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"CREDIT RISK ANALYSIS AND STRUCTURAL MODELS TO DISTRESSED CORPORATE DEBT VALUATION: THE STEFANEL CASE"

RELATORE:

CH.MO PROF. FABIO BUTTIGNON

LAUREANDO/A: GIACOMO MINNUCCI

MATRICOLA N. 1081565

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"If you survive until tomorrow, it could mean that either a) you are more likely to be immortal or b) that you are closer to death"

Nassim Nicholas Taleb, The Black Swan: The Impact of the Highly Improbable

INTRODUCTION

Credit risk is one of the most analysed risk of the market and, at the same time, one of the most difficult to quantify. It affects a big variety of financial instruments, some of which are simple to understand and some other that are relatively new and more complex to analyse (for example credit derivatives). This work gives focus on *non-quoted debt of distressed companies*.

In an economic environment like the Italian, there are a big number of small and medium enterprises (and sometimes even of big size) that rely completely on the banking system to obtain debt financing. In other words, banks have a central and major role in funding firms, which are not accustomed to issue bond in the market. In such an environment, there are many companies that are listed in the stock market but whose debt is private. Is it possible to exploit information from equity markets to value private debt?

The necessity to value private debt arises in case the firm is in distress. Indeed, in this situation debt is usually characterized by a lower market value than its nominal amount and its real value has to be estimated because not directly observable in the market. In addition, despite their deep distress, those companies are characterized by a positive market capitalization.

The intent of this work is twofold: on the one hand it is aimed at developing a model to "roughly" estimate non-quoted debt's market value of distressed firms, and, on the other hand, at explaining the positive market capitalization of such troubled companies.

These two issues are strictly linked and represent the main research questions investigated in this thesis. Said another way, if one applies valuation techniques such as DCF or APV to estimate equity of a firm in crisis, it is possible to get a zero-value (a negative value cannot be obtained because of the limited liability condition). However, it may be possible to observe a positive market capitalization for that company and this could sound counterintuitive. Why do investor still hold shares of a firm whose equity is estimated to be worthless? Why do creditors (banks) allow for restructuring plan granting additional years to the management to turnaround the company? Is market value of debt so low that creditors prefer to accept a restructuring plan instead of liquidating firm's assets?

Those issues could be explained by using option theory. This is why we chose to develop a structural model, that is, a framework that exploits the interdependencies between how company is financed (its capital "*structure*", equity and debt) and the value of its assets.

To do so, in chapter 1, we will start introducing some concepts related to credit risk analysis. Variables such as PD, EAD, LGD will be presented as well as classical techniques to measure creditworthiness of firms like rating process, traditional valuation and Altman Z-score. In addition, more recent and advanced financial instruments to deal with credit risk, such as credit default swap, will be mentioned.

Chapter 2, instead, will go deep in the literature of structural models starting from the basics of Merton (1974). All the other frameworks, that are more recent and more complex, are grounded on intuition of Merton, but they try to increase adherence to reality and practical applicability. We will choose models that are interdependent but, at the same time, diversified from each other to provide a range of different perspectives and solutions as wide as possible.

In chapter 3, our option-pricing framework will be explained, exploiting both Merton's intuition and the functioning of modern and more complex exotic options with barrier. All the procedures and methodologies suggested to estimate the different inputs of the model will be enlightened as well.

Finally, chapter 4 will present the case of Stefanel Group, an Italian fashion group that is experiencing a period of financial crisis but is still characterized by positive market capitalization. Stefanel Group will be used as a real case to apply everything has been said before. In other words, analysis of past performance, synthetic credit rating and Altman Z-score will be performed to build an idea about creditworthiness of the group. After this credit analysis, our structural model, explained in theory in chapter 3, will be applied and its main results and evidence will be discussed.

CHAPTER 1: INTRODUCTORY CONCEPTS ABOUT CREDIT RISK

1.1 Credit Risk, a first glance

Lending is not only a very old financial activity but also a current relevant topic one can read about in newspapers and can hear in television. Banks in Italy and in the rest of the World lent money to people and companies whose financial situation worsened following the financial crisis and its widespread effects in the real economy. In consequence of their (sometimes unwise) activity, banks have found in their portfolio loans that have lost part of their value or have become worthless and that in the jargon are called "non-performing loans".

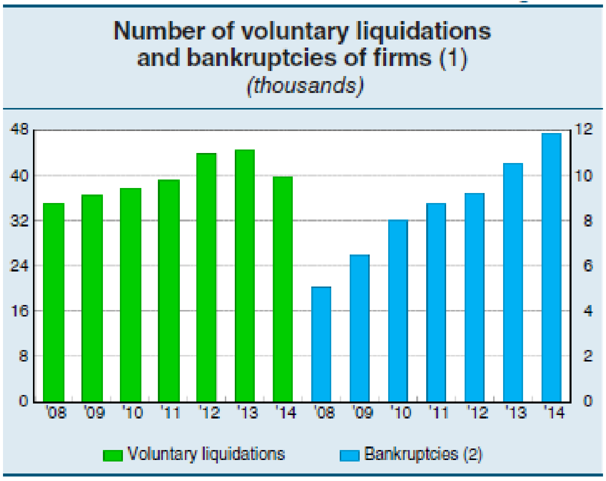
A loan is classified[1] as non-performing when the debtor is failing to make timely payment of principal and interest. Within this class, Bank of Italy, in accordance to AIS/IFRS, has defined four different sub-classes:

- *bad debts*: exposures to insolvent debtors (even if the insolvency has not been declared by a legal court) for which the recovery is doubtful (they are called also as doubtful loans);
- *substandard*: loans to debtors that are experiencing a period of financial difficulty but that are expected to a recover in a reasonable time window;
- *restructured*: are those loans for which the creditor bank has agreed to change (making it less stricter) some terms and conditions of the contracts due to a deterioration of the borrower's financial situation;
- *past-due*: it represents a residual category containing all exposures that cannot be otherwise classified, in particular, all debts that are overdue by over 180 days (for certain kind of exposures the term is 90 days).

Every year, Bank of Italy publics a report providing information about the health of the Italian financial system. In the publication of April, 2015 (regarding 2014), it is possible to find out two interesting pictures where one could notice the surge in bankruptcy of firms (from nearly 5 thousands in 2008 to 12 thousands in 2015) and the upward trend of both non-performing loans and of bad debts. Despite this work aims at analysing and valuing corporate bonds and despite the analysis of Bank of Italy does not make any distinction between debt to households and corporate debt, the two graphs are useful to give readers an idea about how much the topic of credit risk is of current interest.

[1] It is possible to find some information about this classification for example in the website of Intesa San Paolo, in particular in the page: http://www.group.intesasanpaolo.com/portalIsir0/isInvestor/en_FinancialFAQ/Crediti_deteriorati_uk.pdf

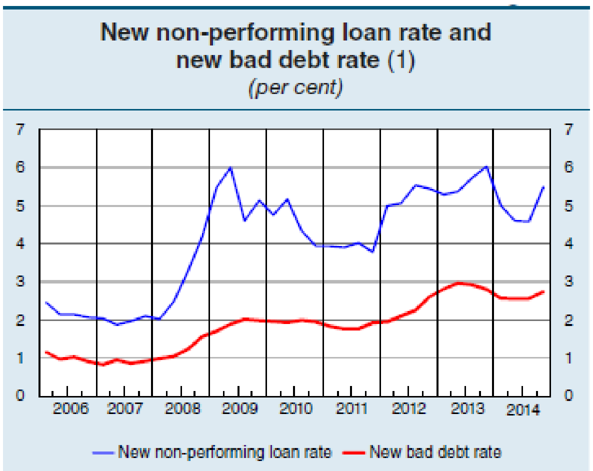
Figure 1 - 1 Number of voluntary liquidations and bankruptcies of firms



Source: Cerved group.
(1) Data for companies that deposited at least one set of financial statements in the 3 years up to the reference date. – (2) Right-hand scale.

Source: Report on Financial Stability N. 1 – 2015 (April 2015). Bank of Italy.

Figure 1 - 2 New non-performing loan rate and new bad debt rate



Source: Central Credit Register.
(1) Annualized quarterly flows of adjusted non-performing loans and adjusted bad debts in relation to the stock of loans at the end of the previous quarter; seasonally adjusted where necessary.

Source: Report on Financial Stability N. 1 – 2015 (April 2015). Bank of Italy.

From those pictures, it is straightforward to understand that every lending activity is subject to credit risk, but when practitioners talk about credit risk what do they exactly refer to?

According to Investopedia.it, it can be defined as "*the risk of loss of a financial reward stemming from a borrower's failure to repay a loan or otherwise meet a contractual obligation*."

Such a description is linked to the concept of *default*, which "always involve the relationship between the debtor firm and the creditor class." (Altman and Hotchkiss 2006, pp.5) and it arises when debtor is not able to meet its obligations, not only about payments but also as regards *covenants*. In the latter case, usually there is a prompt intervention of the management of the debtor company or a new agreement with the creditor.

However, the above-mentioned definition is too much restrictive because default risk could be considered only as a component of credit risk, which in a broader sense includes also the possibility of a reduction in debt's value due to a deterioration in the credit quality of the borrower and not necessarily its default.

In other words, following Fabozzi (2005), credit risk includes two different components that are:

- *default risk*: the possibility that the borrower will not be able to service its debts and obligation and it is linked to the event of default;
- *credit spread risk*: the possibility that credit quality of the issuer gets worse leading to a rise in the credit spread.

When one looks at the yield of different bonds, she or he could notice some differences (spreads) that sometimes may be large. While analysing the yield of a risky bond and that of a risk-free benchmark (e.g. U.S. Treasury Security) with the same maturity, their comparison leads to a difference that represents the premium above the relevant risk-free rate to compensate the riskiness of the investment. That difference is defined as credit spread because it is strictly related to the different credit quality of the issuers. An increase in credit spread means a rise in the perceived riskiness of a bond, a consequent upturn of the yield required by the market and a reduction in the debt value. Said another way, the higher the spread and the lower the price of the debt.

Fabozzi (2005) pointed out that it is important to be careful while talking about credit spread because it does not only include the difference in credit merit of the issuers but also in liquidity risk. Bond markets are known to be not much liquid in the sense that it is not always easy to find a counterparty willing to buy, and this causes a widening in the bid/ask spread. In other words, an investor who wants to sell a bond, sometimes have to accept a lower price than the real intrinsic value, leading to a rise in the spread.

Apart from that, debt could lose value because of fluctuations in the credit spread and this explains the credit spread risk. According to Fabozzi (2005), these fluctuations may be due to both macroeconomic conditions such as business cycle or consumer confidence and issue-specific factors, which are linked to the so-called *downgrade risk or migration risk*.

This "new" risk covers the possibility of a reduction in the credit rating during the term of the bond, indeed a downgrading, which is caused by a worsening in its economic and financial conditions and, obviously, the longer the term of the bond and the higher it is its migration risk.

In conclusion, basically, there are two different meanings of credit risk: one that includes only the default risk and another one that is less restrictive and considers relevant for the analysis not only the extreme event of default but also the possibility of a change in the credit spread.

The latter, in turn, depends on either macroeconomic conditions or downgrade risk. Again, a downgrade implies an increase in the credit spread and a consequent reduction in the market value of the bond.

The two different approaches in defining credit risk were considered also by the Basle Committee in 1999 and lead to two different paradigms for the measurement of *credit loss*, which is the difference between the book value of debt and its future value at the end of a specific time horizon.

It is not in the aim of the work to go deep in the analysis of the two different paradigms, but it is useful just to give a better understanding of the two different meanings of credit risk.

In the first case, default is treated as a binary event and credit loss is a mere consequence of the realization of one of these two possibilities. In case of "no default", credit loss will be zero while in case of "default", it "*would reflect the difference between the bank's credit exposure (the amount it is owed at the time of default) and the present value of future net recoveries (cash payments from the borrower less workout expenses).*" Basel Committee, 1999 and this is Default Mode (DM) Paradigm.

On the other hand, the Marked-To-Market (MTM) Paradigm grounds on the assumptions that debt value is not only affected by default but also by any change in the creditworthiness of the borrower and consequently, it must take into account any downgrading/upgrading made by rating agencies.

In this introduction, some concepts that have been mentioned need to be pointed out for sake of clarity and this is the aim of the following paragraphs.

1.2 Basic components of credit risk

In the previous paragraph, credit loss has been defined as the difference between the *credit exposure* and future value of the debt, but what does credit exposure mean?

Going in order, it represents one of the three main components of the credit risk, which will be presented in what follows.

Probability of Default, PD (Default Risk): it aims at quantifying the probability that debtor fails to make the payment of principal and interests as they become due under the terms of the agreement. Since it is a probability, its value goes from zero to one, $PD \in [0,1]$, and it can be expressed in percentage form.

Exposure at Default EAD: is the quantity of money the bank has lent to borrower plus any accrued interest at the time of default; assuming that default could happen only at interest payment date, among the three, EAD is the only known variable, as pointed out by Guthner (2012).

Loss Given Default LGD: can be intended as the part of the exposure the creditor would lose in case of default of the borrower and it can be expressed either in monetary terms (for example 100m €) or in percentage form (in this case, assuming an EAD of 350m €, the LGD would be equal to $\frac{100}{350} * 100 \approx 28{,}57\%$). It must be pointed out that, like PD, also LGD has both a minimum and a maximum value, $LGD \in [0; 1]$. In the most positive case scenario, lender will recover the entire amount without suffering any loss while, in the worst case scenario, the LGD will be total and the recovered amount will be zero, as stated by Guthner (2012).

Recovered amount is the portion of the debt that avoid the write-off and that the creditor will receive, it is expressed in monetary units (for example 250€). Dividing the Recovered amount by the EAD, one will obtain the *Recovery Rate*, RR (continuing the example, $\frac{250}{350} * 100 \approx 71{,}43\%$) which is complementary to LGD in the sense that $LGD = 1 - Recovery\ Rate$ and in fact $1 - 0{,}7143 \approx 0{,}2857$.

RR and its complementary variable LGD depends on many elements such as debt seniority, credit cycle, industry factors and company-specific factors. One of the most relevant, which is worth to talk about, is the debt *seniority*. Seniority can be seen as a way of ranking the order in which, different loans will be reimbursed by debtor. The basic distinction lays between the two principal rankings: secured and unsecured debt. The former is guaranteed by a collateral (common example is a mortgage where the property of an asset will be transferred to creditor in case of default of debtor) while the latter is not. Within these two rankings, it is possible to find different sub-rankings (e.g. senior secured and junior secured). To conclude, the lower the level of seniority and the lower the recovered amount will be. The presence or the absence of a collateral has an irrelevant effect on the probability of default but heavily influences loss given default differences from a ranking level to another one.

Another element affecting LGD is industry characteristics such as the cyclicality of the business. For industries that are more sensitive to the business cycle, recovery rates will be lower than those of non-cyclical ones. Moreover, within the same industry, LGD varies from a company to another due to specific characteristics.

Both PD and LGD are uncertain variables at the time of the assessment of the credit risk of a loan. It is indeed this uncertainty about the future that let us talking about credit risk. Since they are uncertain, they are random variables, which follow a specific probability distribution and have to be estimated.

It has been already said that default is a binary event. Consequently, PD is a binary variable and the most appropriate probability function is the binomial distribution. Furthermore, also LGD may assume different values in a specific range. According to Guthner (2012), it can be approximated by beta distribution that is a very flexible distribution, defined in the interval [0,1] and shaped according to the value of two parameters α and β which are the exponents of the random variable. Giving specific values to α and β, one can get a shape that is similar to the Normal distribution or, changing them a bit, it is possible to get a skewed bell-shaped curve for example.

Apart from their theoretical probability distribution, they can also computed following two different methods: the *market-price method* and the *ultimate recovery method*. Using either one procedure or another gives rating agencies the possibility to collect historical data about both LGD and RR.

The former is accomplished by computing the difference between face value of the debt (EAD) and the prices of publicly traded bond just after the default. This is a very informative measure but can be observed and used only in case bonds are publicly traded.

The latter consist in valuing the present value of what creditors has received at the end of the bankruptcy process. This is most complicated because the bankruptcy process is long lasting and it can follow two different paths: liquidation or reorganization. In case of liquidation, the recovered amount will be equal to the cash obtained from the sales of company's assets. In case of reorganization, instead, usually the company will issue new bonds and the present value of its cash-flows would represent the sum of money the initial bondholders will recover. This last procedure usually provides higher recovery amounts than the market-price method because, following the default, investors such as pension and mutual funds cannot hold defaulted bonds so that they sell quickly these securities, creating a big downward price impact. On the contrary, in case of reorganization, as time passes, there is higher chance of recovery for the business and, at the same time, higher chances for the firm, its debt and its collateral to be correctly valued in consideration of the upside economic potential.

It is normal that, the actual observed values will deviate from what have been predicted. These differences or deviations, allow the introduction of two additional concepts about loan valuation: *Expected Loss* (EL) and *Unexpected Loss* (UL).

The former represents the long term average loss that an analyst would expect after the estimation of PD and LGD, assuming that EAD is constant. Its general formula is

$$Expexted\ Loss = EAD * PD * LGD. \quad (1.1)$$

Conversely, the latter represents the portion of loss debtor would incur in, due to deviations between realized values (ex-post) and expected values (ex-ante). Its formula is

$$Unexpected\ Loss = EAD * \sqrt{PD^2 * \sigma_{LGD}^2 + LGD^2 * \sigma_{PD}^2 + \sigma_{PD}^2 * \sigma_{LGD}^2} \quad (1.2)$$

Following Guthner (2012), since probability of default is a binomial random variable, its variance can be computed as $\sigma_{PD}^2 = PD * [1 - PD]$ (1.3)

Equation (1.2) and (1.3) can be combined to find a new expression for the unexpected loss

$$Unexpected\ Loss = \sqrt{PD * [1 - PD] * LGD^2 + PD * \sigma_{LGD}^2} \quad (1.4)$$

In order to compute both expected and unexpected loss, an analyst needs first to estimate the variables in formula (1) and (4) which are:

- probability of default;
- loss given default;
- variance of loss given default.

According to Guthner (2012), for the first two variables one can start from data provided by major *rating agencies*: Moody's, Standard & Poor's, Fitch (description of rating process is left to the next paragraph).

Especially Moody's and Standard & Poor's collected historical data about:

- *spot default rate*: representing the yearly percentage of companies that defaulted over the total number of observed companies, classified by rating class;
- *cumulative default rate*: characterizing the sum of the yearly spot rate, also classified by rating class;
- *recovery rates*: defining the percentage of the total exposure that have been recovered by creditor always, divided by rating class.

As readers can easily notice, the concept of estimating spot default rate and cumulative default rate is strictly linked to the credit assessment regularly performed by rating agencies. For now, the tables will be taken as given and, in the next paragraph, the procedure behind the rating procedure will be descripted.

The author suggests to combine and normalize data provided by both the rating agencies from 1920 to current year, into three different tables that then, are used in the estimation process. First of all, a table with historical spot default rates for the first 10 years of a contract, classified by different rating class. Secondly, a table with historical cumulative default rates. Finally, a table with the historical loss given default rates, divided by differentiated by industries and by the position in the capital structure the particular debt contract under investigation has.

Table 1 - 1 Spot default table

S&P	Moody's	1	2	3	4	5	6	7	8	9	10
AAA	Aaa	0.005%	0.013%	0.030%	0.056%	0.091%	0.134%	0.180%	0.224%	0.264%	0.298%
AA+	Aa1	0.017%	0.035%	0.058%	0.098%	0.153%	0.213%	0.272%	0.326%	0.375%	0.420%
AA	Aa2	0.025%	0.051%	0.087%	0.153%	0.228%	0.306%	0.379%	0.446%	0.506%	0.565%
AA−	Aa3	0.035%	0.073%	0.121%	0.195%	0.276%	0.361%	0.447%	0.529%	0.608%	0.686%
A+	A1	0.046%	0.103%	0.173%	0.256%	0.349%	0.448%	0.549%	0.654%	0.760%	0.866%
A	A2	0.065%	0.147%	0.280%	0.431%	0.587%	0.744%	0.903%	1.062%	1.220%	1.379%
A−	A3	0.090%	0.226%	0.454%	0.688%	0.917%	1.145%	1.370%	1.590%	1.809%	2.029%
BBB+	Baa1	0.132%	0.331%	0.681%	1.051%	1.410%	1.755%	2.079%	2.379%	2.663%	2.938%
BBB	Baa2	0.210%	0.538%	1.011%	1.513%	2.008%	2.492%	2.950%	3.373%	3.748%	4.088%
BBB−	Baa3	0.370%	0.924%	1.687%	2.478%	3.244%	3.945%	4.586%	5.182%	5.733%	6.223%
BB+	Ba1	0.623%	1.563%	2.708%	3.965%	5.127%	6.110%	6.950%	7.738%	8.475%	9.132%
BB	Ba2	1.101%	2.730%	4.754%	6.794%	8.458%	9.878%	10.992%	12.034%	13.033%	13.906%
BB−	Ba3	2.165%	4.731%	7.594%	10.419%	12.812%	14.695%	16.172%	17.498%	18.688%	19.775%
B+	B1	3.327%	6.655%	9.764%	12.570%	15.031%	17.221%	19.110%	20.778%	22.297%	23.604%
B	B2	6.074%	11.392%	16.008%	19.821%	22.869%	25.337%	27.336%	29.016%	30.517%	31.862%
B−	B3	9.699%	16.816%	22.597%	27.205%	31.300%	34.517%	37.142%	39.369%	41.198%	42.850%
CCC+	Caa1	14.047%	23.039%	30.352%	36.221%	41.435%	45.462%	48.804%	51.712%	54.227%	56.557%
CCC	Caa2	19.027%	30.267%	37.486%	43.551%	48.888%	53.661%	57.788%	61.524%	64.977%	68.269%
CCC−	Caa3	29.374%	41.209%	49.858%	56.514%	62.208%	67.190%	71.367%	75.060%	78.389%	81.007%
CA	Ca	43.804%	53.008%	59.704%	64.684%	69.091%	72.580%	75.636%	78.422%	80.999%	83.198%
C	C	58.234%	64.807%	69.550%	72.855%	75.974%	77.970%	79.905%	81.783%	83.610%	85.389%
D	D	100.000%	100.000%	100.000%	100.000%	100.000%	100.000%	100.000%	100.000%	100.000%	100.000%

Source: Guthner (2012, pp.102)

Table 1 - 2 Cumulative default table

S&P	Moody's	1	2	3	4	5	6	7	8	9	10
AAA	Aaa	0.005%	0.008%	0.017%	0.026%	0.036%	0.043%	0.046%	0.045%	0.040%	0.034%
AA+	Aa1	0.017%	0.018%	0.023%	0.040%	0.054%	0.060%	0.058%	0.054%	0.050%	0.045%
AA	Aa2	0.025%	0.026%	0.036%	0.065%	0.075%	0.078%	0.073%	0.067%	0.060%	0.059%
AA−	Aa3	0.035%	0.039%	0.048%	0.073%	0.081%	0.085%	0.085%	0.082%	0.079%	0.078%
A+	A1	0.046%	0.057%	0.070%	0.083%	0.093%	0.098%	0.101%	0.105%	0.106%	0.106%
A	A2	0.065%	0.082%	0.133%	0.151%	0.156%	0.158%	0.158%	0.159%	0.159%	0.158%
A−	A3	0.090%	0.136%	0.227%	0.234%	0.229%	0.228%	0.225%	0.220%	0.219%	0.220%
BBB+	Baa1	0.132%	0.199%	0.350%	0.370%	0.360%	0.345%	0.325%	0.300%	0.285%	0.274%
BBB	Baa2	0.210%	0.328%	0.473%	0.502%	0.496%	0.484%	0.458%	0.423%	0.375%	0.341%
BBB−	Baa3	0.370%	0.555%	0.762%	0.791%	0.766%	0.701%	0.641%	0.596%	0.551%	0.491%
BB+	Ba1	0.623%	0.940%	1.145%	1.257%	1.161%	0.983%	0.840%	0.788%	0.738%	0.657%
BB	Ba2	1.101%	1.628%	2.025%	2.040%	1.664%	1.420%	1.114%	1.042%	0.999%	0.873%
BB−	Ba3	2.165%	2.566%	2.863%	2.825%	2.393%	1.883%	1.477%	1.325%	1.190%	1.087%
B+	B1	3.327%	3.328%	3.109%	2.806%	2.461%	2.190%	1.889%	1.668%	1.519%	1.307%
B	B2	6.074%	5.318%	4.616%	3.812%	3.048%	2.468%	1.999%	1.680%	1.501%	1.345%
B−	B3	9.699%	7.117%	5.780%	4.609%	4.095%	3.218%	2.624%	2.227%	1.829%	1.652%
CCC+	Caa1	14.047%	8.992%	7.313%	5.869%	5.214%	4.027%	3.342%	2.909%	2.515%	2.330%
CCC	Caa2	19.027%	11.240%	7.220%	6.065%	5.337%	4.773%	4.127%	3.736%	3.453%	3.293%
CCC−	Caa3	29.374%	11.835%	8.649%	6.656%	5.694%	4.982%	4.177%	3.693%	3.329%	2.618%
CA	Ca	43.804%	9.204%	6.696%	4.981%	4.407%	3.489%	3.056%	2.785%	2.578%	2.199%
C	C	58.234%	6.573%	4.743%	3.305%	3.120%	1.996%	1.934%	1.878%	1.827%	1.779%
D	D	100.000%	0.000%	0.000%	0.000%	0.000%	0.000%	0.000%	0.000%	0.000%	0.000%

Source: Guthner (2012, pp, 103)

Table 1 - 3 Loss given default table

	Automotive	Banking	Construction	Products	Healthcare	Hotels	Industrial
Secured bank loan	36.0%	34.4%	36.8%	36.5%	36.4%	31.1%	34.9%
Equipment trust	39.1%	37.4%	40.0%	39.6%	39.5%	33.7%	37.9%
Senior secured note	49.9%	47.7%	51.0%	50.6%	50.4%	43.1%	48.4%
Unsecured bank loan	57.0%	54.5%	58.3%	57.8%	57.6%	49.2%	55.3%
Senior unsecured note	57.9%	55.4%	59.2%	58.7%	58.5%	50.0%	56.2%
Senior subordinated note	65.9%	63.0%	67.4%	66.8%	66.6%	56.9%	63.9%
Subordinated note	71.0%	67.9%	72.6%	72.0%	71.8%	61.3%	68.9%
Junior subordinated note	77.8%	74.4%	79.6%	78.9%	78.7%	67.2%	75.5%
	Insurance	**Media**	**Misc**	**Oil & Gas**	**Real Estate**	**Retail**	**Steel**
Secured bank loan	36.4%	33.4%	32.7%	30.0%	38.5%	35.4%	39.2%
Equipment trust	39.5%	36.3%	35.5%	32.6%	41.8%	38.5%	42.6%
Senior secured note	50.4%	46.3%	45.3%	41.6%	53.3%	49.1%	54.4%
Unsecured bank loan	57.6%	52.9%	51.8%	47.5%	60.9%	56.2%	62.1%
Senior unsecured note	58.5%	53.7%	52.6%	48.2%	61.9%	57.0%	63.1%
Senior subordinated note	66.6%	61.1%	59.8%	54.9%	70.4%	64.9%	71.8%
Subordinated note	71.8%	65.9%	64.5%	59.2%	75.9%	70.0%	77.4%
Junior subordinated note	78.7%	72.2%	70.7%	64.9%	83.2%	76.7%	84.8%
	Technology	**Telecom**	**Transport Air**	**Transport Ocean**	**Transport Surface**	**Utilites Electric**	**Utilites Gas**
Secured bank loan	38.1%	41.5%	35.5%	33.1%	34.3%	31.7%	26.2%
Equipment trust	41.4%	45.1%	38.5%	35.9%	37.2%	34.4%	28.5%
Senior secured note	52.8%	57.5%	49.2%	45.9%	47.5%	43.9%	36.3%
Unsecured bank loan	60.4%	65.7%	56.2%	52.4%	54.3%	50.2%	41.5%
Senior unsecured note	61.3%	66.8%	57.1%	53.2%	55.1%	50.9%	42.2%
Senior subordinated note	69.7%	76.0%	65.0%	60.5%	62.7%	58.0%	48.0%
Subordinated note	75.2%	81.9%	70.1%	65.3%	67.6%	62.5%	51.7%
Junior subordinated note	82.4%	89.8%	76.8%	71.5%	74.1%	68.5%	56.7%

Source: Guthner (2012, pp.109)

In his book, he presents these three tables whose usage can be easily explained by an example. The procedure to estimate the expected loss of a one-year, secured bank loan in the BB+/Ba1 class, of a company operating in the Media Industry is then straightforward. It is enough to look for the one-year cumulative default rate (that coincides with the spot default rate) of the BB+/Ba1 class that is equal to 0.623% and to the LGD of an secured bank loan in the Media Industry which is 33.4%.

$Expected\ Loss = PD * LGD = 0.623\% * 33.4\% = 0.21\%$

For this kind of loan, an analyst would expect an expected loss of 0.21 percent.

If one would also estimate the unexpected loss, he or she needs also an estimation of the variance of the loss given default, σ^2_{LGD}. Two paths are proposed by Guthner, (2012): it can be estimated starting from historical data, or, conversely, from the beta distribution. He follows the second method arriving to the following table.

Table 1 - 4 LGD versus standard deviation of LGD

LGD	Std Dev of LGD
5.00%	7.4%
10.00%	10.0%
15.00%	11.6%
20.00%	12.8%
25.00%	13.6%
30.00%	14.2%
35.00%	14.6%
40.00%	14.8%
45.00%	15.0%
50.00%	15.1%
55.00%	15.0%
60.00%	14.7%
65.00%	14.5%
70.00%	14.0%
75.00%	13.4%
80.00%	12.6%
85.00%	11.5%
90.00%	9.9%
95.00%	7.3%

Source: Guthner (2012, pp.110)

Given a LGD of 33.4%, its expected standard deviation estimated by the beta distribution is 14.6%.

Now all the data are available for the computation of the unexpected loss following equation (4).

$Unexpected\ Loss = \sqrt{0.623\% * (1 - 0.623\%) * 33.4^2 + 0.623\% * 14.6^2} = 2.86\%$

Looking at the results, one could notice that the expected loss is low but unexpected loss is a bit larger. When moving from a rating class to a riskier one, both EL and UL increase. A senior unsecured debt of an industrial company, with a rating of BB/Ba2 would have an expected loss about 0.62% and an unexpected loss of around 6.07%, Guthner (2012).

What it is important to point out here, is the fact that the loan under investigation is considered as a stand-alone basis and this is the reason why UL may seems quite high. Especially for banks, the loan is usually part of a much wider portfolio, composed of a big number of loans and would benefit from *diversification*.

This simple method is not exempt from shortcomings. Actually, using average spot default rates and average cumulative default rates whose "averages" are computed from historical data can be misleading. Actual default rate may differ from historical average: in a period of recession, the former may be greater than the latter and the contrary may happen during a period of expansion. An analyst must be aware of these problems and should try to find a way to adjust historical averages for current economic condition and this is one of the challenge of credit risk estimation.

1.3 Rating process

The previous paragraph explains which are the main components of credit risk, how they can be estimated and used for the computation of both expected and unexpected losses. It is now the moment to explain in detail what does rating mean and why it is so important.

Every bond issued by a corporation is characterized by a certain level of credit risk and the rating is a way to measure this risk. Credit Rating Agencies (CRA) are companies which professionally and independently from the issuer, assign a rating to bonds according to a pre-defined ranking scale. The "big three" CRAs are: Moody's (1909), Standard & Poor's (1916) and Fitch (1914).

Standard & Poor's defines rating as "*an opinion of the general creditworthiness of an obligor, or of the creditworthiness of an obligor with respect to a particular debt security or other financial obligation, based on relevant risk factors*"[2]. This interesting definition leads to a first important distinction between *issuer credit rating*, that is an assessment of the overall creditworthiness of the company and, *issue-specific credit rating* that on the contrary, assesses the risk of a specific financial obligation and depends on whether the debt is secured or unsecured or whether it is senior or junior.

In other words, a rating is a sequence of symbols (letters or numbers and letters) which summarizes the overall credit merit of a company or the credit risk of a specific debt issuance.

[2] S&P Corporate Ratings Criteria, 1998, pp.3

Figure 1 - 3 Corporate bond credit ratings

Fitch	Moody's	S&P	Summary Description
			Investment Grade
AAA	Aaa	AAA	Gilt edged, prime, maximum safety, lowest risk, and when sovereign borrower considered "default-free"
AA+	Aa1	AA+	
AA	Aa2	AA	High-grade, high credit quality
AA–	Aa3	AA–	
A+	A1	A+	
A	A2	A	Upper-medium grade
A–	A3	A–	
BBB+	Baa1	BBB+	
BBB	Baa2	BBB	Lower-medium grade
BBB–	Baa3	BBB–	
			Speculative Grade
BB+	Ba1	BB+	
BB	Ba2	BB	Low grade; speculative
BB–	Ba3	BB–	
B+	B1		
B	B	B	Highly speculative
B–	B3		
			Predominantly Speculative, Substantial Risk or in Default
CCC+		CCC+	
CCC	Caa	CCC	Substantial risk, in poor standing
CC	Ca	CC	May be in default, very speculative
C	C	C	Extremely speculative
		CI	Income bonds—no interest being paid
DDD			
DD			Default
D	D		

Source: Fabozzi, (2005, pp.328)

Figure 1.3 shows the different ranking levels for each of the three main CRA. As one can easily notice, going from the top to the bottom, credit quality goes down so that AAA bonds are safer than BB bonds and, as a consequence their yield is lower and their price higher. Apart from that, what is crucial is the line dividing investment grade bonds from speculative grade ones. A big number of entities such as pension and mutual funds can invest only in investment grade bonds so that, for an issuer, being downgraded from a BBB-/Baa3 level to a BB+/Ba1 may be relevant because the number of investors who is willing to buy their bonds (and consequently to finance their company) may change significantly. Furthermore, as it has been said before, this kind of downgrading may cause massive sales, leading to a slump in the prices and a surge in the yield requested by the market. Moving from investment grade to speculative level is relevant also because it changes the impact of the different components in the credit risk valuation. On the one hand, for investment grade companies, what matters the most is the probability of default (since PD is low, the role of the LGD is even smaller) while, for speculative bonds characterized by high probability of default, it is the loss given default that drives the valuation of the debt and of its associated expected loss. As a result, junk bonds are more affected by factors that drive LGD such as debt seniority or collateral. In other words, when bonds are rated as junk, their valuation is strongly affected by their ranking in the capital structure (whether they are senior or junior) and by the characteristics of the collateral. The collateral indeed has to be properly valued and has to be enough liquid so that it can be easily transformed into cash in case of default.

The same reasoning holds true when the contract is characterized by a *cross-default provision*. This is a contractual clause establishing that when a company has issued more than one bonds, default in one contract triggers the default of them all. In other words, if a company for example has two bonds outstanding (one senior and another junior), it doesn't matter whether the senior or the junior has defaulted because that event will affect both of them. In such a situation, even thought the two different bonds share the same probability of default, the creditor will incur in different losses according to the characteristics of the two contracts. In general terms, the loss on the senior will be lower than that on the junior so that, ex-ante the senior should have an higher rating than the junior.

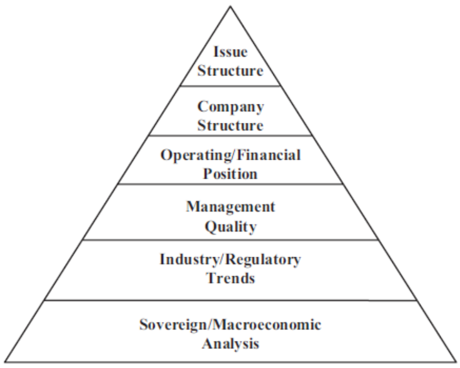
Nonetheless, rating process happens on a continuous basis. Once a credit assessment has been assigned, it could also be reviewed and changed. If a corporation is announced to be on a "rating watch", it means that its rating will be upgraded or downgraded within a short time period (generally no longer than 3 months). On the other hand, "rating outlook" is a forecast of the future credit rating on a longer time horizon.

In conclusion, credit rating, rating watch and rating outlook are information about the credit quality of an issuer or of an issuance that CRAs provide to the market.

After having explained the different rating categories, next step is describing which are the criteria followed by rating agencies to assign and eventually change a credit rating.

