UNIVERSITÀ DEGLI STUDI DI PADOVA

DIPARTIMENTO DI MATEMATICA "TULLIO LEVI-CIVITA"

Corso di Laurea Magistrale in Matematica

Teaching with formularies and its effects on long-term memory and math anxiety

Supervisor: **Francesco Ciraulo**

Student: **Sebastiano Nicola Fortunato Bravi**
Number: 2062762

19 April 2024
Academic year 2023/24

Many's the time I've been stuck in the middle of the stream. ... Which only means that mathematics will never be over and done with. And a good thing too: there will always be plenty to keep us busy.
(The Number Devil in *The Number Devil: A Mathematical Adventure* written by H. M. Enzensberger)

Contents

Introduction

We all know that we can learn something with different intensities.
We can all, from personal experience, distinguish the difference between a concept learnt superficially and one that is reworked more often and learned more deeply. We can see this distinction especially if we think about the school experience where it is usual for those dealing with a subject they do not like, to learn by heart what is necessary to face a test and then actually forget what they have learnt.
For a predominantly notionistic subject such as mathematics, this dynamic is strongly present. Therefore, in the perspective of improving students' learning, experimenting with new educational methods can be a valid means to improve students' school experience.
In this thesis, the investigated method is aimed precisely at avoiding mnemonic learning done in a superficial manner: in fact, it is proposed the use of formularies during the written tests combined with the use of more speculative exercises, called "challenging tasks" to consolidate the students' knowledge in a different manner. In addition to the didactic side, this thesis also focuses on the psychological dimension of learning mathematics, in particular on the anxiety that this subject evokes in students and how it may vary with the introduction of a new method.
In the first chapter of the thesis, we will fully illustrate how memory works, introduce the concept of *math anxiety* and how it affects students, and explain what challenging tasks consist of and how they can be introduced into the classroom.
After providing the right theoretical foundations in the first chapter, the second chapter introduces the study carried out, explains its purpose and presents the data analysis performed.
The results obtained are discussed in the third and final chapter, together

with a discussion on the limitations found in the research.
Summarised in a few lines, the study investigated variations in performance and levels of math anxiety in classes adopting the use of formularies.
In short, the study found a slight drop in assessment anxiety levels in classes that newly adopted the formularies and a general drop in performance in classes adopting this methodology, mainly due to the failure to memorise formulas.
Although the study did not produce the expected results, it opened up numerous hints for future research, emphasising once again that research in education is an active area of study that can and should involve students from different disciplines.

Chapter 1

Theoretical background

In this first chapter, we will provide an overview of the concepts that will help us to fully understand the research. We will explain quickly but effectively how our memory is structured and the processes that involve it. We will introduce the concept of "Math anxiety" and its effects on learning. Finally, we will present "challenging tasks" and how their use can influence pupils' performance and learning.

1.1 Memory: how it is articulated and how it works

Recalling the date of a friend's birthday, performing the right movements in order to stay balanced on a bicycle, tackling a test or an exam that requires knowledge of numerous notions: all these and many more are activities that we can perform thanks to our ***memory***.
The act of memorising is based, according to psychologists, on three distinct processes: the ***encoding*** through which we obtain an initial recording of information, the ***storage*** that allows us to store this information which we are able to reuse thanks to the last process, the ***retrieval*** [8].
For a deeper understanding of how this processes works and what kind of information is processed in each of them, Atkinson and Shiffrin proposed the ***three-system memory theory***. According to this theory, there are three different processes in which information is processed: ***sensory memory***, ***short-term memory*** and ***long-term memory***[8].

Sensory memory

In the instant we are exposed to a stimulus, it is immediately recorded in the *sensory memory*. This information is recorded with a high degree of accuracy at the expense, however, of a very short duration: in fact, depending on the nature of the stimulus, this recording only lasts a few seconds and if the information is not moved to short-term memory then it is forgotten.

Given the varied nature of the stimuli it is more correct to speak of different sensory memories each associated with a different sense: for example for visual stimuli we speak of *iconic memory* while for auditory stimuli of *echoic memory* [8].

In a nutshell, this type of memory functions as a snapshot in which all the information from the various senses is present and this snapshot is immediately replaced by a new one unless it is moved to another type of memory.

Short-term memory and working memory

Given the 'raw' nature contained in the accurate snapshots provided to us by sensory memory, stimuli need to be processed in order to become meaningful and reusable by our mind.

In the first place, such processing is carried out in the *short-term memory* through processes that are not yet well understood and for which there are various hypotheses. What we do know is that the final result is a more elaborated piece of information, and more usable by our mind, that survives for a longer period than sensory memory (most psychologists speak of 15-25 seconds before the information is forgotten) at the cost of having a less precise representation of the stimulus.

Short-term memory has a capacity limited to about seven items called 'chunks' which we can define as a group of stimuli grouped together in a meaningful way: it can be a picture, a word or a sequence of numbers. Better organised stimuli are remembered more efficiently: a group of 21 random letters is more difficult to remember than the same letters arranged in seven groups of three letters each where in the latter case every group would represent a single chunk of information rather than three separate ones.

Short-term memory is not only the place where memories are processed and sent into long-term memory, but it also has a more active task that can involve new stimuli and old memories in processing new thoughts: in this case we speak of this in terms of *working memory*.

Working memory keeps information in an active, ready-to-use state and reprocesses it according to new stimuli or memories brought back to mind. We can give an explanatory example by thinking of when we perform multiplication in our minds: in unison we handle the new stimulus (the new calculation to be done) and already established memories (the multiplication tables) to produce provisional results that need to be added up to obtain the final result.

An increase in usage of working memory correlates with an increase in usage of cognitive resources and a consequent reduction in awareness of our surroundings. We will also see in the next section how stress can negatively impact our working memory.

Long-term memory

Once thoughts have been processed in short-term memory, it is possible for them to move into the *long-term memory*. There are several experimental trials confirming a clear division between short-term and long-term memory: one of these involved brain-damaged patients who could no longer recall new information obtained after the damage but were able to recall memories prior to the damage.[8]

Long-term memory is often divided and distinguished into *procedural memory* (or non-declarative memory) which includes 'how' we perform actions and all the movements involved in performing them and *declarative memory* which contains factual information. Declarative memory itself is divided into *semantic memory* and *episodic memory*: the first one contains our knowledge and the logic that allows us to give further inferences about that knowledge while the second one concerns the events we experienced at a given time and place.

But how can information be transferred from short-term to long-term memory? The transfer is mainly based on *rehearsal*: through repetition the information is consolidated. However, a simple repetition may not be enough: in these cases we speak of ***elaborative rehearsal*** that is a repetition that includes a better organisation of the information, whether or not it is a logical link to other memories or conversion into an image or any other medium. These organisational strategies for better memorising informa-

tion are called *mnemonics.*
Thus the information arrived in long-term memory is not only ‘stacked’ together with our other memories but also forms connections with them on the basis of common characteristics.
For example, let us think of any object present in a class as, for instance, a chalk: the information of the chalk will be connected to all the objects present in a classroom by the simple fact that they share a common space. Or we can link the chalk to all white objects or all tools used for writing. In this way, we create networks of information that are interconnected by common characteristics: those networks are called *semantic networks.*

How we remember and why we forget

After having succeeded in transferring a piece of information into long-term memory, it is essential to be able to reuse it when we need it most: this ability is called ***retrieval***. More precisely, we speak of ***recall*** when we need to retrieve information from our mind and of ***recognition*** when we need to recognise information already stored from a group of several pieces of information.
Sometimes, however, some memories are difficult to retrieve and in order to do so we need an external stimulus that acts as an aid and that in the literature is called ***retrieval clue***: it can be a word, a sound or any other type of stimulus that allows us to retrieve the right memory from the dense archive of information within our mind.
The ***levels of processing theory*** has been proposed to describe the different intensities with which we can remember something. This theory emphasises how differences in the initial processing of a stimulus directly determine our ability to remember it: higher processing corresponds to greater ease of recall.
Starting at a low level of processing, information is not remembered except for its sensory impressions, whereas as we gradually increase the level of processing we begin to abstract these sensory impressions and make sense of them until we reach a maximum level of processing where we are able to fully comprehend the information in a meaningful way and place it in a larger context in which we will find easier to remember it.
In short, the greater is our knowledge of a piece of information, the greater will be both its retention in memory and our ability to remember it.
A further confirmation of this implication is provided by a study [6] in

which three different experiments confirmed the role of working memory consolidation for a better long-term memorisation.

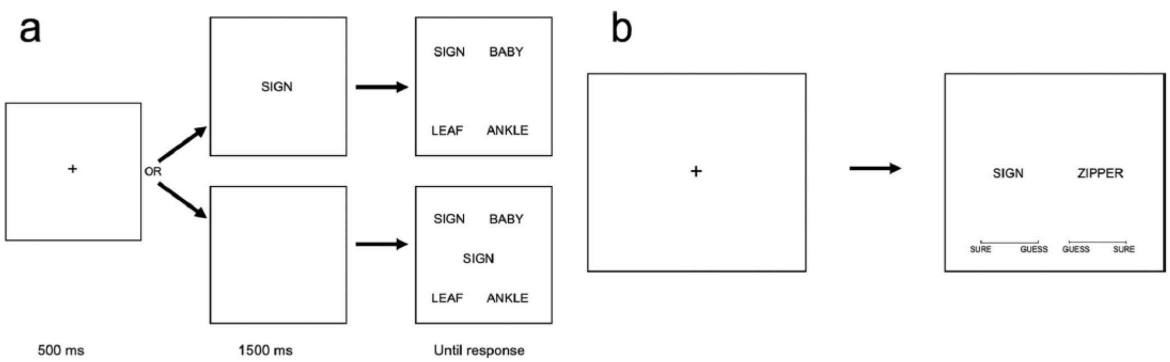


Figure 1.1: Scheme of the two phases of the first experiment in [6]

In the first of these experiments, participants had to perform in the first phase a simple recognition task: given one word in the centre and four others in the corners of the screen, they had to point to the corner with the same word as the centre of the screen.
This first task was proposed in two different ways in the two groups of participants: one group was shown the word to be recognised one and a half seconds before the other words were shown, while another group was shown all the words on the screen at the same time. Following this first task and a short break, both groups were given a task in which they had to remember whether or not the words on the screen were presented in the previous task. The first group in which the stimulus of the word was consolidated for a longer time in working memory because was shown before the other words showed better abilities to remember the words previously encountered in the subsequent test.
Initial processing is also important when such information is forgotten: in fact, a simple error in initial encoding, induced by distraction or other factors, causes a faulty processing that produces less clear memory that tends to be forgotten more easily.
Other factors that contribute to forgetting information are its ***degradation***, which is inversely proportional to how many times we recall this information, and the ***interference*** with other information that we distinguish in *proactive interference* and *retroactive interference* depending on whether the disturbing information was learnt before or after the disturbed

one.
In other cases it may happen that we do not directly forget information but the possible retrieval clues that allowed us to regain it, in this case we speak of *cue-depending forgetting*.

