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Intensional superpositions: exploring comprehension and incomprehension in a neurophysical framework

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Abstract

In this thesis, we present a new logical model based on strong empirical evidence, designed to formalize the comprehension and incomprehension phenomena in natural language-mediated interactions among humans. We called this model Intensional Disjunctive Superposition Evaluation (IDSE). The model builds on formal semantics λ-calculus computation of natural language, where words and sentences are intensionalized as meanings superpositions, using this quantum borrowed concept instead of the Lewisian and Kripkean approach. Superpositions are characterized as possibly infinite always-true disjunctions of intensions, guaranteed by quantum theory experimental evidence applied to brain physiology and cognitive behaviour. λ-terms in superposition are processed by a distributive evaluation operator linked to either a speaker or a listener. This operator outputs an interpretation of the disjunction, restricted by the superposition's design. Distributivity is an important feature that consents us to preserve the Fregean principle of compositionality in our calculus. Finally, we input evaluated λ-terms into an equality check function. If the inputs are equal, the function outputs the comprehension of the λ-term. Otherwise, it outputs incomprehension. The model assumes that quantum mechanics laws govern the electrochemical signals in an individual's nervous system. Therefore, we materialistically define linguistic elements as the cited signals, locating the model in a quantum biological framework which tries to connect theoretical linguistics consolidated tools to the leading research in physics and neuroscience. In particular, we cover various hyperscanning methods, like fMRIs, EEGs and microendoscopic calcium imaging. Moreover, the architecture can be easily detached from its evidential grounding to explore the system's adaptability in abstract theories.

Keywords: Theoretical linguistics, Formal semantics, Quantum mechanics, Neuroscience

Contents

1 Introduction

Misunderstandings have plagued conversations since humans first began exchanging words. At the heart of this issue lies the natural languages' inherent semantical and structural ambiguity, which have persistently challenged our ability to convey ideas with accuracy. These subtle nuances often cloak words and phrases in layers of multiple meanings, leading to confusion, frustration, and sometimes even hilarity in our attempts to connect through language. One of the most fascinating topics in formal semantics and philosophy of language is intensionality, namely how to effectively describe the deep and intricate multi-layered and contextual dependent meanings of words and sentences. Indeed, a lot of logicians interested in linguistic issues, such as Gottlob Frege, Richard Montague, David Lewis and Saul Kripke, to name a few, demonstrated the fruitfulness of logical and mathematical methods in the study of semantics. But, as every good theorist should admit, the powerful formal concretizations of imagination must inevitably be compared with experimental evidence. Therefore, in this work, we propose a formal model for natural language-mediated interactions mirroring the mathematical architecture of Quantum mechanics, its experimental proofs, the physiology of the human brain and the most recent developments of research in neuroscience.

The thesis is organized as follows:

- In Chapter 1, we provide preliminary knowledge about the definition of comprehension and various incomprehension phenomena, building an interdisciplinary branching between linguistic, anthropological and philosophical studies. Furthermore, we introduce foundational concepts such as natural language ambiguity, ignorance, knowability, intension, and a glimpse of our model formalization.

- In Chapter 2, we deliver an overview of the theoretical concepts important to understand our work. We discuss Propositional logic, First-order logic, and lambda calculus as the fundamental building blocks of our structure. Additionally, we examine superposition and vector space concepts borrowed from Quantum mechanics, which form the essential tools for intensionalization. We also explain other Quantum mechanics' main notions and describe how information flows in our brains.

- In Chapter 3, we didactically outline the model through example applications of each element belonging to the system, starting with stating that lexical items and sentences are brain electrochemical signals and formalizing them as superpositions, i.e. vectors made up of unfalsifiable disjunctions of intensions relative to λ-terms. We continue by describing the two operators of the model, namely the distributive evaluation operator K and the identity checker over intensions E. We conclude the Chapter covering various hyperscanning methods, like fMRIs, EEGs and microendoscopic calcium imaging, enlightening the methodological benefits of connecting sciences and the validity of our formalization.

- In Chapter 4, we present the conclusions of our work, as well as some future developments.

1.1 Defining incomprehension

Grice's Cooperative Principle (1975) [42] describes how conversations are cooperative efforts in which participants have a common purpose, namely mutual understanding. It underlines the importance of transmitting the correct meaning during an interaction and provides cooperative maxims of quantity, quality, relation and manner. Grice's work is a great starting point to illustrate how effective communication is vital for fostering mutual understanding, whether in the intimate realm of personal interactions or in exchanges of information for more practical matters. However, misunderstandings can often arise, hindering the ability to communicate correctly. Given our familiarity with the problem at hand, understanding the phenomenon on an intuitive level is an easy task for us as human beings, but this did not stop experts in linguistics, philosophy of language, logic, cognitive science, psychology of language and linguistic anthropology from extensively studying this issue. Several attempts have been made to describe misunderstanding and here we will survey what we consider some of the fundamental scientific contributions accompanied by some clarifying concepts.

1.1.1 What we understood about misunderstandings

In this section, we do a little literature review. We will take a closer look at what other scholars said about our topic. We will be exploring the existing research to see what has already been said and how our work fits into the bigger picture.

Let us begin with 'Mutual misunderstanding - scepticism and the theorizing of language and interpretation' (1992) [84] in which Talbot J. Taylor explores the importance of sceptical doubts about communicational understanding in shaping theories of language and interpretation. Rather than taking for granted the assumption that we understand each other, Taylor argues that these doubts play a crucial role in the development of Western linguistic thought. Through an analysis of rhetorical patterns in the works of John Locke, Jacques Derrida, Gottlob Frege, Jonathan Culler, Noam Chomsky, Ferdinand de Saussure, H. Paul Grice, Michael Dummet, Stanley Fish, Alfred Schutz, Barbara Herrnstein Smith, Harold Garfinkel and others, Taylor demonstrates how scepticism influences disciplines such as critical theory, philosophy of language, communication theory, and linguistics. This study sheds light on the complex interplay between commonsense assumptions and relativism in intellectual discourse, making it relevant to readers across different disciplinary backgrounds. We must also cite 'Introduction: Some questions about misunderstanding' by Marcelo Dascal (1999) [25], where misunderstanding is highlighted as a common issue that can occur during communication, where people fail to reach a shared understanding despite their efforts to communicate effectively. Various reasons for misunderstandings can either work together to make things clearer or create more confusion. Understanding these interactions can help improve communication and reduce misinterpretations. Dascal tries to solve the puzzle by searching for answers to the following questions: How does misunderstanding occur? How often is misunderstanding detected and corrected? How is misunderstanding managed? What are the causes of misunderstanding? If we want more socio-anthropological answers to these questions we can look to 'Language and Social Identity' by John J. Gumperz (1982) [45] 'Directions in Sociolinguistics: The Ethnography of Communication' (1986) [44] also written by Gumperz. Rapidly, few words on ethnomethodology, which is a study field that focuses on intersubjectivity and on how people can know and act within a common world. It also examines how members of different sociocultural contexts, namely contexts coloured by cultural variables, negotiate and achieve a common context. Here we define 'context' with the following general definition that looks at the entry in David Crystal's 'Dictionary of Linguistics and Phonetics' (2008) [21]. The term refers to the non-linguistic elements systematically employed in relation to linguistic units. Similarly, 'situation' is used in this sense, as in the compound 'situational context'. The situational context includes the entire non-linguistic background of a text or utterance, and the surrounding circumstances in which it occurs. It also includes the awareness by the speaker and hearer of what has been said earlier and of any pertinent external beliefs or presuppositions. Therefore, assumptions and inferences play a role too, as people often make them based on their beliefs, leading to misinterpretations of others' intentions and messages. We will not give an extensive dissertation on this concept due to space reasons. For a deep dive into its abyssal complexity, please refer to the dedicated chapter in 'The Oxford Handbook of Pragmatics' (2017) [54]. Back to us, another culture-dependent perspective on communication is 'The Ethnography of Communication: An Introduction' by Muriel Saville-Troike (2008) [77], then expanded by Hymes in 'Foundations in Sociolinguistics: An Ethnographic Approach' (2013) [55] and 'Contextualization and understanding'

(1992) [46] and 'Contextualization revisited' (1992) [47] both by Gumperz. Regarding pragmatic strategies to avoid incomprehension we suggest 'Do you "(mis)understand" what I mean? Pragmatic strategies to avoid cognitive maladjustment' (2011) [85] by Jesús Romero-Trillo and Elizabeth Lenn. The article looks at how people use language strategies to prevent misunderstandings in conversations, especially between speakers of different languages. By studying the use of specific language cues called pragmatic markers, the researchers aim to understand how they help in managing and avoiding misunderstandings during cross-cultural conversations. Another formidable and really useful study is 'The interactional handling of misunderstanding in everyday conversations' by Bazzanella and Damiano (1999) [9]. In this paper, the authors discuss how people handle misunderstandings during conversations using an Italian dataset. They analyze who corrects the misunderstanding (speaker or listener), the different stages of resolving it, and common patterns of how to fix it. The study emphasizes that misunderstanding is a normal part of communication and should be seen as a continuous process that requires interaction and negotiation to reach mutual comprehension. Another interesting work is 'Building True Understanding Via Apparent Miscommunication: A Case Study' by Elda Weizman (1999) [91]. Weizman examines a conversation in a Hebrew short story by Nobel prize laureate Shmuel Yossef Agnon, aiming to demonstrate how apparent miscommunication is used by the speakers to establish an implicit mutual understanding. It introduces a theoretical framework that distinguishes between individual speaker meanings (I-level) and shared direction of the conversation (We-level). The analysis focuses on how miscommunication at the individual level and mutual understanding at the shared level are shaped by the speakers' assessments of their familiarity with certain information.

Although we cannot provide an exhaustive literature review on the topic of incomprehension due to space constraints, we strive for "completeness" even in extreme situations. Therefore, we further suggest: 'Dialogue: An Interdisciplinary Approach' by Marcelo Dascal (1985) [23]; 'Two Modes of Understanding: Comprehending and Grasping' by Marcelo Dascal and Isidoro Berenstein (1987) [26]; 'Repair After Next Turn: The Last Structurally Provided Defense of Intersubjectivity in Conversation' by Emanuel A. Schegloff (1992) [78]; 'Models of interpretation' by Marcelo Dascal (1992) [24]. These works are classics and are great for improving one's understanding of conversation, comprehension, and misunderstandings.

1.1.2 Ambiguity

At this stage, we should be able to identify some key points relating to the problem (cross-culturally analysis, pragmatic markers, adjustment strategies, speakers' intentions etc.), but we find it helpful to continue our journey bringing under the light another tricky concept undoubtedly linked to incomprehension; we are talking about ambiguity.

Ambiguity has been the source of much frustration and amusement for every being who regularly uses language to communicate.

From a historical point of view, in Aristotle's 'Sophistical Refutations' (Hasper 2013) [49], the Greek philosopher identified various fallacies associated with ambiguity. The Stoics, including Chrysippus, were intrigued by ambiguity too (Atherton 1993) [5]. He claimed that every word is ambiguous arguing that words could be understood in many different ways by different people. Philosophers exploring the connection between language and thought, such as Ockham and Frege, debated whether language used in thought could have ambiguous phrases. While Ockham accepted ambiguity in mental sentences but not in mental terms (Spade 2002) [81], Frege (1892) [38] expressed disdain for ambiguity, suggesting it should be avoided, especially in a perfect language. This ambiguity aversion persists today, as formal languages are often employed to clarify ambiguous sentences.

To explore ambiguity we need to identify what carries ambiguity. This leaves various possibilities, including utterances, sentences, discourses, and inscriptions. These distinctions matter: a written sentence can be uttered in multiple ways, influenced by factors like prosody, contexts and so on. Similarly, two written utterances may sound alike but have different spellings, leading to phonological ambiguity without orthographic ambiguity. A key question about ambiguity is how to represent it. Structural ambiguities pose no independent issue, but lexical ambiguities present a real challenge. It may seem like there are many equally valid approaches to solve this problem. For instance, we could represent the meaning of a word like "bank" disjunctively, or we could treat it as multiple words that look and sound alike, perhaps using subscripts. However, both approaches have drawbacks: disjunctive meanings are not unique to ambiguity, and using subscripts does not clarify what they represent. This suggests that the issue has significant implications for how we approach semantics. For further insights into the challenge of representing ambiguity, refer to 'Truth and Meaning' by Donald Davidson (1967) [27], 'Ambiguity, Generality, and Indeterminacy' by Brendan Gillon (1990) [41], and 'How to Think About Meaning' by Paul Saka (2007) [76]. Philosophers of language and linguists use "ambiguity" to denote a particular phenomenon distinct from the broader concept of multiple interpretations. We will try to delineate ambiguity from other common cases that are often mistakenly conflated with it. Let us start with vagueness. Vagueness occurs when a term or sentence lacks precision in its meaning or reference, making it unclear whether something falls within its definition. Unlike ambiguity, which involves multiple possible meanings, vagueness is characterized by borderline cases where it's not clear if something fits the term's definition or not. This lack of clear boundaries can lead to difficulties in defining terms precisely, making them open to interpretation. Then, we have context sensitivity which refers to how the meaning of a word or sentence can change based on the context in which it is used, without changing the word's conventional meaning. For example, the word ”I” (indexical) in the sentence ”I am tired” can shift in reference depending on who is speaking. Unlike ambiguity, which involves multiple meanings of a term, context sensitivity comprises variations in meaning based on the context of the utterance rather than the inherent meanings of the terms themselves. Now we move on under-specification and generality. Suppose Albert owns three dogs named Ginger, Odin, and Igor. One day, he tells his best friend Kurt that he needs to take his dog to the vet because he ate an entire chocolate bar. However, Kurt, who knows about Albert's three dogs, is confused and asks which dog he is referring to. Even though Albert's statement is not ambiguous, it is under-specified, which prompts Kurt to seek more specific information. Lastly, we discuss the phenomenon of transference of sense or reference, which involves situations where the meaning of a term shifts from its usual context to another context. Consider this scenario: Kurt and Albert have an appointment, and Kurt calls Albert to inform him of his location. Kurt states, ”I am parked next to the bank”. ”Parked” refers to the car, not to Kurt, namely the person speaking. This transfer of reference can lead to ambiguity and context sensitivity in language, highlighting the complexity of how words and meanings interact within sentences.

Now that we have clarified what ambiguity is not, we will show how scholars use it in linguistics and the philosophy of language. There are different sources and types of ambiguities and to describe them properly, we need to engage in a brief interlude about modern syntactic and semantic theory, which we will expand through a more precise description in a dedicated section later. In simple terms, modern linguistic theory focuses on understanding how language works, including how sentences are put together. One important aspect of this is studying syntax, which is how words are organized to form meaningful sentences. In current syntactic theory, the main idea is that words, or ”lexicon”, are like building blocks, and we use rules to put these blocks together to make sentences. These sentence structures are called ”Logical Forms” (LF). When we look at sentences in natural language and compare them to these logical forms, we find that there is not always a one-to-one match. In other words, one sentence can correspond to different logical forms. A classic example, the sentence ”every boy kissed a girl” might have two different logical forms, namely ”every boy is such that there is some girl that he likes” (surface scope) and ”some girl is such that she is liked by every boy” (inverse scope). This problem is addressed as scope ambiguities in the literature. Furthermore, some linguists argue that this sentence could have just one logical form, but with different ways of interpreting it. These interpretations might involve different ways of understanding the meanings of quantifiers like "every" and "a". These ideas are discussed in 'Logical Form: Its Structure and Derivation' (May 1985) [36]. In linguistics, there is this common, yet debated, idea that LFs are the input to semantic theory and not the actual sounds or written forms of words. However, even though logical forms may not be ambiguous, the sentences we speak and write often are. Now, logical forms are canonically represented as trees where nodes, the endpoints of the branches, are words. These words are all sorts of linguistic items, which might not always look like the words we are familiar with, and they might not match our usual understanding of words. To put it differently, when we think about how a word sounds or how it is used in different contexts, we might see that it can change its pronunciation or spelling quite a bit. However, it is not as clear whether the basic meaning of the word stays the same when these changes happen. So, it seems that there is a distinction between the actual words we use and the more abstract concepts of linguistic items (called lexemes), which is not good at all. The idea that understanding meaning relies only on LFs is simply unbearable when standing in front of the grand jury of correct reasoning. In 'Simpler Syntax' (2005) [22] Culicover and Jackendoff propose simpler syntactic structures with complex mappings to semantic or conceptual structures. On the other hand, others scholars like Pauline Jacobson in 'Towards a Variable-Free Semantics' (1999) [56] and Maria Bittner in 'Online Update: Temporal, Modal, and de Se Anaphora in Polysynthetic Discourse' (2007) [10], suggest that surface syntax could do most of the work, with intricate semantic theories accounting for the data. This debate extends to whether certain ambiguities are syntactic or semantic.

Now, it is time to show what linguists and philosophers of language address as ambiguities beginning with lexical ambiguities. The lexicon includes entries that sound the same (homophonous) or even spelt the same, but have different meanings and syntactic categories. Take "park" as an example. It is both a verb, as in "I don't know how to park" and a noun, "I am taking my dog for a walk at the park". There are two relevant things to consider about lexical ambiguity. First, we need to decide if treating them as multiple entries in the lexicon with two meanings for ambiguous terms or as part of semantic interpretation with one entry but multiple meanings. Second, we should be mindful of the distinction between words and lexical items due to words that sound the same but are not spelt the same. Let us move on to syntactic ambiguities. They arise when multiple logical forms correspond to the same sentence and there are many types of them like the earlier cited scope ambiguities, movement and binding. Such ambiguities can appear at different levels ranging from entire sentences to smaller phrases. To have unclear phrases that fit into different sentence structures is common. Take ”big house cleaner” it is unclear whether it means ”cleaner of big houses” or ”big cleaner of houses”. This confusion arises because the sentence does not indicate whether ”big” describes ”house” or ”cleaner”. In modern syntax, we would treat this phrase as two different noun phrases. Similarly, a phrase can be unclear between an adjunct and an argument. There can be confusion about who does what in a sentence, especially when there is a lack of information. Consider another classic example in the literature ”Albert saw the man with the telescope”. This sentence can be interpreted in two ways: ”Albert saw a man who was holding a telescope”, or ”Albert saw a man through the lens of a telescope”. The ambiguity arises because ”with the telescope” can modify either ”saw” or ”the man.” Depending on how we understand the relationship between the elements in the sentence, the meaning changes. We also have ambiguities due to multiple connectives and their scopes as in ”She bought a new dress and shoes or a handbag”. Pronouns make up ambiguities too, as in ”she is tall but she is short”. The sentence has a contradictory interpretation in which the two pronouns refer to the same person, but also a consistent one in which they refer to different persons. This problem solution is achieved by treating pronouns like variables, which can be free or bound. Moving forward, we lastly explore pragmatic ambiguity, something we have touched on before. We will only discuss speech acts and truth-conditional pragmatics. Providing a neutral account of the types or interpretations of speech acts is not straightforward, we here use the original formulation provided by Austin (1975) [6] with locutionary acts, namely uttering a phonetically, syntactically and morphologically well-formed expression, illocutionary act, the intention carried by the sentence (what the speaker wants to convey and do with language), and finally perlocutionary act, the effect that the sentence produced in reality. By the way, when we utter a sentence like ”the cops are coming” it can serve as an assertion, a warning, or an expression of relief. Another famous example is ”can you pass the salt?”, which could be a request or a skill check. The last thing we discuss is presuppositions ambiguities. Presuppositions are always true implicit conditions necessary to ensure the logical validity of statements. They are identifiable through linguistic triggers. We briefly examine the case of ”too” in ”I love you too”, pointed out by Kent Bach in ’Semantic Nonspecificity and Mixed Quantifiers’ (1982) [8]. According to Bach, ”I love you too” can mean, at least, one of the following four things: ”I love you (just like you love me)”; ”I love you (just like someone else does)”; ”I love you (and I love someone else)”; ”I love you (as well as bearing some other relationship (i.e. admiring) to you)”. If none of these meanings is true, the sentence is infelicitous.

There are other interesting cases of ambiguity, there are tests to detect ambiguities and these tests have problems, and there are even philosophical issues with ambiguity. We will cover none of these, instead, we invite you to take a look at the Stanford Encyclopedia of Philosophy entry on the topic, edited by Adam Sennet [79]. This source is a formidable tool and the main influence in writing this section.

1.1.3 Ignorance and Incomprehension

Another way to describe incomprehension is by pointing out the differences between this phenomenon and the more extensively studied concept of ignorance.

Over the past years, there has been an increasing interest in the investigation of ignorance, which has led to the discovery of a wide range of interdisciplinary knowledge. This new field of study, Ignorance Studies, has helped to connect various academic domains and fostered collaborations between different research areas such as philosophy, logic, psychology, sociology, linguistics (especially cognitive linguistics), cognitive sciences, economics, neuroscience (Smithson and Pushkarskaya 2015) [71], anthropology (High and Kelly 2012) [52], and gender studies. In this section, we will focus on logical, analytical and cognitive research, under the guidance of Arfini (2021) [3], a more exhaustive review of the main contributions linked to ignorance. There are three main viewpoints to determine the most practical definition of ignorance or a particular type of it. These viewpoints are presented by:

- the standard theory of ignorance
- the new theory of ignorance
- examinations of skepticism and logical inquiries on ignorance

The standard theory of ignorance defines ignorance as the absence or lack of knowledge. This simple definition has been widely adopted because it is conceptually dependent on the tripartite theory of knowledge, which defines knowledge as a true justified belief (Boden 2006) [11]. This theory is used today to define propositional knowledge.

Pierre Le Morvan (2013) [63] was the first philosopher to advocate for the standard theory of ignorance, which posits a clear-cut division between knowledge and ignorance, with no intermediary levels. Many scholars gained this view, particularly those examining the issue of belief justification.

The new theory of ignorance, endorsed by Rik Peels (2011) [70] and other scholars like Alexander Guerrero (2007) [43], openly contrasts the position just outlined. According to this theory, it is more apt to describe ignorance as a lack of true beliefs, shifting the focus of epistemic evaluation from knowledge to belief. So, ignorance is the opposite of true belief rather than knowledge, instead of true but unjustified beliefs being considered ignorant. This definition acknowledges ambiguity between knowledge and ignorance, with no longer an exclusive relationship. This addition enables more flexibility in investigating phenomena, such as misinformation, where it is difficult to distinguish between ignorance and knowledge.

