very intuitive picture of the impact of the sources of flavour violation contained in the soft terms, and allows to transparently display the main dependencies of flavour changing amplitudes on the MSSM parameters. Experimental constraints are then given on the δ's, usually constraining one mass insertion at a time.

MSSM flavour problem

The NP flavour problem manifests itself also in the MSSM, in fact the new sources of flavour violation, arising from the soft terms, generally lead to unacceptably large contributions to FCNC and/or CP-violating observables. Imposing that the supersymmetric contributions to flavour changing processes do not exceed the phenomenological bounds leads to constrains of the form $\delta \ll 1$ (see for example [5, 35]). The most restrictive bounds come from the $K^0 - \overline{K}^0$ mixing, ϵ_K and the radiative muon decay $\mu \to e\gamma$ [36]. The most popular protection mechanisms to suppress such unwanted contributions are

- *Decoupling.* The sfermion mass scale is taken to be very high. This happens for example in Split SUSY [37, 38], where the masses of sfermions are taken to be extremely heavy and only charginos and neutralinos remain relatively light. While in such a setup SUSY is not responsible anymore for the solution of the gauge hierarchy problem, Split SUSY is for example compatible with gauge coupling unification and the observed dark matter abundance.

- *Degeneracy.* The sfermion masses are degenerate to a large extent, leading to a strong GIM suppression [39]. In such a framework the soft squark masses are to a first approximation universal, i.e. proportional to the unit matrix. While the rotation angles that diagonalize the squark mass matrices are generally not small, the suppression of flavour violating amplitudes is due to the near degeneracy of the squark mass eigenstates.

- *Alignment.* The quark and squark mass matrices are aligned [40], i.e. the squark and quark mass matrices are nearly simultaneously diagonal in the super-CKM basis. This lead to the suppression of the flavour-changing gaugino-sfermion-fermion couplings.

- *MFV.* The flavour violating parameters of the MSSM are assumed to respect the MFV hypothesis as outlined in Sec. 3.2.

  Following the MFV rules, the supersymmetry-breaking squark mass terms and the trilinear couplings can be written as [33]

$$\tilde{m}^2_{Q_L} = \tilde{m}^2 \left(a_1 \mathbb{1} + b_1 Y_u Y_u^\dagger + b_2 Y_d Y_d^\dagger + b_3 Y_d Y_d^\dagger Y_u Y_u^\dagger + b_4 Y_u Y_u^\dagger Y_d Y_d^\dagger \right) , \tag{3.15}$$

$$\tilde{m}^2_u = \tilde{m}^2 \left(a_2 \mathbb{1} + b_5 Y_u^\dagger Y_u \right) , \qquad \tilde{m}^2_d = \tilde{m}^2 \left(a_3 \mathbb{1} + b_6 Y_d^\dagger Y_d \right) , \tag{3.16}$$

$$A_u = A \left(a_4 \mathbb{1} + b_7 Y_d Y_d^\dagger \right) Y_u \, , \qquad A_d = A \left(a_5 \mathbb{1} + b_8 Y_u Y_u^\dagger \right) Y_d \, . \tag{3.17}$$

  where a_i and b_i are unknown numerical coefficients, and we omitted higher-order terms in $Y_{u,d}$ (see [41] for the most general expressions).

Chapter 4

Flavour Physics phenomenology

This Chapter is devoted to the low energy observables which enter our numerical analysis. In the first Section we give the prescription for the determination of the Wilson coefficients. In the last three Sections we present the results of our calculations for the most important MSSM contributions to the Wilson coefficients of the selected $\Delta F = 2$, $\Delta F = 1$ and $\Delta F = 0$ transitions. We give both the expressions in the mass eigenstate basis and in the super-CKM basis with the mass insertion approximation. The details of the calculations are reported in Appendices C and D.

4.1 Matching conditions for Wilson coefficients

Flavour Changing Neutral Currents usually involve two vastly different energy scales. The low scale μ_l is set by typical hadronic energy scale of $\mathcal{O}(1\text{GeV})$ while the high scale μ_h is determined by the heavy particles running in the loops. Including QCD corrections then usually leads to large logarithms of the kind $\log\left(\mu_h^2/\mu_l^2\right)$ that might spoil the perturbative expansion in α_s. The most efficient way to deal with this problem is to switch to an effective low energy theory where the heavy particles are integrated out. The basic idea is to set up a theory that only contains the low energy degrees of freedom of the full theory one wants to describe. The effects of the heavy particles are accounted for by modifying the couplings of the light particles and by introducing new interactions in the form of additional higher dimensional local operators, which are built out of the low energy fields and respect all relevant symmetries

$$\mathcal{H}_{\text{eff}} = \sum_i C_i Q_i \,, \tag{4.1}$$

with "couplings" C_i, called *Wilson coefficients.* These higher dimensional operators make the theory nonrenormalizable and one has in principle to include infinitely many operators Q_i and couplings C_i. But still, the theory turns out to be predictive, as long as one considers processes well below the scale of the heavy particles. The higher dimensional operators are suppressed by inverse powers of the scale of the heavy particles μ_h [42]. This power counting in $1/\mu_h$ ensures that only a finite number of operators up to a certain dimension has to be considered in order to achieve a given precision. For FCNC processes it is usually sufficient to include operators up to dimension six. The Wilson coefficients C_i are determined by demanding that the effective theory has to reproduce the results of the full theory for low energy processes. This leads to the so-called *matching conditions.* Considering the amplitude of a process where some initial state $|i\rangle$ goes into some final state $|f\rangle$, with both states only including low energy degrees of freedom, one requires that both the full and the effective theory give the same result at some matching scale μ_0

$$A^{i\to f}_{\rm full} = A^{i\to f}_{\rm eff} = \langle f|\mathcal{H}_{\rm eff}|i\rangle = \sum_i C_i(\mu_0)\,\langle f|Q_i(\mu_0)|i\rangle \ . \tag{4.2}$$

One has to consider as many S matrix elements (amplitudes) as needed, to gather enough information to determine all the Wilson coefficients.

Renormalization group evolution in the effective theory

As usual in quantum field theory, we need to renormalize the local operators Q_i since the Green functions with insertions of Q_i are usually divergent (see Appendix B for the basic facts about the renormalization procedure). Calling $Q_i^{(0)}$ the unrenormalized operators, we can write

$$Q_i^{(0)} = Z_{ij} Q_j \,. \tag{4.3}$$

The renormalization of $Q_i^{(0)}$ commonly involves other operators, so we introduced a renormalization matrix Z_{ij} to account for that. Since the bare operators do not depend on the renormalization scale μ_0, we can obtain the renormalization group equations for the operators Q_i by taking the derivative with respect to μ_0 of equation (4.3). We get

$$\mu_0 \frac{\mathrm{d}}{\mathrm{d}\mu_0} Q_j = -\gamma_{ji} Q_i \,, \qquad \gamma_{ji} = Z^{-1}_{jk}\left(\mu_0 \frac{\mathrm{d}}{\mathrm{d}\mu_0} Z_{ki}\right), \tag{4.4}$$

where γ_{ji} is the so-called *anomalous mass dimension* matrix which has the following perturbative expansion

$$\gamma_{ji}(g) = \frac{g^2}{16\pi^2}\sum_{n=0}^{\infty}\gamma_{ji}^{(n)}\left(\frac{g^2}{16\pi^2}\right)^n . \tag{4.5}$$

Then we need also to renormalize the Wilson coefficients C_i. Since the final amplitude cannot depend on the arbitrarily chosen μ_0, the scale dependence has to cancel between the Wilson coefficients $C_i(\mu_0)$ and the operator matrix elements $\langle f|Q_i(\mu_0)|i\rangle$. It is easy to see that this means that the RGEs for the Wilson coefficients are fully determined by the anomalous mass dimension matrix of the operators Q_i

$$\mu_0 \frac{\mathrm{d}}{\mathrm{d}\mu_0} C_i = C_j \gamma_{ji} \,. \tag{4.6}$$

But as the calculations are performed in perturbation theory, the truncation of the perturbative series actually does introduce a dependence on μ_0 which is of the order of the neglected terms. The question then arises of which value one should choose for the scale.

As mentioned above, QCD corrections typically lead to terms that are enhanced by large logarithms of the kind $\log(\mu_h^2/\mu_l^2)$ in the amplitude of the full theory. In the effective theory the perturbative operator matrix elements $\langle f|Q_i(\mu_0)|i\rangle$ contain terms like $\log(\mu_0^2/\mu_l^2)$, while the Wilson coefficients contain terms with $\log(\mu_h^2/\mu_0^2)$. This corresponds to a factorization of long distance (operator matrix elements) and short distance (Wilson coefficients) contributions.

In order to avoid the large logarithms that appear in the full theory one proceeds in the following way. One first chooses $\mu_0 \approx \mu_h$ in the evaluation of the Wilson coefficients from the matching conditions (4.2). This avoids large logarithms in the Wilson coefficients and is expected to make them as reliable as possible concerning the perturbative expansion. Then the μ_0 dependence of the Wilson coefficients in the effective theory is governed by the RGEs (4.6) and by solving these equations with initial conditions $C_i(\mu_h)$, it is possible to run down the Wilson coefficients to the low scale μ_l. At that scale also the operator matrix elements do not contain large logarithms anymore and they can be evaluated reliably.

As the RGE (4.6) is a system of ordinary linear differential equations, the solution can generically be written in the following way

$$C_i(\mu) = U(\mu, \mu_0)_{ij}\, C_j(\mu_0)\,. \tag{4.7}$$

At leading order the evolution matrix $U(\mu, \mu_0)$ is found to be

$$U(\mu, \mu_0) = \left[\frac{\alpha_s(\mu_0)}{\alpha_s(\mu)}\right]^{\frac{\gamma^{(0)T}}{2\beta_0}}\,. \tag{4.8}$$

With this evolution matrix the running of the Wilson coefficients from the high scale μ_h down to the low scale μ_l can be performed. As in the case of α_s, a resummation of the large logarithms $\log\big(\mu_h^2/\mu_l^2\big)$ is achieved through this evolution. Calculating leading order Wilson coefficients and one loop QCD running amounts to a resummation of terms of the following form

$$\sum_{n=0}^{\infty}\left(\alpha_s \log\frac{\mu_h^2}{\mu_l^2}\right)^n\,. \tag{4.9}$$

This is the so-called leading log approximation (LLA). For the next-to-leading log approximation (NLLA), one has to calculate additional one loop QCD corrections to the Wilson coefficients and the running has to be performed at two loop level, and so forth for higher orders.

4.2 $\Delta F = 2$ processes

The phenomenon of meson-antimeson oscillation, being a flavour changing neutral current (FCNC) process, is very sensitive to heavy degrees of freedom propagating in the mixing amplitude and, therefore, it represents one of the most powerful probes of NP. In K and $B_{d,s}$ systems the comparison of observed meson mixing with the SM prediction has achieved a good accuracy and plays a fundamental role in constraining possible extensions of the SM. The meson-antimeson oscillations are described by the mixing amplitudes $\langle K^0|\mathcal{H}_{\rm eff}^{\Delta F=2}|\overline{K}^0\rangle$. The most general $\Delta F = 2$ effective Hamiltonian has the form

$$\mathcal{H}_{\rm eff}^{\Delta F=2} = \sum_{i=1}^{5} C_i Q_i + \sum_{j=1}^{3} \widetilde{C}_j \widetilde{Q}_j + \text{h.c.}\,, \tag{4.10}$$

with the operators Q_i given, in the case of K^0 mixing, by

$$\begin{aligned}
Q_1 &= (\overline{s}^\alpha \gamma_\mu P_L d^\alpha) \otimes (\overline{s}^\beta \gamma^\mu P_L d^\beta)\,,\\
Q_2 &= (\overline{s}^\alpha P_L d^\alpha) \otimes (\overline{s}^\beta P_L d^\beta)\,,\\
Q_3 &= (\overline{s}^\alpha P_L d^\beta) \otimes (\overline{s}^\beta P_L d^\alpha)\,,\\
Q_4 &= (\overline{s}^\alpha P_L d^\alpha) \otimes (\overline{s}^\beta P_R d^\beta)\,,\\
Q_5 &= (\overline{s}^\alpha P_L d^\beta) \otimes (\overline{s}^\beta P_R d^\alpha)\,,
\end{aligned} \tag{4.11}$$

where α, β are colour indices. The operators $\widetilde{Q}_{1,2,3}$ are obtained from $Q_{1,2,3}$ by the replacement $L \leftrightarrow R$. In the SM only the operator Q_1 is generated, because of the $(V - A)$

structure of the SM charged currents. On the other hand, within the MSSM, all operators typically arise.

The mass difference ΔM_K between the mass eigenstates K_L–K_S and the CP-violating parameter ε_K are given by:

$$\Delta M_K = 2\,\mathrm{Re}\langle K^0|\mathcal{H}_{\mathrm{eff}}^{\Delta S=2}|\bar{K}^0\rangle\ , \tag{4.12}$$

$$\varepsilon_K = \frac{1}{\sqrt{2}\Delta M_K}\,\mathrm{Im}\langle K^0|\mathcal{H}_{\mathrm{eff}}^{\Delta S=2}|\bar{K}^0\rangle\ . \tag{4.13}$$

In the following we determine the leading order results for the Wilson coefficients in (4.10) by calculating the process $d\overline{s} \to \overline{d}s$ both in the MSSM and with the effective Hamiltonian at leading order and applying the matching condition (4.2). The amplitude at the high scale μ_h in the effective theory formalism reads

$$\mathcal{A}_{\mathrm{eff}}^{d\overline{s}\to\overline{d}s} = \langle\overline{d}s|\mathcal{H}_{\mathrm{eff}}^{\Delta F=2}|d\overline{s}\rangle = \sum_{i=1}^{5} C_i(\mu_h)\,\langle\overline{d}s|Q_i(\mu_h)|d\overline{s}\rangle + \sum_{j=1}^{3}\widetilde{C}_j(\mu_h)\,\langle\overline{d}s|\widetilde{Q}_j(\mu_h)|d\overline{s}\rangle\ . \tag{4.14}$$

Effective theory matrix elements

On the effective theory side we need the tree level matrix elements for the process $d\overline{s} \to \overline{d}s$. There are four possible insertions for each of the operators in the effective Hamiltonian, and one has to take into account relative minus signs between the corresponding diagrams (see Appendix C.1). The so obtained matrix elements read

$$\begin{aligned}
\langle\overline{d}s|Q_1|d\overline{s}\rangle =&\ 2(\overline{u}_s^\alpha\gamma^\mu P_L v_d^\alpha)(\overline{v}_s^\beta\gamma_\mu P_L u_d^\beta) - 2(\overline{u}_s^\alpha\gamma^\mu P_L u_d^\alpha)(\overline{v}_s^\beta\gamma_\mu P_L v_d^\beta)\,,\\
\langle\overline{d}s|\widetilde{Q}_1|d\overline{s}\rangle =&\ 2(\overline{u}_s^\alpha\gamma^\mu P_R v_d^\alpha)(\overline{v}_s^\beta\gamma_\mu P_R u_d^\beta) - 2(\overline{u}_s^\alpha\gamma^\mu P_R u_d^\alpha)(\overline{v}_s^\beta\gamma_\mu P_R v_d^\beta)\,,\\
\langle\overline{d}s|Q_2|d\overline{s}\rangle =&\ 2(\overline{u}_s^\alpha P_L v_d^\alpha)(\overline{v}_s^\beta P_L u_d^\beta) - 2(\overline{u}_s^\alpha P_L u_d^\alpha)(\overline{v}_s^\beta P_L v_d^\beta)\,,\\
\langle\overline{d}s|\widetilde{Q}_2|d\overline{s}\rangle =&\ 2(\overline{u}_s^\alpha P_R v_d^\alpha)(\overline{v}_s^\beta P_R u_d^\beta) - 2(\overline{u}_s^\alpha P_R u_d^\alpha)(\overline{v}_s^\beta P_R v_d^\beta)\,,\\
\langle\overline{d}s|Q_3|d\overline{s}\rangle =&\ 2(\overline{u}_s^\alpha P_L v_d^\beta)(\overline{v}_s^\beta P_L u_d^\alpha) - 2(\overline{u}_s^\alpha P_L u_d^\beta)(\overline{v}_s^\beta P_L v_d^\alpha)\,,\\
\langle\overline{d}s|\widetilde{Q}_3|d\overline{s}\rangle =&\ 2(\overline{u}_s^\alpha P_R v_d^\beta)(\overline{v}_s^\beta P_R u_d^\alpha) - 2(\overline{u}_s^\alpha P_R u_d^\beta)(\overline{v}_s^\beta P_R v_d^\alpha)\,,\\
\langle\overline{d}s|Q_4|d\overline{s}\rangle =&\ (\overline{u}_s^\alpha P_L v_d^\alpha)(\overline{v}_s^\beta P_R u_d^\beta) + (\overline{u}_s^\alpha P_R v_d^\alpha)(\overline{v}_s^\beta P_L u_d^\beta)\\
&- (\overline{u}_s^\alpha P_L u_d^\alpha)(\overline{v}_s^\beta P_R v_d^\beta) - (\overline{u}_s^\alpha P_R u_d^\alpha)(\overline{v}_s^\beta P_L v_d^\beta)\,,\\
\langle\overline{d}s|Q_5|d\overline{s}\rangle =&\ (\overline{u}_s^\alpha P_L v_d^\beta)(\overline{v}_s^\beta P_R u_d^\alpha) + (\overline{u}_s^\alpha P_R v_d^\beta)(\overline{v}_s^\beta P_L u_d^\alpha)\\
&- (\overline{u}_s^\alpha P_L u_d^\beta)(\overline{v}_s^\beta P_R v_d^\alpha) - (\overline{u}_s^\alpha P_R u_d^\beta)(\overline{v}_s^\beta P_L v_d^\alpha)\,,
\end{aligned} \tag{4.15}$$

Along this perturbative matrix element, we need to know the *hadronic matrix elements*, i.e. the renormalized matrix elements at the low scale which enter the physical observables in Eqs. (4.12), (4.13). A commonly used parametrization is [43]

$$\langle \bar{K}^0 | \hat{Q}_1(\mu) | K^0 \rangle = \frac{1}{3} M_K f_K^2 B_1(\mu)\,, \tag{4.16}$$

$$\langle \bar{K}^0 | \hat{Q}_2(\mu) | K^0 \rangle = -\frac{5}{24} \left(\frac{M_K}{m_s(\mu) + m_d(\mu)} \right)^2 M_K f_K^2 B_2(\mu)\,, \tag{4.17}$$

$$\langle \bar{K}^0 | \hat{Q}_3(\mu) | K^0 \rangle = \frac{1}{24} \left(\frac{M_K}{m_s(\mu) + m_d(\mu)} \right)^2 M_K f_K^2 B_3(\mu)\,, \tag{4.18}$$

$$\langle \bar{K}^0 | \hat{Q}_4(\mu) | K^0 \rangle = \frac{1}{4} \left(\frac{M_K}{m_s(\mu) + m_d(\mu)} \right)^2 M_K f_K^2 B_4(\mu)\,, \tag{4.19}$$

$$\langle \bar{K}^0 | \hat{Q}_5(\mu) | K^0 \rangle = \frac{1}{12} \left(\frac{M_K}{m_s(\mu) + m_d(\mu)} \right)^2 M_K f_K^2 B_5(\mu)\,, \tag{4.20}$$

where the notation $\hat{Q}_i(\mu)$ denotes the operators renormalised at the scale μ. The so-called Bag parameter B_i parametrize the deviation of the matrix elements from the vacuum insertion approximation. Their values are reported in the Appendix C.1.

The Wilson coefficients at the low scale are

$$C_r(\mu) = \sum_i \sum_s \left(b_i^{(r,s)} + \eta\, c_i^{(r,s)} \right) \eta^{a_i}\, C_s(M_S), \tag{4.21}$$

where, $C(M_S)$ are the Wilson coefficients calculated at the high scale, i.e. the SUSY scale. The parameter η is defined as $\eta = \alpha_s(M_S)/\alpha_s(m_t)$. The magic numbers a_i, b_i and c_i for the running at the low scale $\mu = 2$ GeV are given in the Appendix C.1.

Full theory calculation

We now want to determine the Wilson coefficients in the MSSM. Each of the Wilson coefficients can be decomposed into six different contributions according to the virtual particles that appear in the corresponding box diagrams of the full MSSM

$$C_i = \delta^{SM} C_i + \delta^{H^-} C_i + \delta^{\chi^-} C_i + \delta^{\chi^0} C_i + \delta^{\chi^0 g} C_i + \delta^{g} C_i\,. \tag{4.22}$$

The right hand side of the above expression features Standard Model contributions, contributions that involve charged Higgs particles, chargino contributions, neutralino contributions, contributions with both neutralinos and gluinos and finally pure gluino contributions. In our analysis we will focus only on pure gluino contributions which are depicted in Fig. 4.1.

The neutral K-meson mixing at leading order is purely given by box diagrams with heavy particles in the loop, therefore the momenta of the external particles can be set to zero. The evaluation of the box diagrams is then straightforward, but in the MSSM with general flavour mixing, usually a large number of different Dirac structures appear. These structures have to be projected onto the Dirac structures of the matrix elements (4.15) performing Fierz rearrangements. Due to the Majorana nature of the gluinos the *crossed* diagrams 4.1c and 4.1d feature *clashing fermion lines*. In order to calculate amplitudes for crossed diagrams we follow the prescriptions given in [44]. Our conventions for the charge-conjugation matrix C and charge-conjugate fields, which typically appear in the crossed diagrams, have been reported in Appendix A.2. Feynman rules for the MSSM can be found in [45].

The calculation of diagrams in Fig. 4.1 is done explicitly in Appendix C.2. The amplitudes are then summed together with the proper relative sign (the diagram 4.1a has opposite sign with respect to the other three as explained in the Appendix) and matched with the effective amplitude.

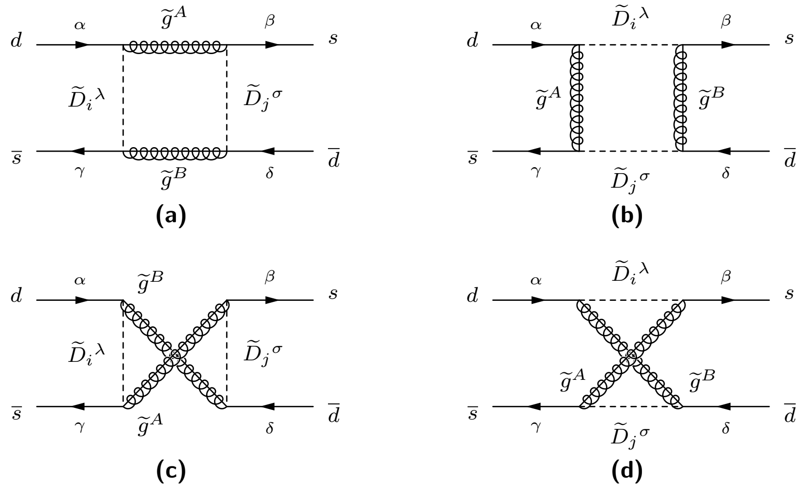


Figure 4.1: Gluino contributions to $K^0 - \overline{K}^0$ mixing

Wilson Coefficients

We get the following results for the gluino contributions to the Wilson coefficients.

$$\begin{aligned}\delta^g C_1 = &-\frac{g_s^4}{16\pi^2}\frac{1}{9}M_{\tilde g}^2 D_0(m_{\tilde D_i}^2, m_{\tilde D_j}^2, M_{\tilde g}^2, M_{\tilde g}^2)(Z_D)_{1i}(Z_D)_{1j}(Z_D^*)_{2i}(Z_D^*)_{2j}\\ &-\frac{g_s^4}{16\pi^2}\frac{11}{9}D_2(m_{\tilde D_i}^2, m_{\tilde D_j}^2, M_{\tilde g}^2, M_{\tilde g}^2)(Z_D)_{1i}(Z_D)_{1j}(Z_D^*)_{2i}(Z_D^*)_{2j}\,,\end{aligned} \tag{4.23}$$

$$\begin{aligned}\delta^g \widetilde{C}_1 = &-\frac{g_s^4}{16\pi^2}\frac{1}{9}M_{\tilde g}^2 D_0(m_{\tilde D_i}^2, m_{\tilde D_j}^2, M_{\tilde g}^2, M_{\tilde g}^2)(Z_D)_{4i}(Z_D)_{4j}(Z_D^*)_{5i}(Z_D^*)_{5j}\\ &-\frac{g_s^4}{16\pi^2}\frac{11}{9}D_2(m_{\tilde D_i}^2, m_{\tilde D_j}^2, M_{\tilde g}^2, M_{\tilde g}^2)(Z_D)_{4i}(Z_D)_{4j}(Z_D^*)_{5i}(Z_D^*)_{5j}\,,\end{aligned} \tag{4.24}$$

$$\delta^g C_2 = -\frac{g_s^4}{16\pi^2}\frac{17}{18}M_{\tilde g}^2 D_0(m_{\tilde D_i}^2, m_{\tilde D_j}^2, M_{\tilde g}^2, M_{\tilde g}^2)(Z_D)_{1i}(Z_D)_{1j}(Z_D^*)_{5i}(Z_D^*)_{5j}\,, \tag{4.25}$$

$$\delta^g \widetilde{C}_2 = -\frac{g_s^4}{16\pi^2}\frac{17}{18}M_{\tilde g}^2 D_0(m_{\tilde D_i}^2, m_{\tilde D_j}^2, M_{\tilde g}^2, M_{\tilde g}^2)(Z_D)_{4i}(Z_D)_{4j}(Z_D^*)_{2i}(Z_D^*)_{2j}\,, \tag{4.26}$$

$$\delta^g C_3 = +\frac{g_s^4}{16\pi^2}\frac{1}{6}M_{\tilde g}^2 D_0(m_{\tilde D_i}^2, m_{\tilde D_j}^2, M_{\tilde g}^2, M_{\tilde g}^2)(Z_D)_{1i}(Z_D)_{1j}(Z_D^*)_{5i}(Z_D^*)_{5j}\,, \tag{4.27}$$

$$\delta^g \widetilde{C}_3 = +\frac{g_s^4}{16\pi^2}\frac{1}{6}M_{\tilde g}^2 D_0(m_{\tilde D_i}^2, m_{\tilde D_j}^2, M_{\tilde g}^2, M_{\tilde g}^2)(Z_D)_{4i}(Z_D)_{4j}(Z_D^*)_{2i}(Z_D^*)_{2j}\,, \tag{4.28}$$

$$\begin{aligned}\delta^g C_4 = &-\frac{g_s^4}{16\pi^2}\frac{7}{3}M_{\tilde g}^2 D_0(m_{\tilde D_i}^2, m_{\tilde D_j}^2, M_{\tilde g}^2, M_{\tilde g}^2)(Z_D)_{1i}(Z_D)_{4j}(Z_D^*)_{2i}(Z_D^*)_{5j}\\ &+\frac{g_s^4}{16\pi^2}\frac{2}{9}D_2(m_{\tilde D_i}^2, m_{\tilde D_j}^2, M_{\tilde g}^2, M_{\tilde g}^2)(Z_D)_{1i}(Z_D)_{4j}\{6(Z_D^*)_{2i}(Z_D^*)_{5j} + 11(Z_D^*)_{2j}(Z_D^*)_{5i}\}\,,\end{aligned} \tag{4.29}$$

$$\begin{aligned}\delta^g C_5 = &-\frac{g_s^4}{16\pi^2}\frac{1}{9}M_{\tilde g}^2 D_0(m_{\tilde D_i}^2, m_{\tilde D_j}^2, M_{\tilde g}^2, M_{\tilde g}^2)(Z_D)_{1i}(Z_D)_{4j}(Z_D^*)_{2i}(Z_D^*)_{5j}\\ &+\frac{g_s^4}{16\pi^2}\frac{10}{9}D_2(m_{\tilde D_i}^2, m_{\tilde D_j}^2, M_{\tilde g}^2, M_{\tilde g}^2)(Z_D)_{1i}(Z_D)_{4j}\{3(Z_D^*)_{2j}(Z_D^*)_{5i} - 2(Z_D^*)_{2i}(Z_D^*)_{5j}\}\,.\end{aligned} \tag{4.30}$$

In the above expressions we left out explicit summation signs in the results. The summation on the indices $i, j = 0, \dots, 6$ is understood. The loop functions $D_0(m_1^2, m_2^2, m_3^2, m_4^2)$ and $D_2(m_1^2, m_2^2, m_3^2, m_4^2)$ are defined in the Appendix.

Exact diagonalization in a two-generation framework

The above coefficients can be expressed in terms of fewer parameter when a specific framework is chosen. Here we give the results for a two-generation framework, that is assuming that the underlying $s \to d$ transition cannot proceed by the double flavour transition $(s \to b) \times (b \to d)$. We also neglect the small Yukawa couplings for the first two generations and therefore the corresponding LR/RL soft terms. Under these assumptions the mass matrix $\mathcal{M}_D^2$ is block diagonal. This implies that also the unitary diagonalization matrix Z_D is block diagonal and can be described by means of two angles θ_L, θ_R and two phases ϕ_L, ϕ_R (see Appendix C.4).