1.2 Math Anxiety

You are in the classroom, a new lesson begins, the teacher enters and starts talking about a new topic. Their way of explaining is calm and does not differ too much from the other teachers' method, yet you begin to feel uneasy. Your performance in their subject is as good as the rest of the other subjects, yet you are not completely at ease. The fact that they is explaining something new and that the blackboard is filled with formulas and symbols that you had never seen before, and the thought of having to learn to use them in the near future, probably in a test, does not make you feel comfortable. The only problem would seem to lie in the subject explained: mathematics.
In 1972, Richardson and Suinn defined ***Math Anxiety*** as something which "*involves feelings of tension and anxiety that interfere with the manipulation of numbers and the solving of mathematical problems in a wide variety of ordinary life and academic situations*" [14].
But how much does this problem affect the population? Various studies aimed at providing an estimate give different results probably due to the variety of ages and populations involved in them; however, even the lowest estimates speak of a percentage between 15 and 20 per cent of the population experiencing a high rate of mathematics anxiety [7] thus highlighting the importance of fully understanding the phenomenon in order to try to solve it.
First of all, it should be specified that the construct of Math Anxiety, although sharing with it many aspects, is to be considered a separate construct from general anxiety: various studies have shown that there are positive correlations between general anxiety and Math Anxiety but have also shown much higher positive correlations between the various questionnaires used to measure Math Anxiety [7] highlighting the common but separate nature of the two constructs. Thus, Math Anxiety is to be considered different from general anxiety for the fact that 'prefers' a single group

of related activities, in this case the manipulation of numbers, symbols and mathematical formulae.
The existence of various forms of anxiety localised to individual activities or subjects, such as reading anxiety, is also based on this distinction. However, despite the existence of various forms of localised anxiety, a 2012 study showed that mathematics anxiety manifests itself on average more severely than its counterparts in other subjects [12].

How to measure Math Anxiety: MARS e AMAS

In order to measure something as subjective as anxiety, we need instruments that convert efficiently our feelings into concrete measurements. Psychometrics comes to our aid, providing us with the means to implement these measurements.
In 1972, Richardson and Suinn constructed the first questionnaire for measuring maths anxiety: the Math Anxiety Rating Scale (MARS) [14]. The MARS questionnaire consists of 98 items in which are described various situations involving the use of mathematical instruments to be rated on a scale from 1 to 5 where a higher value corresponds to a higher level of anxiety.
Higher scores are correlated with higher levels of Math Anxiety. The questionnaire has a test-retest reliability coefficient[1] of 0.85 and an internal consistency reliability coefficient[2] of 0.95: these values indicate not only that the questionnaire is reliable but also that the measurements of the various items are potentially influenced by a single factor, which is probably Math Anxiety [14].
However, despite its high reliability as confirmed by further studies, the MARS has some limitation: the huge amount of items included in the questionnaire make it too long and therefore difficult to administer both in terms of timing and in terms of amount of data to be processed.
In this regard, alternative and abbreviated versions of the MARS were subsequently proposed, including the MAS (Mathematics Anxiety Scale) by

[1]The test-retest reliability coefficient is a value between 0 and 1 used to calculate the reliability of an instrument by repeated testing under the same conditions: the closer this value is to 1, the more reliable the instrument and the resulting measurement. Usually, tests and questionnaires that have a reliability coefficient of at least 0.8 are considered acceptable.

[2]The internal consistency reliability coefficient is a value between $-\infty$ and 1 that indicates the homogeneity in the measurements of the various items in a questionnaire. The higher the value, the more the items in a questionnaire are correlated with each other and possibly influenced by a single factor.

Fennema and Sherman in 1976, the ATMS (Anxiety Toward Mathematics Scale) by Sandman in 1980, the MARS-R (Math Anxiety Rating Scale - Revised) by Plake and Parker in 1982 and the sMARS (Abbreviated Math Anxiety Rating Scale) by Alexander and Martray in 1989 [11].
A further development of the MARS-R resulted in the AMAS (Abbreviated Math Anxiety Scale), which is the shortest questionnaire of all. Indeed, the AMAS consists of only 9 items that require, as in the MARS, to evaluate various situation on a scale from 1 to 5, where a higher value corresponds to a greater degree of anxiety experienced. As the MARS, higher scores on this questionnaire are correlated with a greater degree of experienced Math Anxiety.
Within the AMAS questionnaire, as in the other reworkings of the MARS, it is possible to divide the items into two subsets: the LMA (Learning Math Anxiety) scale and the MEA (Math Evaluation Anxiety) scale, the former aimed at highlighting anxiety related to the actual learning of mathematics while the latter to anxiety about mathematics-related performance. Within the AMAS, the LMA and MEA subsets consist of 5 and 4 items, respectively. A 2003 study by Hopko and colleagues [9], in order to show the reliability of the AMAS questionnaire and its two subsets, calculated not only very good test-retest reliability coefficients (AMAS: 0.85; LMA: 0.78; MEA: 0.83) but also excellent internal consistency reliability coefficients (AMAS: 0.90; LMA: 0.85; MEA: 0.88).
A further study by Primi and colleagues [11] not only provided an Italian translation of the AMAS questionnaire but also extended the sample to include high school students since all previous research had always studied a sample made up of university students. Thus, using a sample of 215 high school students, test-retest reliability coefficients were calculated that were not only very high (AMAS: 0.86; LMA: 0.81; MEA: 0.80) but not too far from the same values calculated on university students.
The research also involves a sample of university students, obtaining values almost identical to those of Hopko's 2003 study.

Gender and age differences and external influences

Among various studies, attention is also paid to possible gender differences in the perception of Math Anxiety.
In fact, some recent studies have shown that in countries where there are no differences in education between genders, performance does not change

between genders, but there are differences in self-assessment and Math Anxiety: girls tend to have higher values of Math Anxiety and a more pessimistic self-assessment than boys.

However, even in other studies where students were provided with methods to manage performance anxiety in general, it was seen that girls in contrast to boys continued to show a correlation between maths anxiety and their own performance. Various hypotheses have been put forward to explain why this difference in perception exists, but studies have not yet shown sufficient evidence to support one at the expense of the others.

One of these hypotheses assumes that this difference is simply a consequence of an already confirmed higher level of anxiety experienced on average by women while another hypothesis attempts to explain this difference as a consequence of gender stereotypes and stereotype threat.

The latter hypothesis supposes that people's desire not to confirm a stereotype that is disadvantageous to them (in this case the stereotype that women are less good at mathematics than men) increases the anxiety and stress experienced precisely in assessments involving this stereotype.

Confirmation in favour of this hypothesis is presented in various studies in which women presented greater levels of stress and consequent drops in performance on the very tests where the intention to investigate this stereotype was made explicit. In these same studies, these increases in anxiety did not occur if the stereotype was not presented or if it was presented with due explanations concerning stereotype threat and its influence on anxiety and performance [7].

Other studies, on the other hand, attempt to identify differences in the perception of Math Anxiety with varying age. In these cases, studies show that maths anxiety increases with age, but again, there are various hypotheses that provide an explanation for this increase.

Among these reasons are: the excessive abstraction and difficulty present in mathematics with advancing age, a stereotypical belief that overestimates the difficulty of the subject and also a positive correlation with an increase in the required capacity of working memory.

However, some researchers highlight a lack of research aimed at younger students and therefore suggest a greater involvement of pupils in the early years of school in the research landscape.

For example, Szczygieł and Pieronkiewicz in a study from 2021 [20] investigated the presence and possible causes of maths anxiety in a sample of 369 children aged around 7 years. In this research, a modified version of the AMAS questionnaire adapted for primary school children was used, administered at the beginning, middle and end of the school year. In addition to the questionnaire, a qualitative individual interview was also carried out to investigate the reasons that triggered anxiety in the children.

The results of the research showed a concordance with the hypothesis that at the beginning of the course the students experience very little if any anxiety, except for a few statistically insignificant cases, but they also noted a slight increase in these measurements with regard to assessment-related anxiety.

Even more interesting were the results of the individual interviews in which more than half of the children cited fear of failure as a reason for Math Anxiety. Other reasons presented by the children, but less frequently, were fear of bad grades (a reason that tripled in frequency over the course of the year to become the second main reason reported at the end of the year with 39 per cent) and fear associated with the very nature of mathematics (on average 23 per cent).

A group of research, on the other hand, has looked for possible causes external to the individual subject and dependent on figures close to them such as parents or teachers themselves.