According to Crouchy, Galai and Mark (2006, pp.237), "the rating process includes quantitative, qualitative and legal analyses" and the main observed factors are resumed in the following pyramid.

Figure 1 - 4 Moody's rating analysis of an industrial company



Source: Crouchy, Galai and Mark, (2005, pp.238)

Starting from the base of the pyramid, what an analyst should consider first are the macroeconomic conditions of the countries where the company operates and, in particular, in which point of the business cycle those countries are. Periods of economic recession could increase the risk related to the credit worthiness of a firm, causing a downgrading. On the

contrary, during period of expansions, probabilities of default are usually lower reflecting a lower credit risk.
Secondly, the industry where issuer competes has to be analysed, especially its cyclicality, as it has been said before. In addition, the regulatory environment has to be taken into account to understand current entry barriers and how regulation may be changed in the future, increasing or decreasing competitive advantages of a company with respect to another.
The quality of the management plays also a relevant role in the credit assessment, because managers take the most important strategic decisions that will affect the future of the firm in terms of profitability and financial strength. Furthermore, they take decisions about pay-out policies (dividend and stock-repurchase) that may affect capital structure.
The most quantitative part of the analysis is represented by the investigation of balance sheet, income statement and cash flow statement, both actual and projected, to understand the historical performance and future prospective. In this context, it is crucial to perform a proper ratio and cash flow analysis. Since this part is in common with the traditional credit analysis, it will be discussed in the next paragraph.
To conclude, it is worth mentioning that on the one hand, CRA have an important role because they increase the transparency and the efficiency of the market providing useful information that investors couldn't have otherwise. On the other hand, they are paid by the same corporations they judge and this creates space for possible conflicts of interest that could lead to misleading credit assessment. The independence of CRA remains an issue for financial market's authorities.
Apart from possible conflict of interests, CRA are not perfect since they may make mistake and an example is represented by the financial crisis of 2008, when they assigned investment-grade rating to subprime mortgage securities that in turn revealed to be junk and worthless.
In addition, generally, credit ratings lag market price in the sense that changes in financial markets happen quickly while it takes time to revise rating. Even within the same rating class, there may be differences in prices and yield because market prices reflects information other than credit and default risk. In conclusion, credit rating is a useful way to measure credit risk, but it is better for an analyst to couple information provided by rating with other kind of analysis such as traditional analysis, credit scoring models and credit risk models.

1.4 Traditional approach to credit risk

Apart from the rating process already described, there are other kind of approaches that can be used by an analyst while dealing with credit risk. Traditional credit analysis and credit scoring models will be presented in what follows and represent the most conventional way to investigate whether a company will be able to meet its future financial obligations. While the former consist of a qualitative and quantitative scrutiny of the company-issuer, the latter, proposed firstly by Altman (1968), is about giving a score to the debtor and, from that score, deduce its probability of default.

Going in order, firstly it will be presented the traditional credit analysis approach. Following Fabozzi (2007), traditional credit analysis has to do with the "four C's of credit":

- Capacity;
- Collateral;
- Covenants;
- Character.

Capacity refers to the capability of debtors to pay back the loan and represent the most quantitative part of the analysis that should be performed by every investors and also by Credit Rating Agencies. It consists mainly in ratio analysis and cash flows forecast.

Talking about ratios, they can be divided in different categories and the most relevant are:

- **liquidity ratios**

$current\ ratio = \frac{total\ current\ assets}{total\ current\ liabilities}$ or better

$quick\ or\ acid - test\ ratio = \frac{cash+governament\ securities+receivables}{total\ current\ liabilities}$

Comparing total current assets and liabilities is always useful to understand the firm's capacity to pay short-term obligations using its current assets without having to ask for additional financing. Quick or acid-test ratio is more meaningful since it excludes inventories from total current assets in a prudential way because inventories usually are the less liquid assets that cannot be sold quickly by the management.

Generally speaking, "*for any liquidity ratio, the higher the value, the better, and the higher the margin of safety a company has to cover short-term obligations*." Guthner (2012, pp.10). Of course, this does not mean that every company with a current ratio lower than one will default on its debt for sure. Current ratio < 1 means that working capital[3] is negative. Occasionally, according to the point of the business cycle in which the company is operating, a negative working capital can be viewed as part of the financing strategy pursued by managers. [Guthner, 2012]. Nonetheless, it can be an initial signal of financial distress that an analyst should account for.

[3] $Working\ capital = total\ current\ assets - total\ current\ liabilities$

- **Financial leverage ratios**

$$debt\ ratio = \frac{total\ debt}{total\ assets}$$

$$debt - to - equity\ ratio = \frac{total\ debt}{total\ equity}$$

It is important for an analyst to look at the degree of leverage used by the managers, to have an insight about which portion of the investments are financed through equity and which through debt and about whether the company has space to issue additional debt. In this context, it is important to point out that there is no magic number regarding the quantity of debt a company should have relative to its total assets or equity, it also depends on the industry and this is why the ratio should be compared with the competitors. Furthermore, maturity is a parameter that plays a significant role in this environment, to investigate whether there is coherence among maturities of liabilities and those of assets.

- **Profitability and coverage ratios**

$$EBIT\ interest\ coverage\ ratio = \frac{EBIT}{annual\ interest\ expanses}$$

$$EBITDA\ interest\ coverage\ ratio = \frac{EBITDA}{annual\ interest\ expanses}$$

These last two ratios are relevant to see whether the company is profitable or not, because it is through the generation of profits (and cash) that it is possible to have enough resources to pay interest annually and to pay back the loan. The usage of EBIT or EBITDA in the numerator depends on whether the effect of non-monetary costs as depreciations and amortization have to be taken into account or not. Anyway, an index lower than one means that the company is not using the borrowed resources in a proper way because the cost of financing the activity is higher than the operating profits from activity itself.

These listed ratios are only a few among the vast amount of choices an analyst has, but they are meaningful to get a first insight of the company situation. Usually, current ratios are compared with those of the previous years and those of the competitors, they help in understanding the current health of the company but this is not enough because they only allow to look backward. The problem is that what happened in the past will not necessarily happen also in the future. Market conditions and consumers' choices change continuously and so do companies. This is the main reason why ratio analysis is usually performed together with cash-flows projections. Investigating current cash-flows statement and future projections is of fundamental importance because at the end of day the profit created by the company should be converted eventually into cash, and, these resources allow the company to service its debt. Statement of cash-flows is interesting because it does not only allow to understand from which activity the cash has been produced (operating activities, investing, financing) but also give the chance to compute a coverage ratio such as, [Fabozzi, 2007]

$$debt\ service\ coverage\ ratio = \frac{free\ operating\ cash\ flow + Interest}{Interest + Annual\ principal\ repayment\ obligation}$$

It gives the possibility to compare the amount of cash generated during the year and the expenses related to the debt service. This ratio has to be coupled with EBITDA or EBIT coverage ratio. While the latter two compare debt service's costs with a profitability variable (EBITDA or EBIT) that will not be necessarily converted into cash, the former makes the comparison more pragmatic, considering the cash available after having paid all the operating costs and investments in both fixed and working capital.

Future projections are instead a way to forecast future possible evolution of the capacity to produce cash and it is also the base for some company valuation techniques such as FCF (Free Cash Flows) models that will be presented in chapter 3.

Looking at the "capacity" means also taking into account a wide variety of qualitative factors that have been previously mentioned such as industry trends, regulatory environment, competitive position, company structure, risk of special events such as mergers, acquisitions or restructuring. [Fabozzi, 2007].

As it has been already said, the presence or the absence of "*collateral*" affects the distinction between secured and unsecured debt and plays a significant role in the valuation of speculative-grade bonds because they are characterized by high probability of default, and, as a consequence, their value is heavily driven by loss given default. Indeed, the existence of a collateral does not influence probability of default at all, whereas it has a relevant impact on loss given default, which is expected to be reduced by an amount equal to the collateral value. This is only one of the difference in the characteristics and in the valuation of speculative-grade bonds with respect to investment-grade ones. A more deepen comparison will be presented in chapter 2.

Apart from that, a collateral could be a real estate pledged to guarantee a mortgage but could also be a personal guarantee. While talking about real estate, it is crucial to have an understating of its liquidity and on the consequent market value.

Covenants represent a crucial variable since they "*deal with limitations and restrictions on the borrower's activities*" [Fabozzi, 2007, pp.432]. They can be defined as contractual provisions that oblige the debtor company to respect or to avoid pre-fixed conditions in the aim to protect creditors from eventual misbehaviour that can reduce the value of existing debt.

Usually they can be split into two different classes: affirmative covenants, which define the behaviour the debtor has to respect or the level that some financial ratios has to maintain; negative covenants, on the other hand, represent activities or operations that are forbidden or strictly limited. Examples of affirmative obligations could be the fulfilment of specific ratios on leverage (Debt/EBITDA), on debt service coverage (Cash-Flows/Debt Service) or on financial charges coverage (EBITDA/interest). Usually, creditor fixes some prudential threshold above or below which those ratios cannot go and timely control its accomplishment. Whereas, negative covenants could be the prohibition to issue additional senior debt or limitations about dividend's payment or stock repurchase. All these decisions in fact may create a shift in cash distribution from debtholders to equityholders or from a class of debtholders to another, leading to a reduction in the current market value of the debt.
Undoubtedly, covenant breach could represent a first signal of deterioration in the credit quality of debtors. Usually, in those situation creditors have to ask for a prompt intervention of the managers to tackle eventual mismanagement problems or sometimes they have to change those covenants with a reformulation of the financing plan, reducing the amount, changing deadlines or the profile of repayment.

Talking about *character*, it refers to the quality of the management of the debtor company and it includes a variety of qualitative aspects about the strategic direction, financial philosophy, conservatism, succession planning and internal control. [Fabozzi, 2007]. In addition, it is fundamental to investigate the corporate governance model defined as the system of rules, obligations, duties of the principal officers; how and who takes the most important decisions and which are the powers of the internal control body. In the recent past, from the biggest financial/accounting scandals such as Parmalat in Italy and Enron in US, it is well known the fundamental role of a balanced system of corporate governance to prevent bad management decisions that can hurt not only the debtor class but also the company as a whole. Investigating deeply those corporate governance features and agency problems goes beyond the scope of the works, readers can find papers about the issue in the note.[4]

1.5 Credit scoring models

The first credit scoring model was proposed by Altman in his work of 1968. His basic idea is simple and the underlying process can be divided into three steps:

[4] Jensen, M.C., Meckling, W.H., 1976. Theory of the Firm: Managerial Behaviour, Agency Costs and Ownership Structure. Journal of Financial Economics, 3 (4), pp. 305-360.

1. Calculating the credit score of a specific issuer;
2. Mapping credit score to a bond rating equivalent;
3. Using some cumulative default probabilities to estimate default probability in a specific time horizon in the future.

Firstly, he implemented a model to assign a number (technically the score) to specific issuers which is $Z = 1.2X_1 + 1.4X_2 + 3.3X_3 + 0.6X_4 + 1.0X_5$, where:

$X_1 = Working\ Capital\ \ / Total\ Assets$

$X_2 = Retained\ Earnings\ / Total\ Assets$

$X_3 = EBIT\ \ / Total\ Assets$

$X_4 = Market\ Value\ of\ Equity\ \ / Book\ Value\ of\ Total\ Liabilities$

$X_5 = Sales\ \ / Total\ Assets$

$Z = Overall\ Index\ or\ Score$

He started dividing companies (observations) in two groups, defaulted and non-defaulted, and then used Multiple Discriminant Analysis (MDA), which is a statistical technique that gives the possibility to classify a company in one of the two groups according to a series of characteristics or variables. He used a sample of 22 different ratios computed from balance sheet and income statement to understand which of them had the most significant contribution in predicting bankruptcy. MDA gave him the possibility to choose the five most relevant ratios, while weights were assigned though a specific computer algorithm.

It is interesting to mention at least two of the five variables of the model, in particular those with the highest and lowest weight: $X_3 = EBIT\ / Total\ Assets$ and $X_4 = Market\ Value\ of\ Equity\ \ / Book\ Value\ of\ Total\ Liabilities$.

EBIT on Total Assets is the most important variable because it measures the capability of the invested capital (assets) to produce earnings, in other words, the profitability of assets. Since usually the purchase of assets is financed with borrowed capital, looking at this productivity measure is meaningful while dealing with credit risk to understand whether the investment policy was appropriate or not. According to Altman [2006, pp.242], this ratio "*consistently, is at least as predictive as cash flow measures*".

The variable with the lowest weigh instead, is represented by the market value of equity divided by the book value of total liabilities. Despite the marginal role, just its presence in a model of 1968 is cutting-edge in certain ways. This ratio, indeed, allows to compare two fundamental capital structure's dimensions and, in particular, it gives an hint about how much the value of the assets (measured as the sum of market value of equity plus book value of total liabilities) can decrease without the firm being insolvent. In other words, until assets value is higher than the book value of total liabilities, company will be solvent and will continue to operate because it is convenient for equityholders, since equity value is positive. This is the basic idea that has been reorganized by Merton, in 1974, in the option-pricing theory model to value corporate debt like a put option on the company assets. Merton Model is the origin of the so-called structural models whose literature will be described in chapter 2. They are named as "*structural*" because they use the most significant dimensions of capital structure (market value of equity and book value of total liabilities as inputs).

Once this Z-Score model has been built, an analyst has nothing but insert the value of the 5 above-mentioned ratios and computing the score of the debtor company. After that computation, the last step is represented by the definition of a threshold below which debtor companies are classified as insolvent. Altman defined two thresholds and three different classes according to the value of Z:

- if Z is higher than 2.675, the company is considered as healthy;
- if Z $\in [1.81; 2.675]$ it is difficult to categorize debtor as safe or risky since the score lies in a "*grey area*" where it is not possible to make a clear credit judgment;
- if Z is lower than 1.81, borrower has to monitor the debtor firm carefully since its creditworthiness is low.

In addition, the model has been also reworked to allow to assign a credit score also to private firms, changing $X_4 = Market\ Value\ of\ Equity\ /\ Book\ Value\ of\ Total\ Liabilities$ with another measure of the firm financial leverage such as $X_4 = Book\ Value\ of\ Equity\ /\ Book\ Value\ of\ Total\ Liabilities$. This adjustment required a new MDA that resulted in a new model with different coefficients as it is shown in the next figure.

Figure 1 - 5 Z-Score private firm model

$$Z' = 0.717\ X_1 + 0.847\ X_2 + 3.107\ X_3 + 0.420\ X_4 + 0.998\ X_5$$

$$X_1 = \frac{\text{Current Assets} - \text{Current Liabilities}}{\text{Total Assets}}$$

$$X_2 = \frac{\text{Retained Earnings}}{\text{Total Assets}}$$

$$X_3 = \frac{\text{Earnings before Interest and Taxes}}{\text{Total Assets}}$$

$$X_4 = \frac{\text{Book Value of Equity}}{\text{Total Equity}}$$

$$X_5 = \frac{\text{Sales}}{\text{Total Assets}}$$

$Z' > 2.90$: Safe Zone
$1.23 < Z' < 2.90$: Gray Zone
$Z' < 1.23$: Distress Zone

Source: Altman and Hotchkiss (2006, pp. 246)

The second step is to map the obtained credit score into a bond rating equivalent, which means to assign a credit rating to companies with a credit scoring in between a pre-specified interval.

The following table describes that process, for example, companies with a Z-Score in the interval between 3.74 and 4.73 will receive A as rating.

Table 1 - 5 Average Z-Scores by S&P bond rating, 1996-2001

	Average Annual Number of Firms	Average Z-Score	Standard Deviation
AAA	66	6.2	2.06
AA	194	4.73	2.36
A	519	3.74	2.29
BBB	530	2.81	1.48
BB	538	2.38	1.85
B	390	1.8	1.91
CCC	10	0.33	1.16
D[a]	244	–0.20	n.a.

[a]Median, based on data from 2000 to 2004.
Source: Compustat data tapes, 1996–2001, author compilation.

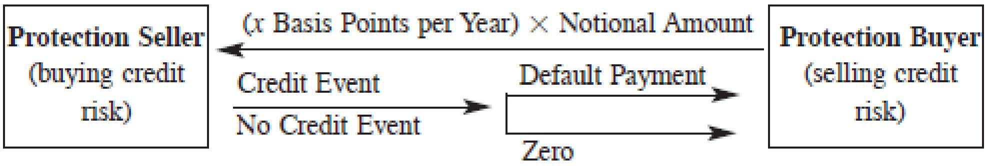
Source: Altman and Hotchkiss (2006, pp.247)

According to the assigned rating, the last step is to assign a default probability to the analysed company. In so doing, one should use tables provided by the major credit rating agencies about mortality rates that shows the marginal and cumulative default rate divided by year of the life of the bond, similar to those used by Guthner.

1.6 Credit Default Swaps (CDSs)

It has been already said that credit risk characterizes every lending activities, so that every lender should be aware of it and of its possible negative consequences. Besides traditional approaches and credit scoring model, and in addition to the more innovative credit modelling techniques to value credit risk such as structural approaches, lenders have the possibility to actively manage, hedge and transfer credit risk through derivative contracts. In the past, hedging activities were represented by means of traditional clauses such as the pledge of a collateral (whose effects have been previously described), guarantees or letters of credit from a third party with a higher creditworthiness, or, finally, the purchase of bond insurance contracts to reduce risk’s exposure. Nowadays, the most frequent way to deal with credit risk is indeed represented by credit derivatives which become common during the 1990s and whose usage skyrocketed in the years of the financial crisis. To understand the reason of this tendency, the structure of the most used derivative contract, Credit Default Swaps (CDSs), will be described.

In general, credit derivatives are divided into two different classes: single-name contracts such as credit default options, CDSs, total rate of return swaps and credit linked notes where underlying assets is represented only by one reference entity; and multi-name contracts that conversely have multiple underlying assets, such as basket default swaps or index swaps.[5]
Particular focus will be given to Credit Default Swaps.
CDSs works as an insurance on the event of default of a specific underlying asset: the "*insured*" or "*protection buyer*" buys insurance by means of selling credit risk to the "*insurer*" or "*protection seller*". The swap provides for two different flows of money between the two counterparties: one for sure and another one only if the "*credit event*" happens. On the one hand, the insured commits himself to pay a periodic premium expressed in number of basis points on the notional amount of the underlying asset that is represented by a bond or a loan issued by a company called "*reference entity*". On the other hand, the insurer has to pay the protection buyer only in case of default of the bond/loan a sum equal to the notional amount minus a pre-fixed recovery amount. This scheme is well summarized by the following figure.
Figure 1 - 6 Typical credit default scheme



Source: Crouchy, Galai, and Mark, 2006, pp. 308.
The amount a counterparty has to pay to the other can be computed in two different ways. There is the possibility for *physical settlement* (or physical delivery), when bonds have to be physically delivered in exchange for a cash payment, and *cash settlement* whereby the credit/debit positions of the counterparties are compensated in the sense that, who is in debit position has to pay the difference to the other with a unique flow of money.
Credit event, which is the condition triggering the default payment of the protection seller, should be clearly defined and represents the key aspect of the CDS contract. (Hull, 2012). Occurrence of credit event has to be declared by a third party in a legal document, for example when the reference entity goes bankrupt or defaults in a scheduled payment, when it is downgraded by a Credit Rating Agency or sometimes when it is subject to restructuring measures, that are the prevailing credit event.
Its maturity can vary from 1 to 10 years (5 years is the most common) while notional amount is usually no lower than 1 million dollars. (Grasselli and Hurd, 2015).

[5] Most of these derivatives contracts are described and explained by Hull (2012).

The main aim of CDSs, as one can easily see, is to hedge credit risk synthetically, selling it to a counterparty who is willing to take the opposite position. In other words, buying the protection is the same as selling short the underlying corporate bond without the involvement of the reference entity. Last sentence shows two of the main advantages of CDS and credit derivative as a whole. As regards to the first advantage, while bonds are less frequently exchanged than equity, and while collaterals are even less liquid, CDS's give lenders and investors in general the possibility to sell credit risk in an easy way, without incurring in too high transactions cost. Furthermore, investors have the chance "*to reverse the skewed profile of credit risk*", The J. P. Morgan Guide To Credit Derivatives (1999, pp. 10[6]). Payoff function of a debtholder is well known: she or he will get the maximum value between zero and the principal plus accrued interests while sharing the risk of a large loss. Conversely, buying a CDS and consequently selling short the bond/loan, gives investors the chance "*to pay a small premium for the possibility of a large gain upon credit deterioration*", The J. P. Morgan Guide To Credit Derivatives (1999, pp. 10). Moreover, a market participant does not need to be a lender to buy CDS but she or he can use credit derivatives with speculative purposes, betting on the default of a specific company or a specific instruments so that, as every derivative contracts, also CDSs could be used both for hedging and speculative reasons. The second advantage of credit derivatives is the lack of involvement of the reference entity. In other words, investors have the possibility to buy and sell CDSs without having any relationship with the company whose debt represents the underlying asset and they can decide the terms of the contract (such as seniority, compensation, maturity) independently of underlying loans, The J. P. Morgan Guide To Credit Derivatives (1999).

Mentioning CDSs in this work is useful because of their important role in pricing corporate risky debts and more generally in determining credit spread. Credit spread can indeed be expressed in two ways: by computing the difference between the YTM of a corporate bond and a risk-free asset (for example U.S. treasury), or directly observing it on the CDSs market; in this case it is called CDS spread. According to the law of one price that rules out any arbitrage opportunity, assuming no liquidity costs and the same maturity for both CDS and corporate bond, the strategy of buying the protection against default with a CDS should have the same cost of investing in corporate bonds. In other words, the price that an investor has to pay to buy a CDS (that are quoted directly in spread), should be approximately the same as the difference between the corporate bond YTM and the risk-free return, indeed, credit spread should be equivalent to CDS spread.

[6] This guide can be found at the website page: http://www.defaultrisk.com/pp_crdrv121.htm.

Three researchers from ECB published a study, in 2009, where they analysed the behaviour of both credit and CDS spread before and during the financial crisis that started to explode in 2007. They empirically discovered that, in the period precedent to 2007, both the spreads shared the same patterns, while, during the crisis, their strict tie was broken and credit spreads greatly increased with respect to CDS spreads. This was due to the search for low-risk investment opportunities by market participants who sold corporate bonds (usually considered as risky) to buy sovereign bonds or to keep cash.

It has been said that, in the period just before the crisis, as it usually happens in stable and low volatility periods, both spreads were characterized by equal behaviours, but truth be honest, CDS spread was slightly higher. In fact, investors who want to be protected against credit risk, usually buy CDS on their markets, which are much more liquid than corporate debt ones, instead of short selling corporate bonds and this made CDS prices to increase with an enlargement of the spreads.

To conclude, having a knowledge about the functioning of credit derivatives is important for an analyst to understand the signals arising from those markets. The advantages of looking at CDS are strictly related to the liquidity of their markets (that are preferred by investors willing to be covered against credit risk) and to the fact that CDSs represent a way of decoupling credit risk from the other risks incorporated in a bond/loan such as liquidity or interest rate risk. Furthermore, while credit spreads have to be derived by computing the difference between YTM and risk-free rate, and consequently are influenced by the choice of the risk-free instrument, CDSs are directly quoted in spread without being affected by anything else, Alexopoulou, Andersson, Georgescu, (2009).

1.7 Conclusion

The main idea of this chapter has been to present some concepts about credit risk to give readers a broad understanding of the topic that will be deeply analysed in what follows.

It is important to know that credit risk includes both default and credit spread risk (which in turn incorporate also migration risk) and that its main components are exposure at default, loss given default and probability of default. Most used techniques to model credit risk have two main aims: on the one hand to price risky debt and, on the other hand, computing its probability of default, which is the most complex variable to estimate among the three above-mentioned. In addition, while assessing the creditworthiness of an issuer, there are important different kind of information coming from the markets and the financial industry that should be processed. The knowledge of how the rating process works and how CRA come up with their judgment is as insightful as a careful observance of credit derivative markets and its related spreads. Moreover, traditional credit analysis and credit scoring models are important as well, since they imply the analysis of a wide variety of quantitative and qualitative information about the issuer company that help in assessing its creditworthiness.

Finally, since the aim of the following chapter is to give a literature review about the evolution of structural models, traditional credit analysis and credit scoring models have been described because they represent the basis where the most advanced credit-modelling techniques are grounded.

CHAPTER 2: CONTINGENT CLAIM ANALYSIS, MERTON AND PRINCIPAL STRUCTURAL MODELS

2.1 Contingent Claim Analysis: an introduction

A contingent claim is a "claim that can be made only if one or more specified outcomes take place" so that it is characterized by uncertainty upon the occurrence of some future events[7].

A derivative is a security whose price depends or derives from one or more underlying assets. The dependence from the fluctuations in the underlying asset brings about reliance on the future that creates uncertainty as well.

Readers can easily notice that the two previous concepts share the characteristic of being uncertain and depending on the future: Contingent Claim Analysis (CCA from now on) puts these two definitions together. In fact, following the website of CFA Institute[8], CCA can be defined as a methodology for the valuation of assets whose value is uncertain and linked to the value of other assets, using the option-pricing theory (options are one of the most common type of derivative indeed).

In other words, if it is possible to characterize the payoff of an asset as a payoff of an option, then it will be easy to apply the Black&Scholes formula to price that asset.

This section is about the application of the CCA for the valuation of risky corporate debt, also called structural models because they exploit the linkage between default risk and capital structure of the firm.

In this sense, the milestone is represented by the Merton Model (1974).

The intuition for that model is the fact that equity can be thought as a call option on the firm's assets. A call option is a derivative that gives the owner the right but not the obligation to buy a security at a pre-specified price (Strike Price, K). If at the maturity T the value of the security is higher than K, then the option will be exercised, otherwise the owner will leave the option expire.

If we suppose that the capital structure of the firm is composed only by equity and one class of debt, it is possible to get an equivalent situation. Equityholders of a company have the right but not the obligation to buy the firm's assets at a strike price equal to the face value of the debt. If at the maturity T of debt the market value of assets is higher than the face value of debt,

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[7] Almost the same definition is provided by different websites as: http://www.nasdaq.com/investing/glossary/c/contingent-claim or http://financial-dictionary.thefreedictionary.com/Contingent+claim.

[8] https://www.cfainstitute.org/learning/products/publications/dig/Pages/dig.v38.n3.35.aspx

equityholders will exercise the option, buying the assets, paying the debtholders and gaining the difference.

In the opposite situation, they will not exercise the option, leaving the company in the hands of debtholders without incurring in any other negative consequences because they will exploit the limited liability condition, avoiding investing additional money.

In other words, it is the same as the company is owned by the debtholders until the maturity T of the debt contract. At the maturity, the decision is up to equityholders either to exercise the call option buying the assets back and consequently paying creditors, or to leave the option expire. In this latter case, the bondholders will get the "ownership" of the assets suffering a loss given by the difference between the face value of the debt and the (lower) market value of the assets.

The previous payoff can be rewritten as $D_T = D - \max[D - A_T, 0]$ in which we can recognize two different components:

1) D, that represents the reward at time T for investing in a risk-free zero-coupon bond with the face value of D;
2) $-\max[D - A_T, 0]$ that is the payoff of a short put options in the company's assets A with a strike price K=D and a maturity T.

Notice that the debtholders' payoff is affected by the value of the put option. In fact, they are selling a put option to shareholders meaning that the higher its value and the lower will be the economic (market) value of the debt. Put differently, the positive benefits following the restructuring plan will be captured by shareholders at the expenses of debt holders.

This is only one of the two possibility to use the option pricing theory to value the debt. Recall in fact that the Merton model exploits the fact that the payoff function faced by equityholders is exactly the same as that of a holder of a long call option with a strike price D (face value of the debt) with a maturity T.

This is precisely the relationship that Merton used in his model (under a specified set of restrictive assumptions that will be listed in the following) to price the corporate liabilities.

Without going deeply in the model, what it is important to notice are the similarities and the differences of the two return distribution.

As stated by Guthner (2012, pp. 47-48) "loans have a predefined limited upside with the potential for a full loss of the investment made" while equityholders "capture all the upside from the unforeseen fortune… (while, in the event of default) will lose all their investment". This is the main difference between equity and debt and the reason why trading strategies of equity-managers are completely different from those of bond-managers.

Despite these differences, it is evident that the value of both debt and equity depends on the market value of the company's assets. The payoff patterns faced by debtholders and shareholders are differently shaped but both depend on the same "underlying asset". This doesn't show only that the Merton model was fundamental to understand the link between the market value of the firm's assets and its equity, as noted by Crosbie and Bohn (2003), but also the fact that the pricing of debt and equity instrument is somehow related.
Going in order, at first the main assumptions, main findings and limitations of the Merton Model will be explained. After that it will be also possible to give a brief look to some other important models that can be found in the really vast literature on the topic.

2.2 Merton Model (1974)

The mathematical rationale behind the model goes beyond the purposes of this work; the main formulas and the main results will be explained without going deeply in the calculation that can be easily found in the original paper.
The assumptions stated by Merton (1974) are:
A.1 no transaction costs, taxes or problems with indivisibilities of assets;
A.2 there are enough investors in the market so that they do not affect the market price of assets with their demand and supply;
A.3 it is possible to lend and borrow at the same interest rate;
A.4 short-selling is allowed;
A.5 it is a continuous time model;
A.6 the irrelevance proposition of Modigliani Miller holds[9];
A.7 the instantaneous riskless interest rate r is known and constant;
A.8 the dynamics for the value of the firm V can be expressed with the following stochastic differential equation $dV = (\alpha V - C)dt + \sigma V dz$, where α is the expected rate of return of the firm, C is the payment in dollars made by the firm in favour of both shareholders and debt holders (e.g., dividends or interest payments), δ^2 is the variance of the return per unit of time and dz is a standard Brownian Motion.
Other important assumptions are:

- the firm being financed only by equity and one class of zero coupon debt;

[9] Modigliani and Miller (1958) stated that the total value of the firms is irrelevant to financing decisions in absence of taxes and bankruptcy costs. In other words, the capital structure does not affect the value creation of the firm but only how that total value is distributed between owners (equityholders) and creditors (bondholders).

- the impossibility to "pay cash dividends or do share repurchase prior to the maturity of the debt", and to "issue any new senior (or of equivalent rank) claims on the firm" Merton (1974, pp.453).

The first four assumptions serve for setting the risk-neutral framework that implies the "no-arbitrage condition".

Obviously, in reality markets are not perfect and frictionless, traders and other participants do not share the same information and there exist exploitable arbitrage opportunities. It results in the fact that real-world probabilities, also called actual probabilities, are usually higher than the risk-neutral ones.

The reason of the last assumption about the impossibility to issue new debt is well explained by Black and Scholes in their work of 1973. When the firm issues new debt (and consequently the face value D increase), keeping the asset's value constant, its market value will increase by a smaller amount because the default risk will also rise and the existing bondholders will suffer the negative effects of a lower probability of being repaid. It is possible to notice that, when a firm increases its indebtedness, there are two opposite effects on the market value of the debt: on the one hand, it will get greater because of the higher face value while on the other hand it will decrease due to the larger probability of default.

Applying the B&S option pricing formula, he got that the value of the equity is exactly the same as of a long position in a call option

$$E\ (V,\tau) = V\mathrm{N}(d_1) - Be^{-r\tau}\mathrm{N}(d_2)$$

where $d_1 = \frac{\ln\left(\frac{V}{B}\right)+(r+\frac{\sigma^2}{2})\tau}{\sigma\sqrt{\tau}}$

and $d_2 = d_1 - \sigma\sqrt{\tau}$;

And that the market value of the debt, that will be called F from now on, is equal to the pricing formula of a put option

$$F[V,\tau] = Be^{-r\tau}\left\{\mathrm{N}\ [h_2(d,\sigma^2\tau)] + \frac{1}{d}\mathrm{N}\ [h_1(d\sigma^2\tau)]\right\}$$

where $d = \frac{Be^{-r\tau}}{V}$ called the "quasi debt-to-equity ratio"[10]

and $h_1(d,\sigma^2\tau) = -[\frac{1}{2}\sigma^2\tau - \log(d)]/\sigma\sqrt{\tau}$

and $h_2(d,\sigma^2\tau) = \ -[\frac{1}{2}\sigma^2\tau + \log(d)]/\sigma\sqrt{\tau}$

The risk premium is

[10] Merton, 1974, defined d as "quasi debt-to-equity ratio" because in the numerator the value of the debt is discounted by the riskless interest rate instead of an appropriate (higher) discount rate causing an upward bias.

$$R(\tau) - r = \frac{-1}{\tau}\log\left\{\mathrm{N}[h_2(d, \sigma^2\tau)] + \frac{1}{d}\mathrm{N}\,[h_1(d\sigma^2\tau)]\right\}$$

where R(τ) is the yield-to-maturity on risky debt provided that the firm does not default.

Finally, the risk-neutral probability that the firm will default by T, is easily computed as $PD = N(-d_2)$.

Despite its extreme simplicity and the impossibility to apply it to a real case without any reworking, the model lays the foundation about the behaviour of:

a. The debt's market value;
b. The risk premium;
c. The risk of the debt relative to the risk of the firm.

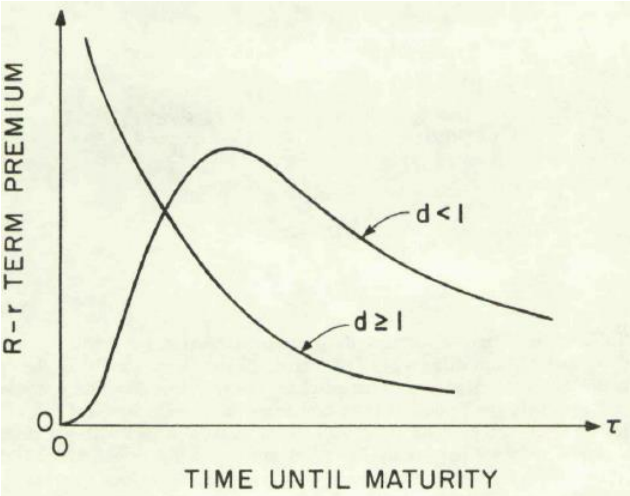
As regards point a, results are straightforward: the economic value of the debt D is positively related to both its the face value and the value of the firm's assets. On the other hand, it is negatively related to time to maturity τ, the volatility of the firm's assets (σ^2) and the riskless interest rate.

Talking about the risk premium, it is interesting to notice its particular relationship with the time to maturity, depending on different levels of d, the quasi debt-to-equity ratio:

a. for low levels of the quasi debt-to-equity ratio ($d<1$), the firm has solid capital structure, and the higher the time to maturity τ and the wider the risk premium will be. This is what one would normally expect: if debt has to be paid far in the future, the uncertainty is high because of the great possibility of negative events for the firm, so that this risk has to be compensated by a higher spread;
b. when $d \geq 1$, then the firm is characterized by an higher probability of default in the near future and correlation between risk premium and time to maturity becomes somehow counterintuitive.

   Indeed an indebted company is characterized by high expectation of default on short-term debt. However, if this company makes it though, it will be expected to recover completely and to be less prone to default in the future. "This is know as *survival bias*, which suggests that the longer a company can go without defaulting, the more likely it is to survive" (Guthner, 2013, pp. 66).

Figure 2 - 1 Relationship between time to maturity and risk premium according to different levels of d



Source: Merton (1974, pp. 459)

Finally, according to Merton, the correct way to measure the debt's risk relative to the risk of the firm is $g[d,T] = \frac{\sigma_y}{\sigma}$ and it is included in the interval $0 \leq g \leq 1$. "The debt of the firm can never be more risky than the firm as a whole, and as a corollary, the equity of a levered firm must always be at least as risky as the firm" Merton (1974, pp. 463).

We can consider the two extreme cases: when $d \to 0$ the value of the debt is equal to risk free bond and its risk is equal to zero, $g[d,T] = 0$; while as $d \to \infty$ the value of the debt nears that of the firm and $g[d,T] = 1$. This sheds light about the connection between the risk characteristic of the debt and the capital structure of the firm.