The gluino corrections to the Wilson coefficients turn out to be

$$\delta^g C_1 = -\frac{g_s^4}{16\pi^2}\frac{1}{9}\left(s_L c_L e^{-i\phi_L}\right)^2\left(M_{\tilde g}^2 B_{qq}(M_{\tilde g}^2, M_{\tilde g}^2) + 11 C_{qq}(M_{\tilde g}^2, M_{\tilde g}^2)\right), \tag{4.31}$$

$$\delta^g \widetilde{C}_1 = -\frac{g_s^4}{16\pi^2}\frac{1}{9}\left(s_R c_R e^{-i\phi_R}\right)^2\left(M_{\tilde g}^2 B_{dd}(M_{\tilde g}^2, M_{\tilde g}^2) + 11 C_{dd}(M_{\tilde g}^2, M_{\tilde g}^2)\right), \tag{4.32}$$

$$\delta^g C_4 = -\frac{g_s^4}{16\pi^2}\frac{1}{3}\left(s_L c_L e^{-i\phi_L}\right)\left(s_R c_R e^{-i\phi_R}\right)\left(7 M_{\tilde g}^2 B_{qd}(M_{\tilde g}^2, M_{\tilde g}^2) - 4 C_{qd}(M_{\tilde g}^2, M_{\tilde g}^2)\right), \tag{4.33}$$

$$\delta^g C_5 = -\frac{g_s^4}{16\pi^2}\frac{1}{9}\left(s_L c_L e^{-i\phi_L}\right)\left(s_R c_R e^{-i\phi_R}\right)\left(M_{\tilde g}^2 B_{qd}(M_{\tilde g}^2, M_{\tilde g}^2) + 20 C_{qd}(M_{\tilde g}^2, M_{\tilde g}^2)\right), \tag{4.34}$$

$$\delta^g C_2 = \delta^g \widetilde{C}_2 = \delta^g C_3 = \delta^g \widetilde{C}_3 = 0\,. \tag{4.35}$$

The $B(M_{\tilde g}^2, M_{\tilde g}^2)$ and $C(M_{\tilde g}^2, M_{\tilde g}^2)$ functions are given in the Appendix.

Mass Insertion expansion

When the sfermion masses are almost degenerate one can obtain the usual coefficients in the mass insertion approximation and expand them around the central value:

$$m_{\tilde D_i}^2 = m_{\tilde D}^2 + \delta m_{\tilde D_i}^2\,. \tag{4.36}$$

We expand also the loop integrals depending on the sfermion masses:

$$\begin{aligned} f(m_{\tilde D_i}^2, m_{\tilde D_j}^2, \dots) \approx{}& f(m_{\tilde D}^2, m_{\tilde D}^2, \dots) + (m_{\tilde D_i}^2 - m_{\tilde D}^2)\left.\frac{\partial f(m_{\tilde D_i}^2, m_{\tilde D}^2, \dots)}{\partial m_{\tilde D_i}^2}\right|_{m_{\tilde D_i}^2 = m_{\tilde D}^2} \\ &+ (m_{\tilde D_j}^2 - m_{\tilde D}^2)\left.\frac{\partial f(m_{\tilde D}^2, m_{\tilde D_j}^2, \dots)}{\partial m_{\tilde D_j}^2}\right|_{m_{\tilde D_j}^2 = m_{\tilde D}^2} + \dots \end{aligned} \tag{4.37}$$

and use the definitions of the mixing matrices Z_D and their unitarity to simplify expression containing combinations of the sfermion mixing angles:

$$Z_D^{ik} Z_D^{jk\star} = \delta^{ij}\,, \qquad Z_D^{ik} Z_D^{jk\star} m_{\tilde D_k}^2 = \mathcal{M}_{\tilde D}^{ij}\,. \tag{4.38}$$

Performing this expansion on the Wilson coefficients eqs. (4.23)-(4.30) we find

$$\delta^g C_1 = -\frac{\alpha_s^2}{m_{\tilde{D}}^2}\frac{1}{36}\Big[4x_{gq}f_6(x_{gq}) + 11\tilde{f}_6(x_{gq})\Big](\delta_d^{LL})_{12}^2\,, \tag{4.39}$$

$$\delta^g \tilde{C}_1 = -\frac{\alpha_s^2}{m_{\tilde{D}}^2}\frac{1}{36}\Big[4x_{gq}f_6(x_{gq}) + 11\tilde{f}_6(x_{gq})\Big](\delta_d^{RR})_{12}^2\,, \tag{4.40}$$

$$\delta^g C_2 = -\frac{\alpha_s^2}{m_{\tilde{D}}^2}\frac{17}{18}x_{gq}f_6(x_{gq})(\delta_d^{RL})_{12}^2\,, \tag{4.41}$$

$$\delta^g \tilde{C}_2 = -\frac{\alpha_s^2}{m_{\tilde{D}}^2}\frac{17}{18}x_{gq}f_6(x_{gq})(\delta_d^{LR})_{12}^2\,, \tag{4.42}$$

$$\delta^g C_3 = +\frac{\alpha_s^2}{m_{\tilde{D}}^2}\frac{1}{6}x_{gq}f_6(x_{gq})(\delta_d^{RL})_{12}^2\,, \tag{4.43}$$

$$\delta^g \tilde{C}_3 = +\frac{\alpha_s^2}{m_{\tilde{D}}^2}\frac{1}{6}x_{gq}f_6(x_{gq})(\delta_d^{LR})_{12}^2\,, \tag{4.44}$$

$$\begin{aligned}\delta^g C_4 = &-\frac{\alpha_s^2}{m_{\tilde{D}}^2}\frac{1}{3}\Big[7x_{gq}f_6(x_{gq}) - \tilde{f}_6(x_{gq})\Big](\delta_d^{LL})_{12}(\delta_d^{RR})_{12}\\ &+\frac{\alpha_s^2}{m_{\tilde{D}}^2}\frac{11}{18}\tilde{f}_6(x_{gq})(\delta_d^{LR})_{12}(\delta_d^{RL})_{12}\,,\end{aligned} \tag{4.45}$$

$$\begin{aligned}\delta^g C_5 = &-\frac{\alpha_s^2}{m_{\tilde{D}}^2}\frac{1}{9}\Big[x_{gq}f_6(x_{gq}) + 5\tilde{f}_6(x_{gq})\Big](\delta_d^{LL})_{12}(\delta_d^{RR})_{12}\\ &+\frac{\alpha_s^2}{m_{\tilde{D}}^2}\frac{5}{6}\tilde{f}_6(x_{gq})(\delta_d^{LR})_{12}(\delta_d^{RL})_{12}\,,\end{aligned} \tag{4.46}$$

where $x_{gq} = M_{\tilde{g}}^2/m_{\tilde{D}}^2$ and the functions $f_6(x)$ and $\tilde{f}_6(x)$ are defined in the Appendix.

4.3 $\Delta F = 1$ processes

The radiative $b \to s\gamma$ decay

We now turn to the MSSM contribution to $\Delta F = 1$ processes, focusing on the magnetic and chromomagnetic operators. The effective Hamiltonian relevant for the $b \to s\gamma$ transition is

$$\mathcal{H}_{\rm eff}^{\Delta F=1} = C_{7\gamma}Q_{7\gamma} + C'_{7\gamma}Q'_{7\gamma} + C_{8g}Q_{8g} + C'_{8g}Q'_{8g}\,, \tag{4.47}$$

where the $Q_{7\gamma}^{(\prime)}$, $Q_{8g}^{(\prime)}$ operators are, in the case of $b \to s\,\gamma$, $b \to sg$,

$$Q_{7\gamma} = \frac{e}{16\pi^2}m_b(\bar{s}\sigma^{\mu\nu}P_R b)F_{\mu\nu}\,, \qquad Q'_{7\gamma} = \frac{e}{16\pi^2}m_b(\bar{s}\sigma^{\mu\nu}P_L b)F_{\mu\nu}\,, \tag{4.48}$$

$$Q_{8g} = \frac{g_s}{16\pi^2}m_b(\bar{s}\,T^A\sigma^{\mu\nu}P_R b)G_{\mu\nu}^A\,, \qquad Q'_{8g} = \frac{g_s}{16\pi^2}m_b(\bar{s}\,T^A\sigma^{\mu\nu}P_L b)G_{\mu\nu}^A\,. \tag{4.49}$$

In order to obtain the Wilson Coefficients one needs to calculate the processes both in the MSSM and with the effective Hamiltonian and apply the matching condition to the amplitudes as in (4.5). In general, the MSSM one-loop contributions to the Wilson coefficients of the magnetic and chromomagnetic operators can be decomposed into SM contributions, charged Higgs contributions, chargino contributions, neutralino contributions and gluino contributions.

$$C^{(\prime)}_{7\gamma/8g} = \delta^{SM} C^{(\prime)}_{7\gamma/8g} + \delta^{H^-} C^{(\prime)}_{7\gamma/8g} + \delta^{\chi^-} C^{(\prime)}_{7\gamma/8g} + \delta^{\chi^0} C^{(\prime)}_{7\gamma/8g} + \delta^{g} C^{(\prime)}_{7\gamma/8g} \,. \tag{4.50}$$

We will concentrate on the gluino contributions only.

Let us now compute the matrix elements of these operators. Taking advantage of the antisymmetry of $\sigma^{\mu\nu}$ we can write

$$F_{\mu\nu}\sigma^{\mu\nu} = (\partial_\mu A_\nu - \partial_\nu A_\mu)\sigma^{\mu\nu} = -2(\partial_\nu A_\mu)\sigma^{\mu\nu} \tag{4.51}$$

and similarly for $G^A_{\mu\nu}\sigma^{\mu\nu}$. This translates to $-2ik_\nu \widetilde{A}_\mu(k)\sigma^{\mu\nu}$ when taking the Fourier transform. After performing the contractions with external fields, we get

$$\langle \gamma s | Q^{(\prime)}_{7\gamma} | b \rangle = -\frac{e}{16\pi^2} m_b \, 2\epsilon^*_\mu \overline{u}_s(p') \, i\sigma^{\mu\nu} k_\nu P_{R(L)} \, u_b(p) \,. \tag{4.52}$$

$$\langle \gamma s | Q^{(\prime)}_{8\gamma} | b \rangle = -\frac{g_s}{16\pi^2} m_b \, 2\epsilon^{*A}_\mu \left(\overline{u}^\beta_s \, T^A_{\beta\alpha} i\sigma^{\mu\nu} P_{R(L)} \, u^\alpha_b \right) k_\nu \,. \tag{4.53}$$

Where $k_\mu = (p - p')_\mu$ for the energy-momentum conservation, p being the four-momentum of the incoming particle and p' that of the outgoing massive particle.

In the MSSM there are three gluino mediated diagrams contributing at one loop order at the process $b \to s\gamma$, which have been depicted in Fig. 4.2. The first one is the correction to the vertex, while the other two are loop corrections to the external legs and are reducible diagrams. The "external leg correction" diagrams do not contribute to the Wilson coefficients. All the three diagrams are divergent but the sum is finite as a consequence of the Ward-Takahashi identity. We show explicitly this cancellation in Appendix. In the case of $b \to sg$ there are four one loop gluino contributions, which are depicted in Fig. 4.3). Only the first two give a contribution to the chromomagnetic operators.

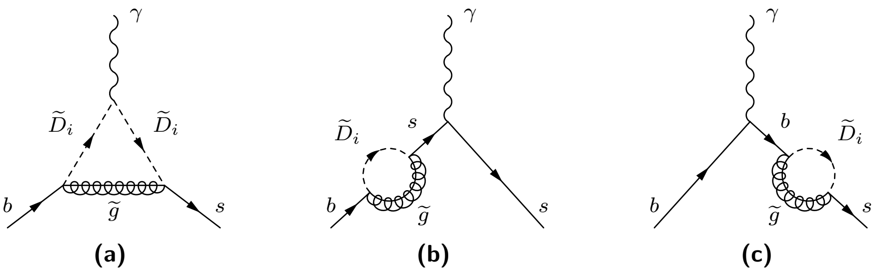


Figure 4.2: Gluino contributions to the magnetic penguins

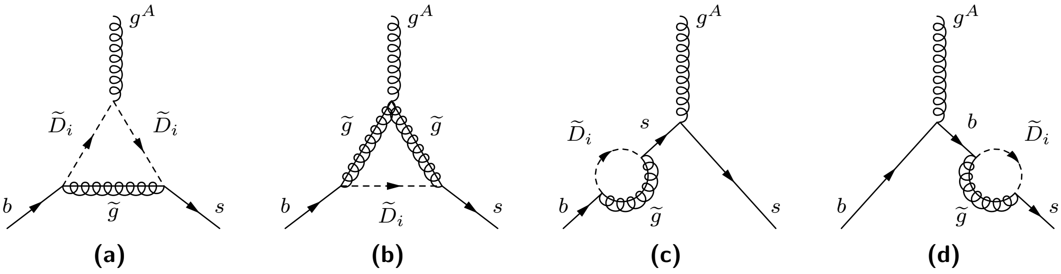


Figure 4.3: Gluino contributions to the chromomagnetic penguins

The Wilson coefficients $\delta^g C_{7\gamma^{(\prime)}}$ and $\delta^g C_{8\gamma^{(\prime)}}$ are obtainded comparing the amplitudes calculated in Appendix D.1 with the matrix elements of the effective operators $Q_{7\gamma}^{(\prime)}$ and $Q_{8\gamma}^{(\prime)}$ in Eqs. (4.52), (4.53). We get

$$\delta^g C_{7\gamma} = \frac{16 g_s^2}{9 M_{\tilde{g}}^2} \left\{ \left((Z_D^*)_{2i} (Z_D)_{3i} + \frac{m_s}{m_b} (Z_D^*)_{5i} (Z_D)_{6i} \right) f_{7\gamma}^{[1]}(\xi_i) - \frac{M_{\tilde{g}}}{m_b} (Z_D^*)_{2i} (Z_D)_{6i} f_{7\gamma}^{[2]}(\xi_i) \right\},$$
$$\delta^g C'_{7\gamma} = \frac{16 g_s^2}{9 M_{\tilde{g}}^2} \left\{ \left((Z_D^*)_{5i} (Z_D)_{6i} + \frac{m_s}{m_b} (Z_D^*)_{2i} (Z_D)_{3i} \right) f_{7\gamma}^{[1]}(\xi_i) - \frac{M_{\tilde{g}}}{m_b} (Z_D^*)_{5i} (Z_D)_{3i} f_{7\gamma}^{[2]}(\xi_i) \right\},$$

$$\delta^g C_{8g} = \frac{4 g_s^2}{3 M_{\tilde{g}}^2} \left\{ \left((Z_D^*)_{2i} (Z_D)_{3i} + \frac{m_s}{m_b} (Z_D^*)_{5i} (Z_D)_{6i} \right) f_{8g}^{[1]}(\xi_i) - \frac{M_{\tilde{g}}}{m_b} (Z_D^*)_{2i} (Z_D)_{6i} f_{8g}^{[2]}(\xi_i) \right\},$$
$$\delta^g C'_{8g} = \frac{4 g_s^2}{3 M_{\tilde{g}}^2} \left\{ \left((Z_D^*)_{5i} (Z_D)_{6i} + \frac{m_s}{m_b} (Z_D^*)_{2i} (Z_D)_{3i} \right) f_{8g}^{[1]}(\xi_i) - \frac{M_{\tilde{g}}}{m_b} (Z_D^*)_{5i} (Z_D)_{3i} f_{8g}^{[2]}(\xi_i) \right\}.$$

We used $\xi_i = m_{\tilde{D}_i}^2 / M_{\tilde{g}}^2$ and the loop functions $f_{7\gamma}^{[1]}(\xi)$, $f_{7\gamma}^{[2]}(\xi)$, $f_{8g}^{[1]}(\xi)$, $f_{8g}^{[2]}(\xi)$ are defined in the Appendix.

The radiative $\mu \to e\gamma$ decay

Within SUSY models, LFV effects relevant to charged leptons originate from any misalignment between fermion and sfermion mass eigenstates. Once non-vanishing LFV entries in the slepton mass matrices are present, LFV rare decays like $\ell_i \to \ell_j \gamma$ are naturally induced by one-loop diagrams with the exchange of gauginos and sleptons.

The decay $\ell_i \to \ell_j \gamma$ is described by the dipole operator and the corresponding amplitude reads

$$T = e m_{\ell_i} \epsilon^{\lambda *} \overline{u}_j (p-q) [i q^\nu \sigma_{\lambda\nu} (A_L P_L + A_R P_R)] u_i(p)\,, \tag{4.54}$$

where p and q are momenta of the leptons ℓ_k and of the photon respectively and $A_{L,R}$ are the two possible amplitudes entering the process. The lepton mass factor m_{ℓ_i} is associated to the chirality flip present in this transition. The decay rate for $\ell_i \to \ell_j \gamma$ is

$$\Gamma(\ell_i \to \ell_j \gamma) = \frac{e^2}{16\pi} m_{\ell_i}^5 (|A_L^{ij}|^2 + |A_R^{ij}|^2). \tag{4.55}$$

Using the the well known tree level formula for the SM decay width $\Gamma(\ell_i \to \ell_j \nu_i \bar{\nu_j}) = G_F^2 m_{\ell_i}^5 / 192\pi^3$ we can write the branching ratio of $\ell_i \to \ell_j \gamma$ as

$$\frac{\mathrm{BR}(\ell_i \to \ell_j \gamma)}{\mathrm{BR}(\ell_i \to \ell_j \nu_i \bar{\nu_j})} = \frac{48\pi^3 \alpha}{G_F^2} (|A_L^{ij}|^2 + |A_R^{ij}|^2)\,. \tag{4.56}$$

Within the MSSM, contributions to the $\mu \to e\gamma$ process arise at one loop from diagrams involving neutralino and charginos in Fig. 4.4. The first three diagrams are the neutralino-slepton vertex correction and the neutralino-slepton external legs corrections. The last three diagrams are the chargino-sneutrino vertex correction and external legs corrections. Each of the coefficients $A_{L,R}$ can be decomposed into two contributions

$$A_{L,R} = A_{L,R}^{(n)} + A_{L,R}^{(c)}\,,$$

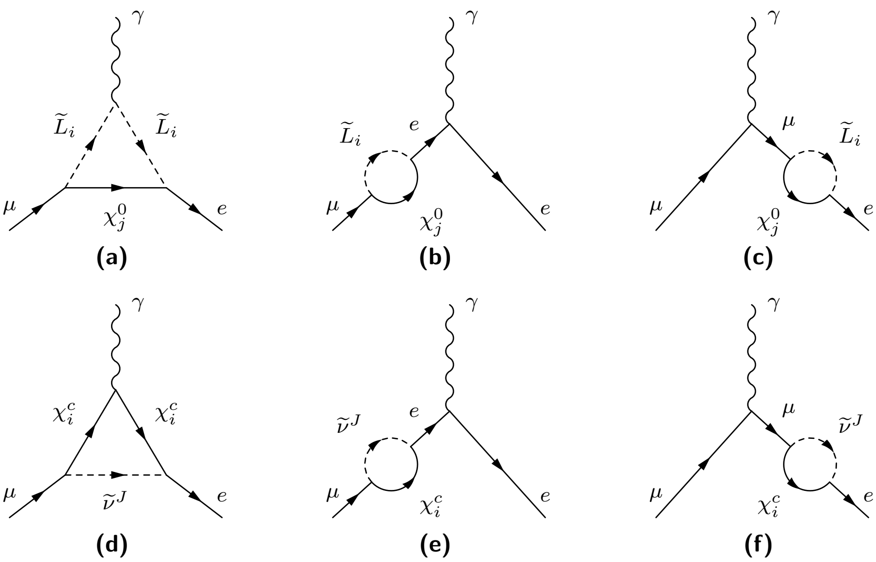


Figure 4.4: Neutralino-slepton and chargino-sneutrino one loop contributions to $\mu \to e\gamma$ process.

where $A^{(n)}_{L,R}$ and $A^{(c)}_{L,R}$ stand for the contributions from the neutralino loops and from the chargino loops, respectively. We calculate them in Appendix D.3. The results are

$$A_L^{21(n)} = -\frac{1}{4\pi^2}\frac{1}{m^2_{\widetilde{L}_i}}\left[\frac{m_{\chi^0_j}}{m_\mu} N_L^{1ji} N_R^{2ji*} f^{[2]}_{7\gamma}(\zeta_{ij}) + N_L^{1ji} N_L^{2ji*} f^{[1]}_{7\gamma}(\zeta_{ji})\right], \tag{4.57}$$

$$A_R^{21(n)} = -\frac{1}{4\pi^2}\frac{1}{m^2_{\widetilde{L}_i}}\left[\frac{m_{\chi^0_j}}{m_\mu} N_L^{1ji} N_R^{2ji*} f^{[2]}_{7\gamma}(\zeta_{ji}) + N_R^{1ji} N_R^{2ji*} f^{[1]}_{7\gamma}(\zeta_{ji})\right], \tag{4.58}$$

and

$$A_L^{21(c)} = -\frac{1}{16\pi^2}\frac{1}{m^2_{\widetilde{\nu}_J}}\left[\frac{m_{\chi_i}}{m_\mu} C_L^{1Ji} C_R^{2Ji*} f_2(\eta_{iJ}) - 4 C_L^{1Ji} C_L^{2Ji*} f^{[1]}_{7\gamma}(\eta_{iJ})\right], \tag{4.59}$$

$$A_R^{21(c)} = -\frac{1}{16\pi^2}\frac{1}{m^2_{\widetilde{\nu}_J}}\left[\frac{m_{\chi_i}}{m_\mu} C_R^{1Ji} C_L^{2Ji*} f_2(\eta_{iJ}) - 4 C_R^{1Ji} C_R^{2Ji*} f^{[1]}_{7\gamma}(\eta_{iJ}),\right] \tag{4.60}$$

where $\zeta_{ji} = m^2_{\chi^0_j}/m^2_{\widetilde{L}_i}$, $\eta_{iJ} = m^2_{\chi_i}/m^2_{\widetilde{\nu}_J}$ and the loop functions as well as the coefficients $N^{Iji}_{L,R}$, $C^{IJi}_{L,R}$ are defined in the appendices.

Besides $\ell_i \to \ell_j\gamma$, there are also other promising LFV channels, such as $\ell_i \to \ell_j\ell_k\ell_k$ and μ-e conversion in nuclei. However, within SUSY models, these processes are typically dominated by the dipole transition $\ell_i \to \ell_j\gamma^*$ leading to the prediction,

$$\begin{aligned}\frac{\mathrm{BR}(\ell_i \to \ell_j\ell_k\bar{\ell}_k)}{\mathrm{BR}(\ell_i \to \ell_j\bar{\nu}_j\nu_i)} &\simeq \frac{\alpha_{el}}{3\pi}\left(\log\frac{m^2_{\ell_i}}{m^2_{\ell_k}} - 3\right)\frac{\mathrm{BR}(\ell_i \to \ell_j\gamma)}{\mathrm{BR}(\ell_i \to \ell_j\bar{\nu}_j\nu_i)}\,, \\ \mathrm{CR}(\mu \to e \text{ in N}) &\simeq \alpha_{\rm em} \times \mathrm{BR}(\mu \to e\gamma)\,,\end{aligned} \tag{4.61}$$

where CR stands for the conversion rate of $\mu \to e$ in Nuclei.

Mass insertion expansion

In the mass insertion approximation and keeping only the $\tan\beta$ enhanced terms we find that

$$
\begin{aligned}
A_L^{ij(n)} &= \frac{\alpha_2}{4\pi}\frac{(m_{LL}^2)_{ij}}{m_L^4}\frac{\mu M_2 t_\beta}{|M_2|^2-|\mu|^2}(f_{2n}(x_{2L})-f_{2n}(x_{\mu L})) \\
&+ \frac{\alpha_Y}{4\pi}\frac{(m_{LL}^2)_{ij}}{m_L^4}\frac{\mu M_1 t_\beta}{|M_1|^2-|\mu|^2}(-f_{2n}(x_{1L})+f_{2n}(x_{\mu L})) \\
&- \frac{\alpha_Y}{4\pi}\frac{M_1}{m_\mu}\frac{(m_{LR}^2)^*_{ji}}{m_L^2-m_R^2}\left[\frac{1}{m_L^2}f_{3n}(x_{1L})-\frac{1}{m_R^2}f_{3n}(x_{1R})\right] \\
&+ \frac{\alpha_Y}{4\pi}\frac{M_1}{m_\mu}\frac{(m_{LL}^2 m_{LR}^2)^*_{ji}}{(m_L^2-m_R^2)m_L^2}\left[\frac{m_L^2}{m_L^2-m_R^2}\left(\frac{1}{m_L^2}f_{3n}(x_{1L})-\frac{1}{m_R^2}f_{3n}(x_{1R})\right)+\frac{2}{m_L^2}f_{2n}(x_{1L})\right] \\
&+ \frac{\alpha_Y}{2\pi}\frac{M_1}{m_\mu}\frac{(m_{LL}^2 m_{LR}^2 m_{RR}^2)^*_{ji}}{(m_L^2-m_R^2)^2 m_L^2 m_R^2}\times \\
&\left[\frac{m_L^2 m_R^2}{m_L^2-m_R^2}\left(\frac{1}{m_L^2}f_{3n}(x_{1L})-\frac{1}{m_R^2}f_{3n}(x_{1R})\right)+\frac{m_R^2}{m_L^2}f_{2n}(x_{1L})+\frac{m_L^2}{m_R^2}f_{2n}(x_{1R})\right],
\end{aligned}
\tag{4.62}
$$

$$
A_L^{ij(c)} = \frac{\alpha_2}{4\pi}\frac{(m_{LL}^2)_{ij}}{m_L^4}\frac{\mu M_2 t_\beta}{|M_2|^2-|\mu|^2}(f_{2c}(x_{2L})-f_{2c}(x_{\mu L}))\,, \tag{4.63}
$$

$$
\begin{aligned}
A_R^{ij(n)} &= \frac{\alpha_Y}{2\pi}\frac{(m_{RR}^2)_{ij}}{m_R^4}\frac{\mu M_1 t_\beta}{|M_1|^2-|\mu|^2}(f_{2n}(x_{1R})-f_{2n}(x_{\mu R})) \\
&- \frac{\alpha_Y}{4\pi}\frac{M_1}{m_\mu}\frac{(m_{LR}^2)_{ij}}{m_L^2-m_R^2}\left[\frac{1}{m_L^2}f_{3n}(x_{1L})-\frac{1}{m_R^2}f_{3n}(x_{1R})\right] \\
&- \frac{\alpha_Y}{4\pi}\frac{M_1}{m_\mu}\frac{(m_{LR}^2 m_{RR}^2)_{ij}}{(m_L^2-m_R^2)m_R^2}\left[\frac{m_R^2}{m_L^2-m_R^2}\left(\frac{1}{m_L^2}f_{3n}(x_{1L})-\frac{1}{m_R^2}f_{3n}(x_{1R})\right)+\frac{2}{m_R^2}f_{2n}(x_{1R})\right] \\
&+ \frac{\alpha_Y}{2\pi}\frac{M_1}{m_\mu}\frac{(m_{LL}^2 m_{LR}^2 m_{RR}^2)_{ij}}{(m_L^2-m_R^2)^2 m_L^2 m_R^2}\times \\
&\left[\frac{m_L^2 m_R^2}{m_L^2-m_R^2}\left(\frac{1}{m_L^2}f_{3n}(x_{1L})-\frac{1}{m_R^2}f_{3n}(x_{1R})\right)+\frac{m_R^2}{m_L^2}f_{2n}(x_{1L})+\frac{m_L^2}{m_R^2}f_{2n}(x_{1R})\right].
\end{aligned}
\tag{4.64}
$$

Here $(m_{LR}^2)_{ii} = m_{\ell_i}(A_i-\mu^* t_\beta)$, $x_{iA} = |M_i^2|/m_A^2$, $x_{\mu A} = |\mu|^2/m_A^2$ with $i = 1, 2$ and $A = L, R$. The explicit expressions for the loop functions are given in Appendix E.

4.4 $\Delta F = 0$ processes

The SM predictions for electric dipole moments are very far from the present experimental sensitivity. Any experimental observation of EDMs would therefore represent a very clean signal of the presence of NP effects. Given that the EDMs are CP-violating but flavour conserving observables, they do not require in principle any source of flavour violation, hence, we refer to them as $\Delta F = 0$ processes. The electron EDMs can be obtained starting from the effective CP-odd Lagrangian

$$
\mathcal{L}_{\rm eff} = -\frac{d_e}{2}\bar\psi_e(F\cdot\sigma)\gamma_5\psi_e + \sum_{i,j} C_{ij}\,(\bar\psi_i\psi_i)(\bar\psi_j i\gamma_5\psi_j) + \ldots, \tag{4.65}
$$

where the coefficients C_{ij} are relative to the dimension-six CP-odd four-Fermi interaction operators. The main obstacles to fully exploit the NP sensitivity of the EDMs is that experimentally, one measures the EDMs of composite systems, as heavy atoms, molecules or the neutron EDM, while the theoretical predictions are relative to the EDMs of constituent particles. Therefore a matching between quarks and leptons EDMs into physical EDMs is necessary and this induces unavoidable uncertainties. The electron EDM is related to the thallium EDM ($d_{\rm Tl}$) as [46]

$$d_{\rm Tl} = -585\, d_e - e\, 43\, {\rm GeV}\, C_S^{(0)} \ , \tag{4.66}$$

where $C_s^{(0)}$ is given by a combination of the coefficients C_{ij}.

The electron EDMs receive contributions by the only neutralino and chargino sectors [47]. They read

$$\{d_e\}_{\chi^0} = -\frac{1}{4\pi^2}\frac{m_{\chi^0_j}}{m^2_{\tilde{L}_i}}\,{\rm Im}\left[N_R^{1ji}N_L^{1ji*}\right] f^{[2]}_{7\gamma}(\zeta_{ij})\,, \tag{4.67}$$

$$\{d_e\}_{\chi^\pm} = -\frac{1}{16\pi^2}\frac{m_{\chi_i}}{m^2_{\tilde{\nu}_J}}\,{\rm Im}\left[C_R^{1Ji}C_L^{1Ji*}\right] f_2(\eta_{iJ})\,, \tag{4.68}$$

where $\zeta_{ji} = m^2_{\chi^0_j}/m^2_{\tilde{L}_i}$, $\eta_{iJ} = m^2_{\chi_i}/m^2_{\tilde{\nu}_J}$. The summation over the indices, each upon its relative range, is understood.