In investigating the role of parental influence there is, for example, the 2015 research by Soni and Kumari [16]. This study used measurements of levels of Math Anxiety and levels of attitude and performance towards mathematics taken on a sample of 596 children from various primary and secondary schools and one parent per pupil. The questionnaires used to measure anxiety were all modified versions of the MARS adapted for primary school students or adolescent students and an abbreviated version for parents (MARS-E, MARS-A, MARS-SV respectively).

The study showed a positive correlation between the values measured in the students with that in their parents, confirming the influence they have on both anxiety and positive attitudes towards mathematics in their children. One of the limitations of the latter study was that it involved only one parent instead of both, cutting out any considerations of gender differences. Such observations were made in a 2020 study by Szczygieł [19].

In this research similar to the previous one, the anxiety and attitude levels

towards mathematics of 241 students in the early years of primary school and their parents who agreed to participate (176 mothers and 51 fathers) were measured.
The results of this study differ from those of the previous studies: in fact, a positive correlation between parents' and children's Math Anxiety was not found except in the case of fathers in the first and third years of primary school.
This irregular pattern (probably due to the low number of participating fathers) suggests, according to the researcher, that parental influence is not as direct as assumed and that therefore parent-child relationships in the influence of anxiety should be further investigated in the future.
The same study also researched the role of Math Anxiety in teachers and its possible influence on students. In agreement with the data collected in previous research, no correlation was found between the two factors.
The only studies that show any influence on students given by teachers show a tenuous negative correlation between teachers' Math Anxiety and pupils' performance [13] and a negative correlation between the Math Anxiety experienced by some teachers and the performance of female pupils only [2].
The latest study proposes as an explanation for this correlation the possible influence of anxiety related to gender stereotypes from teachers to pupils. In support of this, the data collected in the study also show that students displaying internalised agreement with gender stereotypes tend to perform lower in mathematics.
In summary, this last group of research shows that the question of adult influences on children with regard to Math Anxiety is still an area that needs to be investigated in depth.

How to deal with Math Anxiety

Having now established the problem of maths anxiety and its deleterious effects on pupils' self-efficacy and performance, we can only wonder if there are ways to manage it and diminish its negative influences.
Recently, one approach is to use cognitive reappraisal of the causes that trigger Math Anxiety by breaking the vicious cycle in which Math Anxiety induces stress and performance declines that induce higher levels of anxiety. In some research, this reassessment was done using expressive writing techniques that allowed people to remember and put on paper the occa-

sions when anxiety occurred [7].
However, these techniques were found to have a positive impact almost only on subjects with high levels of Math Anxiety, especially in tasks that required high capacities for working memory, and were therefore not a wide-ranging solution.
Another attempt was made with cognitive tutoring: in a research [18], they examined the effects of an eight-week one-to-one mathematical tutoring course. This research showed that thanks to this course, students developed a more positive attitude towards mathematics and there were decreases in the levels of Math Anxiety in both high and low-anxiety students.
Still in its beginnings is research proposing the use of non-invasive brain stimuli such as transcranial electrical stimulation as a potential treatment[7].
Given the premature nature of much research and the few results obtained from it, one wonders whether an ad hoc solution to the problem of Math Anxiety can be found sooner or later, probably not a one-size-fits-all solution, but with cautious optimism, while the research continues, one can however start by changing the classroom context, as Blyth suggests in his article [3].
Creating a safe environment in the classroom where students do not feel judged, where mistakes before being mistakes are cues for learning, where there is proper education on emotions and their control, and where the teacher encourages the spirit of cooperation and stimulates the pupils' desire to learn is a first step to simplify the educational journey of many students and to make Math Anxiety a more manageable problem.

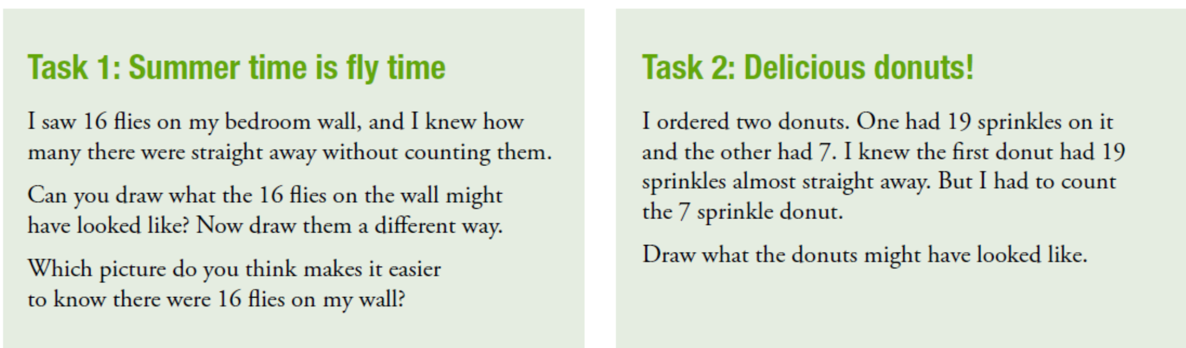
1.3 Challenging tasks

Let's start with a question: what is a ***challenging task***?
A trivial answer could simply be to define a challenging task as a task that is more difficult to perform than others proposed, but in a school context, we can give a more in-depth definition: a challenging task is in fact a more complex exercise that has the task of involving students by presenting multiple levels of difficulty and allowing the formation of connections between mathematical topics.
This type of exercise aims to better involve students and also to encourage

group activity, and to do so it also makes use of ***enabling prompts*** and ***extending prompts***: the first one allow students who would otherwise get stuck while carrying out the exercise to move forward while the second one allow those who finish the exercise early to reflect further on it and thus expand the set of connections and tools useful for solving similar exercises [5].

Some example

Due to the nature of challenging tasks, it is easier to deeply investigate less complex mathematical concepts, presented especially in the early school years.
An example proposed in [15] aimed precisely at primary school students proposed the tasks in Figure 1.2 to investigate not only how we count but also to make students understand how counting more or less ordered objects is different.


Task 1: Summer time is fly time

I saw 16 flies on my bedroom wall, and I knew how many there were straight away without counting them.

Can you draw what the 16 flies on the wall might have looked like? Now draw them a different way.

Which picture do you think makes it easier to know there were 16 flies on my wall?

Task 2: Delicious donuts!

I ordered two donuts. One had 19 sprinkles on it and the other had 7. I knew the first donut had 19 sprinkles almost straight away. But I had to count the 7 sprinkle donut.

Draw what the donuts might have looked like.

Figure 1.2: Picture taken from [15]: both of the tasks focus on counting certain objects (in the first one flies on a wall, in the second one sprinklers on a donut) and how the arrangement of them affects the ease of counting

Speaking instead of 11-12 years students, Papadopoulos in a 2019 study [10] presents an activity to accompany students in the transition between arithmetic and algebra and to analyse what are the relationships between quantities that are also not presented explicitly.
For this purpose, it uses *mobile puzzles*, i.e. collections of objects balanced on hanging scales in which equal shapes correspond to equal weights. The aim of the initial exercise is to identify the weights of the individual shapes once the total weight on the scales or of a single shape has been given, as we can see in Figure 1.3. The activity continues by delving into the

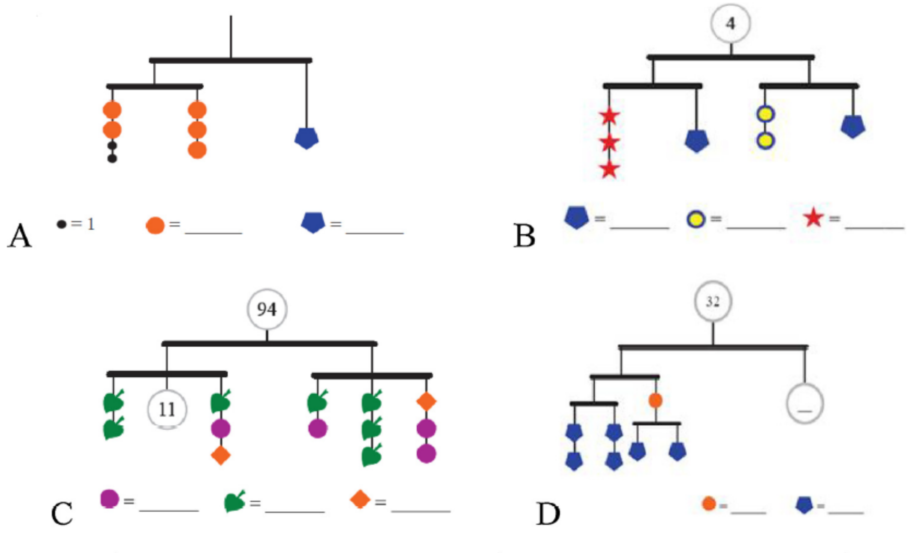


Figure 1.3: Picture taken from [10]: the mobile puzzles consist of different scales where are hanged different shapes

Appendix 1: Quadratics task from the study

A. Can you find some equations of parabolas that:

- Cut across the x-axis twice?
- Cut across the x-axis once?
- Don't cross the x-axis at all?

B. What do you notice about each of the different groups of parabolas you have found?

C. Can you use your previous answers to find the equations of horizontal lines that cut across the parabola $y = x^2 - 2x$ once, twice, or not at all?

Figure 1.4: Picture taken from [22]: an example of task used for the study. In three points the student investigates the number of intersection of parabolas with the main axis

use of moving puzzles and investigating how balances change by moving pieces or adding different scales together. In these questions, students are also constantly asked why they gave their answers and how they arrived at them, thus making them focus more on the reasoning done rather than the result itself. As a final request, students are asked to construct their own mobile puzzle, allowing them to creatively apply the knowledge acquired during the activity [10].

Moving on to the early years of secondary school, in an article Wilkie instead presents an activity aimed at a more complete understanding of quadratic functions [22]. The article investigates not only the results but also the students' reactions to the proposed activity.