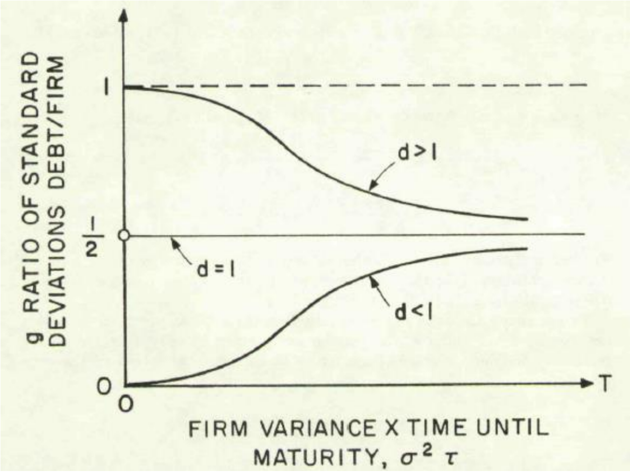
If the company is highly indebted, its probability of default is also great and the risk profile of debt approaches that of the equity. From picture 2.2, it is interesting to observe that as time to maturity gets larger, the risk level decreases: firms whose debt is highly risky are expected to default in the near future, but if they survive, their level of riskiness is expected to be lower as time passes. This is another way to see the survival bias effect (downward slope of the curve for d>1).

On the other hand, when d is really low, the debt is almost riskless and can be discounted using the risk-free rate. As maturity gets larger, the risk level increases and this is show by the upward sloped curve for d<1.

Finally when d=1, which signals that the firm is financed with almost the same quantity of debt and equity, the risk of debt is half of that of the firm independently of σ^2 or of τ.

In conclusion, one should point out that the risk profile of the debt lays between these two extremes according to the fluctuations in the value of d.

Figure 2 - 2 Relationship between capital structure a riskiness of debt relative to riskiness of the firm



Source: Merton (1974, pp. 462)

It is possible to conclude that the model developed by Merton represents a breakthrough in the valuation of corporate debt using option-pricing theory. In addition, it gives insight about the main characteristics of the debt and its behaviour.

Despite its importance, it cannot be used in practice for valuation purposes without any reworking and adjustment because of the strictness of its main assumptions.

In reality, the company has a more complex capital structure than the one assumed by Merton: almost every firm has more than one class of debt with different maturity, different seniority, different covenants, and it is not zero-coupon-bond. Consequently, firms may default not only at the date of maturity, but in whatever moment while, in Merton, the probability of default is zero for each $t < T$ and can be positive only when $t \geq T$. This is confirmed while looking at the term structure implied by Merton (1974) which implies zero default probability for low leveraged companies in the short-term and this is due to the stochastic process governing the asset value dynamic. For high value of V, it is almost impossible that the bankruptcy threshold will be reached. In other words, there will never be an unexpected default. In addition, this stochastic process characterized by the positive drift makes the company assets to increase with time while the amount of debt remains constant (because it is forbidden to issue additional debt) and in that way, in long-term horizon the probability of default is decreasing.

Moreover, the risk-free interest rate and the volatility of the assets' return are not constant for the whole period under analysis. According to Black and Scholes (1973), while the formula assumes a constant variance of the return of the underlying assets, in reality it depends on the value of the assets itself and on the maturity of the option.

In this context, it is worth mentioning the work of Jones at al. (1985) where the authors empirically studied the performances of a simple reworking of the Merton Model observing some systematic mistakes. In particular, they tested if CCA as introduced by Merton, could correctly price risky bonds. To do so, they used CCA analysis to value risky bonds of 15 firms with rated public debt and then compared what they obtained to the respective market values. They first made the test on the whole sample and then they divided it in two sub-samples following a few methods of differentiation: high vs low rated debt, high vs low variance debt, long vs short term debt, high vs low coupon debt, junior vs senior debt.

They got the conclusion that “the model tends to underprice safe bonds and overprice risky bonds in a systematic way” Jones at al. (1985, pp. 256) and they speculated about which assumptions of the model may be violated. In brief, there are three effects to be considered:

1. tax effect: CCA analysis is based on a Modigliani- Miller environment where there are no taxes, while in reality there exist personal taxes affecting the valuation problem. In fact, capital gains are “tax-preferred”, that means taxed at a lower rate than ordinary income (e.g. wages, interest income and coupon). Since debt with higher risk usually have superior coupon, it will also be highly taxed. Consequently, every model assuming a constant tax rate will overprice lower quality debt relative to higher quality one. The same considerations hold true for longer maturity liabilities with respect to shorter maturity ones.
2. constant variance rate effect: they said that it is empirically proven that equity’s variance goes up following a reduction in equity’s value and vice-versa and this may happen also for the firm’s variance. Assuming a constant variance for the assets could be unreasonable and lead to problems in pricing highly risky debt (while for the higher quality, overestimation is not such a problem). As a conclusion, risky debt will be overpriced with respect to safer debt and this also holds for junior bonds relative to senior bonds and so forth;
3. diluition effect: it is not always true that firms cannot pay existing debt by selling their assets or by issuing new bonds with the same priority. The possibility to choose between these two alternatives, gives to shareholders the option to refund (that is the option to pay existing debt issuing new one with same priority, at the expanses of remaining bonds because of this “*diluition effect*”). Since this option has a positive value for shareholders, it implies the overpricing of junior debt relative to senior one or of longer maturity debt relative to shorter maturity one. “In other words, debt can be economically junior either because it is explicitly junior or because it has a relatively longer maturity than other debt. Jones at al. (1985, pp.258).

And these are only three reasons why the Merton model is not workable in practice and need to be adapted and modified.

Indeed, to deal with real cases, it is necessary to use convoluted models and what lots of authors tried to do after 1974, was to relax the assumptions and to add complexity trying to solve at least one of the problems stated above and to make the model fit a more complicated capital structure.

2.3 First Passage Time Model with constant interest rate: Black and Cox Model (1976)

Since the work of Black and Scholes (1973) and Merton (1974), a big number of models have been developed with the aim of increasing the applicability to cases that are similar to reality. As a starting point, it could be important to talk about the paper of Black and Cox (1976), which promptly gave additional sparks in the CCA literature. The authors built a model that shares some assumptions of Merton (1974): they are both in continuous time, rely on Modigliani Miller environment without any taxes or bankruptcy costs and assume a constant risk-free rate r and volatility σ^2.

The model considers a perpetual debt (maturity = ∞) with a constant coupon payment C instead of a zero-coupon bond with a fixed maturity T and these are the main differences from the Merton model. They said that in a discrete-time setting the model should be solved recursively starting from the last payment and going back untile the time when we want to price the debt, but they did not explain any mathematical method to make such calculation. (A way to get to a solution to the discrete-time model has been provided by Geske, 1977).

This model is a first-time passage model because it assumes that default can happen at any time and not only at maturity T as assumed initially by Merton (1974) and the "default time" is modelled as stopping time τ.

Authors assumed three different settings to make the analysis closer to reality: a situation where there are safety covenants to protect debtholders, another one with subordinated bonds and finally one with restrictions on the financing of interest and dividends payments. Going in order, in the first case, they assumed a safety covenant that is a contractual provision that reduces the risk the bondholders incur in, because when the firm value falls below a specific threshold, they have the right to request the bankruptcy of the firm receiving the ownership of the existing assets. In addition, it is assumed that shareholders are entitled to receive a continuous dividend payment proportional to the the value of firm V, aV (where *a* is the proportionality factor).

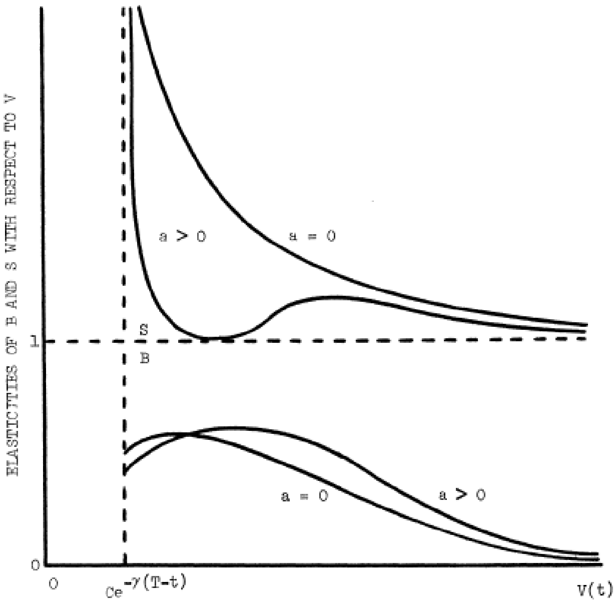
As in Merton, they arrived to a partial differential equation (PDE) that could be solved for B (market value of the debt) only after having specified the boundaries conditions that are:

- $B(V,T) = \min[V,P]$, where P is the promised payment;
- $B\left(Ce^{-\gamma(T-t)},t\right) = Ce^{-\gamma(T-t)}$, where $Ce^{-\gamma(T-t)}$ represents the bankruptcy level.

As the assets reach this level, debt is equal to the bankruptcy level itself, company goes in the hands of debtholders while equity value will be zero. After that, authors assumed $Ce^{-\gamma(T-t)} = \rho P e^{-r(T-t)}$ with $0 \leq \rho \leq 1$, which means that the trigger of the reorganization process (bankruptcy) is given by a fraction of the present value of the promised payment P and they got interesting results.

The value of the debt B depends on the bankruptcy level, which at the same time is strongly affected by the coefficient ρ. They conclude that the higher ρ (which means a greater threshold) and the safer the debt will always be. In other words, the safety covenants provides a floor value for the market value of the debt which cannot go lower even when either σ^2or a goes to infinity. Therefore, the elasticity of the debt relative to the firm (that measures the riskiness of the debt) is characterized by an upper bound and can never go to infinity.

Figure 2 - 3 Elasticities of B and S w.r.t. V for both a=0 and a>0



Source: Black and Cox (1976, pp.358)

From picture 2.3 it is possible to notice that the firm value V cannot go lower than the threshold and that both the elasticities increase as V approaches $Ce^{-\gamma(T-t)}$.

In addition, it is interesting to focus on the elasticity of B with respect to V:

- as one may expect, for high values of V, bondholders have almost the certainty of being repaid so that the elasticity of B relative to V is low, no matter the value of a[11];
- when V is getting lower, the elasticity increases because B strongly depends on the firm value. Since the dividend is proportional to V, it raises the dependence between debt

[11] Conversely, for shareholders, when a=0 there is a convex negative relationship between V and the elasticity of S with respect to V.

and firm value: indeed, in case of a>0, elasticity raises faster and reaches higher level than when a=0;

- as the threshold is approached, the elasticity decreases a bit because bondholders are quite sure about the amount they will receive in case of bankruptcy.

The second settings the authors talked about is the subordinated bonds assuming the presence of two classes of debt: senior and junior with the latter that can be repaid only after the former. Assuming that P and Q are the promised payment of the senior and the junior debt respectively, the new payoff functions is straightforward:

$$Senior\ bond\ B = \begin{cases} V\ if\ V < P; \\ P\ if\ V \geq P. \end{cases}$$

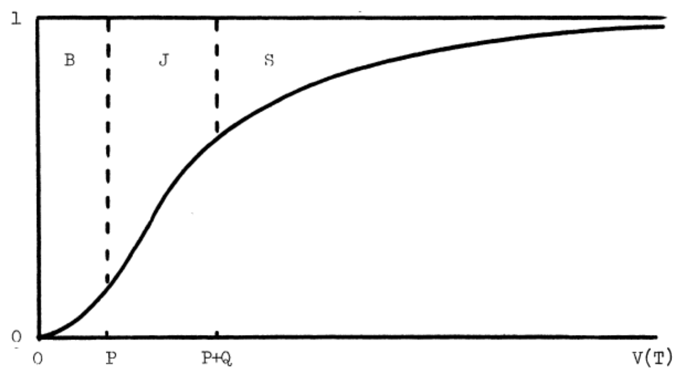
$$Junior\ bond\ J = \begin{cases} 0\ if\ V < P; \\ V - P\ if\ P \leq V \leq P + Q \\ Q\ if\ V > P + Q \end{cases}$$

and finally

$$Stock\ S = \begin{cases} 0\ if\ V \leq P + Q \\ V - P - Q\ if\ V > P + Q \end{cases}$$

The value of these three securities can also be depicted graphically as the areas above the distribution function of the asset's value V after being discounted by $e^{-r(T-t)}$ and their value can be easily computed using integrals.

Figure 2 - 4 Value of the different securities as areas above the distribution function of V(T)



Source: Black and Cox, 1976, pp.360.

In a capital structure with three classes of securities, it is interesting to see that the value of senior bond is higher due to the presence of junior bond. In other words, valuing for example the 60% of the total debt of a company in two different cases, one with only one class of bond and of another one with the remaining 40% of junior bond, we will get a higher value in the second case with both senior and junior debt.

Moreover, the behaviour of the junior debt is somewhat different from the senior counterpart.

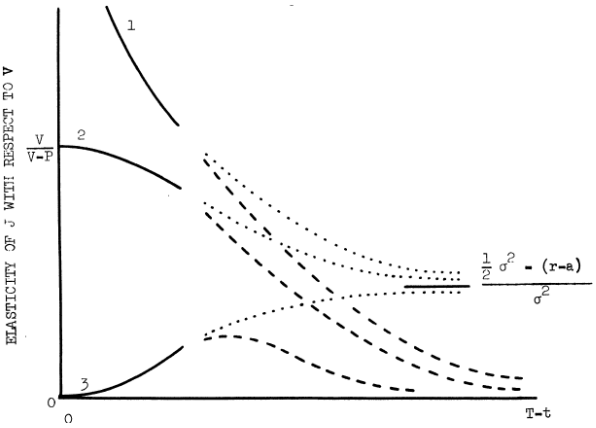
While the latter is always a concave function of V(T) (as V(T) soars, B increases as well but at a decreasing rate), the former initially has a convex relations to the assets value at maturity T, until V(T) reaches the inflection point V* after which their relations becomes concave.
From Merton it has been said that the market value of debt decreases as the assets' volatility goes up and as time to maturity gets larger. On the contrary, for values of V(T) below V* a rise either in σ^2 or in time to maturity, has a positive effect on the market value of the junior bond J. This could lead to two important conclusions:

- when the debt structure is composed of bonds with different priority, it could be possible to have conflict of interest about the riskiness of the projects the company wants to invest in
- the behaviour of the junior debt is in the middle between senior debt and equity depending on the value of V: for $V \to \infty$ the elasticity of J goes to zero and it behaves like a normal debt while for $V \to 0$ the junior debt is highly risky and its behaviour is more similar to the equity.

This could be readily recognized from figure 2.5 where it is possible to see the shape of the elasticity of J with respect to V according to the different value of V itself.

Figure 2 - 5 Elasticities of J w.r.t. V in 3 different situations

1. V<P
2. P≤V≤P+Q
3. V>P+Q



Source: Black and Cox (1976, pp.361)

In the last third case regarding the restrictions on the financing of interests and dividends, that can be paid only by issuing either equity or subordinated debt, the authors found that the market value of debt with these restrictions is always higher than in the case where assets could be sold.
In conclusion, Black and Cox (1976) extended the model of Merton to account for different indenture provisions like the "safety covenants" or the "subordination arrangement" increasing

a little bit the adherence to real cases and above all giving interesting insight about the behaviour and characteristics of bonds in these common situations. Furthermore, they allow for the possibility of an early-default with respect to the maturity of the contract *T*, slightly increasing the short-term probability of default. On the one hand, "safety covenants" is an interesting way to deal with the definition of a default barrier below which default is triggered, while on the other hand, the possibility to issue bonds with different priorities widens the analysis to more complex capital structure.

Unfortunately, it shares with the previous model of 1974 the weaknesses regarding the assumptions about the constant risk-free rate, the constant variance and the absence of taxes and bankruptcy costs. In addition, also the infinite default maturity is far from reality.

2.4 Optimal Capital Structure Models

In this section, two models will be discussed. Both of them can be considered as a way to deal with the estimation of the barrier that brings about default and, in addition, they allow also for the presence of bankruptcy costs and taxes to see how these two variables affect the analysis. Differently from Black and Cox (1976), where the barrier is taken as given, both the models that will be presented consider that it is determined by shareholders in order to maximize their payoff and the event of default is a consequence of their decision.

Nevertheless, while Leland model (1994) considers an infinite maturity, Leland and Toft (1996) allow for a specific expiration of the debt contract at time *T*.

2.4.1 Leland Model (1994)

Leland (1994) contributed significantly to the CCA analysis introducing into the problem of debt valuation two additional aspects of critical relevance:

- the presence of Bankruptcy Cost and of Tax Benefits (that will be called BC and TB respectively);
- the analysis of the optimal capital structure.

Despite he used some assumptions of Merton (1974) and B&C (1976) (the firm's asset are indeed assumed to follow a diffusion process with constant volatility and constant risk-free interest rate), his model increases flexibility and realism of the analysis.

In addition to the two just explained, the main model's assumptions are:

- value of firm's assets is independent of the capital structure (the first Modigliani-Miller proposition about the irrelevance of the capital structure is held true),
- capital structure is assumed to remain constant as well as the face value of the debt;

- time independence: value of corporate securities, both debt and equity, are supposed to depend only on firm's value but not on time[12].

According to the author, this last assumption is critical because it allows to obtain a closed-form solution. Furthermore, it can be justified by referring to two different debt contracts. One with a very long maturity, where the present value of the principal is almost zero, and another one, "when, at each moment, the debt matures but it is rolled over at a fixed interest rate…[and]…this environment bears resemblance to some revolving credit agreements", Leland (1994, pp.1215).

In addition, two different "bankruptcy trigger conditions" have been analysed:

1. **exogenous trigger**: the classical condition when the value of the firm's asset falls below the value of the principal as in the case of a "positive net-worth covenant" (Brennan and Schwartz, 1978 used this bankruptcy trigger, but without getting any closed-form solution);
2. **endogenous trigger**: the firm goes bankrupt when it is no more able to issue equity to pay short-term obligation (assuming that the sale of the company's asset is forbidden).

Starting from the *exogenous* bankruptcy level, the time-independence assumption allowed the derivation of a closed form solution for every security that depends on its instantaneous pay-out $C \geq 0$ financed by equity issuing and on the value of the activities of the firm V.

Denoting V_B as the exogenous trigger, $0 \leq \alpha \leq 1$ as the part of V_B that will be lost in case of bankruptcy and τ as the corporate tax rate, the two boundary conditions for the debt D(V) are:

when $V = V_B, then\ D(V) = (1 - \alpha)V_B$,

and as $V \to \infty, then\ D(V) \to C/r$.

When the asset's value approaches the bankruptcy level, the debt is equal to that level itself minus the cost of bankruptcy, αV_B; on the other hand, as V gets larger, the riskiness decreases and the debt becomes risk-free.

The important intuition of Leland was to identify both bankruptcy cost (BC) and tax benefit (TB) as a time independent claims on the firm value V. In that way, once fixed the appropriate boundaries, it was also possible to obtain closed form solutions for them.

When $V = V_B, then\ BC(V) = \alpha V_B\ , and\ TB(V) = 0$ and as $V \to \infty, then\ BC(V) \to 0\ and\ TB(V) = \tau C/r$, meaning that bankruptcy cost is a decreasing, strictly convex function of V while tax benefit is an increasing, strictly concave function of V. Said another way, as V goes up, the riskiness of the debt reduces and consequently BC tends to zero while TB is valued

[12] The time independence can be seen as coherent with both Merton and B&C models where the debt's maturity was assumed to be infinite.

as a perpetuity discounted at the risk-free rate r. Conversely, when the firm goes bankrupt because the asset's value reaches the trigger V_B, BC becomes sure while there is no tax benefit from the debt.[13]

The solution of debt value that Leland got is almost the same as that of Black and Cox (1976) except for the inclusion of BC and TB.

After those computations, the value of the firm can be easily found by subtracting BC and summing TB to the asset's value V.

The case of *endogenous* trigger is the same as the contract does not provide any protective covenants in favour of debtholders and the firm defaults on the required instantaneous coupon payment if nobody is going to buy its equity. In other words, when equity value becomes null the firms is in bankruptcy. The barrier under which the firm defaults is not defined by any indenture but determined endogenously as a result of a maximization problem by shareholders (to maximize equity value) and chosen as low as possible, provided that for all values $V \geq V_B^{end}$[14], $E(V)$ will be positive.

According to Leland (1994), this new trigger is independent of both the current asset value V and of the bankruptcy cost α; it is proportional to the coupon payment C; and it decreases when the corporate tax rate τ, the risk-free rate r and the riskiness of the firm σ^2 either increase.

Finally, the interest rate paid on the unprotected debt is computed as $R(C/V) = C/D(V)$ and it depends positively on the ratio between the coupon C and the firm value V.

It is interesting to consider that a rise in bankruptcy costs causes a reduction in the value of debt while an increase in corporate tax rate, through its impact on V_B^{end} (that will be lower), has a positive effect on D. However, it is less straightforward the effect on the debt's value when V is close to V_B^{end}. In fact, as V decreases becoming close to the trigger, some relationships may appear to be counterintuitive. For example, D may go down following a rise in the coupon C or it may go up due to an increase in the riskiness of the firm σ^2 or in the risk-free rate r. The explanation for this stands on the effect these variables have on the trigger V_B^{end}: should either σ^2 or r or C get larger, the trigger will go down making the default less imminent and this effect will prevail.

Conclusions previously drawn are only valid for firms whose asset's value is low and close to the bankruptcy level (usually considered highly risky) but not for safe debt (of company with high value of V). Consequently, "the behaviour of "junk" bonds (or "fallen angels") differs

[13] Implicit in this reasoning is the assumption that the firm does not benefit from the tax deductibility of its coupon payments only when it goes bankrupt while, in reality, in order not to have tax benefit, it is enough to have an EBIT lower than the coupon payment.

[14] The superscript "end" is to identify the endogenous trigger from the exogenous one.

significantly from the behaviour of investment-grade bonds when bankruptcy costs and/or taxes are positive." Leland (1994, pp.1223). As a result, the risk structure of interest rate has to be re-written, at least in part: the spread over the risk-free rate is positively related to a rise either in the coupon payment C, in the bankruptcy cost α and in the corporate tax rate τ. However, for junk bonds, it may go down as either the riskiness σ^2 or the risk-free rate r, goes up, for the same reasoning as before.

Recall that the value of debt may eventually decline following a rise in the coupon payment C. This implies that D reaches a maximum $D_{max}(V)$ for a certain coupon $C_{max}(V)$ and, as a consequence, $D_{max}(V)$ can be intended as the firm's debt capacity and both the values can be easily computed.

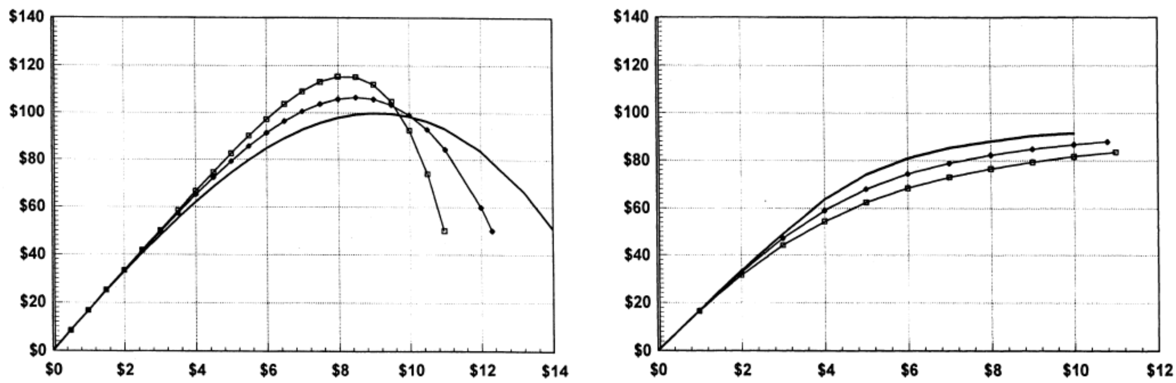
What has not been mentioned yet, but that is critical to point out now, is the fact that the two different bankruptcy conditions, the exogenous trigger ("positive net-worth covenant" that makes debt safer) and the endogenous trigger (unsecured debt) may be considered as proxy for long-term debt and short-term debt, respectively. As long as this approximation holds true, the differences in the behaviour of the variables of the model may be extended to the comparison between long-term and short-term maturity debt.

Giving a brief look at the following four figures taken by Leland (1994, pp.1225-1238) will help in the understanding of the main differences.

The three curves of each graph correspond to different level of asset volatility: σ=0.15 (solid line), σ=0.20 (filled diamond), σ=0.25 (open square). In addition, in each graph the risk-free rate r=0.06, bankruptcy costs α=0.50 and the corporate tax rate τ=0.35.

Figure 2 - 6 Debt value (Y-axis) as a function of coupon (X-axis)

a) unprotected debt b) positive net-worth covenant



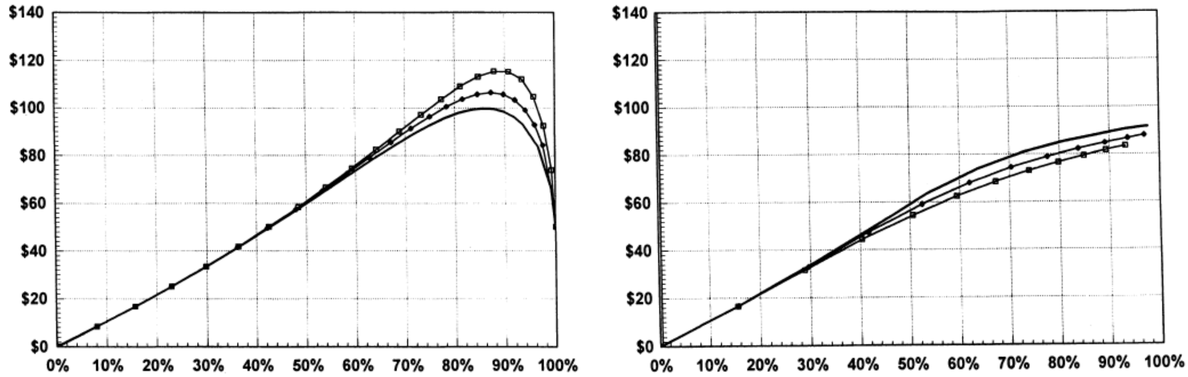
Source: Leland (1994, pp.1225, 1235)

It is possible to notice that the vertex of each curves on part a) represents the maximum debt capacity and the corresponding maximum coupon above which the reversal starts. For protected debt (on the right), results are straightforward in the sense that the debt value increases with C without any reversal.

Finally, when debt is unprotected, riskier firms have the higher maximum debt capacity while for the safety-covenant the opposite is true.

Figure 2 - 7 Debt value (Y-axis) as a function of leverage (X-axis)

a) unprotected debt b) positive net-worth covenant

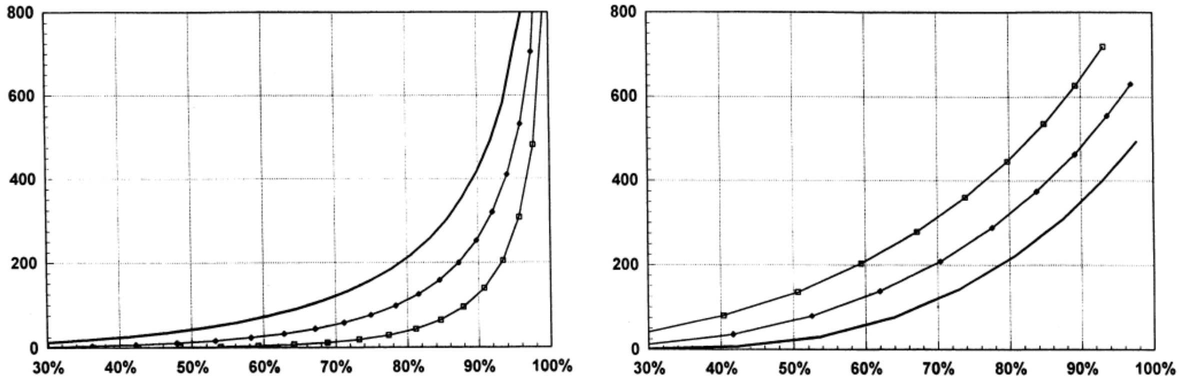


Source: Leland (1994, pp.1227, 1236)

When costs of bankruptcy are positive (α>o), value of unprotected debt can be greater than that of the protected one, especially for high levels of leverage. In case of positive net worth covenant, the bankruptcy trigger is higher so that default will be more likely. Consequently, in this case, bankruptcy cost will be higher and the value of the debt lower.

Figure 2 - 8 Yield spread on debt (Y-axis) as a function of leverage (X-axis)

a) unprotected debt b) positive net-worth covenant

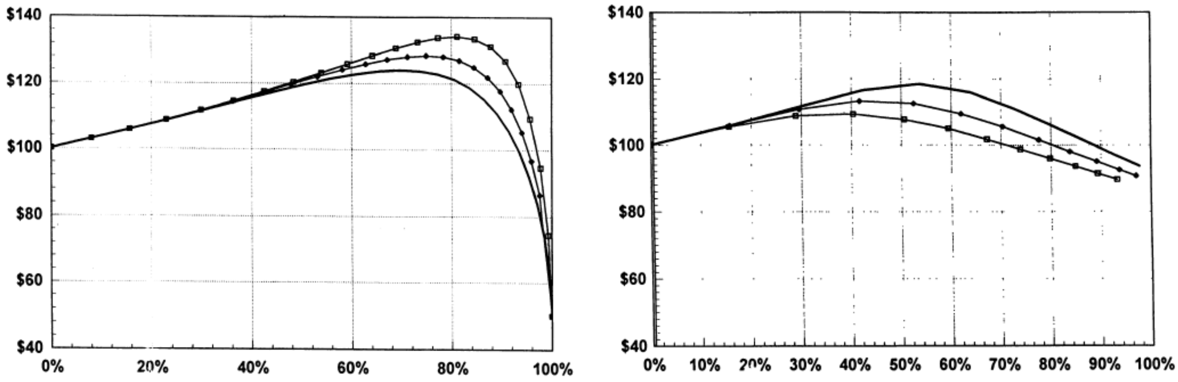


Source: Leland (1994, pp.1229, 1237)

Figure 2.8 shows that the spread on protected debt are higher than those on unprotected ones except when leverage is extremely high. This is a consequence of what has been said in the previous figure. To compensate for the lower value of the protected debt, the spread should be higher than in the unprotected case.

Figure 2 - 9 Total firm value (Y-axis) as a function of leverage (X-axis)

a) unprotected debt   b) positive net-worth covenant



Source: Leland (1994, pp.1231, 1238)

When debt is unprotected, the maximum potential value of the firm is higher than that of the protected one, but the optimal leverage necessary to reach it is also greater. This implies a larger interest rate paid at the optimal level of leverage.

This model is of paramount importance in the CCA analysis because it succeed in finding a closed form solution for the debt including the effects of both bankruptcy costs and tax benefits in the valuation problem. Moreover, it links the valuation framework to the choice of optimal capital structure of the firm and the related agency problems between shareholders and debtholders.

The main shortcoming is the infinite maturity of debt contract under investigation. Despite it could be viewed as an approximation for certain kind of agreement, in reality each contract has a finite maturity and the maturity itself is a relevant factor of the analysis.

2.4.2 Leland and Toft Model (1996)

The work of Leland and Toft (1996) can be interpreted as a continuation of the previous Leland model (1994) to give major insight in the valuation of a debt contract with finite maturity and endogenous bankruptcy and in the analysis of the optimal capital structure and of the connected agency problems.

The two models are strictly related and based upon the same assumptions and the only difference is the finite maturity of the debt contract.

The authors found an equation for the market value of each debt issue that is given by the sum of the discounted expected value of the coupons, of the principal and of portion of asset value that goes to bondholders in case of bankruptcy.

$$d(V;V_B^{end},t) = \int_0^t e^{-rs}\, c(t)\left[1-F\left(s;V,V_B^{end}\right)\right]ds + e^{-rt}p(t)\left[1-F\left(t;V,V_B^{end}\right)\right]$$
$$+\int_0^t e^{-rs}\rho(t)V_B^{end}f\left(s;V,V_B^{end}\right)ds$$

where F(s) can be seen as a synonym of the probability of default.

Then, they assumed the firm to have a *stationary debt structure* where the total principal of all issued bonds P and its maturity T are constant. Said another way, as long as the firm is not in bankruptcy, it will issue new debt continuously, at a rate $p = P/T$ per year, with a maturity T; while, at the same time, an equal portion of previously-issued bonds expired and is paid.

Value of the debt is given by

$$D(V;V_B,T) = \int_{t=0}^{T} d(V;V_B^{end},t)dt$$
$$= \frac{C}{r} + (P - \frac{C}{r})(\frac{1-e^{-rT}}{rT} - I(T)) + \left[(1-\alpha)V_B - \frac{C}{r}\right]J(T)$$

where $I(T) = \frac{1}{T}\int_0^T e^{-rt}F(t)dt$

and $J(T) = \frac{1}{z\sigma\sqrt{T}}(-(\frac{V}{V_B^{end}})^{-a+z}N[q_1(T)]q_1(T) + (\frac{V}{V_B^{end}})^{-a+z}N[q_2(T)]q_2(T))$

and $V_B^{end}$is the endogenous bankruptcy trigger, that is chosen by shareholders in order to maximize both the value of the equity and of the firm subject to the constrain $E(V) \geq 0\ for\ all\ V \geq V_B^{end}$.

It is worth noticing the characteristics of this new trigger: despite the independence of time, it now depends on the maturity T.

Unlike Merton (1974) or Black and Cox (1976), in this case bankruptcy is not an “accounting” event arising whenever the current asset value is lower than the principal, but it is triggered by shareholders according to the previous mentioned maximization problem. In other words, each debt issue has two effects: it leads to an appreciation of the equity value but it also corresponds to a cash-outflow for equityholders.

When $V \geq V_B^{end}$, equity value is positive and for shareholders it is convenient to finance the debt repayment because their expected cash-outflow would be less than the expected equity appreciation. In the alternative case of $V < V_B^{end}$, the opposite is true and default will be triggered by shareholders. Thus, the bankruptcy trigger does not coincide with P.

All of this is very important to understand why and when companies default. In fact, debt with long maturity is usually characterized by a trigger, V_B^{end}, lower than the principal, meaning that the firm will avoid the default despite a negative net worth because it has enough asset (even if at low level) to justify the payment of the coupon.

On the other hand, when maturity is approached ($T \rightarrow 0$), then $V_B \rightarrow P/(1-\alpha)$ which is greater than P when α>0: "thus when debt is short term and α>0, bankruptcy will occur despite net worth being positive", Leland and Toft (1996, pp.994).

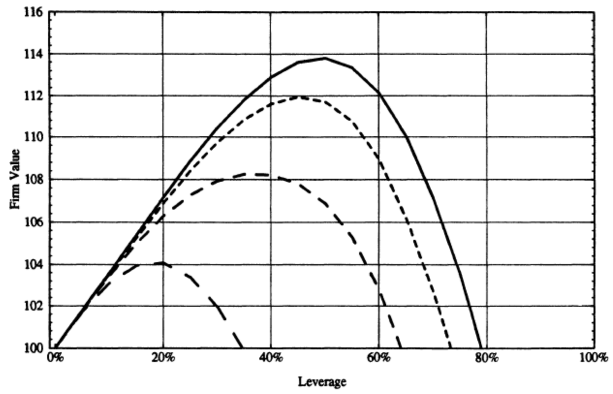
Finally, the total market value of the firm is found as the sum of three components as in Leland (1994): value of the assets plus the value of tax benefits minus bankruptcy costs.

This value is strictly linked to the leverage ratio and to the debt maturity: the further on time the debt expires and the greater will be the optimal leverage as in figure 2.10, with a consequent higher value for the firm.

Figure 2 - 10 Firm value as a function of leverage for different maturities

Legend T = 6 months (long dashed line); T = 5 years (medium dashed line);

T = 20 years (short dashed line); T = ∞ (solid line).



Source: Leland and Toft (1996, pp.997)

Since longer maturity debt has a positive effect on the firm value, readers could question about the reasoning behind the choice of short term financing. One possible answer lies in the agency problems between equityholders and debtholders, especially regarding "*asset substitution*[15]".

Long maturities have a positive effect on the firm value but, at the same time, they have the fallout of increasing the average risk of the firm because of the previously-mentioned asset substitution problem. Consequently, the choice of a right combination between long-term and short-term financing have to take into account this trade-off.

To understand the dimension of the problem, the authors computed the partial derivative of debt and of equity value with respect to the firm volatility σ, for debt contracts of different expirations. They found that:

[15]Asset substitution happens when shareholders have the incentive to increase the average risk of the firm, being in favour of the more risky investment projects. In this way, they transfer to themselves part of the payoff that otherwise would have gone to debtholders.