Both the standard theory of ignorance and the new theory of ignorance refer to the dominant epistemological and philosophy of mind paradigm during the second half of the 20th century (cognitivist, computationalist, and internalist). The theories describe factual beliefs, namely knowing facts, and procedure beliefs, knowing-how, as equivalent even if they are absent. This equivalence does not guarantee openness to externalist philosophical perspectives, which propose distinctions and specific evaluations regarding factual and procedure knowledge. Externalist perspectives are now more widely adopted in cognition studies. Therefore, these theories are now mainly used to develop epistemological rather than cognitive analyses.

Scholars that explore the notion of ignorance, often address the dichotomy between knowledge and ignorance to discuss human epistemological limits. They consider ignorance as some sort of paradoxical entity. For instance, in Firestein's view (2012) [34], ignorance is not only the beginning of the pursuit of knowledge but also what remains at the end of it. Then, if we see ignorance as both the origin and byproduct of what we learn, discussing how we can evaluate it may facilitate us in measuring acquired knowledge.

Discussions within cognitive psychology, which found affinities with theological investigations (Franke 2015) [37] and sceptical philosophical reflections (Rescher 2009) [73], have formulated the idea that ignorance is an indispensable assumption needed to outline a realistic framework of the epistemic possibilities of a human agent (Gigerenzer 2004) [39].

Anyhow, Socrates said that we only know that we don't know and this idea still influences discussions about knowledge today (Ravetz, 2015) [72]. Specifically, recent discussions in epistemology and logic have questioned the traditional separation between ignorance and knowledge. Some research clarified the relationships between ignorance and knowledge in contemporary logic and analytic philosophy (Van Der Hoek and Lomuscio 2004) [86]. Ignorance has been defined as a condition of the agent (Fine, 2018) [33] or as an epistemic action (Kubyshkina and Petrolo 2019) [61]. Some scholars who approach the topic holistically, discuss how ignorance affects human cognition, which the currents of analytic philosophy presented so far do not consider.

Several authors linked the theme of ignorance with important topics in psychological research and cognitive sciences. Specifically, they have examined the connections between ignorance and reasoning errors, such as heuristics, fallacies, cognitive biases, the desire for knowledge and epistemic, cognitive, and moral illusions. Studies on the link between ignorance and reasoning errors (Gigerenzer and Goldstein 1996) [40] analyzed how these reasonings are quick and easy to use because they exploit context-adaptation rules. In other words, humans employ non-deductive reasoning to compensate for a lack of information or other contextual limitations, such as a short amount of reasoning time etc.

Regarding investigating the relationship between ignorance and the will to know, scholars aimed to determine which types of deliberate ignorance can be considered rational or irrational. They tried to establish when it is reasonable to choose not to know something (William 2020) [92]. This led to debates about rational and irrational voluntary ignorance in situation where the cost of acquiring knowledge outweighs its benefits.

Philosophical studies regarding these states aim to investigate why cognitive agents might want to alter their state of knowledge. Scholars who focus on the cognitive and meta-cognitive analysis of subjects' perception of their knowledge have recently begun to explore the perception of ignorance as well. Ignorance and illusions are linked to what one knows about oneself, abilities, and cognitive limitations (Dunning 2011) [30].

At the end of our informative dissertation on ignorance, we can now survey what version of the concept we will use to better characterize incomprehension, by opposition.

Here, we define ignorance as not knowing a piece of information related to the conversational theme, which is a definition stemming from Fine's work; so if Albert ignores ϕ means that Albert does not know ϕ. This concept is dramatically different from the notion of incomprehension; not comprehending ϕ imply to know ϕ and interpreting ϕ as something else. For example, imagine that Kurt invites Albert to go to an art exhibition featuring paintings by Michelangelo Merisi at the town art museum but Albert declines the invitation because he prefers Caravaggio's paintings and knows that there is a Caravaggio exhibition as well. Kurt, who has read Nietzsche and believes that sparing someone from feeling shame is the most human thing to do, decides not to tell his friend that Caravaggio is Merisi's home town name and nickname, and that they are talking about the same exhibition. On the day of the exhibition, Albert realizes his mistake and says to Kurt, "I searched on the Wiki. I am stupid." To which Kurt replies, "No, you are not. You simply did not know."

1.1.4 Comprehension, Knowability and Incomprehension

All this talk about misunderstanding, ambiguity, incomprehension and so on requires an unavoidable clarification about a central concept connected to our main topic. Therefore, in this section, we will survey an explanatory discourse on comprehension, which inevitably recalls the concept of knowability. In doing so, we also try to better shape 'incomprehension' in line with the work we already fostered in this chapter.

Let us start with comprehension, which is definable as ”to seize, grasp, lay hold of, catch [...] with mind or senses, [...] to understand (a person)” or, in a logical approach, ”the sum of the attributes comprehended in a notion or concept; intension”. A cognitive interpretation could be ”the faculty of grasping with the mind, power of receiving and containing ideas, mental grasp”. But then, what do we mean when we say that we know something? Maybe we mean ”to recognize, acknowledge, perceive [...] (a thing or person) as identical with one already perceived or considered; to recognize; to identify”. Hence, something knowable must be ”that may be known; capable of being apprehended, understood, or ascertained”.

All these definitions from the Oxford English Dictionary are similar and ambiguous, just paraphrases of the same background intuition, let alone all the different formalizations in the literature. We don't need that. What we seriously need, as always, is to avoid pointless complications. Here, we will synthesize and formalize these notions to offer a preview of more in-depth explanations presented in the following chapter.

Straight to the point, we consider knowing, comprehending and understanding the same action or operation. We will use them as synonymous during the dissertation. However, our definition is semantically tending towards the logical and philosophical concept of intension. In a broad definition, we say that intention refers to the set of qualities or properties that a word, a sentence, or any other symbol conveys, whether linguistic or not. It is easily comparable to the concept of ”sense” in Frege, the cognitive significance of an expression (the way by which a subject conceives of the denotation/referent of the term in the physical world), and we can also think of it as a more detailed expansion of ”signified” in De Saussure (1916) [67], namely the meaning, the idea or the concept that a sign evokes.

In 1.3 we will briefly expand on the concept of intension, which is foundational for our work. For now, let us say that comprehending, understanding, and knowing an expression is grasping its speaker-contextualized intention, scilicet the one particular meaning among the others that the speaker chose to convey.

1.2 On why developing a formal model about incomprehension

Someone might question the need to develop a coherent explanation for the phenomenon of incomprehension in natural language. One convincing argument for not doing so is the existence of pragmatic or psychological explanations we previously discussed. It is a fact that misunderstanding can arise due to various pragmatic, psychological and cognitive factors that significantly impact the way we interpret and communicate messages. Let us briefly highlight some of these. Factors like tone of voice, body language, and situational context significantly influence how we interpret messages. Language's inherent ambiguity can also cause misinterpretations, as interpretations vary based on individual perspectives and experiences. Perceptual filters, shaped by past experiences and biases, impact how messages are understood. Confirmation bias, where individuals interpret information to confirm their existing beliefs, can worsen misunderstandings by filtering out alternative perspectives. Cognitive overload is another barrier, hindering comprehension when individuals are overwhelmed with information or distractions. Finally, emotional states like anger, fear, or stress can impair cognitive processing, leading to misinterpretations of others' intentions.

Combining these explanations with precise syntactic, morphological, and semantic formalizations might seem to offer a compelling reason for discontinuing our investigations. However, the true effectiveness of a theory relies on its ability to bring together all the elements related to the study object under the same principles.

Imagine the theory as a hotel. Something extremely unusual happens to the scholar of incomprehension: the receptionist assigns him not one room but two rooms, rooms 712 and 713. Room 712 has all the comforts of a luxurious hotel room except for the hairdryer, which lies in room 713; company policy, don't ask. Therefore, assuming that our scholar cares about personal hygiene (which is by no means taken for granted) and has a long stereotypical hippie hairstyle, after taking a shower in room 712, he should leave to go to room 713, where the hairdryer is. What we desire is to have the hairdryer in room 712.

Increased attention to incomprehension and a complete formalization of it could lead to significant theoretical and practical developments in language-related disciplines.

We aim to create a theoretical tool that can be seamlessly incorporated into language-related theories to address the issue of incomprehension. Our purpose is also to ensure that this tool is based on a robust theoretical framework that is grounded in solid evidence. We provide a precise and coherent description of this phenomenon by basing it on biological evidence and the concepts and rigorous quantum mechanical experimental tests. We seek to create a solid starting point for conducting thorough examinations of this problem, starting from a better formalization of words' and sentences' meaning.

1.3 A glimpse of incomprehension formalization

Taking into account what we have discussed in the previous sections it is clear that incomprehension, here comes the obvious, is not a simple issue. It is a gigantic, multi-layered, nightmarish problem with many contributing factors. There are tons of elements that can lead to miscommunication and they can also appear simultaneously. We could embark on a really difficult explanation discussing how ambiguity is linked to incomprehension through a hierarchical conceptual framework, and what factors we should prioritize when trying to address misunderstandings but, we are not going to make things so difficult. We want to highlight that this is a true declaration of intent underlying the work carried out and reported here: we are not going to make things unnecessarily challenging. We suggest a clear and concise definition of this linguistic phenomenon, without sacrificing its nuanced nature to also make room for most of the elements that contribute to its presence.

Let us clarify our definition of incomprehension:

> Incomprehension is a natural language phenomenon that arises during conversations between two rational agents who aim to understand each other while sharing a common linguistic background. An incomprehension occurs when subject A produces the expression ϕ during a conversation, and subject B fails to understand ϕ. To understand ϕ means that the addressee realizes the semantics of ϕ as intended by the speaker.

With 'rational agent' I refer to an individual consistently striving to achieve optimal outcomes based on given premises and information; in other words, a blind observant of the Gricean cooperative principle. Let us consider a scenario where two rational agents, Kurt and Albert, engage in a conversation. Kurt puts forth a proposition ϕ, and Albert, in response, seeks additional clarifications because he struggles to grasp the intended meaning behind Kurt's statement.

An essential point to clarify is that understanding ϕ involves comprehending the nuanced and complete semantics that Kurt wants to communicate through this proposition. It is useful to keep in mind what was said in 1.1.4 about comprehension and intention. Let's say Kurt wanted to convey ϕ as he intended ϕ; we label that ϕ_1. The fact that Albert did not understand equals saying that he did not intend ϕ as ϕ_1 but instead as ϕ_2, or ϕ_3, which are different possible interpretations of ϕ. Even though both Kurt and Albert share the same language and a mutual desire to understand each other, the possibility of incomprehension persists.

The challenge of explaining linguistical misunderstandings among human beings has never been addressed through a formal model grounded in biological evidence and the most rigorous experimental tests found in Quantum Mechanics throughout the history of science. We want to provide a precise and coherent description of the phenomenon to establish a foundational work for conducting more in-depth investigations into the problem.

Here, we want to give you a little taste of what we are going to do in this thesis, a sneak peek at the further formalization that is the main contribution of the work. This way, we can get a better idea of what we need to do next. Suppose Ω is a set, a collection of objects which are technically called 'elements'. To be an element of Ω, an object must be a proposition produced by Kurt and linked to Kurt's unique correct interpretation of that proposition.

1. Kurt puts forward proposition ϕ.
2. Kurt understands proposition ϕ as ϕ_1.
3. ϕ_1 is how proposition ϕ must be interpreted for Kurt to feel understood.
4. Albert interprets proposition ϕ.
5. Albert interprets proposition ϕ as ϕ_2.
6. ϕ_2 is different from ϕ_1.
7. As ϕ_2 differs from ϕ_1, Albert does not grasp ϕ the way Kurts intended him to understand it.

We designate E as a function, namely the 'Correct Interpretation Function'. The Correct Interpretation Function is a binary relation $E \subseteq \Omega \times \{C; \neg C\}$, it signifies that the function has the property that every proposition $\phi \in \Omega$ is related to exactly one element in $\{C; \neg C\}$.

$$\phi = \begin{cases} C & \text{if } E(\phi) = \phi_1 \to \phi \in \Omega \\ \neg C & \text{if } E(\phi) \neq \phi_1 \to \phi \notin \Omega \end{cases}$$

where C represents the 'comprehended' value, and $\neg C$ represents the 'not comprehended' value. The function associates these values based on the conditions specified for proposition ϕ in relation to ϕ_1 and membership in set Ω. Logic operators here adopted will be explained in the next chapter.

2 Theoretical toolkit

We talked about how misunderstanding works, and now we know that it is a ubiquitous phenomenon in communication due to the murky ambiguity inherent in natural language. This chapter will equip us with essential tools to study how the meaning of individual expressions in natural language combine to produce larger meaningful expressions such as sentences and texts. We will use λ-calculus, an elegant notation for working with applications of functions to arguments. However, we will quickly go over some snippets of simpler types of logic, which are its basics, in a cumulative review that will help us understand how the framework fits all together.

In the first section of the chapter, we will discuss the essentials of Propositional Logic, namely the study of sentences meaning and the relationships between them. It examines how certain logical operators, called propositional connectives, influence the truth or assertability of sentences and arguments that we can easily define as sets of sentences. This kind of analysis goes back to Aristotle, who recognized the significance of these connectives. However, it was not until the nineteenth century that propositional logic as a distinct field emerged.

Then, we present First-Order logic or Predicate Logic, which enables a more precise investigation of meaning using existential and universal quantifiers. We continue with λ-calculus and conclude with a description of syntactic structure in linguistics theory. The first part of this chapter is indebted to the books that most influenced the writing of these sections: ”Beginning Logic” by E. J. Lemmon (1965) [64], ”Logica (3/ed)” by Achille Varzi, John Nolt and Dennis Rohatyn (2022) [87] and ”Invitation to formal semantics” by Elizabeth Coppock and Lucas Champollion (2019) [20]. We want to stress that what we deliver are very brief notes of the subjects functional to our particular study.

The second part is a collection of concepts stolen from Quantum Theory - qubit, superposition, vector space and so on - of tremendous importance for our purpose. There is a large volume of published manuals describing Quantum mechanics, but here we cite only two of them, namely what we employed and mixed writing the section: ”Quantum mechanics: the theoretical minimum” by Leonard Susskind and Art Friedman (2014) [83], and ”Mastering Quantum Mechanics: Essentials, Theory, and Applications” (2022) [95] by Barton Zwiebach (2022) [95]. Finally, we describe how information flows in our brain following ’Guyton and Hall Textbook of Medical Physiology’ by John E. Hall and Michael E. Hall [48].

Now, we are ready to start with our preliminary knowledge sections that will help us translate natural language into a formal one.

2.1 Propositional Logic in a nutshell

Logic is the field of study that examines the conditions required for arguments to be considered correct. In other words, it develops methods to distinguish valid arguments from invalid ones and investigates these conditions regardless of the argument subject.

We define an argument as a set of propositions, but we need to restrict propositions to declarative sentences, namely the only type of sentences which can be intuitively categorized as "True" or "False". One of these statements is the conclusion of an argument, while the others are its premises, which provide the reasons to derive the conclusion. When we say that a declarative statement is true or false, we mean that it accurately describes a particular state of affairs. Consider "Kurt did not eat today". It is true if and only if Kurt's belly is empty at the pronunciation time and false otherwise. When we say that an argument is valid we have ascertained that its premises are true as its conclusion. If there is at least one case where the premises of the argument are true and its conclusion is false, then the argument is invalid.

Before continuing, a clarification is necessary. In this section, we will outline how to translate natural language in propositional logic, and we will not explain arguments' evaluation methods because, as we said, we only put forward what is needed to achieve our goal. Now, back to us.

We gently introduce propositional logic by explaining how to define a formal language. We define a formal language as a recursively defined set of strings on a fixed alphabet. A formal language alphabet (or vocabulary) is a finite set of symbols we employ to form language strings. A string is a well-formed sequence of symbols that satisfies the formation rules outlined in the language syntax. Last but not least, a language semantics in which we specify operators' meaning.

Let us build a formal language L. Our language vocabulary is a set having every Roman alphabet upper-case letter as a member. We will use them as variables on propositions of natural language, hence we will call them "propositional variables".

Our language vocabulary also contains logical operators. Every operator is represented by a symbol and its meaning is defined in the language semantics through an interpretation function that assigns it a truth value. Our language syntax comprises the following rules:

- If ϕ is a Roman alphabet upper case letter, then it is a well-formed string.
- If ϕ is a well-formed string, then $\neg\phi$ is a well-formed string.
- If ϕ and ψ are well-formed strings then so are $(\phi \wedge \psi), (\phi \vee \psi), (\phi \rightarrow \psi), (\phi \iff \psi)$.

A methodological note: we use Greek alphabet letters as meta-variables, namely variables standing for objects in our language that serve us to talk about the language itself.

To establish our language semantics we need an interpretation function, therefore we need to clarify its nature describing Cartesian product between sets and what a binary relation is. Suppose Γ and Ω are two non-empty sets with $\alpha \in \Gamma$ and $\beta \in \Omega$, their Cartesian product is the set of pairs: $\Gamma \times \Omega = \{\langle \alpha, \beta \rangle | \alpha \in \Gamma, \beta \in \Omega\}$. This generalizes to more than two sets. A binary relation between Γ and Ω is a subset $\Upsilon \subseteq \Gamma \times \Omega$, that is, a set of pair $\langle \alpha, \beta \rangle$ with $\alpha \in \Gamma$ and $\beta \in \Omega$. A binary relation on Ω is a subset of $\Omega \times \Omega$.

The interpretation function is a binary relation $I \subseteq \{T, F\}$ where T stands for 'True' and F stands for 'False'. Let $[\![\phi]\!]^I$ be the denotation of a well-formed string $\phi \in L$ due the interpretation function I. Here it is our language semantics:

– If ϕ is any propositional letter, then $[\![\phi]\!]^I = I(\phi)$.

– If ϕ is a well-formed string, then $[\![\neg\phi]\!]^I = T$ if $[\![\phi]\!]^I = F$, and F otherwise.

– If ϕ and ψ are well-formed strings, then $[\![\phi \wedge \psi]\!]^I = T$ if $[\![\phi]\!]^I = T$ and $[\![\psi]\!]^I = T$, and F otherwise.

– If ϕ and ψ are well-formed strings, then $[\![\phi \vee \psi]\!]^I = T$ if $[\![\phi]\!]^I = T$ or $[\![\psi]\!]^I = T$ (or both), and F otherwise.

– If ϕ and ψ are well-formed strings, then $[\![\phi \rightarrow \psi]\!]^I = F$ if $[\![\phi]\!]^I = T$ and $[\![\psi]\!]^I = F$, and T otherwise.

– If ϕ and ψ are well-formed strings, then $[\![\phi \iff \psi]\!]^I = T$ if $[\![\phi]\!]^I = T$ and $[\![\psi]\!]^I = T$, or if $[\![\phi]\!]^I = F$ and $[\![\psi]\!]^I = F$, and F otherwise.

where:

– $\neg$, namely the negation symbol, stands for ”not”.

– $\wedge$, conjunction, translates ”and”.

– $\vee$, inclusive disjunction, translates as ”either... or... or both”, or ”and/or” – or indeed as plain ”or”.

– $\rightarrow$, conditional, stands for ”If... then...”.

– $\iff$, biconditional, translates ”If and only if... then...”.

For the sake of completeness, we provide the symbols truth tables, which are basically a sum up of our language semantic conditions in tabular form:

P	$\neg P$
T	F
F	T

Negation

P	Q	$P \wedge Q$
T	T	T
T	F	F
F	T	F
F	F	F

Conjunction

P	Q	$P \vee Q$
T	T	T
T	F	T
F	T	T
F	F	F

(Inclusive) disjunction

P	Q	$P \rightarrow Q$
T	T	T
T	F	F
F	T	T
F	F	T

Implication

P	Q	$P \iff Q$
T	T	T
T	F	F
F	T	F
F	F	T

Biconditional

Let us end this section with a practical examples of translation:

- It is not true that elephants cannot fly. $\rightsquigarrow \neg A$
- Heidegger is a charlatan and a nazist. $\rightsquigarrow A \wedge B$
- My grandma is an excellent chef or she is a drug cartel chief. $\rightsquigarrow A \vee B$
- If Kurt speaks clearly, then Albert understands him. $\rightsquigarrow A \rightarrow B$
- Mary is a great swimmer if and only if she is the next Dalai Lama. $\rightsquigarrow A \iff B$

With A standing for "elephants cannot fly" and the first sentence in the other examples, and B standing for the second one.

2.2 First-order Logic in a nutshell

Quantification theory, another name for First-order logic, explores the logic of arguments involving quantifiers (expressions like "all", "some", and "none", etc.) and explains how these expressions interact with propositional connectives. It is the framework in which all of the deductive reasoning needed in science and mathematics must be conducted.

In this section, we will describe First-order Logic as an extension of Propositional Logic. Therefore, what we have previously discussed regarding Propositional Logic holds to First-order Logic. The only difference is that we will introduce some notation adjustments and additional operators that will pull out the meaning of sentences more precisely.

Regarding vocabulary, in our extended language L_1, we define lower case Roman alphabet letters as non-logical constants. They stand for proper names, hence we call them "individual constants". We can use every lower case letter except x, y, z. They are our language "individual variables", we use them to formalize generic individuals. Upper case Roman alphabet letters in L_1 do not represent propositions as in L. We call them "predicative constants", which are non-logical constants standing for "properties" of individuals, namely unsaturated arguments needing an individual constant to predicate on. Furthermore, we distinguish another kind of predicative constant from properties called "relations", which are predicates saturated by two or more individuals. What we said about x, y, and z, also applies to X, Y, and Z which are "predicative variables". Talking about relations, we introduce a particular kind called "identity", which tells us that any given object can be replaced by another object with the same characteristics. We discuss quantifiers, logical operators that allow us to quantify the number of individuals to whom a specific predicate is attributed. In L_1 we have two quantifiers, the "existential quantifier" and the "universal quantifier". Given a generic non-empty set, the universal quantifier let us apply a predicate to all elements of the set, and the existential quantifier let us apply a predicate to at least one element of the set. Quantifiers can also act on variables, so we classify them as "bound" or "free", depending on whether they are subject to a quantifier or not.