The activity in Figure 1.4 consists of a guided exercise that initially asks students to give several examples of parabolas with different numbers of intersections with the X-axis. The students are then asked to reason about the differences between the three types of parabolas identified earlier, with the aim of making them understand the relationship between the discriminant of a quadratic function and the number of intersections of the parabola with the X-axis. Having reached this point, we are asked to use the knowledge acquired in the first two points to solve a similar problem on a given quadratic function. The study also provides alternative versions of the exercise in 1.4.

Let us conclude with students in their final years of pre-academic studies: the concept of proof is constantly presented to students in schools whose curriculum requires it, and they are often only required to learn the demonstrations presented and to familiarise themselves with the formal mathematical language, yet they are often not allowed to explore the topic freely by the students for obvious reasons of time and for the veiled fear that if asked to demonstrate even a simple thing, the answer would just be a bunch of blank sheets of paper, except for those handed in by high-achieving students.

One school environment where demonstration is explored and tackled is the mathematical olympiads, but the competitiveness may not put all students at ease. We should therefore propose a middle way where without the tension due to competition and by providing the right tools and possibly guiding the student through some of the more delicate steps we open ourselves to a multitude of possible demonstration exercises.

This type of exercise is also sometimes presented in the Italian Maturity Examination, as we can see in Figure 1.5 in one of the problems proposed in the second test for ordinary high-school science proposed in the 2017-2018 school year.

Now, unlike the exercises previously shown, this exercise lacks any intermediate steps as it poses the question by showing only an example and without giving any clue as to any theorems to be used or useful relations to be found. However, this last exercise more than as an example can be useful as a start together with the exercises proposed in the mathematical olympiads for constructing challenging tasks aimed at final year students.

4. Nella figura è evidenziato un punto $N \in \Gamma_1$ e un tratto del grafico Γ_1. La retta normale a Γ_1 in N (vale a dire la perpendicolare alla retta tangente a Γ_1 in quel punto) passa per l'origine degli assi O. Il grafico Γ_1 possiede tre punti con questa proprietà. Dimostra, più in generale, che il grafico di un qualsiasi polinomio di grado $n > 0$ non può possedere più di $2n - 1$ punti nei quali la retta normale al grafico passa per l'origine.

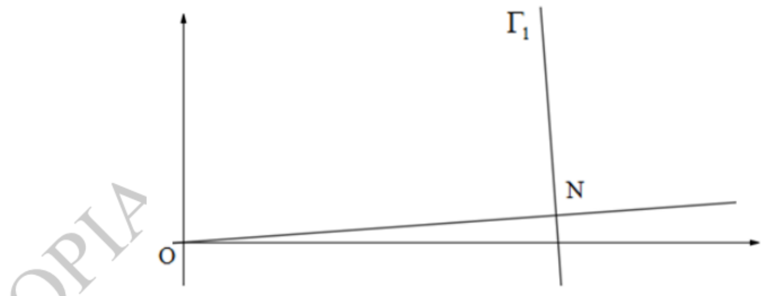


Figure 1.5: Picture taken from MIUR's archive. The exercise was proposed in the exams for scientific high schools in the school year 2018/2019. [4]

Doubts and potentials

When a new type of exercise is proposed, it is natural that doubts arise regarding its use. We present the main doubts about challenging tasks by re-proposing the ones described in the article by Russo and colleagues [15] where they collected at the beginning of the school year through various focus groups the opinions of 102 primary school teachers about the use of challenging tasks in the classroom and then through a follow-up meeting at the end of the school year they also collected any comments or solutions to the problems and doubts they had found.
The first doubt is about the spontaneous assumption that more complicated exercises are only suitable for high-achieving pupils, thus neglecting the rest of the students.
Through meetings, the most useful answer to this doubt is to change one's mindset and have no preconceived notions about the abilities of high and

[4] Site: https://www.istruzione.it/esame_di_stato/201718/Licei.htm
Translation by me: A point is highlighted in the figure $N \in \Gamma_1$ and a section of the graph Γ_1. The line normal to Γ_1 at N (i.e. the perpendicular to the tangent line to Γ_1 in that point) passes through the origin of the axes O. Graph Γ_1 has three points with this property. Prove, more generally, that the graph of any polynomial of degree $n > 0$ cannot possess more than $2n - 1$ points where the line normal to the graph passes through the origin.

low performing pupils, as this can only cause damage.
This change of mindset is supported both by the individual experiences of some teachers who have noticed an improvement in all students in the classroom, and by Zohar and Dori's 2003 study [23].
In this research, four studies in four different schools the use of more demanding requirements in science-related exercises was examined. All four studies presented similar results showing that more demanding exercises improved the performance of all pupils, thus showing that more demanding exercises are beneficial to all types of student. The study tends to point out that this type of exercise does not aim to close the gap between low and high-achieving pupils but to improve all students overall.
The second doubt, however, concerns the right age at which to start using challenging tasks, as using them in the very early years might seem inadvisable due to the fact that some pupils in the first year of school present themselves with little or no acquired knowledge.
Comparing various classes in which teachers started using challenging tasks at different times during the school year, it was noted that caution in presenting pupils with challenging tasks was unnecessary as more exposure to this type of task allows students to get used to it earlier.
In addition, mistakes or misunderstandings by students who are not yet ready to tackle such tasks provide excellent teaching cues as incorrect tasks can be repeated later, allowing those who made mistakes to perform the exercise correctly and those who had solved the task on the first attempt to revise.
The third doubt exposes the problem of the lack of involvement and attention that students might show when faced with a challenging task. The proposed solution is to also use the normal techniques for increasing pupil involvement in challenging tasks in any type of classroom: in the case of primary school students, such techniques include storytelling or the use of pupils' names in problems.
The fourth and final doubt concerns how and when to use the enabling prompts: in addition to having doubts about how long to wait before providing such suggestions, many teachers also questioned whether such aids should not be provided directly to the pupils so as to empower them.
To this problem during the final follow-up meeting, the proposed solution is to provide the enabling prompts to the students right from the start by

advising them when it is appropriate to use them: doing so not only avoids the root of the problem of when to provide these aids but also helps the students to take responsibility and learn based on their own experience of what their actual capabilities are.
Summing it all up in a few lines, we can see that the formulation and use of challenging tasks in the classroom is not an easy task, but with a different didactic approach, aimed more at mastering the tools provided and respecting individual learning times, they prove to be a useful and fruitful teaching tool.

Chapter 2

The present study

In this chapter, we will outline the experimental study carried out for this thesis and deeply describe every aspect of it.
We will start by presenting the activities carried out for the research and then present the hypotheses that this study investigates and the analysis of the data collected to verify these hypotheses

2.1 Development of the research

The activities described here took place in the first half of the 2023/2024 school year. Shortly before the beginning of school activities, teachers were recruited to take part in the project.
This proposal was aimed at mathematics teachers in secondary schools, in this case teachers from scientific high schools.
Three teachers took part in the project for a total of five sample classes: two twelfth grade classes (4B and 4I) and one eleventh grade class (3E) from the scientific high school Eugenio Curiel in Padua and two twelfth grade classes (4D and 4E) from the scientific high school Giacomo Leopardi in Recanati. All the students involved are therefore between 15 and 18 years old.
As the only variation in teaching activities, the use of formularies during the written tests was proposed to the classes involved in order to reduce the students' mnemonic effort during these tests. With this variation, the

classical tests that verify not only the ability to use formulas but also the correct memorisation of them are inadequate.
In this regard, the teachers were introduced to the concept of challenging tasks and this concept was proposed as a solutions for more appropriate tests. Worth mentioning is the fact that classes 4B and 4I had already applied this methodology in previous years with their teacher.
After agreeing on the activities with the teachers, a consent form was sent to the parents of the pupils involved to authorise the collection and use of the anonymous data. All parents authorised the collection of data for a total of 102 students involved in the research.
Once the authorisations had been collected, during the first month of the new school year, the AMAS questionnaire for secondary schools was given to all participating classes for the first measurement of maths anxiety in the sample. A total of 100 pupils participated in this first survey.
Given the status of classes in which the methodology did not in fact change from the previous year, classes 4B and 4I were considered as a separate sample from the other classes. During the first term the sample classes therefore modified their written tests, which included the introduction of formularies. In terms of the type of exercises to be proposed in the tests and which formularies to provide, teachers were given complete freedom to customise and adapt this new methodology with their own teaching style and with the class.
At the end of the first term a common test was organised to check the pupils' knowledge. In addition to the five sample classes, three other classes acting as control classes took part in this test: a twelfth grade and a eleventh grade class from the scientific high school Eugenio Curiel in Padua and a twelfth grade class from the scientific high school Giacomo Leopardi in Recanati.
The tests, included in Appendix A, consisted of classical tests on topics covered approximately at the beginning of the school year by all the classes involved: for the eleventh grade classes, the test topic was straight lines and linear equations, while for the twelfth grade classes, the test topic was goniometric functions and goniometric equations and inequalities.
Since the aim of the final test is to test the students' long-term consolidated skills and knowledge, the topics of the tests was omitted from all students, as well as being chosen among the topics covered in class no less

than a month earlier. To avoid placing the pupils in a stressful condition and considering the conditions of the test, the pupils were assured that the test would not be used in the teachers' evaluations.
A total of eight classes with 150 students participated in this test. The common tests, carried out in the last week of January and the first week of February 2024, were corrected by Dr. Meyer Alena Ramona, external to the research, without giving her any information about which classes were in the sample, all following the correction guidelines in Appendix A.
Shortly afterwards, in the first half of February, all classes in the sample were administered the AMAS questionnaire again. The period of at least three months that elapsed between the two administrations should be sufficient for a correct measurement of any changes in pupils' levels of maths anxiety. A total of 100 pupils also participated in this second survey.
At the end of the activities carried out in class, the teachers of all classes involved provided the students' grades obtained during the first term.
All pupils' data were collected anonymously and by using code to enable the students' grades to be linked with the test taken at the end of the first term.