- for all maturities, as firm is proximate to bankruptcy, $\frac{\partial E}{\partial \sigma}$ becomes positive before $\frac{\partial D}{\partial \sigma}$, meaning that there is a time frame when shareholders have the incentive to increase the riskiness of the firm while debtholders do not. This is when conflicts of interest arises and this range gets larger as T increases;
- differently from long maturities, short deadlines are characterized by lower gains from the greater level of riskiness implying that short-term financing reduces agency problems;
- when default is approached, it may happens that both derivatives have a positive sign and, as a result, there exist a period when both the claimants agree to increase the firm's risk.

Moreover, the choice is made even more complicated by the presence of bankruptcy costs and tax benefits. When the cost of bankruptcy α, are high, lower amount of debt and longer maturities would be preferable because positive effects on the trigger will be higher than the rise in agency costs associated to longer T. The trigger indeed, will be reduced postponing bankruptcy and its related costs.

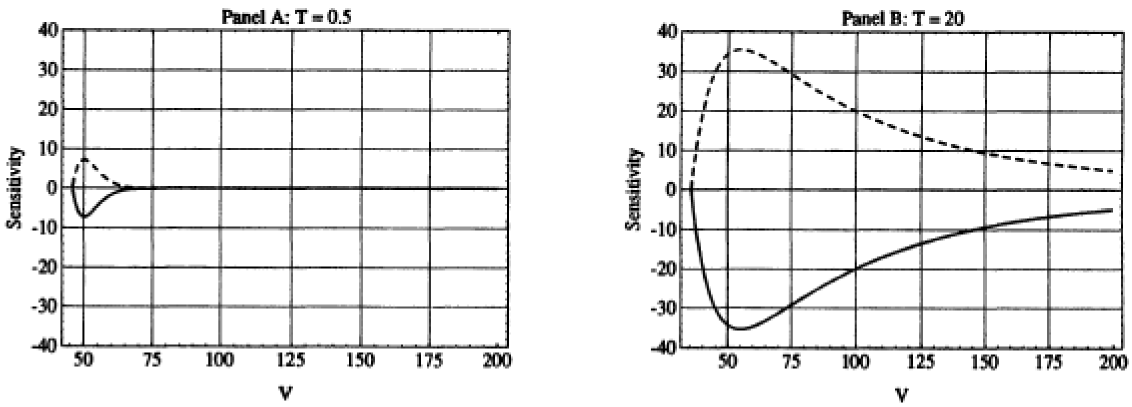
Furthermore, firms with higher risk would be better off choosing lower amount of debt but also shortening the average expiration T because, when σ is already high, the positive effect on firm value is smaller than the negative agency costs effect.

Finally, as tax rate, τ, and bankruptcy costs, α, get smaller, the conflict of interest between the two different classes of claimants is greater because all of the firm value has to be split between them, without any complication related to either BC or TB and this is showed by figure 2.11.

Figure 2 - 11 Effects of an increase in risk on bond and equity values when tax-rate and bankruptcy costs are zero

Legend: Derivative of equity value with respect to the firm risk: dashed line;

Derivative of debt value with respect to the firm risk: solid line.



Source: Leland and Toft (1996, pp.1012).

Talking about debt capacity and debt value, L&T (1996) arrived to the same interesting and striking results of Leland (1994). The latter work discovered this relationship only for long term debt while the conclusions of the former hold true whatever the maturity. In fact, the debt of highly leveraged companies (considered as "junk") increases in value as either r or σ get larger. This follows the diminishing effect the two variables have on the bankruptcy trigger, making default and its related costs less imminent.

The term structure of credit spread they found, and consequently of interest rate, points out the same results as Merton (1974) and Leland (1994) where the spread increases with maturity for all leverages apart from the highest one that are characterized by high risker to default on the short term known as survival bias.

In conclusion, readers can easily understand the importance of such a model, which increases the applicability of debt valuation to more realistic cases with finite maturity introducing some important considerations about duration and convexity.

The main limitations are, in addition to the assumptions of a constant variance σ and risk-free interest rate, the analysis only of the endogenous bankruptcy trigger since it is not suitable for all the types of debt contracts. In the presence of net-worth covenant indeed, the results are no more valid.

2.5 First Passage Time Models with Stochastic Interest Rate

2.5.1 Constant Default Trigger: Longstaff and Schwartz Model (1995)

The model of Longstaff and Schwartz is basically grounded on the Black and Cox assumptions of 1976: classic continuous stochastic process followed by firm assets whose value is independent from the capital structure (as stated by Modigliani and Miller theorem) and whose variance is constant.

Furthermore, differently from Leland (1994) and Leland and Toft (1996), the model assumes an exogenous default trigger, K, in the sense that it is not the result of a maximization problem solved by shareholders, but it is exogenously determined. When assets' value falls below K, the firm enters in a bankruptcy status and this reflects the classical net-worth covenant (also defined as "*stock-based insolvency*"). Nevertheless, according to the authors, this condition may be also seen as "*flow-based insolvency*" because it assumes that the company is no more able to generate enough cash to service the debt given the too low level of assets. In addition, failing to pay back a specific bond leads the company to default in all debt contracts, which is coherent with the cross-default provision. In other words, default in one contract triggers the insolvency of the firm as a whole and each class of bond is then discriminated by a different writedown,

w. This is relevant because it allows for more complex capital structure where there are bonds with different priority. For example, a "lucky" class of debtholders may be subject to w = 0, meaning that they will not incur in any loss, while another class may have to suffer a total loss with zero recovery in case of w = 1. Obviously, these are two extreme cases and the only constraint is that the sum of all the recovered amounts cannot be greater than K.

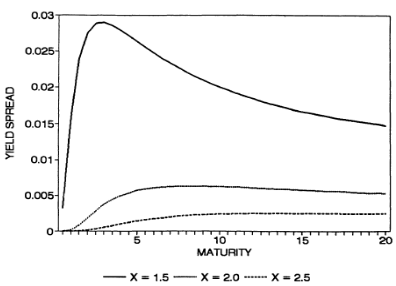
Differently from the framework considered up to now, Longstaff and Schwartz allowed for a stochastic risk-free interest rate that is correlated to the asset's rate of return, and this represents the main innovation of their model. In fact, interest rate dynamics is $dr = (\zeta - \beta r)dt + \eta dZ_2$ where ζ, β and η are constants and $dZ_2$is a standard Brownian Motion whose correlation with dZ_1[16]is given byρdt. Interestingly, stochastic interest rate allows to price both fixed and floating-rate debt with the possibility to compare the behaviour of one with respect to the other and to analyse how firm-specific factors and the interest rate risk affect credit spreads observed in the market.

Going in order, authors first found an expression for Fixed-Rate Debt

$P(X, r, T) = D(r, T) - wD(r, T)Q(X, r, T)$

where $D(r, T) = \exp(A(T) - B(T)r)$ [17] is the value of a riskless discount bond as given by Vasicek Model (1977), w is the writedown and $Q(X, r, T)$ is the risk-neutral probability of default. It is insightful to notice that $P(X, r, T)$ strongly depends on $X = V/K$ which is the ratio between the asset value of the firm and the constant bankruptcy trigger and represents how far from default the company is. As one may expect, as X gets larger, so does the bond price, signalling a decreasing risk (and a decreasing spread). From figure 2.16, it is possible to notice the humped-shape for firms close to default, which again represents the survival bias: since they have low values of X, they are expected to default in the near future, but if the case they survive, they will experience decreasing spreads.

Figure 2 - 12 Credit spread for different values of X



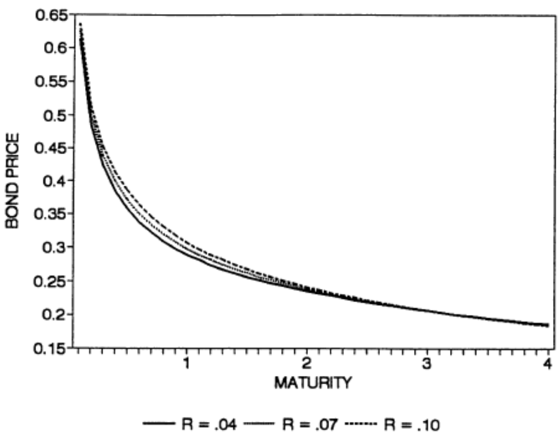
Source: Longstaff and Schwartz (1995, pp. 800)

[16] The dynamic of the assets of the firm is given by $dV = \mu V dt + \sigma V dZ_1$.

[17] For the formulas describing all the terms of the equation, look at pp. 795 and 796 of the original paper.

It is interesting to observe the interrelationship between the risk-free interest rate and bond prices. On the one hand, as one would expect, a rise in *r* leads to a reduction in the value of the bond, but on the other hand, it has also a positive effect on the firm value V which can "*drift away from K at a faster rate*", Longstaff and Schwartz (1995, pp.798). In other words, because of the correlation between the Brownian Motions of the two stochastic process (of *V* and of *r*), a rise in the interest rate has also an indirect effect on bond price, leading to an increase in the firm value and a consequent reduction in the perceived riskiness. Which one of these two effects would prevail depends on the characteristics of the debt. Junk bonds (with value of *X* and *w* close to one) that are very close to default may benefit from a rise in the interest rate as it is shown in figure 2.17 because the positive indirect effect will overcome the negative direct effect.

Figure 2 - 13 Discount bond price as a function of the maturity of bonds, for different levels of risk-free interest rate

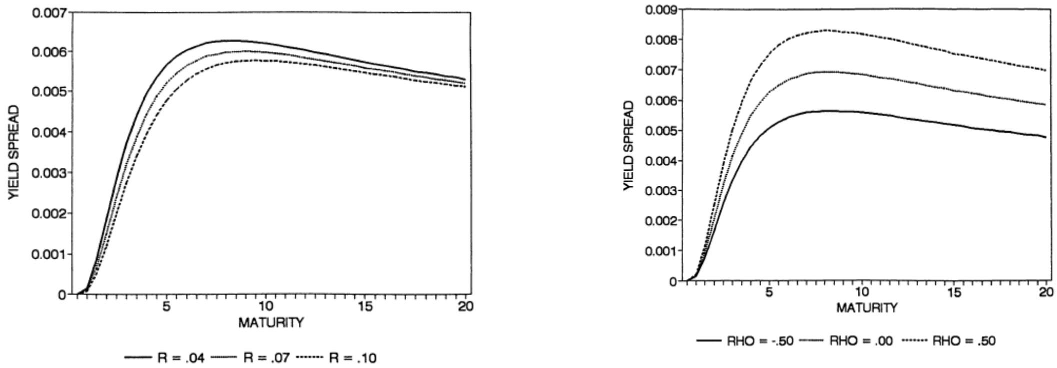


Source: Longstaff and Schwartz (1995, pp.799)

This counterintuitive positive effect of a rise in interest rate can be analysed also under a different perspective. In fact, higher *r* leads to a decline in the yield spread and this effect gets larger with the level of the correlation ρ. This implies that the correlation between the dynamics of interest rate and of the asset value is fundamental in the determination of credit spreads and this explains why, empirically, companies with the same rating have not the same spread. Spreads, indeed, are not only affected by firm-specific variables, but also by the industry where it operates.

Figure 2 - 14 Credit spread for different value of r and ρ

a) different values of *r* b) different values of ρ



Source: Longstaff and Schwartz (1995, pp.802, 803)

After the investigation of the fixed-rate debt framework, authors derived the following formula to price floating-rate coupon payment (indicated with F) at time T, with the floating rate determined at time τ:

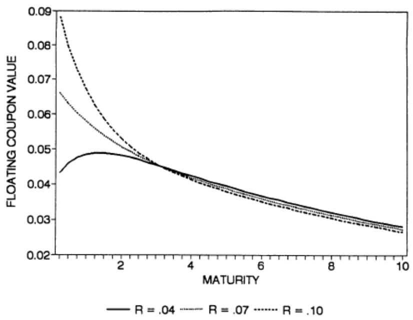
$F(X, r, \tau, T) = P(X, r, T)R(r, \tau, T) + wD(r, T)G(X, r, \tau, T)$

where $P(X, r, T)$ represents the price of the fixed-rate debt multiplied by $R(r, \tau, T)$ which is a function of the floating interest rate and $G(X, r, \tau, T)$ is instead a measure of the risk-neutral probability of default and accounts also for the correlation between the risk-free interest rate and *X*, which again measures how far from default the company is.

As it happens in the fixed-rate framework, also in this situation *X* plays a significant role in debt pricing and its negative relation with bond value is confirmed. What is no more straightforward it is represented by the effects that *T* has on $F(X, r, \tau, T)$.

In the fixed-rate scenario, an increase in the maturity *T* will lead to an increase in the riskiness of the debt and to a consequent reduction in its value, except for junk bonds whose behaviour is particular, while in the floating-rate case, longer maturity may have positive effects on debt price. Since the interest rate dynamics is assumed to be mean-reverting, when *r* is below its long-term average, bond value is expected to rise with *T* and this is shown in figure 2.19 by the solid line representing bond value when the long-run interest rate is equal to 4%.

Figure 2 - 15 Values of floating-rate coupon payments for different values of r



Source: Longstaff and Schwartz (1995, pp. 805).

Finally, interest rate has the same effects on bond price as it does in the fixed-rate framework. Said another way, a rise in *r* may have a positive impact on the value of junk bonds for two

reasons: on the one hand because of its effects on the firm value that makes *X* to increase, drifting away from the bankruptcy trigger *K* and, on the other hand, because of its effect on the ratio *X*. In fact, if the correlation between interest rate and firm value ρ is positive, then an increase in *r* leads also to a rise in *X*, reducing the probability of default.
After having defined the formulas for both fixed and floating-rate, authors made an empirical analysis to investigate whether their conclusions were coherent with empirical data. To do so, they regressed observed changes in yield spreads with respect to variations in risk-free interest rate (using 30-year Treasury yield as a proxy) and with respect to assets return (using equity index return as a proxy). They found a statistically significant negative coefficient for both the variables and, more interestingly, that interest rate movements have a stronger effect than asset-value factors. Their results goes against traditional credit analysis according to which changes in the spread are explained only by changes in the fundamentals, introducing a new perspective in the analysis of bond pricing and yield spreads.
As a conclusion, their work is useful to get insights about the effects of a dynamic interest rate on bond valuation as well as about differences in behaviour between fixed and floating rate coupon. Whereas, as all the other models defined up to now, the model does not allow for a dynamic variance of asset value (that is assumed constant).

2.5.2 Stationary leverage: Collin-Dufresne and Goldstein Model (2001)

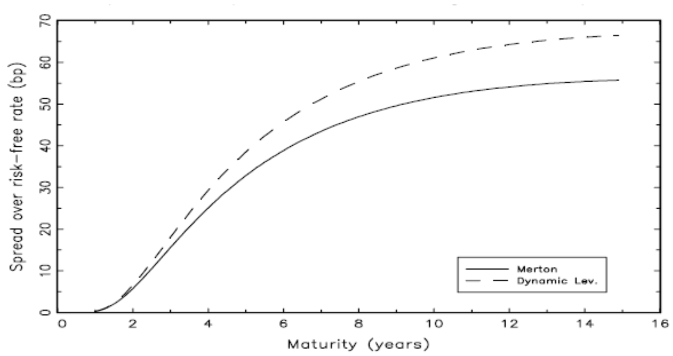
This model can be interpreted as an extension of the one of Longstaff and Schwartz (1995) because both of them share the same assumptions about the evolution of firm assets as well as a non-constant interest rate, which follows a specific dynamic. Nevertheless, this new model does not only account for the current capital structure but also for the capability of the firm to issue new debt with the same priority of the previously issued one, changing, in that way, the proportion between debt and equity and the proportion among the different classes of debt. Indeed, the capital structure of the firm is no more constant but can be changed in order to respect its target level. According to the authors, in fact, "*accounting for the firm's option to issue additional debt in the future significantly increases the predicted credit spread for previously-issued debt*.", Collin-Dufresne, Goldstein (2001, pp.1930).
Doing so, the adherence to reality of the structural analysis has been increased since it can be observed empirically that firms, in general, tend to decide for additional debt issuance coherently with the target capital structure that depends, in turn, on the industry where the firm operates. In other words, managers fix a target capital structure according to average industry level (of course taking into account individual future needs) and then issue and retire debt

accordingly: if current level of debt is lower than the target capital structure, then it will be more likely a new issuance of debt and vice-versa.

A constant bankruptcy threshold, *K*, coupled with the assumption of a Geometric Brownian Motion for the firm asset value (which increases over time) implies a decreasing leverage which is not coherent with the stationary capital structure that has been observed in reality, and this is one of the drawback of Longstaff and Schwartz (1995). This new model, instead, assumes a specific mean-reverting dynamic for the firm leverage, which implies a stationary capital structure and allows also for considerations about the interrelationship between capital structure choices and the current level of risk-free interest rate, *r*.

The model is built step-by-step in the sense that, at first, the authors started from the model of Merton (1974), with the same assumptions (zero-coupon debt with maturity *T*, constant risk-free interest rate *r* among the others). Nevertheless, they generalize the framework allowing for the issuance of new debt with the same maturity and the same priority of the previously issued one and then compared the results on credit spreads. They found that the possibility (option) to issue new debt brings about an increase in the spread, especially for long maturities where structural models works well. This can be observed in the following figure.

Figure 2 - 16 Impact of the option to increase leverage on credit spread



Source: Collin-Dufresne and Goldstein (2001, pp.1933)

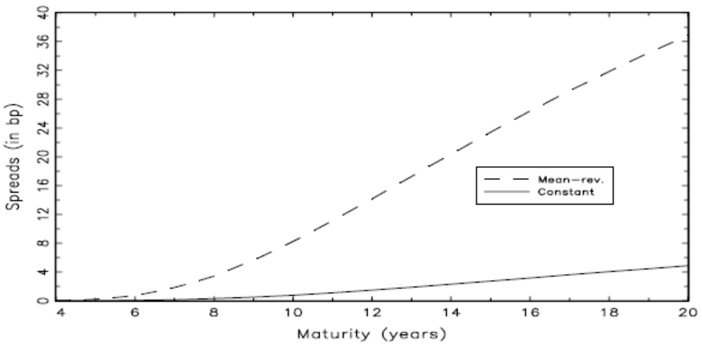
The second step made by the authors was to assume a specific mean-reverting dynamic for the leverage of the firm (which affects also the default threshold that is no more constant), but maintaining the interest rate fixed.

- $dk_t = \lambda(y_t - v - k_t)dt$, which represents the dynamic of the bankruptcy threshold;
- $dl_t = dk - dy$, which represents the dynamic of the leverage.

This aspect can be thought as a way to increase the reality of the model in the sense that companies usually have a target capital structure (and consequently a target level of debt) and their decision about whether to issue new debt depends on the comparison between its current and target levels. When the former is higher than the latter, managers will be more reluctant to issue new debt and vice-versa.

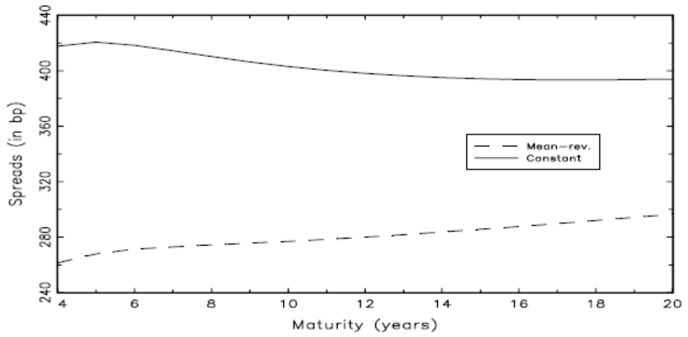
Authors found a closed-form solution for the price of coupon bond that can be imagined as a portfolio of discount bonds, then measured the resulting spreads and finally compared the results obtained in the two frameworks: constant default trigger scenario and the stationary leverage scenario for both investment-grade bonds with low leverage and junk bonds with high level of debt.

Figure 2 - 17 Credit spreads for investment-grade bonds: constant default boundary vs mean-reverting leverage ratio



Source: Collin-Dufresne and Goldstein (2001, pp.1938)

Figure 2 - 18 Credit spread for junk bonds: default boundary vs mean-reverting leverage ratio



Source: Collin-Dufresne and Goldstein (2001, pp.1939)

It can be easily noticed that the mean-reverting leverage framework is characterized by higher spreads for investment-grade bonds especially for long maturities (coherently with the generalization of Merton Model previously seen) and, even more striking, the upward-sloping curve for speculative-grade bonds. This last feature is in contrast with all the models considered up to now, which predicted either a downward-sloping or a humped shaped curve. An upward-sloping curve is coherent with the empirical analysis performed by Helwege an Turner (1999) who found a positive relationship between spreads and maturities for all the bonds, whatever their risk profile. They criticized the previous approaches because of a bias in the analysis. In other words, companies with the same rating may be characterized by different risk and safer companies usually issue debt with longer maturity, and this bias affects the spreads, which appear to be downward-sloping. To avoid the problem, Helwege and Turner, analysed a sample

of bonds with different maturities but issued by the same company finding an upward-sloping curve, and this is confirmed by the model of Collin-Dufresne and Goldstein (2001).
On the one hand, constant default boundary frameworks predict a negligible probability of default for investment grade firms (which will almost never default according to those models) and a great likelihood to go bankrupt in the near future for highly risky firms whose spread is expected to decrease in case they succeed in their turnaround. This happens because default is affected only by the current leverage. On the opposite, the stationary leverage model predicts a larger probability of default for safe firms and an upward-sloping curve also for the spreads of heavily indebted companies because bankruptcy is affected not only by current capital structure but also by the long-term (target) level of leverage, (Collin-Dufresne and Goldstein, 2000).
The last and third step was finally to allow also for a stochastic interest rate to understand how it affects the analysis. In fact, according to Longstaff and Schwartz (1995), credit spreads are decreasing function of interest rate..
To find a closed-form solution for debt price, the following continuous-time stochastic process have been assumed by the authors:

- $dy_t = \left(r_t - \delta - \frac{\sigma^2}{2}\right)dt + \sigma dZ_1(t)$ for the log-firm value $y_t \equiv logV_t$
- $dr_t = k(\theta - r_t)dt + \eta dZ_2(t)$ for the risk-free interest rate, where $dZ_1 * dZ_2 = \rho dt$
- $dk_t = \lambda[y_t - v - \phi(r_t - \theta) - k_t]dt$ which assumes a negative relationship between the default-threshold and the interest rate *r*.

These dynamics permit to define the log-leverage as $l_t = k_t - y_t$ and, applying Ito's Lemma, it becomes $dl_t = \lambda(l^{LR}(r_t) - l_t)dt - \sigma dZ_1(t)$ where l^{LR} represents the long-term target value for the leverage.
Assuming that in case of default bondholders will receive only a fraction (1-*w*) of the principal, the solution for the price of risky discount bond at time *T* is given by

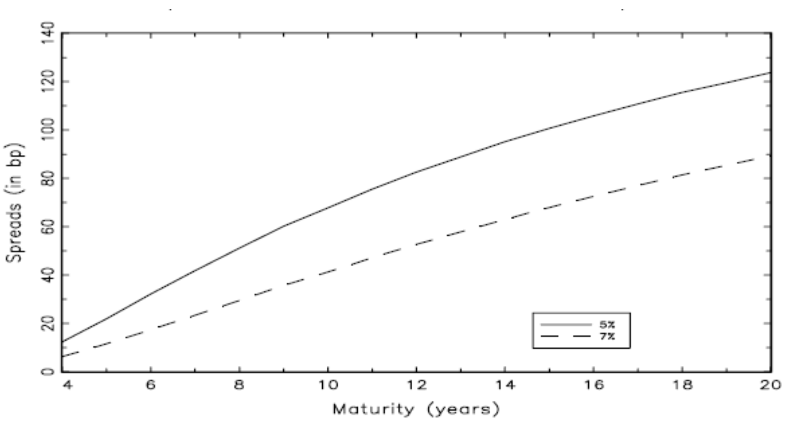
$$P(r_o, l_o) = D(r_0)[1 - wQ(r_0, l_0, T)$$

where $D(r_o) = e^{A(T) - r_0 * B_K(T)}$ as defined by Vasicek (1977) and $Q(r_0, l_0, T)$ represents the risk-neutral probability of default.
Finally, it is possible to compute the yield to maturity $P(r_0, l_0)$=$\sum_{j=1}^{N} Ce^{-Rt_j} + e^{-RT}$ and the consequent credit spread is given by $CS(r_0, l_0) = Y - R$
Analysing the term-structure of credit spreads, they found the same results as before with positively-sloped credit spread curves for both investment-grade and junk bonds, even with more emphasized results with respect to the constant interest rate case, because the stochasticity of *r* leads to an increase in the volatility of the leverage, making the risk to surge. In other words, stationary leverage model is characterized by higher spreads for safe bonds and lower

spreads for junk bonds because credit spread is influenced not only by the current capital structure but also by its target value and, as a consequence, the correlation between current asset value and spread has decreased becoming of minor interest.

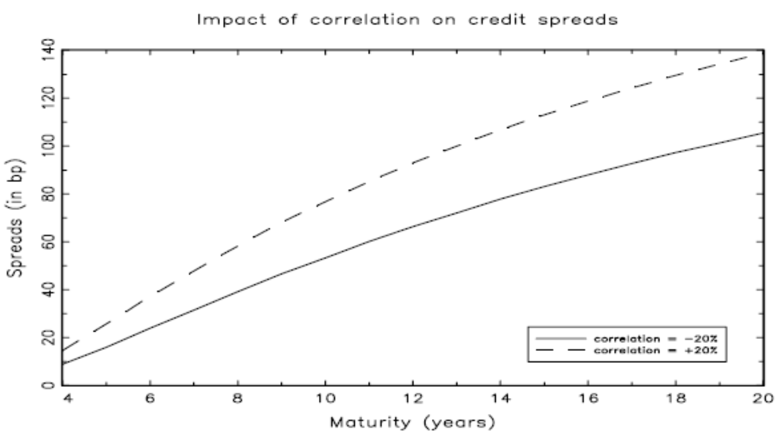
Furthermore, as in Longstaff and Schwartz (1995), there is a negative relationship between spreads and interest rate as it can be observed in the next picture. This is due to the effect that a rise in the interest rate has on the firm value, which drift-away from the bankruptcy threshold. In addition, also the correlation between firm value and interest rate plays an important role because when ρ is negative, a movement of *r* leads to two effects that go in the opposite direction. A fall in the interest rate, for example, on the one hand will make the asset value to rise but, on the other hand, it will have a negative effect on the drift of the underlying assets. As a results, for negative values of ρ (as it is proved by authors who found that the general level of ρ is equal to -0.2), credit spreads will shrink following a reduction in the interest rate.

Figure 2 - 19 Impact of interest rate level on credit spread



Source: Source: Collin-Dufresne and Goldstein (2001, pp.1946)

Figure 2 - 20 Impact of correlation on credit spread



Source: Source: Collin-Dufresne and Goldstein (2001, pp.1947)

To conclude, this model is important because it provides a formula for valuing debt, even considering the option to the managers to issue new debt, allowing them to adjust the firm's capital structure coherently with its target level. Moreover, assuming stochastic processes for firm value, risk-free interest rate and for bankruptcy threshold, authors could analyse also the effects of the correlations among the different variables of the model. In this way, authors found results more coherent with empirical analysis. Usual shortcoming is the constant variance of asset which is not realistic and, despite the adherence with empirical results of Helwege and Turner (1999), the issue about the shape of the term-structure of credit spreads of speculative-grade bonds (whether it is upward, downward-sloping or humped shaped) is still debated.

2.6 Strategic Default Model: Anderson and Sundaresan (1996)

So far, all the investigated models have been elaborated in a continuous-time stochastic framework and only the models of Leland (1994) and Leland&Toft (1996) considered the possibility of an endogenous bankruptcy trigger in the sense that it is determined by shareholders, as a result of a maximization problem. All the others formulations, instead, take default threshold as given and focused their attention on the derivation of a pricing-formula for both risky debt and credit spread without taking into account the possibility that the choices of the different agents may be the results of a strategic decision. Whereas, Anderson and Sundaresan (1996) analysed the debt valuation problem and the determination of credit spread under a new interesting perspective. They consider the problem as a non-cooperative game between shareholders and debtholders where they act in order to maximize their payoff in a discrete time environment with perfect information. At each time, the decision of shareholders affect that of bondholders with the former class of claimants having the "*first move advantage*". The setting is built in a very simple way, where there is one "owner-manager" who is willing to invest in some projects and, to do that, she or he needs to issue new debt for an amount *D*, which requires timely payments of CS_t in each time period *t* until maturity *T*. The project has a value, V_t, which is stochastic and can increase to uV_t or decrease to dV_t according to specific probability $p = \frac{r(1-\beta)-d}{u-d}$, where β represents the pay-out ratio that is, the proportion of the value generated by the project that is not reinvested but used to service the debt, to pay dividends and bankruptcy costs *K*, which are known and constant.

At each time moment, cash-flows produced by the project are given by f_t and the owner/manager chooses a payment $S_{t,}$ that can be equal or lower to the contracted amount CS_t, according to the value of f_t. If equityholders pay what it is owed, the game continues to the next period, while if debtholder receives a smaller amount with respect to CS, then he or she has the possibility to decide whether to accept the payment or to ask for bankruptcy. The problem the agents encounter is different depending on whether the decision has to be taken at time T or at any time before maturity. Whatever the timing, both owner/manager and bondholders take decisions strategically to maximize their own interest in a non-cooperative way.

At time T, owner decides the amount to pay to debtholder according to the value of the project V_T: the payment will be $S_T = CS_T$ in case of sufficiently high project's value and $S_T < CS_T$ otherwise. In this second case, bondholder will accept the payment only if it is greater that what he will otherwise get in case of default: $\max[V_T - K, 0]$.

In other words, when the project is doing well with a high value, the owner/manager will pay the contracted amount CS_T while, in case of underperformance, he or she will choose the level that makes bondholders indifferent between accepting the payment and defaulting. The equilibrium payoffs at time T are indeed dependent from V_T and equal to

$B(V_T) = \min[CS_T, \max[V_T - K, 0]]$ and $E(V_T) = V_T - B(V_T)$

The same happens in every time $t < T$ but with the complication that the agents have to consider also the uncertainty about the possible evolution of the project's value. Both equity and debt, at intermediate time t, depends on whether the firm continues to operate or enters a liquidation procedure.

In case liquidation does not occur, debtor may get CS_t when project performs well or may get a smaller amount that is uncertain and depends on future evolution of V_t, which can either, go up, or down.

As a results, debt value is equal to

$B(V_t) = S(V_t) + \frac{pB(uV_{t)} + (1-p)B(dV_{t)}}{r}$

while, the corresponding equity value is $E(V_T) = f_t - S(V_t) + \frac{pB(uV_{t)} + (1-p)B(dV_{t)}}{r}$.

In this valuation framework, $S(V_t)$ represents the value of the payment and it is given by:

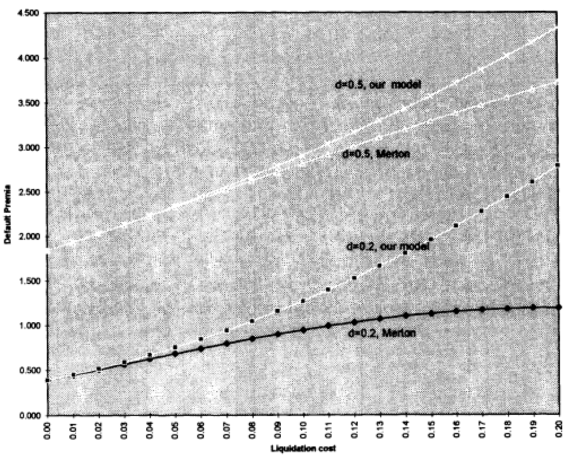
$S(V_t) = \min\{CS_t, \max[0, \max(V_t - K, 0) - \frac{pB(uV_{t)} + (1-p)B(dV_{t)}}{r}]\}$, where r represents 1 + risk-free rate.

In case the company is short on liquidity and therefore defaults, the value of the debt is instead given by: $B(V_t) = \max[0, \min(V_t - K, CS_t + P_t)]$, where P_t is the principal outstanding at time t, while equity is equal to $E(V_t) = V_t - K - B(V_t)$.

As one can easily notice, the equilibrium is split into two different situations: one in case of payment of the owner/manager and another in case of default. In the first scenario, it does not matter whether the amount paid is equal to or lower than the contracted amount CS_t because owner/manager will decide a sum that makes bondholder indifferent between accepting, or asking for default. Debtor consequently will accept the sum of money, allowing the company to continue to operate. Conversely, default will happen only when project underperforms and firm runs out of cash.

After this generalization, the authors applied the model to the case of a zero-coupon bond, with strategic default possibility, for different values of bankruptcy cost, *K,* and then compared the results with an adjusted version in discrete time of Merton model (1974) finding the same results. Credit spreads indeed appears to be positively related to both asset volatility σ^2 and indebtedness of the firm (measured by the quasi debt-to-equity ratio *d*). In addition, also the effect of maturity is the same: credit spread of safe firms increases with maturity, while for heavily indebted companies, the opposite happens and this is in line with the results of previously described models except Collin-Dufresne and Goldstein (2001). Furthermore, credit spread highly increases if positive bankruptcy costs (liquidation cost *K)* are included in the framework as it is shown in the following figure.

Figure 2 - 21 Effects of strategic default on default premium for values of d=0.2 and d=0.5



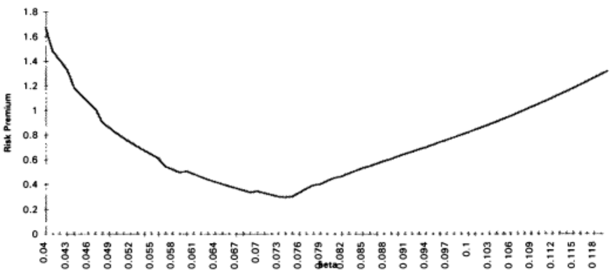
Source: Anderson and Sundaresan (1996, pp. 52)

The figure, which plots the evolution of credit spreads for both strategic default and Merton Model (1974), shows higher values in case of strategic default because when the project underperforms, debtholder expects to receive a lower amount with respect to what contractually are entitled to, and the scope of this loss gets bigger with *K*.

This valuation method could be also extended to the framework of a straight coupon debt. As always, they have found a positive link between credit spread and volatility. Differently from the zero-coupon bond framework, in this case, an increase in leverage may have a positive effect on the value of debt bringing about a reduction in the credit risk because, in specific situations, higher debt reduces the scope for opportunistic payment decisions by the owner/manager.

Furthermore, it is interesting to look at how the spread changes following to modification of the pay-out ratio β that is shown in figure 2.26.

Figure 2 - 22 Effect of beta on risk-premium



Source: Anderson and Sundaresan (1996, pp. 57)

An increase in the pay-out rate, when it is low, has a reducing effect on the credit spreads which gets smaller. In other words, low pay-out means that the company reinvest almost all the resources produced by its investment project and consequently has few cash to pay back the loan and, in this way, the possibility for owner/manager to opportunistically reduce the amount to be paid shrinks. Conversely, when pay-out rate is already high, its increase leads to a rise in the risk-profile of the company because most of the available resources will be paid out in the short-term while the possibilities for future growth will be reduced.

After the definition of the framework to value debt in discrete time allowing for strategic (opportunistic) behaviour of both shareholders and debtholders (especially the former class who has the advantage of the "first move"), the authors discussed how debt's contracts should be designed to minimize inefficiencies according to the possible reactions of both the agents to eventual contractual provisions.

In particular, the owner/manager is expected to choose contractual provisions (regarding final maturity T, coupon payments c, principal value P and grace period g[18]) in order to maximize her or his payoff. Said another way, owner manager has to solve a constrained maximization problem in which she or he maximizes equity value with respect to the four above-mentioned variables (T, c, P, g) subject to the constrained that debt's value B should not be less than the necessary capital, D, to undertake the investment project.

$max_{c,T,P,g}\ E(V_0; \sigma^2, \beta, r, K, \tau)$

subject to $D \leq B(V_0; \sigma^2, \beta, r, K, \tau)$.

According to the authors, this is a second best equilibrium since two different types of inefficiencies may happen: on the one hand, ex-ante, the contracted amount B could be lower than what it is necessary to invest in a positive net-worth project (so-called underinvestment problem) and ex-post the firm may go to forced bankrupt when it runs out of cash.