Now, a quick sum up of L_1's syntax and semantics. Our first-order language syntax comprises the following new rules:

- if ψ is a Roman alphabet lower case letter, then it is a well-formed string.
- if ψ is a well-formed string, then $\Phi(\psi)$ is a well-formed string.
- if ϕ and ψ are well-formed strings, then so are $(\phi = \psi)$, $\forall\psi(\Phi(\psi))$ and $\exists\psi(\Phi(\psi))$.

with Φ as a meta-variable standing for a generic Roman alphabet upper case letter.

Regarding semantics, we simply say that:

- Given a non-empty set Ω, $[\![\forall\psi(\Phi(\psi))]\!]^I = T$ if every $\psi \in \Omega$ has the property Φ, and is Ω, $[\![\forall\psi(\Phi(\psi))]\!]^I = F$ otherwise.
- Given a non empty set Ω, $[\![\exists\psi(\Phi(\psi))]\!]^I = T$ if at least one $\psi \in \Omega$ has the property Φ, and $[\![\exists\psi(\Phi(\psi))]\!]^I = F$ otherwise.

with Ω as a meta-variable standing for a generic non-empty set.

– We define identity through Lebniz's law, also known as "the indiscernibility of identicals". Given two individuals ψ and ϕ, $[\![\psi = \phi]\!]^I = T$ if whatever property ψ has, ϕ as. $[\![\psi = \phi]\!]^I = F$ otherwise.

Some examples:

– Heidegger is a charlatan and a nazist $\rightsquigarrow C(h) \wedge N(h)$.

with h as an individual constant standing for "Heidegger" and C and N as unary predicates respectively standing for "charlatan" and "nazist".

– Every sentence is incomprehensible $\rightsquigarrow \forall x(S(x) \rightarrow I(x))$.

with S and I being unary predicates respectively for "sentence" and "incomprehensible".

– There is a girl who loves mathematics $\rightsquigarrow \exists x(G(x) \wedge L(x, m))$.

  with G as a unary predicate for "girl" and L a binary predicate for "loves".

Finally, we conclude this section by providing some rules of natural deduction calculation, which precisely demonstrates why an argument is valid. Given a certain set of rules Ω, we define a proof as an application of a subset of rules $\Gamma \subseteq \Omega$ based on some assumptions, or premises, on which we base our conclusion. This rules are related to logical operators.

– Assumption rule: it is possible to introduce any well-formed formula at any proof step.

– Conjunction introduction: if ϕ and ψ are true individually, they are also true when considered together. Formally, $\phi, \psi \vdash (\phi \wedge \psi)$.

– Conjunction elimination: if ϕ and ψ are true when considered together, they are also true individually. Formally, $(\phi \wedge \psi) \vdash \phi$ or $(\phi \wedge \psi) \vdash \psi$.

– Disjunction introduction: ϕ is true; therefore the disjunction $(\phi \vee \psi)$ is true. Formally, $\phi \vdash (\phi \vee \psi)$.

– Disjunction elimination: given an well-formed formula $(\phi \vee \psi)$ along with proof of χ from ϕ and a proof of χ from ψ, taken ϕ and ψ as separate assumptions, it is possible to derive χ.

– Conditional proof: given a proof of ϕ from ψ, it is possible to derive $(\psi \rightarrow \phi)$.

- Modus Ponendo Ponens: given ϕ and $(\phi \rightarrow \psi)$, it is possible to derive ψ. Formally, $\phi, (\phi \rightarrow \psi) \vdash \psi$.

- Modus Tollendo Tollens: given $\neg\psi$ and $(\phi \rightarrow \psi)$, it is possible to derive $\neg\phi$. Formally, $\neg\psi, (\phi \rightarrow \psi) \vdash \neg\phi$.

- Reductio Ad Absurdum: if a contradiction derives from ϕ, it is possible to negate ϕ.

Equivalences:

- Double negation (negation elimination): $\neg\neg\phi \equiv \phi$

- Commutativity: $(\phi \wedge \psi) \equiv (\psi \wedge \phi)$; $(\phi \vee \psi) \equiv (\psi \vee \phi)$; $(\phi \iff \psi) \equiv (\psi \iff \phi)$

- Associativity: $\phi \wedge (\psi \wedge \chi) \equiv (\phi \wedge \psi) \wedge \chi$; $\phi \vee (\psi \vee \chi) \equiv (\phi \vee \psi) \vee \chi$

- Distributivity: $\phi \wedge (\psi \vee \chi) \equiv (\phi \wedge \psi) \vee (\phi \wedge \chi)$; $\phi \vee (\psi \wedge \chi) \equiv (\phi \vee \psi) \wedge (\phi \vee \chi)$

- De Morgan's Laws: $\neg(\phi \wedge \psi) \equiv \neg\phi \vee \neg\psi$; $\neg(\phi \vee \psi) \equiv \neg\phi \wedge \neg\psi$

- Absortion laws: $\phi \vee (\phi \wedge \psi) \equiv \phi$; $\phi \wedge (\phi \vee \psi) \equiv \phi$

- Biconditionals' equivalence characterization: $(\phi \iff \psi) \equiv (\phi \rightarrow \psi) \wedge (\psi \rightarrow \phi)$

  with $\vdash$ being the derivation sign.

We unorthodoxically outlined these rules for the sake of brevity and in the name of practicality, basing our selection on on how often they appear in proofs.

2.3 λ-calculus

Thus far, we have built the theory pillars. Now, we are ready to give a precise account of semantics in natural language using the logical representation language called λ-calculus. The mathematician Alonzo Church introduced it in the 1930s as part of his research into the foundations of mathematics (Church, Burge, Enderton 2019) [18]. A few years later, the combined efforts of Stephen Kleene and John Barkley Rosser (1935) [58] gave more insights to the language that, in the '60s, was imported into the linguistics field by Richard Montague and others.

As we said, this formal language is a simple notation for working with applications of functions to arguments, based on a two-type syntax where we define e as the type of individuals and t as the type of truth values. Through type combination, we obtain complex types that allow us to translate numerous natural language expressions.

Before explaining this system in greater detail, let us say more about binary relations. To continue, we need to bring up functions, a special kind of binary relations. We define a function from Γ to Ω as a binary relation $f \subseteq \Gamma \times \Omega$ with the property that every $\alpha \in \Gamma$ is related to exactly one $\beta \in \Omega$. We then write $f(a) = b$. If f is a function from Γ to Ω we write: $f : \Gamma \to \Omega$ where we call the set Γ the domain and the set Ω the codomain of the function.

Now we can really start. We define a universe T of types and for each type $\sigma \in T$, a set of terms $a : \sigma$ of that type. For each type σ a domain D_σ^M of objects of type σ. For each term $a : \sigma$, a denotation $[\![a]\!]^{M,g} \in D_\sigma^M$. As we said, our system has two basic types. e is the type of individuals; t is the type of truth-values. If σ and τ are types, then $\langle \sigma, \tau \rangle$ is a type, namely the type of functions from objects of type σ to objects of type τ.

Here are some complex types:

- $\langle$t,t$\rangle$: type of functions from truth-values to truth-values.
- $\langle$e,t$\rangle$: type of functions from individuals to truth-values (properties).
- $\langle\langle$e,t$\rangle$,$\langle$e,t$\rangle\rangle$: type of functions from properties to properties.
- $\langle\langle$e,t$\rangle, t\rangle$: type of functions from properties to truth-values (2-nd order properties).
- $\langle$e,$\langle$e,t$\rangle\rangle$: type of functions from individuals to properties (binary relations).

For practicality, let us use some abbreviations: et abbreviates the type $\langle e, t \rangle$; eet abbreviates the type $\langle$e,$\langle$e,t$\rangle\rangle$.

In our language, we need a domain D of objects. Given such a domain, we define a domain D_τ as follows:

- $D_e = D$
- $D_t = \{T, F\}$
- $D_{\langle \sigma, \tau \rangle} = \{f \,|\, f : D_\sigma \to D_\tau\}$ namely the set of functions from D_σ to D_τ; e.g. $D_{\langle\langle e,t\rangle,\langle e,t\rangle\rangle}$ contains all functions from properties to properties.

Now, we define the set of expressions that qualify as terms of a given type, abbreviating the statement "a is a term of type τ" with the notation "$a : \tau$". Simultaneously, we define an assignment function g that maps each variable x_τ of type τ to an object $g(x_\tau) \in D_\tau$; so, relative to a model M and an assignment g, we assign to each λ-term of our language a denotation, namely an object in the proper domain (if $a : \tau$, its denotation will be $[\![a]\!]^{M,g} \in D_\tau$).

Let us specify that a model for our typed λ-calculus is a pair $M = \langle D, I \rangle$ where D is a non-empty set providing a domain of individuals, and I is a function mapping each constant $c_\tau : \tau$ to a denotation $I(c_\tau) \in D_\tau$.

– Atoms:

  If x_τ is a variable of type τ, then $x_\tau : \tau$ and $[\![x]\!]^{M,g} = g(x)$.

  If c is a constant of type τ, then $c_\tau : \tau$ and $[\![c]\!]^{M,g} = I(c)$.

– Application:

  If $a : \langle \sigma, \tau \rangle$ and $\beta : \sigma$, then $a(\beta) : \tau$ and $[\![a(\beta)]\!]^{M,g} = [\![a]\!]^{M,g}([\![\beta]\!]^{M,g}) : \tau$

– Abstraction:

  If $a : \tau$ and x_σ is a variable of type σ, then $(\lambda x_\sigma.a) : \langle \sigma, \tau \rangle$ and $[\![\lambda x_\sigma.a]\!]^{M,g}$ is the function taking an object $d \in D_\sigma$ to the object $[\![a]\!]^{M,g[x \to d]}$.

– Identity:

  If $a : \tau$ and $\beta : \tau$, then $(a = \beta) : t$ and

$$[\![(a = \beta)]\!]^{M,g} = \begin{cases} T & \text{if } [\![a]\!]^{M,g} = [\![\beta]\!]^{M,g} \\ F & \text{otherwise} \end{cases}$$

– Negation:

  If $\phi : t$, then $\neg\phi : t$ and

$$[\![\neg\phi]\!]^{M,g} = \begin{cases} T & \text{if } [\![\phi]\!]^{M,g} = F \\ F & \text{if } [\![\phi]\!]^{M,g} = T \end{cases}.$$

– Connectives:

  If $\phi : t$ and $\psi : t$, then $(\phi \wedge \psi) : t$ and

$$[\![\phi \wedge \psi]\!]^{M,g} = \begin{cases} T & \text{if } [\![\phi]\!]^{M,g} = [\![\psi]\!]^{M,g} = T \\ F & \text{otherwise} \end{cases}.$$

If $\phi : t$ and $\psi : t$, then $(\phi \vee \psi) : t$ and

$$[\![\phi \vee \psi]\!]^{M,g} = \begin{cases} F & \text{if } [\![\phi]\!]^{M,g} = [\![\psi]\!]^{M,g} = F \\ T & \text{otherwise} \end{cases}.$$

If $\phi : t$ and $\psi : t$, then $(\phi \rightarrow \psi) : t$ and

$$[\![\phi \rightarrow \psi]\!]^{M,g} = \begin{cases} F & \text{if } [\![\phi]\!]^{M,g} = T \text{ and } [\![\psi]\!]^{M,g} = F \\ T & \text{otherwise} \end{cases}.$$

– Universal Quantifier:

  If $\phi : t$ and x_σ is a variable, then $\forall x_\sigma.\phi : t$, and

$$[\![\forall x_\sigma.\phi]\!]^{M,g} = \begin{cases} T & \text{if } [\![\phi]\!]^{M,g[x \rightarrow d]} = T \text{ for all objects } d \in D_\sigma \\ F & \text{otherwise} \end{cases}.$$

– Existential Quantifier:

  If $\phi : t$ and x_σ is a variable, then $\exists x_\sigma.\phi : t$, and

$$[\![\exists x_\sigma.\phi]\!]^{M,g} = \begin{cases} T & \text{if } [\![\phi]\!]^{M,g[x \rightarrow d]} = T \text{ for some objects } d \in D_\sigma \\ F & \text{otherwise} \end{cases}.$$

We call β-reduction the application of a function described by a λ-expression to an argument. Take $F : et$, $k : e$ and the term $(\lambda x_e.F(x))(k) : t$. If we β-reduce we have $F(k) : t$ and $(\lambda x_e.F(x))(k) \equiv F(k)$, namely they are α-equivalent, which means that the two λ-terms have the same denotation relative to all models and all assignments.

2.3.1 Intensionalizing λ-calculus

Until now, we have done semantics in an extensional setting. We focused our attention on how the denotation of an expression is compositionally determined by its constituents' denotations. However, in natural language, there are many situations where the denotation of a sentence does not depend just on the denotation of its parts, but on their 'sense'. This leads us to intensionality, namely how to deal with meaning in natural language formalizations, one of the most captivating topics in semantics and philosophy of language. In this subsection, we will not explain every attempt to solve the intensionality puzzle, we will only discuss the dominant theories in formal semantics and what originated the debate. To know more about the history and different solutions to this problem, we suggest the rich and crystal-clear Stanford Encyclopedia of Philosophy voice by Melvin Fitting [35].

We start with a quick historical overview of the terminology and the issue. The Port-Royal Logic (Antoine Arnauld, Pierre Nicole 1662) [4] introduced the terms 'denotation' and 'comprehension' to distinguish between referent and meaning. Frege is known for using 'sinn' and 'bedeutung', usually translated as 'sense' and 'reference'. Carnap (1988) [16] eventually settled on 'intension' and 'extension', now widely accepted terms. In the Fregian account, a sign encompasses both a reference and a sense. Additionally, Frege considers something called 'the mode of presentation' of a sign, namely what it is that associates the sign with the things it designates. The notion of a mode of presentation is somewhat obscure, and Frege does not supply further details. Frege further complicates the issue by introducing the concept of the idea associated with a sign, which differs from its sense and reference. Unlike sense and reference, the idea is subjective and can vary from person to person. 'Idea' is another fuzzy concept left not detailed. Frege uses a pretty famous example in the literature, the identity between Hesperus and Phosphorus, respectively 'the evening star' and 'the morning star'. The ancients did not know they were the very same celestial body, the planet Venus. Hesperus and Phosphorus have the same denotation but different senses. The fact that in natural language there are expressions with a sense but no referent, such as the famous hairless king of France in "The present king of France is bald", cost Frege a severe depression due to Bertrand Russell's objection (1905) [75], enlightening that a declarative sentence lacking a referent for its subject cannot be categorized as either true or false, giving birth to his theory of definite descriptions.

Frege outlined a theory of intentionality but did not develop any formal model for dealing with sense, as opposed to reference. Intensional logic emerges from the relationship between Alonzo Church's work and that of Rudolf Carnap. Before building up λ-calculus, Church employs something he just calls "concepts", wherein anything serving as the sense of a name for something can act as a concept for that thing, no further explanations. Church explicitly stated that concepts are possibly uncountable and independent of language. Carnap finally supplied a formal semantic where intensions are identified with detailed model-theoretic entities. Basing his contribution on the precursor of the concept of possible worlds, namely the Wittgensteinian states of affairs, Carnap elaborates a notion of necessity or analytical truth, the L-truth, stating that a sentence is L-true if it holds in every state description. In this framework, intensions are functions on states, while extensions are relative to a single state of affairs. Subsequent researchers, such as Lewis (1986) [65] and Kripke (1980) [60], have expanded upon this, elaborating on modern possible world semantics.

To introduce an intensionalization of our λ-calculus, we cannot avoid some preparatory concepts from David Lewis' work on the notion of 'Possible worlds'. By the way, this is not the place to discuss the metaphysics of possible worlds in any depth; therefore, we cite Lewis for accountability and proceed with a very concise explanation of his definition of a possible world. For Lewis, the actual world is the physical universe as it is, extended through space-time. Essentially, a possible world is a way that things might have been. The world we live in is our most prominent example of what a possible world is. Other possible worlds are no different in kind from our world. Our world does not have any special status that makes it different from other worlds. The only reason that makes this world actual is simply that it is the world that we happen to inhabit. Other worlds and their inhabitants are just as real as we are and in precisely the same sense. According to Lewis, among the many possible worlds, there are worlds described in fiction writing. There is, for example, a world in which Hans Castorp, a young man from Hamburg affected by tuberculosis, is seeking a cure in a sanatorium in Davos, high up in the Swiss Alps. Since this example is taken from the plot of 'Der Zauberberg' by Thomas Mann, the sentence "In the world of Der Zauberberg, Hans Castorp is curing his tuberculosis in a sanatorium in Davos, high up in the Swiss Alps" is true in a world w, if and only if the sentence "Hans Castorp is curing his tuberculosis in a sanatorium in Davos, high up in the Swiss Alps" is true in the world as it described in the novel by Thomas Mann. Now, we extend the discourse to how the concept has been applied in formal semantics. In formal semantics, possible worlds are used to deal with opaque contexts, namely contexts in which substitution of extensional equivalents does not hold. We define the substitution of extensional equivalents as follows: let $A[\_]$ be a linguistic context, an expression with an open slot for a constituent. If b and c have the same denotation, so do $A[b]$ and $A[c]$. When A is a sentence, we should be able to change a constituent by another with the same denotation does not change the truth value of A. Opaque contexts like temporal shifters, intensional verbs, modals, and conditionals displace statement evaluation from the actual world to another world. Taking intensional verbs as an example, possible worlds can help us deal with sentences like "Kurt suspects that Albert is a spy". We will soon see it in detail.

Saul Kripke cloaked his contributions against definite descriptions to the debate on linguistic meaning in uncertainty, stating that he did not sustain any theory about possible worlds or proper names. Kripke generalizes on ordinary uses of proper names and a notion of possible world not dissimilar to the one used by Lewis, but definitely taken with a grain of salt on the theoretical side. Kripke calls Russell's definite descriptions 'flaccid designators', expressions that change denotation depending on the possible world considered. On the other hand, he theorizes that proper names are something dichotomous to definite descriptions and calls them 'rigid designators', namely terms which refer to the same denotation in every possible world. In different possible worlds, the name "Albert Einstein" refers to the same person we all know. However, it is important to note that the name only identifies the person and not their contingent characteristics. So, in another world, Albert Einstein could be something other than a great physicist, such as a camel driver. Kripke justifies this by stating that every object has empirically knowable essential properties, human beings included.

Let us now expand on opaque contexts delving deeper into propositional attitudes verbs and Hintikka's analysis (1969) [53]. Expressions such as "believe", "know", "doubt", "expect", "suspect" and others are typically associated with propositional attitudes, which express relationships between individuals (the attitude holder) and propositions (meanings of sentences). The basic concept is that when we say Kurt believes that Albert is a spy, we are asserting that Kurt considers the proposition "Albert is a spy" to be true. It is important to note that for this statement to be true, it does not necessarily require that Albert is a spy. However, the question arises: in which possible worlds must Albert be a spy for it to be true that Kurt believes he is? We might use the phrase "in the world according to Kurt" as a guide and suggest that Kurt's belief is true if, in the world according to Kurt's beliefs, Albert is indeed a spy. However, this idea needs refinement to accommodate multiple possible worlds consistent with Kurt's beliefs and to link the truth conditions to contingent facts about the world under consideration. In other words, what Kurt believes may vary across different possible worlds. We say that Kurt believes a proposition if and only if that proposition is true in every possible world consistent with his beliefs. If there exists even one world consistent with his beliefs where the proposition is not true, then he entertains the possibility that the proposition may not be true. In such a scenario, we cannot assert that he genuinely believes the proposition.

Everything we said implies we need a device that evaluates λ-terms in a specific world. Previously, our inventory included a domain of entities and a pair of truth values, along with functions operated between entities, truth values, and functions thereof. To intensionalize the lambda calculus, we add possible worlds to our toolkit. Let us assume we have a set W, the set of all possible worlds. W is a vast space as there are numerous ways things could have been different from the way they currently are. Every world in the set has entities as its parts, and some may not exist in other possible worlds. Therefore, each possible world has its unique domain of entities. Nonetheless, we will utilize the grand union of all these world-specific domains of entities as D, the set of all possible individuals. Finally, we use the function I to assign denotations relative to a possible world.

We enrich λ-calculus as follows. We add a new basic type s, in addition to e and t. s is the type of possible worlds. Hence, we have new derived types

- functions from possible worlds to individuals $\rightsquigarrow \langle s, e\rangle$
- functions from possible worlds to truth-values $\rightsquigarrow \langle s, t\rangle$

and new rules

- Intensionalization: if $\phi : \tau$ then ${}^{\wedge}\phi : \langle s, \tau\rangle$
- Extensionalization: if $\phi : \langle s, t\rangle$ then ${}^{\vee}\phi : \tau$

Therefore, we define a model $M = \langle D, W, I\rangle$ with D standing for a non-empty set, namely the domain of individuals; W another non-empty set, the set of possible worlds; I the function that assigns to each constant $c : \tau$ a denotation relative to a possible world $w \in W : I(w, \phi) \in D_\tau$.

We assign to each expression $\phi : \tau$ a denotation $[\![\phi]\!]^{M,w,g} \in D_\tau$ relative to a model M, an assignment g and a possible world $w \in W$.

- Constants: $[\![c]\!]^{M,w,g} = I(w, c)$
- Variables: $[\![x]\!]^{M,w,g} = g(x)$
- Function application and abstraction: as previously stated
- Identity, connectives, quantification: as previously stated
- Intensionalization: if $\phi : \tau$ then $[\![\phi]\!]^{M,w,g} =$ the function $f : W \to D_\tau$ given by: w_x for $[\![\phi]\!]^{M,w_x,g}$
- Extensionalization: if $\phi : \langle s, \tau\rangle$ then $[\![{}^{\vee}\phi]\!]^{M,w,g} = [\![\phi]\!]^{M,w,g}$ for w
- Intension: if $\phi : \tau$ without free variables, then we map w to $[\![\phi]\!]$ and denote this map as $[\![\phi]\!]^{M}$
- Extension: if $\phi : \tau$ without free variables, then we have $[\![\phi]\!]^{M,w}$ in a world w

As an example, let $S : et$ stand for the predicate 'spy'. Then we have that $S : et$ denotes the set of spies in w and ${}^{\wedge}S : \langle s, et\rangle$ stands for the map associating the corresponding set of spies to a world. Let $a : e$ formalize the proper name "Albert". $S(a) : t$ denotes True if Albert is a spy in w, and False otherwise. $^\wedge S(a) : st$ denotes the function that maps a world to the truth value denoted by $S(a)$. This function is the proposition "Albert is a spy".