Mass insertion approximation

In the mass insertion approximation and keeping only the $\tan\beta$ enhanced terms we find that

$$\begin{aligned}
\frac{d_i^{(n)}}{e} &= \frac{\alpha_2}{8\pi}\frac{m_{l_i}}{m_L^2}\frac{{\rm Im}(\mu M_2)}{|M_2|^2-|\mu|^2}t_\beta\left[f_{3n}(x_{2L}) - f_{3n}(x_{\mu L})\right]\\
&+ \frac{\alpha_Y}{4\pi}\frac{m_{l_i}}{m_R^2}\frac{{\rm Im}(\mu M_1)}{|M_1|^2-|\mu|^2}t_\beta\left[f_{3n}(x_{1R}) - f_{3n}(x_{\mu R})\right]\\
&- \frac{\alpha_Y}{8\pi}\frac{m_{l_i}}{m_L^2}\frac{{\rm Im}(\mu M_1)}{|M_1|^2-|\mu|^2}t_\beta\left[f_{3n}(x_{1L}) - f_{3n}(x_{\mu L})\right]\\
&+ \frac{\alpha_Y}{4\pi}\frac{{\rm Im}(M_1 m^2_{LR})_{ii}}{m_L^2-m_R^2}\left[\frac{1}{m_L^2}f_{3n}(x_{1L}) - \frac{1}{m_R^2}f_{3n}(x_{1R})\right]\\
&- \frac{\alpha_Y}{2\pi}\frac{{\rm Im}(M_1 m^2_{LL}m^2_{LR}m^2_{RR})_{ii}}{(m_L^2-m_R^2)^2}\times\\
&\left[\frac{\frac{1}{m_L^2}f_{3n}(x_{1L}) - \frac{1}{m_R^2}f_{3n}(x_{1R})}{m_L^2-m_R^2} + \frac{1}{m_L^4}f_{2n}(x_{1L}) + \frac{1}{m_R^4}f_{2n}(x_{1R})\right]\ ,
\end{aligned} \tag{4.69}$$

$$\frac{d_i^{(c)}}{e} = -\frac{\alpha_2}{8\pi}\frac{m_{l_i}}{m_L^2}\frac{{\rm Im}(\mu M_2)}{|M_2|^2-|\mu|^2}t_\beta\left[f_c^{LR}(x_{2L}) - f_c^{LR}(x_{\mu L})\right]\ , \tag{4.70}$$

where $(m^2_{LR})_{ii} = m_{\ell_i}(A_i - \mu^* t_\beta)$, $x_{iA} = |M_i^2|/m_A^2$, $x_{\mu A} = |\mu|^2/m_A^2$ with $i = 1, 2$ and $A = L, R$. The explicit expressions for the loop functions are given in Appendix E.

Chapter 5

Flavour Physics vs. the Higgs boson mass in SUSY

The first phase of the LHC experiments with the proton-proton runs at 7 and 8 TeV was concluded in December 2012. The accelerator is now shut down till 2015 to allow the energy increase up to 13 and 14 TeV. The main result so far was the discovery of a new particle of mass $M_h \sim 125\,\mathrm{GeV}$ by the ATLAS [25] and CMS [26] LHC collaborations. This new particle appears just as the SM Higgs boson in all its properties.

In Sec. 2.9 we saw that within the MSSM the mass of the lightest Higgs boson is bounded from above. In the limit with large top-squark masses this bound reads

$$m_h^2 \lesssim m_Z^2 \cos^2 2\beta + \frac{3G_F}{\sqrt{2}\pi^2} m_t^4 \left[\log \frac{M_\mathrm{S}^2}{m_t^2} + \frac{X_t^2}{M_\mathrm{S}^2}\left(1 - \frac{X_t^2}{12 M_\mathrm{S}^2}\right)\right] , \tag{5.1}$$

where M_S is the geometrical mean of the stop mass eigenvalues and $X_t = A_t - \mu \cot\beta$, with A_t being the stop mixing parameter. In order to reproduce the observed value for the Higgs mass, there must be a large M_S and/or a large X_t. In the latter case, the correction to the Higgs mass is maximized for $X_t \sim \sqrt{6} M_\mathrm{S}$.

In the following, we concentrate on two different and quite opposite scenarios which are compatible with the Higgs observation.

- *Split SUSY*: If supersymmetry is not connected with the origin of the electroweak scale, it may still be possible that some remnant of the superparticle spectrum survives down to the TeV-scale or below. This is the idea of *Split SUSY* [37, 38] in which the only light superpartners are those needed for Dark Matter and coupling unification, i.e. light gluinos, charginos and neutralinos (also A-terms are small) while all scalars are heavy. In *Split SUSY*, there is almost no-mixing in the stop sector and therefore the measured Higgs mass imposes an upper limit to the large scale of heavy spartners at $10-10^4$ TeV, depending on $\tan\beta$. In this case one clearly loses the ability to address the hierarchy problem within the MSSM. A major advantage of *Split SUSY* is the solution of the SUSY flavour and CP problems. Indeed, even for $\mathcal{O}(1)$ flavour violating mass-insertions and CP phases, the stringent low-energy constraints can be kept under control thanks to the high scale for the scalars. However, since the Higgs mass sets an upper bound to this scale, it is interesting to analyze the interplay of flavour observables and the Higgs mass to probe the SUSY parameter space. Instead of taking a completely anarchic soft-sector, we consider a $U(1)$ flavour model accounting for the SM Yukawas and simultaneously predicting the flavour structure of the soft-sector. Then, we study the low-energy implications for flavour observables monitoring the pattern of flavour violation predicted by the model in question.

- *Large A-terms:* The second scenario we consider is characterized by a large mixing in the stop sector induced by large A-terms while the SUSY scale is kept at the TeV scale. In this case, the mass of the lightest MSSM Higgs meets the experimental value thanks to the second term in the square brackets of Eq. (5.1). In contrast to the *Split SUSY* case, naturalness is not completely spoiled now as the required fine tuning to reproduce the EW scale is around $10^{-2} - 10^{-3}$. On the other hand, the flavour and CP problems constitute a serious challenge of this setup requiring a highly non trivial flavour structure of the soft sector. As a concrete and well-motivated setup, we will consider the Partial Compositeness (PC) paradigm which is able to address the SM flavour problem providing, at the same time, the desired suppression for the flavour violating MIs. Similarly to the *Split SUSY* case, we will study the low-energy signals predicted by the PC scenario unveiling the peculiar pattern of flavour violation.

In the following, we study the flavour structure of the soft terms implied by $U(1)$ flavour models and by the PC paradigm. In these models, the SUSY mediation scale Λ_S is assumed to be above the scale of flavour messengers Λ_F, so that the flavour structure of soft terms at the scale Λ_F is controlled entirely by the flavour dynamics at this scale, irrespectively of their structure at the scale Λ_S.

5.1 $U(1)$ flavour Models

In the simplest realization of these models the flavour symmetry is spontaneously broken by the vev of a single "flavon" field with negative unit charge. Yukawa couplings then arise from higher-dimensional operators that involve suitable powers of the flavon to make the operator invariant under the $U(1)$ symmetry, with some undetermined coefficients that are assumed to be $\mathcal{O}(1)$. The suppression scale is the typical scale of the flavour sector that could correspond to the mass scale of Froggatt-Nielsen messengers in explicit UV completions. The Yukawas then depend only on powers of the ratio ϵ of flavon vev and flavour scale, which typically is taken to be of the order of the Cabibbo angle $\epsilon \sim 0.2$. If we restrict our attention to models where only the matter fields are charged, i.e. $H_u = H_d = 0$, Yukawas are of the (hierarchical) form

$$(y_U)_{ij} \sim \epsilon^{Q_i+U_j}, \qquad\qquad (y_D)_{ij} \sim \epsilon^{Q_i+D_j}, \tag{5.2}$$

where ϵ is a small order parameter and Q_i, U_i, D_i denote the positive $U(1)$ charges of the respective superfields. Using $Q_3 = U_3 = 0$ as suggested by the large top Yukawa, all other charges can be expressed in terms of diagonal Yukawa couplings and CKM matrix elements, giving

$$\epsilon^{Q_i} \sim V_{i3}, \qquad\qquad \epsilon^{U_i} \sim \frac{y_i^U}{V_{i3}}, \qquad\qquad \epsilon^{D_i} \sim \frac{y_i^D}{V_{i3}}. \tag{5.3}$$

The structure of the soft masses as determined by $U(1)$ invariance is given by

$$\tilde{m}_Q^2 \sim \epsilon^{|Q_i-Q_j|}, \qquad\qquad \tilde{m}_U^2 \sim \epsilon^{|U_i-U_j|}, \qquad\qquad \tilde{m}_D^2 \sim \epsilon^{|D_i-D_j|}, \tag{5.4}$$

$$A_U \sim \epsilon^{Q_i+U_j}, \qquad\qquad A_D \sim \epsilon^{Q_i+D_j}, \tag{5.5}$$

so that one obtains for the “mass insertions” (MIs)

$$(\delta^u_{LL})_{ij} \sim \frac{V_{i3}}{V_{j3}}|_{i\leq j}, \qquad (\delta^d_{LL})_{ij} \sim \frac{V_{i3}}{V_{j3}}|_{i\leq j}, \tag{5.6}$$

$$(\delta^u_{RR})_{ij} \sim \frac{y^U_i V_{j3}}{y^U_j V_{i3}}|_{i\leq j}, \qquad (\delta^d_{RR})_{ij} \sim \frac{y^D_i V_{j3}}{y^D_j V_{i3}}|_{i\leq j}, \tag{5.7}$$

$$(\delta^u_{LR})_{ij} \sim \frac{m^U_j A}{\tilde{m}_Q \tilde{m}_U}\frac{V^*_{i3}}{V_{j3}}, \qquad (\delta^d_{LR})_{ij} \sim \frac{m^D_j A}{\tilde{m}_Q \tilde{m}_D}\frac{V^*_{i3}}{V_{j3}}, \tag{5.8}$$

where in LL and RR the $i > j$ entries are obtained by hermitian conjugation, and in LR we introduced a complex conjugation to indicate that the diagonal entries are in general complex.

The major problem with the above flavour structures is to satisfy the constraint from $\epsilon_K \sim (\delta^d_{LL})_{12}(\delta^d_{RR})_{12} \sim m_d/m_s$, which typically requires a SUSY scale of $\mathcal{O}\,(100)$ TeV. To less extent, also $\epsilon'/\epsilon \sim (\delta^d_{LR})_{12(21)}$ (where ϵ'/ϵ accounts for direct CP violation in kaon systems) and the neutron EDM (which is dominantly generated by the down-quark EDM) provide strong bounds on $U(1)$ flavour models. Moreover, the flavour $U(1)$ symmetry does not prevent the existence of flavour-blind CPV phases for the gaugino masses, trilinear terms and the μ-term. Therefore, the SUSY CP problem has to be addressed by some other protection mechanisms in order to make this scenario viable.

Similarly to the quark sector, for the lepton Yukawa couplings we have

$$(y_E)_{ij} \sim \epsilon^{L_i+E_j}, \tag{5.9}$$

where L_i and E_i stand for the $U(1)$ charges of the left-handed and right-handed leptons, respectively. The neutrino sector depends on the origin of neutrino masses. If neutrinos are Dirac, then the Yukawa coupling takes the same form as the charged lepton Yukawa above with $E_j \to N_j$. In this case, the left-handed rotations V_{EL}, V_{NL} for the charged lepton and neutrino sectors, respectively, and therefore the PMNS matrix U, have the same parametric structure

$$(U)_{ij} \sim (V_{EL})_{ij} \sim (V_{NL})_{ij} \sim \epsilon^{|L_i-L_j|}. \tag{5.10}$$

Large neutrino mixing angles can therefore be reproduced by taking small left-handed charge differences $L_i - L_j$. Instead small neutrino masses can be accommodated by taking sufficiently large charges N_i of right-handed neutrinos. A more plausible explanation of light neutrinos can be achieved if they originate from the Weinberg operator

$$\Delta W = \frac{(y_{ll})_{ij}}{\Lambda} L_i L_j H_u H_u\,, \tag{5.11}$$

with a flavour structure determined by the $U(1)$ symmetry

$$(y_{ll})_{ij} \sim \epsilon^{L_i+L_j}. \tag{5.12}$$

In this way the smallness of neutrino masses can be elegantly explained by assuming a large UV scale $v_u/\Lambda \ll 1$, but the prediction for the parametric structure of the left-handed neutrino rotations and therefore for the PMNS matrix does not change, and we still get the result of Eq. (5.10), see e.g. [48]. One possibility for an explicit UV completion is the type-I seesaw mechanism. In this scenario, one adds three heavy right-handed neutrinos and Dirac Yukawa couplings

$$\Delta W = (y_\nu)_{ij} L_i N_j H_u + \frac{1}{2}(M_N)_{ij} N_i N_j, \tag{5.13}$$

with their flavour structure given by

$$(y_\nu)_{ij} \sim \epsilon^{L_i+N_j}\,, \qquad\qquad (M_N)_{ij} \sim M_N \epsilon^{N_i+N_j}. \tag{5.14}$$

Integrating out the right-handed neutrinos generates the Weinberg operator with a coefficient given by

$$\frac{(y_{ll})_{ij}}{\Lambda} = -\frac{1}{2}(y_\nu M_N^{-1} y_\nu^T)_{ij}. \tag{5.15}$$

Note that in the simple $U(1)$ models that we will consider here, the parametric flavour structure of the coefficient of the Weinberg operator is the same as in the effective theory

$$(y_{ll})_{ij} \sim \epsilon^{L_i+L_j}, \tag{5.16}$$

and therefore we recover the same estimate for the PMNS matrix as in Eq. (5.10).

Various $U(1)$ models have been discussed in the literature, see e.g. [49, 50, 51, 52]. There is some ambiguity in the choice of charge assignments, since ϵ is typically not a very small parameter (one has $\epsilon \approx 0.2 \div 0.5$) so that the unknown $\mathcal{O}(1)$ parameters can account for one or two units of charge differences. Here we choose to consider just two representative models that have been presented in Ref. [52] and more carefully analyzed in Ref. [53]. The first one, "Anarchy" which we denote by $U(1)_A$, features degenerate charges of left-handed lepton doublets, so that all mixing angles are predicted to be $\mathcal{O}(1)$. The second one, "Hierarchy" denoted by $U(1)_H$, has non-degenerate charges in order to account for the relative smallness of θ_{13} and $\Delta m^2_{solar}/\Delta m^2_{atm}$. Other models that have been considered in Ref. [51, 52] fall in between these two models for what regards their phenomenological consequences. The charge assignments of the two models are given by

- Anarchy

$$E_i = (3,2,0)\,, \qquad L_i = (L_3, L_3, L_3), \qquad N_i = (0,0,0), \qquad \epsilon_A \approx 0.2\,. \tag{5.17}$$

- Hierarchy

$$E_i = (5,3,0)\,, \quad L_i = (2+L_3, 1+L_3, L_3), \quad N_i = (2,1,0), \quad \epsilon_H \approx 0.3\,. \tag{5.18}$$

For simplicity the expansion parameters are taken here as the central values of the accurate fit in Ref. [53], although there is of course some range due to the unknown order one coefficients. Note the dependence on an overall charge shift L_3 that essentially corresponds to $\tan\beta$.

The expectation for the slepton mass insertions at the flavour scale is given by

$$(\delta^e_{LR})_{ij} \sim \frac{A v_d}{\tilde m_L \tilde m_E} \epsilon^{L_i+E_j}\,, \tag{5.19}$$

$$(\delta^e_{LL})_{ij} \sim \epsilon^{|L_i-L_j|}\,, \qquad\qquad (\delta^e_{RR})_{ij} \sim \epsilon^{|E_i-E_j|}\,. \tag{5.20}$$

As we will see, the electron EDM and $\mu \to e\gamma$ provide the most stringent constraints in the leptonic sector. However, the bounds from the quark sector, especially from ϵ_K, are much stronger than those from the lepton sector.

5.2 Partial Compositeness

Partial Compositeness (PC) is a seesaw-like mechanism that explains the hierarchy among the SM fermion masses by mixing with heavy resonances of a strongly coupled sector. Originally proposed within Technicolor models [54], it has been subsequently applied to extra-dimensional RS models [55, 56] and also in the context of SUSY [57, 58].

The basic assumption is that at the UV cutoff the SM fermions couple linearly to operators of the strong sector that is characterized by the mass scale m_ρ and the coupling $1 \leqslant g_\rho \leqslant 4\pi$. According to the paradigm of Partial Compositeness, in the effective theory below the scale m_ρ of the heavy resonances, every light quark $(q,u,d)_i$ is accompanied by a spurion $\epsilon_i^{q,u,d} \lesssim 1$ that measures its amount of compositeness. The quark Yukawa matrices then take the form

$$(y_U)_{ij} \sim g_\rho \epsilon_i^q \epsilon_j^u, \qquad\qquad (y_D)_{ij} \sim g_\rho \epsilon_i^q \epsilon_j^d, \tag{5.21}$$

which closely resembles the case of a single $U(1)$ flavour model, see Eq. (5.2), with the correspondence

$$\epsilon_i^{q,u,d} \longleftrightarrow \epsilon^{Q_i,U_i,D_i}. \tag{5.22}$$

A slight difference arises from the presence of the coupling g_ρ that can be large in this case. This implies that one can consider also $\epsilon_3^q, \epsilon_3^u < 1$ or equivalently $Q_3, U_3 \neq 0$, since the top Yukawa can arise from strong coupling. One has therefore two more parameters that we choose as ϵ_3^q and ϵ_3^u.

Apart from this issue, there is no difference between a single $U(1)$ and PC for what regards Yukawa couplings, or in general all superpotential terms. The main difference is in the non-holomorphic soft terms, which at the scale m_ρ are expected to be of the form [58]

$$\tilde{m}_Q^2 \sim \mathbf{1} + \epsilon_i^q \epsilon_j^q\,, \tag{5.23}$$

$$\tilde{m}_U^2 \sim \mathbf{1} + \epsilon_i^u \epsilon_j^u\,, \qquad\qquad \tilde{m}_D^2 \sim \mathbf{1} + \epsilon_i^d \epsilon_j^d\,, \tag{5.24}$$

$$A_U \sim g_\rho \epsilon_i^q \epsilon_j^u\,, \qquad\qquad A_D \sim g_\rho \epsilon_i^q \epsilon_j^d\,. \tag{5.25}$$

Therefore we find the following MIs

$$(\delta_{LL}^u)_{ij} \sim (\epsilon_3^q)^2 V_{i3}^* V_{j3}, \qquad\qquad (\delta_{LL}^d)_{ij} \sim (\epsilon_3^q)^2 V_{3i} V_{3j}^*, \tag{5.26}$$

$$(\delta_{RR}^u)_{ij} \sim \frac{y_i^U y_j^U}{V_{i3}^* V_{j3}} \frac{(\epsilon_3^u)^2}{y_t^2}, \qquad\qquad (\delta_{RR}^d)_{ij} \sim \frac{y_i^D y_j^D}{V_{3i} V_{3j}^*} \frac{(\epsilon_3^u)^2}{y_t^2}, \tag{5.27}$$

$$(\delta_{LR}^u)_{ij} \sim \frac{m_j^U A}{\tilde{m}_Q \tilde{m}_U} \frac{V_{i3}^*}{V_{j3}}, \qquad\qquad (\delta_{LR}^d)_{ij} \sim \frac{m_j^D A}{\tilde{m}_Q \tilde{m}_D} \frac{V_{3i}}{V_{3j}^*}. \tag{5.28}$$

Passing to the lepton sector, the Yukawa matrices have the form

$$(y_E)_{ij} \sim g_\rho \epsilon_i^\ell \epsilon_j^e, \tag{5.29}$$

where $\epsilon_i^{\ell,e} \lesssim 1$ measures the amount of compositeness for the leptons. Again, we resemble the case of a single $U(1)$ flavour model, with the correspondence

$$\epsilon_i^{\ell,e} \longleftrightarrow \epsilon^{L_i,E_i}\,. \tag{5.30}$$

As a result, the MIs are expected to take the following form [58]

$$(\delta_{LL}^e)_{ij} \sim \epsilon_i^\ell \epsilon_j^\ell \sim \epsilon^{L_i+L_j}, \qquad (\delta_{RR}^e)_{ij} \sim \epsilon_i^e \epsilon_j^e \sim \epsilon^{E_i+E_j}. \tag{5.31}$$

	MFV	PC	$U(1)$
$(\delta^u_{LL})_{ij}$	$V^*_{i3}V_{j3}y_b^2$	$V^*_{i3}V_{j3}(\epsilon_3^q)^2$	$\frac{V_{i3}}{V_{j3}}\vert_{i\leq j}$
$(\delta^d_{LL})_{ij}$	$V_{3i}V^*_{3j}$	$V_{3i}V^*_{3j}(\epsilon_3^q)^2$	$\frac{V_{i3}}{V_{j3}}\vert_{i\leq j}$
$(\delta^u_{RR})_{ij}$	$y_i^U y_j^U V^*_{i3}V_{j3}y_b^2$	$\frac{y_i^U y_j^U}{V^*_{i3}V_{j3}}(\epsilon_3^u)^2$	$\frac{y_i^U V_{j3}}{y_j^U V_{i3}}\vert_{i\leq j}$
$(\delta^d_{RR})_{ij}$	$y_i^D y_j^D V_{3i}V^*_{3j}$	$\frac{y_i^D y_j^D}{V_{3i}V^*_{3j}}(\epsilon_3^u)^2$	$\frac{y_i^D V_{j3}}{y_j^D V_{i3}}\vert_{i\leq j}$
$(\delta^u_{LR})_{ij}$	$y_j^U V^*_{i3}V_{j3}y_b^2$	$y_j^U \frac{V^*_{i3}}{V_{j3}}$	$y_j^U \frac{V^*_{i3}}{V_{j3}}$
$(\delta^d_{LR})_{ij}$	$y_j^D V_{3i}V^*_{3j}$	$y_j^D \frac{V_{3i}}{V^*_{3j}}$	$y_j^D \frac{V^*_{i3}}{V_{j3}}$

Table 5.1: Parametric suppression for mass insertions in various scenarios. The entries in the $U(1)$ column with $i > j$ are obtained from hermiticity. We neglect powers of y_t.

$$(\delta^e_{LR})_{ij} \sim \frac{v_d A g_\rho}{\tilde{m}_L \tilde{m}_E}\epsilon_i^\ell \epsilon_j^e \sim \frac{m_i^e A}{\tilde{m}_L \tilde{m}_E}\epsilon^{E_j - E_i} \sim \frac{m_j^e A}{\tilde{m}_L \tilde{m}_E}\epsilon^{L_i - L_j}\,. \tag{5.32}$$

In PC, the leading contributions to BR$(\mu \to e\gamma)$ typically arise from $(\delta^e_{LR})_{12}$. Note that in PC the left-handed "charges" L_i are determined from the PMNS matrix analogously to $U(1)$ models only in the case of light Dirac Neutrinos. If instead light neutrinos are Majorana, then the Weinberg operator can arise from a bilinear coupling to the composite sector (instead of linear couplings that resemble the $U(1)$ structure). In this case only the combination $L_i + E_j$ is determined by charged lepton Yukawa couplings, and the constraints from LFV can be significantly relaxed by choosing symmetric charges [58]

$$\epsilon^{L_i} \sim \epsilon^{E_i} \sim \sqrt{\frac{y_i^e}{g_\rho}}. \tag{5.33}$$

This implies

$$\frac{\tilde{m}}{m_\mu}(\delta^e_{LR})_{12} \sim \epsilon^{L_1 - L_2} \sim \sqrt{\frac{m_e}{m_\mu}}\,. \tag{5.34}$$

On the other hand, the predictions for the electron EDM are completely independent of any charge assignments since in PC the diagonal elements of the A-terms are generally complex and therefore the eEDM now provides the strongest constraint on the PC scenario.

5.3 Comparison of $U(1)$ and PC scenarios

We now compare the flavour structure of the soft sector in the $U(1)$ and PC scenarios. In Table 5.1, we summarize the parametric flavour suppression for the quark MIs reporting, as a reference, also the predictions of MFV. As can be seen from this table, the MFV scenario always yields the strongest suppressions and $U(1)$ the weakest. A closer look at the LL/RR and LR MIs of Table 5.1 leads to the following general conclusions:

LL/RR mixing: The $U(1)$ model has a much milder suppression compared to the PC case. In particular, assuming the approximate relation $y_i/y_j \sim (V_{i3}/V_{j3})^2$, it turns

	MFV	PC	$U(1)$	EXP.	OBS.
$\langle\delta^d\rangle^2_{12}$	$y_d y_s \lambda^{10}$	$\frac{y_d y_s}{g_\rho^2}$	$\frac{y_d}{y_s}$	7×10^{-8}	ϵ_K
$\langle\delta^u\rangle^2_{12}$	$y_u y_c \lambda^{10} y_b^4$	$\frac{y_u y_c}{g_\rho^2}$	$\frac{y_u}{y_c}$	1×10^{-5}	$\lvert q/p\rvert$, ϕ_D
$(\delta^u_{LR})_{12}$	$\frac{m_c a_U}{\tilde m^2}\lambda^5 y_b^2$	$\frac{m_c A}{\tilde m^2}\lambda$	$\frac{m_c a_U}{\tilde m^2}\lambda$	2×10^{-3}	Δa_{CP}
$(\delta^d_{LR})_{12}$	$\frac{m_s a_D}{\tilde m^2}\lambda^5$	$\frac{m_s A}{\tilde m^2}\lambda$	$\frac{m_s a_D}{\tilde m^2}\lambda$	4×10^{-5}	ϵ'/ϵ
$(\delta^u_{LR})_{11}$	$\frac{m_u a_U}{\tilde m^2}$	$\frac{m_u a_U}{\tilde m^2}$	$\frac{m_u a_U}{\tilde m^2}$	4×10^{-6}	d_n
$(\delta^d_{LR})_{11}$	$\frac{m_d a_D}{\tilde m^2}$	$\frac{m_d a_D}{\tilde m^2}$	$\frac{m_d a_D}{\tilde m^2}$	2×10^{-6}	d_n

Table 5.2: Predictions for the relevant mass insertions in the scenarios of Table 5.1. Here $\langle\delta^q\rangle^2_{12} \equiv (\delta^q_{LL})_{12}(\delta^q_{RR})_{12}$, $\lambda \approx 0.2$ and we neglect powers of y_t. We denote $a_U \equiv A - \mu^*/\tan\beta$ and $a_D \equiv A - \mu^*\tan\beta$, where the parameter A is defined by $(A_U)_{33} = A\,y_t$ in all scenarios. The experimental bounds (EXP.) refer to the imaginary components of the MIs for $\tilde m = 1$ TeV and are obtained imposing the experimental constraints on the most relevant processes listed in the last column (OBS.).

out that $(\delta^{u,d}_{AA})_{ij} \sim V_{i3}/V_{j3}$ (for $i < j$ and $AA = LL, RR$) in the $U(1)$ case, while $(\delta^{u,d}_{AA})_{ij} \sim V_{i3}V_{j3}$ in the PC case. This higher suppression is reminiscent of what happens in the case of wave function renormalization [59, 60] where the LL and RR MIs depend on the sum of charges instead of their difference in contrast to $U(1)$ models. In the PC case, we have a further suppression of order $(\epsilon_3^{q,u})^2$ for $\delta^{u,d}_{LL,RR}$ which is maximized for maximal strong couplings $g_\rho \sim 4\pi$ as the top mass relation implies that $g_\rho\epsilon_3^q\epsilon_3^u = 1$ with $\epsilon_3^{q,u} < 1$.

LR mixing: PC has the same suppression as $U(1)$ in both the up and down sectors. This results from the proportionality in both scenarios of the A-terms with the corresponding SM Yukawas.

We now analyze the phenomenological implications of the flavour structure of sfermion masses in low-energy processes. In particular, we will distinguish among $\Delta F = 2$, $\Delta F = 1$, and $\Delta F = 0$ processes, where in the latter case we refer to flavour conserving transitions like the EDMs that are still sensitive to flavour effects. Concerning $\Delta F = 2, 1$ transitions, we will focus only on processes with an underlying $s \to d$ or $c \to u$ transition as they put the most stringent bounds to the model in question. The predictions for the most relevant combinations of MIs are summarized in Table 5.2.