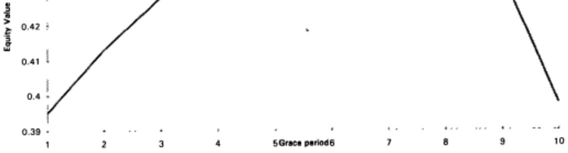
[18] Timely coupons could either include only interest payment or also a part of the principal that is amortized. Grace period is the time frame when coupons contain only interest payment without principal and it goes from the starting date of the contract to the first payment of the amortized principal in a coupon.

At first, the scenario of zero-amortization is considered, meaning that coupons in all the debt's life include only interest payment while the whole principal will be reimbursed at maturity *T*. In this framework, it is insightful to analyse how changes in the pay-out ratio affects the equity value, because β could be used to differentiate between fast-growing and cow companies. Usually, fast-growing companies are those with high growing future potential so that managers choose to reinvest most of the profits (β will be indeed low). With the low amount of resources available to pay-out, those firms will be better of paying low coupons interest rate, because in case of heavy coupon payments, their probability of early-default increases. On the contrary, cow companies with poor potential for future growth are characterized by willingness to pay out sizable amount of cash in order to get the highest possible tax-shield. They are supposed to have considerable liquidity available so that their probability of early default is not significant. This is confirmed by the analysis of the authors who have found a positive relationship between the pay-out ratio and the contractual interest payment while the more cash the company does not reinvest, and the lower the value of equity will be, so that β and equity are negatively correlated.

Furthermore, in case of non-amortizing debt, companies in general will prefer longer maturities because of the heavy burden of the principal reimbursement at time *T* that makes default a possible problem so that the further it is postponed and the better it is. Nonetheless, also in this situation β plays a significant role since companies with a very high pay-out are expected to decrease in value in the future so that they would be better of choosing short-term maturities.

The second scenario considered by authors is the amortized debt with a grace period consisting in the delay in the start of the principal reimbursement. The choice of the appropriate grace period is taken by the owner/manager to maximize equity value and the best solution is shown in the following figure.

Figure 2 - 23 Optimal design of debt contract



Source: Anderson and Sundaresan (1996, pp. 64)

To understand the effects of *g* on equity it is important to remember which effects an increase in the coupon payment has on equity value:

- A positive impact because of the increase in the tax-shield benefits;

- An additional positive effect because the principal that has to be reimbursed at maturity T gets lower, reducing the probability of default;
- But also, a negative effect because of the rise in the probability of defaulting on the intermediate coupon payments.

While the owner/manager chooses the best level of grace period, she or he has to take into account all these eventual trade-off. As a result, too short grace periods are not convenient since firm will not benefit from enough tax-shield advantages while, on the contrary, postponing the beginning of the amortization period provokes a surge in the probability of default because the burden of the principal reimbursement at maturity would be too heavy. In fact, as it is possible to see in the picture, grace period leads to a higher equity value reaching the peak at 7 years and then decreases because the negative effects start to prevail. In addition, cow firms with few investment opportunities and huge cash prefer lower grace period to avoid the bankruptcy risk at maturity.

In conclusion, this work is useful to get an understanding about the strategic reimbursement decisions of shareholders and its consequences on the credit spreads (which increase). It is also interesting because of the analysis of the pay-out rate, β, which allows for differentiations in the behaviour of both fast-growing and cow companies. Finally, the inclusion of the amortized debt gives the possibility to get insights about the choice of the optimal grace period by shareholders.

Nonetheless, the model construction is too simple because it considers only one class of debt with a specific maturity without allowing for more complex capital structure, which, in addition is assumed to remain constant. Furthermore, it does not consider any stochasticity in the interest rate and in the volatility of the asset value. Still, it remains interesting to understand some possible interactions between equityholders and creditors assuming that their choices are a results of a sort of game which in turn represents the negotiation process that all the other models do not consider because they take it as given (exogenous).

2.7 Summary and Conclusions

This literature review about structural models, despite not complete, given the wide numbers of models developed since the intuition of Merton (1974), has the function to give insights about the issue of corporate bond valuation, its complexity and the difficulties to have a model that is at the same time adherent to reality and reliable. The seven models described above are among the most important because they all provide approaches that lead to different valuation formulas and different conclusions. Moreover, they can be used to improve fixed-income analysis with interesting considerations about the behaviour of credit spread and debt value in a variety of scenarios. A summarizing table is presented below where main aspects and characteristics of each model are briefly described to ease the reading. Merton (1974) gave the intuition for the usage of the option-pricing theory to value corporate bonds. Black & Cox (1976) added the possibility for an early default with a first-passage time model where the firm may go bankrupt in each moment (and not only at maturity) whenever assets value falls below a specific threshold (assumed as exogenous in that case). From that moment, academics and practitioners started to focus the attention on this barrier that trigger default. Leland (1994) and Leland & Toft (1996) pushes the analysis even further considering an endogenous barrier determined by equityholders in order to maximize their payoff; they also introduced the presence of both bankruptcy costs and taxes. Longstaff & Schwartz (1996) instead formulated a model to account for the variability of the interest rate, which is assumed to follow a specific dynamics, and this gives the possibility to analyse also the relationship between interest rates and credit spreads. Collin-Dufresne & Goldstein (2001) provided a really interesting model, not only assuming a dynamic interest rate, but also assuming a mean-reverting process for the company's leverage giving the possibility to understand the firm's choices about debt issuance. Finally, despite the simplicity of their model, Anderson & Sundaresan (1996) looked at debt valuation issue under a different perspective, in fact the problem is thought as a non-cooperative game between shareholders and debtholders.

All of this is useful because it allows to understand both strengths and weaknesses of all the approaches and this is the fundamental ground that an analyst should use to build his own model.

Table 2 - 1 Summarizing table of treated structural models

Model	***Debt Structure***	***Default***	***Innovations***
Merton (1974)	zero-coupon bond with finite maturity T	• Default only at maturity T if asset value is below nominal value of principal	• debt valuation and probability of default estimation exploiting similarities with European call option using option-pricing theory
Black & Cox (1976)	coupon bond with infinite maturity $T = \infty$	• exogenous default trigger • default as hitting time; • default may happen in every moment before maturity	• protected vs unprotected debt • senior vs junior debt
Leland (1994)	coupon bond with infinite maturity $T = \infty$	• exogenous vs endogenous default barrier	• introduction of both default cost and tax benefits of debt

		• default as a rational decision of a maximization problem of shareholders	• optimal capita structure consideration
Leland & Toft (1996)	coupon bond with finite maturity T	• endogenous default trigger • default as a rational decision of a maximization problem of shareholders	• introduction of both default cost and tax benefits of debt • optimal capita structure consideration • long-term vs short-term interest rate
Longstaff & Schwartz (1995)	coupon bond with finite maturity T; it allows for more than one class of debt differentiated by different recovery rates	• constant and exogenous default trigger • cross-default provision in case asset value falls below default threshold	• introduction of stochastic interest rate • correlation among interest rate, firm value and default trigger • fixed vs floating interest rate
Collin-Dufresne & Goldstein (2001)	coupon bond with finite maturity T;	• stochastic default trigger that follows a specific dynamic • cross-default provision in case asset value falls below default threshold	• introduction of mean-reverting leverage • dynamic capital structure • stochastic interest rate
Anderson & Sundaresan (1996)	coupon bond with finite maturity T raised by owner-manager of the firm to finance an investment projecte	• default as a result of a non-cooperative game between shareholders and bondholders • endogenous default trigger	• discrete time model • game theory in debt valuation • strategic default

CHAPTER 3: THE MODEL AND ITS MAIN VARIABLES

3.1 Introduction

The previous chapter has been aimed at providing a literature review about the main and most famous models regarding the structural credit modelling. Despite their role in providing many meaningful insights in the credit risk assessment, they are at a theoretical standstill because they rely on too much restrictive assumptions that lack reality and practical applicability. Nevertheless, in recent years academics and practitioners, and in particular companies involved in the credit business such as rating agencies or investment banks, have tried to develop models in the aim of raising the adherence to reality to reach a concrete possible utilization. One of the most famous framework whose usage is widespread in credit industry is the KMV that will be analysed in paragraph 3.2. Afterwards we will focus on the estimation of the most relevant variables of the model such as enterprise value and its volatility. Finally, we will present our model, which exploits the similarities of payoff between shareholders of a company with protected debt and a particular barrier option. It could be considered as a version of KMV, which is in the middle between the classical Merton model and more advanced methodologies that exploit the functioning of exotic options with barrier. In particular, the presence of the barrier gives space to differentiate between senior and junior debt and to discuss possible implications of the bargaining process between the various financers of the company and their specific claims.

3.2 KMV Model (2003)

That operational framework, whose name originates from its authors Kealhofer, McQuown and Vasicek, has been acquired by the agency rating Moody's in 2002 and from that moment has become the Moody's KMV model.

One of the main drawbacks of all the previously mentioned models, as it has been already commented, is represented by the difficulties in modelling correctly the barrier that triggers default, its time dynamic and its relationship with the asset's value. That problem is not present in Merton (1974) because he modelled default as an event that can happen only at maturity T so that the trigger coincides with the strike price of the option, represented by the principal value. Black and Cox (1976) introduced the possibility of early default by assuming a constant trigger, which equals the discounted principal value. Leland (1994) and Leland and Toft (1996) proposed a model with an endogenous bankruptcy trigger that is no more taken as given but it is part of the problem and determined by shareholders to maximize their payoff.

One of the main innovation introduced by KMV is to define the default barrier, called Default Point (DP from now on), as a kind of midpoint between total liabilities and short-term liabilities. Said another way, it has been observed that companies do not always default when asset's value falls below total liabilities but it is important to classify them between short-term and long-term. Should all the liabilities be short-term, the likelihood of default will heavily increase because a big amount of money will become due in a very short time span. Oppositely, the presence of long-term debt that is not immediately due gives the company the possibility to invest and produce cash in the future to meet all the obligations. As a results, to trigger default, asset should touch a value that lies somewhere between total and short-term debt. This is why the default point has been defined as

$$DP = [Short - term\ Liabilities] + [\frac{1}{2} Long - term\ liabilities]$$

So that the net-worth of the company is given by $Market\ value\ of\ Assets - DP$.

This new measure of net-worth can be adjusted for the asset volatility to find out the Distance to Default (also called DD) $DD = \frac{[Market\ Value\ of\ Assets]-DP}{[Market\ Value\ of\ Assets]*[Asset\ volatility]}$.

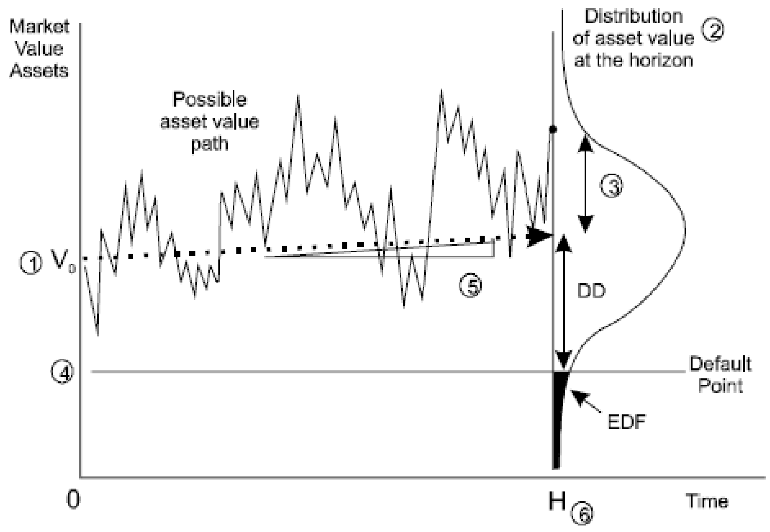
Distance to Default provides a measure of credit risk expressed in number of standard deviations. In other words, if assets value decreases by a number of standard deviations equal to DD, then the company will default. It is a very meaningful information because it provides knowledge about:

- the value of the firm's asset (that has to be correctly measured);
- the capital structure and the leverage of the firms (included in the Default Point);
- the overall business risk according to the industry where the company operates (included in the volatility of the assets, that is another variable that should be correctly estimated). In other words, companies with similar net-worth may be characterized by different probability of default because they have different asset's volatility. Industry also matters since there are sectors whose revenues and cash-flows are more stable (commodities or health business) and can afford to issue more debt while others, such as computer and software business, are riskier so that their debt capacity is limited.

Once the distance to default has been computed, one has only to find a way to attach a reasonable probability of default to each DD. This is done by looking at the Expected Default Frequency (EDF), that is computed by collecting past data about bankruptcies, and provides information about the expected probability of default for the following year.

This procedure is well summarized by the following figure

Figure 3 - 1 Variables of the KMV model



Source: Crosbie and Bohn (2003, pp. 13)

From the picture, it is possible to notice the six fundamental variables that are used in the estimation of the probability of default which are

1- current asset value
2- distribution of the asset value
3- volatility of the asset
4- default point
5- expected rate of growth of the company's assets
6- time horizon, *H*.

where the first four variables of the list are those that have to be estimated affecting the model the most.

The authors' solution for the estimation of default point has been already given, while regarding assets value and their volatility (and consequently their distribution function), they are derived using option-pricing theory. KMV, in fact, could be thought as a modification of Merton model (1974). It grounds on the similarities between the payoff of equity and a call option with a strike price equal to the book value of the debt and uses this fact to compute asset value and its volatility starting from equity value and equity volatility, which are directly observable in the market. Of course, this holds only for public companies whose shares are exchanged in the main financial markets but it can also be used for private firms using data from public competitors. What it is important to highlight here, is the fact that the assessment of the creditworthiness of a company relies on the strong relation between equity and debt. In fact, both equity and debt share the same underlying, which is represented by the assets of the firm and this is important to keep in mind while valuing either equity or debt. The equity market is usually liquid enough to use equity information in the debt valuation process while the reverse is more compelling because corporate bonds are not heavily exchanged. This does not implies the assumption of perfect markets, but that no single investor could do better than the market itself, Crosbie and Bohn (2003).

This is done by solving a system of two equations:

$$equity\ value = option\ function[asset\ value, asset\ volatility, capital\ structure, interest\ rate]$$

$$equity\ volatility = option\ function[asset\ value, asset\ volatility, capital\ structure, interest\ rate]$$

where the only two unknown are asset value and asset volatility.

The advantages of using KMV model are represented not only by the possibility to allow for complex capital structure with short-term, long-term and convertible bonds, but also to allow for dividends payment or any other cash pay-out for other securities. In addition, it gives an intuitive insight about the relationship between equity characteristics and assets characteristics that one should always be careful about. Main drawbacks, as usual, are the assumptions of constant volatility of the assets, which is for sure unreasonable and the usage of the Normal distribution. Differently from the reality, the tails of such distribution are not enough "fat" in the sense that the probability of extreme events is underestimated and this is a problem because, especially in economy and in the financial markets, extremely negative (or positive) events, so-called "black swan" happen when they are not expected.

Despite the similarities, KMV focuses the analysis on the estimation of a reasonable probability of default of the company as a whole, while Merton's purpose was to find out a way to value corporate risky debt. Nonetheless, estimated probability of default can be matched to a specific interest rate that can be subsequently used to give a value to the debt.

3.3 The model

The aim of the model is to put aspects of corporate finance (such as capital structure considerations and company valuation) together with quantitative finance and statistical techniques. This is also what structure models do. In few words, initially, a "company valuation framework" has to be built in order to obtain an estimate for the enterprise value of the company. Subsequently, Newton-Raphson method is performed to compute implied asset volatility. It is well known that a non-performing loan cannot be evaluated at its nominal value due to its low creditworthiness. As credit quality goes down, market value of debt shrinks as well. If the debt is quoted and exchanged on the capital markets, its real value can be easily observed while, in case of non-quoted debt, a proper model should be designed. These are the reasons why, once all the variables have been computed, an option-pricing framework will be set in order to find a way to give a price to distressed debt. Finally, this methodology gives the possibility to discuss the bargaining process between equityholders, senior and junior creditors.

3.3.1 Enterprise Value

For the implementation of the KMV model, Enterprise Value (EV from now on) represents one of the most important variable. There are plenty of business valuation techniques in corporate finance that can be classified, among the others, in asset based approaches, market approaches and income approaches. The most common are income approaches, which ground mainly on the capability of the firm to generate future profits and cash flows. Among the income approaches, DCF is the method usually practitioners and analysts use to come up with a reliable value of the company's assets.

In few words, DCF relies on the (sometimes strong) assumption that the firm's life will be infinite and consists in dividing this infinite-time period in two stages. One explicit forecast period of three to ten years where the analyst makes explicit assumptions about the evolution of the key variables (such as revenues, costs, depreciations&amortization, interest expenses, taxes, working capital needs and capital expenditure).

The second period represents the value of the firm computed over the remaining (infinite) time horizon using a perpetuity formula assuming that key variables will reach and maintain stable levels. Afterwards, both the cash flows of the explicit forecast and the terminal value are discounted using the Weighted Average Cost of Capital (WACC from now on).

$$WACC = \frac{E}{(E+D)} * k_E + \frac{D}{(E+D)} * k_D * (1 - tax\ rate)$$

This embodies both cost of equity and debt, and proportionally weights both costs of capital by the portion of each capital class over the total sources of financing. In addition, it also takes into account the tax shield on debt due to annual interest expenses, which reduce taxable income and consequently makes income taxes lower. The usage of WACC implies the strong (and usually unrealistic) assumption that the capital structure will remain constant over the entire valuation period.

DCF is widespread in finance because it takes into account lots of variables that other models neglect. It starts from NOPAT, sum up non-monetary cost and takes into account the cash outflow/inflow due to annual (positive or negative) investment in both working and fixed capital. Nonetheless, while valuing a distressed company, it has some drawbacks that make it completely unreliable. Damodaran (2009) summarized the main imperfections of such a valuation method while dealing with distress:

- There is the risk that the cash flows in the explicit forecasts may not been materialized, creating an upward bias in the valuation. This is why it is always better to proceed with three different scenarios, a **base case** that follows the assumptions of the reorganization plan, a **worst case** which considers the possibility of a downside in all the key variables (revenues, marginality, profits) and a **best case** taking into account the possibility for an unexpected (and really unlikely) recovery. The values coming from the three different scenarios are then weighted using the corresponding probability of happening to find an average (expected) EV;
- About the 70% of the total EV comes from the terminal value meaning that most of the value depends on the expectation that the company's life will be infinite. Despite reasonable for healthy firms, it is arguable in case of distress;
- The usage of a discount rate that assumes a constant capital structure is controversial because distressed firms are usually characterized by high levels of debt that the management is willing to reduce in the future. In addition, the unbalanced financing structure creates distortions in the estimations of both cost of equity and debt, which surely will not remain constant for the entire valuation period;
- WACC's reliance is also questionable because of the risk of not benefit from the tax shield of debt. Tax shield hypothesis is valid only in case the firm has produced annual positive taxable income and only in case it has not already benefited from other tax shields due to past and current losses;
- last, but not least, WACC is usually computed using book value of both equity and debt. In case of distress, it is common to have a market value of debt, which is well below its book value reflecting the high probability of default. While handling distress, the objective of the analyst is not only to understand the EV, but also to get an estimate of the market value of debt. In other words, debt is not an input of the model, but an incognita itself. All of this creates circularity in the model.

Because of the problems related to the DCF, to reduce the bias in the estimation of EV, following Buttignon (2014), the company will be valued using a mixed method. The Adjusted Present Value (APV from now on) with cash flows discounted by the unlevered cost of capital k_u will be used for the explicit forecast period. Alternatively, the terminal value will be discounted by the usual WACC, relying on the assumptions that at the end of the plan, the company will improve its capital structure and maintain its stable in the future (which will assumed in a conservative way). This mixed version of APV/DCF helps the analyst to make the valuation more adherent to the fast dynamic of capital structure of distressed firms (to whom the normal version of DCF is not suitable at all).

According to the classical theory of corporate finance, capital structure represents only the distribution of cash flows between debtholders and shareholders. However, in the real world, and especially in case of firms in financial troubles debt has two additional effects on the firm value. On the one hand, higher debt could have a positive impact on firm value because of the higher tax benefits. On the other hands, it increases business risk leading to greater financial uncertainty in the future. In other words, companies with high debt face the risk of see source of financing dry because of the excessive risk.

Moreover, one effect may dominate the other according to company's specific characteristics and, as it has been argued by Leland (1994), there is a level of leverage that maximizes the firm value. It means that until a certain inflection point, positive effects of debt will overcome negative ones but, for too high level of liabilities, the opposite would be true.

Should an analyst dealing with distressed firms, APV appears to be more reasonable because it grounds on the separation between the valuation of company's assets and the estimation of the effects of debt. According to Damodaran[19], APV consists in three main steps: valuing the firm as it is unlevered (financed totally with equity and no debt), computing the tax benefits due to tax shield on interest expenses and finally reaching a value of distress costs as the probability of default multiplied by the expected bankruptcy costs (direct and indirect). This work will use a simplified version of APV that considers only the expected tax benefits from borrowing without entering in the complicated topic of the estimation of distress costs.

$$EV = \sum_{t=1}^{T} PV(FCF_t) + PV(Terminal\ Value) + PV(Tax\ shield\ on\ debt)$$

While valuing a company with the APV, the following steps have to be considered:

a) define the explicit forecast period in years (for companies in financial difficulties, the length of the explicit forecast is usually coherent with the reorganization plan);
b) determine the main assumptions about the evolution of the key variables to get FCF of each projected year in each of the three scenarios. Regarding the base case, projections should reflect both what happened in the past and what it is expected for the future. Worst and best case represent instead deviation from the base scenario. The weight of each scenario should be carefully assumed according to macroeconomic conditions, industry characteristics and management quality. In other words, management could have followed too optimistic assumptions also in the base case in order to induce banks to refinance debt so that the weight of base case scenario should not be too high with respect to the worst case;

[19] http://pages.stern.nyu.edu/~adamodar/New_Home_Page/valquestions/apv.htm

c) compute the Terminal Value (TV from now on) according to Koller, Goedhart, Wessels (2010) following to the key value driver formula $TV = \frac{NOPAT*(1-\frac{g}{RONIC})}{WACC-g}$ where $RONIC = \frac{\Delta NOPAT}{\Delta invested\ capital}$

d) discount FCF of the explicit forecast period using the unlevered cost of capital k_u and the TV using WACC.

e) Compute the expected tax benefits deriving from debt.

The model structure for the computation of the yearly FCF of the explicit forecast period using APV is the following:

Figure 3 - 2 FCF computation scheme

FCF computation scheme

Revenues
EBITDA margin
EBITDA = Revenues*EBITDA margin
Depreciation
EBITA = EBITDA-Depreciation
Operating taxes = tax rate*EBITA
NOPAT = EBITA - operating taxes
Delta Net Working Capital (ΔNWC)
Delta Operating Fixed Capital (CAPEX)
Operating FCF = NOPAT+/- ΔNWC +/- CAPEX

In the next chapter, this model will be applied to the real case of Stefanel to get an estimate of the EV of the group. In doing so, all the assumptions about the three scenarios (with a particular focus on both reorganization plan and restructuring agreement) will be described in details as well as all the procedures to evaluate the variables and key value drivers such as long-term rate of growth, RONIC and so forth.

3.3.2 Cost of Capital

The drawbacks of WACC in distressed contexts have been already discussed. Its computation assumes a constant capital structure that is very far from reality in case of troubled firms and that is one of the reason why the usual DCF is not applied.

An important aspect an analyst should consider while valuing a firm, is being coherent between the discount rate used to get the present value of FCF and the way the FCF has been computed. The mixed APV/DCF method divides company's life in two different stages, the explicit forecast period and the remaining infinite life.

- The former is characterized by the major management efforts to improve the capital structure and to reduce debt and that is why it is impossible to use WACC to discount FCF. In this time horizon of three to ten years, there is variability between the proportions of debt and equity on the total sources of financing and this is going to affect the cost of both debt and equity. As it has been already said, the solution to this problem is represented by imaging the company as it was all equity-financed. Only afterwards, the positive benefits of taxes should be accounted for.

- The latter time horizon, which is the infinite period after the conclusion of the reorganization plan, assumes that the firm makes it through and reaches a steady equilibrium with a balanced capital structure.

In order to remain coherent between the way FCF are computed and the rate used to get its present value, both operating cash flows and debt tax-shield in the explicit forecast period should be discounted by a rate that does not take into account the capital structure. In this context, the unlevered cost of capital is the most suitable discount rate since it is computed as the firm has zero debt.

Conversely, in the computation of the terminal value, since it is assumed that the company has recovered and reached a stable state, the interest rate used in the discounting procedure is the usual WACC.

In the end, for the model to be consistent, since firm's life is split in two periods characterized by different assumptions (volatile capital structure in the former, while stable and constant in the latter), two different discount rates has to be computed.

a) *Computation of the unlevered cost of capital*

The unlevered cost of capital is computed using the classical CAPM method with the following equation:

$k_U = r_f + CRP + \beta_U * MRP$

It is the sum of the risk-free rate, the country risk premium and the market risk premium. It is important to point out that the premium for country-risk is equal to zero in the "pure" version of CAPM, but practitioners may decide to set it at non-negative values when countries where the company operates face periods of high uncertainty or instability.

b) *Computation of the weighted average cost of capital*

The formula for the WACC has already been given and in this part it will be explained how the components of that formula are computed.

Cost of capital is found using the classical CAPM equation:

$k_E = r_f + CRP + \beta_E * ERP$

where $\beta_E = \beta_U + (\beta_U - \beta_D) * \frac{D}{E}$.

The debt to equity ratio is computed using a possible forward capital structure at the end of the forecasting period assuming that the company has recovered and consequently that the market value of debt coincides with its book value.

Debt's beta β_D measures the co-movements of debt returns with the market (it is indeed the systematic component of debt) and for healthy companies is simplistically assumed equal to zero. Nevertheless, this is not true for firms with high debt and consequent non-negligible probability of default. For that reason, debt's beta has to be derived from debt's rating. In case debt is not quoted, its rating will be determined synthetically according to the value of some ratios (the procedure will be explained in detail in the next chapter). To each rating class correspond both a specific debt's beta (which measures the systematic risk) and, in addition, a particular credit spread. In turn, credit spread represents a broad premium for both specific and systematic risk. We use debt beta for the computation of the cost of equity while the generic credit spread to obtain the cost of debt. Consequently, k_D is simply computed by summing the risk-free rate to the debt spread (according to the previous synthetic rating estimation), $k_D = r_f + spread.$

3.3.3 Enterprise Value volatility, σ_{EV}

The volatility of assets (or EV volatility) definitely represents the most controversial variable of the model because it is not directly observable in the market so that it has to be derived implicitly. At first, while talking about volatility, it is worth distinguishing between historical and implied one. The former is computed using historical market prices, looking at the variation with respect to their average. Conversely, the latter is an estimate of the expected variability of asset price by the market and, indeed, deduced from market prices. Historical volatility is backward looking. Its usage, indeed, implies the strong (and often unwise) assumption that what happened in the past will repeat also in the future. For example, if the considered period has been characterized by wide price fluctuations, historical volatility will be high and the estimated volatility could be overestimated because it is possible that the future will be less volatile than the past, and vice versa. On the contrary, implied volatility is forward looking because it reflects the market expectation about the future.

Regarding the methodology to compute volatility of EV, it is worth pointing out that most of the times, company assets are financed by a mixture of equity and debt. As a result, asset volatility should be a function of the volatility of both equity and debt. In particular, it could be computed as the average of the volatility of returns of both financing sources, weighted by the proportion of each capital class on the total sources of financing, with the following formula, Guthner (2013):

$\sigma_{EV}^2 = \sigma_E^2 * x_E^2 + \sigma_D^2 * x_D^2 + 2 * \sigma_E * \sigma_D * x_E * x_D * \rho_{E,D}$, where x_E and x_D are the fractions of the firm's capital structure represented by equity and debt, respectively. Both fractions should be computed on a market basis. A weakness of such an approach is that market value of debt is

an incognita. This would lead to a problem of circularity in the model. In other words, in the aim of finding a proper market value of corporate debt, we need to estimate EV volatility that cannot depends on debt market value itself. Another drawback of the formula arises when the company under analysis does not have quoted debt. In this case, it is not possible to estimate variables such as debt volatility σ_D and correlation between returns of debt and equity $\rho_{E,D}$.
Furthermore, such a formula cannot be applied in case of more complex capital structure. Indeed, usually firms have more than one tranche of debt, issued in different time and characterized by different maturity and different seniority. In addition, even equity can be differentiated between common and preferred stocks. It becomes consequently impossible to compute a reliable estimate of equity and debt volatility.
A solution could be to obtain an estimate of the asset volatility exploiting the reverse of the option-pricing formula of Black&Scholes. This allows us to compute asset value and its volatility starting from equity values and equity volatility solving this system of two equations in two unknowns.

$$E_t = EV_t * \exp\left(-\delta(T-t)\right) * N(d_1) - F * \exp(-r(T-t)) * N(d_2)$$

$$\sigma_E = \sigma_{EV} * [\left({EV_t}/{E_t}\right) * N(d_1)$$

The reasoning behind this solution is the same as the one for computing implied volatility in the options market. Looked at another way, practitioners and option's traders do not use B&S formula to obtain options' prices because prices are already available in the market and do not need to be estimated. On the contrary, B&S formula could be practically "reversed" to derive the level of volatility implied by the market. Said another way, B&S formula could be used to estimate implied volatility that makes the B&S price equal to the market price. That specific volatility is the implied level by the market.
The exact same reasoning could be used to obtain volatility of the enterprise value. Said another way, B&S model allows us to get a value of EV volatility that makes B&S equity value equal to its market capitalization. "The volatility of a firm's assets drives the volatility of the firm's equity", Guthner (2013, pp. 139) and, indeed, their relationship can be written as $\sigma_E = \sigma_{EV} * [\left({EV_t}/{E_t}\right) * N(d_1)$ where $N(d_1) = \frac{\partial E}{\partial EV}$ which is the delta of the option. The delta shows that equity volatility is an increasing function of the leverage of the company. In addition, σ_E depends on the sensitivity of the option value (equity value) to changes in the underlying asset (EV). From this relationship, it is possible to obtain the equation for EV volatility. $\sigma_{EV} = (\frac{E}{EV}) * \frac{1}{N(d_1)} * \sigma_E$.

A solution proposed by Loffler and Posh (2007) is to assume $N(d_1) = 1$ to make the system of two equation easier to be solved with an iterative procedure in Excel performed by using the Solver. The idea is to define an error function as

$$estimation\ error = \left(\frac{Estimated\ Equity}{Observed\ Equity} - 1\right)^2 + \left(\frac{Estimated\ Equity\ Volatility}{Observed\ Equity\ Volatility} - 1\right)^2.$$

In other words, at the first iteration, we ask the solver to estimate equity and its volatility starting from initial and arbitrary values of EV and its volatility. After the first iteration, estimated equity value and equity volatility are compared with the observed values in the market, computing the estimation error. The solver continues to compute new values for both estimated equity and estimated equity volatility (changing EV and EV volatility accordingly) until the squared error is minimized. The solution is represented by values for EV and EV volatility that minimize the error function. Said another way, the aim of the iterative procedure is to find a couple of $[EV, \sigma_{EV}]$ that makes the couple $[E, \sigma_E]$ as close as possible to the values observed in the market. In doing so, the squared error function will be minimized.

A criticism to this procedure could be the strong assumption of $N(d_1) = 1$ meaning that each variation of one percent in EV will lead to the same change in equity, which is not always reasonable.

According to Hull (2012), one of the most used procedure to recover implied volatility is the Newton Raphson method. It consists of an iterative procedure that allows to find the approximate solutions of a function in the form of $f(x) = 0$. In other words, given a function $f(x)$, this method gives roots such that $f(x) = 0$[20].

The function of our interest is given by $f(\sigma) = E_{mkt} - E_{BS}(\sigma)$ and the solution we are searching for is the one that assure equality between B&S price and market price. Practically, the analyst needs to start with an initial estimate for EV volatility so that a starting level of B&S price could be computed. Afterwards, with an iterative procedure, the initial guess will be modified until the difference between market and B&S price will be smaller than a pre-defined threshold. This could be easily implemented by a VBA code. The iterative algorithm is represented by the following formula $\sigma_{imp2} = \sigma_{imp1} - \frac{C_{mkt} - C_{BS}(\sigma)}{\frac{\partial C_{BS}(\sigma)}{\partial \sigma}}$. In VBA, we just need to create two different functions that can be used in an Excel worksheet. One function computes B&S price starting from inputs such as underlying asset value, time to maturity, risk-free rate, dividend yield and initial volatility guess. Another function regards the computation of implied

[20] The application of the model in the option-pricing environment can be found for example in the website http://www.risklatte.com/Articles/QuantitativeFinance/QF135.php

volatility following the Newton-Raphson iterative procedure. Code of both functions is presented in figure 3.3 and 3.4 respectively.

Figure 3 - 3 VBA function to compute B&S price for a vanilla option (call and put)

```
Function EuropeanOption(CallOrPut, S, K, v, r, T, q)
Dim d1 As Double, d2 As Double, nd1 As Double, nd2 As Double
Dim nnd1 As Double, nnd2 As Double

d1 = (Log(S / K) + (r - q + 0.5 * v ^ 2) * T) / (v * Sqr(T))
d2 = (Log(S / K) + (r - q - 0.5 * v ^ 2) * T) / (v * Sqr(T))
nd1 = Application.NormSDist(d1)
nd2 = Application.NormSDist(d2)
nnd1 = Application.NormSDist(-d1)
nnd2 = Application.NormSDist(-d2)

If CallOrPut = "Call" Then
  EuropeanOption = S * Exp(-q * T) * nd1 - K * Exp(-r * T) * nd2
Else
  EuropeanOption = -S * Exp(-q * T) * nnd1 + K * Exp(-r * T) * nnd2
End If
End Function
```

Source: http://investexcel.net/implied-volatility-vba/

Figure 3 - 4 VBA function to compute implied volatility through Newton-Raphson method

```
Function ImpliedVolatility(CallOrPut, S, K, r, T, q, OptionValue, guess)
    Dim epsilon As Double, dVol As Double, vol_1 As Double
    Dim i As Integer, maxIter As Integer, Value_1 As Double, vol_2 As Double
    Dim Value_2 As Double, dx As Double

    dVol = 0.00001
    epsilon = 0.00001
    maxIter = 100
    vol_1 = guess
    i = 1
    Do
        Value_1 = EuropeanOption(CallOrPut, S, K, vol_1, r, T, q)
        vol_2 = vol_1 - dVol
        Value_2 = EuropeanOption(CallOrPut, S, K, vol_2, r, T, q)
        dx = (Value_2 - Value_1) / dVol
        If Abs(dx) < epsilon Or i = maxIter Then Exit Do
        vol_1 = vol_1 - (OptionValue - Value_1) / dx
        i = i + 1
    Loop
    ImpliedVolatility = vol_1
End Function
```

Source: http://investexcel.net/implied-volatility-vba/

Once these two functions have been created, they can be used in an Excel spreadsheet where we can insert the value of the different inputs.

The main drawbacks of such a model is that equity volatility is assumed to remain constant over the entire period of valuation and more important, shares of the firm under investigation should be liquid enough. Liquidity could be a relevant issue because if the shares are not exchanged frequently in the market, information of equity value and volatility could be meaningless or misleading.

To make the analysis more precise, it could be better to compare the volatility obtained using the above-mentioned procedure with asset volatility of comparable firms. If our value of implied volatility is lower than the average from competitors, it could be a signal of misleading information from the equity market due to a lack of liquidity.

3.4 Debt Valuation and Probability of Default (PD) estimation

The methodology to compute implied volatility is important because it gives us the price of both call and put options. This lets us come back to the starting point, where we have said that the company's equity could be intended as a call option on the company's assets with a strike price equal to book value of its liabilities. In case of distress, expected equity value computed using APV/DCF model may result equal to zero. Mathematically, equity value could become even negative but it is unreasonable because of the limited liability condition. In particular, during the distressed period, equity could be a deep out of the money option that is unlikely to be exercised at maturity. At the same time, since the underlying asset is characterized by a specific level of volatility, by chance it may happen that it will spike in the future making the option valuable and exercisable at maturity.