As we saw, in formal semantics we standardly accept that proper names are constants standing for individuals (Von Fintel, Heim 2011) [88]. When an intensionalizing λ-term encounters an intensionable one, which contains a proper name in it, what happens to the proper name itself? In intensionalized lambda calculus, proper names and other linguistic items like indexicals, have semantic values which do not differ from world to world. Let us supply an example. In "Kurt suspects that Albert is a spy" $\rightsquigarrow B(k,^S(a))$, the constant a, namely the proper name referring to Albert, must be considered a rigid designator as Kripke defined even if there is no specification about that. Logically, the proper name "Albert" denotes the specific person thought by Kurt when he formulates the sentence.

We purposely did not mention de re and de dicto ambiguities or other compositional translation issues because it is not our intent to discuss such things. This section provides a brief review of how meaning is handled in contemporary formal semantics, relevant to the discussion of these methodologies and concepts in Chapter 3, where intensionalization by possible worlds will be challenged due to some overlooked complications.

2.4 Syntactic structures

The syntax of a natural language is a complex topic. Therefore, we will provide a Chomskyan X-bar theory-based toy syntax for a small fragment of English (Chomsky 1993) [17]. This approach allows us to reach the incomprehension formalization without burning the candle at both ends.

Above all, we need to lay out the principle underneath comprehension in formal semantics theories, the principle of compositionality formulated by Gottlob Frege in "Sense and Denoting".

Let us make Frege speak for himself:

> *It is astonishing what language accomplishes. With a few syllables it expresses a countless number of thoughts, and even for a thought grasped for the first time by a human it provides a clothing in which it can be recognized by another to whom it is entirely new.*
>
> *This would not be possible if we could not distinguish parts in the thought that correspond to parts of the sentence, so that the construction of the sentence can be taken to mirror the construction of the thought* [...]

> *If we thus view thoughts as composed of simple parts and take these, in turn, to correspond to simple sentence-parts, we can understand how a few sentence-parts can go to make up a great multitude of sentences to which, in turn, there correspond a great multitude of thoughts.*
>
> *The question now arises how the construction of the thought proceeds, and by what means the parts are put together* [...]

In other words, the meaning of a complex expression is determined by the meanings of its constituents and the rules used to combine them.

Now, to describe our toy syntax we fix:

- a limited stock of basic building blocks for sentences, namely a lexicon.
- a set of production rules, useful to set up complex expressions.

Our lexicon contains the following syntactic categories, here already translated in λ-terms:

- Proper nouns, labelled PN, like *Kurt* or *Albert*.

  Albert $\rightsquigarrow$ a $(a : e)$

  Kurt $\rightsquigarrow$ k $(k : e)$

- Common nouns, labelled N, like *plumber* or *clown*.

  Plumber $\rightsquigarrow$ P $(P : et)$

  Clown $\rightsquigarrow$ C $(C : et)$

- Adjectives, labelled A, like *silly* or *tricky*.

  Silly $\rightsquigarrow$ S $(S : e)$

  Tricky $\rightsquigarrow$ T $(T : e)$

- Determiners, labelled D, like *the*, *a*, *some*, *no*, *every*.

Every $\rightsquigarrow \lambda Q_{et}\lambda P_{et}.\forall x(Q(x) \to P(x)) : \langle et, \langle et, t\rangle\rangle$

Some $\rightsquigarrow \lambda Q_{et}\lambda P_{et}.\exists x(Q(x) \wedge P(x)) : \langle et, \langle et, t\rangle\rangle$

No $\rightsquigarrow \lambda Q_{et}\lambda P_{et}.\neg\exists x(Q(x) \wedge P(x)) : \langle et, \langle et, t\rangle\rangle$

A $\rightsquigarrow \lambda Q_{et}\lambda P_{et}.\exists x(Q(x) \wedge P(x)) : \langle et, \langle et, t\rangle\rangle$ or $\lambda P_{et}.P \;\; (\lambda P_{et}.P : \langle et, et\rangle)$

The $\rightsquigarrow \lambda P_{et}.\iota x.P(x) : \langle et, e\rangle$

– Intransitive verbs, labelled IV, like *walks* or *dies*.

  Walks $\rightsquigarrow \; W \;\; (W : et)$

  Dies $\rightsquigarrow \; D \;\; (D : et)$

– Transitive verbs, labelled TV, like *loves* or *needs*.

  Loves $\rightsquigarrow \; L \;\; (L : eet)$

  Needs $\rightsquigarrow \; N \;\; (N : eet)$

– Copula, labelled C, *is*.

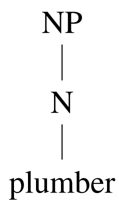
  is $\rightsquigarrow \; \lambda P_{et}.P \;\; (\lambda P_{et}.P : \langle et, et\rangle)$

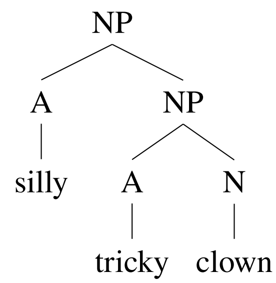
  Copula is a function that denotes a function to itself.

Regarding ”The”, we use the referential analysis approach; definite descriptions like ”The capital of Italy” are compound proper names. The idea is $[\![the\ C]\!] =$ the only object $r \in D$ such that $[\![C]\!](r) = T$. In other words, C is satisfied only by one individual, which in our case is c. We syntactically define ι ruling out: if $\phi : t$ and $x \in Var(e)$, then $\iota x.\phi : e$. The idea can be found in Frege.

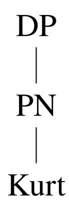
Back to us, we have the following set of production rules and syntactic structures:

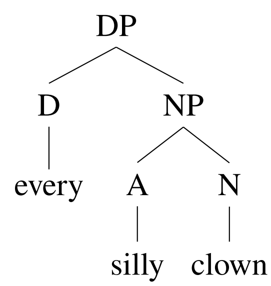
– Noun phrases, labelled NP, follow the rule: NP $\rightarrow$ N | A NP



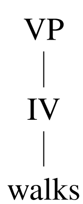


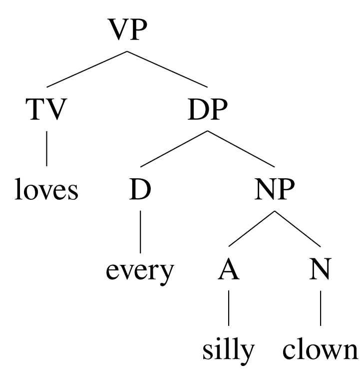
– Determiner phrases, labelled DP, follow the rule: DP $\rightarrow$ PN | D NP

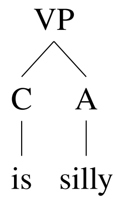


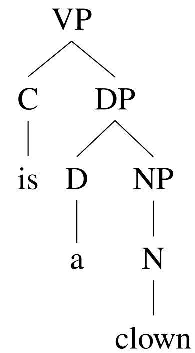


– Verb phrases, labelled VP, follow the rule: VP $\rightarrow$ IV | TV DP | C DP | C A

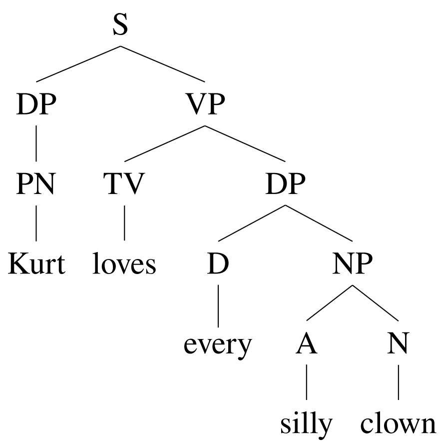


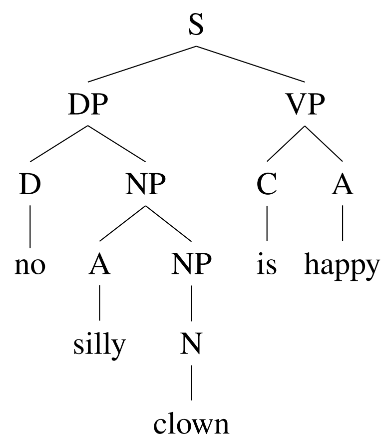






– Sentences, labelled S, follow the rule: S $\rightarrow$ DP VP





Finally, we are ready to define our compositional rules, which are fundamental to assigning a translation to every node in the syntactic trees. From this point forward, we will exclusively present the constituency structure, as the syntactic labels will be irrelevant to our composition. We refer to the end of a branch as a leaf, and each leaf corresponds to the translation of a lexical item.

– Compositional rule 0: leaves

If A is a terminal node associated with a lexical entry P and P $\rightsquigarrow a$, then A $\rightsquigarrow a$.

We assume that non-branching nodes simply inherit the translation of their daughter node.

– Compositional rule 1: non-branching nodes

If A is a node with B as its only daughter and $B \rightsquigarrow a$, then also $A \rightsquigarrow a$.

When a node branches into two daughters its denotation derives from the ”saturation” the denotation of a node with the denotation of the other.

– Compositional rule 2: function application

  If A is a node with two daughters B and C, in either order, and $B \rightsquigarrow \beta : \langle \sigma, \tau \rangle$, and $C \rightsquigarrow \gamma : \langle \sigma \rangle$, then $A \rightsquigarrow \beta(\gamma) : \tau$.

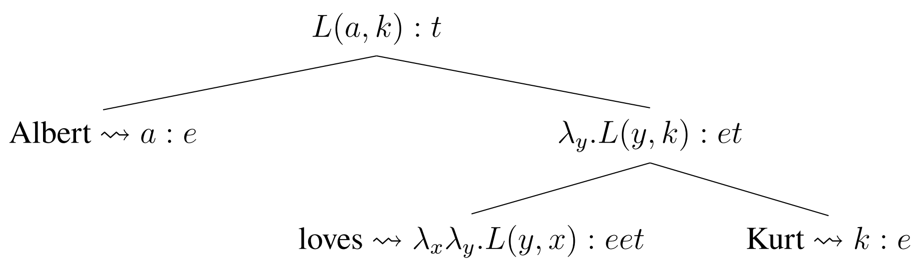
The last composition rule we need is the one concerning two nodes which are sisters in the tree, but neither one denotes a function that has the denotation of the other in its domain, so Function Application cannot be used to combine them.

– Compositional rule 3: predicate modification

  If A is a node with two daughters B and C, and B $\rightsquigarrow \beta : et$, C $\rightsquigarrow \gamma : et$, then A $\rightsquigarrow \lambda x.\beta(x) \wedge \gamma(x) : et$.

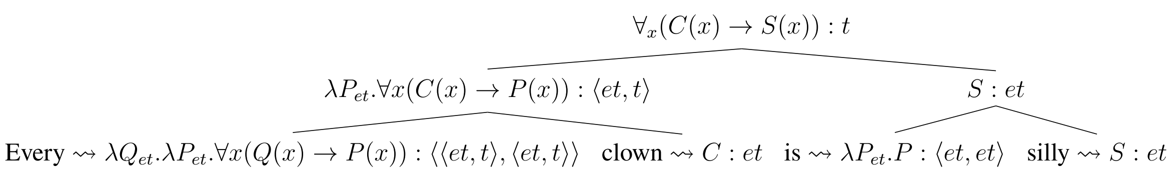
In conclusion, some examples of application:

– Albert loves Kurt



  Calculus: $(\lambda_x \lambda_y.L(y,x))_{eet}(k)_e(a)_e \rightarrow_\beta (\lambda_y.L(y,k))(a)_e \rightarrow_\beta L(a,k) : t$

– Every clown is silly



  Calculus: $(\lambda Q \lambda P.\forall_x(Q(x) \rightarrow P(x)))_{\langle \langle et,t \rangle, \langle et,t \rangle \rangle}(C)_{et}((\lambda P.P)_{\langle et,et \rangle}(S)_{et}) \rightarrow_\beta (\lambda Q \lambda P.\forall_x(Q(x) \rightarrow P(x)))_{\langle \langle et,t \rangle, \langle et,t \rangle \rangle}(C)_{et}(S)_{et} \rightarrow_\beta (\lambda P.\forall_x(C(x) \rightarrow P(x)))_{\langle et,t \rangle}(S)_{et} \rightarrow_\beta \forall_x(C(x) \rightarrow S(x)) : t$

– Some clown is silly

$$\exists_x(C(x) \rightarrow S(x)) : t$$

$$\lambda P_{et}.\exists x(C(x) \rightarrow P(x)) : \langle et, t\rangle \qquad S : et$$

Some $\rightsquigarrow \lambda Q_{et}.\lambda P_{et}.\exists x(Q(x) \rightarrow P(x)) : \langle\langle et, t\rangle, \langle et, t\rangle\rangle$ clown $\rightsquigarrow C : et$ is $\rightsquigarrow \lambda P_{et}.P : \langle et, et\rangle$ silly $\rightsquigarrow S : et$

Calculus: $(\lambda Q\lambda P.\exists_x(Q(x) \rightarrow P(x)))_{\langle\langle et,t\rangle,\langle et,t\rangle\rangle}(C)_{et}((\lambda P.P)_{\langle et,et\rangle}(S)_{et}) \rightarrow_\beta (\lambda Q\lambda P.\exists_x(Q(x) \rightarrow P(x)))_{\langle\langle et,t\rangle,\langle et,t\rangle\rangle}(C)_{et}(S)_{et} \rightarrow_\beta (\lambda P.\exists_x(C(x) \rightarrow P(x)))_{\langle et,t\rangle}(S)_{et} \rightarrow_\beta \exists_x(C(x) \rightarrow S(x)) : t$

– No clown is silly

$$\neg\exists_x(C(x) \rightarrow S(x)) : t$$

$$\lambda P_{et}.\neg\exists x(C(x) \rightarrow P(x)) : \langle et, t\rangle \qquad S : et$$

No $\rightsquigarrow \lambda Q_{et}.\lambda P_{et}.\neg\exists x(Q(x) \rightarrow P(x)) : \langle\langle et, t\rangle, \langle et, t\rangle\rangle$ clown $\rightsquigarrow C : et$ is $\rightsquigarrow \lambda P_{et}.P : \langle et, et\rangle$ silly $\rightsquigarrow S : et$

Calculus: $(\lambda Q\lambda P.\neg\exists_x(Q(x) \rightarrow P(x)))_{\langle\langle et,t\rangle,\langle et,t\rangle\rangle}(C)_{et}((\lambda P.P)_{\langle et,et\rangle}(S)_{et}) \rightarrow_\beta (\lambda Q\lambda P.\neg\exists_x(Q(x) \rightarrow P(x)))_{\langle\langle et,t\rangle,\langle et,t\rangle\rangle}(C)_{et}(S)_{et} \rightarrow_\beta (\lambda P.\neg\exists_x(C(x) \rightarrow P(x)))_{\langle et,t\rangle}(S)_{et} \rightarrow_\beta \neg\exists_x(C(x) \rightarrow S(x)) : t$

– The clown is silly

$$S(\iota x.C(x)) : t$$

$$\iota x.C(x) : e \qquad S : et$$

The $\rightsquigarrow \lambda P_{et}.\iota x.P(x) : \langle et, e\rangle$ clown $\rightsquigarrow C : et$ is $\rightsquigarrow \lambda P_{et}.P : \langle et, et\rangle$ silly $\rightsquigarrow S : et$

Calculus: $(\lambda P\iota x.P(x))_{\langle et,e\rangle}(C)_{et}((\lambda P.P)_{\langle et,et\rangle}(S)_{et}) \rightarrow_\beta (\lambda P\iota x.P(x))_{\langle et,e\rangle}(C)_{et}(S)_{et} \rightarrow_\beta (\iota x.C(x))_e(S)_{et} \rightarrow_\beta S(\iota x.C(x)) : t$

2.5 Quantum Mechanics

Natural language words, and therefore sentences, are multifunctional due to inherent ambiguity in language, so frequently used words have wide distributions of potential meanings. We can narrow down their intended meaning only when they are used in a specific context. However, a reader or listener puts it in his, or her, personal context, which can significantly change the meaning. Traditional linguistics has focused on qualitative explanatory models to explain this sense-making process for decades. We think quantum theory is better suited to capture the inherently human aspects of cognition (Busemeyer, Bruza 2012) [15], (Aerts, Gabora and Sozzo 2013) [1], (Aerts, Sozzo and Veloz 2015) [2] and language understanding (Bruza, Kitto and McEvoy 2008) [13], (Bruza, Kitto, Nelson, McEvoy 2009) [14], (Wang et al. 2016) [90]; looking at some of the latest analyses makes it crystal clear (Surov et al. 2021) [82].

Nowadays, quantum modelling in cognitive science, linguistics (Heunen, Sadrzadeh and Grefenstette 2013) [51] and machine intelligence-related fields (Wang, Melucci and Song 2019) [89] have become more and more popular while still being in a formation state. Here, we will use quantum theory both as a base for the formalization of incomprehension and a validation for biological clues grounding the formalization itself.

Whenever we venture beyond the familiar parameters of ordinary experience, we inevitably encounter concepts that are challenging to visualize. The formulation of a thought, or the transmission of a word-thought from one individual to another, falls into this category. Abstract mathematics becomes necessary to describe such phenomena. We will outline the quantum borrowings required to solve the incomprehension puzzle and explain why we need them.

2.5.1 Wave particle duality

Our tale begins with photons. In 1905, Einstein [31] suggested that energy in a light beam comes in quanta, which are the smallest physical units of energy, to explain the peculiarities of the photoelectric effect. This implies that a light beam is made up of discrete packets of energy. Essentially, Einstein hypothesized that light is composed of particles, each carrying a fixed amount of energy, and he was initially very uneasy with this idea, as it implied that light could behave both as a wave and a particle. He knew that this concept could shatter classical physics and demand an entirely new physical theory. Though he was never a fan of quantum mechanics, his theory on particles of light, later named photons, assisted in developing this new theory. It took physicists until 1925 to acknowledge that light could behave like a particle.

Most sceptics were eventually convinced by the experiments of Arthur Compton in 1923 [19]. Nowadays, scientists around the world routinely manipulate photons in their labs. Visible light photons carry very little energy, and a small laser pulse can consist of billions of them. When we say that light behaves like a particle, we mean a quantum mechanical particle, which is an indivisible unit of energy and momentum.

So, light has a dual nature; it can act both as a particle and as a wave. This means that it can reflect off a mirror like a particle and it can bend and spread out like a wave when it passes through a prism. We can observe how light waves interfere with each other through carefully conducted experiments. Surprisingly, also particles can behave like waves. In quantum mechanics, everything has a wave nature, including light, particles, and even macroscopic objects like tennis balls and people, albeit in an unobservable way.

One way to observe the wave nature of light is through diffraction, which is when light bends around the sharp edges of objects or creates patterns when passing through narrow gaps. The double-slit experiment in modern physics demonstrates that light and particles act as both waves and particles. This experiment was first performed in 1801 by Thomas Young [94] to demonstrate the wave behaviour of visible light.

In 1927, Davisson and Germer [28], and independently George Paget Thomson and his research student Alexander Reid (Navarro 2010) [69], demonstrated that electrons also exhibit the same behaviour, which was later extended to atoms and molecules. In this experiment, laser light passes through two slits in a wall and spreads out like a wave as it passes through. The waves coming from each slit interfere with each other, creating a pattern of light and dark spots on a wall on the other side.

This happens because when two crests or two troughs of a wave meet, they add together, but when a crest and a trough meet, they cancel out. If we have a powerful electron beam, we can make the same experiment by throwing electrons at the wall. Doing so, we can see that instead of two spots directly behind the slits or randomly distributed spots, an interference pattern will appear just like with light. This indicates that electrons behave like waves and interfere with each other.

Furthermore, if we slow down the experiment and send one electron at a time, each electron will pass through and hit the wall at a single point. Over time, we will observe that the same interference pattern appears, even though no other electrons are passing through at the time to create interference. This suggests that each electron interferes with itself. In quantum theory, even something we think of as an individual object, like an electron, has an essential wavelike nature and acts as a wave when it is moving through space. It is worth noting that the particle nature only appears when we make a measurement, and we will explain it shortly.

2.5.2 Linearity

Linearity is a fundamental aspect of quantum mechanics. In any physical theory, we have a set of equations of motion (EOM) that describe the behaviour of a physical system. These equations are expressed as mathematical functions in terms of dynamic variables, which are the variables that we seek to determine because they are associated with observations. The values of dynamic variables, or values derived from them, can be compared to the outcomes of an experiment to evaluate the accuracy of the theory.

A linear theory has multiple solutions. Specifically, if x and y are both solutions of the equations of motion of a system, then we can form a third solution, which is the combination of x and y. This means that if we have two solutions to the equations of motion, we can combine them to create a new solution.

Maxwell's theory of electromagnetism (1865) [66] is one of the most well-known examples of linear theories. Essentially, the theory relies on a set of data $(\vec{E}, \vec{B}, \rho, \vec{J})$ that is valued as a solution only if it satisfies Maxwell's equations, namely a set of equations for the electromagnetic field, charged density, and current density. The vector $\vec{E}$ stands for 'electric field', $\vec{B}$ represents magnetic field, ρ stands for charge density, and $\vec{J}$ represents current density.