2.2 The purpose of the research

The main purpose of the experiment just described is to provide an initial investigation into the use of formularies in schools.
The first hypothesis we will propose is that this method may have a positive impact with regard to maths anxiety.
For a student, knowing that he or she has an extra tool that avoids mnemonic effort and getting used to the different types of exercises that challenging tasks propose could bring control and lower levels of maths anxiety, especially that experienced during tests.
To this purpose, we will see through the data collected via the AMAS questionnaire whether such a decrease is present. Given the presence of classes in which the use of the formularies had already been introduced a year ago we will divide the sample classes into two groups and thus have surveys of anxiety levels taken one year and one and a half years after the adoption of the formularies.
Since the two groups are formed by different populations, the data col-

lected will not provide us with a precise evolution of the trend in maths anxiety, but it will give us an excellent overview on which to make theories about this evolution. In addition to this we will investigate any gender differences in the levels of anxiety detected and see if they are in line with the differences already highlighted in the literature.
A further hypothesis that this research proposes is that the adoption of this method may improve long-term memorisation of concepts and thus contribute to better scholastic performance.
The use of formularies during more difficult and speculative exercises may in fact constitute a better consolidation in working memory than simply memorising formulas, especially in cases where students only memorise and use them on the day before the test. As seen in [6] this better consolidation would lead to better retrieval capacity in long-term memory.
Not anticipating the topic on the common test actually made it possible to collect a snapshot of the students' memorised knowledge.
In addition to this, if the decrease in anxiety levels were confirmed, this would have a direct positive effect on student performance, as stated in [7].
Furthermore, the simple addition of the challenging tasks in the tests would also bring benefits in terms of the student's ability to adapt when faced with a problem. The common test therefore aims to show a general picture of the memorised skills of the classes and to see any differences between the sample group and the control group.
In investigating the results of the test, it will be seen whether any differences occur in all pupils or only in those with a particular school performance. Furthermore, given the nature of the sample divided into two schools, we will see if any regional differences arise.
In summary, the objectives of the research are as follows:

- To investigate the anxiety levels of the two groups and to see if any differences are present both between them and with data found in the literature. To do so we will use the data collected through the AMAS questionnaires.

- To investigate the performance of the pupils in the sample, focusing on their ability to memorise long-term mathematical concepts. To do this we will use the results of the common tests together with the grades obtained by the students during the first term.

2.3 Data analysis

In this section, we will give a complete and comprehensive exposition and analysis of the data collected during the experiment.
The data obtained via the AMAS questionnaire and the data from the final test and evaluations will be treated separately.
We will discuss the results while the conclusions will be treated in the next chapter

AMAS questionnaire data

As mentioned previously, 100 students participated in both AMAS questionnaire administrations.
We shall call "Group A" the one formed by classes in which the use of formularies had just been introduced while we shall call "Group B" the one formed by classes in which such formularies had been in use for a year.
To distinguish the first from the second survey we will speak of "Group A-1" and "Group A-2" and the same with the second group.
Excluding incorrectly completed questionnaires we thus have the following groups (the fact that in some groups the number of males and females does not equal the total is due to some who refrained from providing the gender):

- **Group A-1** composed of 56 students of which 26 were male and 27 female.
- **Group A-2** composed of 57 students of which 28 were male and 28 female.
- **Group B-1** composed of 39 students of which 19 were of the male gender and 19 of the female gender.
- **Group B-2** composed of 40 students of which 20 were of the male gender and 20 of the female gender.

The scores obtained by each group are displayed in Table 2.1.
To compare the various groups with each other, we check whether the data meet the normality condition using the Shapiro-Wilk test, which is suitable for small samples.

Score AMAS

Group		N. students	Mean	Median	S.D
Group A-1	Total	56	25.66	27	5.28
	Male	26	23.69	24.5	5.02
	Female	27	27.25	28	4.98
Group A-2	Total	57	24.85	26	5.36
	Male	28	23.67	24	5.17
	Female	28	26.17	27	5.38
Group B-1	Total	40	23.97	23.5	5.99
	Male	20	21.95	21	6.73
	Female	19	26	25	4.57
Group B-2	Total	40	26.6	26	6.7
	Male	20	23.9	25	5.15
	Female	20	29.3	30	7.09

Table 2.1: AMAS scores of both groups in the two surveys. For each group, we also see the values separated by gender and calculate the mean, median and standard deviation (S.D.).

These tests verified that only data from a few groups satisfied the normality condition. Therefore, a non-parametric test such as the Wilcoxon test was used to compare the groups.
An initial comparison can be made using the graph in 2.1. In particular, we considered the comparison between group A-1 and group A-2 to identify the hypothetical sudden drop in Maths Anxiety levels and compared them to groups B-1 and B-2 respectively.
The p-values of the respective tests are as follows: between group A-1 and A-2 we have p=0.377; between group A-1 and B-1 we have p=0.088, between group A-2 and B-2 we have p=0.269.
Since no test resulted in a p-value of less than 0.05, the null hypothesis cannot be excluded, that is the hypothesis that there are no statistically significant differences between the various groups. It should therefore be emphasised that the average lowering of AMAS scores found in group A is not statistically significant even when compared with the raising of the same values in group B.

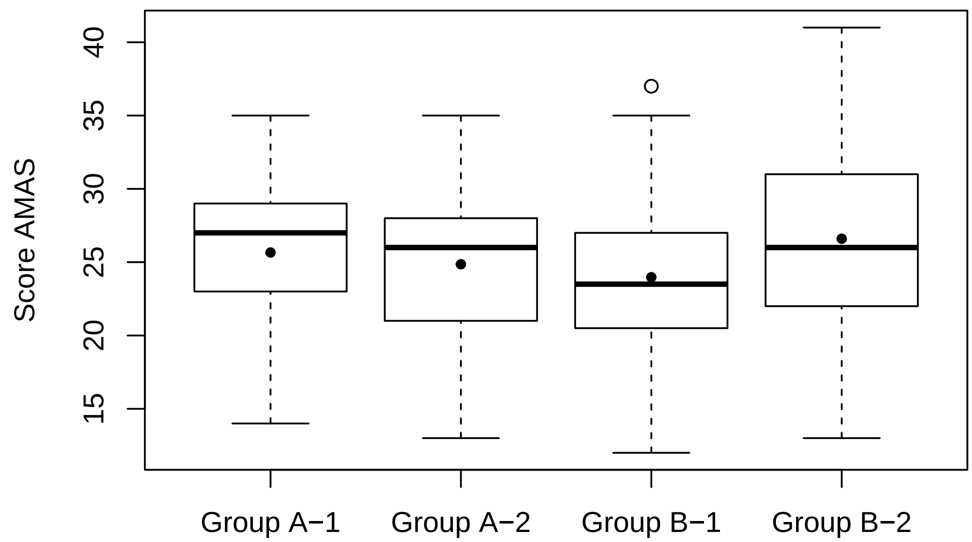


Figure 2.1: Boxplot of the AMAS scores of the four groups: for each group, both the median (the line) and the mean (the dot) are highlighted.

The same results were obtained by performing the Wilcoxon test between the male and female subgroups in each group, thus obtaining p-values greater than 0.05. The same tests showed for each group the difference due to gender already identified in the literature.
In fact, Wilcoxon tests performed between the male and female subgroups in each group show p-values less than 0.05: 0.01 for group A-1, 0.048 for group A-2, 0.02 for group B-1 and 0.023 for group B-2.
As can be seen from Table 2.1, this relevant difference is disadvantageous for female students, who on average have higher levels of Math Anxiety than their peers.
Given the lack of relevant differences in the AMAS scores of the groups, we focused on the EMA and LMA scores in case there were differences in only one of the two subscales of the questionnaire.

Score LMA

Group		**N. students**	**Mean**	**Median**	**S.D**
Group A-1	Total	56	10.94	11.5	3.37
	Male	26	10.19	10	3.39
	Female	27	11.48	12	3.13
Group A-2	Total	57	10.66	10	3.33
	Male	28	10.39	9.5	3.66
	Female	28	11	11	3.05
Group B-1	Total	40	9.45	8.5	3.27
	Male	20	8.75	8	3.4
	Female	19	10.26	10	3.1
Group B-2	Total	40	10.85	10	4.2
	Male	20	9.35	9.5	2.62
	Female	20	12.35	12.5	4.96

Table 2.2: LMA scores of of both groups in the two surveys. For each group, we also see the values separated by gender and calculate the mean, median and standard deviation.

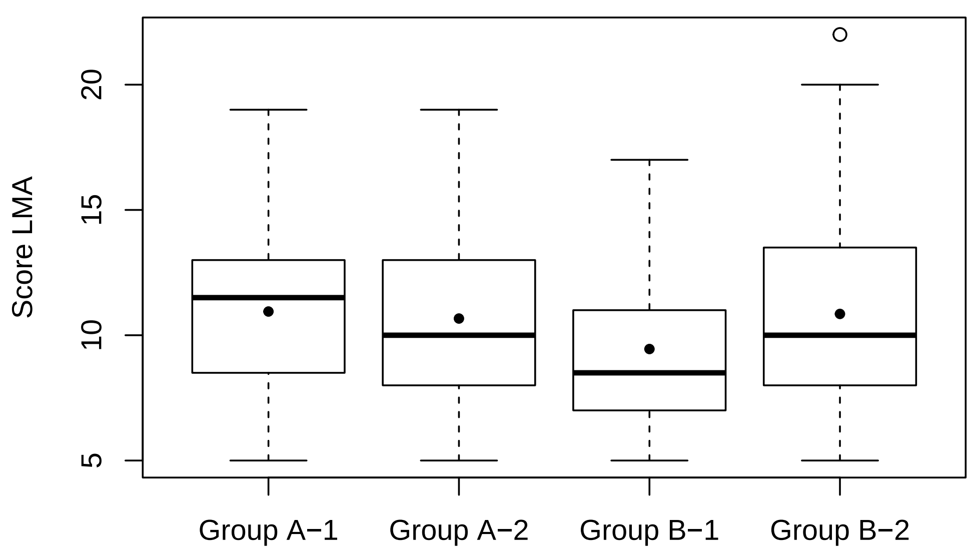


Figure 2.2: Boxplot of the LMA scores of the four groups: for each group, both the median (the line) and the mean (the dot) are highlighted.