This is also what creditors of a distressed company hope. In other words, they have lent money to a company that started to underperform, having troubles in repaying the debt back. Once financial difficulties have started, creditors usually face two alternatives. On the one hand, they can allow for a debt restructuring in the hope of a recovery that is as much unlikely as desirable to obtain all the initial scheduled payments. On the other hand, they can claim for liquidation, which leads to assets sales. The problem with the latter solution is the negligible sum of money that is usually recovered in the liquidation procedure. This is the reason why creditors (banks) prefer to allow for a debt restructuring (accompanied by a feasible industrial plan).

This possibility for an increase in equity value is the leading factor that makes market capitalization of a distressed firm positive. This is implicit in the reorganization plan. To put it simply, giving additional breathing space for a possible turnaround in the future, creditors grants a call option to shareholders. "If events evolve positively (better than the plan), shareholders will be able to enjoy the upside while if events are negative, at worst, E will be zero", Buttignon (2012, pp.5). If equity value benefits from this call option, the reverse of the coin is represented by a reduction in debt value for an amount equal to the put option value. Subtracting the value of the put option to the nominal NFP provides a first rough estimate of debt market value. It is a rough estimate because it reflects all the limitations of B&S formula, like the assumptions of only one class of debt with the same maturity and the constant volatility, which are far from reality.

In addition, the application of B&S formula gives the possibility to directly estimate the probability of default which is equal to $PD = N(d_2)$.

The model is characterized by the following main assumptions:

1. EV of the firm follows a Geometric Brownian Motion with the following process:

$$EV_t = EV_0 * e^{[\left(r-\delta-\frac{\sigma^2}{2}\right)*t+\sigma*Z_t} \text{ with } Z_t \sim N(0,t) \text{ and}$$

$$EV_T = EV_t * e^{[\left(r-\delta-\frac{\sigma^2}{2}\right)*(T-t)+\sigma*(T-t)*Z_t} \text{ with } Z_t \sim N(0,T).$$

2. Volatility of the EV is assumed to be constant over the entire valuation period;
3. Time to maturity can be assumed equal to the length of the restructuring agreement;
4. Payout rate δ assumed equal to zero because the company is distressed and not expected to distribute anything to shareholders, but to reinvest all the resources produced in the year;

Speaking of probability of default, it is worth mentioning that this situation is the same as what in stochastic calculus is called hitting time. Following Jeanblanc (2006), given EV_t dynamics, first hitting time of DP is denoted as

$T_{DP}(EV) = \inf\{t \geq 0: EV_t = DP\} = \inf\{t \geq 0: X_t = \frac{1}{\sigma}\ln\left(\frac{DP}{X}\right)\}$ where $X_t = v * t + W_t$ and $v = \frac{\mu}{\sigma} - \frac{\sigma}{2}$ and $\mu = r - \delta$.

Then, $T_{DP}(EV) = T_\alpha(X)$ where $\alpha = \frac{1}{\sigma}\ln(^{DP}/_{x})$.

It follows that for $DP < x$ (or $\alpha < 0$), $P(T_{DP}(EV) < t) = P(T_\alpha(X) < t) = 1 - P(m_t^X \geq \alpha) = N\left(\frac{\alpha - v*t}{\sqrt{t}}\right) + e^{2*v*\alpha} * N(\frac{v*t+\alpha}{\sqrt{t}})$ that represents a closed-form formula for the computation of default.

A similar approach is the one of the rating agency Fitch, which uses a barrier option model. Equity is still assumed as a call option on company assets with a strike price equal to the KMV's default point. The difference lays on the type of call option. Said another way, rather than a plain vanilla, equity is modelled as a barrier option, in particular as a down-and-out call. This kind of exotic option is said to be path-dependent, Hull (2012). Indeed, if in any moment the underlying assets hits the barrier (that has to be lower than the strike price) the option becomes worthless even in case asset will be higher than the strike price at maturity. It means that in case the underlying does not hit the barrier, option payoff is the same as a plain vanilla call. Otherwise, payoff is zero.

This increases flexibility in the model because there is the possibility to define a barrier in addition to the exercise price and because default could not only happen at maturity but also in any moment prior to expiration date.

The functioning of this option seems to be suitable for equity valuation. If EV hits a default barrier, company will default, equity will become worthless and this may happen before the scheduled maturity. Equity payoff is:

$$E_t = \begin{cases} 0 \; if \; EV_t \leq H, for \; at \; least \; one \; t \leq T \\ Plain \; vanilla \; call \; payoff \; if \; EV_t > H, \; \forall t \leq T \end{cases}.$$

The formula for the computation of the value of this call option (equity value) is provided by Hull (2012), but also by Fitch itself in its quantitative research report of 2007[21]. Fitch provides also a formula to obtain expected probability of default so that we will use their notation for convenience.

$$E_t = EV_t * \left\{N(x^+) - [\frac{H}{EV_t}]^{\frac{2*r}{\sigma^2}+1} * N(y^+)\right\} - e^{-r(T-t)} * K * \{N(x^-) - [\frac{H}{EV_t}]^{\frac{2*r}{\sigma^2}-1} * N(y^-)\}.$$

H represents the exogenous barrier, $x^{\pm} = \frac{ln(EV_t) - \ln(K) + (r \pm \frac{1}{2\sigma^2})(T-t)}{\sigma\sqrt{T-t}}$ and $y^{\pm} = \frac{2*\ln(H) - ln(EV_t) - \ln(K) + (r \pm \frac{1}{2\sigma^2})(T-t)}{\sigma\sqrt{T-t}}$.

Probability of default is given by $PD_t = N(-x^-) + [\frac{H}{EV_t}]^{\frac{2*r}{\sigma^2}-1} * N(y^-)$.

Because of its higher risk of being worthless, a down-and-out call usually has a lower value than a plain vanilla, unless the underlying asset has really high values. Modelling equity as a plain vanilla means that its value always profits from a rise in volatility of the underlying, while this happens only in certain circumstances in case of a down-and-out call. Indeed, higher volatility could be good because EV may increase up to the strike price but, at the same time, it could also go down and touch the barrier, making equity value equal to zero. This could be reasonable in reality because volatility is a double-hedged sword that could help the management (and equityholders) to boost returns, but at the expense of a higher total risk that could lead to the financial collapse of the firm.

Apart from these considerations, in their research report, Fitch's analysts suggest to fix the barrier to the same level of default point. In this way, they allow the company to default in any point between the start of the contract and its expiration whenever EV becomes lower than DP. Therefore, we expect a higher probability of default with respect to the normal plain vanilla call.

[21] Liu, B., Kocagil, A. E., Gupton, G. M., 2007. Fitch Equity Implied Rating and Probability of Default Model. New York.

Nonetheless, setting the barrier $H = DP$ is possible only when the starting EV is higher than DP itself. Said another way, if initial EV is higher than DP, it makes sense to value equity as a down-and-out call both mathematically and practically. Equity is expected to have a positive value only in case EV never touches the default threshold.

On the other hand, if the starting EV is lower than or really close to DP (and it usually happens in case of distress), it makes no sense to fix $H = DP$ because in this case equity will always be worthless. In the exact moment when equity value rises above the strike price allowing equityholders to profitably exercise the option, it also touches the barrier making the option itself worthless. It sounds as a contradiction[22].

In our opinion, the down-and-out option could be used to exploit the possibility to set the barrier at a different level with respect to the strike price.

The barrier could have two different interpretations: one the hand, it could be thought just as a general protection covenant (benefited for all the shareholders) that once hit by EV, it leads the company to default; on the other hand, it could be seen as the amount of senior debt.

In particular, it could be interesting to set H equal to the value of senior bonds. In other words, we assume that senior bonds are protected by a contractual clause according to which, in case EV shrinks below the value of senior bonds itself, senior bondholders will claim for the company default and its consequent liquidation. Said another way, the down-and-out option (DO option from now on) allows us to differentiate between two different classes of debt. Prior to maturity, it does not matter whether EV will fluctuate above or below the DP, but the only aspect bondholders care about is represented by the comparison between EV and the value of senior debt. If the former touches the latter, default is triggered and company is in bankruptcy, even if EV will be above DP at maturity. Otherwise, if EV never touches H, the call option will be exercised by shareholders, at maturity in case of $EV > DP$.

When $H = Senior\ Debt$, equity is expected to have a lower value than the case of only one debt category. In addition, we expect a higher probability of default in case of the down-and-out call. The reduction in equity value due to contractual clause protecting senior debt is represented by the difference between $Equity_{plain\ vanilla\ call\ option} - Equity_{down-and-out\ call\ option}$. The other side of the coin is the higher total market value of the debt.

It is possible to represent payoff function of both senior and junior creditors as follow:

$$Senior\ Bond\ (S = H) = \begin{cases} EV_t\ , if\ EV_t \leq H\ for\ at\ least\ one\ t \leq T \\ \min(EV_T, \quad S), if\ \ EV_t > H\ \forall t \end{cases}$$

[22] This will be shown in details in chapter 4 when the model will be practically applied.

$$Junior\ Bond\ (J) = \begin{cases} 0, if\ EV_t \leq H\ for\ at\ least\ one\ t \leq T \\ \quad if\ EV_t > H\ \forall t \end{cases}$$

To make it straight, we are saying that the presence of the barrier gives the possibility to think at equity as a down-and-out call option rather than a plain vanilla one. This reduces equity value on the one hand, and rises debt value on the other.

In particular, we expect that increasing the level of the barrier will lead to two main consequences:

1. Lower equity value: it means that when a big portion of debt is secured, equityholders are giving up part of their privileges in favour of senior bondholders. It is possible to see it as a bargaining process between equityholders and bondholders;
2. Higher probability of default, showing that sometimes for bondholders is better to have a default.

Point one refers to the linkage between the variables of B&S pricing formula and the bargaining process during distress between the two main classes of financers. On the one hand, shareholders gain from the restructuring plan granted by bondholders. Being similar to a call option, equity benefits from the upside potential in case of a positive scenario while debt suffers from a reduction equal to the put option value that represents the difference between its nominal and (the lower) market value. Furthermore, the limited liability condition encourage managers (who are supposed to act on shareholders' behalf) to invest in risky project to raise EV volatility. On the other hand, bondholders have the possibility to protect themselves asking for being more safeguarded. In our model, this protection is represented by a contractual clause (the barrier) that trigger default once hit by EV.

Second point is somehow linked to this last sentence. The level of the barrier affects heavily PD meaning that sometimes creditors prefer that companies go bankrupt instead of survive. The higher the barrier and the higher the level of EV in case of default, granting greater expected revenues on asset sales. More precisely, senior creditors benefits from the higher probability of default.

The examination of this and more aspects will be continued in chapter 4, where our reasoning will be explained in light of the numerical and mathematical results.

3.5 Conclusion

In conclusion, our structural model could be though as a hybrid between the classical Merton framework and the modern exotic option with barrier. Merton assumed only one class of debt with the same maturity and modelled default like an event that could happen only at maturity, exploiting the similarities with payoff of a long position in a European call option. It is important to remember that Merton developed its framework in 1974 exploiting all the financial tools that were available at that time. Besides, the world of finance is a fast-paced environment characterized by a continuous research for financial innovation to increase flexibility (and sometimes complexity) of products. Exotic options are said to be path-dependent, in the sense that their payoff does not only depend on state variables at maturity but also on their behaviour in the meanwhile. This is suitable to model default as an event (like in real World) that may happen in every moment between stipulation and maturity. Furthermore, the presence of barrier lets us include protected debt in our analysis, giving also space for a discussion about the bargaining process between shareholders, senior and junior bondholders in case of restructuring process.

In addition, it exploits the advantages of the KMV. In particular, the fact that companies usually default when their assets value is somewhere in the middle between short-term and total liabilities. This is the reason why we opted for a strike price equal to the KMV's default point. While applying such a model to a real case, it is very important to correctly estimate inputs like enterprise value and its volatility. Those variables are crucial because their estimation requires assumptions that could affect final results and this is why we will try to explain as clearly as possible all the reasoning behind our choices in next chapter.

To conclude, all of this is aimed at answering to the two main research questions of the work. On the one hand, it allows to explain positive market capitalization of distressed companies. In other words, despite the poor economic performances and despite the heavy amount of debt, investors continue to either buy or hold shares, allowing equity to be non-negative in the market. This could be justified by the possibility of a turnaround in the future due to additional time that is bought by management through the reorganization plan agreed with creditors.

On the other hand, the model gives an idea about how to "roughly" estimate market value of non-quoted corporate debt in case of distress. Indeed, distressed debt is usually characterized by a market value that is lower than its nominal amount. In addition, this reduced market value is not directly observable, but has to be estimated.

Information about debt's real value coupled with credit analysis are valuable to any investor like current and future potential shareholders and current and future potential creditors to understand not only the actual company's creditworthiness but also its future prospective.

CHAPTER 4: THE STEFANEL CASE

4.1 Introduction

The following chapter is about the analysis of the Stefanel S.p.A. case. The company is the industrial holding of the Stefanel Group ("SG" from now on). SG is an Italian designer, manufacturer and retailer of both men's and women's apparel whose collections are distributed in Italy and internationally under its own brand Stefanel through its wide and consolidated "shop-network" which counts 141 shops in Italy, 211 in rest of Europe and 52 in rest of the World, in 2014. In total, Stefanel can rely on 404 shops, of which 167 are Directly Operated Stores (DOS).

Stefanel only represents one of the two main business units. The other major business unit is Interfashion, which designs, produces and distributes women's clothes for both own and licensed brands. The business unit Interfashion is completely managed by the company Interfashion S.p.A, which is, in turn, a subsidiary one hundred percent owned by SG.

This chapter will give, at first, a brief but complete presentation of the group's structure and history in paragraph 4.2.1 and then, it will provide an analysis of historical performances of the last five years (from 2010 to 2014) which covers almost all the company's crisis. In addition, it will be performed also a credit analysis with a computation of the Altman score as well as a synthetic credit rating measurement to give insight about the financial distress in which the company lays. Afterwards, the chapter will continue with a description of the main events that has happened in the years 2015 and 2016, that represents the main reason why this case has been chosen for this thesis. Finally, the model for both company and debt valuation that has been presented in the previous chapter will be applied to the Stefanel case.

4.2 Group overview

4.2.1 Group's structure and history

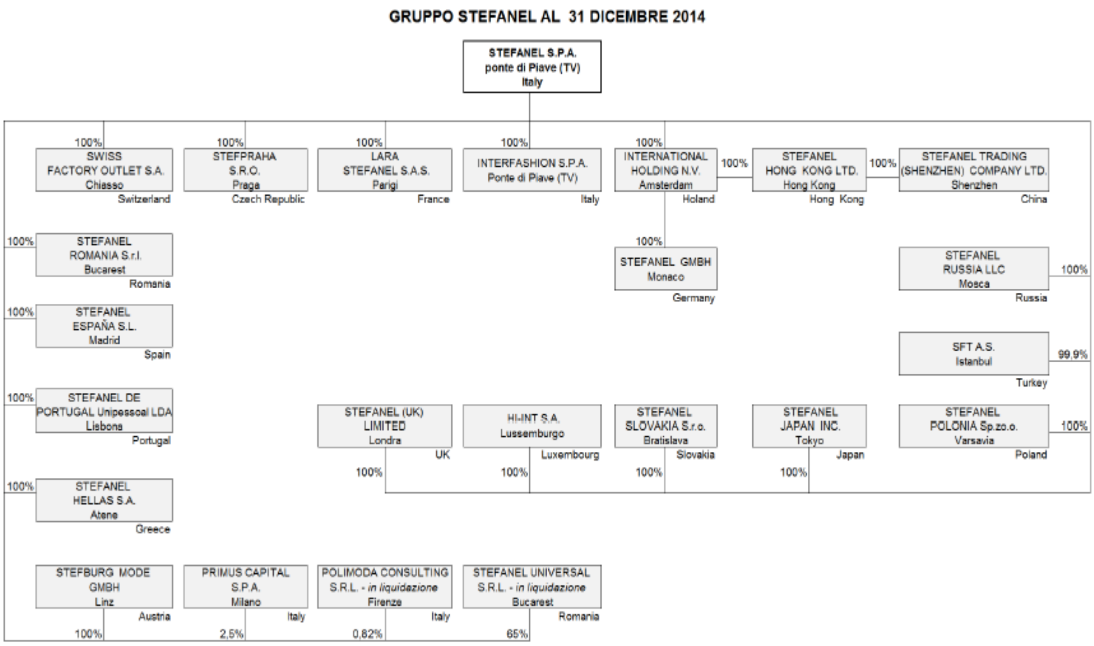
The establishment of the brand and of the company as a whole are strictly linked to Carlo Stefanel. He was a small businessman who first worked in the spinning mill of his aunt and then started to produce knitwear in his factory, the "Maglificio Piave" in 1959 in the Italian region of Veneto (one of the most famous Italian fashion and design district known for famous brands as Benetton and Diesel, which also represent the main competitors).

In the 1960s, "Maglificio Piave" only produced knitwear for the wholesales market. In 1970, Giuseppe Stefanel, Carlo's son, took on the management of the company and started recognizing the company's potentiality to launch its own brand characterized by its unique design. Since then, they started to produce a variety of knitwear, capturing at first the attention of Italian consumers and then, of international ones. This is demonstrated by the fact that the opening of the first branded shop in Italy, in Siena in 1980, was immediately followed by the first opening outside Italy, in Paris in 1982. The 1983 is a very important year because the company's name has been switched from "Maglificio Piave" to Stefanel.

The 1980s was a period of wide expansion for the company. A big number of stores was opened in Italy, Europe and also in North America (especially in USA). In additions, through a series of acquisitions and strategic alliances, the business started to be diversified, not only focused on the production of knitwear but also jeans and sportwear in general. This expansion process was financed by both internal growth and by the decision of becoming public, being quoted in the Italian Stock Exchange, in 1987. 1990 was the year of the acquisition of Interfashion, which still represents one of the two main business unit of the group as it was already mentioned.

Afterwards, also in the new Century the company sought new opportunities to boost sales and expand its business and to reposition the brand (from a medium to a higher segment) with the launch of the clothing brand HIGH by Interfashion S.pA. and, by the license agreement with a famous Italian fashion designer Antonio Marras for the brand I'm Isola Marras. In doing so, in 2003, Stefanel family increased its own direct ownership in the company increasing their ownership to more than 57,5 percent. In 2011, they launched online store for the brand Stefanel. All these operation contributed to the current group structure depicted in the following picture.

Figure 4 - 1 Group structure



Source: Company's Consolidated Financial Statement

Currently, Stefanel S.p.A is the industrial holding of the group composed by companies that mainly operate in the apparel business, focusing on the design, the production as well as the distribution of clothes.

The mission of the company could be abridged by the following sentence "*Fashion First. Italian Always. Quality Forever*." Through the years, Carlo Stefanel and his son Giuseppe have tried to make the brand known for its dependability and dynamism. They always managed to satisfy the evolving consumers' needs and taste as well as to make versatile collections with an always recognizable style. This could have happened only with the support of a solid industrial and business scheme always aimed at efficiency and quality.

The main steps of the company establishment and growth have been summarized in the following figure.

Figure 4 - 2 Company timeline

1947	Carlo Stefanel began producing and selling wool in local markets of Veneto
1959	Carlo Stefanel established his own clothing company, "Maglificio Piave" supplying wholesaleers in Italy
1970	Giuseppe Stefanel, Carlo's son took on the management of the company
1980	The first direct shop was opend in Siena
1982	The first shop outside Italy was opened in Paris
1983	Stefanel became the company name
1985	Company increased fourfold its turnover and the number of stores
1987	Quotation at the Milan Stock Exchange
1990	Interfashion acquisition
1990s	Years of joint ventures with Onward Kashiyama to open new stores in Japan, South Korea, Taiwan; with Calvin Klein. New opening also in North America.
2002	Acquisition of 50% of Nuance Group to boost sales
2003	Giuseppe Stefanel incremented its stake into the group to more than 57,5%
2007	Launching of the the brand HIGH
2008	Speed up of the repositioning process for Stefanel
2011	Launch of the online store for Stefanel

Source: http://fashiongear.fibre2fashion.com/brand-story/stefanel;

http://www.encyclopedia.com/topic/Stefanel_SpA.aspx

4.2.2. The crisis and the current situation

First signals of a potential crisis started in the middle of the two thousands due to the increasing competition in the apparel market, especially in the middle segment (which is characterized by low entry barriers), and due to the difficulties in implementing the strategic plan of Stefanel brand repositioning toward a higher segment (but still with accessible prices).

The 2007 was the first year characterized by a negative result of 27 million euro after a 2006 with a positive but low profit of 1 million euro. This was mainly due to a big reduction in EBITDA that reflected higher costs of the shop networks and higher investment in advertisement and promotion. This period of first difficulties for the firm coincided with the start of the global financial crisis and the consequent deep slowdown of consumptions in all over the world. It was in these years that profitability problems were also followed by financial uncertainties about the availability of financial resources to implement the group's strategic plan over the medium long-term.

The 2010 was the year of first intervention in the capital structure, which was unbalanced and characterized by high leverage and heavy financial costs. Following these problems, the first restructuring agreement with banks was signed on the 26th April and it was also accompanied by a capital increase. At the same time, following the art. 2446 of Italian Civil Code, part of the equity of the company was used to cover current and retained losses.

In June 2011, a new restructuring agreement was signed with banks because some of the contractual covenants were not respected by the company. Unfortunately, the profitability and financial conditions of the company did not improve and in June 2014, a new restructuring agreement and a new restructuring plan for the period 2014-2017 has been signed.

The agreement will expire on the 31st December 2017 and is characterized by the following main contractual terms:

- Freezing of the reimbursement of the long-term financial principal until the 30/06/2016 (reimbursement that will consequently start from December 2016);
- Confirmation of the short-term operating financing line until the 31/12/2017;
- Check, on biannual basis, on covenant respect, regarding EBITDA, Net Financial Position (NFP) and shareholder's equity. The breach of only one of these covenants will lead to the need of a reformulation of the agreement with banks.

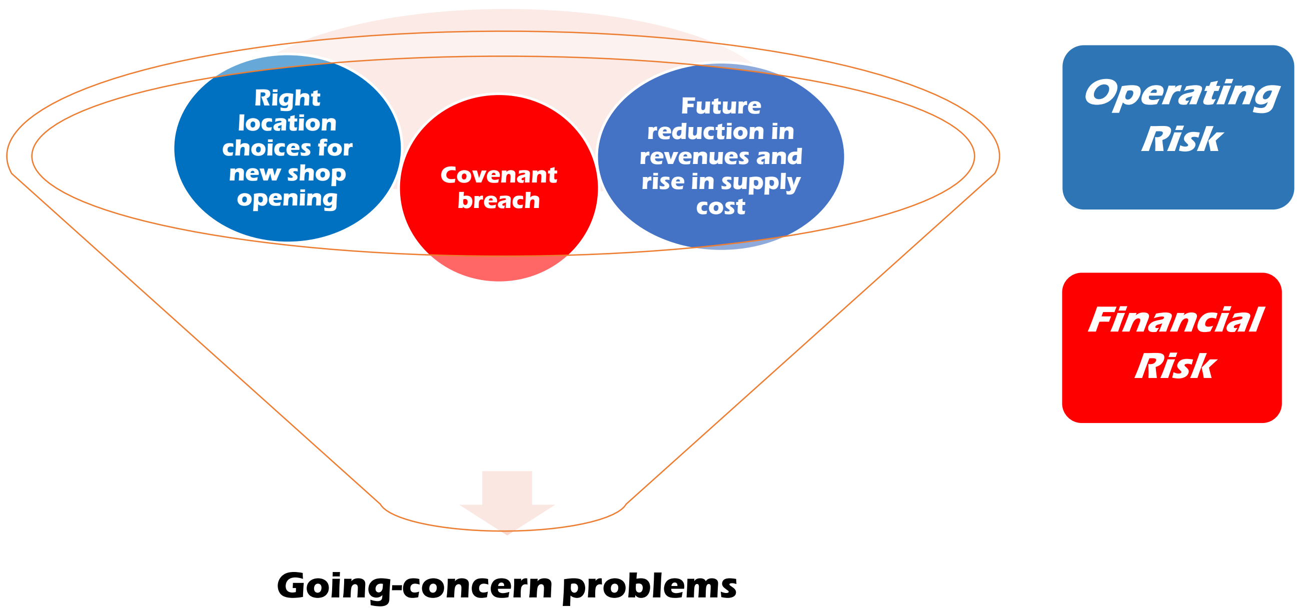
The industry plan for the period 2014-2017 has the following main objectives:

- As regard the business unit Stefanel, conclusion of brand repositioning (toward a medium-high level) and relaunching with an improvement of style-lines, as well as advertisement cost rationalization and shop refurbishment;

- Concerning the business unit Interfashion, continuing the development of the brand HIGH with particular focus on foreign markets boosting sales through the web and new monobrand shops;
- Increase in the like-for-like sales in all the geographical areas where the company operates as well as a rise in the sales from online shops;
- Improvement of gross industrial margin thanks to new and better sourcing and initial mark-up strategies;
- Development and consolidation of the network of both DOS and franchising monobrand shops, with the main focus on growing and emerging markets where the company already has presence (to avoid too high investment in unknown areas).

Main risks and uncertainties linked to the industrial plan 2014-2017 are, instead, summarized by the following figure.

Figure 4 - 3 Stefanel risks and uncertainties



Source: Group's financial statement

In this context, it is worth pointing out that this part of the work has been written during the end of March and April 2016 so that it is good to add the information available in that months. At the beginning of March 2016, the group has communicated that in the financial year 2015 there has been another covenant breach and consequently the society calendar has to be re-scheduled with the approval of consolidate financial statements being delayed to the end of May. As a result, the fear for future risks and uncertainties has been materialized soon in 2015. In the meanwhile, the management is committed to prepare a new industrial plan (for the period 2016-2019) and to negotiate a new (the fourth) restructuring agreement with the financing banks, aimed at prolonging the freezing of debt principal reimbursement as well as maintaining the short-term credit lines to finance daily operations.

Reading newspapers, it has emerged that the company is continuing in the implementation of strategic plan to boost sales, following the policy of closing underperforming shops and opening new ones in emerging markets. The group decided for the close of a shop in Palermo, but at the same time for the new opening of a monobrand flagship store in Dubai (in the "Dubai Mall", the biggest shopping mall in the World) and of a monobrand store in Oslo (in the "Oslo City", the most important shopping mall in Scandinavia).

Since the financial statement for 2015 has not been approved yet, 2015 will not be considered as an actual year but as the first year of financial projections. In doing so, both the information about events in 2016 and those available from the strategic plan 2014-2017, will be put together and used in the elaboration of the financial projections for the estimation of the Enterprise Value of SG.

Before starting with the application of the model described in the previous chapter, an economic, financial, and credit analysis of the company for a five years period (2010-2014) will be performed to understand how the company tried to react to the crisis.

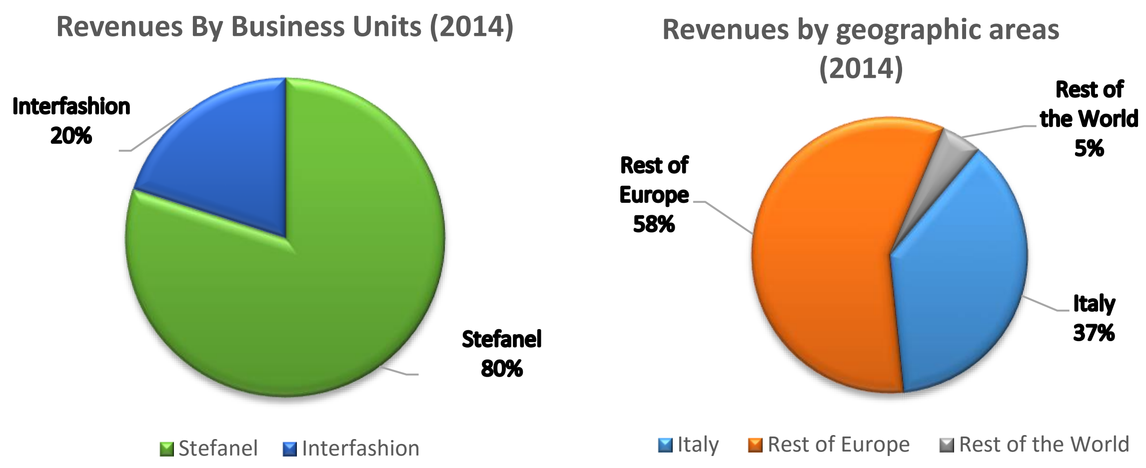
4.3 Economic, Financial and Credit Analysis (2010-2014)

The aim of this paragraph is to elaborate an analysis of the last 5 years performance of SG. The period of 2010-2014 has been chosen because it is characterized by the deepening of the financial crisis and by several attempts of the management to succeed in the company's turnaround through three different strategic plans and reorganization agreements to obtain some additional financial breathing space.

To perform this analysis, balance sheet and income statement have been restated each year into a more suitable scheme that makes it easier the investigation of company's performances and the computation of free cash-flows. Furthermore, main ratios already described in the section about the four's Cs of credit, especially about Capacity, will be presented to understand the firm profitability, liquidity, coverage and leverage situation. In addition, a synthetic rating will be computed, as well as the Z-score to see how the company's creditworthiness has changed during this period of crisis. Usually, the analysis is enriched through the comparison between the performance of the firm under investigation and its competitors but this part has been omitted to save time that will be dedicated to the model application.

Every company's equity research starts with a look at the revenues, their composition and their evolution. In 2014, revenues were composed as follow.

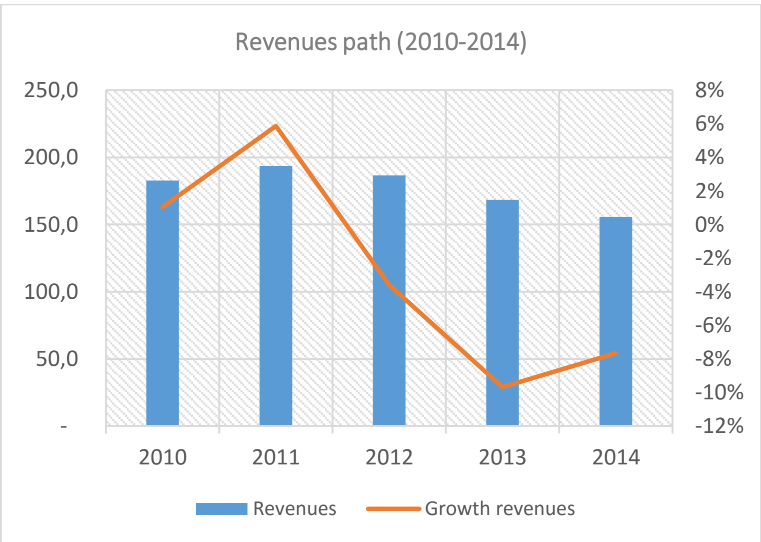
Figure 4 - 4 Revenues by business units and by geographic areas (2014)



Source: company's financial statement

From the sale of the stake in Nuance in 2010, the group is composed by two business units, Stefanel and Interfashion, and is it possible to notice the considerable predominance of the first on company's sales. SG aims at focusing principally on the historic brand Stefanel and not at diversifying the business. This strategy could be a two-edged sword, because all the management effort will be concentrated on Stefanel, exploiting all the potential of the brand but with risk of being too much dependent on it. Geographically, in the past, the company relied a lot on the Italian market, which still cover a big portion of the total sales, but the relevance of the group on the rest of Europe is becoming considerable, while its presence in the rest of the World remains low.

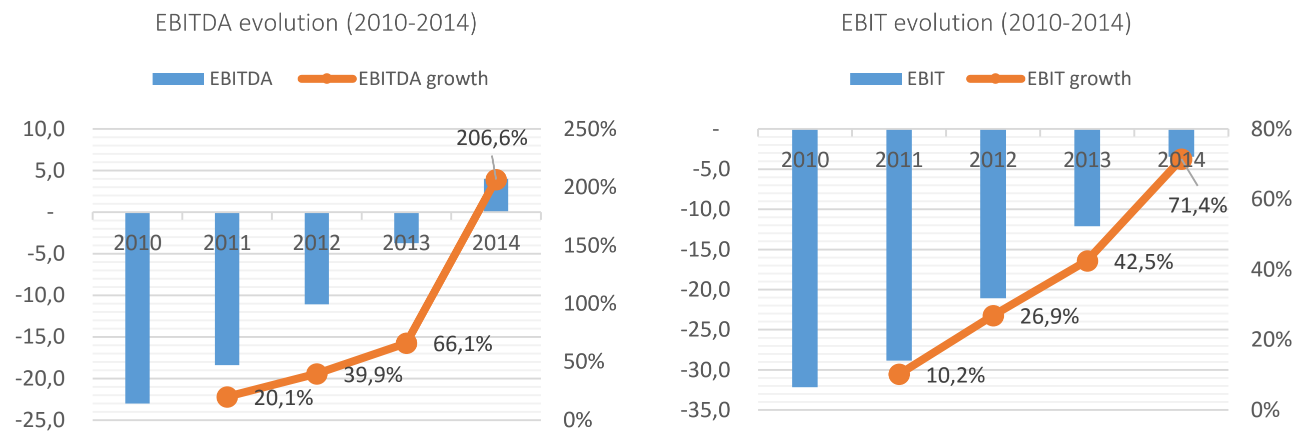
Figure 4 - 5 Revenues path (2010-2014)



Source: Company's financial statement

In the last 5 years, revenues has decreased from about 180 million euro to 155 million and this reduction is due especially to 2013 (more than 10% reduction) and to 2014 (about 8% reduction).

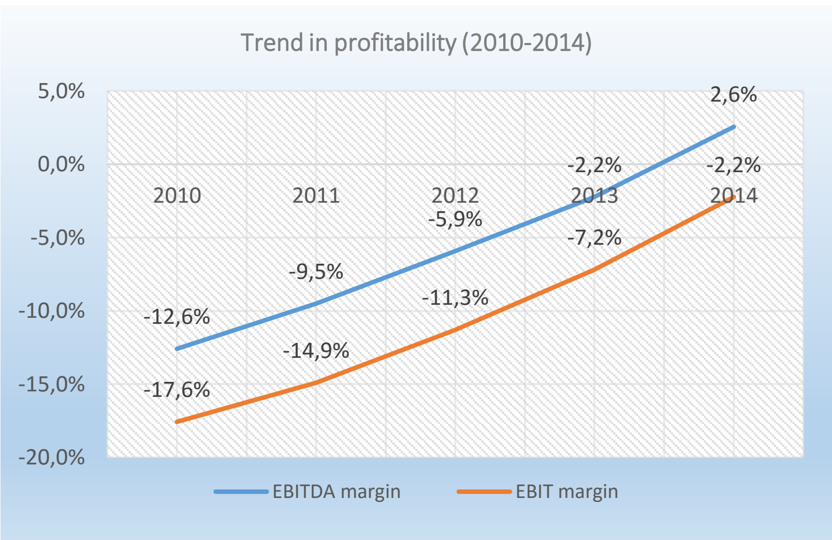
Figure 4 - 6 EBITDA evolution and EBIT evolution (2010-2014)



Source: company's financial statement

Looking at the evolution of both EBITDA and EBIT, one could easily notice how the trend is opposite with respect the one of revenues. EBITDA rises from about -25 million euro to a positive level about 2 million euro, while EBIT, despite remaining negative, rises from more than -30 million to about -5 million, with a 25 million improvement in 5 years. Costs have decreasing more than the revenues and that is the reason for the improvement in operating margins. This is a signal that the company is reducing the cost structure and is pushing down operational and advertisement costs to increase marginality and improve efficiency. This is also captured by the rising trend in profitability as it is shown in the following figure.

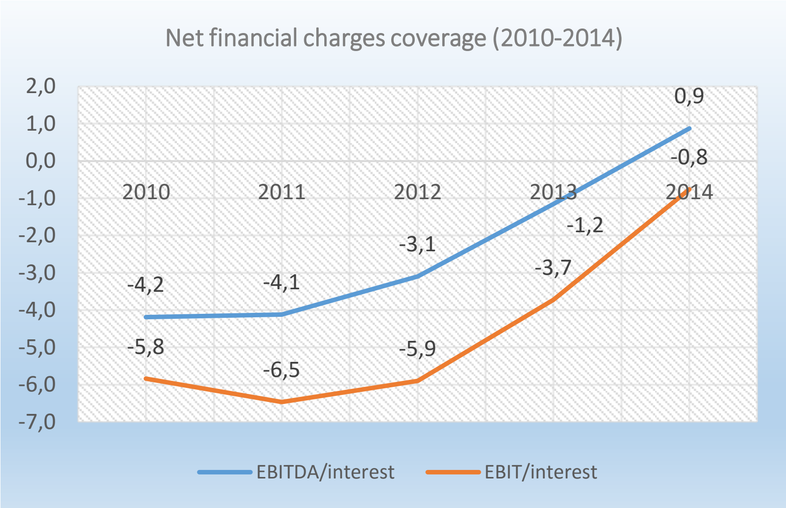
Figure 4 - 7 Trend in profitability (2010-2014)



Source: company's financial statement

After this brief look at profitability, the analysis switches to coverage, comparing operating results such as EBITDA and EBIT (representing the investment-side of the management) with interest expenses that the firm has to face because of the big amount of debt (representing the financing-side of the managerial activity).