Linearity refers to the property of the system where if we multiply each member of our supposed solution set by a real number α, then $(\alpha\vec{E}, \alpha\vec{B}, \alpha\rho, \alpha\vec{J})$ is also a solution, with $\alpha\vec{E}$ representing the new electromagnetic field, $\alpha\vec{B}$ as the new magnetic field, and so on. Additionally, linearity implies that if we have two solutions $(\vec{E}_1, \vec{B}_1, \rho_1, \vec{J}_1)$ and $(\vec{E}_2, \vec{B}_2, \rho_2, \vec{J}_2)$, then their sum $(\vec{E}_1 + \vec{E}_2, \vec{B}_1 + \vec{B}_2, \rho_1 + \rho_2, \vec{J}_1 + \vec{J}_2)$ is also a valid solution. In other words, if we have one solution, we can scale it by a number while still retaining its validity as a solution and, if we have two solutions, the result of their combination is also a solution.

It is necessary to clarify exactly what is meant by 'linear equation'. Let us try to get across the concept avoiding unnecessary details. A linear equation can be loosely described as an equation of the form $L(u) = 0$ where the term L is defined as a 'Linear operator' and u is generally understood to mean an 'unknown element'. While u is commonly a vector in the field of physics, it can be everything needed. For L to be a linear operator, it must satisfy the conditions $L(\alpha u) = \alpha L(u)$ and $L(u_1 + u_2) = L(u_1) + L(u_2)$. Consequently, $L(\alpha u_1 + \beta u_2) = L(\alpha u_{)}) + L(\beta u_2) = \alpha L(u_1) + \beta L(u_2)$. If u_1 and u_2 are solutions, meaning $L(u_1) = L(u_2) = 0$, then $\alpha u_1 + \beta u_2$ is also a solution. This can be proven by showing that if $L(u_1) = 0$ and $L(u_2) = 0$, then $L(\alpha u_1 + \beta u_2) = 0$.

2.5.3 Qubits

Let us introduce some other fundamental concepts. A system comprises various states, representing configurations in which the system can be. We define a state space as the set of all possible configurations that the system can be in. Think about one of the simplest systems available: the toss of a coin. The only elements belonging to the system are the coin and the state space of the system, namely the set $\{H, T\}$ with 'H' standing for 'Head' and 'T' standing for 'Tail'. Replacing the coin with the 'Qubit' and renaming H and T respectively as 1 and -1, makes our 'coin-system' become the basic quantum system. Exactly like the binary bit is the fundamental unit of information in classical or traditional computing, a qubit, or quantum bit, is the fundamental unit of information in quantum theory. Therefore, a qubit is a system which can be in two states $\{1, -1\}$; it means that when a qubit is measured is either 1 or -1. We can also describe it with a mathematical degree of freedom ω. So ω can be $\omega = 1$ or $\omega = -1$. We can also symbolically describe it as $\uparrow$ and $\downarrow$.

To recap:

$$H \equiv (\omega = 1) \equiv \uparrow$$

$$T \equiv (\omega = -1) \equiv \downarrow$$

Notice we are only outlining a notation for abstract concepts, with no implications about geometrical or physical instances. To proceed with our explanation, we have to imagine an experiment. It involves the qubit system and another system called 'Apparatus'. The Apparatus is an abstract machine that enables us to conduct measurements on qubits; we are not going to describe how the detector works, it is not relevant to our experiment, we simply note that it is an up-oriented black box with a screen and a wire. When a qubit is connected to the wire, the Apparatus measures it and report the measurement result on the screen; it senses if a qubit is $\uparrow$ (1) or $\downarrow$ (-1).

Our experiment is to determine what state a qubit is in. When we start the experiment we don't know the qubit state and the Apparatus is in a neutral state, or a blank state. We measure the qubit and on the detector's screen appears the result of the measurement. Let us suppose that the result is 1. The first time we measure a qubit we say that the measurement prepared the qubit in a state, namely the result of the measurement. After that, we quickly reset the apparatus, repeat the measurement and obtain a result of 1 once more. This is crucial: the illustrated experiment can be confirmed.

Now let us do a new experiment. We take a qubit and we measure/prepare it; the result of this first measurement is 1. After a few more measurements, consistently yielding a result of 1, we are now confident that the state of the qubit is 1. As a next step we turn over the Apparatus, repeat the measurement and obtain -1 as a result. It appears that the qubit exhibits a sense of directionality in space and, due to this, we might suspect that it is a vector. We might also think that the Apparatus has a sense of directionality too. Thus, moving the machine allows us to measure the value of the component of the qubit-vector along the direction of the Apparatus itself.

By placing the apparatus on its side, if the qubit is a classical vector, the horizontal component of the qubit-vector should be 0. However, after preparing the qubit with $\omega = 1$, when we measure it with the apparatus placed on its side, the measurement result is not 0 but 1. Therefore, the qubit-vector is not classical.

Let us summarize:

> We prepare the qubit with an up-oriented Apparatus. We note $\omega = 1$ and repeat the measurement.
>
> If the Apparatus is up-oriented (z-axis), $\omega = 1$.
>
> If the Apparatus is down-oriented (z-axis), $\omega = -1$.
>
> If the Apparatus is side-oriented (90° angle; x-axis), $\omega = 1$.

Suppose we iterate the operation numerous times, consistently following the same procedure: beginning with A along the z-axis, prepare $\omega = 1$; rotate the apparatus so that it is oriented along the x-axis; measure ω. The repeated experiment produces a random sequence of plus-ones and minus-ones. Through multiple repetitions, we observe that the occurrences of $\omega = 1$ and $\omega = -1$ events are statistically equal. In other words, the average value of ω is zero. Instead of yielding the classical result, where the component of ω along the x-axis is zero, we discover that the average of these repeated measurements is zero.

If we repeat the entire process rotating the Apparatus towards any arbitrary direction along the unit vector n̂, we get $\omega = \pm 1$ instead of the classical result $\omega = \cos\theta$ (with θ being the angle between n̂ and z). Nevertheless, the average value is $\cos\theta$. The situation is obviously more general and valid for every orientation.

Through these experiments, we aim to emphasize that quantum mechanical systems lack determinism, resulting in statistically random outcomes. However, with repeated experiments, average quantities may approximate the predictions of classical physics, to some extent.

Turning our attention to ”measurement,” another essential clarification is needed. In both classical physics and quantum mechanics, the use of an apparatus to record the outcome of an experiment unavoidably alters the measured system. In classical physics, this modification is considered trivial. However, in quantum mechanics, any interaction that is strong enough to measure one aspect of a system is inherently strong enough to disturb some other aspect of the same system. As a result, it becomes impossible to extract information about a quantum system without simultaneously influencing something else. In practice: if we measure a qubit with the Apparatus oriented upwards, record the result, and then repeat the measurement with the Apparatus on its side, when we reposition the Apparatus upwards, we will not obtain the same result as in the initial measurement. we will better explain this phenomenon in 2.5.6.

Time to unveil what is going on here. Qubits are the most basic quantum systems and they can be in both states 1 and -1 simultaneously. In the earlier part of this section, we described them abstractly. Now, we will consider qubits as spins. A spin is an intrinsic property of elementary particles, and thus of composite particles, that makes the particle behave as we observed in the experiment with the qubits and the apparatus. This property states that particles possess angular momentum irrespective of their rotation around other particles. Electrons are elementary particles with negative electric charge. They are fundamental constituents of atoms and play a crucial role in determining the electronic structure and chemical behaviour of matter. Electrons behave like qubits due to their spin.

2.5.4 Quantum Logic

It is imperative to bear in mind that the logic behind quantum systems differs significantly from that of classical systems. It is crucial to recognize these differences to gain a better understanding of quantum systems. In this regard, we will begin by reminding the differences between the inclusive and exclusive disjunctive operator, alongside elements of set theory. The space of states of a classical system is a set of possible states. We can define a proposition A as a subset of the state space; namely the subset of elements for which the proposition A turns out to be True. $\neg A$ refers to the subset of elements not included in the subset A. Adapting what was said to our qubit is not difficult. Since there are only two states, 1 and -1, ”the state is 1” represents the subset of state 1, while the negation of this proposition represents the subset of state -1. We observe distinct differences between classical and quantum logic in combining statements with $\wedge$ and $\vee$. In classical logic, $A \wedge B$ represents the intersection of subsets A and B; the intersection $A \cap B$ refers to the subset where the statement $A \wedge B$ is true. As we previously said: if ϕ and ψ are well-formed strings, then $[\![\phi \wedge \psi]\!]^I = T$ if $[\![\phi]\!]^I = T$ and $[\![\psi]\!]^I = T$, and F otherwise.

We want to remind that propositional logic employs a specific type of $\vee$ called the ’inclusive’

$\vee$. To clarify its nature it is better to first explain its counterpart, the 'exclusive' $\vee$. The 'exclusive' $\vee$ finds its way into our everyday language. For instance, when we say "I am either poor or rich," it implies that we cannot be both at the same time. In other words, these conditions cannot coexist. In classical logic, $\vee$, as brought out before, observe the following rule: If ϕ and ψ are well-formed strings, then $[\![\phi \vee \psi]\!]^I = T$ if $[\![\phi]\!]^I = T$ or $[\![\psi]\!]^I = T$ (or both), and F otherwise. So, we can surprisingly affirm that the statement 'I am either poor or rich' is true, even if both 'I am poor' and 'I am rich' are true. This may seem counterintuitive, but it is the case. The inclusive $\vee$ can also be defined as the union of two subsets $A \cup B$. Now, let us apply the concepts we just outlined to create "and" and "or" statements regarding qubits. We aim to comprehend how to verify them and what it means to do so. Let A be the proposition $\omega_z = 1$ and B be the proposition $\omega_x = 1$. We can determine if A is True or False by measuring the qubit after turning the Apparatus upward, and B by turning it on its side. Before we delve into the experiment to shed light on the correct evaluation of these statements in quantum mechanics, let's analyze them in a classical framework.

Let's imagine that an experimental physicist's ghost lives in our laboratory. Without our knowledge, this ghost prepared the qubit with $\omega_x = 1$. It is worth noting that the ghost is just an escamotage to speed up the test; his actions are only relevant to that. If we measure ω_z first, we can determine whether $A \vee B$ is True, under these circumstances: when we measure the qubit using the Apparatus pointing upwards, we obtain a value of $\omega_z = 1$, and we get it because of our ghost. We can halt the experiment since we are using an inclusive $\vee$ we already know that $A \vee B$ is True. If we had measured B first, it wouldn't have affected the outcome. This is because, in the classical framework, a measurement doesn't exert any influence on subsequent measurements.

When we adopt a Quantum Mechanics framework, we discover that the truth of $A \vee B$ can only be defined based on the order in which we determine A or B. To clarify, the sequence of determining A or B is critical to defining the truth of the statement.

Another fundamental difference between quantum logic and classical logic is the failure of the propositional distributive law in the former. In classical logic, as we know, the distributive law is a fundamental principle that states that for any propositions P, Q, and R, $(P \wedge Q) \vee R$ is logically equivalent to $(P \vee R) \wedge (Q \vee R)$. This law is a key component of many logical inferences in classical logic. However, in quantum logic, the distributive law does not hold in all cases. Specifically, there are certain cases where $(P \vee Q) \wedge R$ cannot be expressed as $(P \vee R) \wedge (Q \vee R)$. This is because, in quantum logic, the logical connectives are not only truth-functional, meaning that the truth value of a compound proposition depends on more than just the truth values of its components; we already noted that with the order of measurements.

2.5.5 Superposition

If you have heard about quantum mechanics, you may be familiar with Schrödinger's cat, a famous thought experiment in which a cat is believed to be both alive and dead simultaneously, which is known as a quantum superposition. The concept of Schrödinger's cat relates to a real phenomenon in quantum mechanics, where a particle can appear to be doing two contradictory things at the same time. The double-slit experiment, discussed in section 2.5.1, is an excellent example of this. In this experiment, an electron appears to pass through two slits in a screen at the same time. We can explain this by describing the electron as a wave, and we can also say that the electron was in a superposition of all the possible paths. In fact, if we add up all those paths, we have something that looks like a wave. Superpositions are just a way of talking about quantum uncertainty, which is a fundamental concept that we will clarify later. In quantum mechanics, we cannot know any property of a particle with 100% certainty. However, as we saw in section 2.5.2, we can observe or measure a particle in superposition. When we do this, we see the electron from the double-split experiment splashing against the wall in a single spot. So it is like, if we measure a particle, we "destroy" its superposition state. We can say that any isolated quantum system is always in a superposition until the moment it interacts with the world outside, namely, until the moment it is measured. If we conduct an experiment inside a sealed box involving quantum uncertainty, the result is in a superposition until the box is opened and observed. Schrödinger and Einstein discussed this idea and came up with the famous example of an experiment involving a cat inside a box. They did this to demonstrate the absurdity of the superposition concept, as both were sceptical about it.

In this experiment, a perfectly healthy and hungry cat is placed in a sealed box with a radioactive atom that has an equal probability of both decaying and not decaying between the beginning and end of the experiment. If the atom decays, it triggers a Geiger counter inside the box, which is attached to a machine that pulls out cat kibbles if activated. I edited the details of the experiment because I am very against animal violence even if I am allergic to cats. If we do not open the box, we do not know the state of the atom, so it is in a superposition of intact and decayed. This means the Geiger counter is both triggered and not, which means the machine is both off and on, and the cat's belly is both empty and full of kibbles. Opening the box is the only thing that settles the cat in one state or another. The cat is both hungry and not. Schrödinger and Einstein considered this to be complete nonsense. Just think about the cat himself being an observer for the purpose of forcing the particle into a single state, as is, for that matter, the Geiger counter and the kibble machine. How quantum states transition from superpositions to something more definite is still an open question, as is why particles can be in superpositions, but large objects like animals cannot. Schrödinger's cat paradox brings to light the mysterious aspects of quantum theory.

As was mentioned in the previous subsection regarding linearity, the property allows us to accept the sum of two solutions representing physical realities as a new and valid solution. What we described through the measurements of qubits/spins with the apparatus is an example of a quantum superposition measurement. Linearity in quantum mechanics guarantees that adding states is a sensible thing to do. By writing our physical state as sums of other states, we can learn about the properties of our state. Let us be completely clear. When we are not measuring a qubit, it is in a superposition, which is the combination of the two possible states of the measurement, 1 and -1. As said in the previous subsection 2.5.2, qubits and spins have the same behaviour. Let us measure the spin of an electron along the z-axis. The spin can be either 1 or -1 when measured. Assuming that we are dealing with vectors and using Paul Dirac's notation we can write $|1; z\rangle$ and $|-1; z\rangle$ as the only two possible results of the measurement or states of the system. If $|1; z\rangle$ and $|-1; z\rangle$ are possible quantum states, thanks to linearity, we can build a new quantum state which would be the following superposition: $|\psi\rangle = |1; z\rangle + |-1; z\rangle$. Our electron is in the superposition state $|\psi\rangle = |1; z\rangle + |-1; z\rangle$ and that represent two possible outcomes with equal probabilities when measured. To estimate probabilities, an experiment must be repeated multiple times. Imagine a large group of electrons, all in the same state $|\psi\rangle = |1; z\rangle + |-1; z\rangle$, exactly as we said for the qubits experiment. As we measure their spin along z, one at a time, we find about half of them spinning up along z and the other half spinning down along z. We cannot predict which option will be realized for a particular electron until we measure it. It is difficult to imagine the state of the electron in superposition, but one may try as follows. An electron in this state is simultaneously able to be spinning up along z and spinning down along z. So, once measured the electron must immediately "choose" one of the two options, and we always find electrons either spinning up or spinning down.

2.5.6 Uncertainty Principle

In this subsection, we discuss uncertainty, an essential property of the quantum world.

Suppose Kurt, Albert and Eldrick start playing golf. Albert hits the ball with his golf club and scores a hole-in-one. Kurt, with his eyes wide open and filled with incredulity, immediately asks his friend how he made it. Alberts brings out his pocket a sheet full of calculations, saying that if he does precise measurements of the direction of the force, the shape of the ball, the air currents of the golf course etc. it is easy to do a masterstroke. After hearing Albert's words, Eldrick dramatically walks a little bit in the direction of the hole, collapses to the ground and starts crying, screaming that his entire life is a lie. Kurts slowly approaches his emotionally destroyed friend and then he whispers to him "He is just a lucky guy, his calculi are wrong. He did not count the hidden variables". Wink.

Before the development of quantum mechanics, scientists thought that the laws of physics governing our universe were deterministic, in other words, entirely predictable. When quantum mechanics knocked at their doors, everything changed. Uncertainty makes quantum mechanics fundamentally different from classical physics. In quantum mechanics we can still make predictions, but, even if we start with perfectly clean and complete data, those predictions are always going to have some uncertainty built in. This quantum fuzziness is well explained in Heisenberg's uncertainty principle [50].

According to the uncertainty principle, it is impossible to determine both the exact position and velocity of a particle, such as an electron or a photon, with complete accuracy. The more precisely we determine the particle position, the less we can know about its speed, and vice versa. In other words, if we know where something is, we cannot perfectly know how fast it is moving, and backwards. Recalling observations in 2.5.3, one way to think about the uncertainty principle is as a measurement problem. Every time we measure a qubit via apparatus, we are interacting with the qubit and inevitably changing the properties of the qubit. Consider the measuring of a particle which does not emit visible light, namely light that our eyes cannot detect. If the particle is not emitting visible light we measure it by first bouncing light off it, and when we do that, the tiny bit of light we use gives the particle a tiny bit of energy and thus changes its momentum, its mass times its speed. No matter how many methods we try, there is no way to measure the properties of an observable without interacting with it, which changes its properties. Another way to think about uncertainty is as a consequence of the fact that the particles we are trying to measure act like waves, which, by their nature, are not something that can exist in a single point. Take the simplest kind of wave, a sine wave. It is not localized and extends infinitely in both directions. While it has a well-defined frequency that is related to its momentum, it lacks a specific position. If we want to have the position of a wave we need to combine waves of various frequencies that add together in some place and cancel out in others. Only then we can create a wave packet that exists in a small region. Furthermore, we can make the region smaller by adding more and more waves of different frequencies. However, the more we try to squeeze the position to a single point, the more the momentum spreads out because we are adding more frequencies.

Einstein hated this idea and he was quoted as saying that God does not play dice with the universe and there is no uncertainty but only hidden variables that we are not yet able to measure.

2.5.7 Vector spaces

The set of possible states represents the space of states for a classical system, and the logic of classical physics follows Boolean principles, which seems like a natural choice with no other alternative in sight. However, we found out that our world functions quite differently. In quantum mechanics, the space of states for a quantum system is not a regular mathematical set, but rather a vector space. The relationships between the constituents of a vector space, namely vectors, differ substantially from those between the elements of a set, and the logic of propositions is significantly different. In this section, we describe vector spaces. We define a vector as an abstract mathematical item we can characterize as needed, and as the component of a vector space.

- Let Ω be a vector space.
- If $\phi \in \Omega$ then ϕ is a vector. Following Paul Dirac's notation [29], we write ϕ as $|\phi\rangle$, and we name it "ket-vector".

The following are the axioms we will use to define the vector space of states of a quantum system (z and w are complex numbers):

- Ket-vectors are summable. The result of the sum is a ket vector.

  $|A\rangle + |B\rangle = |C\rangle$

- Ket-vectors sum is commutative.

  $|A\rangle + |B\rangle = |B\rangle + |A\rangle$

- ket-vectors sum is associative.

  $\{|A\rangle + |B\rangle\} + |C\rangle = |A\rangle + \{|B\rangle + |C\rangle\}$

- There is a unique vector 0 such that adding it to any ket-vector results in the same ket-vector.

  $|A\rangle + 0 = |A\rangle$

- Given any ket-vector $|A\rangle$, there is a unique ket-vector $(-|A\rangle)$ such that

  $|A\rangle + (-|A\rangle) = 0$

- Multiplying any ket-vector $|A\rangle$ and any complex number z will result in a new ket-vector. Furthermore, it is important to note that the process of scalar multiplication is linear.

  $|zA\rangle = z|A\rangle = |B\rangle$

- When performing ket-vector multiplication by complex numbers, the distributive property holds true.

  $z\{|A\rangle + |B\rangle\} = z|A\rangle + z|B\rangle$

  $\{z + w\}|A\rangle = z|A\rangle + w|A\rangle$

It is important to provide a brief explanation of complex numbers after invoking them. This will help us to clarify what has been discussed and to proceed with what comes next. We will use a straightforward characterization.

Imaginary numbers are a mathematical concept that expands the real number system to include the square root of negative numbers. The imaginary unit, represented by the symbol i, is defined as the square root of -1. Mathematically, $i^2 = -1$.

By utilizing the imaginary unit i, we can form complex numbers, which have both a real part and an imaginary part. A complex number is expressed in the form $a + ib$, where a represents the real part, b represents the imaginary part, and i represents the imaginary unit.

We denote the set of complex numbers by $\mathbb{C}$, and it includes both real numbers (where the imaginary part is 0) and purely imaginary numbers (where the real part is 0).

Now, let us discuss complex conjugates because understanding them is vital for anyone dealing with complex numbers. We can obtain a complex conjugate z^* of a complex number z by simply changing the sign of the imaginary part while leaving the real part untouched. The notation $a - ib$ represents the complex conjugate of $a + ib$.

A short wrap-up:

> A complex number z is typically written in the form $z = a + ib \in \mathbb{C}$
>
> where a and b are real numbers ($a, b \in \mathbb{R}$)
>
> The real part of z is a. Hence, we write $Re(z) = a$
>
> The imaginary part of z is b. Hence, we write $Im(z) = b$. Note that $b \in \mathbb{R}$ but $Re(ib) = 0$ and $Im(ib) \neq 0$.

The complex conjugate of z is $z^* = a - ib$.

We need to report two important and useful identities: we define the norm of a complex number as the square root of $a^2 + b^2$.

$\|\mathbf{z}\| = \sqrt{a^2 + b^2}$, so $\|\mathbf{z}\|^2 = a^2 + b^2$ which is actually euqal to $z \cdot z^*$

Therefore $\|\mathbf{z}\| \in \mathbb{R}$

Consider an angle θ subtended at a unit radius of 1 in a complex plane. We define a complex plane as a two-dimensional plane consisting of complex numbers, which are represented using a Cartesian coordinate system. The horizontal axis on this plane, known as the real axis, represents real numbers, while the vertical axis, known as the imaginary axis, represents imaginary numbers.