Group		Score EMA N.Students	Mean	Median	S.D.
Group A-1	Total	56	14.71	15	2.82
	Male	26	13.5	14	2.83
	Female	27	15.77	16	2.45
Group A-2	Total	57	14.19	15	2.95
	Male	28	13.28	12.5	2.66
	Female	28	15.17	15	2.99
Group B-1	Total	40	14.52	15	3.28
	Male	20	13.2	13	3.63
	Female	19	15.73	16	2.28
Group B-2	Total	40	15.75	16	2.89
	Male	20	14.57	15	2.78
	Female	20	16.95	17.5	2.45

Table 2.3: EMA scores of of both groups in the two surveys. For each group, we also see the values separated by gender and calculate the mean, median and standard deviation.

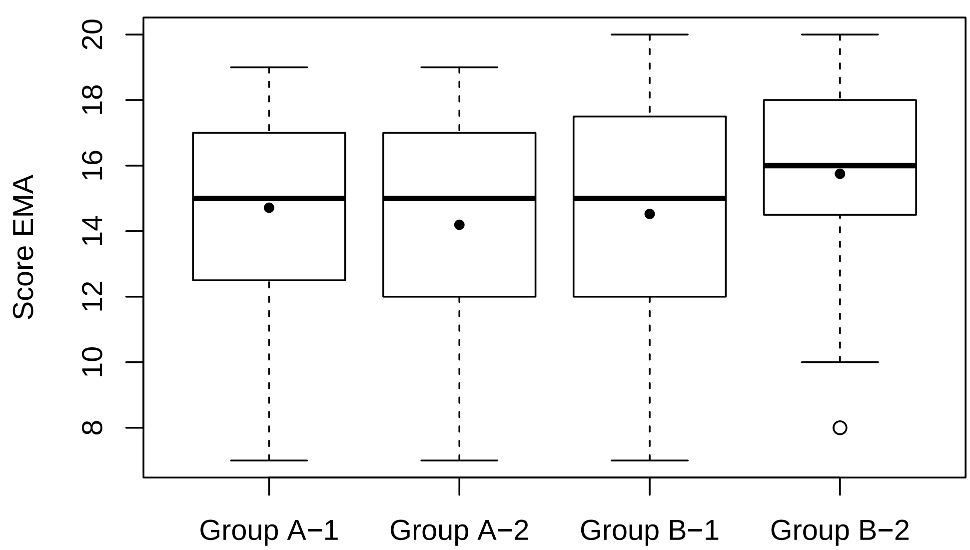


Figure 2.3: Boxplot of the LMA scores of the four groups: for each group, both the median (the line) and the mean (the dot) are highlighted.

For these datasets, normality tests were carried out again and also in this case, given the multiple samples that did not satisfy the normality hypothesis, it was decided to use the Wilcoxon test to make the same comparisons. Amongst all, only two tests reported a p-value less than 0.05: that of the LMA scores between groups A-1 and B-1 (p= 0.024) and that of the EMA scores between groups A-2 and B-2 (p= 0.012).
These differences are also evident from Figures 2.2 and 2.3 where the B-1 group in the first case and the A-2 group in the second differ from the other groups.
Investigating more closely these two comparisons shows that in the first case there are no gender-related differences whereas in the second case the situation is different: the Wilcoxon test performed between the EMA scores of the two subgroups divided by gender in fact show that the two male groups are not statistically different (p= 0.116) while the two female groups are statistically different (p= 0.027).
The female EMA scores are further illustrated in the following graph:

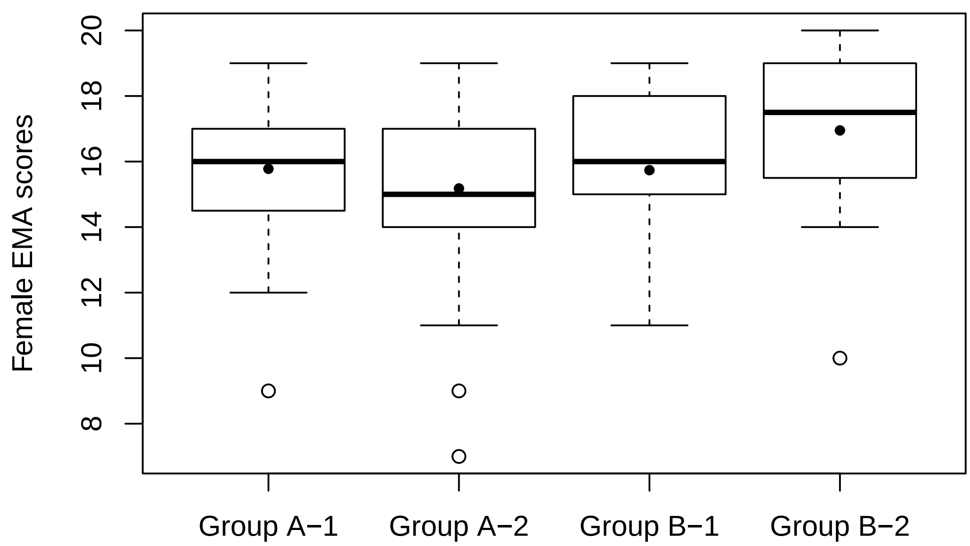


Figure 2.4: EMA scores of the female subgroups: both the median (the line) and the mean (the dot) are highlighted for each group.

Common tests data

As mentioned above, the common tests involved 8 classes with a total of 150 pupils. Given the small number of pupils in grade 11, they will be treated separately.
Each class was further divided into two groups: using the average of the grades obtained in this term, it was possible to divide the pupils into high-performing and low-performing pupils.
The average scores obtained in the tests of the various groups are shown in the following table:

Group		**N.Students**	**Mean**	**S.D.**
12th grade test	Total	73	46.92%	18.33%
	H. P.	37	56.58%	17.23%
	L. P.	36	37%	13.65%
12th grade control	Total	40	56.73%	18.78%
	H. P.	20	68.46%	12.3%
	L. P.	20	45%	16.83%
11th grade test	Total	21	70.72%	18.7%
	H. P.	11	76.26%	17.46%
	L. P.	10	64.62%	18.97%
11th grade control	Total	16	86.80%	7.94%
	H. P.	8	88.42%	8.55%
	L. P.	8	85.18%	7.47%

Table 2.4: Common test scores. Of each group, we see separate values for high-performing (H.P.) and low-performing (L.P.) students. Of the scores, in percentage form, we calculate the mean and standard deviation (S.D.).

For each group, the Shapiro-Wilk normality test was performed. These tests showed that both test and control twelfth grade group satisfied the normality test while there were different results for the eleventh grade groups.
Therefore to compare the twelfth-grade groups we can use a parametric test such as Student's t-test while for the eleventh-grade classes we will use Wilcoxon's test. We will now focus on the twelfth-grade classes.

The t-test between the two groups reports a p-value of 0.009 so the two groups are statistically different and as can be seen from the table 2.4, the control group obtained better scores on average.
Another interesting finding is that the t-tests performed between every male and female subgroups do not show any statistically significant differences in performance.
Investigating further, differences can be seen in the tests performed on the high- and low-performance subgroups. The p-values obtained show that the low-performance groups are not statistically different ($p = 0.07$) while the two high-performance groups are ($p = 0.004$).
Focusing the tests further on this group reveals, unlike the other groups previously involved, a difference due to gender: the two male high-performance groups in fact show no statistically significant differences ($p = 0.056$) whereas the two female groups do ($p = 0.032$).
Therefore, the differences are more pronounced in the high-performance groups, showing a sharper difference in female students.
As an additional visualisation, the boxplots of the test and control group and their high-performance subgroups are shown below.
Concerning the two eleventh-grade classes involved, the data are similar: in fact, the two groups show a statistically significant difference ($p = 0.002$) again in favor of the control class. In this case too, no gender differences related to performance were shown in these two classes.
Dividing the classes into the two high and low performance groups shows a different result from that obtained in the twelfth grade classes: the two high performance groups in fact are not statistically different ($p = 0.06$) while the two low performance groups are ($p = 0.008$).
Given the low number of students, it was not considered useful to investigate gender differences within the subgroups.
However, the data related to the differences between the high and low performing groups in these two classes will be considered to be ignored both because of the numerical gap between these classes and the sample of twelfth grade classes and because in both classes students with an educational debt in mathematics could not participate in the test for organizational reasons, as they were engaged in a recovery test.
Therefore, from the case study consisting of the two eleventh grade classes we can still see how in general the control classes performed better than

the test one.