Figure 4 - 8 Net financial charges coverage (2010-2014)

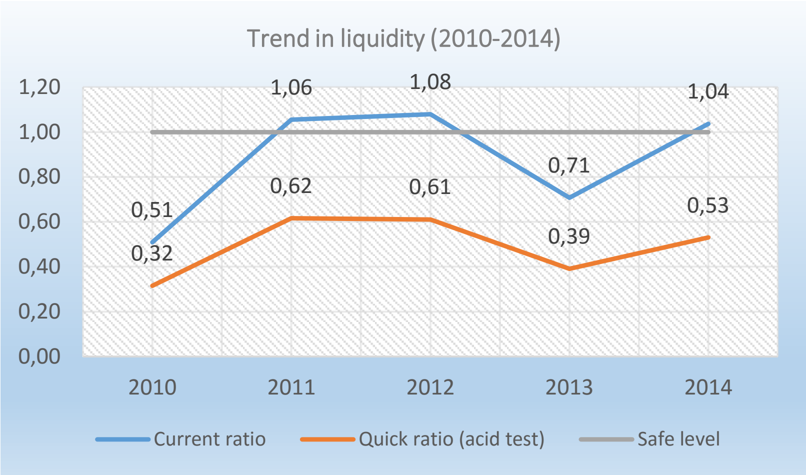


Source: company's financial statement

As it is possible to notice, negative operating results affect the coverage ratio, which is negative in all the period except for 2014 when EBITDA became positive. This means that the company is not creating operating profits that allow to afford debt's costs. In other words, what the company is creating with its operating business is far less than the costs related to the financing of that business itself. If this is expected to continue also in the future, it means that the company will continue to pay interest on debt and, at the same time, destroying resources. This is unsustainable. Despite the negative values, trends in coverage ratios reflect the positive trends in profitability with EBITDA in 2014 able to cover almost the 90% of debt's costs. This allows an analyst to be less pessimistic about future sustainability of the business.

Switching the analysis from income statement to balance sheet, it is worth giving a look at the liquidity situation of the firm. Both current ratio and quick ratio has been computed and figure 4.9 shows how those ratios have not a precise trend. It was already mentioned that it is more insightful to look at the quick ratio rather than the current ratio because the former excludes inventories from the numerator allowing to compare the current assets that are really liquid with current liabilities. Quick ratio results to be far below the safe level of one, meaning that in this 5 years window the company never had enough resources to pay back liabilities expiring within one year. This could be one of the reasons why the terms of financing with banks has been negotiated three times in 5 years. The company needed money to sustain the business continuity in the short-term.

Figure 4 - 9 Trend in liquidity (2010-2014)



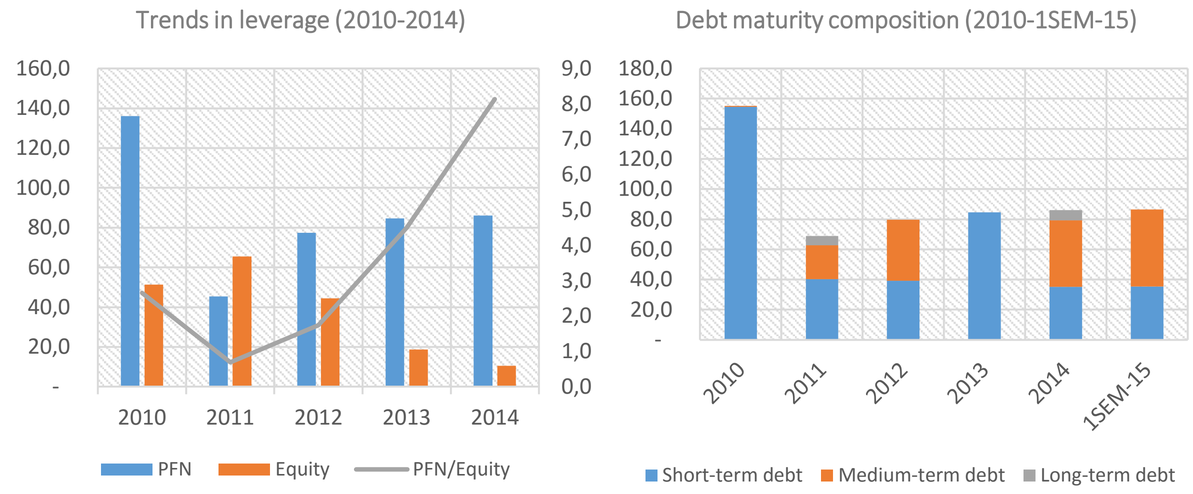
Source: company's financial statement

Looking at leverage, the situation is not any better. Although the three debt restructuring agreements, Net Financial Position is slightly rising year by year. This is accompanied by a reduction in the shareholder's equity due to its usage to absorb both retained and current losses. This is related to article 2446 of Italian Civil Code, which obliges managers to decide for an equity reduction to cover losses, when those losses represents more than one third of the total value of equity.

The contemporaneous surge in NPF and shrinkage in Equity, resulted in a quite dramatic rise in leverage ratios and this is where major problems of SG lay and the main reason of the continuing interventions in debt restructuring, as it is possible to notice from picture 4.10.

From the same figure, it is possible to see the evolution of the maturity composition of debt and the effects of debt restructurings. The problem of the firm is linked to the shortening of maturities that leads to debt's unsustainability. In 2010, all the financial debt was short-term making it impossible for the company to pay it back. The restructuring agreement of 2011 allowed for a wide cut in debt's absolute value together with a maturity reschedule characterized by a massive reduction in short-term debt with an increase in medium-term debt. Despite with lower absolute value, the same situation happened in 2013 and led to the obvious need for the restructuring agreement of 2014 (this time without any debt cut).

Figure 4 - 10 Trends in leverage and debt maturity composition (2010-2014)



Source: company's financial statement

To conclude this part, it has been already said that every restructuring agreement was characterized by covenants, regarding EBITDA, NFP and shareholders' equity. In our opinion, the covenant breach is not related to EBITDA but to the capital structure of the firm. In other words, despite revenues downturn, management has focused their own efforts in successfully reducing operating costs leading to a positive trend in operating marginality. The missing piece of the puzzle is a likewise outstanding strategy in reducing leverage. A capital increase could be important to reduce the ratio between NFP and Equity and, at the same time, it has to be coupled with rising sales and positive profits to create retained earnings that could be used to finance investments projected by the industrial plan. Without a change of gear, there is the risk to continue to absorb equity in covering losses and to finance operations with bank's debt. This will rise financing costs and leverage. There is also uncertainty about the availability of financial resources from banks, which are not willing to concede money with too long maturity (after a restructuring, maturity are shifted from short to medium-term but almost never to long-term) and this a signal of a low expected creditworthiness of the group.

Despite SG has not any bonds quoted in the market and consequently there is no an accepted rating, an analyst could still come up with a measure of its credit quality using alternatively or, better, complementarily, a synthetic credit rating and a credit score computed using Altman Model.

At first, Altman model has been applied using the following equation:

$$Z = 1.2X_1 + 1.4X_2 + 3.3X_3 + 0.6X_4 + 1.0X_5$$

Where

$X_1 = Working\ Capital\ \ / Total\ Assets$

$X_2 = Retained\ Earnings\ / Total\ Assets$

$X_3 = EBIT \ / Total\ Assets$

$X_4 = Market\ Value\ of\ Equity \ / Book\ Value\ of\ Total\ Liabilities$

$X_5 = Sales \ / Total\ Assets$

$Z = Overall\ Index\ or\ Score$

Table 4 - 1 Altman Z-Score computation (2010-2014)

EXHIBIT 7A STEFANEL:		Z-SCORE				
	Weight	2010	2011	2012	2013	2014
Working capital/Total Assets	1,2	0,11	0,11	0,22	0,22	0,24
Retained Earnings/Total Assets	1,4	0,00	0,00	0,00	0,01	(0,04)
EBIT/Total Assets	3,3	(0,15)	(0,13)	(0,10)	(0,07)	(0,02)
N° shares outstanding 31/12/n		84.527	84.527	84.527	84.527	84.527
Price 31/12/n		2,0265	0,4950	0,1490	0,3660	0,2926
Equity Mkt Value		171.293	41.841	12.594	30.937	24.732
Equity Mkt Value/Liabilities Book Value	0,6	1,2	0,8	0,2	0,3	0,3
Sales/Total Assets	1,0	0,8	0,8	0,9	1,0	0,9
SCORE		1,2	1,0	0,9	1,2	1,3

Looking at the individual ratios, the relationship between sales and total assets remained almost constant confirming that the company's troubles do not come from a profitability point of view but especially from the unbalanced capital structure. This can be noticed by looking at the ratio of EBIT on current asset that improved year by year (reflecting positive trend in operating margin). Conversely, market value of equity and the book value of liabilities worsened from 1.2 to 0.3 signaling a big reduction in the market share price coupled with rising liabilities.

In chapter one, it has been said that when Z-Score is lower than 1.81, the company is considered as very risky and this is what happens for SG. After a worsening in both 2011 and 2012, the index started to improve but it is still very low and this is a signal of poor credit quality coherently with what happened in reality.

After that, a synthetic measure of rating has been computed. To do so, three different ratios has been used:

- <u>*NFP/EBITDA*</u>: it measures the ability of the company to sustain the debt. In other words, it can be used as a proxy of how many years the company will take to pay the debt back, assuming that both numerator and denominator will remain stable in the future; it ignores the effects of all the variables that are not considered in the EBITDA computation such as interest, taxes, depreciation and amortization. High values signal a low creditworthiness and correspond to lower credit rating.
- <u>*EBITA/interest expenses*</u>: it is a coverage ratio and, as the others do, it measures the ability of the company to generate earnings (after having paid all the operating costs except for amortization) to cover interest expenses. A value higher than one could mean that, but not for sure, the company can afford to pay interest on debt. There is not

complete safety because of the presence of high amortization, which may undermine financial durability.

- *FCF/NFP*: it is an alternative way to measure the leverage of the firm. The higher its value and the better it is for the firm.

The value of each ratio has been equally weighted to reach a synthetic score and to that score has been assigned a rating according to table 4.3.

In the case of SG, for the year of 2014, NFP/EBITDA is extremely high signaling a low creditworthiness. To sustain such a high level of net debt, the company has to increase its level of EBITDA in the future. Otherwise, a debt reduction would be preferable. If the rating is computed only relying on that ratio, it would be equal to CCC.

EBITA on interest expenses is negative because of the negative value of EBITA itself, signaling again a CCC rating.

FCF over NFP is the only one of the three ratios that provides some better news, flagging a B rating. This is due to positive operating FCF generated in 2014. Despite the company has not produced positive earnings (in 2014 the company had a loss), summing up the non-monetary costs and considering also the reduction in operating working capital and the really small impact of CAPEX, a positive cash flow has been generated by the operating business.

Summing up the values of these three ratios, an analyst would come up with a weighted score of 7.2, which leads to an average rating of CCC signaling a very low credit standing of the company, which is considered as a speculative-grade investment with a high probability of default, or at least of covenant breach. This is coherent with what happened in realty and what has been measured by using Altman Z-Score.

Table 4 - 2 Synthetic rating computation (2014)

EXHIBIT 7B STEFANEL:		SYNTHETIC RATING COMPUTATION	
	Weight		2014
NFP/EBITDA			21,56
RATING 1	0,33		C-CCC
EBITA/Interest expenses			(0,76)
RATING 2	0,33		C-CCC
FCF/NFP			5,8%
RATING 3	0,33		B
Weighted Score			7,2
SYNTHETIC RATING			C-CCC

Table 4 - 3 Credit rating reference

Debt/EBITDA			EBITA/Interest expense			FCF/Debt		
from	to	Rating	from	to	Rating	from	to	Rating
0,0	0,4	AAA	12,5	100,0	AAA	154,6%	1000,0%	AAA
0,4	1,0	AA	9,5	12,5	AA	42,5%	154,6%	AA
1,0	1,5	A	4,5	9,5	A	30,9%	42,5%	A
1,5	2,3	BBB	4,0	4,5	BBB	14,1%	30,9%	BBB
2,3	3,0	BB	3,0	4,0	BB	7,8%	14,1%	BB
3,0	5,4	B	2,0	3,0	B	2,1%	7,8%	B
5,4	100,0	C-CCC	0,0	2,0	C-CCC	0,0%	2,1%	C-CCC

Synthetic Score		Rating	Synthetic score (avg)
from	to		
0,0	0,5	AAA	0,25
0,5	1,0	AA	0,75
1,0	1,5	A	1,25
1,5	2,5	BBB	2,00
2,5	3,5	BB	3,00
3,5	4,5	B	4,00
4,5	5,0	C-CCC	4,75

Source: Buttignon F., 2015. Case study.

4.4: Model Application

4.4.1 Enterprise Value (EV) computation

The first variable we focus on is the EV that is computed, as explained in chapter 3, using a mixed APV/DCF method that allows the analyst to account for a volatile capital structure, especially in the period of explicit forecasts. For sake of clarity, the procedure divided into 5 steps from a) to e) previously described will be followed.

a) Define the length of the explicit forecast period

Despite 2015 is an actual year, it is considered as the first projected year since the final report has not been approved yet at the date this thesis has been written. To maintain consistency with actual data, information from intermediate financial report of 2015 is used to build expectations about the final projected 2015. The length of the first stage of explicit forecasts is assumed to be 5 years, from 2015 to 2019 because 2019 is projected to be the last year of the new industrial plan (2016-2019). It allows us to have coherence between the restructuring process and our forecast period.

b) Define the assumptions about the key variables in all the three scenarios

This represents one of the trickiest part since the few assumptions of the industrial plan provided in the financial statement of the firm have to be translated into real numbers.

First of all specific probability weights have to be assigned to each scenarios according to the subjective view of the analyst. Base scenario is the one with the highest probability of 45% and then, instead of assigning an equal percentage to worst and best case, we opted for a skewed probability distribution with a 30% and 25% likelihood for the worst and for the best case, respectively. This allows us to account for the possibility of too optimistic assumptions of the management in their projections that can lead to an upward bias in the estimated EV.

Thereupon, assumptions about the base case are carefully deduced from the industrial plan while worst and best scenarios are defined as divergences from the base case.

b1) Define the base case

Revenues are assumed to grow at a low but positive rate that peaks in 2017 and then remains almost stable because of the strategy of Stefanel's brand repositioning and relaunching. The expected positive trend in revenues, coupled with a slight reduction in operating costs lead to an improvement in operating margin (EBITDA margin rises to almost 6% at the end of the projections). This is consistent with the industrial plan.

Regarding depreciation and amortization (D&A from now on), we opted for the capital maintenance assumption[23]: coherently with the distressed state of the group that cannot afford to invest too many resources, D&A is set equal to the annual CAPEX in order to maintain fixed capital constant to a suitable level to sustain revenue growth.

Net working capital (NWC) is assumed to slightly rise in absolute value to meet the increasing level of sales. Despite this, its percentage on sales marginally shrinks indicating improving relationship of the firm with both customers and suppliers.

Table 4 - 4 FCF computation (Base case - Plan)

number in /€000	2013	2014	2015	2016	2017	2018	2019
	Actual	Actual	Expected	Expected	Expected	Expected	Expected
EXHIBIT 12 STEFANEL: Reduced FCF Computation (BASE CASE-PLAN)							
Revenues	**168.517,0**	**155.609,0**	**157.165,1**	**159.522,6**	**163.032,1**	**166.292,7**	**169.286,0**
Revenues growth		-7,66%	1,00%	1,50%	2,20%	2,00%	1,80%
EBITDA		**3.995,0**	**6.023,3**	**7.446,0**	**8.658,3**	**9.896,9**	**10.075,1**
EBITDA margin		2,57%	3,83%	4,67%	5,31%	5,95%	5,95%
Total D&A	**(8.372,0)**	**(7.457,0)**	**(6.027,1)**	**(6.027,1)**	**(6.027,1)**	**(6.027,1)**	**(6.027,1)**
D&A on operating fixed capital		14,85%	12,00%	12,00%	12,00%	12,00%	12,00%
EBITA		**(3.462,0)**	**(3,8)**	**1.418,9**	**2.631,2**	**3.869,8**	**4.047,9**
Tax rate on EBITA		15,8%	25137,4%	-44,2%	-20,4%	-3,7%	-39,8%
Operating taxes		547,0	966,8	626,7	537,0	144,2	1.612,9
NOPLAT		**(2.914,8)**	**1.214,4**	**2.045,1**	**3.168,0**	**4.014,0**	**5.660,5**
NWC		38.812,0	40.468,3	41.057,6	41.119,9	41.918,7	41.805,6
NWC on sales		24,94%	25,75%	25,74%	25,22%	25,21%	24,70%
Delta NWC			1.656,3	589,4	62,2	798,9	(113,1)
Operating Fixed Capital		50.226,0	50.226,0	50.226,0	50.226,0	50.226,0	50.226,0
Operating Fixed Capital on sales		32,28%	31,96%	31,49%	30,81%	30,20%	29,67%
Delta Operating Fixed Capital (CAPEX)			0,0	0,0	0,0	0,0	0,0
Operating Invested Capital		89.038,0	90.694,3	91.283,6	91.345,9	92.144,7	92.031,6
Delta Operating Capital			(1.656,3)	(589,4)	(62,2)	(798,9)	113,1
Operating FCF			(441,9)	1.455,8	3.105,8	3.215,1	5.773,5

b2) Define worst and best case

In the best case, revenues are expected to increase at a higher rate than the base one signaling better performances and a full success in the reorganization strategy, with a revenue growth rate of 5% and EBITDA margin of almost 9%. Conversely, the worst scenario should be the picture

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[23] Practitioners usually distinguish between two possible assumptions about CAPEX: fixed capital maintenance in which CAPEX's function is to offset amortization and depreciation, or growing fixed capital where CAPEX is set to high and rising levels to expand the production capacity.

of a failing plan, with a zero rate of growth in revenues and constant operating margins. From the analysis of past performances, it emerged an improvement in the operating efficiency despite negative trend in revenues and this is why we have decided to confirm this fact also in worst scenario.

As in the base case, D&A are assumed to remain constant during the entire projected period. On the contrary, CAPEX is expected to follow different trends in all the scenarios. In the best one, operating fixed capital increases year by year to meet rising sales that require new investment to enlarge production capacity. The incidence of CAPEX on total sales remains constant. Rising CAPEX is also coupled with soaring net working capital.

On the other hand, on the pessimistic side, the ratio between CAPEX and sales is predicted to increase, signaling a lower rate of return of new investments. In other words, managers are supposed to increase a bit the amount of invested resources, but those investments do not have the planned positive impact on sales. Net working capital, instead, follows the mild and stable constant trend of revenues.

All these assumptions are reflected by the following cash flow projections.

Table 4 - 5 FCF computation (Best case)

number in /€000	2013 Actual	2014 Actual	2015 Expected	2016 Expected	2017 Expected	2018 Expected	2019 Expected
EXHIBIT 12 STEFANEL: Reduced FCF Computation (BEST CASE)							
Revenues	**168.517,0**	**155.609,0**	**157.943,1**	**161.891,7**	**168.043,6**	**176.445,8**	**185.268,1**
Revenues growth		-7,66%	1,50%	2,50%	3,80%	5,00%	5,00%
EBITDA		**3.995,0**	**8.686,9**	**11.332,4**	**13.611,5**	**15.880,1**	**16.674,1**
EBITDA margin		2,57%	5,50%	7,00%	8,10%	9,00%	9,00%
Total D&A		(7.457,0)	(6.027,1)	(6.027,1)	(6.027,1)	(6.027,1)	(6.027,1)
D&A on operating fixed capital		14,85%	12,00%	11,71%	11,39%	10,88%	10,39%
EBITA		**(3.462,0)**	**2.659,8**	**5.305,3**	**7.584,4**	**9.853,0**	**10.647,0**
Tax rate on EBITA		15,8%	20,0%	20,0%	20,0%	20,0%	20,0%
Operating taxes		547,0	(532,0)	(1.061,1)	(1.516,9)	(1.970,6)	(2.129,4)
NOPLAT		**(2.915,0)**	**2.127,8**	**4.244,2**	**6.067,5**	**7.882,4**	**8.517,6**
NWC		38.812,0	40.275,5	41.120,5	42.515,0	44.464,3	46.502,3
NWC on sales		24,94%	25,50%	25,40%	25,30%	25,20%	25,10%
Delta NWC			**1.463,5**	**845,0**	**1.394,5**	**1.949,3**	**2.037,9**
Operating Fixed Capital		50.226,0	50.225,9	51.481,6	52.933,7	55.404,0	57.988,9
Operating Fixed Capital on sales		32,28%	31,80%	31,80%	31,50%	31,40%	31,30%
Delta Operating Fixed Capital (CAPEX)			**(0,1)**	**1.255,6**	**1.452,2**	**2.470,2**	**2.584,9**
Operating Invested Capital		89.038,0	90.501,4	92.602,1	95.448,8	99.868,3	104.491,2
Delta Operating Capital			(1.463,4)	(2.100,6)	(2.846,7)	(4.419,5)	(4.622,9)
Operating FCF			**664,4**	**2.143,6**	**3.220,8**	**3.462,9**	**3.894,7**

Table 4 - 6 FCF computation (Worst case)

number in /€000	2013 Actual	2014 Actual	2015 Expected	2016 Expected	2017 Expected	2018 Expected	2019 Expected
EXHIBIT 12 STEFANEL: Reduced FCF Computation (WORST CASE)							
Revenues	**168.517,0**	**155.609,0**	**156.387,0**	**157.169,0**	**157.954,8**	**158.270,7**	**158.429,0**
Revenues growth		-7,66%	0,50%	0,50%	0,50%	0,20%	0,10%
EBITDA		(1.719,0)	4.691,6	5.658,1	6.950,0	7.913,5	7.921,5
EBITDA margin		-1,10%	3,00%	3,60%	4,40%	5,00%	5,00%
Total D&A		**(7.457,0)**	**(6.027,1)**	**(6.027,1)**	**(6.027,1)**	**(6.027,1)**	**(6.027,1)**
D&A on operating fixed capital		-14,8%	-12,0%	-11,9%	-11,6%	-11,6%	-11,5%
EBITA		**(9.176,0)**	**(1.335,5)**	**(369,0)**	**922,9**	**1.886,4**	**1.894,3**
Tax rate on EBITA		15,8%	15,8%	15,8%	15,0%	14,0%	13,0%
Operating taxes		(1.449,8)	211,0	58,3	(138,4)	(264,1)	(246,3)
NOPLAT		**(10.625,8)**	**(1.124,5)**	**(310,7)**	**784,5**	**1.622,3**	**1.648,1**
NWC		38.812,0	39.096,8	39.292,2	39.172,8	39.251,1	39.290,4
NWC on sales		24,94%	25,00%	25,00%	24,80%	24,80%	24,80%
Delta NWC			**284,8**	**195,5**	**(119,4)**	**78,3**	**39,3**
Operating Fixed Capital		50.226,0	50.043,9	50.765,6	51.809,2	52.071,1	52.598,4
Operating Fixed Capital on sales		32,28%	32,00%	32,30%	32,80%	32,90%	33,20%
Delta Operating Fixed Capital (CAPEX)			**(182,1)**	**721,7**	**1.043,6**	**261,9**	**527,4**
Operating Invested Capital		89.038,0	89.140,6	90.057,8	90.982,0	91.322,2	91.888,8
Delta Operating Capital			(102,6)	(917,2)	(924,2)	(340,2)	(566,6)
Operating FCF			(1.227,1)	(1.227,9)	(139,7)	1.282,1	1.081,5

The main difference between best and worst case does not lay on different investment policy followed by managers, but on the results of the investment plan. As it has been already mentioned, in the worst scenario, managers are expected to invest in fixed capital despite the negative effects of the plan in revenues and marginality, precisely in the aim to overturn those trends. Contrarily, in the best case, reorganization plan reaches positive outcomes to such an extent that additional investment are required.

As a consequence of our assumptions, FCF in the worst scenario are negative for the first three years and start to recover in 2018. In the best case, FCF are positive but not much different than the base framework reflecting the phenomenon of liquidity absorption due to the new investment required to face soaring sales. All these projected cash flows are discounted by the unlevered cost of capital, whose computation will be explained in the paragraph about costs of capital.

The higher EV in best case depends mainly on the assumptions about the key value drivers in the computation of the terminal value, which represents the next step.

c) - d) Computation of the terminal value (TV)

As said before, TV is computed applying the *key value driver formula* that takes into account NOPAT, long-term rate of growth, return of new invested capital (RONIC that is calculated starting from ROIC) and the weighted average cost of capital (WACC).

The assumptions that make a scenario different from the others regards only two variables:

- long-term rate of growth *g*, which also affect both NOPAT and capital expenditure. Indeed, both the two variables, for the TV formula, are computed multiplying their projected value of 2019 by *(1+g)*. The rate of growth is assumed equal to 1% in the base case, 1.6% in the best case and only 0.1% in the worst scenario. The reason of this low values in all the three scenario reflects the needs to be prudent because TV usually impacts on about the 70% of the total projected value of a company. Even if the firm will recover from distress, it would be unreasonable to assume a rate of growth in revenues higher than the expected rate of growth of the countries where it operates.

- RONIC, which measures the return on additional invested capital and differs from ROIC if new investment of the last year have produced a higher/lower return with respect to the historical average rate of return. It depends on *g*, WACC, and on invested capital of the year and is assumed equal to 6%, 8,3% and 1,8% in the base, best and worst case, respectively. For cautionary reasons, it should not be assumed higher than WACC and we allow for this optimist assumption only in the best case.

Finally, each TV is discounted back to 2014 using WACC, which is assumed the same for all the three scenarios. Calculation of the costs of capital will be covered in paragraph 4.4.2.

Table 4 - 7 Terminal value computation in three scenarios

EXHIBIT 13 STEFANEL: Continuing value						
	BASE CASE		BEST CASE		WORST CASE	
	2019E	Base for CV	2019E	Base for CV	2019E	Base for CV
g		1,0%		1,60%		0,1%
WACC (continuing value)		7,2%		7,2%		7,2%
NOPLAT	4.763,9	4.811,5	8.517,6	8.653,9	1.648,1	1.649,7
Invested capital	96.016,6	96.976,8	104.491,2	106.163,0	91.888,8	91.980,7
ROIC						
RONIC		5,0%		8,3%		1,8%
Continuing value (KVD formula)	62.118,3		124.679,1		21.941,2	

e) Computation of expected tax benefit deriving from debt

Tax benefit are computed starting from the projected interest rate the company is expected to pay on debt, considering an assumed tax shield rate of 25%. Those flows are discounted using the unlevered cost of capital. Interest rate in the best case are 5% lower than the base scenario due to the reduced perceived riskiness of the firm. For the opposite reason, in the worst case are assumed to be 5% higher.

It is important to mention that not all the companies benefit from the tax shield on debt but only those that have a sufficient earning capacity. Said another way, if a company does not produce enough operating profit to cover interest expenses, there will be no benefits deriving from tax shield on debt. In order to account for this in the valuation process, an analyst should look at operating profit (EBITA in our case) comparing it with interest expenses. On the one hand, if EBITA is negative there will be no tax shield on debt. On the other hand, if EBITA is positive but lower than interest expenses, tax shelter will be limited to the part of EBITA used to pay for financial costs. Finally, only in case of EBITA higher than interest expense the tax shield is full.

Table 4 - 8 Computation of tax-shield value in the three scenarios

EXHIBIT 14B STEFANEL: **Debt tax shield value	BASE CASE				
number in /€000	2015	2016	2017	2018	2019
Interest expense	(4.133,5)	(4.148,4)	(4.102,7)	(3.996,5)	(5.897,2)
Tax shield rate	25%	25%	25%	25%	25%
EBITA	(3,8)	1.418,9	2.631,2	3.869,8	4.047,9
Tax shield flow	0,0	354,7	657,8	967,4	1.012,0
Cost of capital (Ku)	8,23%	8,23%	8,23%	8,23%	8,23%
Discount factor	0,92	0,85	0,79	0,73	0,67
Debt tax shield value (per year)	**0,0**	**258,5**	**322,8**	**272,9**	**140,1**
Total	**994,4**				

EXHIBIT 14B STEFANEL: **Debt tax shield value	BEST CASE				
number in /€000	2015	2016	2017	2018	2019
Interest expense	(3.926,8)	(3.941,0)	(3.897,6)	(3.796,7)	(5.602,4)
Tax shield rate	25%	25%	25%	25%	25%
EBITA	2.659,8	5.305,3	7.584,4	9.853,0	10.647,0
Tax shield flow	664,9	985,3	974,4	949,2	1.400,6
Cost of capital (Ku)	8,23%	8,23%	8,23%	8,23%	8,23%
Discount factor	0,92	0,85	0,79	0,73	0,67
Debt tax shield value (per year)	**614,38**	**718,05**	**478,20**	**267,78**	**193,92**
Total	**2.272,3**				

EXHIBIT 14B STEFANEL: **Debt tax shield value	WORST CASE				
number in /€000	2015	2016	2017	2018	2019
Interest expense	(4.340,2)	(4.355,9)	(4.307,9)	(4.196,3)	(6.192,1)
Tax shield rate	25%	25%	25%	25%	25%
EBITA	(1.335,5)	(369,0)	922,9	1.886,4	1.894,3
Tax shield flow	0,0	0,0	230,7	471,6	473,6
Cost of capital (Ku)	8,23%	8,23%	8,23%	8,23%	8,23%
Discount factor	0,92	0,85	0,79	0,73	0,67
Debt tax shield value (per year)	**0,0**	**0,0**	**113,2**	**133,1**	**65,6**
Total	**311,9**				

Finally, the value of FCF in the explicit forecast period (discounted by unlevered cost of capital) are summed with the discounted terminal value (computed using WACC) and with the total value of debt tax shield (discounted by unlevered cost of capital) as it is shown in the appendix 4.6. Doing so, the analyst will get three different amounts for the total enterprise value (one for each scenario) that should be weighted by the probability defined at the beginning of the analysis.

At the end of this procedure, the average enterprise value should be compared with the total net financial position. In case the latter is greater than the former, equity value is predicted to be equal to zero. This comparison is shown in the following table.

Table 4 - 9 Average enterprise value and comparison with net financial position

EXHIBIT 15 STEFANEL: Value of equity				
number in /€000	2014			
	BASE CASE	BEST CASE	WORST CASE	AVERAGE
Enterprise Value	60.877,9	108.200,0	22.909,2	
Weight	45%	25%	30%	
Weighted EV	27.395,0	27.050,0	6.872,8	**61.317,8**
Net financial position and debt equivalents	89.197,0	89.197,0	89.197,0	**89.197,0**
Minority interest	224,0	224,0	224,0	**224,0**
Equity Value	**0,0**	**18.779,0**	**0,0**	**0,0**
Number of share outstanding (million)				84.529,0
Price per share (2014.12.31)				0,293
Equity market value				24.733,2

As it is possible to notice, the average EV is lower than the book value of liabilities, implying a zero predicted equity value. Only in the best scenario, equity value is expected to be positive. This is a clear signal about the distress status of the firm. Financiers have to hope for the complete recovery as expected in the best scenario. Otherwise, looking at the market equity value it is possible to notice how this is positive (about 24 million euros) and quite high with respect what has been found. This striking divergence between the market and our model, which represents one of the main issue of this thesis, will be discussed and explained later in paragraph 4.5.

4.4.2 Cost of Capital $K_U$and $WACC$ computation

As it has been said in chapter 3, the valuation framework divides company's life in two periods characterized by two different assumptions. In the explicit forecast period, characterized by variability in the capital structure, the unlevered cost of capital should be used imaging the company as financed only by equity in the aim of avoiding bias in the valuation due to the volatile capital structure. Conversely, in the infinite time horizon, terminal value is computed assuming a constant capital composition. This constant capital structure is reflected in the WACC.

a) Unlevered cost of capital computation

$k_U = r_f + CRP + \beta_U * MRP$ that becomes $k_U = 0{,}91\% + 2{,}40\% + 0{,}82 * 6{,}00\%$ and results in a cost of capital of around 8,23%.

As a proxy for r_f, the 10-years IRS quote (EURIRS) has been chosen at the date of 31st December 2014, while both CRP and β_U has been taken by Damodaran website. In particular, unlevered beta is the one for the apparel industry in Europe while the country risk premium is the one of Italy, since it is the country where Stefanel Group is established and where a big portion of sales is concentrated. The market risk premium has been fixed to 6% as suggested by Fernandez, Aguirreamalloa, Avendano (2012).

b) WACC computation

$WACC = \frac{E}{(D+E)} * k_E + \frac{D}{(D+E)} * k_D * (1 - tax\ rate)$ that becomes $WACC = (25\% * 14{,}75\%) + [75\% * 6{,}41\% * (1 - 27{,}50\%)]$ that gives a weighted average cost of capital of around 7,17%, where k_E is computed applying CAPM, while $k_D$is obtained by adding a credit spread to the risk-free interest rate.

The most relevant assumption about that computation is represented by the capital structure because in the moment of the valuation, the company is troubled and debt represents most part of the financing resources. In particular, at the date of 31st December 2014, the net financial position has a nominal value of around 86 million euro while the market equity value is 24,7 million euro. In April 2015, equity is equal to only 12,9 million euro. This means the market has discounted equity value in light of the dim economic and financial performances.

For valuation purposes, we have assumed that, at the end of plan, in 2019, the group has a slightly more balanced financial structure with a reduction in debt level and a consequent better rating. To be more precise, it has been assumed that debt represents the 75% of total source of financing with a B rating, while equity corresponds to the remaining 25%. Despite conservative, this scenario implies positive results of the plan in the capital rebalancing and, as it has been said while performing financial analysis, this is the most desirable aspect for the company, to recover from its economic and financial difficulties. Regarding the cost of equity, it is obtained applying the CAPM with the

- above-mentioned risk-free rate of 0.91%;
- above mentioned country risk premium of 2,4%;
- equity risk premium equal to 5,5% that characterizes a mature equity market[24];
- beta of the equity, computed as $\beta_E = \beta_U + \frac{D}{E} * (\beta_U - \beta_D)$ where $\frac{D}{E}$ is assumed equal to 300% (coherently with the fact that D represents the 75% of the total financial resources), β_U is equal to 0.82 as before and β_D is assumed equal to 0.40 according to Table 4.9 for debt with a B rating. This calculation results in a beta of equity equal to 2.08.

Conversely, the cost of debt is obtained adding the credit spread (characterizing a company whose debt has a B rating) to the risk-free rate. This procedure results in a k_D equal to 6.41%.
It is possible to notice that both the costs of capital are relevant and this reflects the highly perceived riskiness of the firm. This happens despite the current macroeconomic environment that is characterized by heavy and non-conventional expansionary measures followed by ECB (this is reflected by a long-term risk-free rate below 1%). The fact that the final WACC is not incredibly high is only due to the greater impact of k_D with respect to k_E.

Table 4 - 10 Spread and beta for different ratings

Spread and beta for different ratings

Rating	Spread	Beta Debt
AAA	0,65%	0,05
AA	1,15%	0,07
A	1,40%	0,10
BBB	2,50%	0,14
BB	4,25%	0,28
B	5,50%	0,40
C-CCC	8,75%	0,80

Source: Buttignon F., 2015. Case study

4.4.3 Volatility computation

As previously mentioned, the procedure for the estimation of the volatility of the EV is the Newton Raphson method implemented by using VBA and an Excel spreadsheet. The procedure is the same used by traders while estimating the implied volatility of an option based on its price. The most important inputs of the model are the annualized equity volatility and EV (that is the result of the APV/DCF valuation model).

[24]Data about equity premium, or Market Risk Premium (MRP) are provided each year by the study of Fernandez, Linares and Acin.