$\Delta F = 2$ *processes:* the relevant processes here are $K^0 - \bar K^0$ and $D^0 - \bar D^0$ mixings. As it is well known, these processes are mostly sensitive to the combinations of MIs $(\delta^d_{LL})_{12}(\delta^d_{RR})_{12}$ and $(\delta^u_{LL})_{12}(\delta^u_{RR})_{12}$, respectively. In the $U(1)$ case, it turns out that $(\delta^d_{LL})_{12}(\delta^d_{RR})_{12} \sim m_d/m_s \approx 0.05$, which implies a very heavy SUSY spectrum given the model-independent bound from ϵ_K that requires $\mathrm{Im}[(\delta^d_{LL})_{12}(\delta^d_{RR})_{12}] \lesssim 10^{-7}$ $(\tilde m/1\,\mathrm{TeV})$. The $D^0 - \bar D^0$ bounds are automatically satisfied after imposing that from ϵ_K. The situation greatly improves in the PC case where we have $(\delta^d_{LL})_{12}(\delta^d_{RR})_{12} \sim \frac{m_d m_s}{g_\rho^2 v^2}\tan^2\beta \approx 5\times10^{-9}\frac{\tan^2\beta}{g_\rho^2}$. For moderate/small values of $\tan\beta$

	$U(1)_A$	$U(1)_H$	PC	EXP.	OBS.
$\frac{\tilde{m}}{m_e}\mathrm{Im}(\delta_{LR})_{11}$	1	1	1	1×10^{-1}	d_e
$\frac{\tilde{m}}{m_\mu}(\delta_{LR})_{12}$	1	ϵ_H	$\sqrt{\frac{m_e}{m_\mu}}$	3×10^{-2}	$\mu\to e\gamma$
$\frac{\tilde{m}}{m_\mu}(\delta_{LR})_{21}$	ϵ_A	ϵ_H^2	$\sqrt{\frac{m_e}{m_\mu}}$	3×10^{-2}	$\mu\to e\gamma$
$(\delta_{LL})_{12}$	1	ϵ_H	$\frac{\sqrt{y_e y_\mu}}{g_\rho}$	$\frac{1\times10^{-2}}{\tan\beta}$	$\mu\to e\gamma$
$(\delta_{RR})_{12}$	ϵ_A	ϵ_H^2	$\frac{\sqrt{y_e y_\mu}}{g_\rho}$	$\frac{4\times10^{-2}}{\tan\beta}$	$\mu\to e\gamma$

Table 5.3: Predictions for the leptonic mass insertions in SUSY models with an underlying $U(1)$ flavour model and PC. The experimental bounds (EXP.) refer to $\tilde{m} = 1$ TeV and are obtained imposing the experimental constraints on the most relevant processes listed in the last column (OBS.).

and considering $\mathcal{O}(1)$ unknowns, the PC scenario is viable for TeV scale soft masses. Yet, in PC it is easy to generate sizable NP effects for ϵ_K (but not for $B_{d,s}$ mixing) which can improve the UT fit [61].

$\Delta F = 1$ *processes:* The most constraining process of this sector is ϵ'/ϵ, which provides the model-independent bound $\mathrm{Im}(\delta^d_{LR})_{12} \lesssim 4\times 10^{-5}(\tilde{m}/1\text{ TeV})$. Such an upper bound can be saturated in PC and $U(1)$ models where $(\delta^d_{LR})_{12} \sim (A/\tilde{m})\times(m_s\lambda/\tilde{m})$. Imposing the vacuum stability condition $A/\tilde{m} \lesssim 3$, it turns out that $(\delta^d_{LR})_{12} \lesssim 4\times 10^{-5}(1\text{ TeV}/\tilde{m})$.

$\Delta F = 0$ *processes:* Hadronic EDMs constrain the MIs $(\delta^{d,u}_{LR})_{11}$. Imposing the experimental bound on the neutron EDM [62], we find that $\mathrm{Im}(\delta^d_{LR})_{11} \lesssim 2\times 10^{-6}(\tilde{m}/1\text{ TeV})$ and $\mathrm{Im}(\delta^u_{LR})_{11} \lesssim 4\times 10^{-6}(\tilde{m}/1\text{ TeV})$. In $U(1)$ and PC models, assuming $A/\tilde{m} \simeq 1$ and the PDG values for $m_{u,d}$ [63], it turns out that $(\delta^d_{LR})_{11} \sim 3\times 10^{-6}(1\text{ TeV}/\tilde{m})$ and $(\delta^u_{LR})_{11} \sim 1\times 10^{-6}(1\text{ TeV}/\tilde{m})$, which are somewhat in tension with the hadronic EDM bounds especially in the down sector.

In Tab. 5.3, we summarize the predictions for the leptonic MIs most relevant for phenomenology in various models: $U(1)_A$ (first column), $U(1)_H$ (second column), and PC (last column). Similarly to the quark sector, comparing the flavour structure of the soft sector of $U(1)$ and PC scenarios, the most prominent feature is the higher suppression for off-diagonal sfermion masses in the LL and RR sectors in the PC case. Again, the LR sector has the same parametric structure, since in both scenarios the A-terms are proportional to the SM Yukawas.

5.4 Numerical analysis

We are ready now to analyze the predictions of the SUSY scenarios presented in the previous sections. Concerning the flavour observables, we focus only on transitions involving light generations, i.e. $s \to d$ and $\mu \to e$ transitions, as they turn out to be the most sensitive ones to the considered scenarios. Moreover, we take into account also the electron

EDM constraint but not the neutron EDM one as the latter is much less constraining than the former.

In the plots of Fig. 5.1, we show the regions in the $\tan\beta - \tilde{m}$ plane allowed by the Higgs mass measurement and by the flavour observables. The upper plot refers to the *Split SUSY* scenario supplemented by a $U(1)$ flavour model, taking for the SUSY masses the values $M_1 = M_2 = \mu = A = 500$ GeV and $M_g = 3$ TeV. The lower plot stands for the *large A-terms* scenario with an underlying PC paradigm. Here we choose a degenerate spectrum $\tilde{m} = M_1 = M_2 = M_g = \mu = A$.

Among the most prominent differences emerging from a comparison of the two plots are the different ranges for $\tilde{m}$ selected by the Higgs mass measurement $m_h = (125.5 \pm 2)$ GeV. In particular, since in the *Split SUSY* case there is no mixing in the stop sector, much larger values for $\tilde{m}$ are required compared to the case of *large A-terms* where we assume $A/\tilde{m} = 1$.

Moreover, the $\tan\beta$ dependence of the Higgs mass is mainly driven by its tree level contribution, accounted for by the first term of Eq. (5.1), which is maximized for $\tan\beta \to \infty$. In practice, m_h is rather insensitive to $\tan\beta \gtrsim 10$ values, as correctly reproduced by fig. 5.1.

We pass now to the results relative to flavour observables. In the $U(1)$ case ϵ_K, describing CP violating effects in $K^0 - \bar{K}^0$ mixing, turns out to be the most constraining observable. Indeed, the leading effect for ϵ_K arises from the MI combination $\epsilon_K \sim (\delta^d_{LL})_{12}(\delta^d_{RR})_{12}$ which, in the $U(1)$ case, is of order $(\delta^d_{LL})_{12}(\delta^d_{RR})_{12} \sim m_d/m_s \approx 0.05$. This implies a very heavy SUSY spectrum given the model-independent bound $\mathrm{Im}[(\delta^d_{LL})_{12}(\delta^d_{RR})_{12}] \lesssim 10^{-7}\ (\tilde{m}/1\,\mathrm{TeV})$.

By contrast, in the PC case, we have $(\delta^d_{LL})_{12}(\delta^d_{RR})_{12} \sim \frac{m_d m_s}{g_\rho^2 v^2} \tan^2\beta \approx 5 \times 10^{-9} \frac{\tan^2\beta}{g_\rho^2}$, from which we trace back the dependence on g_ρ and $\tan\beta$ shown in the lower plot of fig. 5.1. In particular, for moderate/small values of $\tan\beta$ and $g_\rho > 1$, the PC scenario is viable for TeV scale soft masses.

Concerning LFV processes, we remind first their model-independent correlation $\mathrm{BR}(\mu \to eee) \sim \alpha_{em}\mathrm{BR}(\mu \to e\gamma)$ and $\mathrm{CR}(\mu + N \to e + N) \sim \alpha_{em}\mathrm{BR}(\mu \to e\gamma)$. In the upper plot of fig. 5.1, we consider the model $U(1)_H$. In this case, the dominant effects to LFV transitions arise from the $\tan\beta$-enhanced amplitudes induced by the MI $(\delta_{LL})_{12} \sim \epsilon_H \sim 0.3$. On the other hand, in the PC case, the leading contributions come from the $\tan\beta$-independent amplitude proportional to the MI $(\delta_{LR})_{12} \sim (\delta_{RL})_{12} \sim \sqrt{m_e/m_\mu}$. The above considerations explain the $\tan\beta$-dependence and relative size for LFV signals shown by fig. 5.1.

Finally, we comment on the electron EDM. In the $U(1)_H$ case, the leading effects arise from $\tan\beta$-enhanced amplitudes induced by $SU(2)$ interactions and they are proportional to the CP violating phase $arg(M_2\mu)$. In contrast, in the PC setup, the A-terms are assumed to be the only sources of CP violation and the corresponding amplitudes leading to the electron EDM are induced only by $U(1)$ interactions and turn out to be $\tan\beta$-independent.

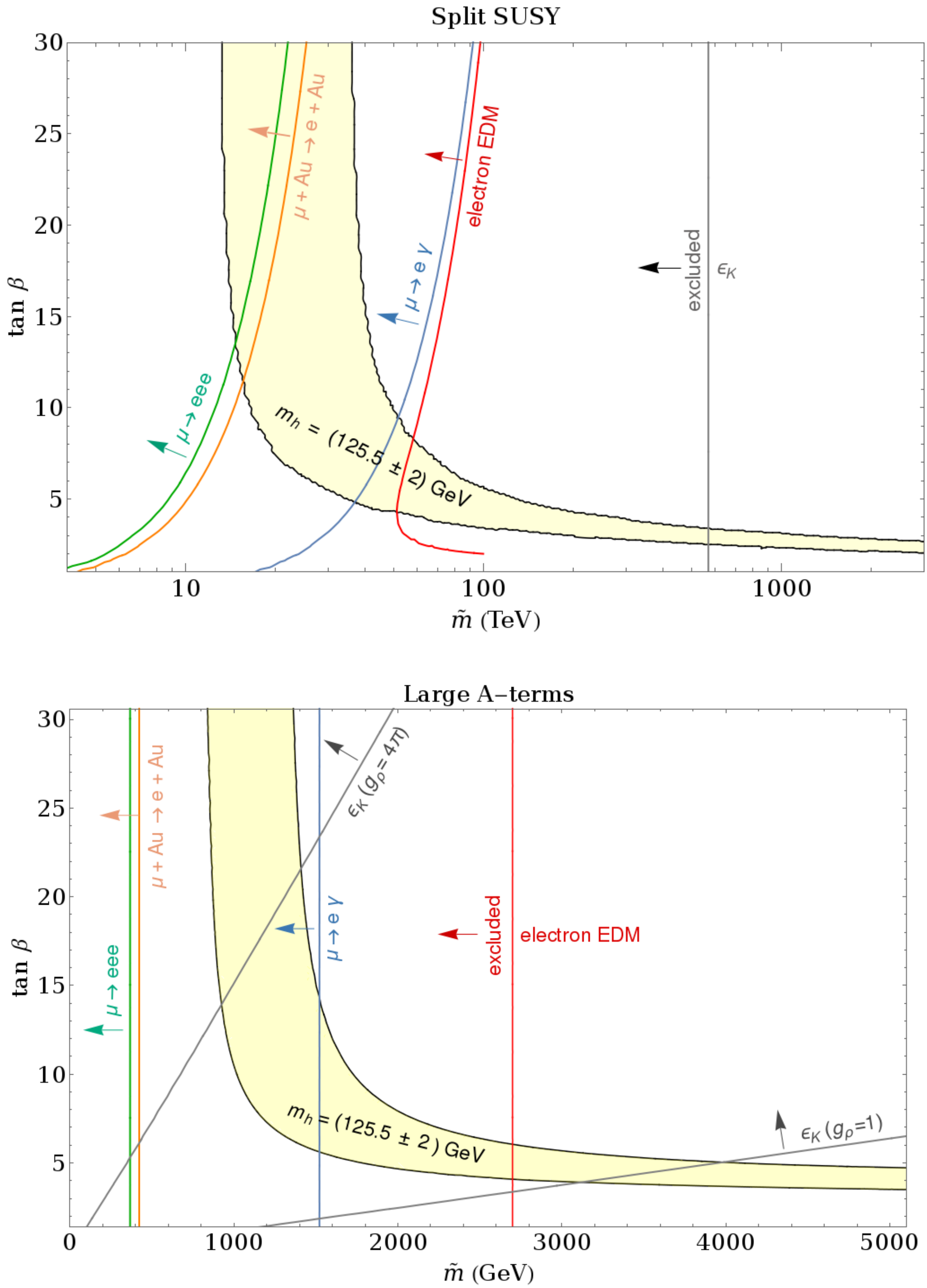


Figure 5.1: Predictions for the Higgs mass m_h and various flavour observables in the $\tan\beta - \tilde{m}$ plane. Upper plot: *Split SUSY* scenario supplemented by a $U(1)$ flavour model assuming $M_1 = M_2 = \mu = A = 500$ GeV and $M_g = 3$ TeV. Lower plot: *large A-terms* scenario with an underlying PC paradigm for a degenerate spectrum $\tilde{m} = M_1 = M_2 = M_g = \mu = A$.

Conclusions

Particle physics entered a new era with the discovery of the Higgs boson. Now that the Higgs boson has been discovered, naturalness becomes another pressing question waiting for the final answer at the LHC14. If new dynamics is present around the TeV scale, as needed to explain naturally the smallness of the electro-weak scale, one would expect too large contributions to flavour transitions mediated by the new physics states unless some protection mechanism is at work. Therefore, the possibility of finding new physics at the LHC is closely related to the existence of a mechanism in flavour physics. The indirect searches are therefore complementary to the direct searches for new particles at the LHC but they can in principle be sensitive to much shorter length scales than the latter experiments.

From the experimental side, there is an extraordinary ongoing activity at the i) LHCb, the B physics experiment at LHC; ii) the MEG experiment searching for Lepton flavour Violation (LFV) in $\mu \to e\gamma$, iii) the experiments looking for the Electric Dipole Moment (EDM) of the neutron at the ILL & PSI, iv) the searches for the rare Kaon decays $K \to \pi\nu\bar{\nu}$ at NA62 & JPARC. Therefore, it is extremely interesting to investigate the interplay of the high energy frontier (direct searches) with the high intensity frontier (indirect searches) in the light of the latest experimental results. This last point should be understood as an attempt to answer the following two questions: if new particles are discovered at ATLAS/CMS, what can the flavour measurements tell us about their properties and the very high energy theory that determines them? And similarly, if deviations from the CKM predictions are found in the flavour measurements, can ATLAS/CMS provide further complementary information that will close in on the source of these deviations?

In this thesis, we have carried out the above program in the context of supersymmetric scenarios. In particular, we have studied some selected phenomenological aspects related to flavour physics, revisited in the light of the LHC results after the 8 TeV run. Specifically, the lack of evidence for new coloured particles up to the TeV scale and the discovery of a scalar boson with a mass of about 125 GeV and similar properties (up to the current experimental uncertainties) to those of the Higgs boson of the Standard Model provide non-trivial information on the mass spectrum of supersymmetric models. In order to reproduce the observed value for the Higgs mass, there must be a large mass for the scalar partners of top quark (stops) and/or a large mixing among them. Therefore, we have focused on two different and quite opposite scenarios which are compatible with the Higgs observation.

- *Split SUSY*: the only light superpartners are those needed for Dark Matter and coupling unification, i.e. gluinos, charginos and neutralinos while all scalars are heavy [37, 38]. In Split SUSY, there is almost no-mixing in the stop sector and the measured Higgs mass imposes an upper limit to the large scale of heavy spartners at $10 - 10^4$ TeV.

- *Large A-terms:* all superpartners lie at the TeV scale and the Higgs boson mass is accounted for by a large stop mixing.

In the case of *Split SUSY*, one clearly looses the ability to address the hierarchy problem within the MSSM. By contrast, in the *large A-terms* scenario, naturalness is not completely spoiled as the required fine tuning to reproduce the EW scale is around $10^{-2} - 10^{-3}$.

A major advantage of *Split SUSY* is that the SUSY flavour and CP problems are significantly relaxed thanks to the high scale for the scalars. However, since the Higgs mass sets an upper bound to this scale, it is interesting to analyze the interplay of flavour observables and the Higgs mass to probe the SUSY parameter space. On the other hand, the flavour and CP problems constitute a serious challenge of the *large A-terms* scenario since all superpartners lie at the TeV scale. Therefore, a highly non trivial flavour structure of the soft sector is required in this case.

Concerning the flavour structure of the soft-sector (which is a necessary input in order to make predictions in flavour physics), we have assumed the predictions implied by popular and well motivated models addressing the favor hierarchy of quarks and leptons. In particular, we have focused on the simplest flavour models based on a $U(1)$ symmetry (which has recently received renewed attention [52, 53] after the reactor neutrino angle θ_{13} turned out to be sizable) and on models satisfying the Partial Compositeness (PC) paradigm. In these models, the SUSY mediation scale Λ_F is assumed to be above the scale of flavour messengers Λ_F, so that the flavour structure of soft terms at the scale Λ_F is controlled entirely by the flavour dynamics at this scale, irrespectively of their structure at the scale Λ_S. Then, we have studied the low-energy implications predicted by the above scenarios unveiling their peculiar pattern of flavour violation.

In the following, we summarize our main findings starting with the flavour structure of the soft sector.

- The non-holomorphic flavour violating soft masses in the LL and RR sectors δ^{ij}_{LL} and δ^{ij}_{RR}, respectively, are much less suppressed in $U(1)$ models then in PC.
- The non-holomorphic flavour violating soft masses in the LR sector δ^{ij}_{LR} have the same suppression in PC and $U(1)$ cases. This results from the proportionality in both scenarios of the A-terms with the corresponding SM Yukawas.

Concerning the phenomenological implications in low-energy processes, we have found that

- processes with underlying flavour transitions relative to light generations, i.e. $s \to d$, $c \to u$, and $\mu \to e$ transitions, are the most sensitive low-energy channels to probe the models in question (see Table 5.2, 5.3).
- $U(1)$ models give (experimentally) testable predictions in the case of Split-SUSY while they are excluded for TeV scale SUSY scenarios such as the large A-terms setup.
- Models based on the PC paradigm give negligible flavour effects in the context of Split-SUSY while they predict sizable and experimentally visible effects in the large A-terms scenario.
- The pattern of flavour violation predicted by $U(1)$ and PC models is different and therefore experimentally distinguishable.
- The observed Higgs boson mass selects regions of the SUSY parameter space which are complementary to those probed by low-energy signals. Therefore, their combined analysis can help in shedding light on the particular SUSY model at work.

In conclusion, we have explored virtual effects of new particles on low energy observables, which could provide crucial information about the presence and properties of new particles that so far escaped the direct search at the LHC. We have found that flavour observables and the Higgs boson mass are highly complementary in probing supersymmetric scenarios. As such our results as well as the motivation for low energy experiments become even more robust.

Appendix A

Notations and conventions

Natural units are used in which both the speed of light and the Planck constant are set to one, $c = \hbar = 1$. Four vector indices are represented by letters from the middle of the Greek alphabet $\mu, \nu, \rho, \ldots$ and go from 0 to 3. The contravariant four-vector position is defined as $x^\mu = (t, \vec{x})$. The space-time metric is

$$\eta_{\mu\nu} = \mathrm{diag}(+1, -1, -1, -1)\,. \tag{A.1}$$

A.1 Basic facts about Lie algebras

An infinitesimal element g of a Lie Group can be written as

$$g(\alpha) = 1 + i\alpha^a T^a + \mathcal{O}\big(\alpha^2\big)\,. \tag{A.2}$$

The coefficients of the infinitesimal group parameters α^a are Hermitian operators T^a, called *generators* of the symmetry group. The commutation relations of the operators T^a are

$$\Big[T^a, T^b\Big] = i f^{abc} T^c\,, \tag{A.3}$$

where f^{abc} are called *structure constants* and for compact Lie groups can be chosen to be totally antisymmetric. The invariants $C(r)$ and $C_2(r)$ of the representation r are defined by

$$\mathrm{Tr}\Big[T^a T^b\Big] = C(r)\delta^{ab}\,, \qquad T^a T^a = C_2(r)\mathbb{1}\,. \tag{A.4}$$

These are related by

$$C(r) = \frac{d(r)}{d(G)} C_2(r) \tag{A.5}$$

where $d(r)$ is the dimension of the representation and G refers to the adjoint representation. Traces and contractions of the T^a can be evaluated using the above identities and their consequences:

$$\begin{aligned} T^a T^b T^a &= \left[C_2(r) - \tfrac{1}{2}C_2(G)\right] T^b\,, \\ f^{acd} f^{bcd} &= C_2(G)\delta^{ab}\,, \\ f^{abc} T^b T^c &= \tfrac{1}{2} i C_2(G) T^a\,. \end{aligned} \tag{A.6}$$

For $SU(N)$ groups, the fundamental representation is denoted by N, and we have

$$C(N) = \frac{1}{2}\,, \quad C_2(N) = \frac{N^2-1}{2N}\,, \quad C(G) = C_2(G) = N\,. \tag{A.7}$$

The following relation, satisfied by the matrices of the fundamental representation of $SU(N)$, is also very helpful:

$$(T^a)_{ij}T^a_{kl} = \frac{1}{2}\left(\delta_{il}\delta_{kj} - \frac{1}{N}\delta_{ij}\delta_{kl}\right) . \tag{A.8}$$

A.2 Dirac, Weyl, Majorana spinors

Gamma matrices satisfy the Clifford algebra

$$\{\gamma^\mu, \gamma^\nu\} = 2\eta^{\mu\nu} . \tag{A.9}$$

We define

$$\gamma_5 \equiv i\gamma_0\gamma_1\gamma_2\gamma_3, \quad \gamma_5^\dagger = \gamma_5, \quad (\gamma_5)^2 = \mathbb{1} . \tag{A.10}$$

The left and right helicity projectors are given by

$$P_L = \frac{1}{2}(\mathbb{1} - \gamma_5), \quad P_R = \frac{1}{2}(\mathbb{1} + \gamma_5) . \tag{A.11}$$

For our purposes it is convenient to use the specific representation of the gamma matrices given in 2×2 blocks by

$$\gamma^0 = \begin{pmatrix} 0 & \mathbb{1} \\ \mathbb{1} & 0 \end{pmatrix}, \quad \gamma^i = \begin{pmatrix} 0 & \sigma^i \\ -\sigma^i & 0 \end{pmatrix}, \quad \gamma^5 = \begin{pmatrix} -\mathbb{1} & 0 \\ 0 & \mathbb{1} \end{pmatrix} . \tag{A.12}$$

Where σ^i are the Pauli matrices given by

$$\sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} , \tag{A.13}$$

and satisfy the following commutation and anticommutation relations

$$\left[\sigma^i, \sigma^j\right] = 2i\epsilon^{ijk}\sigma^k, \quad \left\{\sigma^i, \sigma^j\right\} = 2\delta^{ij} . \tag{A.14}$$

With the definitions $\sigma^\mu = \left(\mathbb{1}, \sigma^i\right)$, $\overline{\sigma}^\mu = \left(\mathbb{1}, -\sigma^i\right)$ the gamma matrices can be written as

$$\gamma^\mu = \begin{pmatrix} 0 & \sigma^\mu \\ \overline{\sigma}^\mu & 0 \end{pmatrix} . \tag{A.15}$$

With this choice, a four-component Dirac spinor can be written in terms of 2 two-component, complex, anticommuting objects ξ_α and $(\chi^\dagger)^{\dot\alpha} \equiv \chi^{\dagger\dot\alpha}$ with two distinct types of spinor indices $\alpha = 1, 2$ and $\dot\alpha = 1, 2$

$$\psi = \begin{pmatrix} \xi_\alpha \\ \chi^{\dagger\dot\alpha} \end{pmatrix} . \tag{A.16}$$

The vertical positions of the dotted and undotted spinor indices are important; the matrices $(\sigma^\mu)_{\alpha\dot\alpha}$ and $(\overline{\sigma}^\mu)^{\dot\alpha\alpha}$ carry indices with vertical positions as indicated. Our conventions for two-component spinors are essentially those of [16]. The conjugate of a Dirac spinor $\overline{\psi} = \psi^\dagger\gamma^0$ reads

$$\overline{\psi} = \begin{pmatrix} \chi^\alpha & \xi^\dagger_{\dot\alpha} \end{pmatrix} . \tag{A.17}$$

Undotted (dotted) indices from the beginning of the Greek alphabet are used for the first (last) two components of a Dirac spinor. The field ξ is called a *left-handed Weyl spinor* and $\chi^\dagger$ is a *right-handed Weyl spinor*. The names fit, because

$$P_L\psi = \begin{pmatrix} \xi_\alpha \\ 0 \end{pmatrix}, \qquad P_R\psi = \begin{pmatrix} 0 \\ \chi^{\dagger\dot{\alpha}} \end{pmatrix}. \tag{A.18}$$

The Hermitian conjugate of any left-handed Weyl spinor is a right-handed Weyl spinor and vice versa:

$$\psi^\dagger_{\dot{\alpha}} \equiv (\psi_\alpha)^\dagger = (\psi^\dagger)_{\dot{\alpha}}, \qquad (\psi^{\dagger\dot{\alpha}})^\dagger = \psi^\alpha. \tag{A.19}$$

Therefore, any particular fermionic degrees of freedom can be described equally well using a left-handed Weyl spinor (with an undotted index) or by a right-handed one (with a dotted index). By convention, all names of fermion fields are chosen so that left-handed Weyl spinors do not carry daggers and right-handed Weyl spinors do carry daggers, as in eq. (A.16). We use the following definition for $\sigma^{\mu\nu}$ and $\overline{\sigma}^{\mu\nu}$

$$\sigma^{\mu\nu} = \frac{i}{4}(\sigma^\mu\overline{\sigma}^\nu - \sigma^\nu\overline{\sigma}^\mu), \qquad \overline{\sigma}^{\mu\nu} = \frac{i}{4}(\overline{\sigma}^\mu\sigma^\nu - \overline{\sigma}^\nu\sigma^\mu). \tag{A.20}$$

The spinor indices are raised and lowered using the antisymmetric symbol

$$\epsilon^{12} = -\epsilon^{21} = \epsilon_{21} = -\epsilon_{12} = 1, \qquad \epsilon_{11} = \epsilon_{22} = \epsilon^{11} = \epsilon^{22} = 0, \tag{A.21}$$

according to

$$\xi_\alpha = \epsilon_{\alpha\beta}\xi^\beta, \qquad \xi^\alpha = \epsilon^{\alpha\beta}\xi_\beta, \qquad \chi^\dagger_{\dot{\alpha}} = \epsilon_{\dot{\alpha}\dot{\beta}}\chi^{\dagger\dot{\beta}}, \qquad \chi^{\dagger\dot{\alpha}} = \epsilon^{\dot{\alpha}\dot{\beta}}\chi^\dagger_{\dot{\beta}}. \tag{A.22}$$

This is consistent since $\epsilon_{\alpha\beta}\epsilon^{\beta\gamma} = \epsilon^{\gamma\beta}\epsilon_{\beta\alpha} = \delta^\gamma_\alpha$ and $\epsilon_{\dot{\alpha}\dot{\beta}}\epsilon^{\dot{\beta}\dot{\gamma}} = \epsilon^{\dot{\gamma}\dot{\beta}}\epsilon_{\dot{\beta}\dot{\alpha}} = \delta^{\dot{\gamma}}_{\dot{\alpha}}$. The relations between σ^μ and $\overline{\sigma}^\mu$ are

$$\sigma^\mu_{\alpha\dot{\alpha}} = \epsilon_{\alpha\beta}\epsilon_{\dot{\alpha}\dot{\beta}}\overline{\sigma}^{\mu\,\dot{\beta}\beta}, \qquad \overline{\sigma}^{\mu\,\dot{\alpha}\alpha} = \epsilon^{\alpha\beta}\epsilon^{\dot{\alpha}\dot{\beta}}\sigma^\mu_{\beta\dot{\beta}}, \tag{A.23}$$

$$\epsilon^{\alpha\beta}\sigma^\mu_{\beta\dot{\alpha}} = \epsilon_{\dot{\alpha}\dot{\beta}}\overline{\sigma}^{\mu\dot{\beta}\alpha}, \qquad \epsilon^{\dot{\alpha}\dot{\beta}}\sigma^\mu_{\alpha\dot{\beta}} = \epsilon_{\alpha\beta}\overline{\sigma}^{\mu\dot{\alpha}\beta}. \tag{A.24}$$

As a convention, repeated spinor indices contracted like

$$^\alpha{}_\alpha \qquad \text{or} \qquad {}_{\dot{\alpha}}{}^{\dot{\alpha}} \tag{A.25}$$

can be suppressed. In particular,

$$\xi\chi \equiv \xi^\alpha\chi_\alpha = \xi^\alpha\epsilon_{\alpha\beta}\chi^\beta = -\chi^\beta\epsilon_{\alpha\beta}\xi^\alpha = \chi^\beta\epsilon_{\beta\alpha}\xi^\alpha = \chi^\beta\xi_\beta \equiv \chi\xi \tag{A.26}$$

with, conveniently, no minus sign in the end. [A minus sign appeared in eq. (A.26) from exchanging the order of anticommuting spinors, but it disappeared due to the antisymmetry of the ϵ symbol.] Likewise, $\xi^\dagger\chi^\dagger$ and $\chi^\dagger\xi^\dagger$ are equivalent abbreviations for $\chi^\dagger_{\dot{\alpha}}\xi^{\dagger\dot{\alpha}} = \xi^\dagger_{\dot{\alpha}}\chi^{\dagger\dot{\alpha}}$, and in fact this is the complex conjugate of $\xi\chi$:

$$\xi^\dagger\chi^\dagger = \chi^\dagger\xi^\dagger = (\xi\chi)^*. \tag{A.27}$$

In a similar way, one can check that

$$\xi^\dagger\overline{\sigma}^\mu\chi = -\chi\sigma^\mu\xi^\dagger = (\chi^\dagger\overline{\sigma}^\mu\xi)^* = -(\xi\sigma^\mu\chi^\dagger)^* \tag{A.28}$$

stands for $\xi^\dagger_{\dot\alpha}(\overline{\sigma}^\mu)^{\dot\alpha\alpha}\chi_\alpha$, etc. The anti-commuting spinors here are taken to be classical fields; for quantum fields the complex conjugation in the last two equations would be replaced by Hermitian conjugation in the Hilbert space operator sense.