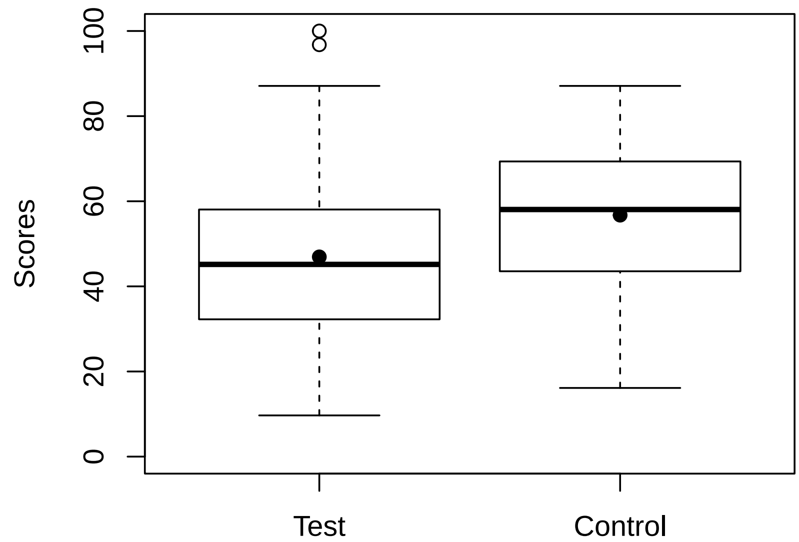


Figure 2.5: Twelfth grade scores: the median (the line) and the mean (the dot) for both the test and control groups were highlighted

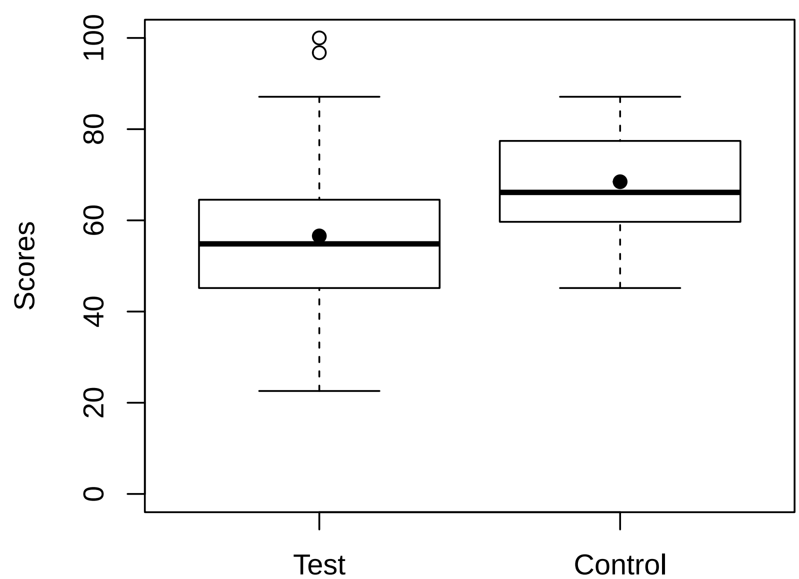


Figure 2.6: Scores of the two high-performing groups in the twelfth grades: the median (the line) and the mean (the dot) for both the test and control groups were highlighted

Brief qualitative analysis of common tests

Since we did not use a standardized test to verify students' memorized knowledge, a qualitative analysis done by focusing on individual exercises is useful for further investigation about students' performance.
In fact, some exercises were designed specifically to test whether knowledge of a specific mathematical formula was present or not: in fact, in the case of

the test for eleventh graders, exercise 4 required knowledge of the formula for point-line distance, while in the test for twelfth graders, the second and third questions in exercise 2 required knowledge of the duplication formula and the prostapheresis formula, respectively.
In both cases, the average scores obtained by the test groups are lower compared to the control groups. In the case of the eleventh-grade classes, out of the 3 points available in the exercise mentioned above, the average score of the control group (2.37) is almost double that of the test group (1.26).
For the twelfth-grade classes, where the number of formulas to learn was higher, the difference is even more pronounced: out of 8 points available in the exercises mentioned above, the average score of the control group (3.27) is almost two and a half times that of the test group (1.36).
It's worth mentioning that several students, especially those belonging to the test classes, recognized the formula to use in the specific exercise but reported not remembering it either by writing it directly on the papers or mentioning it verbally during the common tests.
Therefore, it can be assumed that the lack of explicit request for memorization of formulas in the tests taken during the term negatively affected effective memorization.
Another type of exercises were aimed instead at testing students' ability to solve exercises that required more reasoning.
In the case of the test for the eleventh-grade classes, we are referring to exercises 5 and 6, while in the test for the twelfth-grade classes, we are talking about exercise 4.
For the eleventh-grade classes, out of 7 points available in the mentioned exercises, the average score of the control group (5.31) is higher compared to the test group (3.54).
However, this difference is not observed among the twelfth-grade classes, where out of 6 points available in the mentioned exercise, the average score of the control class (2.27) is slightly lower than the average of the test classes (2.52).
Considering the data from the 6 twelfth-grade classes to be more relevant compared to the only 2 eleventh-grade classes involved, which therefore constitute a too small sample, we can attempt to say that the adoption of formulas and the addition of challenging tasks has not altered the students'

reasoning abilities.
Therefore, considering the results confirmed by the data analysis performed, we can venture the hypothesis that such a difference in performance disadvantageous to the test group is mostly caused by the failure to memorize the necessary formulas rather than in a difference more intrinsically related to the reasoning abilities of the pupils involved.

Chapter 3

Concluding remarks

3.1 Research results

As already explained above, the research aims to provide an initial investigation into the use of formularies and how it affects students in terms of both performance and levels of maths anxiety.
From the point of view of performance, the results do not coincide with those expected: from the scores on the common tests, not only do we not see better results among the students in the test group compared to those in the control group, but we even see on average a decline that an additional qualitative analysis attributes to the non-use of the formulas that had to be memorised.
A further interesting observation is how this decline mostly involves the better half of the class: in this case we can assume that the non-utilisation of the formulas is more or less unchanged in the low-performing students. The slight gender-related difference found only in the scores of the latter group can be attributed to the higher level of maths anxiety experienced by girls on average.
Therefore, we can conclude that in the classroom context, the use of formularies paired with more speculative exercises does not allow formulas to be memorized in a more efficient manner than in a class that adopts a classic methodology.
More articulate results appeared instead in the study of math anxiety levels. The increase in EMA scores found during the second survey in the

group that had adopted the forms in the year prior to the study, especially when compared with the slight decline in the group that had just adopted the forms, can be interpreted in two ways: a first hypothesis states that the use of the forms may in the long run lead to an increase in math anxiety, especially in the evaluation-related anxiety experienced by female students; a second hypothesis, on the other hand, predicts that the trend in anxiety levels detected in the group that had already been using the forms for a year was normal and in line with normal fluctuations in anxiety levels experienced by a class.
In support of the latter hypothesis is the fact that the second survey was carried out immediately after the end of the first term, a period known to have a schedule full of tests and questions in which therefore it does not seem unlikely to detect an increase in evaluation anxiety levels. Further support for the second hypothesis is the fact that it is unlikely that a newly adopted methodology would present a sudden deterioration after a year and a half from its adoption. In summary, this second hypothesis predicts that the adoption of the formularies after one year does not change anxiety levels, especially evaluation anxiety which returns subject to the normal fluctuations given by the school schedule.
The lack of change in anxiety levels detected in the group that had just adopted the formularies would therefore be attributable to the "novelty" factor that the adoption of a new methodology brings with it, especially if this methodology directly changes the way written tests are carried out.
In order to confirm either of these two hypotheses, further research is therefore required, either one analysing the evolution over several years of the levels of maths anxiety in a class that adopts the use of the forms or one that tries to analyse any fluctuations in the levels of maths anxiety in a sample of normal classes.
At the moment, as there is no similar research in the literature, we keep the interpretation of the results open.

3.2 Research limits

The study presented here encountered several difficulties during its conduct.
The first obstacle encountered was the unwillingness of teachers to try a

new methodology: in fact, among the mathematics teachers from three different institutions only three agreed.

The Italian legislation regarding the use of formularies during the state exams played a key role in shortening the time available for the study.

The classes involved in fact after the experiment have returned to the classical methodology or are in any case obliged to return to it within the next few years since the state exam forbids the use of formularies . Therefore, it is common for those adopting different methodologies to return to classical teaching strategies at least a year before the state exam.

Therefore, the tight timeframe did not allow for long-term data collection that could allow for a complete evolutionary framework.

However, another limitation of the research is the age of the students: being students who between primary and secondary school have by now attended at least 10 years of school adopting classical methodologies a change such as the introduction of the formularies could be disadvantageous more because it is something anomalous in relation to the schooling already done rather than an inherent flaw in the method.

Therefore, as suggested in [15] a good approach would be to introduce the challenging tasks and the first simple formularies already among primary school students. A further obstacle is the lack of some research in the literature that could give a full understanding especially of the role that math anxiety may play in students at different stages of their schooling.

However, this last obstacle may be an opportunity as it provides many insights for future researches, which are also possible thanks to the willingness and help of teachers who are open to research, such as those who collaborated in this study.

Appendix A

In this appendix, a translated by me version of the common tests conducted with the classes are attached. The evaluation guidelines are as follows:

Evaluation of common tests for eleventh-grade classes

- Exercise 1: 1 point for each correct answer (maximum 5 points)
- Exercise 2: 3 points for each equation (maximum 6 points) distributed as follows:
  - 1 point if the step for the point in the plane is explained
  - 1 point if the calculations to find the value of q are present
  - 1 point if all calculations performed are correct
- Exercise 3: 3 points for each equation (maximum 6 points) distributed as follows:
  - 1 point if there's the idea of inserting the point into the line
  - 1 point if correct calculations and considerations on the slope are present
  - 1 point if the calculations performed are correct

  If the student provides only the correct equation, give 1 point.
- Exercise 4: 3 total points distributed as follows:
  - 1 point if the equation of the line passing through the points is present

  - 1 point if the correct formula for calculating the distance point-line is present
  - 1 point if the calculations performed are correct

- Exercise 5: 4 total points distributed as follows:
  - 1 point if the idea of putting the lines in explicit form is shown
  - 1 point if calculations equating the slope coefficients are present
  - 1 point if considerations about the values to have intersecting lines are present
  - 1 point if the calculations performed are correct
- Exercise 6: 3 total points distributed as follows:
  - 1 point if the system of the two lines is present
  - 1 point if the idea of setting the ordinates equal to zero is present
  - 1 point if the calculations performed are correct

Evaluation of common tests for twelfth-grade classes

- **Exercise 1:** 1 point for each correct answer (maximum 4 points)
- **Exercise 2:** 4 points for each equation (maximum 12 points) distributed as follows:
  - 2 points for the correct use of trigonometric formulas and equation manipulation (for 2.a, the use of substitution and solving the resulting equations; for 2.b, the use of the double-angle formula and substitution; for 2.c, the use of the sum-to-product formula and rearrangement useful for solving the equation)
  - 1 point for the correct writing of the solution considering periodicities
  - 1 point if the calculations performed are correct
- **Exercise 3:** 3 points for each inequality (maximum 9 points) distributed as follows:

  - 1 point for the correct manipulation of the inequality
  - 1 point for the correct representation of the solution's periodicity
  - 1 point if all calculations performed are correct

- **Exercise 4:** 3 points for each equation (maximum 6 points) distributed as follows:
  - 2 points for graphs drawn correctly
  - 1 point for the correct considerations made about the number of solutions

At the discretion of the examiner, fractions of points may also be used for intermediate evaluations.