In order to compute σ_E, we downloaded from the website Yahoo Finance the daily stock quote of Stefanel S.p.A. in the Milan Stock Exchange (ISIN: IT0004607518) from 02 August 2010 to the 31st December 2014. This time span guarantees almost 4 years and a half of observations and it is about to represent the period in which the group has been in financial troubles. Indeed, in April 2010, the first restructuring agreement was signed signaling the outbreak of the crisis. Furthermore, in the same period of 2010, also a capital injection was approved and implemented. Once we got the stock quotes, the computation of daily compounded returns is straightforward using the formula $Return_t = \ln(\frac{Stock\ quote_t}{Stock\ quote_{t-1}})$. Daily variance of returns is easily found using the function DEV.ST that computes standard deviation obtaining a value of around 3.34%. Finally, this daily variance is converted into a yearly one, using the formula $yearly\ variance = daily\ variance * \sqrt{252}$[25] that is equal to almost 54.6%. During this observed time-period, equity markets of all European countries (and especially of countries like Italy) has been characterized by high fluctuations of stock quotes due for example to the Sovereign Debt Crisis. Those macroeconomic factors coupled with company specific problems such as decline in earnings and profitability and the increase in leverage have brought about the high equity volatility. Given this high level of σ_E we expect a great level also for σ_{EV}. Enterprise Value used in this model is equal to 61,2 million euro that is the result of our mixed valuation framework. The additional inputs are represented by risk-free rate, which is assumed equal to 0.91% (that is the 10-years European Interest Rate Swap Quote at 31/12/2014), dividend yield equal to zero because of the distressed situation of SG that cannot afford to distribute any resources, and time to maturity (equal to 4.5 years, which represents the length of the considered time horizon from August 2010 to December 2014). As a strike price, we decide not to use the entire nominal NFP but the default point DP represented by short-term liabilities plus half of long-term liabilities for the same reason as KMV model.

After having inserted all these inputs in the Excel spreadsheet, we have just to use the functions previously created in VBA to compute option price (B&S equity value) and implied volatility and we find a volatility of EV equal to around 47,72%.

A potential problem of Newton method is its sensibility to the starting point, which in our case is represented by asset volatility computed as $\sigma_{EV} = \sigma_E * \left({}^{A}/_{E}\right) = 21.8\%$. To check the robustness of the model, we tried to see how implied EV volatility moves according to changing the starting point. We fond that finale σ_{EV} remains constant when modifying the initial value in the interval $\pm 5\%$.

[25] 262 is commonly used in finance as the number of trading days in a year.

Once robustness has been checked, volatility should be confronted with comparable companies. It is possible to notice that this estimate is in line with the average standard deviation of firm assets for apparel industry, as measured by Damodaran, which is equal to 47,41%. This average is computed using a big sample of companies in the apparel sector, of all over the World and of different dimension. For example from Italy, the sample includes companies such as Moncler, OVS, Brunello Cucinelli, Aeffe, Piquadro.[26]

Despite we were interested only in the call option (that represents equity), we computed the price of both call and put. It is possible to notice that estimated implied volatility makes the B&S equity value exactly equal to the market capitalization and, in addition, the "put-call parity" holds. We insert the figure of the sheet where B&S price and implied volatility have been obtained.

Figure 4 - 11 Implied EV volatility computation sheet

STEFANEL		
Parameters	**Values**	
Option Value (Equity Market Value)	24.733	
Asset Price (EV)	61.317	
Strike Price (Short-Term NFP +1/2Long-Term NFP)	60.617	
Risk-Free Rate	0,91%	
Time to Maturity (lenght of restructuring plan)	4,5	
Dividend Yield	0,00%	
Guess Value for Volatility (initial volatility)	21,81%	
Equity volatility (historical)	54%	
Results	**Call**	**Put**
Implied Volatility (EV implied volatility)	**47,72%**	**54,92%**
Back-solved Option Value (B&S equity value)	24.733	24.733

To conclude, asset volatility is the most controversial and tricky variable to estimate and we are aware of the fact that the solution of our model could seem too naïve, but it is not. Alternatively, asset volatility could be computed reducing the FCF model in few main variables (revenues, operating costs, depreciation and amortization, operating taxes, variation in net working capital and capital expenditure). For each variable, a specific stochastic process (with specific parameters and correlations) have to be estimated and a Monte Carlo simulation have to be run. Finally, the analyst will obtain a sequence of EVs for which it is possible to compute its probability distribution and its variance. However, this procedure is costly, risky and time consuming because it requires a big number of assumptions about the behavior of several variables and about their interconnections. In addition, each stochastic process has its own parameters that should be inferred, making the estimation even more subject to bias. On the

[26] http://pages.stern.nyu.edu/~adamodar/v.

contrary, relying on data provided by the market, contributes to lower bias in the estimation avoiding lose too much time in making assumptions over assumptions.

4.4.4 Practical Debt Valuation and Probability of Default (PD) estimation

As described in chapter 3, once both EV and its volatility has been estimated, next step of our formulation consists of obtaining the debt value and its related probability of default.

Going in order, the assumptions behind the model are:

- EV follows a Geometric Brownian Motion with a path $EV_T = EV_t * e^{[\left(r-\delta-\frac{\sigma^2}{2}\right)*(T-t)+\sigma*(T-t)*Z_t}$;
- initial EV is the one that results from the APV/DCF model, equal about to 61,3 million euro
- volatility is approximately equal to 48% as estimated in the previous paragraph;
- risk-free interest rate is equal to 0.91% as usual;
- pay-out rate, as already pointed out in chapter three, is assumed equal to zero because of the distressed conditions of the group, which is expected to reinvest all the resources produced during the year to finance their operations;
- time to maturity *(T-t)* is assumed equal to 5 years since both the new restructuring and industrial plans are expected to end in 2019;
- The NFP of the company (at the date of interest 31st December 2014) is showed in table 4.10.

Table 4 - 11 NFP Composition (31/12/2014)

NFP Composition (31/12/2014)

Non current and current payables to banks	31/12/2014		
Item	**Maturity < 1Y**	**1Y<Maturity<5Y**	**Maturity>5Y**
Bank overdraft	8.986,0	0,0	0,0
Fair value of exchange rate derivative	0,0	0,0	0,0
Bank financing(*)	25.916,0	44.046,0	6.950
Accrued expenses	215,0	0,0	0,0
Financial leasing	2,0	0,0	0,0
Total	**35.119,0**	**44.046,0**	**6.950,0**

* Medium/Long Term Bank financing description *(data from the company's balance sheet) 31/12/2014*			
Bank financing		**1Y<Maturity<5Y**	**Maturity>5Y**
Pool Tranche A		20.830,0	3.372,0
Pool Tranche B		5.383,0	851,0
Pool Tranche C		4.037,0	638,0
Banca Pop. Verona		469,0	0,0
Mediocredito FVG		3.209,0	478,0
Unicredit		2.360,0	376,0
Intesa - Cassa Risparmio del Veneto		2.360,0	376,0
Intesa - Cassa Risparmio di Venezia		675,0	107,0
Monte dei Paschi - Antonveneta		2.429,0	387,0
Monte dei Paschi di Siena		675,0	107,0
EFI Banca		607,0	97,0
BNL BNP		1.012,0	161,0
Total		**44.046,00**	**6.950**

While computing implied volatility (considering a time horizon of 4.5 years to remain consistent with the time horizon used for equity price observations), we get a call of about 24.7 million euro, which represents the value transferred from bondholders to shareholders due to the restructuring plan. PD associated to the debt is equal to 67%.

If we use a time horizon of 1.5 year, we could see the predictive power of the model confronting ex-post the B&S predicted equity and the actual observed value in the market. In other words, market value of equity expected within 18 months from the end of 2014 computed by B&S formula, could be compared to observed market capitalization in the middle of 2016. In doing so, we get an call option value (equity value) of 14.7 million euro, which is not far from the real value of 12.9 million euro observed in the market at the date of 1st April 2016. In addition, it is worth highlighting the heavy slump in equity market capitalization that, in one year and a half, dropped from 24.7 million euro to 12.9 million euro. This is a clear signal of the increasing perceived riskiness of the investors in financial markets.

If we increase the time horizon from 1 to 5 years (which is the expected length of the restructuring plan, from 2015 to 2019), equity value rises to around 25.9 million euro with a probability of default of 68.4%. This growth of about 13 million euro is represented by the additional breathing space that bondholders granted to shareholders. Put it simply, despite the accounting value of equity equal to zero (from APV/DCF model), bondholders have given 5 additional years to turnaround the company and shareholders will benefit entirely from a possible positive scenario (due the unlimited upside of their payoff). This is the reason of the rise in equity value.

A rough estimate of market value of default point is given by its nominal value minus the put option. Afterwards, this market value has to be redistributed in order to get the final and total debt market value. This can be easily performed using proportions. The procedure is resumed in the following five steps.

1) $Total\ nominal\ value\ of\ NFP\ 86{,}2\ million =$

$$\begin{cases} Short-term = 35{,}2\ million \\ Medium\ and\ Long-term = 51\ million \end{cases}$$

2) $$DP\ 60{,}6\ million = \begin{cases} Short-term = 35{,}2\ million \\ \left(\frac{1}{2}\right) Medium\ and\ Long\ term = 25{,}4\ million \end{cases}$$

3) $Mkt\ value\ of\ DP = Nominal\ Value\ of\ DP - Put\ Option\ Value\ (with\ a\ 5-years\ horizon) = 60{,}6 - 22.6 = 38\ million$

4) $$Mkt\ value\ of\ DP\ 38\ million = \begin{cases} Short-term = 22\ million \\ \left(\frac{1}{2}\right) Medium\ and\ Long-term = 16\ million \end{cases}$$

$$5) Total\ mkt\ value\ of NFP = \begin{cases} Short-term = 22\ million \\ Medium\ and\ Long-term = 32\ million \end{cases} = 54\ million$$

It means that the expected reduction in debt value is equal to $86{,}1\ million - 54\ million = 32{,}1\ million$.

If on the one hand creditors allow for additional five years of time, on the other hand they will ask for a guarantee, represented by the level of the barrier.

Assuming that half debt is senior we can set $H = 42{,}5\ million\ euro$ and we apply the down-and-out call option pricing formula, still with a time horizon of 5 years, obtaining a call value of around 15 million, which is 14 million lower with respect the case of no guarantee.

Here we explain why we opted for a barrier that is lower than DP. If we decided to set $H = DP = 60{,}6\ million$, the value of the corresponding down-and-out call is predicted to be zero, which makes no sense. This happens because the starting value for the underlying asset (EV) of 61,2 million euro is almost the same as the strike/barrier. In case of a small decrease in EV, barrier will be hit leading to a zero equity value. Infeasibility is confirmed by probability of default that is higher than 100%. This is the main reason we opted for a different (and lower) barrier. This makes the formula mathematically feasible and the reasoning behind the math adherent to reality.

As we already said in the previous chapter, the barrier can be thought in two different ways. On the one hand, it could be a net-worth covenant as the one assumed by Leland (1994). This means a general protection to all the debtholders in case debt shrinks below a certain threshold defined contractually in the covenant.

More interestingly, on the other hand, the barrier could be assumed equal to the value of senior bond. This lets us differentiating between senior and junior bonds. Senior bonds are protected in the sense that EV cannot fall below their value (principal). This implies that, in case of default, they will always be satisfied by selling company's asset out. In this way, in case of default prior to maturity, senior bondholders will receive the repayment of the principal but not of the interests. This happens because EV cannot go lower than senior principal amount. On the contrary, junior bondholders face the risk of losing everything (principal and interests).

Apart from the consideration about the possible meanings of the barrier, what it is important, it is the different value in the call option and in the probability of default, as it is possible to notice in table 4.12.

Table 4 - 12 Different equity interpretation

Table 4.12: Different Equity interpretation

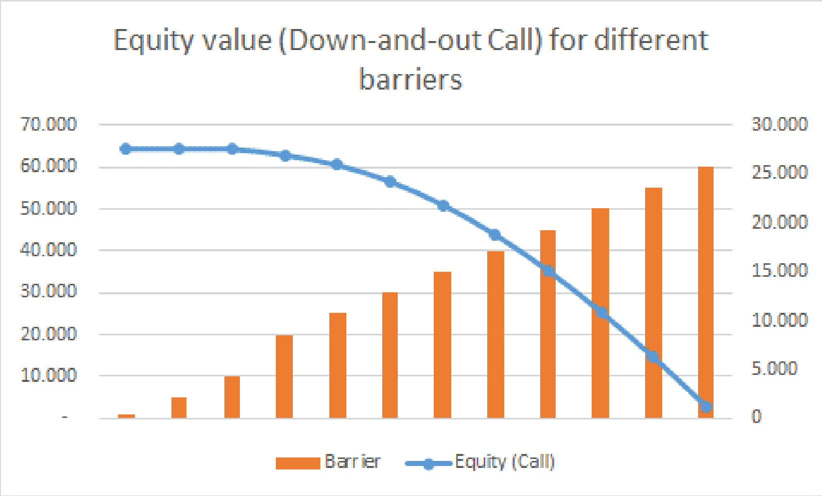
Option type	Plain Vanilla Call	Down-and-out Call
Characteristics	One class of debt	Two classes of debt
	No debt protection	Senior debt protected by covenant
Variables		
Underlying asset - EV	61.317	61.317
Strike price (K) - Default Point (DP)	60.617	60.617
Barrier (senior debt value)	-	42.500
EV volatility	47,72%	47,72%
Time to maturity (restructuring plan lenght in years)	5	5
Risk-free rate	0,91%	0,91%
Dividend yield	0%	0%
Results		
Call option value	**25.906**	**16.757**
Default Probability (PD)	**68,48%**	**86,73%**

The usage of down-and-out call leads to a reduction in equity value of almost 10 million and a rise in the probability of default of around 1800 basis points (18%). The lower equity value could be seen a result of the bargaining process between shareholders and bondholders. Indeed, following APV/DCF model, equity is worthless but, in reality, it had a market capitalization of 24.7 million thanks to bondholders who have approved another restructuring plan of five years. This is the “put option effect” that is exploited by shareholders. Prolonging the time horizon from 1 to 5 years, the option value rises because there are four more years to exploit volatility: in other words, the longer the time to maturity, and the higher possibility for the option to expire “in the money”. On the other hand, the barrier acts as a protection either for all the bondholders (in case of H as a positive net worth covenant) or only for senior bondholders (in case of H as senior bond nominal value).

We assumed the barrier equal to half of the nominal value of NFP. It could be thought as the net worth covenants or as the nominal value of senior bond. Either way, the put option effect becomes close to zero meaning that the barrier represents a clear signal of the contractual compensation that bondholders ask in exchange for the longer time horizon granted to shareholders.

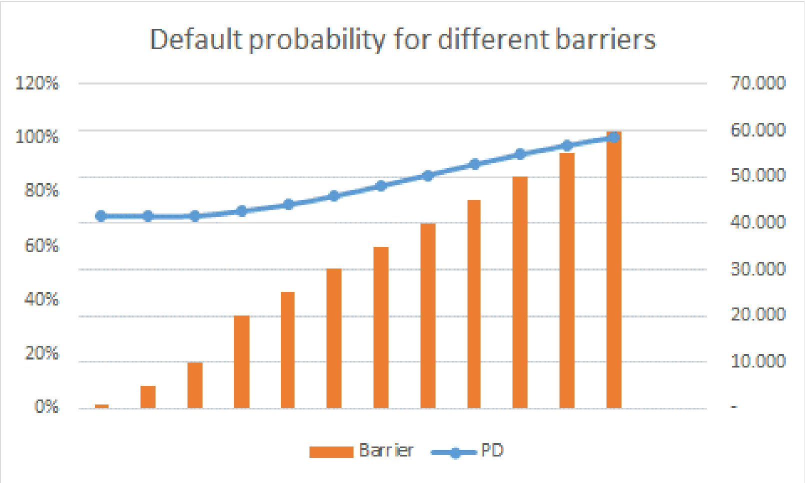
From figure 4.12, it is possible to see the behavior of equity in this situation. When the barrier (measured in the X-axis on the left) is low, from 1 to 20 million, equity (measured on the X-axis on the right) is not so affected and its value remains almost equal to the plain vanilla call case. On the contrary, as the default barrier rises, equity starts to decrease dramatically. In case of a barrier of 60 million, which is really close to the starting EV, equity is worthless and the corresponding PD is equal to 100%, as we already said. In conclusion, equity is maximized in absence of default barrier and starts to decrease in case the default trigger is raised.

Figure 4 - 12 Equity value (down-and-out call option) for different barriers



On the contrary, default probability has the exact opposite trend. Indeed, as threshold goes up, PD increases as well, as it is shown in figure 4.13. For low barriers, PD is the same as the plain vanilla call.

Figure 4 - 13 Default probability (down-and-out call option) for different barriers



It could seem a contradiction that as bondholders protection rises, probability of default increases as well. Given the high EV volatility, when the starting EV is close to the barrier, there is a high possibility that the former hits the latter leading to company default.

In case the barrier represents senior debt, the higher the amount of senior debt on total NFP and the higher the barrier will be, increasing the probability of default. The higher PD comes at the expense of junior creditors. In that situation, senior debtholders are protected because, despite the high PD, in case of default, company assets will be sold on the market. Indeed, company assets could be thought as a collateral on the senior debt. If collateral is valuable and its market liquid enough, senior bondholders will receive the whole principal, losing only the interest. On the contrary, junior bondholders will be reimbursed only on a residual basis. In that way, their payoff profile is similar to that of shareholders.

The choice of barrier level is crucial and it is the result of the bargaining process not only between shareholders and bondholders but also between senior and junior bondholders.

Appendix to chapter 4

Appendix 4 - 1 Reorganized balance sheet, Invested Capital (2010-2014)

number in /€000

EXHIBIT 1 STEFANEL: Reorganized Balance-Sheet (Invested Capital)	2010	2011	2012	2013	2014
€ million					
Working cash*	9.142,8	9.679,4	9.330,3	8.760,0	12.960,0
Trade receivables	24.098,0	25.463,0	41.171,0	32.200,0	24.406,0
Inventories	44.488,0	54.838,0	51.448,0	46.246,0	47.365,0
Trade payables	(59.639,0)	(68.922,0)	(58.722,0)	(50.998,0)	(45.855,0)
Trade working capital	**18.089,8**	**21.058,4**	**43.227,3**	**36.208,0**	**38.876,0**
Other operating current assets	20.045,0	18.467,0	14.262,0	12.869,0	12.172,0
Other operating current liabilities	(14.091,0)	(14.384,0)	(11.810,0)	(10.451,0)	(12.236,0)
Other current assets and liabilities	**5.954,0**	**4.083,0**	**2.452,0**	**2.418,0**	**(64,0)**
Net working capital	**24.043,8**	**25.141,4**	**45.679,3**	**38.626,0**	**38.812,0**
Tangible assets	39.262,0	38.064,0	32.796,0	27.025,0	23.075,0
Operating intangibles	45.255,0	41.669,0	35.130,0	31.389,0	27.151,0
Total operating fixed capital	**84.517,0**	**79.733,0**	**67.926,0**	**58.414,0**	**50.226,0**
Operating receivables and other non-current assets	10.591,0	10.705,0	10.577,0	5.809,0	6.739,0
Operating deferred-tax assets/(liabilities)	543,0	1.943,0	1.912,0	1.790,0	671,0
Operating non-current liabilities	(5.035,0)	(200,0)	0,0	0,0	(432,0)
Operating provisions	(346,0)	(2.169,0)	(2.396,0)	(3.118,0)	(3.489,0)
Total other non-current operating assets and liabilities	**5.753,0**	**10.279,0**	**10.093,0**	**4.481,0**	**3.489,0**
Invested capital excluding goodwill and similar intangibles	**114.313,8**	**115.153,4**	**123.698,3**	**101.521,0**	**92.527,0**
Goodwill and similar intangibles	0,0	0,0	0,0	0,0	0,0
Deferred tax asset/(liabilities) on similar intangibles	0,0	0,0	0,0	0,0	0,0
Goodwill and other similar intangibles	**0,0**	**0,0**	**0,0**	**0,0**	**0,0**
Invested capital including goodwill and similar intangibles	**114.313,8**	**115.153,4**	**123.698,3**	**101.521,0**	**92.527,0**
Non-operating current assets	75.569,0	0,0	0,0	3.445,0	2.328,0
Other non-operating current liabilities	0,0	0,0	0,0	0,0	0,0
Non-operating non-current assets	381,0	244,0	680,0	759,0	702,0
Non-operating deferred-tax assets/(liabilities)	2.009,0	2.009,0	2.009,0	2.051,0	4.453,0
Non-operating non-current liabilities	0,0	0,0	0,0	0,0	0,0
Non-operating assets	**77.959,0**	**2.253,0**	**2.689,0**	**6.255,0**	**7.483,0**
Total funds invested	**192.272,8**	**117.406,4**	**126.387,3**	**107.776,0**	**100.010,0**

Appendix 4 - 2 Reorganized balance sheet, Sources of Financing (2010-2014)

number in /€000

EXHIBIT 2 STEFANEL: Reorganized Balance-Sheet (Source of Financing)	2010	2011	2012	2013	2014
€ million	€ 2.010,00	2011	2012	2013	2014
Excess cash	(19.192,3)	(23.381,6)	(2.363,8)	0,0	0,0
Long-term borrowings	(3,0)	28.617,0	40.481,0	0,0	50.996,0
Adjustment on short term borrowings	627,0	0,0	0,0	0,0	0,0
Short-term borrowings	154.607,0	40.248,0	39.202,0	84.590,0	35.119,0
Net financial position	**136.038,8**	**45.483,4**	**77.319,3**	**84.590,0**	**86.115,0**
Provision for employee benefit	3.058,0	4.570,0	3.847,0	3.784,0	2.624,0
Non-operating provisions	1.654,0	1.457,0	286,0	425,0	458,0
Debt equivalents	**4.712,0**	**6.027,0**	**4.133,0**	**4.209,0**	**3.082,0**
Net financial position and debt equivalents	**140.750,8**	**51.510,4**	**81.452,3**	**88.799,0**	**89.197,0**
Minority interests	**189,0**	**325,0**	**460,0**	**179,0**	**224,0**
Shareholders' equity	**51.330,0**	**65.571,0**	**44.475,0**	**18.798,0**	**10.589,0**
Total source of financing	**192.269,8**	**117.406,4**	**126.387,3**	**107.776,0**	**100.010,0**

Appendix 4 - 3 Reorganized income statement (2010-2014)

number in /€000

EXHIBIT 3 STEFANEL: Reorganized Income Statement	2010	2011	2012	2013	2014
€ million	2010	2011	2012	2013	2014
Revenues	**182.855,0**	**193.588,0**	**186.605,0**	**168.517,0**	**155.609,0**
Costs of sales	(76.724,0)	(84.531,0)	(84.722,0)	(76.027,0)	(62.818,0)
Other operating costs	(3,0)	(2,0)	(1,0)	0,0	0,0
Adversting & Promotion	(15.012,0)	(15.237,0)	(11.116,0)	(5.605,0)	(3.710,0)
Operating cost (personnel and D&A excluded)	(73.279,0)	(71.290,0)	(63.617,0)	(58.801,0)	(55.311,0)
Personnel expenses	(40.864,0)	(40.935,0)	(38.217,0)	(31.832,0)	(29.775,0)
EBITDA	**(23.027,0)**	**(18.407,0)**	**(11.068,0)**	**(3.748,0)**	**3.995,0**
Depreciation	(7.381,0)	(7.580,0)	(7.361,0)	(6.110,0)	(5.226,0)
Amortization of operating intangibles	(1.735,0)	(2.876,0)	(2.659,0)	(2.262,0)	(2.231,0)
Total D&A	(9.116,0)	(10.456,0)	(10.020,0)	(8.372,0)	(7.457,0)
EBITA	**(32.143,0)**	**(28.863,0)**	**(21.088,0)**	**(12.120,0)**	**(3.462,0)**
Amortization of assets similar to goodwill	0,0	0,0	0,0	0,0	0,0
EBIT	**(32.143,0)**	**(28.863,0)**	**(21.088,0)**	**(12.120,0)**	**(3.462,0)**
Impairment losses	(682,0)	(7.752,0)	(3.434,0)	(3.287,0)	(1.166,0)
Non-recurring and extraordinary items	1.027,0	55.755,0	8.562,0	(1.921,0)	1.669,0
Interest income (expense) from investments	25,0	6,0	(123,0)	(394,0)	(70,0)
Exchange rate (losses) gains	1.850,0	(1.195,0)	(14,0)	(1.149,0)	(603,0)
Interest (expense) income	(5.501,0)	(4.465,0)	(3.573,0)	(3.254,0)	(4.579,0)
Net financial result	**(3.651,0)**	**(5.660,0)**	**(3.587,0)**	**(4.403,0)**	**(5.182,0)**
EBT	**(35.424,0)**	**13.486,0**	**(19.670,0)**	**(22.125,0)**	**(8.211,0)**
Taxes	120,0	1.073,0	(414,0)	(1.719,0)	547,0
Group Net Income	**(35.304,0)**	**14.559,0**	**(20.084,0)**	**(23.844,0)**	**(7.664,0)**
Minority result	(84,0)	(175,0)	(122,0)	(112,0)	(150,0)
Net Income	**(35.388,0)**	**14.384,0**	**(20.206,0)**	**(23.956,0)**	**(7.814,0)**

Appendix 4 - 4 NOPAT calculation (2010-2014)

number in /€000

EXHIBIT 4 STEFANEL: NOPLAT calculation					
€ million	2010	2011	2012	2013	2014
EBITA	**(32.143,0)**	**(28.863,0)**	**(21.088,0)**	**(12.120,0)**	**(3.462,0)**
Operating taxes*	3.746,5	17.023,6	1.288,0	(3.548,9)	(364,2)
NOPLAT	**(28.396,5)**	**(11.839,4)**	**(19.800,0)**	**(15.668,9)**	**(3.826,2)**

Appendix 4 - 5 Free cash flow calculation (2010-2014)

EXHIBIT 5 STEFANEL: Free Cash Flow Calculation				
number in /€000	2011	2012	2013	2014
NOPLAT	**(11.839,4)**	**(19.800,0)**	**(15.668,9)**	**(3.826,2)**
Amortization of operating intangibles	2.876,0	2.659,0	2.262,0	2.231,0
Depreciation	7.580,0	7.361,0	6.110,0	5.226,0
Gross cash flow	**(1.383,4)**	**(9.780,0)**	**(7.296,9)**	**3.630,8**
Change in operating working capital	(1.097,6)	(20.537,9)	7.053,3	(186,0)
Net capital expenditures	(5.672,0)	1.787,0	1.140,0	731,0
Change in other operating assets and liabilities	(4.526,0)	186,0	5.612,0	992,0
Gross investment	**(11.295,7)**	**(18.564,9)**	**13.805,3**	**1.537,0**
Free cash flow before goodwill and similar intangibles	**(12.679,1)**	**(28.344,8)**	**6.508,3**	**5.167,8**
Investments in goodwill and other intangibles	0,0	0,0	0,0	0,0
Free cash flow after goodwill and similar intangibles	**(12.679,1)**	**(28.344,8)**	**6.508,3**	**5.167,8**
Investments in non-operating assets	75.706,0	(436,0)	(3.566,0)	(1.228,0)
Impairment losses	(7.752,0)	(3.434,0)	(3.287,0)	(1.166,0)
Non-recurring and extraordinary items	55.755,0	8.562,0	(1.921,0)	1.669,0
Interest income (expense) from investments	6,0	(123,0)	(394,0)	(70,0)
Non-operating taxes	(15.950,6)	(1.702,0)	1.829,9	911,2
Change in debt equivalents	1.315,0	(1.894,0)	76,0	(1.127,0)
Non-operating cash flow	**109.079,4**	**973,0**	**(7.262,1)**	**(1.010,8)**
Cash available to investors	**96.400,4**	**(27.371,9)**	**(753,8)**	**4.157,0**
Net financial result	(5.660,0)	(3.587,0)	(4.403,0)	(5.182,0)
Change in minority interests	(39,0)	13,0	(393,0)	(105,0)
Change in shareholders' equity	(143,0)	(890,0)	(1.721,0)	(395,0)
Decrease (increase) in net financial position	**90.558,4**	**(31.835,9)**	**(7.270,8)**	**(1.525,0)**
Beginning net financial position	**136.038,8**	**45.483,4**	**77.319,3**	**84.590,0**
Ending net financial position	**45.480,4**	**77.319,3**	**84.590,0**	**86.115,0**

Source: Stefanel financial statement and own compilation

Appendix 4 - 6 EV computation in the three scenarios

EXHIBIT 14A STEFANEL: APV valuation		BASE CASE					
number in /€000	2014	2015	2016	2017	2018	2019	Base for CV
NOPLAT		963,0	2.045,6	3.168,2	4.014,0	4.763,9	4.811,5
Operating invested capital variation		(2.152,27)	(589,37)	(62,21)	(798,86)	113,08	(960,17)
Operating free cash flows (FCF)		(1.189,3)	1.456,2	3.106,0	3.215,2	4.876,9	3.851,3
Cost of capital		8,23%	8,23%	8,23%	8,23%	8,23%	7,17%
Discount factor		0,92	0,85	0,79	0,73	0,67	0,71
Present value of $FCF_{2015-2019}$		(1.098,8)	1.243,2	2.449,9	2.343,2	3.284,0	
Present Value of FCF in the forecasting period	8.221,5						
Continuing Value	43.933,3						43.933,3
BEV before debt tax shield and other items	52.154,8						
Tas shield on debt **	994,4						
Non-recurrent and extraordinary items value***	245,6						
Business Enterprise Value (BEV)	53.394,9						
Non-operating assets (NOA) value	7.483,0						
Enterprise Value (EV)	60.877,9						

EXHIBIT 14A STEFANEL: APV valuation		BEST CASE					
number in /€000	2014	2015	2016	2017	2018	2019	Base for CV
NOPLAT		2.127,8	4.244,2	6.067,5	7.882,4	8.517,6	8.653,9
Operating invested capital variation		(1.463,42)	(2.100,64)	(2.846,70)	(4.419,55)	(4.622,88)	(1.671,86)
Operating free cash flows (FCF)		664,4	2.143,6	3.220,8	3.462,9	3.894,7	6.982,0
Cost of capital		8,23%	8,23%	8,23%	8,23%	8,23%	7,20%
Discount factor		0,92	0,85	0,79	0,73	0,67	0,71
Present value of $FCF_{2015-2019}$		613,9	1.830,0	2.540,5	2.523,7	2.622,6	
Present Value of FCF in the forecasting period	10.130,8						
Continuing Value	88.068,3						88.068,3
BEV before debt tax shield and other items	98.199,0						
Tas shield on debt **	2.272,3						
Non-recurrent and extraordinary items value***	245,6						
Business Enterprise Value (BEV)	100.717,0						
Non-operating assets (NOA) value	7.483,0						
Enterprise Value (EV)	108.200,0						

EXHIBIT 14A STEFANEL: APV valuation		WORST CASE					
number in /€000	2014	2015	2016	2017	2018	2019	Base for CV
NOPLAT		(1.124,5)	(310,7)	784,5	1.622,3	1.648,1	1.649,7
Operating invested capital variation		(102,62)	(917,21)	(924,15)	(340,23)	(566,61)	(1,65)
Operating free cash flows (FCF)		(1.227,1)	(1.227,9)	(139,7)	1.282,1	1.081,5	1.648,1
Cost of capital		8,23%	8,23%	8,23%	8,23%	8,23%	7,20%
Discount factor		0,92	0,85	0,79	0,73	0,67	0,71
Present value of $FCF_{2015-2019}$		(1.133,8)	(1.048,3)	(110,2)	934,4	728,2	
Present Value of FCF in the forecasting period	(629,7)						
Continuing Value	15.498,4						15.498,4
BEV before debt tax shield and other items	14.868,7						
Tas shield on debt **	311,9						
Non-recurrent and extraordinary items value***	245,6						
Business Enterprise Value (BEV)	15.426,2						
Non-operating assets (NOA) value	7.483,0						
Enterprise Value (EV)	22.909,2						

CONCLUSION AND DISCUSSION

In conclusion, at first, chapter one describes the main aspects of credit risk, including probability of default estimation, rating process, and credit analysis through Altman Z-score model. Then, in chapter 2, most relevant structural models for corporate debt valuation that it is possible to find in the literature have been described. Finally, in chapter 3, a structural model has been developed exploiting the similarities between equity and a down-and-out call option. This option is said to be path dependent in the sense that its price depends on the path of the underlying asset, which has to be monitored for all the time until expiration, from emission to maturity. In particular, it is characterized by a barrier, which makes the option worthless in case it is hit by the underlying. In the opposite situation, when underlying is above that threshold for the entire contractual time horizon, option payoff is the same as a plain vanilla.

These characteristics allowed us to use the option pricing formula to value equity in case of a protected debt. The barrier, indeed, could be intended either as a net-worth covenant that protects the whole debt or, as the amount of senior debt whose principal reimbursement is somehow guaranteed. In this second situation, the choice of the barrier could be seen as the result of the bargaining process between shareholders, senior and junior bondholders.

Afterwards, in the last chapter, the model has been applied to the real case of Stefanel Group, which is an Italian Apparel group that is experiencing a period of deep crisis since 2010. On April 2015, the approval of the Financial Report of the group has been postponed because of a covenant breach that has led to the need of both a new restructuring and industrial plans.

At first, we made an economic and financial analysis of the historical performance of the group, trying to understand the main reasons of the crisis.

In our opinion, distress is a result of different factors such as the reduced profitability (decreasing revenues) and bad management of capital structure characterized by too heavy leverage. Financial troubles have been inflated by the ongoing global financial crisis and European sovereign debt crisis that have seriously undermined consumer’s confidence all over the World, with particular negative impact on Italy (country where the group is headquartered and most of its sales are derived from). Despite the improvements in marginality, SG needs to boost sales and reduce leverage in future. Otherwise, on the one hand there is the risk of continuously eroding equity to absorb operating losses while, on the other hand, SG faces the possibility of a funding rationing by banks due to higher perceived riskiness.

Subsequently, we computed a synthetic rating and a creditworthiness measure through Altman Z-score, which show an enduring financial distressed situation for the group. The results of both methods displayed coherent results with a really low Z-score and a likewise low credit rating CCC that classifies the group as speculative investment.

Finally, to apply the option pricing formula, we estimated the main variables of the models such as the enterprise value (though a cash flows model which mixes Adjusted Present Value and Discounted Cash Flow) and its volatility (though the Raphson-Newton iteration implemented in VBA). In particular, EV was the result of a weighted average of three different scenarios, signaling an equity value equal to zero due to heavy distress. Nonetheless, market capitalization of the group was around 24 million euro (at 31/12/2014) and this positive value can be explained by the theory of options. Despite the distressed situation, on April 2016, managers of Stefanel were negotiating the terms of a new debt restructuring with main financing banks, buying additional time to turnaround the group. As we already said, equity will benefit from the restructuring plan because of the unlimited upside potential of its payoff. Said another way, equityholders have the possibility to gain from longer time horizon granted by creditors. While current equity value is approximately zero, if the best scenario will become reality within five years, equity could be positive and high. This is also what bondholders hope, because it would mean a complete reimbursement of their credit without any loss. Nevertheless, creditors want to protect themselves. In particular, they want to avoid obtaining an unsatisfactory sum of money in case the worst scenario will happen because of EV erosion due to investments in risky and unprofitable projects. Indeed, the purpose of the barrier is to prevent company assets value from shrinking below a safe level and this is the reasoning behind a net-worth covenant.

In case the barrier represents the nominal value of senior bonds, it is the dividing line between senior and junior bond. Senior bondholders could be thought as being "safe" because in case EV falls below nominal senior debt, they will receive the principal losing only interests. On the contrary, junior bondholders will be reimbursed only if EV never shrinks below the barrier itself.

In conclusion, despite EV at 31/12/2014 is estimated to be worthless, positive market capitalization of SG at the same date could be explained by option theory. In practice, equity of Stefanel could be though as a deep out-of-the money option that currently is not worth to be exercised by shareholders. This does not mean that at the end of the new restructuring plan (December 2019), SG will recover standing in a safer and more stable financial situation, obtaining higher credit rating. This could happen thanks to the high volatility of enterprise value. Nevertheless, high volatility could be a double-hedged sword because it could also lead asset value to touch the barrier leading to a covenant breach and a consequent default prior to maturity and this is signaled by the high probability of default (estimated in the case of barrier option) of 86%.

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