Some other identities that will be useful below include:

$$\xi\sigma^\mu\overline{\sigma}^\nu\chi \;=\; \chi\sigma^\nu\overline{\sigma}^\mu\xi \;=\; (\chi^\dagger\overline{\sigma}^\nu\sigma^\mu\xi^\dagger)^* = (\xi^\dagger\overline{\sigma}^\mu\sigma^\nu\chi^\dagger)^*, \tag{A.29}$$

and the Fierz rearrangement identity:

$$\chi_\alpha\,(\xi\eta) = -\xi_\alpha\,(\eta\chi) - \eta_\alpha\,(\chi\xi), \tag{A.30}$$

and the reduction identities

$$\sigma^\mu_{\alpha\dot\alpha}\,\overline{\sigma}^{\dot\beta\beta}_\mu \;=\; 2\delta^\beta_\alpha\delta^{\dot\beta}_{\dot\alpha}, \tag{A.31}$$
$$\sigma^\mu_{\alpha\dot\alpha}\,\sigma_{\mu\beta\dot\beta} \;=\; 2\epsilon_{\alpha\beta}\epsilon_{\dot\alpha\dot\beta}, \tag{A.32}$$
$$\overline{\sigma}^{\mu\dot\alpha\alpha}\,\overline{\sigma}^{\dot\beta\beta}_\mu \;=\; 2\epsilon^{\alpha\beta}\epsilon^{\dot\alpha\dot\beta}, \tag{A.33}$$
$$\left[\sigma^\mu\overline{\sigma}^\nu + \sigma^\nu\overline{\sigma}^\mu\right]_\alpha{}^\beta \;=\; 2\eta^{\mu\nu}\delta^\beta_\alpha, \tag{A.34}$$
$$\left[\overline{\sigma}^\mu\sigma^\nu + \overline{\sigma}^\nu\sigma^\mu\right]^{\dot\beta}{}_{\dot\alpha} \;=\; 2\eta^{\mu\nu}\delta^{\dot\beta}_{\dot\alpha}, \tag{A.35}$$
$$\overline{\sigma}^\mu\sigma^\nu\overline{\sigma}^\rho \;=\; \eta^{\mu\nu}\overline{\sigma}^\rho\eta^{\nu\rho}\overline{\sigma}^\mu - \eta^{\mu\rho}\overline{\sigma}^\nu - i\epsilon^{\mu\nu\rho\kappa}\overline{\sigma}_\kappa, \tag{A.36}$$
$$\sigma^\mu\overline{\sigma}^\nu\sigma^\rho \;=\; \eta^{\mu\nu}\sigma^\rho\eta^{\nu\rho}\sigma^\mu - \eta^{\mu\rho}\sigma^\nu i\epsilon^{\mu\nu\rho\kappa}\sigma_\kappa, \tag{A.37}$$

where $\epsilon^{\mu\nu\rho\kappa}$ is the totally antisymmetric tensor with $\epsilon^{0123} = +1$.
Computations of cross-sections and decay rates generally require traces of alternating products of σ and $\overline{\sigma}$ matrices

$$\mathrm{Tr}[\sigma^\mu\overline{\sigma}^\nu] = \mathrm{Tr}[\overline{\sigma}^\mu\sigma^\nu] = 2\eta^{\mu\nu}\,, \tag{A.38}$$
$$\mathrm{Tr}[\sigma^\mu\overline{\sigma}^\nu\sigma^\rho\overline{\sigma}^\kappa] = 2\left(\eta^{\mu\nu}\eta^{\rho\kappa} - \eta^{\mu\rho}\eta^{\nu\kappa} + \eta^{\mu\kappa}\eta^{\nu\rho} + i\epsilon^{\mu\nu\rho\kappa}\right)\,, \tag{A.39}$$
$$\mathrm{Tr}[\overline{\sigma}^\mu\sigma^\nu\overline{\sigma}^\rho\sigma^\kappa] = 2\left(\eta^{\mu\nu}\eta^{\rho\kappa} - \eta^{\mu\rho}\eta^{\nu\kappa} + \eta^{\mu\kappa}\eta^{\nu\rho} - i\epsilon^{\mu\nu\rho\kappa}\right)\,. \tag{A.40}$$

Charge conjugate fields

For four-component spinors, we introduce the charge-conjugate fields

$$\chi^c = C\overline{\chi}^T \qquad \overline{\chi}^c = -\chi^T C^{-1}\,. \tag{A.41}$$

The charge conjugate matrix C fulfills

$$C^\dagger = C^{-1}\,, \qquad C^T = -C\,, \qquad C\Gamma_i^T C^{-1} = \eta_i\Gamma_i \tag{A.42}$$

(no summation over i), with

$$\eta_i = \begin{cases} \;\;1 & \text{for} \quad \Gamma_i = 1,\, \gamma_5,\, \gamma_\mu\gamma_5 \\ -1 & \text{for} \quad \Gamma_i = \gamma_\mu,\, \sigma_{\mu\nu} \end{cases} \tag{A.43}$$

Majorana four-component spinors satisfy the condition

$$\psi = \psi^c \equiv C\left(\overline{\psi}\right)^T \tag{A.44}$$

where C is the charge conjugation matrix. We choose

$$C = -i\gamma^2\gamma^0\,, \tag{A.45}$$

so that in two-component spinor notation, the conjugate of Dirac spinor reads

$$\psi^C = -i\gamma^2\psi^*\,, \quad \text{hence} \quad \psi = \begin{pmatrix} \psi_L \\ -i\overline{\sigma}^2\psi_L^* \end{pmatrix}\,, \tag{A.46}$$

while a Majorana spinor is written as

$$\psi = \begin{pmatrix} \xi_\alpha \\ -i(\overline{\sigma}_2)^{\dot{\alpha}\alpha}\,\xi_\alpha \end{pmatrix} \tag{A.47}$$

The u and v spinors for either Dirac or Majorana fermions are related via

$$u(k,s) = C\overline{v}^T(k,s) \qquad v(k,s) = C\overline{u}^T(k,s) \tag{A.48}$$

where $s = \pm 1/2$ labels spin.

A.3 Fierz Identities

Here we state the Fierz transformations for the Dirac structures that appear in our calculations. These identities are valid for *commuting* spinors a, b, c and d.

$$(\overline{a}\gamma_\mu P_{L/R}b)(\overline{c}\gamma_\mu P_{L/R}d) = -(\overline{a}\gamma_\mu P_{L/R}d)(\overline{c}\gamma_\mu P_{L/R}b)\,, \tag{A.49}$$

$$(\overline{a}\gamma_\mu P_{L/R}b)(\overline{c}\gamma_\mu P_{R/L}d) = 2(\overline{a}P_{R/L}d)(\overline{c}P_{L/R}b)\,, \tag{A.50}$$

$$(\overline{a}P_{L/R}b)(\overline{c}P_{R/L}d) = \frac{1}{2}(\overline{a}\gamma_\mu P_{R/L}b)(\overline{c}\gamma_\mu P_{L/R}b)\,, \tag{A.51}$$

$$(\overline{a}P_{L/R}b)(\overline{c}P_{L/R}d) = \frac{1}{2}(\overline{a}P_{L/R}d)(\overline{c}P_{L/R}b) + \frac{1}{8}(\overline{a}\sigma^{\mu\nu}P_{L/R}d)(\overline{c}\sigma_{\mu\nu}P_{L/R}b)\,, \tag{A.52}$$

$$(\overline{a}\sigma^{\mu\nu}P_{L/R}b)(\overline{c}\sigma_{\mu\nu}P_{L/R}d) = 6(\overline{a}P_{L/R}d)(\overline{c}P_{L/R}b) - \frac{1}{2}(\overline{a}\sigma^{\mu\nu}P_{L/R}d)(\overline{c}\sigma_{\mu\nu}P_{L/R}b)\,. \tag{A.53}$$

In the calculation of the "crossed diagrams" charged conjugated spinors appear. Therefore we also state needed identities for them. Again these relations hold for commuting spinors a and b.

$$\overline{a}^c P_{L,R} b^c = -\overline{b} P_{L,R} a \tag{A.54}$$

$$\overline{a}^c \gamma^\mu P_{L,R} b^c = \overline{b}\gamma^\mu P_{R,L} a \tag{A.55}$$

$$\overline{a}^c \sigma^{\mu\nu} P_{L,R} b^c = \overline{b}\sigma^{\mu\nu} P_{L,R} a \tag{A.56}$$

A.4 Gordon Identities

$$\overline{u}_J(p')(p+p')^\mu u_I(p) = (m_I + m_J)\overline{u}_J(p')\gamma^\mu u_I(p) + \overline{u}_J(p')i\sigma^{\mu\nu}(p-p')_\nu u_I(p) \tag{A.57}$$

$$\begin{aligned}\overline{u}_J(p')(p+p')^\mu P_{L,R}u_I(p) = {}& m_I\,\overline{u}_J(p')\,\gamma^\mu P_{R,L}\,u_I(p) + m_J\,\overline{u}_J(p')\,\gamma^\mu P_{L,R}\,u_I(p) \\ &+ \overline{u}_J(p')i\sigma^{\mu\nu}(p-p')_\nu P_{L,R}u_I(p)\end{aligned} \tag{A.58}$$

Appendix B

Theoretical concepts and tools

We outline here the basic concepts and tools representing the theoretical background of the loop diagram calculations. This includes the following topics: regularization, renormalization and renormalization group equations. Very detailed discussions can be found for example in [64, 65].

B.1 Dimensional regularization

Loop integrals often feature divergences. In order to deal with the divergent quantities one first has to regularize the theory to have an explicit parametrization of the singularities. As a next step one has to perform a renormalization procedure and absorb left over divergences by redefining the quantities in the Lagrangian. These renormalized quantities will then depend on a renormalization scale and this dependence is determined by the renormalization group equations (RGEs).

Here we deal with the first step, outlining the main facts about regularization. Let us take as an example the self energy diagram in Fig. B.1 for the down quark self energy. Calculating such a diagram one encounters the logarithmic divergent integral

$$\int \frac{\mathrm{d}^4 q}{(2\pi)^4} \frac{1}{[q^2 - m^2_{\widetilde{D}_i}][(q+p)^2 - M^2_{\widetilde{g}}]} \xrightarrow{q\to\infty} \int \frac{\mathrm{d}^4 q}{(2\pi)^4} \frac{1}{q^4} \,. \tag{B.1}$$

To regularize this divergent integral one can for example put a cutoff on the integration

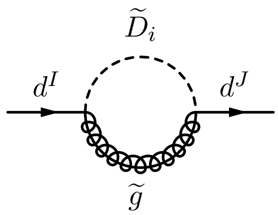


Figure B.1: Example of a down quark self energy diagram

momentum q. The problem with cutoff regularization schemes is that they explicitly break translation and also gauge invariance. A very popular regularization scheme which preserves these symmetries is dimensional regularization (DREG). The basic idea is to evaluate divergent integrals in whatever dimension $d = 4 - 2\epsilon$ needed to render them finite. This leads to analytic expressions in ϵ, where the divergences appear as poles at

$\epsilon = 0$. Generically, the integral (B.1) then reads

$$\int \frac{\mathrm{d}^d q}{(2\pi)^d} \frac{1}{[q^2 - m^2_{\widetilde{D}_i}][(q+p)^2 - M^2_{\widetilde{g}}]} = \frac{A}{\epsilon} + B + \mathcal{O}(\epsilon) \tag{B.2}$$

Techniques for evaluating d dimensional integrals are well established. A systematic treatment to one loop can be found for example in the appendix of [66].

One important issue concerning dimensional regularization is how the Dirac algebra and especially γ_5 is treated in d dimensions. In a commonly used scheme, called Naive Dimensional Regularization (NDREG), the gamma matrices obey the usual anticommutation relation

$$\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu} \tag{B.3}$$

where $g^{\mu\nu}$ is a d-dimensional metric tensor such that

$$g^{\mu\nu} = g^{\nu\mu}, \qquad g^{\mu\rho} g_\rho{}^\nu = g^{\mu\nu}, \qquad g_\mu{}^\mu = d\,. \tag{B.4}$$

The matrix γ_5 is taken to anticommute with all other gamma matrices

$$\{\gamma^\mu, \gamma_5\} = 0\,. \tag{B.5}$$

Unfortunately, this kind of regularization leads to inconsistencies [67] when traces like $\mathrm{Tr}(\gamma_5\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma)$ coming from "closed odd parity fermion loops" have to be evaluated. Moreover NDREG introduces a spurious violation of supersymmetry [68], because going to d dimensions leads to a mismatch between the numbers of gauge boson degrees of freedom (which are now d dimensional vectors) and their superpartners. In NDREG, supersymmetric relations between dimensionless coupling constants (*supersymmetric Ward identities*) are therefore not explicitly respected by radiative corrections involving the finite parts of one-loop graphs and by the divergent parts of two-loop graphs.

In order to preserve supersymmetry one may use another set of manipulation rules [68, 69, 70], known as regularization by dimensional reduction (DRED). In this regularization the only objects that are taken to be d dimensional are the coordinates x^μ and the momenta p^μ, whereas all other tensors stay in four dimensions. The gamma matrices $\widetilde{\gamma}^\mu$ and $\widetilde{\gamma}_5$ obey the usual anticommutation relations

$$\{\widetilde{\gamma}^\mu, \widetilde{\gamma}^\nu\} = 2\eta^{\mu\nu}, \qquad \{\widetilde{\gamma}^\mu, \widetilde{\gamma}_5\} = 0\,. \tag{B.6}$$

with the four dimensional metric tensor $\eta^{\mu\nu}$. As the d-dimensional metric tensor $\widetilde{g}^{\mu\nu}$ still appears in the evaluation of one loop integrals, one needs a rule for combining g and $\widetilde{g}$. The prescription adopted in DRED reads

$$\eta^{\mu\rho} g_\rho{}^\nu = g^{\mu\nu}\,. \tag{B.7}$$

For our calculations the above issues turn out not to play any role. Although single diagrams with self-energies or vertex corrections usually do feature divergences, these divergences cancel out once all diagrams are added to give physical amplitudes. The limit $\epsilon \to 0$ can then be taken without problems and both NDREG and DRED lead to the same results. For the actual calculation we have chosen to use NDREG.

B.2 Renormalization

Let us consider a Lagrangian that depends on fields f_0, masses m_0 and dimensionless couplings g_0

$$\mathcal{L}(f_0, m_0, g_0)\,, \tag{B.8}$$

where the subscript "0" denotes the *bare* (i.e. unrenormalized) quantities. The theory resulting from eq. (B.8) might lead to a number of divergent amplitudes. In order to eliminate these divergences we renormalize the bare fields and parameters inside the Lagrangian redefining them in the following way

$$f_0 = Z_f^{1/2} f, \qquad m_0 = Z_m m, \qquad g_0 = \mu^\epsilon Z_g g\,, \tag{B.9}$$

where we introduced the so-called *renormalization scale* μ which has mass dimension 1. One can easily convince oneself that the introduction of this renormalization scale has the effect of keeping the couplings g dimensionless once one goes into d dimensions. As the value of μ can in principle be chosen arbitrarily, physical observables cannot depend on this scale. A straightforward implementation of renormalization is then provided by the *counterterm* method. One simply reexpresses the original Lagrangian (B.8) in terms of the new renormalized quantities defined in (B.9)

$$\mathcal{L}(f_0, m_0, g_0) = \mathcal{L}'(f, m, g) = \mathcal{L}(f, m, g) + \mathcal{L}_c(f, m, g)\,. \tag{B.10}$$

The last equality can be seen as a definition of the counterterm Lagrangian $\mathcal{L}_c(f, m, g)$, that arises when the redefinitions (B.9) are performed in the following way

$$\begin{gathered} f_0 = Z_f^{1/2} f = f - (1 - Z_f^{1/2}) f, \qquad m_0 = Z_m m = m - (1 - Z_m) m\,, \\ g_0 = \mu^\epsilon Z_g g = \mu^\epsilon g - \mu^\epsilon (1 - Z_g)\,, \end{gathered} \tag{B.11}$$

and the Lagrangian $\mathcal{L}'$ is expanded accordingly. The counterterms, the Lagrangian $\mathcal{L}_c$ consists of, can formally be treated as additional interactions, which depend on the renormalization constants Z_i. These constants are chosen such that they cancel the $1/\epsilon^n$ divergences which arise when one does calculations based on $\mathcal{L}(f, m, g)$. This of course requires that also the Z_i are divergent. In full generality they can be written in the following way

$$Z_i = 1 + \sum_{k=1}^{\infty} \frac{1}{\epsilon^k} Z_{i,k}(g)\,. \tag{B.12}$$

The Z_i constants are obviously not unique and can be chosen accordingly to different renormalization schemes, for example in the *minimal subtraction* (MS) scheme they are chosen so that only the $1/\epsilon^n$ divergences are removed. One freedom that still remains lies in the definition of the renormalization scale μ. In principle it can be multiplied by an arbitrary constant, $\mu \to c\mu$. The following relation

$$\mu^2 \to \overline{\mu}^2 = \mu^2 e^{\gamma_E - \log 4\pi}\,, \tag{B.13}$$

defines the modified minimal subtraction $\overline{\text{MS}}$ scheme [71], which has the advantage that constant terms $\gamma_E - \log 4\pi$, which appear in the evaluation of one loop integrals, are effectively subtracted with the $1/\epsilon$ divergences.

B.3 Renormalization Group Equations

The introduction of the renormalization scale μ has important consequences. As bare quantities like g_0 and m_0 do not depend on μ

$$\frac{\mathrm{d}g_0}{\mathrm{d}\log\mu} = 0\,, \qquad \frac{\mathrm{d}m_0}{\mathrm{d}\log\mu} = 0\,, \tag{B.14}$$

one finds the following renormalization group equations that determine the μ dependence of the renormalized couplings and masses

$$\frac{\mathrm{d}g}{\mathrm{d}\log\mu} = \beta(g) - \epsilon g\,, \qquad \frac{\mathrm{d}m}{\mathrm{d}\log\mu} = -\gamma_m(g)m(\mu)\,. \tag{B.15}$$

The beta function $\beta(g)$ and the anomalous dimension of the mass $\gamma_m(g)$ are defined in the following way

$$\beta(g) = -g\frac{1}{Z_g}\frac{\mathrm{d}Z_g}{\mathrm{d}\log\mu}\,, \qquad \gamma_m(g) = \frac{1}{Z_m}\frac{\mathrm{d}Z_m}{\mathrm{d}\log\mu}\,, \tag{B.16}$$

and have the following perturbative expansion

$$\beta(g) = -\frac{g^3}{16\pi^2}\sum_{n=0}^{\infty}\beta_n\left(\frac{g^2}{16\pi^2}\right)^n\,, \qquad \gamma_m(g) = \frac{g^2}{16\pi^2}\sum_{n=0}^{\infty}\gamma_m^{(n)}\left(\frac{g^2}{16\pi^2}\right)^n\,. \tag{B.17}$$

Standard Model results for the beta function of the strong coupling $\beta(g_3)$ and for the anomalous dimension of the quark masses $\gamma_m(g_3)$ up to four loop in QCD are collected in [72]. Two loop results for all couplings and mass parameters in the MSSM can be found in [73], for example. The leading order results for the QCD β function and the anomalous dimension of the quark masses in the $\overline{\mathrm{MS}}$ scheme read

$$\beta_0 = 11 - \frac{2}{3}n_f\,, \qquad \gamma_m^{(0)} = 8\,, \tag{B.18}$$

with n_f the number of "active" quark flavours at the scale μ. Solving the renormalization group equations (B.15) to one loop order yields the well known expressions for the "running" $\alpha_s = g_3^2/4\pi$ and the "running" quark masses in the $\overline{\mathrm{MS}}$ scheme:

$$\alpha_s(\mu) = \frac{\alpha_s(\mu_0)}{1 - \beta_0\frac{\alpha_s(\mu_0)}{4\pi}\log\left(\frac{\mu_0^2}{\mu^2}\right)}\,, \tag{B.19}$$

$$m(\mu) = m(\mu_0)\left[\frac{\alpha_s(\mu)}{\alpha_s(\mu_0)}\right]^{\frac{\gamma_m^{(0)}}{2\beta_0}}\,. \tag{B.20}$$

It is well known - and obvious by expanding equation (B.19) in $\alpha_s(\mu_0)$ - that these expressions automatically sum up leading logarithms ln (μ_0^2/μ^2) to all orders in perturbation theory.

Appendix C

Amplitudes for $K^0 - \overline{K}^0$ mixing

C.1 Effective theory calculation

On the effective theory side we need the tree level matrix elements for the process $d\overline{s} \to \overline{d}s$. They can be written as[1]

$$\langle \overline{d}s|Q_i|d\overline{s}\rangle = \langle 0|d_d b_s\, Q_i\, b_d^\dagger d_s^\dagger|0\rangle \tag{C.1}$$

There are four possible insertions for each of the operators in the effective Hamiltonian (4.10). For example the matrix element for the fifth operator Q_5 is

$$\begin{aligned}\langle \overline{d}s|Q_5|d\overline{s}\rangle = &+ b_s\,(\overline{s}^\alpha P_L d^\beta)\, d_d\, d_s^\dagger\,(\overline{s}^\beta P_R d^\alpha)\, b_d^\dagger \\ &+ d_s^\dagger\,(\overline{s}^\alpha P_L d^\beta)\, b_d^\dagger\, b_s\,(\overline{s}^\beta P_R d^\alpha)\, d_d \\ &- b_s\,(\overline{s}^\alpha P_L d^\beta)\, b_d^\dagger\, d_s^\dagger\,(\overline{s}^\beta P_R d^\alpha)\, d_d \\ &- d_s^\dagger\,(\overline{s}^\alpha P_L d^\beta)\, d_d\, b_s\,(\overline{s}^\beta P_R d^\alpha)\, b_d^\dagger\,,\end{aligned} \tag{C.2}$$

where the relative signs arise with the permutation of the creation and annihilation operators $d_d b_s b_d^\dagger d_s^\dagger$. The contractions with the external operators are obtained as

$$\begin{aligned}\psi\, b^\dagger &= \langle 0|\psi(x) b^\dagger(p,s)|0\rangle &&\longrightarrow && u(p,s)\\ b\,\overline{\psi} &= \langle 0|b(p,s)\overline{\psi}(x)|0\rangle &&\longrightarrow && \overline{u}(p,s)\\ \overline{\psi}\, d^\dagger &= \langle 0|\overline{\psi}(x) d^\dagger(p,s)|0\rangle &&\longrightarrow && \overline{v}(p,s)\\ d\,\psi &= \langle 0|d(p,s)\psi(x)|0\rangle &&\longrightarrow && v(p,s)\,.\end{aligned} \tag{C.3}$$

In order to apply Eq. (C.3), we have to exchange the operators of the contractions in Eq. (C.2) appearing as $d^\dagger\,\overline{\psi}$ and $\psi\, d$ leading to an additional minus sign for each of these contractions.

The matrix element for the operator Q_5 is then

$$\begin{aligned}\langle \overline{d}s|Q_5|d\overline{s}\rangle = &(\overline{u}_s^\alpha P_L v_d^\beta)(\overline{v}_s^\beta P_R u_d^\alpha) + (\overline{u}_s^\alpha P_R v_d^\beta)(\overline{v}_s^\beta P_L u_d^\alpha)\\ &- (\overline{u}_s^\alpha P_L u_d^\beta)(\overline{v}_s^\beta P_R v_d^\alpha) - (\overline{u}_s^\alpha P_R u_d^\beta)(\overline{v}_s^\beta P_L v_d^\alpha)\,.\end{aligned} \tag{C.4}$$

The matrix elements for the other operators are obtained in the same way and the results are reported in the text.

[1] We indicate with $b_d^\dagger$, $b_s^\dagger$ the creation operators for d and s-type quarks and with $d_d^\dagger$ $d_s^\dagger$ the creation operators for antiquarks.

Bag Parameters and magic numbers The numerical value of the Bag parameters for the hadronic matrix elements and the magic numbers for the evolution of the Wilson Coefficients at the scale $\mu = 2\,\mathrm{GeV}$ have been taken from [43]:

$$B_1(\mu) = 0.60(6)\,, \qquad B_2(\mu) = 0.66(4)\,, \qquad B_3(\mu) = 1.05(12)\,,$$
$$B_4(\mu) = 1.03(6)\,, \qquad B_5(\mu) = 0.73(10)\,,$$

$$a_i = (0.29, -0.69, 0.79, -1.1, 0.14)$$

$$\begin{aligned}
b_i^{(11)} &= (0.82, 0, 0, 0, 0), & c_i^{(11)} &= (-0.016, 0, 0, 0, 0),\\
b_i^{(22)} &= (0, 2.4, 0.011, 0, 0), & c_i^{(22)} &= (0, -0.23, -0.002, 0, 0),\\
b_i^{(23)} &= (0, -0.63, 0.17, 0, 0), & c_i^{(23)} &= (0, -0.018, 0.0049, 0, 0),\\
b_i^{(32)} &= (0, -0.019, 0.028, 0, 0), & c_i^{(32)} &= (0, 0.0028, -0.0093, 0, 0),\\
b_i^{(33)} &= (0, 0.0049, 0.43, 0, 0), & c_i^{(33)} &= (0, 0.00021, 0.023, 0, 0),\\
b_i^{(44)} &= (0, 0, 0, 4.4, 0), & c_i^{(44)} &= (0, 0, 0, -0.68, 0.0055),\\
b_i^{(45)} &= (0, 0, 0, 1.5, -0.17), & c_i^{(45)} &= (0, 0, 0, -0.35, -0.0062),\\
b_i^{(54)} &= (0, 0, 0, 0.18, 0), & c_i^{(54)} &= (0, 0, 0, -0.026, -0.016),\\
b_i^{(55)} &= (0, 0, 0, 0.061, 0.82), & c_i^{(55)} &= (0, 0, 0, -0.013, 0.018).
\end{aligned}$$

C.2 Full theory calculation

We write here the amplitudes corresponding to the diagrams in Fig. 4.1. In the following we leave out explicit summation signs: it is always understood that one has to sum over all internal indices along the corresponding range depending on the particle the index belongs to.

$$\begin{aligned}
i\mathcal{M}_a = \int \frac{\mathrm{d}^4q}{(2\pi)^4} &\overline{u}_s^\beta \Big\{ i\sqrt{2}g_s[-(Z_D^*)_{2j}P_R + (Z_D^*)_{5j}P_L]T^A_{\beta\sigma}\Big\} \frac{i(\not{q} + M_{\tilde{g}})}{q^2 - M_{\tilde{g}}^2} \Big\{ i\sqrt{2}g_s T^A_{\lambda\alpha}[-(Z_D)_{1i}P_L + (Z_D)_{4i}P_R]\Big\} u_d^\alpha \\
&\overline{v}_s^\gamma \Big\{ i\sqrt{2}g_s[-(Z_D^*)_{2i}P_R + (Z_D^*)_{5i}P_L]T^B_{\gamma\lambda}\Big\} \frac{i(\not{q} + M_{\tilde{g}})}{q^2 - M_{\tilde{g}}^2} \Big\{ i\sqrt{2}g_s T^B_{\sigma\delta}[-(Z_D)_{1j}P_L + (Z_D)_{4j}P_R]\Big\} v_d^\delta \\
&\frac{i}{q^2 - m^2_{\tilde{D}_i}} \frac{i}{q^2 - m^2_{\tilde{D}_j}}\,,
\end{aligned}$$

$$\begin{aligned}
i\mathcal{M}_b = \int \frac{\mathrm{d}^4q}{(2\pi)^4} &\overline{u}_s^\beta \Big\{ i\sqrt{2}g_s[-(Z_D^*)_{2i}P_R + (Z_D^*)_{5i}P_L]T^B_{\beta\lambda}\Big\} \frac{i(\not{q} + M_{\tilde{g}})}{q^2 - M_{\tilde{g}}^2} \Big\{ i\sqrt{2}g_s T^B_{\sigma\delta}[-(Z_D)_{1j}P_L + (Z_D)_{4j}P_R]\Big\} v_d^\delta \\
&\overline{v}_s^\gamma \Big\{ i\sqrt{2}g_s[-(Z_D^*)_{2j}P_R + (Z_D^*)_{5j}P_L]T^A_{\gamma\sigma}\Big\} \frac{i(\not{q} + M_{\tilde{g}})}{q^2 - M_{\tilde{g}}^2} \Big\{ i\sqrt{2}g_s T^A_{\lambda\alpha}[-(Z_D)_{1i}P_L + (Z_D)_{4i}P_R]\Big\} u_d^\alpha \\
&\frac{i}{q^2 - m^2_{\tilde{D}_i}} \frac{i}{q^2 - m^2_{\tilde{D}_j}}\,,
\end{aligned}$$

$$\begin{aligned}
i\mathcal{M}_c = \int \frac{\mathrm{d}^4q}{(2\pi)^4} &\overline{v}_s^\gamma \Big\{ i\sqrt{2}g_s[-(Z_D^*)_{2i}P_R + (Z_D^*)_{5i}P_L]T^A_{\gamma\lambda}\Big\} \frac{i(\not{q} + M_{\tilde{g}})}{q^2 - M_{\tilde{g}}^2} C\Big\{ i\sqrt{2}g_s T^A_{\beta\sigma}[-(Z_D^*)_{2j}P_R + (Z_D^*)_{5j}P_L]\Big\}^T C^{-1} v_s^\beta \\
&\overline{u}_d^\delta\, C\Big\{ i\sqrt{2}g_s[-(Z_D)_{1j}P_L + (Z_D)_{4j}P_R]T^B_{\sigma\delta}\Big\}^T C^{-1} \frac{i(\not{q} + M_{\tilde{g}})}{q^2 - M_{\tilde{g}}^2} \Big\{ i\sqrt{2}g_s T^B_{\lambda\alpha}[-(Z_D)_{1i}P_L + (Z_D)_{4i}P_R]\Big\} u_d^\alpha \\
&\frac{i}{q^2 - m^2_{\tilde{D}_i}} \frac{i}{q^2 - m^2_{\tilde{D}_j}}\,,
\end{aligned}$$

$$
\begin{aligned}
i\mathcal{M}_d = \int \frac{\mathrm{d}^4q}{(2\pi)^4}\overline{v}_s^\gamma \Big\{i\sqrt{2}g_s[-(Z_D^*)_{2j}P_R+(Z_D^*)_{5j}P_L]T^A_{\gamma\sigma}\Big\}\frac{i(-\not{q}+M_{\tilde g})}{q^2-M^2_{\tilde g}}C\Big\{i\sqrt{2}g_sT^A_{\beta\lambda}[-(Z_D^*)_{2i}P_R+(Z_D^*)_{5i}P_L]\Big\}^T C^{-1}v_s^\beta \\
\overline{u}_d^\delta\, C\Big\{i\sqrt{2}g_s[-(Z_D)_{1j}P_L+(Z_D)_{4j}P_R]T^B_{\sigma\delta}\Big\}^T C^{-1}\frac{i(\not{q}+M_{\tilde g})}{q^2-M^2_{\tilde g}}\Big\{i\sqrt{2}g_sT^B_{\lambda\alpha}[-(Z_D)_{1i}P_L+(Z_D)_{4i}P_R]\Big\}u_d^\alpha \\
\frac{i}{q^2-m^2_{\tilde D_i}}\frac{i}{q^2-m^2_{\tilde D_j}}\,.
\end{aligned}
$$

Here the transposition is intended over the spinorial indices (it does not affect the colour indices). By use of Eqs. (A.43) and (A.48) one easily finds that $CP^T_{L/R}C^{-1}=P_{L/R}$. Please note that the first gluino propagator inside $\mathcal{M}_d$ features a momentum with opposite sign with respect to the second propagator. This is due to the opposite direction of the momentum flow with respect to the chosen orientation of the fermion chain.