We know that $Re(z) = \cos\theta$ and $Im(z) = \sin\theta$ so $z = \cos\theta + i \cdot \sin\theta$ which is equal to $e^{i\cdot\theta}$.

In classical physics, complex numbers are often employed as auxiliary tools, providing mathematical conveniences in some calculations. However, they are not inherently relevant to observables (electric fields, positions, velocities etc.) because they are real quantities. On the other hand, in quantum mechanics, observables are represented by complex numbers. To extract meaningful information from these complex numbers, we perform calculations involving their squared norms, which are proportional to probabilities (Born 1926) [12].

We can now go back to our vectors. As we have seen complex numbers exhibit a dual representation, the complex conjugate numbers. Similarly, a complex vector space possesses a dual counterpart, the complex conjugate vector space. Each ket-vector $|A\rangle$ corresponds to a "bra"-vector in the dual space, denoted as $\langle A| \in \Omega$.

Now we briefly define inner products of bras and kets, employing expressions like $\langle B|A\rangle$ to construct bra-kets. An inner product is a way to multiply vectors together in a vector space, resulting in a scalar. Basically, it is a generalization of the dot product which plays an indispensable role in the mathematical framework of quantum mechanics and vector spaces.

Another fundamental operation we need to discuss is the tensor product. Consider two vector spaces Γ and Ω, with $\gamma \in \Gamma$ and $\omega \in \Omega$. We define their tensor product, which creates new vector space analogous to the multiplication of integers, as follows: the tensor product of γ and ω is $\gamma \otimes \omega$, which is an element of $\Gamma \otimes \Omega$. It is a bi-linear operation, meaning that it is linear in each of its arguments. In quantum mechanics, the tensor product is used to represent complex systems composed of multiple subsystems or parts, where each subsystem corresponds to a vector space.

Both bra and ket-vectors conform to the same set of axioms. However, when dealing with the relationship between the two, it's important to keep two things in mind:

– Consider $\langle A|$ as the bra-vector corresponding to the ket-vector $|A\rangle$, and $\langle B|$ the bra-vector corresponding to the ket-vector $|B\rangle$. Then the bra-vector corresponding to $|A\rangle + |B\rangle$ is $\langle A| + \langle B|$.

– Consider $z|A\rangle$. It is false to state that the bra-vector corresponding to $z|A\rangle$ is $\langle A|z$. We need to call upon complex conjugates on this one. Therefore, the bra-vector corresponding to $z|A\rangle$ is $\langle A|z^*$.

Reminding what we previously said in 2.5.3, we can now explain how to formally describe a qubit.

The states of a quantum system form a complex vector space; in fact, the space of states of a qubit forms a two-dimensional vector space. Through the measurements previously described, we know that if we measure a qubit with the apparatus vertically and up-ward oriented, we observe a state of the system which can be 1 or -1. These states are vectors in a vector space and we call 1 $|u\rangle$ with u standing for 'up', and -1 $|d\rangle$ with d standing for 'down'. Given that we are dealing with a two-dimensional vector space, any vector in that space can be written as a linear combination of $|u\rangle$ and $|d\rangle$.

2.5.8 Schrödinger's equation

In this dissertation, we previously pointed out that quantum mechanics is a linear theory. Now, we need both an equation of motion (EOM) and a dynamic variable. The EOM we use is the Schrödinger's equation, which was introduced in 1925. This equation describes the behaviour of a dynamic variable known as the wave function ψ, which can depend on time and other factors. The wave function is a fundamental concept in quantum mechanics, and it allows us to calculate the probability of finding a particle in a particular state.

The Schrödinger equation is a partial differential equation, and it takes the form $i\hbar\frac{\partial}{\partial t}\psi = \hat{H}\psi$, where i is the imaginary unit ($i = \sqrt{-1}$), $\hbar$ is the reduced Planck constant, and $\hat{H}$ is the Hamiltonian operator.

The Hamiltonian operator represents the total energy of a system. It typically consists of two terms: the kinetic energy term, which is related to momentum, and the potential energy term, which depends on the position of particles in the system. However, note that the specific form of the Hamiltonian depends on the system under consideration. For example, in the case of a single particle in one dimension, the Hamiltonian operator is given by

$\hat{H} = -\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} + V(x)$, where $\hbar$ is the reduced Planck constant, m is the mass of the particle, and $V(x)$ is the potential energy function.

When the Hamiltonian operator acts on a wavefunction ψ, it yields another wavefunction, typically denoted as $\hat{H}\psi = E\psi$, where E is an eigenvalue, a scalar associated with the energy of the system. The wavefunction ψ corresponding to a specific eigenvalue E is called an eigenfunction of the Hamiltonian operator, which is a linear operator on a function space. Basically, the Hamiltonian operator acts on a wavefunction ψ and outputs the total energy E of ψ, if ψ is an eigenfunction of $\hat{H}$.

We have also a partial derivative with respect to time of ψ. In mathematics, the term 'derivative' refers to the rate of change of a function with respect to a variable. Partial derivatives are useful in analyzing surfaces for maximum and minimum points and give rise to partial differential equations. Schrödinger's equation can be applied to the simplest quantum system, namely a single spin, and to more complex systems.

We included this section solely to fully understand how particles and systems are mathematically characterized; in our formalization, we don't need one-to-one translations of the formula to achieve our goal.

2.6 Brain and thoughts

In the following section, we begin to establish our theory on solid biological foundations by delving into the workings of human brain and cognition. Later we will link them to the quantum concepts already outlined.

The nervous system is a very complex organ system and, for brevity's sake, we will omit the description of the peripheral nervous system (PNS) here. The brain governs our thoughts, memories, emotions, touch, motor skills, vision, breathing, temperature, hunger, and every other process that regulates our body. The brain and the spinal cord, which extends from it, jointly constitute the central nervous system (CNS). The brain is contained within the cranial cavity of the skull, and the spinal cord is contained within the vertebral cavity of the vertebral column. It is an oversimplification to say that the CNS is what is inside these two cavities and the PNS is outside of them, but that is one way to start thinking about it and also a useful heuristic.

The human brain weighs around 3 pounds on average and comprises 60% fat, while the remaining 40% is a mixture of water, protein, carbohydrates, and salts. It is not a muscle and contains blood vessels and various types of nerves, including neurons and glial cells. Glial cells are essential components of the nervous system that provide structural support to neurons and facilitate their functions. These cells form a framework of tissue that serves as a platform for neurons to carry out their activities. The neuron is more functionally important than the glia for the communicative function of the nervous system. Neurons are specialized cells with a cell body called soma. They have extensions known as processes. The primary process in every neuron is the axon, which is the fibre that connects a neuron with its target; a bundle of axons, or fibers, found in the CNS, is called a tract. Another type of process that originates from the soma is called the dendrite, which receives inputs from other neurons. A localized collection of neuron cell bodies in the CNS is referred to as a nucleus.

There are two distinct regions in the central nervous system: grey matter and white matter. In the brain, grey matter refers to the darker, outer portion, while white matter describes the lighter, inner section beneath. However, in the spinal cord, the order is reversed, with white matter located on the outside and grey matter positioned within. Grey matter contains numerous cell bodies and dendrites, and White matter contains many axons. White matter is white because axons are insulated by a lipid-rich substance called myelin.

The nervous system receives information about the environment around us (sensation) and generates responses to that information (motor responses). It can be split into regions that are responsible for sensation (sensory functions) and for the response (motor functions). However, there is a third function that needs to be considered, integration. Sensory input needs to be integrated with other sensations, as well as with memories, emotional states, or learning (cognition). Some regions of the nervous system are called integration or association areas. Integration combines sensory perceptions and higher cognitive functions to produce a response.

Let us delve deeper into these functions and provide a more detailed description of each one:

– Sensation: the primary function of the nervous system is to receive information from the environment to gain input about what is happening outside or sometimes inside the body. The nervous system's sensory functions detect a change from a state of balance or a particular event in the environment, known as a stimulus. We commonly think of the "big five" senses: taste, smell, touch, sight, and hearing. There are actually more senses than just those, but that list represents the major senses. Those five are all senses that receive stimuli from the outside world, and of which there is conscious perception. Additional sensory stimuli might be from inside the body, such as the stretch of an organ wall.

– Response: when our sensory organs perceive stimuli, the nervous system responds by producing a reaction. This response is not limited to muscle movements but also includes the control of all three types of muscle tissue contraction. Additionally, responses also include the neural control of glands in the body. Responses can be categorized as either voluntary (contraction of skeletal muscles) or involuntary (contraction of smooth muscles, regulation of cardiac muscle, activation of glands). The somatic nervous system governs voluntary responses, while the autonomic nervous system regulates involuntary responses.

– Integration: sensory structures receive stimuli, and this information is processed within the nervous system. The received stimuli are compared, or integrated, with other stimuli, memories of prior stimuli, or the individual's state at a specific moment. This comparison process ultimately determines the specific response that will be generated.

Enhancing our comprehension of neuronal functioning will facilitate establishing connections between neurons and their roles within our theoretical framework, so let us provide a more detailed description. Neurons are responsible for the computation and communication that the nervous system provides. However, they would not be able to perform their function without glial support. They are electrically active and release chemical signals to target cells. Their unique three-dimensional structure allows for an extensive network of connections within the nervous system. As we already said, the main part of a neuron is the soma, namely the cell body, which contains the nucleus and most of the major organelles. Neurons have many extensions of their cell membranes, which are generally referred to as 'processes'. They are usually described as having a single axon, a fibre that emerges from the cell body and can branch repeatedly to communicate with many target cells. It propagates the nerve impulse, which is communicated to one or more cells. The neuron's other processes are dendrites, extensions that receive information from other neurons at distinct contact points known as synapses. Dendrites typically feature extensive branching, providing sites for interaction with other neurons, letting them communicate with the soma. Information, starting from the dendrites, flows through a neuron and arrives down the axon, crossing the cell body. This directional flow gives polarity to the neuron, meaning that the information flows in this specific direction. Where the axon emerges from the cell body, there is a region known as the axon hillock, which is a tapering of the cell body toward the axon fibre. Within this region, the cytoplasm changes to a solution of limited components called axoplasm. The axon hillock marks the start of the axon and, because of that, it is also designed as 'the initial segment'. Many axons are wrapped by an insulating substance called myelin, made from glial cells. There are gaps in the myelin covering of an axon. Each gap is called a 'node of Ranvier'.

They are fundamental for electrical signals to travel down the axon. The length of the axon between each gap, which is wrapped in myelin, is an 'axon segment'. At the end of the axon is the 'axon terminal', where there are several branches extending toward the target cell, each of which ends in an enlargement called a 'synaptic end bulb'. These bulbs make the connection with the target cell at the synapse.

We can classify neurons due to their shape, distinguishing between unipolar cells, bipolar cells, and multipolar cells. Unipolar cells have one process that includes both the axon and dendrite. Bipolar cells have two processes, the axon and a dendrite. Multipolar cells have more than two processes, the axon and two or more dendrites.

Now that we have covered the basics of neurons, let us explore how they communicate. To do that, we need to understand what action potentials are. To quickly recap: neurons are made up of three main parts. The dendrites, small branches receiving signals from other neurons, the soma, or cell body, housing essential organelles like the nucleus, and the axon which is intermittently wrapped in fatty myelin. Dendrites receive signals from other neurons through neurotransmitters. When these neurotransmitters bind to receptors on the dendrite, they function as a chemical signal. The binding of neurotransmitters opens ion channels, enabling charged ions to flow in and out of the cell, converting the chemical signal into an electrical signal. When the combined effect of multiple dendrites receiving inputs at the same time alters the overall electrical charge of the cell significantly, it initiates an action potential. Action potentials are electrical signals that rapidly travel down the axons, triggering the release of neurotransmitters on the other end and further relaying the signal. The cell has an electric charge because of the different concentrations of ions on the inside versus the outside of the cell; there are more $Na+$ (sodium ions), $Cl-$ (chloride ions), and $Ca2+$ (calcium ions) on the outside, and more $K+$ (potassium ion) and $A-$ (negatively charged anions), on the inside of the cell. The distribution of these ions gives the cell a net negative charge of approximately -65 millivolts relative to the external environment. This is referred to as the neuron's resting membrane potential. When a neurotransmitter binds to a dendrite receptor, a ligand-gated ion channel opens up, allowing specific ions to flow through, depending on the channel. The term 'ligand-gated' indicates that the gate responds to a ligand, which in this context is a neurotransmitter. Let us make clear what we just explained: consider a ligand-gated Na+ ion channel as an example. When it opens, it allows Na+ flow into the cell. The additional positive charge that enters makes the cell less negative and therefore less 'polar'. Gaining positive charge is called 'depolarization'. Neurotransmitters usually activate various ligand-gated ion channels simultaneously. This activation can lead to the inflow of ions such as sodium and calcium, and the outflow of other ions like potassium. Finally, when everything is tallied, if the net result is an influx of positive charge, we have an excitatory postsynaptic potential (EPSP). Conversely, if only ligand-gated Cl- ion channels are opened, there is a net influx of negative charge that creates an inhibitory postsynaptic potential (IPSP); this makes the cell potential more negative, leading to 'repolarization'.

A single EPSP or IPSP causes an irrelevant change in the resting membrane potential. However, if an adequate number of EPSPs occur across multiple dendritic sites they can collectively push the membrane potential to a specific threshold value, typically around -55 mV, triggering the opening of voltage-gated sodium channels at the axon hillock. These voltage-gated channels open in response to a voltage change and, when these open, sodium rushes into the cell. Nearby voltage-gated sodium channels open due to the change in membrane potential and this sets off a chain reaction that propagates down the entire length of the axon, culminating in the generation of our action potential. When this happens we say that the neuron has 'fired.' After a substantial influx of sodium has rushed across the neuronal membrane, the cell becomes positevely charged relative to the outside (reaching approximately $+40$ mV). The depolarization terminates when the sodium channel stops allowing sodium to flow into the cells, which is a process called 'inactivation'. This inactivated state is specific of the voltage-gated sodium channel. This gate stops sodium influx shortly after depolarization and remains in the closed state until the cell repolarizes. Once the cell repolarizes, the channel enters the closed state again and the inactivation gate resets, allowing the influx of sodium once more. Additionally, there are potassium voltage-gated channels alongside the sodium voltage-gated channels. Potassium voltage-gated channels respond slowly and remain closed until the sodium channels have opened and undergone inactivation, because they don't have a separate inactivation gate. Consequently, following the initial influx of sodium into the cell, potassium exits the cell down its own electrochemical gradient. This process removes positive charge and diminishes the impact of sodium depolarization. Potassium voltage-gated channels respond slowly and remain closed until the sodium channels have opened and undergone inactivation, because they do not have a separate inactivation gate. Consequently, following the initial influx of sodium into the cell, potassium exits the cell down its own electrochemical gradient. This process removes positive charge and diminishes the impact of sodium depolarization. These potassium channels stay open for slightly longer so that there is a timeframe when there is a net outflux of positive ions that leads to a more negative membrane potential (repolarization). Throughout the repolarization phase, the cell relies on the sodium-potassium pump, an active transporter that expels three sodium ions from the cell and transports two potassium ions into it. This period coincides with the absolute refractory period of the cell: sodium channels are inactivated and will not respond to any amount of stimuli. The absolute refractory period prevents action potentials from occurring too closely in time, ensuring that the action potential continues to propagate in a single direction. The sodium-potassium pump, in conjunction with the prolonged opening of potassium channels, induces a brief phase of overcorrection, causing the neuron to become hyperpolarized in relation to its resting potential. During this period, sodium channels return to their initial closed state, while potassium channels remain open for a brief period. At this stage, we are in the relative refractory period since the sodium channels are closed but can be activated. However, because the potassium channels are still open and we are in a hyperpolarized state, a strong stimulus is required to do so. In the end, the potassium channels close and the neuron restores its resting membrane potential.

3 Model and example application

The primary contribution of this work lies in the introduction of a new logical model that can seamlessly integrate into any syntactic and semantic framework. This model proposes a new way to formalize words' and sentences' intensions and an explanation for the phenomenon of incomprehension.

We provide a precise outline of the model, grounded in the assumption that the logical-mathematical tools employed in its formalization are intuitively effective in describing the elements and phenomena they represent. At the same time, we will justify the employed concepts by relating them to physical and neurological observations and experimentation, building up a materialistic view of cognitive activity and human language-mediated interaction based on the scientific method and knowledge at our disposal today.

3.1 Intensional Disjunctive Superposition Evaluation (IDSE)

At the beginning of 2.6, we discussed how quantum mechanics' theoretical concepts and structures appear adequate for brain function descriptions. Quantum biology (Lambert et al. 2013) [62] is an interdisciplinary field that explores applications of quantum mechanics principles and phenomena to biological systems, including investigations on quantum processes in neural systems and exploring potential quantum effects in cognition, information processing and consciousness. The difficulty lies in finding good evidence for it (Koch and Hepp 2006) [59]. It is not the century discovery that sometimes even the scientific method produces a tower of Babel, especially in interdisciplinary studies. Nevertheless, we believe that continuous comparisons between purely theoretical investigations and evidence-based research are the key to solving highly complex problems. Therefore, we provide an extremely interdisciplinary model to give an account for comprehension and incomprehension in natural language interactions, which is holistically coherent to the mixed nature of theoretical linguistics itself.

This model implies a more complex in-depth conceptualization of words and sentences as analysis units. Given the premises, we are now ready to set out our model starting with introducing words and sentences.

3.1.1 Words and sentences as superpositional λ-terms

Kurt and Albert sit at their favourite bar, enjoying a sunny spring morning. Kurt reads the newspaper while Albert ferociously devours a gigantic creamy croissant. "Come on, Kurt!

Order something, for God's sake!" murmured Albert with his mouth full, only to be met with an annoyed snort followed by a question. "Heard about last night's party at Werner's? I think Bill really loves Monica." firmly says Kurt behind his grey word-marked curtains. "What?! What do you mean by "loves"? What happened?! Bill is Hillary's husband!" Albert screams, spitting some croissant pieces in Kurt's direction (a completely ineffective try to scratch the wall of text). "As a friend, you silly whiskered drama king." calmly specifies Kurt. Albert takes a moment to compose himself, and after a long pause, he whispers, "I love Monica." Kurt lowers the sentences-made drawbridge and, letting appear a wry smile on his face, says: "Like a friend?".

Let us analyze this little conversation. What happens is, without any doubt, an incomprehension due to contextual elements and semantic ambiguity of the word 'love', which inevitably makes unclear the entire sentence to which it belongs.

We believe the best way to introduce our model is by showcasing its fundamental components, words and sentences. In this quantum biological language-related framework, we must buy the idea that all biological organisms must obey the laws of non-classical physics, which is not difficult if we believe in the scientific method and think there are atoms in all the matter in our universe, our body included. Every cell in our body is composed of atoms, including neurons. Classical physics laws cannot describe atomic structures. If they could, electrons would be spherical objects that circle the atomic nucleus, emitting radiation, losing energy, and collapsing into the nucleus. It would mean that atoms could not exist. Subatomic particles, such as electrons, are also non-classical structures. In Quantum Mechanics, electrons are positioned in 'shells' around the nucleus, which prevents them from emitting radiation. This is a vital feature that contributes to the stability and existence of atoms. Therefore, since cells are made of atoms, we can reasonably assume that they exhibit quantum behaviours and are governed by quantum laws.

In 2.7, we pointed out how neurons work, describing our brain processes as exchanges of information, namely electrochemical signals occurring in an individual's nervous system. We are used to defining our cognitive process as 'thoughts', whether we are talking about emotions, decision-making and all sorts of concepts. Therefore, we can easily define words' and sentences' meanings as electrochemical processes in this pure scientific and materialistic view.

Regarding formalization, we generally define words and sentences as semantical qubits in meaning-superposition, building on the natural language translations provided by first-order logic and lambda calculus we outlined in sections 2.2 and 2.3. Furthermore, what follows will not invalidate the computation of meaning yielded by the formal semantics toy system we illustrated in Chapter 2. The last clarification before the general definition is how we adapt superposition (2.6.5) in our model. In the formal frameworks used in this thesis (propositional logic, first-order logic, and lambda calculus), the sum operator is not available. Therefore, we decide not to include it in our model. Instead, we represent superpositions as inclusive 'or' statements. This means that the statements are true if at least one disjoint result is true, and false otherwise. Considering the scenario where all the meaning-disjoints are false, we find that this never occurs when dealing with the meanings of words and sentences. From a functional and pragmatic perspective, there are no meaningless words or sentences, making it impossible for the superposition to result in being false. Given these necessary premises, now we can provide a general definition of words and sentences.

We say that every well-formed formula ϕ in typed λ-calculus is a word, except for those of type t. Every word is in a superposition of its possible meanings.

Formally:

$$\forall_\phi(\phi_{\sigma \neq t} \to |\phi_\sigma\rangle \in W \subseteq L)$$

with

$$|\phi_\sigma\rangle = \upsilon_x|\phi_\sigma\rangle \vee \upsilon_y|\phi_\sigma\rangle \; ...$$

$[\![\phi \vee \psi]\!]^I = T$ if $[\![\phi]\!]^I = T$ or $[\![\psi]\!]^I = T$ (or both), and F impossible.

with σ standing for any possible type in λ-calculus, L being a vector space, namely the vector space of every λ-term and W standing for the subset of L comprising every λ-term excluded those of type t.

Besides, we have υ_x and υ_y, which are fundamental elements of the system we will explain as soon after the definition of sentences.

We say that every well-formed formula ϕ in typed λ-calculus of type t is a sentence. Every sentence is in a superposition of its possible meaning.

Formally:

$$\forall_\phi(\phi_t \to |\phi_t\rangle \in S \subseteq L)$$

with

$$|\phi_t\rangle = \upsilon_x|\phi_t\rangle \vee \upsilon_y|\phi_t\rangle \; ...$$

$[\![\phi \vee \psi]\!]^I$ as in word's definition.