COMMON MATH TEST (Third classes)

1) Indicate whether the following statements are True (T) or False (F)

a) The two straight lines of equation **y=3x+1** and **y+3x-2=0** are parallel ____
b) The distance between points **A(6;6)** and **B(0;-2)** is **10** ____
c) The straight line parallel to the x-axis through point **A(6;6)** has equation **y=6** ____
d) The straight line of equation **y=4x-1** meet the y-axis in the point **A(0;1/4)** ____
e) The two straight lines of equation **y=-2x+3** and **y=1/2x+1** are perpendicular ____

2) Write the equation of the straight lines passing through the given point A with the given angular coefficient *m*

- **A(-1; 2)** *m* = 3 ______________________________
- **A(4/3; 5/2)** *m* = -1 ______________________________

3) Given the stright line 2x+2y-1=0 write the equation of the perpendicular and the parallel straight lines passing through the point A(-4; -3). (write only the answer)

Parallel straigth line: ______________________________

Perpendicular straight line: ______________________________

4) Calculate the distance of P(0; 6) from the straight line passing through points A(2; 3) and B(1/2; 1)

Distance: ______________________________

5) Discuss for which values of a the straight lines r: 2ax+2y-3=0 and s: (a-2)y-x+2=0 are parallel or incident

____

____

6) Find for which value of k the straight lines of equations (k-1)x+2ky+6=0 and x-y+2=0 intersect on the x-axis

____

____

CLASS: ____ GENDER: _____

COMMON MATHEMATICS TEST (Fourth Classes)

1) Indicate whether the following statements are True (T) or False (F)

a) $\sin(\pi + \alpha) = \sin(-\alpha)$ ____
b) If $\cos(\alpha)=\cos(\beta)$ then we have $\alpha = \beta$ ____
c) Per ogni angolo α vale l'espressione $\sin(\alpha)+\cos(\alpha) = 1$ ____
d) L'equazione $\cos(\alpha) = 2$ non ha soluzione ____

2) Solve the following equations

- $2\sin^2(x) + \sin(x) = 0$ ________________________________
- $\cos(2x) + 3\cos(x) = 1$ ________________________________
- $\sin(4x) + \sin(6x) = \cos(x)$ ________________________________

3) Solve the following disequations

- $2\sin(x)+\sqrt{(2)}>0$
- $\tan(x)-\sqrt{3}>0$
- $|\cos(x)|<1$

4) Use the graphical method to determine the number of solutions of the following equations.

- $|\sin(x)|-x^2+1=0$ ________________________________
- $\sqrt{x}=\tan(\frac{x}{2})$ ________________________________

CLASS____ GENDER____

APPENDIX A

Bibliography

[1] Ashcraft M. H., Moore A. M. (2009) *Mathematics Anxiety and the Affective Drop in Performance.* Journal of Psychoeducational Assessment, 27(3), 197-205. https://doi.org/10.1177/0734282908330580

[2] Beilock S., Gunderson E., Ramirez G. Levine S. C. (2010) *Female teachers' math anxiety affects girls' math achievement.* Proceedings Of The National Academy Of Sciences. 107, 1860-1863. https://www.pnas.org/doi/abs/10.1073/pnas.0910967107

[3] Blyth J. (2022) *Math Anxiety: Finding Solutions to a Multifaceted Problem* BU Journal of Graduate Studies in Education, Volume 14, Issue 3

[4] Caviola S., Primi C., Chiesi F. Mammarella I. (2017) *Psychometric properties of the Abbreviated Math Anxiety Scale (AMAS) in Italian primary school children.* Learning And Individual Differences. 55 pp. 174-182. http://dx.doi.org/10.1016/j.lindif.2017.03.006

[5] Cheeseman J., Downton A., Livy S. (2017) *Investigating Teachers' Perceptions of Enabling and Extending Prompts* in 40 years on: We are still learning! Proceedings of the 40th Annual Conference of the Mathematics Education Research Group of Australasia (pp. 141-148)

[6] Cotton K., Ricker T. J. (2021) *Working memory consolidation improves long-term memory recognition.* Journal of Experimental Psychology: Learning, Memory, and Cognition, 47(2), 208–219. https://doi.org/10.1037/xlm0000954

[7] Dowker A., Sarkar A., Looi C. Y. (2016) *Mathematics Anxiety: What Have We Learned in 60 Years?.* Frontiers in psychology, 7, 508. https://doi.org/10.3389/fpsyg.2016.00508

[8] Feldman R. S. (2011) *Understanding Psychology, 10th ed.* New York: McGraw Hill Education.

[9] Hopko D. R., Mahadevan R., Bare R. L., Hunt M. K. (2003) *The Abbreviated Math Anxiety Scale (AMAS): construction, validity, and reliability.* Assessment. 10(2):178-182. doi:10.1177/1073191103010002008

[10] Papadopoulos I. (2019) *Using mobile puzzles to exhibit certain algebraic habits of mind and demonstrate symbol-sense in primary school students.* The Journal Of Mathematical Behavior. 53 pp. 210-227. https://doi.org/10.1016/j.jmathb.2018.07.001

[11] Primi C., Busdraghi C., Tomasetto C., Morsanyi K., Chiesi F. (2014) *Measuring math anxiety in Italian college and high school students: Validity, reliability and gender invariance of the Abbreviated Math Anxiety Scale (AMAS)* Learning And Individual Differences. 34 pp. 51-56. https://doi.org/10.1016/j.lindif.2014.05.012.

[12] Punaro L., Reeve R. (2012) *Relationships between 9-Year-Olds'Math and Literacy Worries and Academic Abilities* Child Development Research. (10) DOI:10.1155/2012/359089

[13] Ramirez G., Hooper S. Y., Kersting N. B., Ferguson R., Yeager D. (2018) *Teacher Math Anxiety Relates to Adolescent Students' Math Achievement* AERA Open, 4(1). DOI:10.1177/2332858418756052

[14] Richardson F. C., Suinn R. M. (1972) *The Mathematics Anxiety Rating Scale: Psychometric Data* Journal of Counseling Psychology, 19(6), 551–554. DOI:10.1037/h0033456

[15] Russo J., Bobis J., Downton A., Hughes S., Livy S., McCormick M., Sullivan P. (2019) *Teaching with challenging tasks in the first years of school: What are the obstacles and how can teachers overcome them?* Australian Primary Mathematics Classroom 24(1)

[16] Soni A., Kumari S. (2015) *The Role of Parental Math Anxiety and Math Attitude in Their Children's Math Achievement* International Journal of Science and Mathematic Education 15, 331–347 DOI 10.1007/s10763-015-9687-5

[17] Sullivan P., Mornane A. (2014) *Exploring teachers' use of, and students' reactions to, challenging mathematics tasks* Mathematics Education Research Journal 26, 193–213. DOI:10.1007/s13394-013-0089-0

[18] Supekar K., Iuculano T., Chen L., Menon V. (2015) *Remediation of Childhood Math Anxiety and Associated Neural Circuits through Cognitive Tutoring* Journal of Neuroscience 9 September, 35 (36) 12574-12583 DOI:10.1523/JNEUROSCI.0786-15.2015

[19] Szczygieł M. (2020) *When does math anxiety in parents and teachers predict math anxiety and math achievement in elementary school children? The role of gender and grade year* Social Psychology of Education 23, 1023–1054. https://doi.org/10.1007/s11218-020-09570-2

[20] Szczygieł M. and Pieronkiewicz B. (2021) *Exploring the nature of math anxiety in young children: Intensity, prevalence, reasons* Mathematical Thinking and Learning, 24(3), 248–266. DOI:10.1080/10986065.2021.1882363

[21] Vogel S., Schwabe L. (2016) *Learning and memory under stress: implications for the classroom* NPJ science of learning, 1, 16011. DOI:10.1038/npjscilearn.2016.11

[22] Wilkie K.J. (2016) *Using challenging tasks for formative assessment on quadratic functions with senior secondary students* The Australian Mathematics Teacher vol.72 no.1

[23] Zohar A., Dori Y.J. (2003) *Higher Order Thinking Skills and Low-Achieving Students: Are They Mutually Exclusive?* Journal of the Learning Sciences, 12(2), 145–181. https://doi.org/10.1207/S15327809JLS1202_1

Credits

First of all, I would like to thank my family and friends for this thesis work, for the support that was never lacking and for the good experiences I had.
I thank my supervisor Francesco Ciraulo, not only for his willingness to assist me in this work but also for always giving me the opportunity to write this thesis as independently and productively as possible.
I thank Professors Alberto Branciari, Marco Neri Da Re and Alessandra Pavan for their valuable collaboration during the research period.
A warm thank you to all the students from the classes who participated in every phase of the experiment without whom this thesis would not exist.
Last but not least, I would like to thank the University of Padua for the years of enjoyable study.