As one can see, peculiar products of the Gell-Mann matrices appear in the amplitudes, with two different structures, one for the straight diagrams and one for the crossed diagrams. By use of equation (A.8) and carefully summing over repeated indices we get

$$
(T^A_{\beta\sigma}T^A_{\lambda\alpha})(T^B_{\gamma\lambda}T^B_{\sigma\delta}) = \frac{1}{4}\left(\frac{7}{3}\delta_{\beta\alpha}\delta_{\gamma\delta}+\frac{1}{9}\delta_{\gamma\alpha}\delta_{\beta\delta}\right), \tag{C.5}
$$

for the straight diagrams and

$$
(T^A_{\beta\lambda}T^A_{\gamma\sigma})(T^B_{\sigma\delta}T^B_{\lambda\alpha}) = \frac{1}{4}\left(\frac{10}{9}\delta_{\beta\alpha}\delta_{\gamma\delta}-\frac{2}{3}\delta_{\beta\delta}\delta_{\gamma\alpha}\right) \tag{C.6}
$$

for the crossed ones. There are two non-vanishing integrals which have to be evaluated which can be found in Appendix E. We repeat here their definition

$$
\int\frac{\mathrm{d}^4q}{(2\pi)^4}\frac{1}{(q^2-M^2_{\tilde g})^2(q^2-m^2_{\tilde D_i})(q^2-m^2_{\tilde D_j})} \equiv \frac{i}{16\pi^2}D_0(m^2_{\tilde D_i},m^2_{\tilde D_j},M^2_{\tilde g},M^2_{\tilde g})\,, \tag{C.7}
$$

$$
\int\frac{\mathrm{d}^4q}{(2\pi)^4}\frac{q_\mu q_\nu}{(q^2-M^2_{\tilde g})^2(q^2-m^2_{\tilde D_i})(q^2-m^2_{\tilde D_j})} \equiv \frac{i\eta_{\mu\nu}}{16\pi^2}D_2(m^2_{\tilde D_i},m^2_{\tilde D_j},M^2_{\tilde g},M^2_{\tilde g})\,. \tag{C.8}
$$

In order to keep the expressions in a tolerably compact form, we separate the amplitudes in pieces with a given combination of the projection operators $P_{L/R}$. Each time the first (second) projector in the square brackets is selected we are referring to a left (right) handed spinor or antispinor. We will name the so obtained amplitudes with superscripts referring to the chirality of the ingoing and outgoing particles. For example $\mathcal{M}^{(LL)(LL)}$ is the amplitude for the process $d_L\,\overline{s}_L\to\overline{d}_L\,s_L$, while $\mathcal{M}^{(LR)(RL)}$ is the amplitude for the process $d_L\,\overline{s}_R\to\overline{d}_R\,s_L$ and so on. The identities needed to bring the amplitudes to a form corresponding to (4.15) can be found in the Appendix A.3.

Amplitudes for $d_L\,\overline{s}_L\to\overline{d}_L\,s_L$

$$
\begin{aligned}
\mathcal{M}_a^{(\mathrm{LL})(\mathrm{LL})} &= \frac{g_s^4}{16\pi^4}D_2(m^2_{\tilde D_i},m^2_{\tilde D_j},M^2_{\tilde g},M^2_{\tilde g})(Z_D)_{1i}(Z_D)_{1j}(Z_D^*)_{2i}(Z_D^*)_{2j}\times \\
&\times\left\{\frac{7}{3}(\overline{u}_s\gamma^\mu P_Lu_d)(\overline{v}_s\gamma_\mu P_Lv_d)-\frac{1}{9}(\overline{u}_s\gamma^\mu P_Lv_d)(\overline{v}_s\gamma_\mu P_Lu_d)\right\}
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{M}_b^{(\mathrm{LL})(\mathrm{LL})} &= \frac{g_s^4}{16\pi^4}D_2(m^2_{\tilde D_i},m^2_{\tilde D_j},M^2_{\tilde g},M^2_{\tilde g})(Z_D)_{1i}(Z_D)_{1j}(Z_D^*)_{2i}(Z_D^*)_{2j}\times \\
&\times\left\{\frac{7}{3}(\overline{u}_s\gamma^\mu P_Lv_d)(\overline{v}_s\gamma_\mu P_Lu_d)-\frac{1}{9}(\overline{u}_s\gamma^\mu P_Lu_d)(\overline{v}_s\gamma_\mu P_Lv_d)\right\}
\end{aligned}
$$

$$\mathcal{M}_c^{(\mathrm{LL})(\mathrm{LL})} = \frac{g_s^4}{16\pi^4} M_{\tilde{g}}^2 D_0(m^2_{\widetilde{D}_i}, m^2_{\widetilde{D}_j}, M^2_{\tilde{g}}, M^2_{\tilde{g}})(Z_D)_{1i}(Z_D)_{1j}(Z_D^*)_{2i}(Z_D^*)_{2j} \times \\ \times \left\{ \frac{5}{9}(\overline{u}_s\gamma^\mu P_L v_d)(\overline{v}_s\gamma_\mu P_L u_d) + \frac{1}{3}(\overline{u}_s\gamma^\mu P_L u_d)(\overline{v}_s\gamma_\mu P_L v_d) \right\}$$

$$\mathcal{M}_d^{(\mathrm{LL})(\mathrm{LL})} = \frac{g_s^4}{16\pi^4} M_{\tilde{g}}^2 D_0(m^2_{\widetilde{D}_i}, m^2_{\widetilde{D}_j}, M^2_{\tilde{g}}, M^2_{\tilde{g}})(Z_D)_{1i}(Z_D)_{1j}(Z_D^*)_{2i}(Z_D^*)_{2j} \times \\ \times \left\{ -\frac{1}{3}(\overline{u}_s\gamma^\mu P_L v_d)(\overline{v}_s\gamma_\mu P_L u_d) - \frac{5}{9}(\overline{u}_s\gamma^\mu P_L u_d)(\overline{v}_s\gamma_\mu P_L v_d) \right\}$$

Amplitudes for $d_R\,\overline{s}_R \to \overline{d}_R\,s_R$. The amplitudes $M_\alpha^{(\mathrm{RR})(\mathrm{RR})}$ with $\alpha = a,b,c,d$ are obtained by the corresponding amplitudes $\mathcal{M}_\alpha^{(\mathrm{LL})(\mathrm{LL})}$ after the replacements

$$P_L \to P_R\,, \qquad (Z_D)_{1k} \to (Z_D)_{4k}\,, \qquad (Z_D)_{2k} \to (Z_D)_{5k}\,.$$

Amplitudes for $d_L\,\overline{s}_R \to \overline{d}_L\,s_R$

$$\mathcal{M}_a^{(\mathrm{LR})(\mathrm{LR})} = \frac{g_s^4}{16\pi^4} M_{\tilde{g}}^2 D_0(m^2_{\widetilde{D}_i}, m^2_{\widetilde{D}_j}, M^2_{\tilde{g}}, M^2_{\tilde{g}})(Z_D)_{1i}(Z_D)_{1j}(Z_D^*)_{5i}(Z_D^*)_{5j} \times \\ \times \left\{ \frac{7}{3}(\overline{u}_s^\alpha P_L u_d^\alpha)(\overline{v}_s^\beta P_L v_d^\beta) + \frac{1}{9}(\overline{u}_s^\alpha P_L u_d^\beta)(\overline{v}_s^\beta P_L v_d^\alpha) \right\}$$

$$\mathcal{M}_b^{(\mathrm{LR})(\mathrm{LR})} = \frac{g_s^4}{16\pi^4} M_{\tilde{g}}^2 D_0(m^2_{\widetilde{D}_i}, m^2_{\widetilde{D}_j}, M^2_{\tilde{g}}, M^2_{\tilde{g}})(Z_D)_{1i}(Z_D)_{1j}(Z_D^*)_{5i}(Z_D^*)_{5j} \times \\ \times \left\{ \frac{7}{3}(\overline{u}_s^\alpha P_L v_d^\alpha)(\overline{v}_s^\beta P_L u_d^\beta) + \frac{1}{9}(\overline{u}_s^\alpha P_L v_d^\beta)(\overline{v}_s^\beta P_L u_d^\alpha) \right\}$$

$$\mathcal{M}_c^{(\mathrm{LR})(\mathrm{LR})} = \frac{g_s^4}{16\pi^4} M_{\tilde{g}}^2 D_0(m^2_{\widetilde{D}_i}, m^2_{\widetilde{D}_j}, M^2_{\tilde{g}}, M^2_{\tilde{g}})(Z_D)_{1i}(Z_D)_{1j}(Z_D^*)_{5i}(Z_D^*)_{5j} \times \\ \times \left\{ -\frac{10}{9}(\overline{u}_s^\alpha P_L v_d^\alpha)(\overline{v}_s^\beta P_L u_d^\beta) + \frac{10}{9}(\overline{u}_s^\alpha P_L u_d^\beta)(\overline{v}_s^\beta P_L v_d^\alpha) \right. \\ \left. + \frac{2}{3}(\overline{u}_s^\alpha P_L v_d^\beta)(\overline{v}_s^\beta P_L u_d^\alpha) - \frac{2}{3}(\overline{u}_s^\alpha P_L u_d^\alpha)(\overline{v}_s^\beta P_L v_d^\beta) \right\}$$

$$\mathcal{M}_d^{(\mathrm{LR})(\mathrm{LR})} = \frac{g_s^4}{16\pi^4} M_{\tilde{g}}^2 D_0(m^2_{\widetilde{D}_i}, m^2_{\widetilde{D}_j}, M^2_{\tilde{g}}, M^2_{\tilde{g}})(Z_D)_{1i}(Z_D)_{1j}(Z_D^*)_{5i}(Z_D^*)_{5j} \times \\ \times \left\{ +\frac{2}{3}(\overline{u}_s^\alpha P_L v_d^\alpha)(\overline{v}_s^\beta P_L u_d^\beta) - \frac{2}{3}(\overline{u}_s^\alpha P_L u_d^\beta)(\overline{v}_s^\beta P_L v_d^\alpha) \right. \\ \left. - \frac{10}{9}(\overline{u}_s^\alpha P_L v_d^\beta)(\overline{v}_s^\beta P_L u_d^\alpha) + \frac{10}{9}(\overline{u}_s^\alpha P_L u_d^\alpha)(\overline{v}_s^\beta P_L v_d^\beta) \right\}$$

Amplitudes for $d_R\,\overline{s}_L \to \overline{d}_R\,s_L$. The amplitudes $M_\alpha^{(\mathrm{RL})(\mathrm{RL})}$ with $\alpha = a,b,c,d$ are obtained by the corresponding amplitudes $\mathcal{M}_\alpha^{(\mathrm{LR})(\mathrm{LR})}$ after the replacements

$$P_L \to P_R\,, \qquad (Z_D)_{1k} \to (Z_D)_{4k}\,, \qquad (Z_D)_{5k} \to (Z_D)_{2k}\,.$$

Amplitudes for $d_L\,\overline{s}_R \to \overline{d}_R\,s_L$

$$\begin{aligned}\mathcal{M}_a^{(\mathrm{LR})(\mathrm{RL})} &= \frac{g_s^4}{16\pi^4} D_2(m^2_{\widetilde{D}_i}, m^2_{\widetilde{D}_j}, M^2_{\widetilde{g}}, M^2_{\widetilde{g}})(Z_D)_{1i}(Z_D)_{4j}(Z^*_D)_{5i}(Z^*_D)_{2j}\times \\ &\times \left\{\frac{14}{3}(\overline{u}_s^\alpha P_R v_d^\beta)(\overline{v}_s^\beta P_L u_d^\alpha) + \frac{2}{9}(\overline{u}_s^\alpha P_R v_d^\alpha)(\overline{v}_s^\beta P_L u_d^\beta)\right\}\end{aligned}$$

$$\begin{aligned}\mathcal{M}_b^{(\mathrm{LR})(\mathrm{RL})} &= \frac{g_s^4}{16\pi^4} M^2_{\widetilde{g}} D_0(m^2_{\widetilde{D}_i}, m^2_{\widetilde{D}_j}, M^2_{\widetilde{g}}, M^2_{\widetilde{g}})(Z_D)_{1i}(Z_D)_{4j}(Z^*_D)_{2i}(Z^*_D)_{5j}\times \\ &\times \left\{\frac{1}{9}(\overline{u}_s^\alpha P_R v_d^\beta)(\overline{v}_s^\beta P_L u_d^\alpha) + \frac{7}{3}(\overline{u}_s^\alpha P_R v_d^\alpha)(\overline{v}_s^\beta P_L u_d^\beta)\right\}\end{aligned}$$

$$\begin{aligned}\mathcal{M}_c^{(\mathrm{LR})(\mathrm{RL})} &= \frac{g_s^4}{16\pi^4} D_2(m^2_{\widetilde{D}_i}, m^2_{\widetilde{D}_j}, M^2_{\widetilde{g}}, M^2_{\widetilde{g}})(Z_D)_{1i}(Z_D)_{4j}(Z^*_D)_{5i}(Z^*_D)_{2j}\times \\ &\times \left\{\frac{4}{3}(\overline{u}_s^\alpha P_R v_d^\beta)(\overline{v}_s^\beta P_L u_d^\alpha) - \frac{20}{9}(\overline{u}_s^\alpha P_R v_d^\alpha)(\overline{v}_s^\beta P_L u_d^\beta)\right\}\end{aligned}$$

$$\begin{aligned}\mathcal{M}_d^{(\mathrm{LR})(\mathrm{RL})} &= \frac{g_s^4}{16\pi^4} D_2(m^2_{\widetilde{D}_i}, m^2_{\widetilde{D}_j}, M^2_{\widetilde{g}}, M^2_{\widetilde{g}})(Z_D)_{1i}(Z_D)_{4j}(Z^*_D)_{2i}(Z^*_D)_{5j}\times \\ &\times \left\{\frac{20}{9}(\overline{u}_s^\alpha P_R v_d^\beta)(\overline{v}_s^\beta P_L u_d^\alpha) - \frac{4}{3}(\overline{u}_s^\alpha P_R v_d^\alpha)(\overline{v}_s^\beta P_L u_d^\beta)\right\}\end{aligned}$$

Amplitudes for $d_R\,\overline{s}_L \to \overline{d}_L\,s_R$**.** The amplitudes $M_\alpha^{(\mathrm{RL})(\mathrm{LR})}$ with $\alpha = a,b,c,d$ are obtained by the corresponding amplitudes $\mathcal{M}_\alpha^{(\mathrm{LR})(\mathrm{RL})}$ after the replacements $P_L \leftrightarrow P_R$.

Amplitudes for $d_R\,\overline{s}_R \to \overline{d}_L\,s_L$

$$\begin{aligned}\mathcal{M}_a^{(\mathrm{RR})(\mathrm{LL})} &= \frac{g_s^4}{16\pi^4} M^2_{\widetilde{g}} D_0(m^2_{\widetilde{D}_i}, m^2_{\widetilde{D}_j}, M^2_{\widetilde{g}}, M^2_{\widetilde{g}})(Z_D)_{1i}(Z_D)_{4j}(Z^*_D)_{2i}(Z^*_D)_{5j}\times \\ &\times \left\{\frac{1}{9}(\overline{u}_s^\alpha P_R u_d^\beta)(\overline{v}_s^\beta P_L v_d^\alpha) + \frac{7}{3}(\overline{u}_s^\alpha P_R u_d^\alpha)(\overline{v}_s^\beta P_L v_d^\beta)\right\}\end{aligned}$$

$$\begin{aligned}\mathcal{M}_b^{(\mathrm{RR})(\mathrm{LL})} &= \frac{g_s^4}{16\pi^4} D_2(m^2_{\widetilde{D}_i}, m^2_{\widetilde{D}_j}, M^2_{\widetilde{g}}, M^2_{\widetilde{g}})(Z_D)_{1i}(Z_D)_{4j}(Z^*_D)_{5i}(Z^*_D)_{2j}\times \\ &\times \left\{\frac{14}{3}(\overline{u}_s^\alpha P_R u_d^\beta)(\overline{v}_s^\beta P_L v_d^\alpha) + \frac{2}{9}(\overline{u}_s^\alpha P_R u_d^\alpha)(\overline{v}_s^\beta P_L v_d^\beta)\right\}\end{aligned}$$

$$\begin{aligned}\mathcal{M}_c^{(\mathrm{RR})(\mathrm{LL})} &= \frac{g_s^4}{16\pi^4} D_2(m^2_{\widetilde{D}_i}, m^2_{\widetilde{D}_j}, M^2_{\widetilde{g}}, M^2_{\widetilde{g}})(Z_D)_{1i}(Z_D)_{4j}(Z^*_D)_{2i}(Z^*_D)_{5j}\times \\ &\times \left\{-\frac{20}{9}(\overline{u}_s^\alpha P_R u_d^\beta)(\overline{v}_s^\beta P_L v_d^\alpha) + \frac{4}{3}(\overline{u}_s^\alpha P_R u_d^\alpha)(\overline{v}_s^\beta P_L v_d^\beta)\right\}\end{aligned}$$

$$\begin{aligned}\mathcal{M}_d^{(\mathrm{RR})(\mathrm{LL})} &= \frac{g_s^4}{16\pi^4} D_2(m^2_{\widetilde{D}_i}, m^2_{\widetilde{D}_j}, M^2_{\widetilde{g}}, M^2_{\widetilde{g}})(Z_D)_{1i}(Z_D)_{4j}(Z^*_D)_{5i}(Z^*_D)_{2j}\times \\ &\times \left\{-\frac{4}{3}(\overline{u}_s^\alpha P_R u_d^\beta)(\overline{v}_s^\beta P_L v_d^\alpha) + \frac{20}{9}(\overline{u}_s^\alpha P_R u_d^\alpha)(\overline{v}_s^\beta P_L v_d^\beta)\right\}\end{aligned}$$

Amplitudes for $d_L\,\bar{s}_L \to \bar{d}_R\,s_R$. The amplitudes $M_\alpha^{(\mathrm{LL})(\mathrm{RR})}$ with $\alpha = a, b, c, d$ are obtained by the corresponding amplitudes $\mathcal{M}_\alpha^{(\mathrm{RR})(\mathrm{LL})}$ after the replacements $P_L \leftrightarrow P_R$.

C.3 Determination of the relative sign

In order to get the complete amplitude for two-gluino box diagrams we must sum the four contribution with the proper relative sign coming from the Wick contractions. Adopting the conventions of [44] we write the matrix element corresponding to the Feynman diagrams in Fig. 4.1 as

$$\langle 0| d_d b_s\, \mathrm{T}\{(\bar{s}\,\Gamma_s\lambda)(\bar{\lambda}\,\Gamma_d\,d)(\bar{s}\,\Gamma_s\lambda)(\bar{\lambda}\,\Gamma_d\,d)\}\, b_d^\dagger d_s^\dagger |0\rangle \ , \tag{C.9}$$

where λ is the Majorana field describing gluinos, while $\Gamma_{d,s}$ denotes generic fermionic interactions connecting d and s quarks to gluino and include Dirac matrices, coupling constants and squark fields. The T-product is then evaluated according to the Wick theorem. The sign emerging in the amplitudes is due to the permutation of the creation and annihilation operators among themselves (the reordering of the interaction Lagrangians does not yield minus signs). Crossed diagrams are obtained when the gluino fields are contracted as $\lambda\lambda$ and $\bar{\lambda}\bar{\lambda}$. In order to write them in the usual form we use the identity $(\chi_1\Gamma\chi_2) = (\overline{\chi_2}^c\Gamma'\chi_1^c)$, where $\Gamma' = C\Gamma^T C^{-1}$ and χ^c denotes the conjugate of the spinor χ. We get the following signs for the matrix elements

$$\begin{aligned}
-b_s\,(\bar{s}\,\Gamma_s\lambda)(\bar{\lambda}\,\Gamma_d\,d)\,b_d^\dagger\,d_s^\dagger\,(\bar{s}\,\Gamma_s\lambda)(\bar{\lambda}\,\Gamma_d\,d)\,d_d \quad &\to \quad -\mathcal{M}_a \\
+b_s\,(\bar{s}\,\Gamma_s\lambda)(\bar{\lambda}\,\Gamma_d\,d)\,d_d\,d_s^\dagger\,(\bar{s}\,\Gamma_s\lambda)(\bar{\lambda}\,\Gamma_d\,d)\,b_d^\dagger \quad &\to \quad +\mathcal{M}_b \\
+d_s^\dagger\,(\bar{s}\,\Gamma_s\lambda)(\bar{\lambda}^c\,\Gamma_s'\,s^c)\,b_s\,d_d\,(\bar{d}^c\,\Gamma_d'\lambda^c)(\bar{\lambda}\,\Gamma_d\,d)\,b_d^\dagger \quad &\to \quad +\mathcal{M}_{c,d}
\end{aligned}$$

We did not write explicitly the contractions among scalar fields since they do not change the overall sign. The amplitudes $\mathcal{M}_c$ and $\mathcal{M}_d$ share the same fermion lines and differ only for the scalar fields contractions so they have the same sign. The complete amplitude for two-gluino box diagrams is then given by $-\mathcal{M}_a + \mathcal{M}_b + \mathcal{M}_c + \mathcal{M}_d$.

C.4 Exact diagonalization in a two-generation framework

In order to keep the notation light we adopt here the following convention for the down squark mass eigenvalues

$$Z_D^\dagger \mathcal{M}_D^2 Z_D = \mathrm{diag}(m^2_{\tilde{q}_1}, m^2_{\tilde{q}_2}, m^2_{\tilde{q}_3}, m^2_{\tilde{d}_1}, m^2_{\tilde{d}_2}, m^2_{\tilde{d}_3})\,, \tag{C.10}$$

in place of the usual $m^2_{\tilde{D}_i}$ $i = 1, \ldots, 6$.

We work in a two generation framework, neglecting also the LR/RL terms. Under these assumptions the mass matrix for the down squarks is block diagonal

$$\mathcal{M}_D^2 = \begin{pmatrix} \mathcal{M}_{LL}^2 & \mathcal{M}_{LR}^2 \\ \mathcal{M}_{LR}^{2\,\dagger} & \mathcal{M}_{RR}^2 \end{pmatrix} \approx \begin{pmatrix} M_{LL}^2 & 0 & \cdots & 0 \\ 0 & m^2_{\tilde{q}_3} & \ddots & \vdots \\ \vdots & \ddots & M_{RR}^2 & 0 \\ 0 & \cdots & 0 & m^2_{\tilde{d}_3} \end{pmatrix} . \tag{C.11}$$

Here M^2_{LL} and M^2_{RR} are hermitian 2×2 matrices and can be diagonalized by two unitary matrices U_L and U_R defined as

$$U_L^\dagger M^2_{LL} U_L = \mathrm{diag}(m^2_{\tilde q_1}, m^2_{\tilde q_2})\,, \qquad U_R^\dagger M^2_{RR} U_R = \mathrm{diag}(m^2_{\tilde d_1}, m^2_{\tilde d_2})\,, \tag{C.12}$$

$$U_L = \begin{pmatrix} c_L & -s_L e^{-i\phi_L} \\ s_L e^{i\phi_L} & c_L \end{pmatrix}, \qquad U_R = \begin{pmatrix} c_R & -s_L e^{-i\phi_R} \\ s_R e^{i\phi_R} & c_R \end{pmatrix}. \tag{C.13}$$

Where $c_N = \cos\theta_N$, $s_N = \sin\theta_N$, $N = L, R$. The squark squared masses $m^2_{\tilde q_{1,2}}$ and $m^2_{\tilde d_{1,2}}$ are then the eigenvalues

$$m^2_{\tilde f_{1,2}} = \frac{m^2_{11} + m^2_{22} \pm \sqrt{\left(m^2_{11} - m^2_{22}\right)^2 + 4\left|m^2_{12}\right|^2}}{2}\,, \qquad f = q, u \tag{C.14}$$

where m^2_{ij} stands for $\left(M^2_{LL}\right)_{ij}$ or $\left(M^2_{RR}\right)_{ij}$ when $f = q, u$ respectively.

The unitary matrix Z_D which diagonalize the down squark mass matrix $Z_D^\dagger \mathcal{M}^2_D Z_D = \mathrm{diag}(m^2_{\tilde q_1}, \ldots, m^2_{\tilde d_3})$ is block diagonal. The only non-vanishing entries are

$$\begin{array}{llll} (Z_D)_{11} = c_L & (Z_D)_{12} = -s_L e^{-i\phi_L} & (Z_D)_{44} = c_R & (Z_D)_{45} = -s_R e^{-i\phi_R} \\ (Z_D)_{21} = s_L e^{i\phi_L} & (Z_D)_{22} = c_L & (Z_D)_{54} = s_R e^{i\phi_R} & (Z_D)_{55} = c_R \\ & (Z_D)_{33} = 1 & (Z_D)_{66} = 1 & \end{array} \tag{C.15}$$

Using this two-generation framework can express the Wilson coefficients for (Eqs. (4.23)-(4.30)) as functions of the mixing angles θ_L, θ_R and the phases ϕ_L, ϕ_R. We compute, as a representative example, the following quantity appearing in $\delta^g C_4$

$$\begin{aligned} \sum_{i,j=0}^{6} & D_0(m^2_{\tilde D_i}, m^2_{\tilde D_j}, M^2_{\tilde g}, M^2_{\tilde g})(Z_D^*)_{2i}(Z_D)_{1i}(Z_D^*)_{5j}(Z_D)_{4j} = \\ & +D_0(m^2_{\tilde q_1}, m^2_{\tilde d_1}, M^2_{\tilde g}, M^2_{\tilde g})(Z_D^*)_{21}(Z_D)_{11}(Z_D^*)_{54}(Z_D)_{44} \\ & +D_0(m^2_{\tilde q_1}, m^2_{\tilde d_2}, M^2_{\tilde g}, M^2_{\tilde g})(Z_D^*)_{21}(Z_D)_{11}(Z_D^*)_{55}(Z_D)_{45} \\ & +D_0(m^2_{\tilde q_2}, m^2_{\tilde d_2}, M^2_{\tilde g}, M^2_{\tilde g})(Z_D^*)_{22}(Z_D)_{12}(Z_D^*)_{55}(Z_D)_{45} \\ & +D_0(m^2_{\tilde q_2}, m^2_{\tilde d_1}, M^2_{\tilde g}, M^2_{\tilde g})(Z_D^*)_{22}(Z_D)_{12}(Z_D^*)_{54}(Z_D)_{44} \\ & = \left[s_L c_L e^{-i\phi_L}\right]\left[s_R c_R e^{-i\phi_R}\right] B_{qd}(M^2_{\tilde g}, M^2_{\tilde g})\,. \end{aligned} \tag{C.16}$$

Where in the first line we wrote only the terms not involving null elements of Z_D, then we substituted the values of the matrix elements as in Eq. (C.15). The function $B_{qd}(M^2_{\tilde g}, M^2_{\tilde g})$ is given in Appendix E.