Now, we introduce two kinds of new types, the secondary grade type relative to the collection of all possible meanings of a linguistic item (wide intension) and the secondary grade type relative to every possible single meaning of a linguistic item (singular intension). We assign to each word $|\phi_{\sigma\neq t}\rangle$ and each sentence $|\phi_t\rangle$ the secondary grade type of wide intensions c and write $|\phi_\sigma\rangle_c$. We assign to every possible meaning of a word and a sentence the secondary grade of singular intensions υ and write $|\phi_\sigma\rangle_c = \upsilon_x|\phi_\sigma\rangle_v \vee \upsilon_y|\phi_\sigma\rangle_v$ ...

We define υ_x and υ_y as meaning coefficients, namely the particular natural language semantic meaning of the related lambda term.

To give an example application of what said so far, let us reduce the vast semantic of 'love' to the meanings employed by Kurt and Albert during their breakfast.

Consider 'love' to be comprehensible as 'love as friends' and 'love as lovers'; our model word's formalization is

$$|(\lambda y \lambda x.L(x,y))_{eet}\rangle_c = \upsilon_f|(\lambda y \lambda x.L(x,y))_{eet}\rangle_v \vee \upsilon_l|(\lambda y \lambda x.L(x,y))_{eet}\rangle_v$$

with $\upsilon_f|(\lambda y \lambda x.L(x,y))_{eet}\rangle_v \vee \upsilon_l|(\lambda y \lambda x.L(x,y))_{eet}\rangle_v$ standing for 'love as friends or love as lovers'.

Proper names like 'Bill' and 'Monica' are formally translated as

$$\text{Bill} \rightsquigarrow |b_e\rangle_c = \upsilon_x|b_e\rangle_v \vee \upsilon_y|b_e\rangle_v ...$$
$$\text{Monica} \rightsquigarrow |m_e\rangle_c = \upsilon_x|m_e\rangle_v \vee \upsilon_y|m_e\rangle_v ...$$

with every disjunct being a specific individual named 'Bill' or 'Monica' due to homonymy.

Finally, a brief formal summary:

- Syntax. Two kinds of objects: types and intensions. We recursively define:
  - a universe T of types
  - for each type $\sigma \in T$, a set of intensions $\phi : \sigma$ of that type
- Semantics.
  - for each type σ, a domain D^M_σ of objects of type σ
  - for each term $\phi : \sigma$, a denotation $[\![\phi]\!]^{M,g} \in D^M_\sigma$
- Types.
  - υ is a type (the type of singular intensions)
  - c is a type (the type of wide intensions)

3.1.2 Vector space L

In our model, we propose a light characterization of the concept of vector space by applying some restrictions on the rule set previously outlined in 2.6.7 and adapting λ-calculus calculation rules from 2.4.

Let L be a vector space, hence if $\phi \in L$, ϕ is a vector and we write it as $|\phi\rangle$, with ϕ being a variable over λ-terms. We define a vector space through the following axioms:

- $L = \{\phi_\sigma \mid \phi_\sigma$ is a λ-term of the form $|\phi_\sigma\rangle_\omega = \upsilon_x|\phi_\sigma\rangle_\delta \vee \upsilon_y|\phi_\sigma\rangle_\delta ...\}$
- If $\phi_{\langle\sigma,\tau\rangle} \in L$ and $\psi_\sigma \in L$ are well-formed formula in in typed λ-calculus, then $(|\phi_{\langle\sigma,\tau\rangle}\rangle_\omega)(|\psi_\sigma\rangle_\omega) \rightarrow_\beta |\theta_\tau\rangle_\omega$ by function application, with $(|\phi_{\langle\sigma,\tau\rangle}\rangle_\omega)(|\psi_\sigma\rangle_\omega) \equiv |\theta_\tau\rangle_\omega$ by α-equivalence.
- If $\phi_{\langle\sigma,\tau\rangle} \in L$ and $\psi_{\langle\sigma,\tau\rangle} \in L$ are well-formed formula in in typed λ-calculus, then $|(|\phi_{\langle\sigma,\tau\rangle}\rangle_\omega)(|\psi_{\langle\sigma,\tau\rangle}\rangle_\omega) \rightarrow_{p.m} |(|\phi_{\langle\sigma,\tau\rangle}\rangle_\omega \wedge |\psi_{\langle\sigma,\tau\rangle}\rangle_\omega)\rangle_\omega$ by predicate modification, with $(|\phi_{\langle\sigma,\tau\rangle}\rangle_\omega)(|\psi_{\langle\sigma,\tau\rangle}\rangle_\omega) \equiv |(|\phi_{\langle\sigma,\tau\rangle}\rangle_\omega \wedge |\psi_{\langle\sigma,\tau\rangle}\rangle_\omega)\rangle_\omega$ by α-equivalence.

We will now provide an example application based on the sentence "Bill loves Monica" from the previous subsection, with the proper names' semantics also reduced.

Given

- Bill $\rightsquigarrow |b_e\rangle_c = \upsilon_x|b_e\rangle_v \vee \upsilon_y|b_e\rangle_v$
- Monica $\rightsquigarrow |m_e\rangle_c = \upsilon_x|m_e\rangle_v \vee \upsilon_y|m_e\rangle_v$
- Loves $\rightsquigarrow |(\lambda y\lambda x.L(x,y))_{eet}\rangle_c = \upsilon_f|(\lambda y\lambda x.L(x,y))_{eet}\rangle_v \vee \upsilon_l|(\lambda y\lambda x.L(x,y))_{eet}\rangle_v$

we have the following syntactic tree

$|(L(|b\rangle,|m\rangle))_t\rangle_c$

Bill $\rightsquigarrow |b_e\rangle_c$ $\qquad$ $|(\lambda x.L(x,|m\rangle))_{et}\rangle_c$

loves $\rightsquigarrow |(\lambda y\lambda x.L(x,y))_{eet}\rangle_c$ $\qquad$ Monica $\rightsquigarrow |m_e\rangle_c$

Calculus:$(|(\lambda y\lambda x.L(x,y))_{eet}\rangle_c)(|m_e\rangle_c)(|b_e\rangle_c) \rightarrow_\beta (|\lambda x.L(x,|m\rangle)_{et}\rangle_c)(|b_e\rangle_c) \rightarrow_\beta |(L(|b\rangle,|m\rangle))_t\rangle_c$

with $|(L(|b\rangle,|m\rangle))_t\rangle_c = (\upsilon_f|L(\upsilon_x|b\rangle \vee \upsilon_y|b\rangle, \upsilon_x|m\rangle \vee \upsilon_y|m\rangle)\rangle)_v \vee (\upsilon_l|L(\upsilon_x|b\rangle \vee \upsilon_y|b\rangle, \upsilon_x|m\rangle \vee \upsilon_y|m\rangle)\rangle)_v$

Finally, an example of predicate modification.

Given

- Smart $\rightsquigarrow |(\lambda x.S(x))_{et}\rangle_c = \upsilon_x|\lambda x.S(x)\rangle_v \vee \upsilon_y|\lambda x.S(x)\rangle_v$
- Boy $\rightsquigarrow |(\lambda x.B(x))_{et}\rangle_c = \upsilon_x|\lambda x.B(x)\rangle_v \vee \upsilon_y|\lambda x.B(x)\rangle_v$

we have the following syntactic tree

$|(\lambda x.S(x) \wedge B(x))_{et}\rangle_c$

Smart $\rightsquigarrow |(\lambda x.S(x))_{et}\rangle_c$ Boy $\rightsquigarrow |(\lambda x.B(x))_{et}\rangle_c$

Calculus: $(|(\lambda x.S(x))_{et}\rangle_c)(|(\lambda x.B(x))_{et}\rangle_c) \to_{p.m.} |(\lambda x.S(x) \wedge B(x))_{et}\rangle_c$

with $|(\lambda x.S(x) \wedge B(x))_{et}\rangle_c = \lambda x.((\upsilon_x|\lambda x.S(x)\rangle_v \vee \upsilon_y|\lambda x.S(x)\rangle_v)) \wedge \lambda x.((\upsilon_x|\lambda x.B(x)\rangle_v \vee \upsilon_y|\lambda x.B(x)\rangle_v))$

3.1.3 Operator K

In this sub-section, we will discuss one of the primary operators of our model. We define K as a distributive operator such that $K_z : |\phi_\sigma\rangle_c \in L \to \{1, 0\}$, which first outputs every possible combination of values based on the intensional superposition design, and then selects one line of evaluation, which is the interpretation of the term according to z. In other words, K takes terms of type c as input and gives the evaluation of the term. We choose to employ 1 and 0 as intensional evaluation values to keep up with good notation, and do not generate confusion using the same values of terms of type t. We formally write:

- $(K_z(|\phi_{\langle\sigma,\tau\rangle}\rangle_\omega))(K_z(|\psi_\sigma\rangle_\omega)) \to_\beta K_z(|\psi_\tau\rangle_\omega)$
- $K_z(|\phi_\sigma\rangle_\omega) = K_z(\upsilon_y|\phi_\sigma\rangle_\delta \vee \upsilon_x|\phi_\sigma\rangle_\delta) = [\![\upsilon_x|\phi_\sigma\rangle_\delta \vee \upsilon_y|\phi_\sigma\rangle_\delta]\!]^{K_z}$

  with

  $$[\![\upsilon_x|\phi_\sigma\rangle_\delta \vee \upsilon_y|\phi_\sigma\rangle_\delta]\!]^I = \begin{cases} F & \text{impossible } [\![\upsilon_x|\phi\rangle_\sigma]\!]^I = [\![\upsilon_y|\phi\rangle_\sigma]\!]^I = F \text{ never happens} \\ T & \text{otherwise} \end{cases}$$

  and

  $$[\![\upsilon_x|\phi\rangle_\sigma \vee \upsilon_y|\phi\rangle_\sigma]\!]^{K_z} = \begin{cases} 1\,1 & [\![\upsilon_x|\phi_\sigma\rangle_\delta]\!]^{K_z} = [\![\upsilon_y|\phi_\sigma\rangle_\delta]\!]^{K_z} = 1 \\ 1\,0 & [\![\upsilon_x|\phi_\sigma\rangle_\delta]\!]^{K_z} = 1 \;\; [\![\upsilon_y|\phi_\sigma\rangle_\delta]\!]^{K_z} = 0 \\ 0\,1 & [\![\upsilon_x|\phi_\sigma\rangle_\delta]\!]^{K_z} = 0 \;\; [\![\upsilon_y|\phi_\sigma\rangle_\delta]\!]^{K_z} = 1 \end{cases}$$

We give an example application using the sentence ”Bill loves Monica” with the assumed intensional duality of each term.

– $K_z(|m\rangle_e) = [\![\upsilon_1|m\rangle \vee \upsilon_2|m\rangle]\!]^{K_z}$

$$[\![\upsilon_1|m\rangle \vee \upsilon_2|m\rangle]\!]^I = \begin{cases} [\![\upsilon_1|m\rangle]\!]^I = T \text{ and } [\![\upsilon_2|m\rangle]\!]^I = T \\ [\![\upsilon_1|m\rangle]\!]^I = T \text{ and } [\![\upsilon_2|m\rangle]\!]^I = F \\ [\![\upsilon_1|m\rangle]\!]^I = F \text{ and } [\![\upsilon_2|m\rangle]\!]^I = T \end{cases}$$

$$[\![\upsilon_1|m\rangle \vee \upsilon_2|m\rangle]\!]^{K_z} = \begin{cases} [\![\upsilon_1|m\rangle]\!]^{K_z} = 1 \text{ and } [\![\upsilon_2|m\rangle]\!]^{K_z} = 1 \\ [\![\upsilon_1|m\rangle]\!]^{K_z} = 1 \text{ and } [\![\upsilon_2|m\rangle]\!]^{K_z} = 0 \\ [\![\upsilon_1|m\rangle]\!]^{K_z} = 0 \text{ and } [\![\upsilon_2|m\rangle]\!]^{K_z} = 1 \end{cases}$$

– $K_k(|b\rangle_e) = [\![\upsilon_1|b\rangle \vee \upsilon_2|b\rangle]\!]^{K_z}$

$$[\![\upsilon_1|b\rangle \vee \upsilon_2|b\rangle]\!]^I = \begin{cases} [\![\upsilon_1|b\rangle]\!]^I = T \text{ and } [\![\upsilon_2|b\rangle]\!]^I = T \\ [\![\upsilon_1|b\rangle]\!]^I = T \text{ and } [\![\upsilon_2|b\rangle]\!]^I = F \\ [\![\upsilon_1|b\rangle]\!]^I = F \text{ and } [\![\upsilon_2|b\rangle]\!]^I = T \end{cases}$$

$$[\![\upsilon_1|b\rangle \vee \upsilon_2|m\rangle]\!]^{K_z} = \begin{cases} [\![\upsilon_1|b\rangle]\!]^{K_z} = 1 \text{ and } [\![\upsilon_2|b\rangle]\!]^{K_z} = 1 \\ [\![\upsilon_1|b\rangle]\!]^{K_z} = 1 \text{ and } [\![\upsilon_2|b\rangle]\!]^{K_z} = 0 \\ [\![\upsilon_1|b\rangle]\!]^{K_z} = 0 \text{ and } [\![\upsilon_2|b\rangle]\!]^{K_z} = 1 \end{cases}$$

– $K_z(|\lambda y \lambda x.L(x,y)\rangle_{eet}) = [\![\upsilon_f|\lambda y \lambda x.L(x,y)\rangle_{eet} \vee \upsilon_l|\lambda y \lambda x.L(x,y)\rangle_{eet}]\!]^{K_z}$

$$[\![\upsilon_f|\lambda y \lambda x.L(x,y)\rangle \vee \upsilon_l|\lambda y \lambda x.L(x,y)\rangle]\!]^I = \begin{cases} [\![\upsilon_f|\lambda y \lambda x.L(x,y)\rangle]\!]^I = T \text{ and } [\![\upsilon_l|\lambda y \lambda x.L(x,y)\rangle]\!]^I = T \\ [\![\upsilon_f|\lambda y \lambda x.L(x,y)\rangle]\!]^I = F \text{ and } [\![\upsilon_l|\lambda y \lambda x.L(x,y)\rangle]\!]^I = T \\ [\![\upsilon_f|\lambda y \lambda x.L(x,y)\rangle]\!]^I = T \text{ and } [\![\upsilon_l|\lambda y \lambda x.L(x,y)\rangle]\!]^I = F \end{cases}$$

$$[\![\upsilon_f|\lambda y\lambda x.L(x,y)\rangle \vee \upsilon_l|\lambda y\lambda x.L(x,y)\rangle]\!]^{K_z} = \begin{cases} [\![\upsilon_f|\lambda y\lambda x.L(x,y)\rangle]\!]^{K_z} = 1 \text{ and } [\![\upsilon_l|\lambda y\lambda x.L(x,y)\rangle]\!]^{K_z} = 1 \\ [\![\upsilon_f|\lambda y\lambda x.L(x,y)\rangle]\!]^{K_z} = 1 \text{ and } [\![\upsilon_l|\lambda y\lambda x.L(x,y)\rangle]\!]^{K_z} = 0 \\ [\![\upsilon_f|\lambda y\lambda x.L(x,y)\rangle]\!]^{K_z} = 0 \text{ and } [\![\upsilon_l|\lambda y\lambda x.L(x,y)\rangle]\!]^{K_z} = 1 \end{cases}$$

$$\begin{array}{ccc} & K_z((|L(K_z(|b\rangle), K_z(|m\rangle)))_t\rangle_c) & \\ K_z(|b_e\rangle_c) & & K_z(|\lambda x.L(x, K_z(|m\rangle))_{et}\rangle_c) \\ & K_z((|\lambda y\lambda x.L(x,y))_{eet}\rangle_c) & K_z(|m_e\rangle_c) \end{array}$$

Calculus: $(K_z((|\lambda y\lambda x.L(x,y))_{eet}\rangle_c))(K_z(|m_e\rangle_c))(K_z(|b_e\rangle_c)) \rightarrow_\beta$
$\rightarrow_\beta (K_z((|\lambda x.L(x, K_k(|m\rangle)))_{et}\rangle_c))(K_z(|b_e\rangle_c)) \rightarrow_\beta$
$\rightarrow_\beta K_z((|L(K_z(|b\rangle), K_z(|m\rangle)))_t\rangle_c)$

The distributivity of K also grants recursive intensional interpretation as in

$$K_z((|\lambda x.L(x, K_z(|m\rangle)))_{et}\rangle_c)$$

which reflects the possibility of a word's resemantization due to the encounter with another word.

We can express straightforwardly superpositional intention evalations using matrices.

$$[\![\upsilon_1|m\rangle_v \vee \upsilon_2|m\rangle_v]\!]^{K_z} = \begin{bmatrix} 1 & 1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$[\![\upsilon_1|b\rangle_v \vee \upsilon_2|b\rangle_v]\!]^{K_z} = \begin{bmatrix} 1 & 1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$[\![\upsilon_f|\lambda y\lambda x.L(x,y)\rangle \vee \upsilon_l|\lambda y\lambda x.L(x,y)\rangle]\!]^{K_z} = \begin{bmatrix} 1 & 1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Note that what is relevant to us is the single disjoints' value. Now, we can compute and visualize the semantics of Kurt's interpretation of "Bill loves Monica"

$$K_k(|L(K_k(|b\rangle_c), K_k(|m\rangle_c))\rangle_c) = [\![\upsilon_f|L([\![\upsilon_1|b\rangle \vee \upsilon_2|b\rangle]\!]^{K_k}, [\![\upsilon_1|m\rangle \vee \upsilon_2|m\rangle]\!]^{K_k}))\rangle \vee \upsilon_l|L([\![\upsilon_1|b\rangle \vee \upsilon_2|b\rangle]\!]^{K_k}, [\![\upsilon_1|m\rangle \vee \upsilon_2|m\rangle]\!]^{I}))\rangle]\!]^{K_k} = 1\ 1\ 0\ 1\ 0\ 0\ 1\ 0\ 1\ 0$$

with

$$K_k(|b_e\rangle_c) = [\![\upsilon_1|b\rangle_v \vee \upsilon_2|b\rangle_v]\!]^{K_k} = \begin{bmatrix} 1 & 1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$K_k(|m_e\rangle_c) = [\![\upsilon_1|m\rangle_v \vee \upsilon_2|m\rangle_v]\!]^{K_k} = \begin{bmatrix} 1 & 1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$K_k((|\lambda y\lambda x.L(x,y))_{eet}\rangle_c) = [\![\upsilon_f|\lambda y\lambda x.L(x,y)\rangle_v \vee \upsilon_l|\lambda y\lambda x.L(x,y)\rangle_v]\!]^{K_k} = \begin{bmatrix} 1 & 1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$$

and Albert's interpretation

$$K_a(|L(K_a(|b\rangle_c), K_a(|m\rangle_c))\rangle_c) = [\![\upsilon_f|L([\![\upsilon_1|b\rangle \vee \upsilon_2|b\rangle]\!]^{K_a}, [\![\upsilon_1|m\rangle \vee \upsilon_2|m\rangle]\!]^{K_a}))\rangle \vee \upsilon_l|L([\![\upsilon_1|b\rangle \vee \upsilon_2|b\rangle]\!]^{K_a}, [\![\upsilon_1|m\rangle \vee \upsilon_2|m\rangle]\!]^{K_a}))\rangle]\!]^{K_a} = 0\ 1\ 0\ 1\ 0\ 1\ 1\ 0\ 1\ 0$$

with

$$K_a(|b_e\rangle_c) = [\![\upsilon_1|b\rangle_v \vee \upsilon_2|b\rangle_v]\!]^{K_a} = \begin{bmatrix} 1 & 1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$K_a(|m_e\rangle_c) = [\![\upsilon_1|m\rangle \vee \upsilon_2|m\rangle]\!]^{K_a} = \begin{bmatrix} 1 & 1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$K_a(|(\lambda y\lambda x.L(x,y))_{eet}\rangle_c) = [\![\upsilon_f|\lambda y\lambda x.L(x,y)\rangle_v \vee \upsilon_l|\lambda y\lambda x.L(x,y)\rangle_v]\!]^{K_a} = \begin{bmatrix} 1 & 1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Therefore, the interpretations $K_k(|L(K_k(|b\rangle_c), K_k(|m\rangle_c))\rangle_c) = [\![\upsilon_f|L([\![\upsilon_1|b\rangle \vee \upsilon_2|b\rangle]\!]^{K_k}, [\![\upsilon_1|m\rangle \vee \upsilon_2|m\rangle]\!]^{K_k}))\rangle \vee \upsilon_l|L([\![\upsilon_1|b\rangle \vee \upsilon_2|b\rangle]\!]^{K_k}, [\![\upsilon_1|m\rangle \vee \upsilon_2|m\rangle]\!]^{I}))\rangle]\!]^{K_k} = 1\ 1\ 0\ 1\ 0\ 0\ 1\ 0\ 1\ 0$ and $K_a(|L(K_a(|b\rangle_c), K_a(|m\rangle_c))\rangle_c) = [\![\upsilon_f|L([\![\upsilon_1|b\rangle \vee \upsilon_2|b\rangle]\!]^{K_a}, [\![\upsilon_1|m\rangle \vee \upsilon_2|m\rangle]\!]^{K_a}))\rangle \vee \upsilon_l|L([\![\upsilon_1|b\rangle \vee$

$\upsilon_2|b\rangle]\!]^{K_a}, [\![\upsilon_1|m\rangle \vee \upsilon_2|m\rangle]\!]^{K_a}))]\!]^{K_a} = 0\ 1\ 0\ 1\ 0\ 1\ 1\ 0\ 1\ 0$ are manifestly different for $K_k(|\lambda y \lambda x.L(x,y)\rangle_c)$ and $K_a(|\lambda y \lambda x.L(x,y)\rangle_c)$.

3.1.4 Operator E

At the very beginning of this work, precisely in section 1.3, we introduced a simple function called E. This function takes an interpretation of a word or a proposition as input. If the interpretation matches the speaker's interpretation, namely the correct interpretation in the conversational exchange, the function outputs comprehension, otherwise it outputs incomprehension. Here, we will re-define it, concluding our model formalization.

Let E be a function that takes as input two terms of type c processed by K and compares them. If the two terms are identical, then E outputs the comprehension value C, otherwise the incomprehension value $\neg C$.