Expression for the mixing angles

Here we derive a useful expression for the combination $s_L c_L e^{-i\phi_L}$. We start writing the mass matrix M^2_{LL} as

$$M^2_{LL} = U_L \begin{pmatrix} m^2_{\tilde q_1} & 0 \\ 0 & m^2_{\tilde q_2} \end{pmatrix} U_L^\dagger \tag{C.17}$$

and we compute the following difference

$$U_L \begin{pmatrix} m^2_{\tilde q_1} & 0 \\ 0 & m^2_{\tilde q_2} \end{pmatrix} - \begin{pmatrix} m^2_{\tilde q_1} & 0 \\ 0 & m^2_{\tilde q_2} \end{pmatrix} U_L = s_L(m^2_{\tilde q_1} - m^2_{\tilde q_2}) \begin{pmatrix} 0 & e^{-i\phi_L} \\ e^{i\phi_L} & 0 \end{pmatrix}. \tag{C.18}$$

We then take the expression for $U_L \cdot \mathrm{diag}(m^2_{\tilde{q}_1}, m^2_{\tilde{q}_2})$ from the above equation and substitute it into Eq. (C.17). We get

$$M^2_{LL} = \begin{pmatrix} m^2_{\tilde{q}_1} & 0 \\ 0 & m^2_{\tilde{q}_2} \end{pmatrix} + s_L(m^2_{\tilde{q}_1} - m^2_{\tilde{q}_2}) \begin{pmatrix} s_L & c_L e^{-i\phi_L} \\ c_L e^{i\phi_L} & -s_L \end{pmatrix} . \tag{C.19}$$

The equality for the second element in the first row reads

$$\left(M^2_{LL}\right)_{12} = \left(m^2_{\tilde{q}_1} - m^2_{\tilde{q}_2}\right) s_L c_L e^{-i\phi_L} . \tag{C.20}$$

The same result is valid for M^2_{RR} with the substitutions $L \to R$, $q \to d$. We then have the following relations

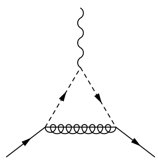
$$s_L c_L e^{-i\phi_L} = \frac{\left(M^2_{LL}\right)_{12}}{m^2_{\tilde{q}_1} - m^2_{\tilde{q}_2}} , \qquad s_R c_R e^{-i\phi_R} = \frac{\left(M^2_{RR}\right)_{12}}{m^2_{\tilde{d}_1} - m^2_{\tilde{d}_2}} . \tag{C.21}$$

Appendix D

Amplitudes for the (chromo)magnetic operators

D.1 Full theory calculation for $b \to s\gamma$ process

We calculate the one-loop gluino contributions to the Wilson coefficients $C_{7\gamma}^{(\prime)}$ for the process $b \to s\gamma$. The three diagrams which are involved have been depicted in Fig. 4.2. These diagrams feature loops with heavy virtual particles but we can not set the external momenta to zero since we have an external massless particle. We will perform an expansion in $m_{b,s}/M_{\widetilde{g}}$ and neglect terms of the second order. The amplitude for the first diagram reads



$$= \quad \overline{u}_s^{\beta}(p')\Big[i\Gamma^{\mu}_{bs\gamma}(k)\delta_{\beta\alpha}\Big]u_b^{\alpha}(p)\,\epsilon^*_{\mu}(k)\,, \tag{D.1}$$

$$\begin{aligned} i\Gamma^{\mu}_{bs\gamma}(k)\delta_{\beta\alpha} = \int \frac{\mathrm{d}^4 q}{(2\pi)^4} & i\sqrt{2}g_s[-(Z_D^*)_{2i}P_R + (Z_D^*)_{5i}P_L]T^A_{\beta\lambda}\frac{i(\not{q} + M_{\widetilde{g}})}{q^2 - M_{\widetilde{g}}^2} \\ & i\sqrt{2}g_s[-(Z_D)_{3i}P_L + (Z_D)_{6i}P_R]T^A_{\lambda\alpha} \\ & \frac{i}{(p-q)^2 - m^2_{\widetilde{D}_i}}\frac{i}{(p'-q)^2 - m^2_{\widetilde{D}_i}} i\frac{e}{3}(p + p' - 2q)^{\mu}\,. \end{aligned} \tag{D.2}$$

A few comments are in order. Since the amplitude features a logarithmically divergent part a regularization procedure is understood. We choose the dimensional regularization procedure: we perform the integrals in a dimension $d = 4 - 2\varepsilon$ spacetime and then we take the limit for $\varepsilon \to 0$. We did not write it explicitly. We also omitted the little imaginary part in the propagators but it is always understood. The sum over the index i is implicitly assumed.

Using equation (A.8) one finds

$$T^A_{\beta\lambda}T^A_{\lambda\alpha} = \frac{1}{2}\left(\delta_{\alpha\beta}\delta_{\lambda\sigma} - \frac{1}{3}\delta_{\beta\sigma}\delta_{\lambda\alpha}\right)\delta_{\lambda\sigma} = \frac{4}{3}\delta_{\alpha\beta}\,. \tag{D.3}$$

The amplitude can be split in two parts involving different Dirac structures and integrals as:

$$
\begin{aligned}
i\Gamma^{\mu}_{bs\gamma}(k) = &+ \frac{8}{9} e g_s^2 M_{\tilde{g}} \int \frac{\mathrm{d}^4 q}{(2\pi)^4} \frac{(p+p'-2q)^{\mu}}{[(p-q)^2 - m^2_{\widetilde{D}_i}][(p'-q)^2 - m^2_{\widetilde{D}_i}][q^2 - M^2_{\tilde{g}}]} \times \\
&\qquad \times [(Z_D^*)_{5i}(Z_D)_{3i} P_L + (Z_D^*)_{2i}(Z_D)_{6i} P_R] \\
&- \frac{8}{9} e g_s^2 \int \frac{\mathrm{d}^4 q}{(2\pi)^4} \frac{q_{\nu}(p+p'-2q)^{\mu}}{[(p-q)^2 - m^2_{\widetilde{D}_i}][(p'-q)^2 - m^2_{\widetilde{D}_i}][q^2 - M^2_{\tilde{g}}]} \times \\
&\qquad \times \gamma^{\nu}[(Z_D^*)_{2i}(Z_D)_{3i} P_L + (Z_D^*)_{5i}(Z_D)_{6i} P_R] \,.
\end{aligned} \tag{D.4}
$$

In order to calculate the integrals we write the denominator making use of the *Feynman paramters*. It then reads

$$
\frac{1}{[(p-q)^2 - m^2_{\widetilde{D}_i}][(p'-q)^2 - m^2_{\widetilde{D}_i}][q^2 - M^2_{\tilde{g}}]} = 2\int_0^1 \mathrm{d}x \int_0^x \mathrm{d}y \, \frac{1}{[(q - p'x - (p-p')y)^2 - \Omega)]^3} \,,
$$

with Ω being a function of x, y, defined as

$$
\begin{aligned}
\Omega &= M^2_{\tilde{g}} + (m^2_{\widetilde{D}_i} - M^2_{\tilde{g}})x - m_s^2 x - (m_b^2 - m_s^2)y + m_s^2(x+y)^2 + m_b^2 y^2 + 2(p\cdot p')(xy - y^2)\,, \\
\Omega &= M^2_{\tilde{g}}[1 + (\xi_i - 1)x + \ldots]\,.
\end{aligned} \tag{D.5}
$$

In (D.5) we used $\xi_i = m^2_{\widetilde{D}_i}/M^2_{\tilde{g}}$ and the dots stands for terms of order $m^2_{b,s}/M^2_{\tilde{g}}$ which are neglected. We can now shift q in the integrals as $q \to q + p'x + (p-p')y$ and perform the integrals in the momentum space dropping all terms linear with q. The integration in the y and x variables is then trivial.

$$
\begin{aligned}
\Gamma^{\mu}_{bs\gamma}(k) = &+ \frac{8}{9}\frac{e g_s^2}{M^2_{\tilde{g}}}(p+p')^{\mu} M_{\tilde{g}}[f_2(\xi_i) - f_1(\xi_i)](z_{53}P_L + z_{26}P_R) \\
&+ \frac{8}{9}\frac{e g_s^2}{M^2_{\tilde{g}}}\frac{1}{2}(p+p')^{\mu}(p+p')_{\nu} f_2(\xi_i)\gamma^{\nu}(z_{23}P_L + z_{56}P_R) \\
&- \frac{8}{9}\frac{e g_s^2}{M^2_{\tilde{g}}}\frac{1}{3}\left[p^{\mu}(2p+p')_{\nu} + p'^{\mu}(2p'+p)_{\nu}\right] f_3(\xi_i)\gamma^{\nu}(z_{23}P_L + z_{56}P_R) \\
&+ \frac{8}{9} e g_s^2 \left[\frac{1}{2}\left(\frac{1}{\varepsilon} - \gamma_E + \log 4\pi - \log M^2_{\tilde{g}}\right) - f_0(\xi_i)\right]\gamma^{\mu}(z_{23}P_L + z_{56}P_R)\,.
\end{aligned} \tag{D.6}
$$

The last line of eq. (D.6) contains the divergent part of the diagram which behaves like $1/\varepsilon$ for $\varepsilon \to 0$. Note the fact that the last line involves the logarithm of $M^2_{\tilde{g}}$, a dimensionful quantity. This is a well-known peculiarity of DREG and arises since the scale of the logarithm is hidden in the $1/\varepsilon$ term [66]. The definitions for the functions $f_a(\xi_i)$, $a = 0,..,3$ can be found in the Appendix. We recall that $\gamma_E \approx 0.5772$ is the Euler–Mascheroni constant. In Eq. (D.6) we used the following notation

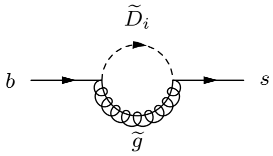
$$
z_{IJ} = \frac{1}{16\pi^2}\sum_i (Z_D^*)_{Ii}(Z_D)_{Ji}\,. \tag{D.7}
$$

We can now simplify the expression in eq. D.6 using the equations of motion for external spinors:

$$
\not{p}\, u_b(p) = m_b u_b(p)\,, \qquad \overline{u}_s(p')\not{p}' = \overline{u}_s(p) m_s\,. \tag{D.8}
$$

$$
\begin{aligned}
\Gamma^{\mu}_{bs\gamma}(k) = &+ \frac{8}{9}\frac{eg_s^2}{M_{\widetilde{g}}^2}(p+p')^{\mu} M_{\widetilde{g}}[f_2(\xi_i) - f_1(\xi_i)](z_{53}P_L + z_{26}P_R) \\
&- \frac{8}{9}\frac{eg_s^2}{M_{\widetilde{g}}^2}(p+p')^{\mu}\frac{f_3(\xi_i) - f_2(\xi_i)}{2}[(m_s z_{23} + m_b z_{56})P_L + (m_s z_{56} + m_b z_{23})P_R] \\
&- \frac{8}{9}\frac{eg_s^2}{M_{\widetilde{g}}^2}k^{\mu}\frac{f_3(\xi_i)}{3}[(-m_s z_{23} + m_b z_{56})P_L + (-m_s z_{56} + m_b z_{23})P_R] \\
&+ \frac{8}{9}eg_s^2\left[\frac{1}{2}\left(\frac{1}{\varepsilon} - \gamma_E + \log 4\pi - \log M_{\widetilde{g}}^2\right) - f_0(\xi_i)\right]\gamma^{\mu}(z_{23}P_L + z_{56}P_R)\,.
\end{aligned}
\tag{D.9}
$$

Diagrams in Figs. 4.2b and 4.2c involve external legs corrections and are reducible. One can therefore separately evaluate the diagram corresponding to $b \to s$ self energy and then plug the result in the complete diagrams.



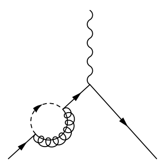
$$
= \quad \overline{u}_s^{\beta}(p)\, i\Sigma(\not{p})\, \delta_{\beta\alpha} u_b^{\alpha}(p)\,, \tag{D.10}
$$

$$
\begin{aligned}
i\Sigma(\not{p})\,\delta_{\beta\alpha} = \int \frac{\mathrm{d}^4 q}{(2\pi)^4} & i\sqrt{2}g_s(-(Z_D^*)_{2i}P_R + (Z_D^*)_{5i}P_L)T^A_{\beta\lambda}\frac{i(\not{q} + M_{\widetilde{g}})}{q^2 - M_{\widetilde{g}}^2} \\
& i\sqrt{2}g_s(-(Z_D)_{3i}P_L + (Z_D)_{6i}P_R)T^A_{\lambda\alpha}\frac{i}{(q-p)^2 - m^2_{\widetilde{D}_i}}\,.
\end{aligned}
\tag{D.11}
$$

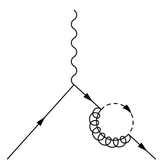
We use the relation $T^A_{\beta\lambda}T^A_{\lambda\alpha} = 4\delta_{\beta\alpha}/3$ and we perform the integral in dimensional regularization.

$$
\begin{aligned}
i\Sigma(\not{p}) = &- \frac{8}{3}g_s^2 M_{\widetilde{g}}\left[\left(\frac{1}{\varepsilon} - \gamma_E + \log 4\pi - \log M_{\widetilde{g}}^2\right) + \frac{\xi_i}{1-\xi_i}\log\xi_i + 1\right](z_{53}P_L + z_{26}P_R) \\
&+ \frac{8}{3}g_s^2\not{p}\left[\frac{1}{2}\left(\frac{1}{\varepsilon} - \gamma_E + \log 4\pi - \log M_{\widetilde{g}}^2\right) - f_0(\xi_i)\right](z_{23}P_L + z_{56}P_R)\,.
\end{aligned}
\tag{D.12}
$$

Using this result we can now easily write the amplitudes for the diagrams in Figs. 4.2b and 4.2c as

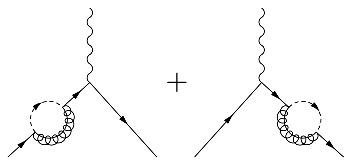


$$
= \quad \overline{u}_s(p')\left(i\frac{e}{3}\gamma^{\mu}\right)\frac{i(\not{p} + m_s)}{p^2 - m_s^2} i\Sigma(\not{p}) u_b(p)\,\epsilon^*_{\mu}\,, \tag{D.13}
$$



$$
= \quad \overline{u}_s(p') i\Sigma(\not{p}')\frac{i(\not{p}' + m_b)}{p'^2 - m_b^2}\left(i\frac{e}{3}\gamma^{\mu}\right) u_b(p)\,\epsilon^*_{\mu}\,. \tag{D.14}
$$

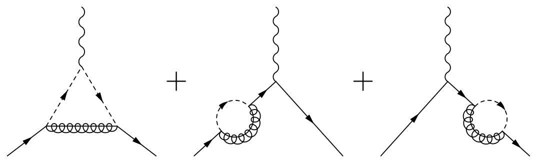
Applying the equations of motion Eq. (D.8) for the external spinors and summing the two diagrams yields



$$=$$

$$-i\frac{8}{9}eg_s^2\left[\frac{1}{2}\left(\frac{1}{\varepsilon}-\gamma_E+\log 4\pi-\log M_{\tilde{g}}^2\right)-f_0(\xi_i)\right]\overline{u}_s(p')\gamma^\mu(z_{23}P_L+z_{56}P_R)\,u_b(p)\,\epsilon^*_\mu\,. \tag{D.15}$$

The expression in eq. (D.15) is exactly what we need to cancel the divergence coming from the last line of eq. (D.9). This is the expected result and it is a consequence of the Ward-Takahashi identities. The total amplitude is then



$$=$$

$$\begin{aligned}
&-i\frac{16}{9}\frac{eg_s^2}{M_{\tilde{g}}^2}\,2\epsilon^*_\mu(p+p')^\mu\,\overline{u}_s(p')\left[f_{7\gamma}^{[1]}(\xi_i)(m_b z_{56}+m_s z_{23})-M_{\tilde{g}}f_{7\gamma}^{[2]}(\xi_i)z_{53}\right]P_L\,u_b(p) \\
&-i\frac{16}{9}\frac{eg_s^2}{M_{\tilde{g}}^2}\,2\epsilon^*_\mu(p+p')^\mu\,\overline{u}_s(p')\left[f_{7\gamma}^{[1]}(\xi_i)(m_b z_{23}+m_s z_{56})-M_{\tilde{g}}f_{7\gamma}^{[2]}(\xi_i)z_{26}\right]P_R\,u_b(p) \\
&\quad -i\frac{8}{9}\frac{eg_s^2}{M_{\tilde{g}}^2}\epsilon^*_\mu k^\mu\,\frac{f_3(\xi_i)}{3}\,\overline{u}_s(p')[(m_b z_{56}-m_s z_{23})P_L+(m_b z_{23}-m_s z_{56})P_R]u_b(p)\,,
\end{aligned} \tag{D.16}$$

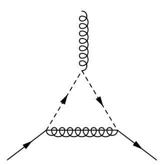
where we defined the functions

$$\begin{aligned}
f_{7\gamma}^{[1]}(\xi_i) &= \frac{1}{8}[f_3(\xi_i)-f_2(\xi_i)] = -\frac{\xi_i\log\xi_i}{8(1-\xi_i)^4}+\frac{\xi_i^2-5\xi_i-2}{48(1-\xi_i)^3}\,, \\
f_{7\gamma}^{[2]}(\xi_i) &= \frac{1}{4}[f_2(\xi_i)-f_1(\xi_i)] = -\frac{\xi_i\log\xi_i}{4(1-\xi_i)^3}-\frac{\xi_i+1}{8(1-\xi_i)^2}\,.
\end{aligned}$$

Note that the last line in eq. (D.16) vanishes for real photons since $\epsilon^*_\mu k^\mu = 0$ on shell. The last step consists in using the Gordon identity (A.58) to rewrite the total amplitude.

D.2 Full theory calculation for $b \to sg$ process

We calculate the one-loop gluino contributions to the Wilson coefficients $C_{8g}^{(\prime)}$ for the process $b \to sg$. The four diagrams which are involved have been depicted in Fig. 4.3. The amplitude for the first diagram (Fig 4.3a) reads



$$= \quad \overline{u}_s^\beta(p')\left[i\Gamma^\mu_{bsg}(k)T^A_{\beta\alpha}\right]u_b^\alpha(p)\,\epsilon^{*A}_\mu(k)\,, \tag{D.17}$$

$$
\begin{aligned}
i\Gamma^{\mu}_{bsg}(k)T^{A}_{\beta\alpha} = \int \frac{\mathrm{d}^4 q}{(2\pi)^4} & i\sqrt{2}g_s[-(Z_D^*)_{2i}P_R + (Z_D^*)_{5i}P_L]T^{B}_{\beta\sigma}\frac{i(\not{q}+M_{\widetilde{g}})}{q^2 - M^2_{\widetilde{g}}} \\
& i\sqrt{2}g_s[-(Z_D)_{3i}P_L + (Z_D)_{6i}P_R]T^{B}_{\lambda\alpha} \\
& \frac{i}{(p-q)^2 - m^2_{\widetilde{D}_i}}\frac{i}{(p'-q)^2 - m^2_{\widetilde{D}_i}}\left[-ig_s T^{A}_{\sigma\lambda}(p+p'-2q)_{\mu}\right].
\end{aligned} \tag{D.18}
$$

Using equation (A.8) one finds

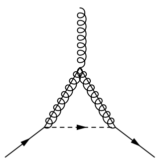
$$
(T^{B}_{\beta\sigma}T^{B}_{\lambda\alpha})T^{A}_{\sigma\lambda} = \frac{1}{2}\left(\delta_{\alpha\beta}\delta_{\lambda\sigma} - \frac{1}{3}\delta_{\beta\sigma}\delta_{\lambda\alpha}\right)T^{A}_{\sigma\lambda} = -\frac{1}{6}T^{A}_{\beta\alpha}. \tag{D.19}
$$

From now on the calculation proceed in the same way as that in eq. (D.2). The terms in the amplitude we are interested in are those proportional to $(p+p')_{\mu}$ which give the term $\sigma^{\mu\nu}k_{\nu}$ after using the Gordon identity (A.58).

$$
\begin{aligned}
\Gamma^{\mu}_{bsg}(k) = & + \frac{g_s}{3M^2_{\widetilde{g}}}(p+p')^{\mu}\left[(f_2(\xi_i) - f_1(\xi_i))M_{\widetilde{g}}z_{53} - \frac{f_3(\xi_i) - f_2(\xi_i)}{2}(m_b z_{56} + m_s z_{23})\right]P_L \\
& + \frac{g_s}{3M^2_{\widetilde{g}}}(p+p')^{\mu}\left[(f_2(\xi_i) - f_1(\xi_i))M_{\widetilde{g}}z_{26} - \frac{f_3(\xi_i) - f_2(\xi_i)}{2}(m_b z_{23} + m_s z_{56})\right]P_R \\
& + \dots,
\end{aligned} \tag{D.20}
$$

where the dots refer to terms proportional to k^{μ} and γ^{μ} as in eq. (D.9) which include a divergent part (afterwards cancelled by adding the other diagrams).

We then proceed with next diagram (Fig 4.3b), which corresponds to



$$
= \quad \overline{u}^{\beta}_{s}(p')\left[i\Gamma'^{\mu}_{bsg}(k)T^{A}_{\beta\alpha}\right]u^{\alpha}_{b}(p)\,\epsilon^{*A}_{\mu}(k), \tag{D.21}
$$

$$
\begin{aligned}
i\Gamma'^{\mu}_{bsg}(k)T^{A}_{\beta\alpha} = \int \frac{\mathrm{d}^4 q}{(2\pi)^4} & i\sqrt{2}g_s[-(Z_D^*)_{2i}P_R + (Z_D^*)_{5i}P_L]T^{C}_{\beta\lambda} \\
& \frac{i(\not{p}' - \not{q} + M_{\widetilde{g}})}{(p'-q)^2 - M^2_{\widetilde{g}}}\left(g_s f^{BCA}\gamma^{\mu}\right)\frac{i(\not{p} - \not{q} + M_{\widetilde{g}})}{(p-q)^2 - M^2_{\widetilde{g}}} \\
& i\sqrt{2}g_s[-(Z_D)_{3i}P_L + (Z_D)_{6i}P_R]T^{B}_{\lambda\alpha}\frac{i}{q^2 - m^2_{\widetilde{D}_i}},
\end{aligned} \tag{D.22}
$$

where f^{BCA} are the structure constants of the $SU(3)$ group which satisfy the relations $\left[T^B, T^C\right] = if^{BCA}T^A$. We now make use of the identity in Eq. (A.6) which, in the case of the $SU(3)$ group, reads

$$
2f^{BCA}T^{C}T^{B} = -i3T^{A}. \tag{D.23}
$$

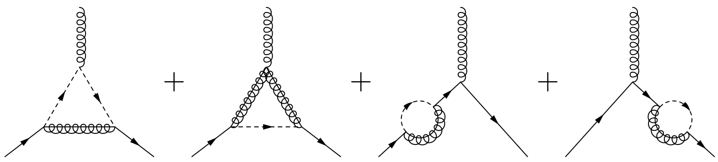
The calculation than proceed in the usual way. One needs to recast the denominator with the Feynman parameters as done in eq. (D.1):

$$
\frac{1}{[(p-q)^2 - M^2_{\widetilde{g}}][(p'-q)^2 - M^2_{\widetilde{g}}][q^2 - m^2_{\widetilde{D}_i}]} = 2\int_0^1 \mathrm{d}x \int_0^x \mathrm{d}y\,\frac{1}{[(q - p'x - (p-p')y)^2 - \Omega')]^3},
$$

where we have $\Omega' = M_{\tilde{g}}^2[\xi_i + (1-\xi_i)x + \ldots]$ neglecting terms of order $m_{b,s}^2/M_{\tilde{g}}^2$. One then has to perform the shift in the momentum, do the integrations in the q, y, x variables and use the equations of motion for the external fermions. In the following we write only the terms proportional to $(p+p')^\mu$

$$\begin{aligned}\Gamma'^{\mu}_{bsg}(k) = &-\frac{3g_s^3}{M_{\tilde{g}}^2}(p+p')^\mu\left[\hat{f}_2(\xi_i)M_{\tilde{g}}z_{53} + \frac{\hat{f}_3(\xi_i)-\hat{f}_2(\xi_i)}{2}(m_b z_{56} + m_s z_{23})\right]P_L \\ &-\frac{3g_s^3}{M_{\tilde{g}}^2}(p+p')^\mu\left[\hat{f}_2(\xi_i)M_{\tilde{g}}z_{26} + \frac{\hat{f}_3(\xi_i)-\hat{f}_2(\xi_i)}{2}(m_b z_{23} + m_s z_{56})\right]P_R \\ &+\ldots\,.\end{aligned} \tag{D.24}$$

The functions $\hat{f}_a(\xi)$, $a = 0,\ldots,3$ satisfy $\hat{f}_a(\xi) = \xi f_a(1/\xi)$ and are listed in the Appendix E. Since diagrams 4.3c and 4.3d only produce terms proportional to $\gamma^\mu\epsilon_\mu^{*A}$ we already have all the pieces contributing to the chromomagnetic operator which can be obtained summing the terms proportional to $(p+p')^\mu$ in the amplitudes (D.17) and (D.21)



$$=$$

$$\begin{aligned}&-i\frac{4}{3}\frac{g_s^3}{M_{\tilde{g}}^2}\,2\epsilon_\mu^{*A}(p+p')^\mu\,\overline{u}_s(p')\Big[f_{8g}^{[1]}(\xi_i)(m_s z_{23} + m_b z_{56}) - M_{\tilde{g}} f_{8g}^{[2]}(\xi_i) z_{53}\Big]P_L\,u_b(p) \\ &-i\frac{4}{3}\frac{g_s^3}{M_{\tilde{g}}^2}\,2\epsilon_\mu^{*}(p+p')^\mu\,\overline{u}_s(p')\Big[f_{8g}^{[1]}(\xi_i)(m_s z_{56} + m_b z_{23}) - M_{\tilde{g}} f_{8g}^{[2]}(\xi_i) z_{26}\Big]P_R\,u_b(p) \\ &+\ldots\,,\end{aligned} \tag{D.25}$$

where we wrote explicitly only the terms contributing to the chromomagnetic operator and we used the following definitions for the functions $f_{8g}^{[1]}(\xi_i)$ and $f_{8g}^{[2]}(\xi_i)$

$$\begin{aligned}f_{8g}^{[1]}(\xi_i) &= \frac{1}{16}\Big[f_3(\xi_i) - f_2(\xi_i) + 9\Big(\hat{f}_3(\xi_i) - \hat{f}_2(\xi_i)\Big)\Big] = \frac{\xi_i(9\xi_i - 1)\log\xi_i}{16(1-\xi_i)^4} + \frac{19\xi_i^2 + 40\xi_i - 11}{96(1-\xi_i)^3}\,, \\ f_{8g}^{[2]}(\xi_i) &= \frac{1}{8}\Big[f_2(\xi_i) - f_1(\xi_i) - 9\hat{f}_2(xi_i)\Big] = \frac{\xi_i(9\xi - 1)\log\xi_i}{8(1-\xi_i)^3} + \frac{13\xi_i - 5}{8(1-\xi_i)^2}\,.\end{aligned}$$

We can now use the Gordon identity (A.58) in order to compare the amplitude with the effective matrix elements.

D.3 Full theory calculation for $\mu \to e\gamma$ process

The one loop MSSM contributions to the decay $\mu \to e\gamma$ can be divided in neutralino and chargino contributions. The diagrams involved are depicted in Fig. 4.4. Only the first and the fourth diagrams contribute to the coefficients of interest.

Neutralino contributions

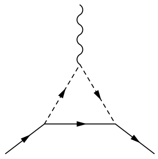
Here we analyse the neutralino contributions to the amplitude for the decay $\mu \to e\gamma$. Accordingly to the notation introduced in Sec. 2.9 – which essentially is that of [45] except for the sign of the Yukawa matrices Y_e, Y_d – the Lagrangian for the interaction lepton-slepton-neutralino is given as

$$\mathcal{L}_{e\chi^0\widetilde{L}} = \bar{e}^I(N_L^{Iji}P_L + N_R^{Iji}P_R)\chi_j^0\widetilde{L}_i^- + \text{h.c.}\,, \tag{D.26}$$

with the coefficients given in terms of the diagonalization matrices

$$\begin{aligned} N_L^{Iji} &= \frac{-e\sqrt{2}}{c_W} Z_L^{(I+3)i*} Z_N^{1j} - y_e^I Z_L^{Ii*} Z_N^{3j}\,, \\ N_R^{Iji} &= \frac{e}{\sqrt{2}s_W c_W} Z_L^{Ii*}(Z_N^{1j*}s_W + Z_N^{2j*}c_W) - y_e^I Z_L^{(I+3)i*} Z_N^{3j*}\,. \end{aligned} \tag{D.27}$$

With these definitions we can compute the diagram in Fig. 4.4d which consists in a loop formed by two charged slepton propagators and one neutralino propagator. The amplitude then reads



$$= \quad \overline{u}_e(p')\left[i\Gamma^\lambda_{\mu e\gamma}(k)\right]u_\mu(p)\,\epsilon^*_\lambda(k)\,, \tag{D.28}$$

$$\begin{aligned} i\Gamma^\lambda_{\mu e\gamma}(k) = \int \frac{\mathrm{d}^4 q}{(2\pi)^4} i(N_L^{1ji}P_L + N_R^{1ji}P_R)\frac{i(\not{q} + m_{\chi_j^0})}{q^2 - m^2_{\chi_j^0}} i(N_L^{2ji*}P_R + N_R^{2ji*}P_L) \times \\ \times \frac{i}{(p-q)^2 - m^2_{\widetilde{L}_i}} \frac{i}{(p'-q)^2 - m^2_{\widetilde{L}_i}} ie(p+p'-2q)^\lambda\,. \end{aligned} \tag{D.29}$$

This integral is similar to the one we calculate for the $b \to s\gamma$ process, namely $\Gamma_{bs\gamma}$, but involves a more complicated structure of diagonalizing matrices. The steps one has to perform in order to evaluate $\Gamma^\lambda_{\mu e\gamma}$ are the same as in section D.1 therefore we avoid to write them again. This time we neglect terms of the second order in $m_\mu/m_{\widetilde{L}_i}$ and we define the ratio $\zeta_{ji} = m^2_{\chi_j^0}/m^2_{\widetilde{L}_i}$.