Therefore, we say

$$K_z(|\phi_\sigma\rangle_\omega) = K_u(|\phi_\sigma\rangle_\omega) \iff [\![\upsilon_x|\phi_\sigma\rangle_\delta \vee \upsilon_x|\phi_\sigma\rangle_\delta]\!]^{K_z} = [\![\upsilon_x|\phi_\sigma\rangle_\delta \vee \upsilon_x|\phi_\sigma\rangle_\delta]\!]^{K_u}$$

In other words, the two λ-terms are equal if they have the same intensional sequence.

We formally define E:

$$E(K_z(|\phi_\sigma\rangle_\omega), K_u(|\phi_\sigma\rangle_\omega)) = \begin{cases} C & \iff K_z(|\phi\rangle_\sigma) = K_u(|\phi\rangle_\sigma) \\ \neg C & \text{otherwise} \end{cases}$$

Going back to the conversation between Kurt and Albert in 3.1.1, we have:

$$E(K_k(|L(K_k(|b\rangle), K_k(|m\rangle))\rangle), K_a(|L(K_a(|b\rangle), K_a(|m\rangle))\rangle)) = \neg C$$

because

$$1\ 1\ 0\ 1\ 0\ 0\ 1\ 0\ 1\ 0 \neq 0\ 1\ 0\ 1\ 0\ 1\ 1\ 0\ 1\ 0$$

Finally, we have a fully computable λ-calculus-based model that ensures us an explanation of incomprehension phenomena in natural language through a re-evaluation of superpositions as always-true disjunctions. In the next section, we will discuss how we can strengthen this model by grounding it in neurobiological and quantum evidence.

3.2 Intensional Superpositions

In subsection 2.3.1, we elaborated on how semanticists approach the meaning of words and sentences. We also briefly touched upon the debate surrounding intension in the philosophy of language field. Here, we demonstrate the importance of our intentional superpositions in effectively resolving semantic issues that arise during interactions. It is clear that possible worlds are useful in dealing with spatial-temporal dislocations allowing us to formalize contextual pieces of information, which contribute to the meaning of a statement. However, this is only a fragment of information which contributes to the understanding of the complex semantics of an expression and does not exhaust its meaning. Take as an example indexicals and proper names. In contemporary modern semantics, we treat them as linguistic items with no changeable meaning in every possible world. This approach contradicts one foundational property of natural languages, namely the property that allows us to talk about language through language itself. In our account, every word has at least two meanings, one of these meanings is the meta-linguistic meaning which makes it possible to express statements about its role as a linguistic item in the sentence, its grammatical features and so on. Regarding proper names, we recognize the effectiveness of the Kripkean functional view, where they are used in every possible world to always designate the same individual, but this again is not relevant when contextualized in interaction among human beings. It is possible, as in the conversation between Albert and Kurt, that there are homonyms among a group of known people and that the listener could interpret the name as a pointer to another referent. Every theory previously outlined fails to give a practical account of language. In other words, formal semantics does not collocate language in interaction, only computes it in the solipsistic act of formulating sentences with meanings already clarified by the speaker who does not interact with anyone, failing to give an account for incomprehension, for example. We think that our contribution resonates with Hintikka's work (2.3.1), namely that propositional attitudes and specific word meanings are bound to speakers' mental states or interpretations. Our choice to define words and sentences equally as intensional superpositions do not conflict with the process of intensionalization through possible worlds because they contribute differently in shaping meaning. Possible worlds enrich expression meaning with contextual information, our formal tool mirrors its very functional nature, namely the polysemy that leads to ambiguity and incomprehensions in communication. Another similar concept is "idea" in Frege, namely the intuition there are subjective sentences' and words' meanings, which find a representation in possibly infinite disjunctive superpositions.

A potential critique from classical semanticists might be that the formalization we propose is unnecessary, given that standard formal semantics can handle multiple meanings of words as multiple extensions of a single linguistic element. For instance, consider the verb 'loves' L in the sentence 'Bill loves Monica'. In classical semantics, if 'loves' has two possible meanings, 'love as friends' L_f and 'love as lovers' L_l, these meanings could be represented as a set of extensions $\{L_f, L_l\}$. When Albert applies the operator K to L, we get $K_a(L) = L_l$, and when Kurt applies K to L, we get $K_k(L) = L_f$. However, this approach faces significant issues. By definition, an extension must correspond to something in the physical world. Classical semanticists would need to demonstrate that 'love as friends' and 'love as lovers' have unequivocal physical counterparts. Our framework suggests that such distinctions could correspond to electrochemical patterns in the brains of Albert and Kurt. However, classical semantics does not assume that meanings of words and sentences are physical objects, namely manifestations of neuronal activity.

Furthermore, our framework focuses on multiple intensions rather than multiple extensions. Correcting the classical semantics perspective, L_f and L_l should be viewed as two intensions of L. The challenge lies in formalizing how both meanings remain valid until context or usage determines which one applies. The formalization proposed by classical semantics does not give an account for contemporary validity and arbitrary assignment of meanings to signs. Stating that L_f and L_l are two possible valid intensions of L does not imply that they are both valid at the same time when not used by a speaker because the relationship between these two intensions is not formally defined. To illustrate this concept more effectively, consider a generic vocabulary headword 'love'. In the absence of context or specific usage, all potential intensions of 'love' are listed and are contemporary valid.

Let us summarize the benefits of having a tool that supports information formalized using possible worlds. We said that temporal shifters, intensional verbs, modals, conditionals and other linguistic elements displace the evaluation of certain expressions, translating them into a state of affairs that does not correspond to the spatial and temporal coordinates in which the statement is pronounced. We said that this kind of intensionalization contributes to sentences' meaning with information induced by the contexts, but does not formally specify the particular sense of the terms employed and does not show the natural rich polysemy of the terms. Some of these terms, such as properties in λ-calculus, do not enjoy the possibility of being fully shaped by the context, therefore intensionalization by possible worlds is not able to characterize their meaning. Moreover, we said that Kripkean vision of proper names as rigid designators succeeds in the formalization of the intuition that if a speaker predicates about a certain individual using his proper name to designate the referent, he predicates on the same referents when the sentence is intensionalized. But, again, this does not mirror all the possible meanings of a proper name, which vary on the denotation, namely the numerous homonyms. Finally, we said that formal semantics incapsulate language in a solipsistic exercise of computation, forgetting that language must be always analyzed with its purpose in mind, correct communication, and with its innate enemies, such as incomprehension due to polysemy and so on. This is why intensional superposition could be an important innovation in language formalization, it solves all these issues showing through a simple formal tool and a pair of operators that is possible to give an account to all the complex problems emerging from natural ambiguity of human language.

3.3 Brain waves synchronicity

We are finally closing the circle. We started our journey by assuming that neuro-behavioural evidence is connectable to quantum experimented rules that dominate reality out of sight. In the previous section, we outlined a formal model which mirrors the physical and physiological knowledge at our disposal. Here, we will investigate the recent experimentations in collective neuroscience, showing how effective our model is.

Neuroscientists usually study one brain at a time to observe how neurons fire during certain activities, such as reading or playing a video game. However, a complete understanding of the social functions of our brains requires a broader picture that reflects the dynamic nature of social interactions, as the emergent neural properties that arise from multiple individuals as a single integrated system. Researchers are increasingly incorporating this reality into their studies. A foundational discovery from early studies is that during conversation or shared experiences, the brain waves of individuals synchronize. Neurons in corresponding regions of the different brains fire simultaneously, producing similar patterns. Higher-order brain areas synchronize during more challenging tasks, such as making meaning out of something seen or heard. Scholars are finding synchronicity not only in humans but also in other species, and they are analyzing its rhythm, timing, and fluctuations to gain deeper insights into it.

In the following subsections, we will briefly present different studies and methods to clarify what we have said here.

3.3.1 Functional Magnetic Resonance Imaging hyperscanning

With hyperscanning, we refer to a methodology for measuring the neural substrates of human social interaction first ideated by neuroscientist Read Montague et al. (2002) [68]. The experiment took place at the Baylor College of Medicine in Houston. Montague put two subjects in separate fMRI scanners and recorded their brain activity while participating in a competitive game. The primary objectives of this experiment were to showcase the possibility of tracking concurrent brain activity in two individuals and to pinpoint any technical challenges involved. Since then, the field has improved at hyperscanning with fMRI and expanded to other kinds of technology, which we outline later.

Given that conversation is a primary means of social influence, and its effects on brain activity remain unknown, a recent study (Sievers et al. 2020) [80] explores how group discussions aimed at consensus influence brain activity, fostering neural alignment even beyond the topics directly addressed. In the study experiment, participants engaged in conversations to agree on interpretations of ambiguous movie clips, where the sound was muted, eliminating music, dialogue, and contextual cues that could otherwise influence the understanding of the narrative. Brain scans later revealed similar brain activity among groups that agreed on a conclusive interpretation. A Quantum theory-based analysis of empirical evidence relaying on hyper-scanning by Salomon Rettig (2008) [74] reasonably highlighted issues relating to the uncertainty principle (2.6.6) and the apparatus dependence of measurements (2.6.3). Rettig asks if the merging of two distinct neurological systems detects an objective reality, or creates it through measurement. This question is relevant since is not possible to ignore that the brain processes observed, namely the making out of semantic ambiguities, are electrochemical signalling regulated by quantum laws.

3.3.2 Electroencephalography hyperscanning

In a 2006 study by Babiloni et al. [7], the simultaneous recording of hemodynamic or neuroelectric activity of the brain is hyperscanned through electroencephalography (EEG) performed on a group of people engaged in cooperative games. EEG is another scanning method, which focuses on timing by measuring the speed and sequence of brain activity, prioritizing the ”when” over the ”where” revealed by fMRI. EEG also indicates the relative pace of different brain waves or oscillations. Brain waves fluctuate in cycles, ranging from fast to slow.

The five main types of brain waves, alpha, beta, gamma, delta, and theta, are categorized based on their oscillation rate, each representing different brain states.

- δ waves, ranging from 0.5 to 4 Hz (Hertz) (1 Hz equals 1 oscillation per second), typically signify deep, restful sleep.
- θ waves signify deeps relaxation in a wake state and range from 4 to 8 Hz.
- α waves stand for passive attention during deep relaxation, ranging from 8 to 12 Hz.
- β waves, 13 to 30 Hz, and γ waves, 30 to 100 Hz, are fast and choppy, associated with awake and conscious activity.

Research findings demonstrate causal connections between the prefrontal regions of individuals when engaging in cooperative games across various frequency ranges.

3.3.3 Microendoscopic Calcium Imaging

As we said in 2.7, to initiate action potentials activity, multiple ion channels on the membrane have to open to allow the positive ions to enter the neuron. One type of this ion is the calcium ion. Imagine that you are measuring the calcium ion concentration inside the neuron; when the membrane potential increases, so does the calcium ion concentration, which drops down very slowly. We can harness the change in the calcium ion concentration to look at potential increases. One way to do it is by employing a molecule that can bind to calcium ions and become fluorescent; in other words, when the molecule absorbs light of a shorter wavelength, it emits light of a longer wavelength. Therefore, when a neuron is active, the calcium ion concentration inside the neuron increases, and it glows. We can put calcium indicators into neurons by directly loading them into the neuron using a microinjection pipette, or we can genetically modify neurons, making them able to produce calcium indicators. EEG and fMRI cannot provide the high spatial resolution guaranteed by calcium ion imaging. Microendoscopic calcium imaging allows us to record neuronal activity from multiple neurons simultaneously. UCLA researcher Weizhe Hong et al. (2019) [57] observed that the brains of pairs of mice synchronize during social situations. The synchronized activity arose during various types of social behaviour, and the level of synchronization actually predicted how much the animals would interact. They used microendoscopic calcium imaging to record thousands of neurons in the dorsomedial prefrontal cortex (dmPFC) of pairs of mice engaged in social interactions. Their study provides conclusive evidence for interbrain activity correlations in interacting mice, as well as a cellular-level neural basis underlying this phenomenon. The research also identifies a critical role for interbrain synchrony in coordinating and facilitating social interaction.

3.3.4 Electrochemical patterns semantical mapping

Simultaneous recordings from multiple human subjects using non-invasive techniques such as fMRI and EEG have uncovered notable patterns of interbrain neural activity coupling during language-mediated social interactions. However, there is still much uncertainty about the mechanisms underlying interbrain synchrony during social engagement. Additionally, it is unclear how synchrony originates from individual neurons and neuronal populations, partly because the spatial resolution of current recording techniques in humans is limited and cannot capture single-cell activity. Calcium imaging has only been employed on human induced pluripotent stem cells (hiPSC), reprogrammable cells that can be transformed in every type of body tissue, even in neural cells (Yamanaka 2007) [93]. A study conducted by Estévez-Priego E. et al. (2023) [32] used calcium fluorescence imaging to track spontaneous neuronal activity in hiPSC-derived human and rat primary cultures. The researchers compared the dynamic and functional behavior of these cultures as they matured and found that hiPSC-derived cultures are excellent models for investigating the development of neuronal assemblies.

Our take on the complexity of how meaning can be understood by scientific analysis led us to elaborate a new kind of formal tool, namely the superposition as always-true disjunction, to accommodate the recent evidence in the leading research we have outlined. We think that linguistics and neuroscience could benefit from comparing results to how a large part of the scientific community thinks that sub-atomical particles work, in other words, how reality works. It is unmistakable that what we see through fMRIs, EEGs and microendoscopic calcium imaging are electrons and chemical reagents standing for what we commonly call ”thinking”. Therefore, we sustain that formalizing linguistic elements and intensions echoing the experimental observations we recollected, could guide the disciplines to gain more insights into this enormous problem. Brain synchronicity happening when people coomunicate to make out meanings is a fact, and it is here to stay. So why not improve experimental tests through quantum mechanics knowledge?

Maybe, one day, we will be able to unpack words' and sentences' meanings and clearly see them as different patterns of neural activity, formalizing them as disjuncts of intensional superposition.

4 Conclusion and further suggestions

In this thesis, we presented ”IDSE”, a model that shows the possibility of giving an account of incomprehension phenomena through a new formalization of words’ and sentences’ meaning, which effortlessly integrates into standard intensional semantics. We defined intensions as superpositions, namely possibly infinite and always valid disjunctive strings in which each disjoint represents a meaning. This solution allows us to formalize polysemy due to the arbitrariness of the link between sign and meaning in natural languages, which often leads to incomprehension during interactions. The model includes an evaluation operator that determines the values of the single disjoints in the superpositions, which are then processed by an equality check function that outputs comprehension if the inputs are the same and incomprehension otherwise. The architecture is placed within a materialist framework asserting that electrochemical signals in the brain are intensions conveyed through language, dominated by quantum laws. They are detectable via neuroscience hyper-scanning techniques, including fMRIs, EEGs, and microendoscopic calcium imaging, which testify that neural activity synchronizes between subjects aligning the semantic values of expressions during a conversation. This model overcomes the lack of a correct description of intensional information not originating in the conversational context. In contemporary formal semantics, contextual meanings are provided through possible worlds. However, as we showed, the actual intensional semantics approach fails to exhaust the complete intension of a linguistic object and does not mirror the arbitrary link between sign and meaning.

IDSE is a fully computable model complementary to the actual state of the art in formal semantics, based on the Fregean principle of compositionality and λ-calculus. It also enriches the debate by expanding on concepts like ”sense” and ”idea” by Frege and on semantical implications of mental states as possible worlds in Hintikka’s suggestions for prepositional attitudes verbs.

Furthermore, future works will regard the use of IDSE in other theoretical linguistic frameworks involving not only syntax and semantics but also morphology or phonetics, with the possible choice to detach the model from the physical framework outlined here, to explore its potential in more abstract environments.

A Appendix A

The architecture of IDSE is presented in its bare bone structure.

General architecture of IDSE

Syntax

- Two kinds of objects: types and intensions. We recursively define:
  - a universe T of types
  - for each type $\sigma \in T$, a set of intensions $\phi : \sigma$ of that type

Semantics

- for each type σ, a domain D_σ^M of objects of type σ
- for each term $\phi : \sigma$, a denotation $[\![\phi]\!]^{M,g} \in D_\sigma^M$

Types

- Basic types
  - v is a type (the type of singular intensions)
  - c is a type (the type of wide intensions)
- Inductive clause
  - if σ, τ are types and $\phi : \sigma$, $(\phi_x \vee \phi_y \ldots) : \tau$.

Domains

- $D_v = \{1, 0\}$ (we identify 1 with 'selected' and 0 with 'unselected')
- $D_c = \{C, \neg C\}$ (we identify C with 'comprehension' and $\neg C$ with 'incomprehension')

Intensions

- For each type σ, our model contains a (possibly infinite) set of intensions of type σ
- We define a single intension as a disjunct $\phi : \upsilon$ of a wide intension, namely a disjunction $(\phi_x \vee \phi_y \,...) : c$

  We write a single intension as $\upsilon_x|\phi\rangle_v$, with υ_x being the meaning coefficient, the coefficient that characterize that singular intension

  We write a wide intension as $|\phi\rangle_c = \upsilon_x|\phi\rangle_v \vee \upsilon_y|\phi\rangle_v \,...$

Operators

- K_z is a function mapping each disjoint $\upsilon_x|\phi\rangle_v$ of $|\phi_\sigma\rangle_c$ to a denotation $K_z(\upsilon_x|\phi\rangle_v) \in D_v$ (evaluation of expressions). After mapping, K_z selects one line of evaluation, which is the interpretation of the term according to z.
- E is an equality check function that takes as inputs two terms of type c that have already been processed by K. If the terms are equal, E outputs C. If they are not equal, it outputs $\neg C$. Two wide intensions are considered equal if they have the same evaluation by K

Note: if the model is employed in a typed theory, c and v become secondary grade types.

B Appendix B

We provide an overview of the main concepts within the framework we presented.

1. Brain and brain processes exhibit quantum properties and are ruled by quantum laws

- Atoms are the basic building blocks of matter.
  - Explanation: Atoms are the basic units of matter. All ordinary matter is made up of atoms, which consist of protons, neutrons and electrons.
- At the atomic and subatomic level, matter exhibits quantum behavior.
  - Explanation: Quantum mechanics is the branch of physics that describes the behavior of matter and energy at the atomic and subatomic scales. It provides a mathematical framework for describing the wave-particle duality of matter.
- If something is made of matter, then it exhibit quantum properties and is ruled by quantum mechanics laws.
  - Explanation: Since matter is composed of atoms, and atoms exhibit quantum behavior according to experimental evidence in quantum mechanics, any object made of matter must inherently follow the principles and laws of quantum mechanics at the atomic and subatomic levels.
- The brain is made of matter.
  - Explanation: The brain is a physical organ made up of biological tissues, cells, and molecules. It is composed of matter.
- The brain exhibit quantum properties and is ruled by quantum mechanics laws. ■
  - Explanation: Since we derived that if something is made of matter, then it is ruled by quantum mechanics law, we conclude that since the brain is made of matter, it must follow the principles and laws of quantum mechanics at the atomic and subatomic levels. Therefore, brain processes, which involve the interactions of atoms and molecules within the brain, are governed by quantum mechanics.

2. Thoughts are electrochemical patterns in the brain

- The brain is made up of neurons.
  - Explanation: Neurons are specialized cells in the brain that are responsible for transmitting information.
- Neurons communicate with each other through electrochemical signals.
  - Explanation: Neurons transmit information through electrical impulses along their axons and release neurotransmitters at synapses. These chemicals trigger electrical changes in neighboring neurons.
- Thoughts arise from the activity of neurons in the brain.
  - Explanation: Thoughts, perceptions, and cognitive processes are generated by the interconnected activity of neurons in various regions of the brain.
- Thoughts are electrochemical patterns in the brain. ■
  - Explanation: Since neurons communicate using electrical impulses and chemical neurotransmitters, and thoughts arise from neuronal activity, they are patterns of electrochemical activity in the brain.

2.1. Electrochemical patterns/thoughts exhibit quantum properties and are ruled by quantum laws

- Brain processes are ruled by quantum laws.
- Electrochemical patterns / thoughts are brain processes.
- Electrochemical patterns / thoughts exhibit quantum properties and are ruled by quantum laws. ■

3. Intensions are superpositions

- Superposition is a fundamental principle of quantum mechanics.

  – Explanation: In quantum mechanics, the principle of superposition allows for the coexistence of multiple states of a system until it is measured. When a system is measured, the superposition collapses into a definite state.

- Intension, or the semantic content of words and sentences, is represented in the brain as patterns of neuronal activity.

  – Explanation: Language processing is a brain process.

- Intensions are superpositions. ■

  – Explanation: If brain processes exhibit quantum properties and processing meaning is a brain process, then the representation of meaning in the brain (intensions) must exhibit characteristics of quantum superposition.

4. The need for a new kind of intensionalization in formal semantics

- Language is inherently ambiguous, words and sentences can have wide distributions of potential meanings.

  – Explanation: The relationship between the signifier (the physical form of the sign, such as the sound pattern of a spoken word or the sequence of letters in a written word) and the signified (its sense; the concept or meaning associated with the sign) is arbitrary, which means there is no inherent connection between this two components. This arbitrariness leads to natural language ambiguity and polysemy, namely the possibility that a signifier could correspond to multiple signified or intensions. The ambiguity inherent in natural language is the primary cause of incomprehensions during interactions.

- Standard intensionalization by possible worlds struggles to capture semantic ambiguity inherent in language.

  – Explanation: Standard intensional semantics represents meaning by assigning truth values to propositions in different possible worlds. While this approach can handle certain types of semantic ambiguity, such as modal and propositional attitudes, it struggles to capture the full range of polysemy found in natural language.

- There is a need for a new kind of intensionalization in formal semantics that better captures the ambiguous nature of language and provides a formalization of sentences and words with multiple meanings. ■

  – Explanation: Since intensionalization by possible worlds fails in capturing the semantic richness of words and sentences and there is no correct formalization of words and sentences, such that it mirrors polysemy, a new method is needed to address this limitation.

4.1 IDSE and Intensional Superpositions help explain incomprehensions and provide a suitable explanation for intensions

- The model presented in the thesis has the advantage of adhering to reality by describing meanings as superpositions, as they are electrochemical signals in the brain.

- The tool of superpositional intentions effectively describes the polysemous nature of words and the wide semantics of sentences in natural language.

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