The vertex function $\Gamma^\lambda_{\mu e\gamma}$ turns out to be

$$\begin{aligned} \Gamma^\lambda_{\mu e\gamma}(k) = -\frac{1}{4\pi^2}\frac{e}{m^2_{\widetilde{L}_i}}(p+p')^\lambda \Big[\Big(m_{\chi_j^0} N_L^{1ji}N_R^{2ji*} f_{7\gamma}^{[2]}(\zeta_{ji}) + m_\mu N_L^{1ji}N_L^{2ji*} f_{7\gamma}^{[1]}(\zeta_{ji})\Big)P_L \\ + \Big(m_{\chi_j^0} N_R^{1ji}N_L^{2ji*} f_{7\gamma}^{[2]}(\zeta_{ji}) + m_\mu N_R^{1ji}N_R^{2ji*} f_{7\gamma}^{[1]}(\zeta_{ji})\Big)P_R\Big] + \dots\,. \end{aligned} \tag{D.30}$$

The loop functions are listed in the Appendix E. Here we wrote only terms contributing to the magnetic operator and we neglected terms proportional to $m_e/m_{\chi_j^0}$ since m_e is so small that can be set to zero to our purposes.

After using the Gordon Identities in Eq. (A.58) we can compare the above amplitude with the one in Eq. (4.54). We obtain the following neutralino contributions to the coefficients A_L, A_R

$$A_L^{21(n)} = -\frac{1}{4\pi^2}\frac{1}{m^2_{\widetilde{L}_i}}\left[\frac{m_{\chi_j^0}}{m_\mu}N_L^{1ji}N_R^{2ji*} f_{7\gamma}^{[2]}(\zeta_{ji}) + N_L^{1ji}N_L^{2ji*} f_{7\gamma}^{[1]}(\zeta_{ji})\right], \tag{D.31}$$

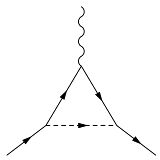
$$A_R^{21(n)} = -\frac{1}{4\pi^2}\frac{1}{m^2_{\widetilde{L}_i}}\left[\frac{m_{\chi_j^0}}{m_\mu}N_R^{1ji}N_L^{2ji*} f_{7\gamma}^{[2]}(\zeta_{ji}) + N_R^{1ji}N_R^{2ji*} f_{7\gamma}^{[1]}(\zeta_{ji})\right]. \tag{D.32}$$

Chargino contributions

We now focus on the chargino contributions to the amplitude for the decay $\mu \to e\gamma$. Accordingly to our notation, we parametrize the Lagrangian for the interaction lepton-sneutrino-chargino as

$$\begin{aligned} \mathcal{L}_{e\chi\tilde{\nu}} &= \bar{e}^I (C_L^{IJi} P_L + C_R^{IJi} P_R) \chi_i^c \tilde{\nu}^J + \text{h.c.} \,, \\ C_L^{IJi} &= y_e^I Z_-^{2i} Z_\nu^{IJ} \,, \qquad C_R^{IJi} = -\frac{e}{s_W} Z_+^{1i*} Z_\nu^{IJ} \,. \end{aligned} \tag{D.33}$$

The diagram in Fig. 4.4d consists in a loop formed by two chargino propagators and one neutral slepton propagator and corresponds to



$$= \quad \overline{u}_e(p') \Big[i\Gamma'^{\lambda}_{\mu e\gamma}(k) \Big] u_\mu(p)\, \epsilon^*_\lambda(k) \tag{D.34}$$

$$\begin{aligned} i\Gamma'^{\lambda}_{\mu e\gamma}(k) = \int \frac{\mathrm{d}^4 q}{(2\pi)^4} \big[i(C_L^{1Ji} P_L + C_R^{1Ji} P_R) \big] \frac{i(\not{p}' - \not{q} + m_{\chi_i})}{(p'-q)^2 - m_{\chi_i}^2} (ie\gamma^\mu) \\ \frac{i(\not{p} - \not{q} + m_{\chi_i})}{(p-q)^2 - m_{\chi_i}^2} \big[i(C_L^{2Ji*} P_R + C_R^{2Ji*} P_L) \big] \frac{i}{q^2 - m_{\tilde{\nu}_J}^2} \,. \end{aligned} \tag{D.35}$$

The summation over the indices i, J is understood. The integral in the above amplitude has the same structure of the one we computed for Γ'_{bsg}. The result is

$$\begin{aligned} \Gamma'^{\lambda}_{\mu e\gamma}(k) = -\frac{1}{16\pi^2} \frac{e}{m_{\tilde{\nu}_J}^2} (p+p')^\lambda \Big[\Big(m_{\chi_i} C_L^{1Ji} C_R^{2Ji*} f_2(\eta_{iJ}) - 4 m_\mu C_L^{1Ji} C_L^{2Ji*} f_{7\gamma}^{[1]}(\eta_{iJ}) \Big) P_L \\ + \Big(m_{\chi_i} C_R^{1Ji} C_L^{2Ji*} f_2(\eta_{iJ}) - 4 m_\mu C_R^{1Ji} C_R^{2Ji*} f_{7\gamma}^{[1]}(\eta_{iJ}) \Big) P_R \Big] + \dots \,, \end{aligned} \tag{D.36}$$

where again we wrote only terms contributing to the magnetic operator. We also defined the ratio $\eta_{iJ} = m_{\chi_i}^2 / m_{\tilde{\nu}_J}^2$. One can then use the Gordon Identities in Eq. (A.58) and compare the obtained amplitude with the one in Eq. (4.54). The results for the chargino contributions are then

$$A_L^{21(c)} = -\frac{1}{16\pi^2} \frac{1}{m_{\tilde{\nu}_J}^2} \left[\frac{m_{\chi_i}}{m_\mu} C_L^{1Ji} C_R^{2Ji*} f_2(\eta_{iJ}) - 4 C_L^{1Ji} C_L^{2Ji*} f_{7\gamma}^{[1]}(\eta_{iJ}) \right] , \tag{D.37}$$

$$A_R^{21(c)} = -\frac{1}{16\pi^2} \frac{1}{m_{\tilde{\nu}_J}^2} \left[\frac{m_{\chi_i}}{m_\mu} C_R^{1Ji} C_L^{2Ji*} f_2(\eta_{iJ}) - 4 C_R^{1Ji} C_R^{2Ji*} f_{7\gamma}^{[1]}(\eta_{iJ}) \right] . \tag{D.38}$$

Appendix E

Loop functions

E.1 Loop functions for $K^0 - \overline{K}^0$ mixing

$$\int \frac{\mathrm{d}^4 q}{(2\pi)^4} \frac{1}{(q^2 - M_{\tilde{g}}^2)^2 (q^2 - m_{\tilde{D}_i}^2)(q^2 - m_{\tilde{D}_j}^2)} \equiv \frac{i}{16\pi^2} D_0(m_{\tilde{D}_i}^2, m_{\tilde{D}_j}^2, M_{\tilde{g}}^2, M_{\tilde{g}}^2)\,, \tag{E.1}$$

$$\int \frac{\mathrm{d}^4 q}{(2\pi)^4} \frac{q_\mu q_\nu}{(q^2 - M_{\tilde{g}}^2)^2 (q^2 - m_{\tilde{D}_i}^2)(q^2 - m_{\tilde{D}_j}^2)} \equiv \frac{i\eta_{\mu\nu}}{16\pi^2} D_2(m_{\tilde{D}_i}^2, m_{\tilde{D}_j}^2, M_{\tilde{g}}^2, M_{\tilde{g}}^2)\,, \tag{E.2}$$

$$D_0(m_1^2, m_2^2, m_3^2, m_4^2) = \frac{m_1^2 \log m_1^2}{(m_4^2 - m_1^2)(m_3^2 - m_1^2)(m_2^2 - m_1^2)} + \{1\leftrightarrow 2\} + \{1\leftrightarrow 3\} + \{1\leftrightarrow 4\}\,, \tag{E.3}$$

$$D_2(m_1^2, m_2^2, m_3^2, m_4^2) = \frac{m_1^4 \log m_1^2}{4(m_4^2 - m_1^2)(m_3^2 - m_1^2)(m_2^2 - m_1^2)} + \{1\leftrightarrow 2\} + \{1\leftrightarrow 3\} + \{1\leftrightarrow 4\}\,. \tag{E.4}$$

Two generation framework

The loop function in the two generation framework are $(a, b = q, d)$

$$B_{ab}(M^2, M^2) = D_0(m_{\tilde{a}_1}^2, m_{\tilde{b}_1}^2, M^2, M^2) - D_0(m_{\tilde{a}_1}^2, m_{\tilde{b}_2}^2, M^2, M^2) + \{1 \leftrightarrow 2\}\,, \tag{E.5}$$

$$C_{ab}(M^2, M^2) = D_2(m_{\tilde{a}_1}^2, m_{\tilde{b}_1}^2, M^2, M^2) - D_2(m_{\tilde{a}_1}^2, m_{\tilde{b}_2}^2, M^2, M^2) + \{1 \leftrightarrow 2\}\,. \tag{E.6}$$

Mass insertion approximation

$$f_6(x) = \frac{6(1+3x)\log x + x^3 - 9x^2 - 9x + 17}{6(x-1)^5}\,, \tag{E.7}$$

$$\tilde{f}_6(x) = \frac{6x(1+x)\log x - x^3 - 9x^2 + 9x + 1}{3(x-1)^5}\,. \tag{E.8}$$

E.2 Loop functions for (chromo)magnetic coefficients

$$
\begin{aligned}
f_0(\xi) &= \int_0^1 \mathrm{d}x \log\left(1+(\xi-1)x\right)x = \frac{1}{4(1-\xi)^2}(-3+4\xi-\xi^2-4\xi\log\xi+2\xi^2\log\xi)\,,\\
f_1(\xi) &= \int_0^1 \mathrm{d}x\,\frac{x}{1+(\xi-1)x} = \frac{1}{(1-\xi)^2}(-1+\xi-\log\xi)\,,\\
f_2(\xi) &= \int_0^1 \mathrm{d}x\,\frac{x^2}{1+(\xi-1)x} = \frac{1}{2(1-\xi)^3}(-3+4\xi-\xi^2-2\log\xi)\,,\\
f_3(\xi) &= \int_0^1 \mathrm{d}x\,\frac{x^3}{1+(\xi-1)x} = \frac{1}{6(1-\xi)^4}(-11+18\xi-9\xi^2+2\xi^3-6\log\xi)\,.
\end{aligned}
$$

The functions $\hat{f}_a(\xi)$, $a=0,\ldots,3$ are obtained as $\hat{f}_a(\xi)=\xi f_a(1/\xi)$. Their explicit expressions are

$$
\begin{aligned}
\hat{f}_0(\xi) &= \frac{1}{4(1-\xi)^2}(-1+4\xi-3\xi^2+2\xi^2\log\xi)\,,\\
\hat{f}_1(\xi) &= \frac{1}{(1-\xi)^2}(1-\xi+\xi\log\xi)\,,\\
\hat{f}_2(\xi) &= \frac{1}{2(1-\xi)^3}(1-4\xi+3\xi^2-2\xi^2\log\xi)\,,\\
\hat{f}_3(\xi) &= \frac{1}{6(1-\xi)^4}(2-9\xi+18\xi^2-11\xi^3+6\xi^3\log\xi)\,.
\end{aligned}
$$

$$
f_{7\gamma}^{[1]}(\xi_i) = \frac{1}{8}[f_3(\xi_i)-f_2(\xi_i)] = -\frac{\xi_i\log\xi_i}{8(1-\xi_i)^4}+\frac{\xi_i^2-5\xi_i-2}{48(1-\xi_i)^3}\,, \tag{E.9}
$$

$$
f_{7\gamma}^{[2]}(\xi_i) = \frac{1}{4}[f_2(\xi_i)-f_1(\xi_i)] = -\frac{\xi_i\log\xi_i}{4(1-\xi_i)^3}-\frac{\xi_i+1}{8(1-\xi_i)^2}\,, \tag{E.10}
$$

$$
\begin{aligned}
f_{8g}^{[1]}(\xi_i) &= \frac{1}{16}\Big[f_3(\xi_i)-f_2(\xi_i)+9\Big(\hat{f}_3(\xi_i)-\hat{f}_2(\xi_i)\Big)\Big]\\
&= \frac{\xi_i(9\xi_i-1)\log\xi_i}{16(1-\xi_i)^4}+\frac{19\xi_i^2+40\xi_i-11}{96(1-\xi_i)^3}\,,
\end{aligned} \tag{E.11}
$$

$$
\begin{aligned}
f_{8g}^{[2]}(\xi_i) &= \frac{1}{8}\Big[f_2(\xi_i)-f_1(\xi_i)-9\hat{f}_2(xi_i)\Big]\\
&= \frac{\xi_i(9\xi-1)\log\xi_i}{8(1-\xi_i)^3}+\frac{13\xi_i-5}{8(1-\xi_i)^2}\,.
\end{aligned} \tag{E.12}
$$

Mass insertion approximation

$$f_{1n}(x) = \frac{-17x^3 + 9x^2 + 9x - 1 + 6x^2(x+3)\ln x}{24(1-x)^5}\,, \tag{E.13}$$

$$f_{2n}(x) = \frac{-5x^2 + 4x + 1 + 2x(x+2)\ln x}{4(1-x)^4}\,, \tag{E.14}$$

$$f_{3n}(x) = \frac{1 + 2x\ln x - x^2}{2(1-x)^3}\,, \tag{E.15}$$

$$f_{1c}(x) = \frac{-x^3 - 9x^2 + 9x + 1 + 6x(x+1)\ln x}{6(1-x)^5}\,, \tag{E.16}$$

$$f_{2c}(x) = \frac{-x^2 - 4x + 5 + 2(2x+1)\ln x}{2(1-x)^4}\,, \tag{E.17}$$

$$f_c^{LR}(x) = \frac{-3 + 4x - x^2 - 2\log x}{(1-x)^3}\,. \tag{E.18}$$

Bibliography

[1] F. Mandl and G. Shaw, *Quantum Field Theory*. A Wiley-Interscience publication. John Wiley & Sons, 2010.

[2] T. Cheng and L. Li, *Gauge Theory of Elementary Particle Physics*. Oxford science publications. Clarendon Press, 1984.

[3] N. Cabibbo, "Unitary Symmetry and Leptonic Decays," *Phys.Rev.Lett.* **10** (1963) 531–533.

[4] M. Kobayashi and T. Maskawa, "CP Violation in the Renormalizable Theory of Weak Interaction," *Prog.Theor.Phys.* **49** (1973) 652–657.

[5] A. Masiero, S. Vempati, and O. Vives, "Flavour physics and grand unification," *Les Houches Summer School on Theoretical Physics: Session 84: Particle Physics Beyond the Standard Model* (2005) 1–78, `arXiv:0711.2903 [hep-ph]`.

[6] R. Mohapatra and A. Smirnov, "Neutrino Mass and New Physics," *Ann.Rev.Nucl.Part.Sci.* **56** (2006) 569–628, `arXiv:hep-ph/0603118 [hep-ph]`.

[7] M. Gonzalez-Garcia and M. Maltoni, "Phenomenology with Massive Neutrinos," *Phys.Rept.* **460** (2008) 1–129, `arXiv:0704.1800 [hep-ph]`.

[8] Y. Grossman, "TASI 2002 lectures on neutrinos," `arXiv:hep-ph/0305245 [hep-ph]`.

[9] G. Altarelli, "Concluding talk: fundamental lessons and challenges from neutrinos," `arXiv:1312.1107 [hep-ph]`.

[10] G. Fogli, E. Lisi, A. Marrone, D. Montanino, A. Palazzo, *et al.*, "Global analysis of neutrino masses, mixings and phases: entering the era of leptonic CP violation searches," *Phys.Rev.* **D86** (2012) 013012, `arXiv:1205.5254 [hep-ph]`.

[11] G. F. Giudice, "Naturally Speaking: The Naturalness Criterion and Physics at the LHC," `arXiv:0801.2562 [hep-ph]`.

[12] G. 't Hooft, C. Itzykson, A. Jaffe, H. Lehmann, P. Mitter, *et al.*, "Recent Developments in Gauge Theories. Proceedings, Nato Advanced Study Institute, Cargese, France, August 26 - September 8, 1979," *NATO Sci.Ser.B* **59** (1980) pp.1–438.

[13] J. Polchinski, "Effective field theory and the Fermi surface," `arXiv:hep-th/9210046 [hep-th]`.

[14] S. Weinberg, *The quantum theory of fields. Vol. 3: Supersymmetry*. Cambridge University Press, 2000.

[15] J. Wess and J. Bagger, *Supersymmetry and Supergravity.* Princeton University Press, 1992.

[16] S. P. Martin, "A Supersymmetry primer," `arXiv:hep-ph/9709356 [hep-ph]`.

[17] J.-P. Derendinger, "Lecture notes on globally supersymmetric theories in four-dimensions and two-dimensions," *Published in Corfu 1989, Elementary Particle Physics* (1990) 111–243.

[18] I. J. Aitchison, "Supersymmetry and the MSSM: An Elementary introduction," `arXiv:hep-ph/0505105 [hep-ph]`.

[19] M. Drees, "An Introduction to supersymmetry," `arXiv:hep-ph/9611409 [hep-ph]`.

[20] S. R. Coleman and J. Mandula, "All Possible Symmetries of the S Matrix," *Phys.Rev.* **159** (1967) 1251–1256.

[21] R. Haag, J. T. Lopuszanski, and M. Sohnius, "All Possible Generators of Supersymmetries of the s Matrix," *Nucl.Phys.* **B88** (1975) 257.

[22] H. E. Haber and R. Hempfling, "Can the mass of the lightest Higgs boson of the minimal supersymmetric model be larger than m(Z)?," *Phys.Rev.Lett.* **66** (1991) 1815–1818.

[23] J. R. Ellis, G. Ridolfi, and F. Zwirner, "Radiative corrections to the masses of supersymmetric Higgs bosons," *Phys.Lett.* **B257** (1991) 83–91.

[24] Y. Okada, M. Yamaguchi, and T. Yanagida, "Upper bound of the lightest Higgs boson mass in the minimal supersymmetric standard model," *Prog.Theor.Phys.* **85** (1991) 1–6.

[25] **ATLAS Collaboration** Collaboration, G. Aad *et al.*, "Observation of a new particle in the search for the Standard Model Higgs boson with the ATLAS detector at the LHC," *Phys.Lett.* **B716** (2012) 1–29, `arXiv:1207.7214 [hep-ex]`.

[26] **CMS Collaboration** Collaboration, S. Chatrchyan *et al.*, "Observation of a new boson at a mass of 125 GeV with the CMS experiment at the LHC," *Phys.Lett.* **B716** (2012) 30–61, `arXiv:1207.7235 [hep-ex]`.

[27] P. P. Giardino, K. Kannike, I. Masina, M. Raidal, and A. Strumia, "The universal Higgs fit," `arXiv:1303.3570 [hep-ph]`.

[28] M. Misiak, S. Pokorski, and J. Rosiek, "Supersymmetry and FCNC effects," *Adv.Ser.Direct.High Energy Phys.* **15** (1998) 795–828, `arXiv:hep-ph/9703442 [hep-ph]`.

[29] G. Isidori, Y. Nir, and G. Perez, "Flavor Physics Constraints for Physics Beyond the Standard Model," *Ann.Rev.Nucl.Part.Sci.* **60** (2010) 355, `arXiv:1002.0900 [hep-ph]`.

[30] L. Wolfenstein, "Parametrization of the Kobayashi-Maskawa Matrix," *Phys.Rev.Lett.* **51** (1983) 1945.

[31] **CKMfitter Group** Collaboration, J. Charles *et al.*, "CP violation and the CKM matrix: Assessing the impact of the asymmetric B factories," *Eur.Phys.J.* **C41** (2005) 1–131, `arXiv:hep-ph/0406184 [hep-ph]`. Online update at `http://ckmfitter.in2p3.fr/`.

[32] **UTfit Collaboration** Collaboration, M. Bona *et al.*, "The 2004 UTfit collaboration report on the status of the unitarity triangle in the standard model," *JHEP* **0507** (2005) 028, `arXiv:hep-ph/0501199 [hep-ph]`.

[33] G. D'Ambrosio, G. Giudice, G. Isidori, and A. Strumia, "Minimal flavor violation: An Effective field theory approach," *Nucl.Phys.* **B645** (2002) 155–187, `arXiv:hep-ph/0207036 [hep-ph]`.

[34] L. J. Hall, V. A. Kostelecky, and S. Raby, "New Flavor Violations in Supergravity Models," *Nucl.Phys.* **B267** (1986) 415.

[35] M. Ciuchini, E. Franco, D. Guadagnoli, V. Lubicz, M. Pierini, *et al.*, "D - $\bar{D}$ mixing and new physics: General considerations and constraints on the MSSM," *Phys.Lett.* **B655** (2007) 162–166, `arXiv:hep-ph/0703204 [hep-ph]`.

[36] W. Altmannshofer, A. J. Buras, S. Gori, P. Paradisi, and D. M. Straub, "Anatomy and Phenomenology of FCNC and CPV Effects in SUSY Theories," *Nucl.Phys.* **B830** (2010) 17–94, `arXiv:0909.1333 [hep-ph]`.

[37] N. Arkani-Hamed and S. Dimopoulos, "Supersymmetric unification without low energy supersymmetry and signatures for fine-tuning at the LHC," *JHEP* **0506** (2005) 073, `arXiv:hep-th/0405159 [hep-th]`.

[38] G. Giudice and A. Romanino, "Split supersymmetry," *Nucl.Phys.* **B699** (2004) 65–89, `arXiv:hep-ph/0406088 [hep-ph]`.

[39] S. Dimopoulos and H. Georgi, "Softly Broken Supersymmetry and SU(5)," *Nucl.Phys.* **B193** (1981) 150.

[40] Y. Nir and N. Seiberg, "Should squarks be degenerate?," *Phys.Lett.* **B309** (1993) 337–343, `arXiv:hep-ph/9304307 [hep-ph]`.

[41] G. Colangelo, E. Nikolidakis, and C. Smith, "Supersymmetric models with minimal flavour violation and their running," *Eur.Phys.J.* **C59** (2009) 75–98, `arXiv:0807.0801 [hep-ph]`.

[42] T. Appelquist and J. Carazzone, "Infrared singularities and massive fields," *Phys. Rev. D* **11** (May, 1975) 2856–2861. `http://link.aps.org/doi/10.1103/PhysRevD.11.2856`.

[43] M. Ciuchini, V. Lubicz, L. Conti, A. Vladikas, A. Donini, *et al.*, "Delta M(K) and epsilon(K) in SUSY at the next-to-leading order," *JHEP* **9810** (1998) 008, `arXiv:hep-ph/9808328 [hep-ph]`.

[44] A. Denner, H. Eck, O. Hahn, and J. Küblbeck, "Feynman rules for fermion-number-violating interactions," *Nucl. Phys. B* **387** no. CERN-TH-6549-92, (Jun, 1992) 467–481. 14 p.

[45] J. Rosiek, "Complete set of Feynman rules for the MSSM: Erratum," `arXiv:hep-ph/9511250 [hep-ph]`.

[46] Z. W. Liu and H. P. Kelly, "Analysis of atomic electric dipole moment in thallium by all-order calculations in many-body perturbation theory," *Phys. Rev. A* **45** (Apr, 1992) R4210–R4213. `http://link.aps.org/doi/10.1103/PhysRevA.45.R4210`.

[47] J. Hisano, M. Nagai, and P. Paradisi, "Flavor effects on the electric dipole moments in supersymmetric theories: A beyond leading order analysis," *Phys.Rev.* **D80** (2009) 095014, `arXiv:0812.4283 [hep-ph]`.

[48] G. Altarelli and F. Feruglio, "Discrete Flavor Symmetries and Models of Neutrino Mixing," *Rev.Mod.Phys.* **82** (2010) 2701–2729, `arXiv:1002.0211 [hep-ph]`.

[49] N. Haba and H. Murayama, "Anarchy and hierarchy," *Phys.Rev.* **D63** (2001) 053010, `arXiv:hep-ph/0009174 [hep-ph]`.

[50] G. Altarelli and F. Feruglio, "Models of neutrino masses and mixings," *New J.Phys.* **6** (2004) 106, `arXiv:hep-ph/0405048 [hep-ph]`.

[51] W. Buchmuller, V. Domcke, and K. Schmitz, "Predicting θ_{13} and the Neutrino Mass Scale from Quark Lepton Mass Hierarchies," *JHEP* **1203** (2012) 008, `arXiv:1111.3872 [hep-ph]`.

[52] G. Altarelli, F. Feruglio, I. Masina, and L. Merlo, "Repressing Anarchy in Neutrino Mass Textures," *JHEP* **1211** (2012) 139, `arXiv:1207.0587 [hep-ph]`.

[53] J. Bergstrom, D. Meloni, and L. Merlo, "A Bayesian comparison of U(1) lepton flavour models," *Phys.Rev.* **D89** (2014) 093021, `arXiv:1403.4528 [hep-ph]`.

[54] D. B. Kaplan, "Flavor at SSC energies: A New mechanism for dynamically generated fermion masses," *Nucl.Phys.* **B365** (1991) 259–278.

[55] T. Gherghetta and A. Pomarol, "Bulk fields and supersymmetry in a slice of AdS," *Nucl.Phys.* **B586** (2000) 141–162, `arXiv:hep-ph/0003129 [hep-ph]`.

[56] K. Agashe, G. Perez, and A. Soni, "Flavor structure of warped extra dimension models," *Phys.Rev.* **D71** (2005) 016002, `arXiv:hep-ph/0408134 [hep-ph]`.

[57] Y. Nomura, M. Papucci, and D. Stolarski, "Flavorful supersymmetry," *Phys.Rev.* **D77** (2008) 075006, `arXiv:0712.2074 [hep-ph]`.

[58] B. Keren-Zur, P. Lodone, M. Nardecchia, D. Pappadopulo, R. Rattazzi, *et al.*, "On Partial Compositeness and the CP asymmetry in charm decays," *Nucl.Phys.* **B867** (2013) 394–428, `arXiv:1205.5803 [hep-ph]`.

[59] E. Dudas, G. von Gersdorff, J. Parmentier, and S. Pokorski, "Flavour in supersymmetry: Horizontal symmetries or wave function renormalisation," *JHEP* **1012** (2010) 015, `arXiv:1007.5208 [hep-ph]`.

[60] S. Davidson, G. Isidori, and S. Uhlig, "Solving the flavour problem with hierarchical fermion wave functions," *Phys.Lett.* **B663** (2008) 73–79, `arXiv:0711.3376 [hep-ph]`.

[61] E. Lunghi and A. Soni, "Possible Indications of New Physics in B_d -mixing and in sin(2 beta) Determinations," *Phys.Lett.* **B666** (2008) 162–165, `arXiv:0803.4340 [hep-ph]`.

[62] C. Baker, D. Doyle, P. Geltenbort, K. Green, M. van der Grinten, *et al.*, "An Improved experimental limit on the electric dipole moment of the neutron," *Phys.Rev.Lett.* **97** (2006) 131801, `arXiv:hep-ex/0602020 [hep-ex]`.

[63] **Particle Data Group** Collaboration, K. Nakamura *et al.*, "Review of particle physics," *J.Phys.* **G37** (2010) 075021.

[64] G. Buchalla, A. J. Buras, and M. E. Lautenbacher, "Weak decays beyond leading logarithms," *Rev.Mod.Phys.* **68** (1996) 1125–1144, `arXiv:hep-ph/9512380 [hep-ph]`.

[65] A. J. Buras, "Weak Hamiltonian, CP violation and rare decays," `arXiv:hep-ph/9806471 [hep-ph]`.

[66] M. Peskin and D. Schroeder, *An Introduction to Quantum Field Theory.* Advanced book classics. Addison-Wesley Publishing Company, 1995.

[67] P. Breitenlohner and D. Maison, "Dimensional Renormalization and the Action Principle," *Commun.Math.Phys.* **52** (1977) 11–38.

[68] W. Siegel, "Supersymmetric Dimensional Regularization via Dimensional Reduction," *Phys.Lett.* **B84** (1979) 193.

[69] D. Stockinger, "Regularization by dimensional reduction: consistency, quantum action principle, and supersymmetry," *JHEP* **0503** (2005) 076, `arXiv:hep-ph/0503129 [hep-ph]`.

[70] D. Stockinger, "Regularization of supersymmetric theories: Recent progress," *Nucl.Phys.Proc.Suppl.* **157** (2006) 136–140.

[71] W. A. Bardeen, A. Buras, D. Duke, and T. Muta, "Deep Inelastic Scattering Beyond the Leading Order in Asymptotically Free Gauge Theories," *Phys.Rev.* **D18** (1978) 3998.

[72] K. Chetyrkin, J. H. Kuhn, and M. Steinhauser, "RunDec: A Mathematica package for running and decoupling of the strong coupling and quark masses," *Comput.Phys.Commun.* **133** (2000) 43–65, `arXiv:hep-ph/0004189 [hep-ph]`.

[73] S. P. Martin and M. T. Vaughn, "Two loop renormalization group equations for soft supersymmetry breaking couplings," *Phys.Rev.* **D50** (1994) 2282, `arXiv:hep-ph/9311340 [hep-ph]`.